Universal Turing Machines

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Language of a TM

Defn. Language of a TM M is

$$\mathcal{L}(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}$$

$$\begin{cases} w \in \mathcal{L}(M) \rightarrow M \text{ accepts } w \\ w \notin \mathcal{L}(M) \rightarrow \begin{cases} M \text{ rejects } w \\ M \text{ infinite loops on } w \end{cases} \end{cases}$$

Recognizability Lang is recognizable if it is lang of a TM

TM M where $\mathcal{L}(M) = L$ is recognizer for L

RE – set of all recognizable languages

→ approach to solve via exhaustive search

Decidability Lang is decidable if $\exists$ TM M where $\mathcal{L}(M) = L$

i.e. $w \in L \rightarrow M$ accepts w

$w \notin L \rightarrow M$ rejects w

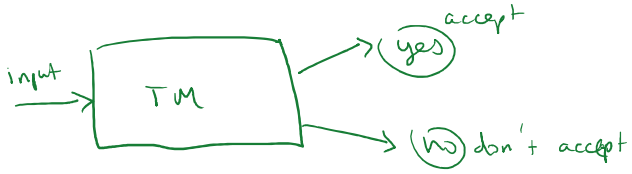
R – set of all decidable langs

problems that can "definitely be solved" by a computer

→ approach to solve w/o having to do exhaustive search

Strings, Language, Encodings

Decision problem. Goal - provide yes/no answer



Encoding Can represent anything w/ bit string

$\langle obj \rangle$ = some bit string representing obj

$\langle a, b, c, d, \ldots n \rangle$ = single string encoding all these objs

Emergent Properties

Defn. Property arising out of smaller pieces assembled to system that doesn't exist in the pieces

ex. Neurons in a brain

→ In TM's and equivalents, can use to show some problems not solvable

→ Universality - single computing device can do all computation

→ Self-reference - can ask q's about own behavior

Universal TM

Thm. There is a turing machine U_{TM} (Universal TM) that when run on input of the form $\langle M, w \rangle$ ($M \to$ TM, $w \to$ str) simulates M run on w and exhibits exact behavior of M on w (accepts, loops, rejects)

$$\mathcal{L}(U_{TM}) = \{ \langle M, w \rangle \mid M \text{ is TM}, \; w \in \mathcal{L}(M) \}$$

(acceptance language of Turing machines A_{TM})

$A_{TM} \in RE$