Binary Relations

Binary Relations

Defn. A binary relation is a relation that focuses on two objects.

OR: A predicate R that can be applied to ordered pairs of elements over a given set.

Ex.
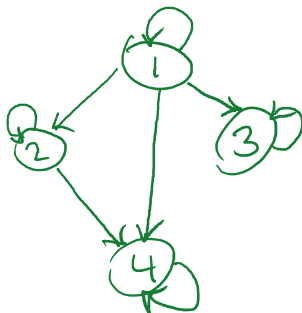$$A \subseteq B$$
$$x < y$$
$$x \equiv y \pmod{k}$$

General notation.

$$a \ R \ b$$
$$\text{or} \quad a \ \not{R} \ b$$

Visualizing relations. Draw set elements and draw arrows between elements where aRb is true

ex. $a \mid b$ (a divides b) over $\{1, 2, 3, 4\}$

Capturing structure - Partitions

Defn. A partition of a set is a way of splitting up the set into disjoint, nonempty subsets so that every element belongs to exactly one subset.

$\rightarrow$ Two sets S and T are <u>disjoint</u> if $S \cap T = \varnothing$.

Classifying relations.

$$\forall a \in A \quad a R a \qquad \qquad \text{reflexive}$$

$$\forall a, b \in A \quad a R b \rightarrow b R a \qquad \text{symmetric}$$

$$\forall a, b, c \in A \quad a R b \wedge b R c \rightarrow a R c \qquad \text{transitive}$$

All these rules must apply for partitions

Equivalence Relations.

Defn. An equivalence relation is a relation that is reflexive, symmetric, and transitive.

ex. $\quad x = y$

$\quad x \equiv_k y$

Proofs. To prove R is an equivalence relation, prove R is reflexive, symmetric, and transitive.