# First-Order Logic

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## First-Order Logic

**Defn.** First-order logic is a logical system for reasoning about properties of objects.

Augments logical connectives from propositional logic with:

- → predicates (describe object properties)
- → functions (map objects to one another)
- → quantifiers (allow reasoning about multiple objects)

**Ex.** Likes(You, Eggs) ∧ Likes(You, Tomatoes) → Likes(You, Shakshuka)

Constant symbols are objects, not propositions

Predicates take constant symbols as input and yield booleans

**Ex.** $>$ $<$ $=$ $\subseteq$ $\in$

Functions take constant symbols/objects and yield objects

**The existential quantifier**

A statement of the form

$\exists x.$ Some-formula

is true if for some choice of $x$, Some-formula is true when $x$ is plugged into it.

ex. $\exists x.$ Even($x$) ∧ prime($x$)

Note: has precedence just below ¬

use parentheses!

The universal quantifier

A statement of the form

$\forall x.$ some-formula

is true if for <u>every</u> choice of $x$, some-formula is true when $x$ is plugged into it.

ex. $\forall x. (x \in \mathbb{N} \rightarrow (\text{Even}(x) \leftrightarrow \text{Even}(x^2)))$