Graphs and the Pigeonhole Principle

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The Pigeonhole Principle

Thm. If m objects are distributed into n bins and $m > n$, then at least one bin will contain two objects.

Proof. Assume for the sake of contradiction that for some m, n where $m > n$, there is a way to distribute m items into n bins where each bin has at most one item.

Number the bins $1, 2, 3, \ldots n$ and let x_i denote the number of objects in bin i. There are m objects in total, so $m = \sum_{i=1}^{n} x_i$.

Since each bin has at most one element, we know $x_i \leq 1$ for each i. Therefore $m = \sum_{i=1}^{n} x_i \leq n$. This means $m \leq n$, but we assumed $m > n$, contradicting are initial assumption.

As such, with m objects and n bins where $m > n$, there must be one bin with at least two members.

The Generalized Pigeonhole Principle

Thm. If m objects are distributed into $n > 0$ bins, then some bin will contain at least $\lceil m/n \rceil$ objects.