ElGamal Encryption (1982)

PKE from D-H.

Ingredients: G: FCG of order q w/gen $g \in G$.

(E_s, D_s): sym. cipher over (K, M, C)

$H: G^2 \to K$: hash fn.

Scheme:

Gen: $\alpha \xleftarrow{R} \mathbb{Z}_q$, $h := g^\alpha$, $[sk := \alpha,\ pk := h \in G]$

$E(pk, m)$: $\beta \xleftarrow{R} \mathbb{Z}_q$, $u := g^\beta$, $v := h^\beta = g^{\alpha\beta}$

$\quad \Vert$

$\; h \in G$

$K := H(u, v) \in K$ ← derived from D-H secret v

$c := E_s(k, m)$

output (u, c)

$D(sk, (u, c))$: $\quad v = u^\alpha = g^{\alpha\beta}$

$\Downarrow$

α

$k = H(u, v) \in K$

$m = D_s(k, c)$

output m

Performance: enc: 2 exp in G

1 sym enc

dec: 1 exp in G

1 sym dec

As a standard: ECIES (ell. curve. enc. system)

Security.

Thm. 1. (Gen, E, D) is semsec (eavesdropping)

assuming (1) CDH holds in (G, g)

(2) (E_s, D_s) is semsec

(3) H is a secure key derivation fn (preserves entropy in v)

Thm. 2. (Gen, E, D) is CCA secure.

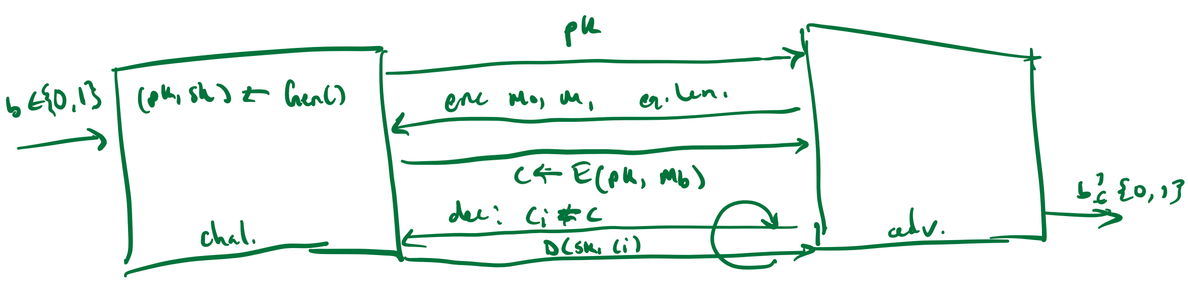
tampering

assuming (1) Interactive D-H assumption holds (stronger than CDH)

(2) (E_s, D_s) provides A.E.

(3) H is a "random oracle" (ideal hash fn)

CCA security



Trapdoor functions (TDF)

Def. Tuple of eff. algs (Gen, F, F^{-1})

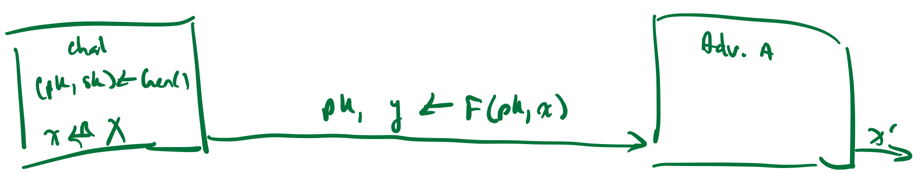
Gen: rand. alg outputs key pair (pk, sk)

$F(pk, \cdot)$ det. alg that defines an fn $X \to Y$

$F^{-1}(sk, \cdot)$ defines a fn $Y \to X$ that inverts $F(pk, \cdot)$

$\forall (pk, sk)$ def by Gen, $\forall x \in X$: $F^{-1}(sk, F(pk, x)) = x$

Security. (Gen, F, F^{-1}) is secure if it is a one-way fn:
can be evaluated, but not inverted w/o sk



Def. $Gen(F, F^{-1})$ is secure if for all eff. A:

$$Adv_{OW}[A, F] = Pr[x = x'] < \text{negligible.}$$

PKE from TDF.

$(Gen\ F, F^{-1})$: secure TDF $X \to Y$

(E_s, D_s) sym. auth. enc. over (K, M, C)

$H: X \to K$ hash fn

$\Rightarrow (Gen, E, D)$:

Gen: same as TDF gen

$E(pk, m)$: $x \xleftarrow{R} X$, $y \leftarrow F(pk, x)$
$k \leftarrow H(x)$, $c \leftarrow E_s(k, m)$
output (y, c)

$D(sk, (y, c))$

$x \leftarrow F^{-1}(sk, y)$

$k \leftarrow H(x)$, $m \leftarrow D_s(k, c)$

output m

Thm. If (Gen, F, F^{-1}) is a secure TDF, (E_s, D_s) provides AE, $H: X \to K$ is a random oracle, then it is CCA-secure.

RSA

Trapdoor permutation

Let $N = pq$ where p, q prime

Gen(): choose random distinct primes $p, q \approx 1024$ bits

set $N = pq$

choose ints e, d s.t. $ed = 1 \pmod{\varphi(N)}$

output $pk = (N, e)$, $sk = (N, d)$

$F(pk, x): \mathbb{Z}_N^* \to \mathbb{Z}_N^*$; $RSA(x) = x^e$ in $\mathbb{Z}_N$

$F^{-1}(sk, y) = y^d$; $y^d = RSA(x)^d = x^{ed} = x^{k\varphi(N)+1} = (x^{\varphi(N)})^k x = x$

RSAe Assumption

RSA w/ exp e. is a one-way permutation

For all eff. algs A:

$$\Pr[A(N, e, y) = y^{1/e}] < \text{negligible}$$

where $p, q \xleftarrow{R}$ n-bit primes, $N \leftarrow pq$, $y \xleftarrow{R} \mathbb{Z}_N^*$

PKE

(E_s, D_s): sym. enc. scheme providing AE

$H: \mathbb{Z}_n \to \mathbb{K}$ where $\mathbb{K}$ is key space of (E_s, D_s)

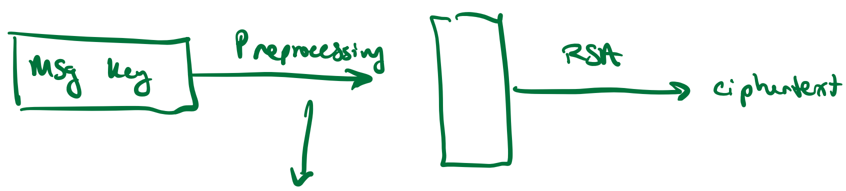
Gen(): generate RSA params $pk = (N, e)$, $sk = (N, d)$

$E(pk, m)$:
1. choose random x in $\mathbb{Z}_n^*$
2. $y \leftarrow RSA(x) = x^e$, $k \leftarrow H(x)$
3. output $(y, E_s(k, m))$

$D(sk, (y, c))$: output $D_s(H(RSA^{-1}(y)), c)$

RSA in practice

Never use textbook RSA



PKCS1 v1.5 mode 2

16 bits

02	random pad	00	msg

Resulting value is RSA encrypted

Widely deployed (e.g. HTTPS - TLS 1.2)

Is RSA a one-way permutation?

To invert w/o d: attacker must compute

x from $c = x^e \pmod N$

Best algo:

1. factor N (hard) ← easy on a quantum computer
2. compute e^{th} roots mod p, q (easy)

Note: if we use small ($\sim$128 bit) sk:

RSA is very insecure (d can be recovered from N, e)

However, making e small is ok (min $e=3$, usually $e = 65537$)

Asymmetry of RSA: fast enc slow dec

ElGamal: approx same time for both

Why is RSA dying?

Key lengths: security of PK system should be comparable to security of sym. cipher

Cipher key size	RSA mod size	ECC mod size
80 bits	1024 bits	160 bits
128 bits	3072 bits	256 bits
256 bits (AES)	15360 bits	512 bits

Also - very vulnerable to side-channel attacks.