Nonregular Languages

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Nonregular Languages

→ Regular languages correspond to problems solvable w/ finite memory

→ Nonregular languages – problems cannot be solved w/ finite memory

→ To prove language is nonregular, prove there is no DFA for it

Distinguishability. Two strings $x \in \Sigma^*$, $y \in \Sigma^*$ are distinguishable relative to L if there exists $w \in \Sigma^*$ if exactly one of xw and yw is in L.

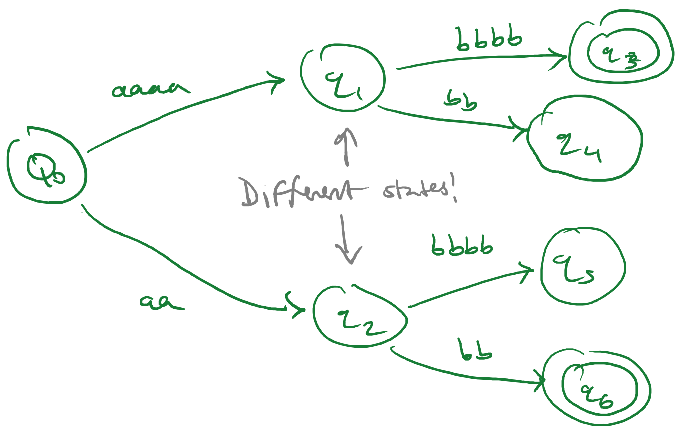
$x \not\equiv_L y$

Thm. Let L be an arbitrary language over Σ. Let $x, y \in \Sigma^*$ be strings where $x \not\equiv_L y$. Then if D is any DFA for L, then D must end up in different states when run on inputs x and y.

Ex. $E = \{a^n b^n \mid n \in \mathbb{N}\}$

$\{a^0 b^0, a^1 b^1, a^2 b^2, \ldots\}$

ε ab aabb



This DFA must keep a^2 and a^4 separate

$a^2 \not\equiv_L a^4$

because $a^2b^2 \in E$ but $a^4b^2 \notin E$

similarly $a^i \not\equiv_L a^j$ where $i \neq j$

and must end up in different state

Proof of irregularity: Suppose contr E regular, let there be k states in E's DFA. Consider $a^0, a^1, \ldots a^k$ – $k+1$ strs but k states, so two must end up in same same state q_i.

But bc these are distinguishable, this is not possible, so contr – E not regular

$\Sigma = \{a, b, c\}$

Another example $EQ = \{wcw \mid w \in \{a,b\}^*\}$

i.e., must remember the first string

Proof template: Suppose DFA exists w/ k states, consider $k+1$ strings, so pigeonhole principle means some x, y end up in same state

but $x \not\equiv_L y$, so contradiction

prove this

Myhill-Nerode Theorem

Thm. If L is a language and S is a distinguishing set for L that contains infinitely many strings, then L is not regular.