

CS 255: Course Info

Intro. to Cryptography

Web page: `cs255.stanford.edu`

Textbook + mooc available online

Assignments: 4 HWs, 2 programming projects

late days: 3 free late days

TA email: `cs255ta@stanford.edu`

Sections: Friday, recommended

Other security courses:

- CS155 → CS356
- CS255 → CS355
- CS251 (Fall)

Goals: how crypto works and how to use it correctly

Cryptography is everywhere!

→ This Zoom room: TLS encryption

Crypto in use

- TLS, SSH
 - pub-key (ElGamal, RSA)
Certificates
 - ① session setup
Secret key initialization btwn server, browser
 - ② use shared key to encrypt traffic
symmetric cipher
For privacy, data integrity
- Secure chat: Signal, iMessage
- at-rest (filesystem) encryption: dmecrypt, FileVault, BitLocker
- wireless: 802.11 (WEP, WEP2, WEP3), 4G, 5G, Bluetooth
- User auth: password management, 2-factor auth (WebAuthn)
- Payments: credit card → Apple Pay, Blockchain
- other applications: elections, auctions

Things to remember

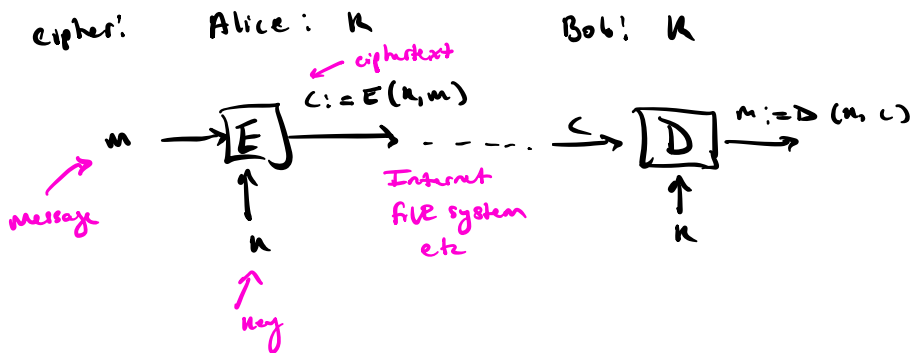
→ In cryptography, no security through obscurity

Goal: cryptosystem should be secure even if source code is open
only secret: short key (128 bits, 16 bytes)

Course organization

- ① using a shared key for confidentiality + data integrity
- ② session setup using public key encryption, digital signatures
- ③ Protocol: session setup, zero knowledge proofs, etc

Symmetric Encryption



Def! A cipher def. over (K, M, C) is a pair of "efficient" algorithms (E, D)

$$E: K \times M \rightarrow C \quad D: K \times C \rightarrow M$$

$$\text{s.t. } \forall m \in M, \forall K \in K: D(K, E(K, m)) = m$$

"efficient": polynomial time in message length, or runs in 1ms

note: enc. alg. E can be randomized

dec. alg. D is always deterministic

← run same alg twice,
may get diff outputs on
same inputs

⇒ No security requirements! (yet)

oldest cipher: substitution

$$\text{key: } K = \begin{bmatrix} a \rightarrow m \\ b \rightarrow g \\ c \rightarrow w \\ \vdots \\ z \rightarrow a \end{bmatrix} \quad |K| = 26! \approx 2^{88}$$

enc. of plaintext $m = "bcza"$ is $C = E(K, m) = "gnom"$

Caesar cipher (no key): shift by 3

$$\begin{bmatrix} a \rightarrow d \\ b \rightarrow e \\ c \rightarrow f \\ \vdots \\ z \rightarrow c \end{bmatrix}$$

To break: ① Freq. of English letters

"e" : 12.7.10 of the time

"t" : 9.1.10

"a" : 8.1.10

⋮

"z"

"x"

② Freq. of pairs of letters (digrams)

"th", "he", "in", "an" ...

③ Trigrams

"the" ...

\Rightarrow Obtain entire key

\Rightarrow ciphertext only attack!

A "secure" cipher: one-time pad (OTP)
Vernam, 1917

$$M = C = \{0,1\}^n \text{ (all } n\text{-bit strings)}$$

$$K = \{0,1\}^n$$

secret key = random bit string as long as the messages

$$C := E(K, m) = K \oplus m$$

$$D(K, c) = K \oplus c$$

$$\Rightarrow D(K, E(K, m)) = K \oplus (K \oplus m) \\ = (K \oplus K) \oplus m = 0 \oplus m = m$$

ex. msg: 0100110
key: 1101100

CT: 1001010

Very fast enc./dec. !!!

Problem: very long keys!

(hard to use - key sharing method can be used to share message)

Goal: keys are 128 or 256 bits

Is OTP a "secure" cipher?

What is a "Secure" cipher?

Shannon 1949

Goal: a cipher is secure if ciphertext reveals no "info" about plaintext

Def: a cipher (E, D) over K, M, C has perfect secrecy if

$$\forall m_0, m_1 \in M \quad (\text{len}(m_0) = \text{len}(m_1)), \quad \forall c \in C$$

$$\Pr[E(k, m_0) = c] = \Pr[E(k, m_1) = c]$$

where k is uniform in K ($k \leftarrow K$)

Given intercepted ciphertext c , attacker can't tell
if msg is m_0 or m_1 ,

($\forall m_0, m_1 \Rightarrow$ attacker learns "nothing" about message)

\Rightarrow if $\text{len}(m_0) \neq \text{len}(m_1)$, no guarantee

\Rightarrow a cipher with perfect secrecy need not hide the message length

\uparrow problems

Theorem: OTP has perfect secrecy

Proof: $\forall m \in M, c \in C$

$$\begin{aligned} \Pr[E(k, m) = c] &= \frac{|\{k \in K \text{ where } E(k, m) = c\}|}{|K|} \\ &= \frac{|\{k \in K \text{ where } k \oplus m = c\}|}{2^n} \end{aligned} \quad \left| \begin{array}{l} \text{if } k \oplus m = c \\ \text{then } k = m \oplus c \\ \Rightarrow \text{one key where } \\ k \oplus m = c \end{array} \right.$$

$$\text{So } \forall m \in M, c \in C: \Pr[E(k, m) = c] = \frac{1}{2^n}$$

So this holds.

\Rightarrow No CT only attacks on OTP!

Bad news tho!

Every cipher (E, D) over (K, M, C) w/ perfect secrecy
must also satisfy $|K| \geq |M|$
 $\Rightarrow \text{len}(K) \geq \text{len}(M)$