# First-Order Logic Continued

Sunday, September 27, 2020 3:10 PM

## Aristotelian Forms

"All A's are B's"

$\forall x. \, (A(x) \to B(x))$

"Some A's are B's"

$\exists x. \, (A(x) \land B(x))$

"No A's are B's"

$\forall x. \, (A(x) \to \neg B(x))$

"Some A's aren't B's"

$\exists x. \, (A(x) \land \neg B(x))$

## Translation exercises

Ex 1. Person(p) - p is a person

Loves(x, y) - x loves y

"every person loves someone else"

$\forall x. \, Person(x) \to (\exists y. \, (Person(y) \land Loves(x, y) \land x \neq y))$

"there is a person that everyone else loves"

$\exists x. \, (Person(x) \land (\forall y. \, ((y \neq x \land Person(y)) \to Loves(y, x))))$

Ex 2. Set(s) - s is a set

$x \in y$ - x is an element of y

"the empty set exists"

$\exists s. \, (set(s) \land \neg(\exists x. \, x \in s))$

$\exists s. \, (set(s) \land (\forall x. \, x \notin s))$

# Negating statements

**Table.**

| | True | False |
|---|---|---|
| $\forall x. P(x)$ | For any $x$, $P(x)$ | For some $x$, $\neg P(x)$ |
| $\exists x. P(x)$ | For some $x$, $P(x)$ | For any $x$, $\neg P(x)$ |
| $\forall x. \neg P(x)$ | For any $x$, $\neg P(x)$ | For some $x$, $P(x)$ |
| $\exists x. \neg P(x)$ | For some $x$, $\neg P(x)$ | For any $x$, $P(x)$ |

**Conclusion.**

$$\neg(\forall x. P(x)) \equiv \exists x. \neg P(x)$$

$$\neg(\exists x. P(x)) \equiv \forall x. \neg P(x)$$

i.e., push the negation across the quantifier, then flip the quantifier

**Equivalences.**

$$\neg(p \land q) \equiv p \rightarrow \neg q$$

$$\neg(p \rightarrow q) \equiv p \land \neg q$$

$\rightarrow$ used w/ $\forall$

$\land$ used w/ E

When negating, flip connectives in accordance.

**Ex.** Negate

$$\exists S. \left(Set(S) \land \forall x. \neg(x \in S)\right)$$

Result:

$$\forall S. \left(Set(S) \rightarrow \exists x. (x \in S)\right)$$

## Restricted Quantifiers

### Quantifying over sets.

The notation $\forall x . \in S . P(x)$ means that "for any $x$ that is an element of set $S$, $P(x)$ holds".
(Vacuously true if $S$ empty)

The notation $\exists x \in S . P(x)$ means that "there exists an element $x$ of set $S$ such that $P(x)$ holds".
(false if $S$ empty)

Note: Do not use other variations of this syntax!

## Expressing uniqueness.

Ex. $\text{WayToFindOut}(w)$ – $w$ is a way to find out

"there is only one way to find out"

$$\exists w . (\text{WayToFindOut}(w) \wedge (\forall x . \text{WayToFindOut}(x) \to (w = x)))$$