Set Theory

Wednesday, September 16, 2020 12:28 PM

"Laws" of Computer Science

Guiding questions

→ "What problems can be solved w/ computers?" ②

→ "Why are some problems harder to solve than others?" ③

→ "How can we be certain in our answers to these questions?" ①

Intro to Set Theory

→ Groups of things; i.e. "CS103 Students"

Defn. A set is an unordered collection of discrete objects which may be anything, including other sets.

$S = \{\text{penny, nickel, dime, quarter}\}$

Two sets are equal if their elements are the same ignoring order.

Sets cannot contain duplicates - they are ignored

The objects of a set are called its elements.

$\text{penny} \in S$

$\text{dollar coin} \notin S$

Sets can contain any number of elements

$\{\} = \emptyset$

Infinite Sets.

Natural #'s

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

Integers

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots\}$

Real numbers $\mathbb{R}$

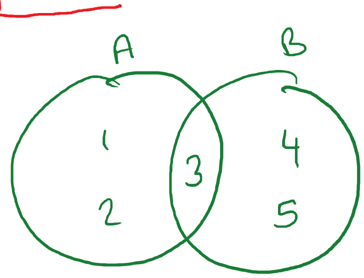
Set-builder notation

Set of even natural numbers.

$\{x \mid x \in \mathbb{N},\ x \equiv 0 \pmod 2\}$

$\{0, 2, 4, 6, \ldots\}$

Combining Sets.



$A = \{1, 2, 3\}$
$B = \{3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5\}$ Union

$A \cap B = \{3\}$ Intersection

$A - B = \{1, 2\}$ Difference

$A \,\Delta\, B = \{1, 2, 4, 5\}$ Symmetric Difference

Subsets and Power sets.

Defn. A set S is a subset of a set T (i.e., $S \subseteq T$) if all elements of S are elements of T.

Ex. $\{1, 2\} \subseteq \{1, 2, 3, 4\}$

$\mathbb{N} \subseteq \mathbb{Z}$

$\emptyset \subseteq S$ for any S

$S \subseteq S$ for any S

Defn. The power set of a set S, i.e. $\mathcal{P}(S)$, is the set of all subsets of S.

Ex. $\mathcal{P}(\{1, 2\}) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

$\mathcal{P}(\emptyset) = \{\emptyset\}$

Cardinality.

Defn. The cardinality of a set S, i.e. $|S|$, is the number of elements within S.

Ex. $|\{1, 2\}| = 2$

$|\emptyset| = 0$

Cardinality of $\mathbb{N}$. $|\mathbb{N}| = \aleph_0$ (aleph-zero)

Another defn. Two sets have the same cardinality if there exists a way to pair each of their elements with no element left uncovered.

there exists a way to pair each of their elements with no element left uncovered.

Ex. $|\mathbb{N}| = |\mathbb{Z}|$

Cantor's theorem. For any set S, $|S| < |\wp(S)|$

due to diagonalization.