

STATS 202A Final Homework due Thursday 5pm

Note: Please write R code, but please use C++ to implement all the loops. Please do not use built-in random number generators.

Problem 1: Write random number generators for uniform and standard normal distributions. You can generate T numbers from each random number generator, and plot the histogram.

Problem 2: Let $\pi(x) \propto \exp\{-x^2/(2\sigma^2)\}$ with x being integers. Implement the Metropolis algorithm to sample from π . You can start from $X_0 = 0$, and then generate X_1, X_2, \dots, X_T . Plot the histogram of X_{B+1}, \dots, X_T .

Problem 3: Let $p(x, y)$ be the bivariate normal distribution with the following density function

$$p(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{x^2 + y^2 - 2\rho xy}{2(1-\rho^2)}\right].$$

Starting from (X_0, Y_0) , use the Gibbs sampler to generate (X_t, Y_t) for $t = 1, 2, \dots, T$. Repeat it M times, so that we get M parallel chains $(X_t^{(m)}, Y_t^{(m)}), t = 1, \dots, T, m = 1, \dots, M$, as if M people are moving independently according to the Gibbs sampler. For each t , draw a scatterplot of the M people: $(X_t^{(m)}, Y_t^{(m)}), m = 1, \dots, M$, which is a snapshot of the M people at time t . See if you can make a movie of the change of the scatterplot over time. Try different starting values of (X_0, Y_0) (e.g., $(0, 0)$, $(-10, -10)$, etc.), and different ρ (e.g., .5, .9, .99 etc.)