Question No.2.

Refer the Linear programming problem from Question No. 1. Solve this LP problem using the Table 1. If a variable $x_6 \geq 0$ is added to the problem with:

(a) $A^6 = [2, 1]^T$, coefficient of x_6 in $z = -1$.

$A^{(6)*} = B^{-1}A6$

$B = \begin{bmatrix} 2 & -6 \\ 3 & -3 \end{bmatrix}$ $\quad B^{-1} = \frac{1}{|B|} \text{Adj.} B.$

$= \frac{1}{12}\begin{bmatrix} -3 & 6 \\ -3 & 2 \end{bmatrix}$

$B^{-1} = \frac{1}{12}\begin{bmatrix} -3 & 6 \\ -3 & 2 \end{bmatrix}$

$A^{(6)*} = \frac{1}{12}\begin{bmatrix} -3 & 6 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$= \frac{1}{12}\begin{bmatrix} -6+6 \\ -6+2 \end{bmatrix}$

$= \frac{1}{12}\begin{bmatrix} 0 \\ -4 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ -1/3 \end{bmatrix}$

$c_6^* = C_6 - C_B A^{*(6)}$

$= -1 - [-2 \;\; 5]\begin{bmatrix} 0 \\ -1/3 \end{bmatrix}$

$= -1 - (0 - 5/3)$

$= -1 + 5/3$

$= 2/3.$

(b) $A^{(6)} = [2, 1]^T$, coefficient of x_6 in $z = -2$.

$A^{(6)*} = \begin{bmatrix} 0 \\ -1/3 \end{bmatrix}$

$c_6^* = C_6 - C_B A^{*(6)}$

$= -2 - (0 - 5/3)$

$= -2 + 5/3$

$= -1/3$

(c) $A^{(6)} = [8, 6]^T$, coefficient of x_6 in $z = -9$.

$$A^{(6)*} = B^{-1}A_6 = \frac{1}{12}\begin{bmatrix} -3 & 6 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ 8 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 12 \\ -12 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C_6^* = C_6 - C_B A^{*(6)} = -9 - [-2 \quad 5]\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -9 - (-2-5) = -9 - (-7) = -9 + 7 = -2$$

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	0	1/2	-1/4	7/2	1	1	6
x_1	1	1/6	-1/4	3/2	0	-1	1
	0	1/6	3/4	5/2	0	-2	-14
x_6	0	1/2	-1/4	7/2	1	1	6
x_1	1	2/3	-1/2	5	1	0	7
	0	7/6	1/4	19/2	2	0	-2

$z_{min} = -2$. at $(7, 0, 0, 0, 0, 6)$

(d) $A^{(6)} = [8, 6]^T$, coefficient of x_6 in $z = -11$

$$A^{(6)*} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C_6^* = C_6 - C_B A^{*(6)} = -11 - (-2-5) = -11 - (-7) = -11 + 7 = -4$$

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	0	1/2	-1/4	7/2	1	1	6
x_1	1	1/6	-1/4	3/2	0	-1	1
	0	1/6	3/4	5/2	0	-4	-14

x_6	0	1/2	-1/4	7/2	1	1	6
x_1	1	2/3	-1/2	5	1	0	7
	0	13/6	-1/4	33/2	4	0	10

$Z_{min} = 10$ at $(7, 0, 0, 0, 0, 6)$.

Question No. 2.

Bhutan furniture House company makes tables, chairs, and book shelves, along with other items. The information is tabulated as follows.

	Product			
	Tables	Chairs	Book shelves	Availability
Wood	10	3	10	100
Labour (hrs)	5	5	5	6.
Profit (Nu).	200	150	150	

Maximize. $200x_1 + 150x_2 + 150x_3$

S.T $\quad 10x_1 + 3x_2 + 10x_3 \leq 100$

$5x_1 + 5x_2 + 5x_3 \leq 6.$

$x_1, x_2 \ldots x_5 \geq 0.$

(a) Solve the given LP model to obtain the optimal product mix.

Minimize $\quad -200x_1 - 150x_2 - 150x_3$

ST $\quad 10x_1 + 3x_2 + 10x_3 + x_4 = 100$

$5x_1 + 5x_2 + 5x_3 + x_5 = 6.$

$x_1, x_2 \ldots x_5 \geq 0.$

	x_1	x_2	x_3	x_4	x_5	
x_4	10	3	10	1	0	100
x_5	5	5	5	0	1	60
	-200	-150	-150	0	0	0
x_1	1	3/10	1	1/10	0	10
x_5	0	7/2	0	-1/2	1	10
	0	-90	50	20	0	2000
x_1	1	0	1	1/7	-3/35	64/7
x_2	0	1	0	-1/7	2/7	20/7
	0	0	50	50/7	180/7	15800/7

$z_{min} = \frac{-15800}{7}$, $z_{max} = \frac{15800}{7}$ at $(64/7, 20/7, 0)$.

(b) Determine the sensitivity limits for the following. within which the product mix remain optimal:

(1) Available wood.

$b = \begin{bmatrix} 100+\lambda \\ 60 \end{bmatrix}$ $\quad C_B = \begin{bmatrix} -200 & -150 \end{bmatrix}$ $\quad B^{-1} = \frac{1}{|B|} \text{Adj } B.$

$B = \begin{bmatrix} 10 & 3 \\ 5 & 5 \end{bmatrix}$ $\quad |B| = 50 - 15 = 35$

$B^{-1} = \frac{1}{35}\begin{bmatrix} 5 & -3 \\ -5 & 10 \end{bmatrix}$

$$b^* = B^{-1} b$$

$$= \frac{1}{35}\begin{bmatrix} 5 & -3 \\ -5 & 10 \end{bmatrix}\begin{bmatrix} 100+\lambda \\ 60 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 500+5\lambda-180 \\ -500-5\lambda+600 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 320+5\lambda \\ 100-5\lambda \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 320+5\lambda \\ 100-5\lambda \end{bmatrix} = \begin{bmatrix} 64/7 + 1/7\lambda \\ 20/7 \cancel{+} - 1/7\lambda \end{bmatrix} \geq 0$$

$320 + 5\lambda \geq 0$ $\quad$, $\quad$ $100 - 5\lambda \geq 0$

$5\lambda \geq -320$ $\quad\quad$ $-5\lambda \geq -100$

$\lambda \geq -64$ $\quad\quad$ $\lambda \leq 20.$

$-64 \leq \lambda \leq 20$

$$z^* = z_0 - C_B b^*$$
$$= 0 - \begin{bmatrix} -200 & -150 \end{bmatrix} \begin{bmatrix} 64/7 + 1/7\lambda \\ 20/7 - 1/7\lambda \end{bmatrix}$$
$$= 0 - \frac{1}{35} \begin{bmatrix} -200 & -150 \end{bmatrix} \begin{bmatrix} 320 + 5\lambda \\ 100 - 5\lambda \end{bmatrix}$$
$$= -\frac{1}{35}\left(-64000 \cancel{+} -1000\lambda + -15000 + 750\lambda\right)$$
$$= -\frac{1}{35}\left(-79000 - 250\lambda\right)$$
$$= \frac{79000}{35} + \frac{250\lambda}{35}$$
$$= \frac{15800}{7} + \frac{50}{7}\lambda \quad \text{at } -64 \leq \lambda \leq 20.$$

(ii) Available labour (hrs).

$$b = \begin{bmatrix} 100 \\ 60+\lambda \end{bmatrix}$$

$$b^* = B^{-1}b$$
$$= \frac{1}{35}\begin{bmatrix} 5 & -3 \\ -5 & 10 \end{bmatrix}\begin{bmatrix} 100 \\ 60+\lambda \end{bmatrix}$$
$$= \frac{1}{35}\begin{bmatrix} 500 - 180 - 3\lambda \\ -500 + 600 + 10\lambda \end{bmatrix}$$
$$= \frac{1}{35}\begin{bmatrix} 320 - 3\lambda \\ 100 - 10\lambda \end{bmatrix} \geq 0.$$

$$320 - 3\lambda \geq 0$$
$$-3\lambda \geq -320$$
$$\lambda \leq 320/3$$
$$\lambda \leq 106\tfrac{2}{3}$$

,

$$100 - 10\lambda \geq 0$$
$$-10\lambda \geq -100$$
$$\lambda \leq 100/10$$
$$\lambda \leq 10$$

$\therefore \lambda \leq 10$

$$z_0^* = z_0 - C_B b^*$$

$$= 0 - \frac{1}{35}\begin{bmatrix} -200 & -150 \end{bmatrix}\begin{bmatrix} 320-3\lambda \\ 100-10\lambda \end{bmatrix}$$

$$= -\frac{1}{35}\left(-64000 + 600\lambda + -15000 + 1500\lambda\right)$$

$$= -\frac{1}{35}\left(-79000 + 2100\lambda\right)$$

$$= \frac{15800}{7} - \frac{420}{7}\lambda$$

$$= \frac{15800}{7} - 60\lambda$$

Ⓒ Determine the new optimal solution with the following alterations:

(1) Wood availability is decreased to 90 board feet.

$$b = \begin{bmatrix} 90 \\ 60 \end{bmatrix}$$

$$b^* = B^{-1}b$$

$$= \frac{1}{35}\begin{bmatrix} 5 & -3 \\ -5 & 10 \end{bmatrix}\begin{bmatrix} 90 \\ 60 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 450-180 \\ -450+600 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} 270 \\ 150 \end{bmatrix}$$

$$= \begin{bmatrix} 54/\cancel{35}\,7 \\ 30/7 \end{bmatrix}$$

$$z_0^* = z_0 - C_B b^*$$

$$= 0 - \begin{bmatrix} -200 & -150 \end{bmatrix}\begin{bmatrix} 54/7 \\ 30/7 \end{bmatrix}$$

$$= -\left(-\frac{10800}{7} - 4500/7\right)$$

$$= \frac{15300}{7} \quad \text{at } (54/7,\ 30/7,\ 0)$$

(ii) Available labour is increased to 100 hours.

$$b = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

$$b^* = B^{-1} b$$
$$= \frac{1}{35} \begin{bmatrix} 5 & -3 \\ -5 & 10 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$
$$= \frac{1}{35} \begin{bmatrix} 500-300 \\ -500+1000 \end{bmatrix}$$
$$= \frac{1}{35} \begin{bmatrix} 200 \\ 500 \end{bmatrix}$$
$$= \begin{bmatrix} 40/7 \\ 100/7 \end{bmatrix}$$

$$z_0^* = z_0 - C_B b^*$$
$$= 0 - \begin{bmatrix} -200 & -150 \end{bmatrix} \begin{bmatrix} 40/7 \\ 100/7 \end{bmatrix}$$
$$= -\left(\frac{-8000}{7} - \frac{15000}{7}\right)$$
$$= \frac{23000}{7} \text{ hn.}$$

(d) Determine the total profit the company makes a profit of Nu. 180 per table and Nu. 100 per chair.

$$Z = 180x_1 + 100x_2 + 150x_3$$

$$c = [-180 \quad -100 \quad -150 \quad 0 \quad 0].$$

$$C_B = [-180 \quad -100]$$

$$c^* = c - C_B b^*$$
$$= [-180 \quad -100 \quad -150 \quad 0 \quad 0] - [-180 \quad -100] A^*$$
$$= [-200 \quad -150 \quad -150 \quad 0 \quad 0] + [+20 \quad 50 \quad 0 \quad 0 \quad 0] - \{[-200 \quad -150] + [20$$
$$A^*$$
$$= [-200 \quad -150 \quad -150 \quad 0 \quad 0] - [-200 \quad -150] A^* + [20 \quad 50 \quad 0 \quad 0 \quad 0] - [20 \quad 50] A^*$$
$$= [0 \quad 0 \quad 50 \quad 50/7 \quad 180/7] + [20 \quad 50 \quad 0 \quad 0 \quad 0] - [20 \quad 50] \begin{bmatrix} 64/7 \\ 20/7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 50 & 50/7 & 180/7 \end{bmatrix} + \begin{bmatrix} 20 & 50 & 0 & 0 & 0 \end{bmatrix} - \cancel{1280} \begin{bmatrix} 20 & 50 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1/7 & -3/35 \\ 0 & 1 & 0 & -1/7 & 2/7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 50 & 50/7 & 180/7 \end{bmatrix} + \begin{bmatrix} 20 & 50 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 20 & 50 & 20 & \frac{20-50}{7} & \frac{-60}{35} + \frac{100}{7} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 50 & 50/7 & 180/7 \end{bmatrix} + \begin{bmatrix} 20 & 50 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 20 & 50 & 20 & -\frac{30}{7} & \frac{440}{35} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 50 & 50/7 & 180/7 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -20 & \frac{30}{7} & -\frac{440}{35} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 50 & 50/7 & 180/7 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -20 & \frac{30}{7} & -\frac{88}{7} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 30 & \frac{80}{7} & \frac{92}{7} \end{bmatrix}$$

$$z_0^* = z_0 - c_B b^*$$

$$= 0 - \begin{bmatrix} -180 & -100 \end{bmatrix} \begin{bmatrix} 64/7 \\ 20/7 \end{bmatrix}$$

$$= -\left[-\frac{11520}{7} - \frac{2000}{7} \right]$$

$$= -\left(-\frac{13520}{7} \right)$$

$$= \frac{13520}{7}$$

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Question 4.

A company sells two products A and B. The selling price and incremental cost information is as follows:

	Product A	Product B
selling price (Nu)	60	40
Incremental cost (Nu.)	30	10
Incremental profit (Nu.)	30	30

The two products are produced in a common production process and are sold in two diff. markets. The production process has a capacity of 30,000 labour hours. It takes three hours to produce a unit of A and one hour to produce a unit B. The market has been surveyed and the company officials feel that the maximum number of units of A that can be sold is 8000; and that of B is 12000 units.

(a) Determine the optimal product mix.

Maximize $30x_1 + 30x_2$

$$\text{s.t} \begin{cases} 3x_1 + x_2 \leq 30000 \\ x_1 \leq 8000 \\ x_2 \leq 12000 \\ x_1, x_2 \geq 0. \end{cases}$$

Minimize $-30x_1 \cancel{+3} -30x_2$

$$\text{s.t} \begin{cases} 3x_1 + x_2 + x_3 = 30000 \\ x_1 + x_4 = 8000 \\ x_2 + x_5 = 12000 \end{cases}$$

$$x_1, x_2 \ldots x_5 \geq 0.$$

	x_1	x_2	x_3	x_4	x_5	
x_3	3	1	1	0	0	30000
x_4	1	0	0	1	0	8000
x_5	0	1	0	0	1	12000
	-30	-30	0	0	0	0
x_3	0	1	1	-3	0	6000
x_1	1	0	0	1	0	8000
x_5	0	1	0	0	1	12000
	0	-30	0	30	0	240000
x_2	0	1	1	-3	0	6000
x_1	1	0	0	1	0	8000
x_5	0	0	-1	3	1	6000
	0	0	30	-60	0	420000
x_2	0	1	0	0	1	12000
x_1	1	0	1/3	0	-1/3	6000
x_4	0	0	-1/3	1	1/3	2000
	0	0	10	0	20	540000

$Z_{min} = -540000 \Rightarrow Z_{max} = 540,000$ at $(6000, 12000)$

(b) Perform the sensitivity analysis if the following changes are incorporated in the original problem:

(i) Maximum no. of units of A can be sold is actually 9000

$x_1 \leq 9000$

$$b = \begin{bmatrix} 30000 \\ 9000 \\ 12000 \end{bmatrix}$$

$C_B = \begin{bmatrix} -30 & -30 & 0 \end{bmatrix}$

$\text{Adj } B = \begin{bmatrix} 0 & -1 & -1 \\ 0 & 0 & -3 \\ 3 & 1 & 1 \end{bmatrix} \Rightarrow$

$= \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 3 \\ 3 & -1 & 1 \end{bmatrix} \Rightarrow$

$= \begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} \text{Adj } B$

$= \frac{1}{3} \begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}$

$b = \begin{bmatrix} 30000 \\ 9000 \\ 12000 \end{bmatrix}$

$b^* = B^{-1} b$

$= \frac{1}{3} \begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 30000 \\ 9000 \\ 12000 \end{bmatrix}$

$= \frac{1}{3} \begin{bmatrix} 36000 \\ 18000 \\ 9000 \end{bmatrix}$

$= \begin{bmatrix} 12000 \\ 6000 \\ 3000 \end{bmatrix}$

$z_0^* = z_0 - \begin{bmatrix} -30 & -30 & 0 \end{bmatrix} b^*$

$= 0 - \begin{bmatrix} -30 & -30 & 0 \end{bmatrix} \begin{bmatrix} 12000 \\ 6000 \\ 3000 \end{bmatrix}$

$= 0 - (-360000 - 180000 + 0]$

$= 540,000$

(ii) Maximum no. of units of B can be sold is actually 13 000

$$b = \begin{bmatrix} 30000 \\ 8000 \\ 13000 \end{bmatrix}$$

$$b^* = B^{-1} b$$

$$= \frac{1}{3}\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 30000 \\ 8000 \\ 13000 \end{bmatrix}$$

$$= \frac{1}{3}\left\{\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}\left\{\begin{bmatrix} 30000 \\ 8000 \\ 12000 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix}\right\}\right\}$$

$$= \frac{1}{3}\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 30000 \\ 8000 \\ 12000 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 12000 \\ 6000 \\ 2000 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 3000 \\ -1000 \\ 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 12000 \\ 6000 \\ 2000 \end{bmatrix} + \begin{bmatrix} 1000 \\ -1000/3 \\ 1000/3 \end{bmatrix}$$

$$= \begin{bmatrix} 13000 \\ 17000/3 \\ 10000/3 \end{bmatrix}$$

$$z_0^* = z_0 - C_B b^*$$

$$= 0 - \begin{bmatrix} -30 & -30 & 0 \end{bmatrix}\begin{bmatrix} 13000 \\ 17000/3 \\ 10000/3 \end{bmatrix}$$

$$= 0 - (-390000 - 510000/3)$$

$$= 0 - (-390000 - 170000)$$

$$= 560{,}000$$

(iii) Labour hours available is 31000 instead of 30,000 hours as in the base case.

$$b = \begin{bmatrix} 31000 \\ 8000 \\ 12000 \end{bmatrix}$$

$$b^* = B^{-1}b.$$

$$= \frac{1}{3}\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 31000 \\ 8000 \\ 12000 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}\left\{\begin{bmatrix} 30000 \\ 8000 \\ 12000 \end{bmatrix} + \begin{bmatrix} 1000 \\ 0 \\ 0 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 12000 \\ 8000 \\ 3000 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & -1 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1000 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 12000 \\ 6000 \\ 3000 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 0 \\ 1000 \\ -1000 \end{bmatrix} = \begin{bmatrix} 12000 \\ 19000/3 \\ 8000/3 \end{bmatrix}$$

$$z_0^* = z_0 - C_B b^*$$

$$= 0 - \begin{bmatrix} -30 & -30 & 0 \end{bmatrix}\begin{bmatrix} 12000 \\ 19000/3 \\ 8000/3 \end{bmatrix}$$

$$= 0 - (-360000 - 190000)$$

$$= 550000$$

Question No.5.

Solve the linear programming problem.

$$\max.\quad z = 11x_1 + 4x_2 + x_3 + 15x_4$$

$$\text{S.T}\quad 3x_1 + x_2 + 2x_3 + 4x_4 \leq 28,$$

$$8x_1 + 2x_2 - x_3 + 7x_4 \leq 50,$$

$$x_1, x_2, \ldots x_4 \geq 0.$$

Table 2: Initial & Final Tableau.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	3	1	2	4	1	0	28
x_6	8	2	-1	7	0	1	50
	11	4	1	15	0	0	0
x_4	-2	0	5	1	2	-1	6
x_2	11	1	-18	0	-7	4	4
	-3	0	-2	0	-2	-1	-106.

(a) $2x_1 - x_2 - 5x_3 \geq 12$

checking if soln point. satisfies the new constraint.

$(0, 4, 0, 6)$.

$$0 - 4 - 0 \ngeq 12$$

Introduce the new slack x_7 in the new constraint.

$$-2x_1 + x_2 + 5x_3 + x_7 = \cancel{12} -12$$

Eliminate x_2.

$$-2x_1 + (-11x_1 + 18x_3 + 7x_5 - 4x_6) + 5x_3 + x_7 = -12 - 4$$

$$-13x_1 + 23x_3 + 7x_5 - 4x_6 + x_7 = -16.$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	-2	0	5	1	2	-1	0	6
x_2	11	1	-18	0	-7	4	0	4
x_7	-13	0	23	0	7	-4	1	-16
	+13	0	2	0	2	1	0	106
x_4	0	0	19/13	1	12/13	-5/13	-2/13	110/13
x_2	0	1	19/13	0	-14/13	8/13	11/13	-124/13
x_1	1	0	-23/13	0	-7/13	4/13	-1/13	16/13
	0	0	95/13	0	47/13	1/13	3/13	1330/13

~~New optimal point is at (16/13, -124/1~~