

### Suffix Array:

```
int wa[mx],wb[mx],wv[mx],Ws[mx];
//(1-indexed) sa[i] = starting position (0...n-1) of ith
lexicographically smallest suffix in s
//(0-indexed) Rank[i] = lexicographical rank of s[i....n-1] ((i+1)th
suffix by position)
//LCP[i] = longest common prefix of sa[i] & sa[i-1]
int sa[mx],Rank[mx],LCP[mx];
int cmp(int *r,int a,int b,int l) {return r[a]==r[b] &&
r[a+l]==r[b+l];}
//Suffix Array (O(nlogn))
//m = maximum possible ASCII value of a string character
(alphabet size)
//also, m = maximum number of distinct character in string
(when compressed)
void buildSA(string s,int* sa,int n,int m){
    int i,j,p,*x=wa,*y=wb,*t;
    for(i=0; i<m; i++) Ws[i]=0;
    for(i=0; i<n; i++) Ws[x[i]]=s[i]++;
    for(i=1; i<m; i++) Ws[i]+=Ws[i-1];
    for(i=n-1; i>=0; i--) sa[--Ws[x[i]]]=i;
    for(j=1,p=1; p<n; j<=1,m=p){
        for(p=0,i=n-j; i<n; i++) y[p++]=i;
        for(i=0; i<n; i++) if(sa[i]>=j) y[p++]=sa[i]-j;
        for(i=0; i<n; i++) wv[i]=x[y[i]];
        for(i=0; i<m; i++) Ws[i]=0;
        for(i=0; i<n; i++) Ws[wv[i]]++;
        for(i=1; i<m; i++) Ws[i]+=Ws[i-1];
        for(i=n-1; i>=0; i--) sa[--Ws[wv[i]]]=y[i];
        for(t=x,x=y,y=t,p=1,x[sa[0]]=0,i=1; i<n; i++){
            x[sa[i]]=cmp(y,sa[i-1],sa[i],j) ? p-1 : p++;
        }
    }
}
//Kasai's LCP algorithm (O(n))
void buildLCP(string s,int *sa,int n){
    int i,j,k=0;
    for(i=1; i<=n; i++) Rank[sa[i]]=i;
    for(i=0; i<n; i++) LCP[Rank[i+1]]=k;
    for(k?k--:0, j=sa[Rank[i]-1]; s[i+k]==s[j+k]; k++){
        return;}
}
// Pattern Subtring hisbe ace kina
bool Pattern(string &text,string &pat)
{ int lo=1,hi=text.size();
  while(lo<=hi)
  { int mid=(lo+hi)/2;
    int ok=0;
    for(int i=0;i<pat.size();i++)
    { if(text[i+sa[mid]]>pat[i]) {ok=1;break;}
      if(text[i+sa[mid]]<pat[i]) {ok=-1;break;}
    }
    if(!ok) return true;
    if(ok>0) hi=mid-1;
    else lo=mid+1;
  }
  return false;
}
```

```
}
pair<int,int> Patterntern_occurence(string Text ,string Pattern)
{
    int n=Text.size();
    int m=Pattern.size();
    int be=1,en=n;
    while(be<en)
    { int mid = (en+be)/2;
      int ok=0;
      for(int i=0;i<m;i++){
        if(Text[i+sa[mid]]>Pattern[i]){ok=1;break;}
        if(Text[i+sa[mid]]<Pattern[i]){ok=-1;break;}
      }
      if(ok+1) en=mid;
      else be=mid+1;
    }
    bool ok = 1;
    for(int i=0;i<m;i++) if(Text[i+sa[be]]!=Pattern[i]){ok=0;break;}
    if(!ok) return {-1,-1};
    pair<int,int> re;
    re.first=be;
    be=1,en=n;
    while(be<en)
    { int mid = (en+be)/2;
      int ok=0;
      for(int i=0;i<m;i++){
        if(Text[i+sa[mid]]>Pattern[i]){ok=1;break;}
        if(Text[i+sa[mid]]<Pattern[i]){ok=-1;break;}
      }
      if(ok>0) en=mid;
      else be=mid+1;
    }
    ok = 1;
    for(int i=0;i<m;i++) if(Text[i+sa[en]]!=Pattern[i]){ok=0;break;}
    if(!ok) en--;
    re.second=en;
    return re;
}
/// this is for LCP from index i to index j.
/// just run a query from min(Rank[i-1],Rank[j-1])+1 to
max(Rank[i-1],Rank[j-1])
int ST[mx][22];
int Jump_LOG[mx];
void Build_Sparse(int n)
{ for(int i=1;i<=n;i++)ST[i][0]=LCP[i];
  for(int i=2;i<=n;i++){Jump_LOG[i]=Jump_LOG[i-1]+!(i&(i-1));
    for(int j=1;(1<j)<=n;j++){
      for(int i=1;(i+(1<j)-1)<=n;i++){
        ST[i][j]=min(ST[i][j-1],ST[i+(1<j)-1]][j-1]);
      }
    }
  }
}
int query(int i,int j){
    int boro_lav=Jump_LOG[j-i+1];
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return min(ST[i][boro_lav],ST[j-
(1<<boro_lav)+1][boro_lav]);
}
void solve(){
    buildSA(s,sa,n+1,130); //Important
    buildLCP(s,sa,n);
    for(int i=1;i<=n;i++) cout<<sa[i]<<" "; cout<<endl;
    for(int i=0;i<n;i++) cout<<Rank[i]<<" "; cout<<endl;
    for(int i=1;i<=n;i++) cout<<LCP[i]<<" ";
    pair<int,int>re=Patterntern_occurence(s,t);
    if(re.second==1)printf("0\n");
    else printf("%d\n",re.second-re.first+1 );
}

```

#### Aho Corasick:

```

struct Aho_Corasick{
    int Trie[mx][27],Suffix_Link[mx];
    vector<int> Mark[mx];
    int Node;
    void Init() {
        fill(Trie[0],Trie[0]+26,-1);
        Mark[0].clear();
        Node=0;
    }
    void Insert(char ch[],int idx) {
        int len=strlen(ch);
        int cur=0;
        for(int i=0;i<len;i++){
            int val=ch[i]-'a';
            if(Trie[cur][val]==-1) {
                Trie[cur][val]=++Node;
                fill(Trie[Node],Trie[Node]+26,-1);
                Mark[Node].clear();
            }
            cur=Trie[cur][val];
        }
        Mark[cur].push_back(idx);
    }
    void Cal_Suffix_Link() {
        queue<int>q;
        Suffix_Link[0]=0;
        for(int i=0;i<26;i++){
            if(Trie[0][i]!=-1){
                q.push(Trie[0][i]);
                Suffix_Link[Trie[0][i]]=0;
            }
            else Trie[0][i]=0;
        }
        while(!q.empty()){
            int u=q.front();
            q.pop();
            for(int v: Mark[Suffix_Link[u]]){
                Mark[u].push_back(v);
            }
            for(int i=0;i<26;i++) {
                if(Trie[u][i] != -1) {

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                Suffix_Link[Trie[u][i]] = Trie[Suffix_Link[u]][i];
                q.push(Trie[u][i]);
            }
        }
        else
            Trie[u][i] = Trie[Suffix_Link[u]][i];
    }
}
}Automata;
/// Pattern Occurence Count
int cnt[mx];
void Count_Pattern(char ch[]){
    int cur=0;
    int len=strlen(ch);
    for(int i=0;i<len;i++) {
        int val=ch[i]-'a';
        cur= Automata.Trie[cur][val];
        for(int id: Automata.Mark[cur])cnt[id]++;
    }
}

```

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void solve(){
    char ch1[1000005],ch[mx];
    scanf("%d%s",&n,ch1);
    Automata.Init();
    for(int i=0;i<n;i++){
        scanf("%s",ch);
        Automata.Insert(ch,i);
    }
    Automata.Cal_Suffix_Link();
    Count_Pattern(ch1);
    /// print Occurence Frequency
    for(int i=0;i<n;i++){
        printf("%d\n",cnt[i]);
        cnt[i]=0;
    }
}

```

#### Hashing:

```

/*backup prime 307,367,1040160883,1066517951
,1e9+7,1e9+9,1072857881,1000004249 */
struct Hash_dui{
    ll base,mod;
    int sz;
    vector<int>Rev,Forw,P;
    Hash_dui(){
        Hash_dui(const char* s,ll b,ll m)
        {
            sz=strlen(s),base=b,mod=m;

            Rev.resize(sz+2,0),Forw.resize(sz+2,0),P.resize(sz+2,1);
            for(int i=1;i<=sz;i++)P[i]=(base*P[i-1])%mod;
            for(int i=1;i<=sz;i++)Forw[i]=(Forw[i-1]*base+(s[i-1]-
'a'+1))%mod; /// digit hole s[i-1]-'0'
            for(int i=sz;i>=1;i--)Rev[i]=(Rev[i+1]*base+(s[i-1]-
'a'+1))%mod; ///alphabet hole s[i-1]-'a'
        }
    }
}

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void Single_char_ad(char cc) {
    P.push_back((P.back()*base)% mod);
    Forw.push_back((Forw.back()*base+(cc-'a'+1))% mod);
}

inline int Range_Hash(int l,int r) {
    int re_hash=Forw[r+1]-((ll)P[r-l+1]*Forw[l]%mod);
    if(re_hash<0)re_hash+=mod;
    return re_hash;
}

inline int Reverse_Hash(int l,int r) {
    int re_hash=Rev[l+1]-((ll)P[r-l+1]*Rev[r+2]%mod);
    if(re_hash<0)re_hash+=mod;
    return re_hash;
}

};

struct Hash_Main
{
    Hash_dui h1,h2;
    Hash_Main(){}
    Hash_Main(const char* s){
        h1=Hash_dui(s,1949313259, 2091573227);
        h2=Hash_dui(s,1997293877, 2117566807);
    }
    void Char_Add(char cc){
        h1.Single_char_ad(cc);
        h2.Single_char_ad(cc);
    }
    inline ll Range_Hash(int l,int r) /// O base index
    {
        return
        ((ll)h1.Range_Hash(l,r)<<32)^h2.Range_Hash(l,r);
    }
    inline ll Reverse_Hash(int l,int r) /// O base index
    {
        return
        ((ll)h1.Reverse_Hash(l,r)<<32)^h2.Reverse_Hash(l,r);
    }
};

void solve(){
    int n;
    scanf("%d%s",&n,ch);
    string re=ch;
    Hash_Main h_ek(ch);
    ll h1=h_ek(l,r)//O base
}

Manachers:
int oddPlen[mx],evenPlen[mx];
void Manachers(){
    int l=0,r=-1;
    for(int i=0;i<n;i++) {
        int k=(i>r)?1:min(oddPlen[l+r-i],r-i+1);
        while(k<=i && i+k<n && ch[i-k]==ch[i+k]) k++;
        oddPlen[i]=k--;
        if(i+k>r){
            l=i-k;

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            r=i+k;
        }
    }
    l=0,r=-1;
    for(int i=0;i<n;i++){
        int k=(i>r)?0:min(evenPlen[l+r-i+1],r-i+1);
        while(k+1<=i && i+k<n && ch[i-k-1]==ch[i+k])k++;
        evenPlen[i]=k--;
        if(i+k>r){
            l=i-k-1;
            r=i+k;
        }
    }
}

void solve()
{
    Manachers();
    for(int i=0;i<n;i++)printf("%d %d\n",oddPlen[i]*2-1,evenPlen[i]*2);
}

Pi Table / Prefix Functions:
vector<int> Create_Pi_Table(const char* s){
    int sz=strlen(s);
    vector<int>pi(sz);
    for(int i=1;i<sz;i++){
        int j=pi[i-1];
        while(j>0 && s[i]!=s[j])j=pi[j-1];
        if(s[j]==s[i])j++;
        pi[i]=j;
    }
    return pi;
}

void solve(){
    vector<int> pi=Create_Pi_Table(ch);
    for(int i=0;i<n;i++)printf("%d\n",pi[i] );
}

HLD(value in edge):
vector<pair<int,int>>g[mx];
int par[mx],sub_sz[mx];
int Head[mx],st[mx],sesh[mx];
int Rin[mx]; /// Segment Tree er init ye Tree[bode]=ar[Rin[be]]
likte hobe
int T;
using namespace Segment_Tree;
void sz_dfs(int u,int p){
    sub_sz[u]=1;
    par[u]=p;
    for(auto &v: g[u]){
        if(v.first==p)continue;
        sz_dfs(v.first,u);
        sub_sz[u]+=sub_sz[v.first];
        if(sub_sz[v]>sub_sz[g[u][0].first])swap(v,g[u][0]);
    }
}

void hld_dfs(int u,int p,int cost){
    st[u]++;T;

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Rin[st[u]]=u;
ar[st[u]]=cost; /// node ye nai , sgement tree build array
for(auto v:g[u]) {
    if(v.first==p)continue;
    Head[v.first]= (v.first==g[u][0].first ? Head[u]:v.first);
    hld_dfs(v,u,v.second);
}
}
sesh[u]=T;
}
void hld_build(int root){
    T=0;
    Head[root]=root;
    sz_dfs(root,root);
    hld_dfs(root,root,0);
}
bool ls_it_parent(int p,int u){
    return st[p]<=st[u] && sesh[u]<=sesh[p];
}
int path_query(int u,int v){
    int re=-inf;
    while(1){
        if(ls_it_parent(Head[u],v))break;
        re=max(re,query(1,1,n,st[Head[u]],st[u])); ///
for sum we will do just add all query sum
        u=par[Head[u]];
    }
    swap(u,v);
    while(1){
        if(ls_it_parent(Head[u],v))break;
        re=max(re,query(1,1,n,st[Head[u]],st[u])); ///
for sum we will do just add all query sum
        u=par[Head[u]];
    }
    if(st[u]>st[v])swap(u,v);
    re=max(re,query(1,1,n,st[u]+1,st[v])); /// node hole
st[u] theke start
    return re;
}
void path_update(int u,int v,int val){
    while(1){
        if(ls_it_parent(Head[u],v))break;
        Rupdate(1,1,n,st[Head[u]],st[u],val);
        u=par[Head[u]];
    }
    swap(u,v);
    while(1){
        if(ls_it_parent(Head[u],v))break;
        Rupdate(1,1,n,st[Head[u]],st[u],val);
        u=par[Head[u]];
    }
    if(st[u]>st[v])swap(u,v);
    Rupdate(1,1,n,st[u]+1,st[v],val); /// node hole st[u]
theke start
}
void update_subtree(int u,int val){

```

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Rupdate(1,1,n,st[u]+1,sesh[u],val);
}
1D Sparse Table:
int ST[mx][MAX_logN];
int Jump_LOG[mx];
void Build_Sparse(){
    for(int i=1;i<=n;i++)ST[i][0]=ar[i];
    for(int i=2;i<=n;i++){
        Jump_LOG[i]=Jump_LOG[i-1]+!(i&(i-1));
        for(int j=1;(1<<j)<=n;j++){
            for(int i=1;(i+(1<<j)-1)<=n;i++){
                ST[i][j]=min(ST[i][j-1],ST[i+(1<<j-1)][j-1]);
            }
        }
    }
}
int query(int i,int j){
    int boro_lav=Jump_LOG[j-i+1];
    return min(ST[i][boro_lav],ST[j-
(1<<boro_lav)+1][boro_lav]);
}
2D Sparse (Rectangle):
int ST[mx][mx][MAX_logN][MAX_logN];
void Build_2D_Sparse(){
    for(int i=1;i<=n;i++){
        for(int j=1;j<=n;j++){
            ST[i][j][0][0]=ar[i][j];
        }
        for(int l=1;(1<<l)<=n;l++){
            int pre=1<<(l-1);
            for(int j=1;j+pre<=n;j++){
                ST[i][j][0][l]=min(ST[i][j][0][l-1],ST[i][j+pre][0][l-1]);
            }
        }
        for(int l=1;(1<<l)<=n;l++){
            int pre=1<<(l-1);
            for(int i=1;i+pre<=n;i++){
                for(int k=0;(1<<k)<=n;k++){
                    for(int j=1;j<=n;j++){
                        ST[i][j][l][k]=min(ST[i][j][l-1][k],ST[i+pre][j][l-1][k]);
                    }
                }
            }
        }
    }
}
int query(int i,int j,int p,int q) {
    int boro_jum1=log2(p-i+1);
    int boro_jum2=log2(q-j+1);
    int pre1=1<<boro_jum1;
    int pre2=1<<boro_jum2;
    int re1=min(ST[i][j][boro_jum1][boro_jum2],ST[i][q-
pre2+1][boro_jum1][boro_jum2]);
    int re2=min(ST[p-
pre1+1][j][boro_jum1][boro_jum2],ST[p-pre1+1][q-
pre2+1][boro_jum1][boro_jum2]);
}

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        return min(re1,re2);
    }
2D Sparse (Square):
int ST[mx][mx][MAX_logN];
void Build_2D_Sparse(){
    for(int l=0;(1<=l)<=n;l++){
        for(int i=1;i+(1<=l)-1<=n;i++){
            for(int j=1;j+(1<=l)-1<=n;j++){
                if(l==0)ST[i][j][l]=ar[i][j];
                else{
                    int pre=1<=(l-1);
                    int val1=min(ST[i][j][l-1],ST[i+pre][j][l-1]);
                    int val2=min(ST[i][j+pre][l-1],ST[i+pre][j+pre][l-1]);
                    ST[i][j][l]=min(val1,val2);
                }
            }
        }
    }
    int query(int i,int j,int sz){
        int boro_lav=log2(sz);
        int pre=1<=(boro_lav);
        int val1=min(ST[i][j][boro_lav],ST[i+sz-pre][j][boro_lav]);
        int val2=min(ST[i][j+sz-pre][boro_lav],ST[i+sz-pre][j+sz-pre][boro_lav]);
        return min(val1,val2);
    }
MO:
namespace MO{
    const int N=100005;
    const int Q=100005;
    int ar[N],BlockId[N],ans[Q];
    bool vis[N];
    struct node {
        int l,r,id;
        node(){}
        node(int l,int r,int id){
            this->l=l;
            this->r=r;
            this->id=id;
        }
        bool operator < (const node& u) {
            int a=BlockId[l],b=BlockId[u.l];
            if(a==b){
                return (a & 1 ? (r > u.r) : (r < u.r));
            }
            else return a<b;
        }
    }query[Q];
    int boro=0;
    int cnt[mx],cnt_tot[mx];
    void check(int pos) {
        if(vis[pos]){
            cnt_tot[cnt[ar[pos]]]--;
            cnt[ar[pos]]--;
            if(cnt[ar[pos]]cnt_tot[cnt[ar[pos]]]++);
            if(cnt_tot[boro]==0)boro--;
        }
        else{

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            if(cnt[ar[pos]]cnt_tot[cnt[ar[pos]]]--);
            cnt[ar[pos]]++;
            cnt_tot[cnt[ar[pos]]]++;
            if(cnt_tot[boro+1])boro++;
        }
        vis[pos]^=1;
    }
}
using namespace MO;
void solve(){
    int q;
    boro=0;
    scanf("%d%d",&n,&q);
    int sz=sqrt(n);
    for(int i=1;i<=n;i++){
        BlockId[i]=i/sz;
        vis[i]=false;
        scanf("%d",&ar[i]);
    }
    memset(cnt,0,sizeof(cnt));
    memset(cnt_tot,0,sizeof(cnt_tot));
    for(int i=1;i<=q;i++){
        int x,y;
        scanf("%d%d",&x,&y);
        query[i]=node(x,y,i);
    }
    sort(query+1,query+q+1);
    int left=query[1].l;
    int right=left-1;
    for(int i=1;i<=q;i++){
        node Now=query[i];
        while(left<Now.l)check(left++);
        while(left>Now.l)check(--left);
        while(right<Now.r)check(++right);
        while(right>Now.r)check(right--);
        ans[Now.id]=boro;
    }
}
MO's On tree:
int n,m,ii,k,LOG;
int depth[mx];
int par[mx][25];
namespace MO{
    const int N=100005;
    const int Q=100005;
    int ar[N],br[N],BlockId[N],ans[Q];
    bool vis[N];
    struct node {
        int l,r,id,lca;
        node(){}
        node(int l,int r,int lca,int id){
            this->l=l;
            this->r=r;
            this->lca=lca;
            this->id=id;
        }
    }

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    bool operator < (const node& u) {
        int a=BlockId[l],b=BlockId[u.l];
        return (a==b)?(r<u.r):a<b;
    }
}query[Q];
int re=0,sz;
int cnt[100005];
void check(int pos) {
    if(vis[pos]){
        if(cnt[ar[pos]]==1)re--;
        cnt[ar[pos]]--;
    }
    else{
        if(cnt[ar[pos]]==0)re++;
        cnt[ar[pos]]++;
    }
    vis[pos]^=1;
}
vector<int> g[N];
int Euler[2*N],st[N],en[N],Time;
void dfs(int u,int p,int lvl) {
    st[u]=++Time;
    Euler[Time]=u;
    par[u][0]=p;
    depth[u]=lvl;
    for(int v:g[u]) {
        if(v==p)continue;
        dfs(v,u,lvl+1);
    }
    en[u]=++Time;
    Euler[Time]=u;
}
}
using namespace MO;
void init(int root){
    dfs(root,-1,1);
    for(int j=1;j<LOG;j++) {
        for(int i=1;i<=n;i++) {
            if(par[i][j-1]!=-1) {
                par[i][j]=par[par[i][j-1]][j-1];
            }
            else par[i][j]=-1;
        }
    }
}
int lca(int u,int v){
    if(depth[u]<depth[v])swap(u,v);
    int log=1;
    while(1) {
        int next=log+1;
        if(depth[u]<(1<<next))break;
        log++;
    }
    for(int i=log;i>=0;i--) {
        if(depth[u]-(1<<i)>=depth[v]) {

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            u=par[u][i];
        }
    }
    if(u==v)return u;
    for(int i=log;i>=0;i--) {
        if(par[u][i]!=-1 && par[u][i]!=par[v][i]) {
            u=par[u][i];
            v=par[v][i];
        }
    }
    return par[v][0];
}
void solve(){
    int q;
    scanf("%d%d",&n,&q);
    LOG=log2(n)+1;
    Time=0;
    re=0;
    sz=sqrt(n);
    for(int i=1;i<=n;i++)
        scanf("%d",&ar[i]),br[i]=ar[i],BlockId[i]=i/sz,vis[i]=false,cnt[i]=0;
    // Compressing Coordinates . its a alternative of map
    sort(br+1,br+n+1);
    k = unique(br+1,br+n+1)-br-1;
    for(int i=1;i<=n;i++)
        ar[i]=lower_bound(br+1,br+k+1,ar[i])-br;
    for(int i=1;i<=n;i++){
        int x,y;
        scanf("%d%d",&x,&y);
        g[x].push_back(y);
        g[y].push_back(x);
    }
    init(1);
    for(int i=1;i<=q;i++){
        int x,y;
        scanf("%d%d",&x,&y);
        if(st[x]>st[y])swap(x,y);
        int p=lca(x,y);
        if(x==p)query[i]=node(st[x],st[y],-1,i);
        else query[i]=node(en[x],st[y],p,i);
    }
    sort(query+1,query+1+q);
    int left=query[1].l;
    int right=left-1;
    for(int i=1;i<=q;i++){
        node Now=query[i];
        while(left<Now.l)check(Euler[left++]);
        while(left<Now.l)check(Euler[--left]);
        while(right<Now.r)check(Euler[++right]);
        while(right>Now.r)check(Euler[right--]);
        if(Now.lca!=-1)check(Now.lca);
        ans[Now.id]=re;
        if(Now.lca!=-1)check(Now.lca);
    }
    for(int i=1;i<=q;i++)printf("%d\n",ans[i]);
}

```

```
for(int i=1;i<=n;i++)g[i].clear();
}
```

### Trie (max min xor subarray):

```
int Trie[mx][2];
int End[mx];
int ar[50005];
int Trie[50000*32][2];
int n,ii,st=1;
void Insert(int val){
    int cur=1;
    for(int i=31;i>=0;i--){
        int bit=0;
        if(((1<<i) & val))bit=1;
        if(Trie[cur][bit]==0)Trie[cur][bit]=++st;
        cur=Trie[cur][bit];
    }
    End[cur]=val;
}
int query_min(int val){
    int cur=1;
    for(int i=31;i>=0;i--){
        int bit=0;
        if(((1<<i) & val))bit=1;
        if(Trie[cur][bit])cur=Trie[cur][bit];
        else if(Trie[cur][bit^1])cur=Trie[cur][bit^1];
    }
    return End[cur]^val;
}
int query_max(int val){
    int cur=1;
    for(int i=31;i>=0;i--){
        int bit=0;
        if(((1<<i) & val))bit=1;
        if(Trie[cur][bit^1])cur=Trie[cur][bit^1];
        else if(Trie[cur][bit])cur=Trie[cur][bit];
    }
    return End[cur]^val;
}
void solve(){
    int suffix=0;
    int re_min=INT_MAX,re_max=0;
    Insert(0);
    for(int i=1;i<=n;i++){
        scanf("%d",&ar[i]);
        suffix^=ar[i];
        re_min=min(re_min,query_min(suffix));
        re_max=max(re_max,query_max(suffix));
        Insert(suffix);
    }
}
```

### SegTree :

#### Bracket Sequence:

```
struct info{
    int open,close,ans;
};
```

```
info Merge(info a,info b){
    info re;
    int valid=min(a.open,b.close);
    re.open=a.open+b.open-valid;
    re.close=a.close+b.close-valid;
    re.ans=a.ans+b.ans+valid;    /// this code works for maximum
length of correct bracket sequence in l to r range
    return re;
}
```

#### Kth element merge sort tree:

```
int query(int node,int be,int en,int l,int r,int k){
    if(be==en)return seg[node][0];
    int pos
=upper_bound(seg[node*2+1].begin(),seg[node*2+1].end(),r)
-lower_bound(seg[node*2+1].begin(),seg[node*2+1].end(),l);
    int mid=(be+en)/2;
    if(pos>=k){
        return query(node*2+1,be,mid,l,r,k);
    }
    else return query(node*2+2,mid+1,en,l,r,k-pos);
}
```

#### Delete Type Id Found:

```
int id_query(int node,int be,int en,int pos){
    if(be==en)return be;
    int mid=(be+en)/2;
    if(Present[node*2]>=pos){
        return id_query(node*2,be,mid,pos);
    }
    else return id_query(node*2+1,mid+1,en,pos-
Present[node*2]);
}
```

#### Rang max subarray / suffix-prefix sum:

```
struct info{
    ll max_pref,max_suf,ans,sum;
    void Merge(info p1,info p2){
        sum=p1.sum+p2.sum;
        max_pref=max(p1.max_pref,p1.sum+p2.max_pref);
        max_suf=max(p2.max_suf,p2.sum+p1.max_suf);
        ans=max(max(p1.ans,p2.ans),p1.max_suf+p2.max_pref);
    };
    void Relax(int node,int be,int en){
        if(!cur[node])return;
        Tree[node].sum=Lazy[node]*(en-be+1);
        Tree[node].max_pref=max(0LL,Tree[node].sum);
        Tree[node].max_suf=max(0LL,Tree[node].sum);
        Tree[node].ans=max(0LL,Tree[node].sum);
        if(be!=en){
            Lazy[node*2]=Lazy[node];
            Lazy[node*2+1]=Lazy[node];
            cur[node*2]=true;
            cur[node*2+1]=true;
        }
        cur[node]=false;
        Lazy[node]=0;
    }
}
```

### Centroid Decomposition:

```
int dis[18][mx],re[mx],vis[mx];
int p[mx],sub[mx],lvl[mx];
vector<int>g[mx],ng[mx];
/* p[u] = parent of u in centroid tree
dis[x][u] = distance from u to a parent of u at level x of centroid
tree
if u is in subtree of centroid c, then dis[lvl[c]][u] = dist(c, l)
If (x, y) edge exist, then x must be in g[y] and y must be in g[x]*/
/* we can do more pre work in dfs function*/
void dfs(int l,int u,int par){
    if(par!=-1)dis[l][u]=dis[l][par]+1;
    for(int v:g[u])
        if(v!=par && !vis[v])dfs(l,v,u);
}
int centroid(int u,int par,int r){
    for(int v:g[u])
        if(v!=par && !vis[v] && sub[v]>r)return centroid(v,u,r);
    return u;
}
void pre_cal(int u,int par){
    sub[u]=1;
    for(int v:g[u])
        if(v!=par && !vis[v])pre_cal(v,u),sub[u]+=sub[v];
}
void decompose(int u,int par){
    pre_cal(u,-1);
    int tem=centroid(u,-1,sub[u]>>1);
    vis[tem]=1,p[tem]=par,lvl[tem]=0;
    if(par!=-1)lvl[tem]=lvl[par]+1,ng[par].push_back(tem);
    dfs(lvl[tem],tem,-1);
    for(int v:g[tem])
        if(v!=par && !vis[v])decompose(v,tem);
}
void update(int u){
    for(int v=u;v!=-1;v=p[v])re[v]=min(re[v],dis[lvl[v]][u]);
}
int query(int u){
    int ans=1e9;
    for(int v=u;v!=-1;v=p[v])
        ans=min(ans,re[v]+dis[lvl[v]][u]);
    return ans;
}
int lca(int u,int v){
    if(lvl[u]<lvl[v])swap(u,v);
    while(lvl[u]>lvl[v])u=p[u];
    while(u!=v && p[u]!=-1)u=p[u],v=p[v];
    return u;
}
int dist(int u,int v){
    int lc=lca(u,v);
    return dis[lvl[lc]][u]+dis[lvl[lc]][v];
}
int GetRoot(int u){
    while(p[u]!=-1)u=p[u];
}
```

```
return u;
}
// for all pair
void update(int u,int p){
    int val=dis[lvl[p]][u];
    for(int i=0;i<20;i++){
        cnt[i][chk(val,i)]++;
    }
    for(int v:ng[u])update(v,p);
}
void query(int u,int p){
    int val=dis[lvl[p]][u]^ar[p];
    for(int i=0;i<20;i++){
        ans+=cnt[i][!chk(val,i)]*(1LL<<i);
    }
    for(int v:ng[u])query(v,p);
}
void Go_Ahead(int u){
    memset(cnt,0,sizeof(cnt));
    for(int i=0;i<20;i++)cnt[i][chk(ar[u],i)]++;
    for(int v:ng[u]){
        query(v,u);
        update(v,u);
    }
    ans+=ar[u];
    for(int v:ng[u])Go_Ahead(v);
}
// at first call decompose(1,-1)
Dinic:
const ll eps = 0;
struct edge {
    int a, b;
    ll cap,flow;
    int yo, x, y;
};
struct Dinic {
    int s,t,d[mx], ptr[mx] ;
    //int Id[mx][mx];
    vector<edge>e;
    vector<int>g[mx];
    void init() {
        e.clear();
        memset(d,0,sizeof(d));
        for(int i = 0; i < mx ; i++)g[i].clear();
        // for(int i=0;i<mx;i++)
        // {
        //     for(int j=0;j<mx;j++)
        //         {
        //             Id[i][j]=0;
        //         }
        //     }
        // }
}
void addEdge(int a,int b,ll cap, int x = -1, int y= -1) {
    edge e1 = { a, b, cap, 0, 1, x, y };
    edge e2 = { b, a, 0, 0, 0, x, y };
}
```



```
// Id[a][b]=e.size();
g[a].push_back((int)e.size());
e.push_back(e1);
// Id[b][a]=e.size();
g[b].push_back((int)e.size());
e.push_back(e2);
}
bool bfs() {
    queue < int > Q;
    Q.push(s);
    memset(d,-1,sizeof(d));
    d[s]=0;
    while (!Q.empty()) {
        int u=Q.front();
        Q.pop();
        for(int i=0; i<g[u].size(); i++) {
            int id=g[u][i];
            int v=e[id].b;
            // printf("%d %d %0.3lf\n",u,v,e[id].cap,e[id].flow);
            if(d[v]==-1&&e[id].flow<e[id].cap) {
                Q.push(v);
                d[v]=d[u]+1;
            }
        }
        return d[t]!=-1;
    }
    ll dfs(int u,ll flow) {
        if (flow<=eps) return 0;
        if ( u==t ) return flow;
        for(int& i = ptr[u]; i<g[u].size(); i++) {
            int id = g[u][i];
            int v = e[id].b;
            if ( d[v] != d[u]+1 ) continue;
            ll pushed = dfs(v,min (flow,e[id].cap-e[id].flow));
            //cout << "pushed " << pushed << endl;
            if (pushed>eps) {
                e[id].flow+=pushed;
                e[id^1].flow-=pushed;
                return pushed;
            }
        }
        return 0;
    }
    ll dinic() {
        ll flow = 0;
        while(true) {
            if(!bfs()) break;
            memset(ptr, 0, sizeof(ptr));
            while (true){
                ll pushed = dfs(s,INF);
                if(pushed<=eps)break;
                flow += pushed;
            }
        }
        return flow;
    }
}
```

```
}
};
Upper Lower Limit:
Dinic dc;
int x,y;
struct tem{
    int u,v,a,b;
};
vector<tem>ed;
ll func(ll val){
    dc.init();
    dc.s=n+1;
    dc.t=n+2;
    /// for upperbound(0,val)
    // dc.addEdge(y,n+3,val); /// sink to super super source
    // dc.addEdge(n+1,x,0); /// sink to source
    // dc.addEdge(n+3,n+2,0); /// super super source to super sink
    // dc.addEdge(n+3,x,val); /// super super source to source
    /// for lowerbound(val,inf)
    dc.addEdge(y,n+3,INF); /// sink to super super source
    dc.addEdge(n+1,x,val); /// sink to source
    dc.addEdge(n+3,n+2,val); /// super super source to super sink
    dc.addEdge(n+3,x,INF); /// super super source to source
    for(auto it:ed){
        dc.addEdge(n+1,it.v,it.a);
        dc.addEdge(it.u,n+2,it.a);
        dc.addEdge(it.u,it.v,it.b-it.a);
    }
    return dc.dinic();
}
void solve(){
    scanf("%d%d",&n,&m);
    scanf("%d%d",&x,&y);
    dc.addEdge(y,x,INF);
    dc.s=n+1;
    dc.t=n+2;
    ll val=0;
    ll en=0;
    for(int i=1;i<=m;i++){
        int u,v,a,b;
        scanf("%d%d%d%d",&u,&v,&a,&b);
        ed.push_back({u,v,a,b});
        val+=a;
        en+=b;
        dc.addEdge(n+1,v,a);
        dc.addEdge(u,n+2,a);
        dc.addEdge(u,v,b-a);
    }
    if(dc.dinic()<val){
        printf("0\n");
        return;
    }
    ll be=val;
    ll re=be;
}
```

```

while(be<=en){
    ll mid=(be+en)/2;
    ll have=func(mid);
    if(have>=mid+val) {
        re=mid;
        be=mid+1;
    }
    else en=mid-1;
}
printf("%lld\n",re);
}

Hopcroft_Karp:
#define mx 40005
#define INF (1<<28)
struct Hopcroft_Karp
{
    vector<int> g[mx];
    int n, m, Matching[mx], Distance[mx];
    // n: number of nodes on left side, nodes are
    // numbered 1 to n
    // m: number of nodes on right side, nodes are
    // numbered n+1 to n+m
    void init(int num){

for(inti=0;i<=num;i++)Matching[i]=0,Distance[i]=0,g[i].clear();
}

    void addEdge(int u,int v) {
        g[u].push_back(v);
    }

    bool bfs() {
        int i, u, v, len;
        queue<int> q;
        for(i=1; i<=n; i++) {
            if(Matching[i]==0) {
                Distance[i] = 0;
                q.push(i);
            }
            else Distance[i] = INF;
        }
        Distance[0] = INF;
        while(!q.empty()) {
            u = q.front(); q.pop();
            if(u!=0) {
                for(int v:g[u]) {
                    if(Distance[Matching[v]]==INF) {
                        Distance[Matching[v]] = Distance[u] + 1;
                        q.push(Matching[v]);
                    }
                }
            }
        }
        return (Distance[0]!=INF);
    }

    bool dfs(int u) {
        int i, v, len;

```

```

        if(u!=0) {
            for(int v:g[u]) {
                if(Distance[Matching[v]]==Distance[u]+1) {
                    if(dfs(Matching[v])) {
                        Matching[v] = u;
                        Matching[u] = v;
                        return true;
                    }
                }
            }
            Distance[u] = INF;
            return false;
        }
        return true;
    }

    int hopcroft_karp() {
        int Matchinging = 0, i;
        while(bfs())
            for(i=1; i<=n; i++)
                if(Matching[i]==0 && dfs(i))
                    Matchinging++;
        return Matchinging;
    }
};

Hopcroft_Karp hk;

Hungarian (visit all node with minimum cost):
#define INF 1e18
pair<ll,vector<int>> hungarian(vector<vector<ll>>mat,int f,int
sz){
    vector<int>par(sz+1,0),way(sz+1,0),match(sz+1,0);
    vector<bool>vis(sz+1,0);
    vector<ll>U(sz+1,0),V(sz+1,0),MinV(sz+1,0);
    for(int i=1;i<=sz;i++) {
        for(int j=1;j<=sz;j++) {
            mat[i][j]*=f;
        }
    }
    int a,b,d;
    ll r,w;
    for(int i=1;i<=sz;i++){
        par[0]=i;
        b=0;
        for(int j=1;j<=sz;j++)MinV[j]=INF,vis[j]=0;
        do{
            vis[b]=1;
            a=par[b],d=0,w=INF;
            for(int j=1;j<=sz;j++) {
                if(!vis[j]) {
                    r=mat[a][j]-U[a]-V[j];
                    if(r<MinV[j])MinV[j]=r,way[j]=b;
                    if(MinV[j]<w)w=MinV[j],d=j;
                }
            }
        }
        for(int j=0;j<=sz;j++) {
            if(vis[j])U[par[j]]+=w,V[j]-=w;

```

```

        else MinV[j]=-w;
    }
    b=d;
}
while(par[b]!=0);
do{
    d=way[b];
    par[b]=par[d],b=d;
}
while(b!=0);
}
for(int j=1;j<=sz;j++)match[par[j]]=j;
return {-f*V[0],match};
}
// called hungarain(mat,1,n)
Min Cost Max Flow:
typedef long long T1;//for cost
typedef long long T2;//for flow
const int maxn = 20100;
const T1 INF = 1e12;
const T2 inf = 1e12;
const T1 eps = 0;
struct Edge {
    int from, to;
    T2 cap, flow;
    T1 cost;
};
int n,m,k,ii;
struct MCMF { //0-indexed
    int n, m, s, t;
    vector<Edge> edges;
    vector<int> G[maxn];
    int p[maxn],inq[maxn];
    T1 d[maxn];
    T2 a[maxn];
    void init() {
        for(int i = 0; i < n; i++) G[i].clear();
        edges.clear();
    }
    void AddEdge(int from,int to,T2 cap,T1 cost) {
        edges.push_back((Edge){from, to, cap, 0, cost});
        edges.push_back((Edge){to, from, 0, 0, -cost});
        m = edges.size();
        G[from].push_back(m-2);
        G[to].push_back(m-1);
    }
}
pair<T1,T2> Mincost() { //bellmanFord
    T1 tot_cost = 0;
    T2 tot_flow = 0;
    while(true) {
        for(int i = 0; i < n; i++) d[i] = INF;
        d[s] = 0;
        p[s] = 0;
        a[s] = inf;
        bool up=true;

```

```

        while(up) {
            up=false;
            for(int u = 0; u < n; u++) {
                if(d[u]-INF>=-eps)continue;
                for(int j:G[u]) {
                    Edge &e=edges[j];
                    if(e.cap > e.flow && d[e.to] > d[u] + e.cost+eps) {
                        d[e.to] = d[u] + e.cost;
                        p[e.to] = j;
                        a[e.to] = min(a[u], e.cap - e.flow);
                        up=true;
                    }
                }
            }
        }
        if(abs(d[t]-INF)<=eps)break;
        tot_cost += (T1)d[t] * a[t];
        tot_flow += (T2)a[t];
        int u = t;
        while(u != s) {
            edges[p[u]].flow += a[t];
            edges[p[u]^1].flow -= a[t];
            u = edges[p[u]].from;
        }
    }
    return {tot_cost,tot_flow};
}
pair<T1,T2> Mincost2() { //SPFA
    T1 tot_cost = 0;
    T2 tot_flow = 0;
    while(true) {
        for(int i = 0; i < n; i++) d[i] = INF;
        memset(inq, 0, sizeof(inq));
        d[s] = 0;
        inq[s] = 1;
        p[s] = 0;
        a[s] = inf;
        queue<int> Q;
        srand(time(NULL));
        Q.push(s);
        while(!Q.empty()) {
            int u = Q.front();
            Q.pop();
            inq[u] = 0;
            for(int i = 0; i < G[u].size(); i++) {
                Edge& e = edges[G[u][i]];
                if(e.cap > e.flow && d[e.to] > d[u] + e.cost+eps) {
                    d[e.to] = d[u] + e.cost;
                    p[e.to] = G[u][i];
                    a[e.to] = min(a[u], e.cap - e.flow);
                    if(!inq[e.to]) {
                        Q.push(e.to);
                        inq[e.to] = 1;
                    }
                }
            }
        }
    }
}

```

```

    }
}
if(abs(d[t]-INF)<=eps)break;
tot_cost += (T1)d[t] * a[t];
tot_flow += a[t];
int u = t;
while(u != s) {
    edges[p[u]].flow += a[t];
    edges[p[u]^1].flow -= a[t];
    u = edges[p[u]].from;
}
}
return {tot_cost,tot_flow};
}
} mcmf;

```

### Covering Problems Solvable in Polynomial Time

- Minimum Edge Cover in General Graph
  - Smallest set of edges where each vertex is end-point of at least one edge
  - V - matching (if edge cover exists)
- Minimum Path Cover (Vertex Disjoint) in DAG
  - Minimum number of vertex disjoint paths that visit all nodes
- Minimum Path Cover (Vertex not-disjoint) in General Graph
  - Minimum number of paths that visit all nodes

### Covering Problems Solvable in Polynomial Time

- Maximum Independent Set in Bipartite Graph
  - Largest set of nodes who do not have any edge between themselves
  - Solution: V - Max Matching
- Minimum Vertex Cover in Bipartite Graph
  - Smallest set of nodes where at least one end-point of each edge is present
  - Solution: Max Matching

#### Kuhn:

```

struct BPM{
bool Done[mx];
vector<int>g[mx];
int mach[mx];
void addEdge(int u,int v) {
    g[u].push_back(v);
}
void init() {
    for(int i=0;i<mx;i++)g[i].clear();
}
bool Tem_Matching(int u){
    for(int i=0;i<(int)g[u].size();i++){
        int v=g[u][i];
        if(Done[v]) continue;
        Done[v] = true;
        if(mach[v]==-1 || Tem_Matching(mach[v])) {
            mach[v] = u;
            return true;
        }
    }
    return false;
}
}

```

```

int Max_Matching(int num) {
    // Be Careful with this section. when passin num.
    memset(mach,-1,sizeof(mach));
    int re = 0;
    for(int i=1;i<=num;i++) {
        memset(Done,false,sizeof(Done));
        if(Tem_Matching(i)) re++;
    }
    return re;
}
};

```

#### LCA(value on edge):

```

int par[mx][20];
ll ans[mx][20];
int depth[mx],LOG;
vector<pair<int,ll>>g[mx];
void dfs(int u,int p,int lvl){
    par[u][0]=p;
    depth[u]=lvl;
    for(auto it:g[u]) {
        int v=it.first;
        ll w=it.second;
        if(v==p)continue;
        ans[v][0]=w;
        dfs(v,u,lvl+1);
    }
}
void init(int root){
    dfs(root,-1,1);
    for(int j=1;j<LOG;j++) {
        for(int i=1;i<=n;i++) {
            if(par[i][j-1]!=-1){
                par[i][j]=par[par[i][j-1]][j-1];
                ans[i][j]=max(ans[i][j-1],ans[par[i][j-1]][j-1]);
            }
            else par[i][j]=-1;
        }
    }
}
ll query(int u,int v){
    if(u==v)return 0;
    if(depth[u]<depth[v])swap(u,v);
    int diff=depth[u]-depth[v];
    ll re=0;
    for(int i=LOG-1;i>=0;i--) {
        if(diff>=(1<<i)) {
            diff-=(1<<i);
            re=max(re,ans[u][i]);
            u=par[u][i];
        }
    }
    if(u==v)return re;
    for(int i=LOG-1;i>=0;i--) {
        if( par[u][i]!=par[v][i]){
            re=max({re,ans[u][i],ans[v][i]});

```

```

        u=par[u][i];
        v=par[v][i];
    }
}
re=max({re,ans[u][0],ans[v][0]});
return re;
}
int dist(int u,int v){
    return depth[u]+depth[v]-2*depth[lca(u,v)];
}
int kth_parent(int u,int k){
    for(int i=LOG-1;i>=0;i--) {
        if(k>=(1<<i)) {
            k-=(1<<i);
            u=par[u][i];
        }
    }
    if(u==-1)return u;
}
return u;
}
}
solve(){
for(int i=1;i<=n;i++){
    g[i].clear();
    for(int j=0;j<LOG;j++)ans[i][j]=0,par[i][j]=-1;
}
LOG=log2(n)+1;
}

```

#### LCA(value in node):

//dfs function ye ans[u][0] line likha jabe nah

// init function same

// query function er sesh ye ei 3 line likhbo

```

re=max(re,ans[u][0]);
re=max(re,ans[v][0]);
re=max(re,ans[par[v][0]][0]);
for(int i=1;i<=n;i++){
    scanf("%d",&ar[i]);
    ans[i][0]=ar[i];
}

```

#### DSU:

```

int Size[mx];
int Findparent(int x){
    return (x==parent[x])?x:(parent[x]=Findparent(parent[x]));
}
void Union(int x,int y){
    int px=Findparent(x);
    int py=Findparent(y);
    if(px==py)return;
    if(Size[px]>Size[py]) {
        Size[px]+=Size[py];
        parent[py]=px;
    }
    else {
        Size[py]+=Size[px];
        parent[px]=py;
    }
}

```

```

void initialize(){
    for(int i=0;i<=n;i++)parent[i]=i,Size[i]=1;
}
Bellman Ford:
vector<Edge>E;
ll dist[100];
bool bellman_ford(){
    // here i can start from 1 .if given that stating node i can set
    dist[src]=0
    for(int i=1;i<=n;i++)dist[i]=10000000;
    dist[1]=0;
    for(int i=1;i<=n;i++){
        for(Edge it: E)
            if(dist[it.v]>dist[it.u]+it.w)
                dist[it.v]=dist[it.u]+it.w;
    }
    for(Edge it:E)
        if(dist[it.v]>dist[it.u]+it.w)return true;//negative cycle
    return false;
}
Floyed Warshal:
for(int i=1;i<=n;i++){
    for(int j=1;j<=n;j++){
        if(i==j || dis[i][j]>0)continue;
        dis[i][j]=1e18;
    }
}
for(int l=1;l<=n;l++){
    for(int i=1;i<=n;i++){
        for(int j=1;j<=n;j++){
            dis[i][j]=min(dis[i][j],dis[i][l]+dis[l][j]);
        }
    }
}
Articulation Point:
vector<int>g[mx];
int articular_point[mx];
int st[mx],low[mx];
int Time=1;
int dfs(int u,int p){
    st[u]=low[u]=Time++;
    int child=0;
    for(auto it:g[u]) {
        if(it==p)continue;
        if(st[it]==0) {
            child++;
            dfs(it,u);
            if(st[u]<=low[it])articular_point[u]=1;
            low[u]=min(low[u],low[it]);
        }
        else low[u]=min(low[u],st[it]);
    }
    return child;
}
void solve(){
    for(int i=1;i<=n;i++) {
        if(st[i])continue;
        articular_point[i]=(dfs(i,-1)>1);
    }
}

```

```

}
Articulations Bridge:
vector<int>g[mx];
vector<pair<int,int>>Bridge;
int st[mx],low[mx];
int Time=1;
void dfs(int u,int p){
    st[u]=low[u]=Time++;
    int child=0;
    for(auto it:g[u]) {
        if(it==p)continue;
        if(st[it]==0){
            dfs(it,u);
            if(st[u]<low[it])Bridge.push_back({u,it});
            low[u]=min(low[u],low[it]);
        }
        else low[u]=min(low[u],st[it]);
    }
}
void solve(){
    for(int i=1;i<=n;i++) {
        if(st[i])continue;
        dfs(i,-1);
    }
}
Strongly Connected Component:
vector<int>g[mx],g_rev[mx],st(mx),en(mx),component[mx],option,visit;
vector<pair<int,int>>dekhi;
int node,edge,cnt,tem;
int mp[mx];
void dfs1(int u){
    visit[u]=true;
    st[u]=++cnt;
    for(auto it:g[u]) {
        if(visit[it])continue;
        dfs1(it);
    }
    en[u]=++cnt;
}
void dfs2(int u){
    visit[u]=true;
    component[cnt].push_back(u);
    for(auto it:g_rev[u]) {
        if(visit[it])continue;
        dfs2(it);
    }
}
void clean(){
    for(int i=1;i<=node+2;i++) {
        g[i].clear();
        g_rev[i].clear();
        component[i].clear();
    }
    option.clear();

```

```

    cnt=0;
    st.clear();
    en.clear();
    dekhi.clear();
    memset(mp,0,sizeof(mp));
}
void solve(){
    scanf("%d%d",&node,&edge);
    for(int i=1;i<=edge;i++) {
        int u,v;
        scanf("%d%d",&u,&v);///directed graph
        g[u].push_back(v);
        g_rev[v].push_back(u);
        mp[u]++;
        mp[v]++;
    }
    visit.assign(node+2,false);
    for(int i=1;i<=node;i++) {
        if(visit[i]==true || mp[i]==0)continue;
        dfs1(i);
    }
    for(int i=1;i<=node;i++) {
        if(visit[i]==true && mp[i])dekhi.push_back({en[i],i});
    }
    sort(dekhi.begin(),dekhi.end());
    reverse(dekhi.begin(),dekhi.end());
    visit.assign(node+2,false);
    cnt=1;
    for(int i=0;i<dekhi.size();i++) {
        int pos=dekhi[i].second;
        if(visit[pos] || mp[pos]==0)continue;
        dfs2(pos);
        cnt++;
    }
    for(int i=1;i<cnt;i++) {
        for(auto it:component[i]) {
            cout<<it<<" ";
        }
        cout<<endl;
    }
}
Matrix Expo:
#define MAX 105
#define ll long long int
const ll MOD = 1e9 + 7;
const ll MOD2 = MOD * MOD * 3;
inline ll bigMod(ll a,ll b){
    ll res=1;
    while(b){
        if(b&1) res=(res*a)%MOD;
        a=(a*a)%MOD; b>>=1;
    }
    return res;
}
inline ll inv(ll n) {return bigMod(n,MOD-2);}
inline ll Mul(ll a,ll b) {return (a*b)%MOD;}

```

## JnU-The\_Last\_Phase

```
inline ll Div(ll a, ll b) {return Mul(a, inv(b));}
```

```
/* 1 base row column index */
```

```
struct Matrix{
    int row, col;
    ll m[MAX][MAX];
    Matrix() {memset(m, 0, sizeof(m));}
    void Set(int r, int c) {row = r; col = c;}
    Matrix(int r, int c) {memset(m, 0, sizeof(m)); Set(r, c);}
    void normalize(){
        for(int i=1; i<=row; i++){
            for(int j=1; j<=col; j++){
                m[i][j] %= MOD;
                if(m[i][j] < 0) m[i][j] += MOD;
            }
        }
    }
};
```

```
Matrix Multiply(Matrix A, Matrix B){
    Matrix ans(A.row, B.col);
    for(int i=1; i<=A.row; i++){
        for(int j=1; j<=B.col; j++){
            ans.m[i][j]=0;
            ll sm = 0;
            for(int k=1; k<=A.col; k++){
                sm += (A.m[i][k] * B.m[k][j]);
                if(sm >= MOD2) sm -= MOD2;
            }
            ans.m[i][j] = sm % MOD;
        }
    }
    return ans;
}
```

```
Matrix Power(Matrix mat, ll p){
    Matrix res(mat.row, mat.col);
    Matrix ans(mat.row, mat.col);
    int n = ans.row;
    for(int i=1; i<=n; i++){
        for(int j=1; j<=n; j++){
            ans.m[i][j]=0;
            res.m[i][j]=mat.m[i][j];
        }
        ans.m[i][i]=1;
    }
    while(p){
        if(p&1) ans=Multiply(ans, res);
        res=Multiply(res, res);
        p=p/2;
    }
    return ans;
}
```

## // Gaussian Elimination Offline

```
ll a[MAX], n; //0 base index
```

```
ll maxxor(){
    int r = 0; ll ret = 0;
    for(int c = 63; c >= 0; c--){
```

```
        int idx = -1;
        for(int i = r; i < n && idx < 0; i++){
            if(a[i] >> c & 1) idx = i;
        }
        if(idx == -1) continue;
        swap(a[r], a[idx]);
        for(int i = 0; i < n; i++) if(i != r)
            if(a[i] >> c & 1) a[i] ^= a[r];
        r++;
    }
    for(int i = 0; i < n; i++) ret = max(ret, ret ^ a[i]);
    return ret;
}
```

**Inclusion Exclusion:**

```
/// koto gulo number ace[1,n] jara a1 or a2 or a3...,am dara divide
```

```
/// if m=3 and 3 values are a1,a2,a3 then
```

```
/// |a1 U a2 U a3| = |a1| + |a2| + |a3| - |a1 union a2| - |a2 union a3| - |a1 union a3| + |a1 union a2 union a3|
```

```
/// if number of cadidate is odd do add or do subtract
```

```
/// time complexity 2^m.
```

```
/// for better perform use recursive
```

```
void func(int idx, int cnt, ll lcm){
    if(lcm > n) break;
    if(idx == m) {
        if(cnt == 0) return;
        if(cnt & 1) re1 += n / lcm;
        else re1 -= n / lcm;
        return;
    }
    func(idx+1, cnt+1, (lcm * ar[idx]) / __gcd(lcm, (ll)ar[idx]));
    func(idx+1, cnt, lcm);
}
```

```
void solve(){
    scanf("%lld%d", &n, &m);
    for(int i=0; i<m; i++) scanf("%d", &ar[i]);
    ///using bitmask
    for(int i=1; i<(1<<m); i++){
        ll lcm=1;
        int cnt=0;
        for(int j=0; j<m; j++){
            if(i & (1<<j)) {
                cnt++;
                lcm = (lcm * ar[j]) / __gcd(lcm, (ll)ar[j]);
                if(lcm > n) break;
            }
        }
        if(cnt & 1) re += n / lcm;
        else re -= n / lcm;
    }
}
```

**Linear sieve:**

```
bitset<mx> is_composite;
vector<int> prime;
int phi[mx], mobius[mx];
void seive(int n){
```

## JnU-The\_Last\_Phase

```

phi[1]=mobius[1]=1;
for(int i=2;i<=n;i++){
    mobius[i]=1;
    if(!is_composite[i]){
        prime.push_back(i);
        phi[i]=i-1;        ///i is prime
    }
    for(int j=0;j<prime.size() && i*prime[j]<=n;j++){
        is_composite[i*prime[j]]=true;
        if(i%prime[j]==0){
            phi[i*prime[j]]=phi[i]*prime[j]; ///prime[j] divides i
            break;
        }
        else {
            phi[i*prime[j]]=phi[i]*phi[prime[j]]; ///prime[j] do not
            divide i
        }
    }
}
for(int val:prime){
    int temp=val*val;
    if(temp>n)break;
    for(int j=temp;j<=n;j+=temp)mobius[j]=0;
}
for(int val:prime){
    for(int j=val;j<=n;j+=val)mobius[j]*=-1;
}
}

```

**Eulor totient:**

```

int phi[mx];
void eulor_totient(int n){
    for(int i=2; i<=n; i++) phi[i]=i;
    for(int i=2;i<=n;i+=2){
        phi[i]>>=1;
    }
    for(int i=3; i<=n; i+=2){
        if(phi[i]==i){
            phi[i]--;
            for(int j=2*i; j<=n; j+=i)
                phi[j]-=(phi[j]/i);
        }
    }
}

```

**CRT:**

```

ll ar[mx],br[mx];
struct GCD_type { ll x, y, d; };
GCD_type ex_GCD(ll a, ll b){
    if (b == 0) return {1, 0, a};
    GCD_type pom = ex_GCD(b, a % b);
    return {pom.y, pom.x - a / b * pom.y, pom.d};
}
ll normalize(ll val,ll mod){
    val%=mod;
    if(val<0)val+=mod;
    return val;
}

```

```

}
void solve(){
    ll ans=br[1]; /// here br remainder
    ll lcm=ar[1];
    bool f=true;
    for(int i=2;i<=n;i++){
        auto pom=ex_GCD(lcm,ar[i]);
        ll x1=pom.x;
        ll d=pom.d;
        if((br[i]-ans)%d!=0){
            f=false;break;
        }
        ans=ans+x1*(br[i]-ans)/d%(ar[i]/d)*lcm;
        ans=normalize(ans,lcm*ar[i]/d);
        lcm=(lcm*ar[i])/__gcd(lcm,ar[i]);
    }
    if(f)printf("%lld %lld\n",ans,lcm); /// here is the smallest
    answer .next xth answer will be ans+x*lcm where x=[1,2,...]
}

```

**Extended Euclidean (inverse):**

```

int Extended_Euclidean(int a,int b,int &x,int &y)
{
    if(b==0){
        x=1;y=0;
        return a;
    }
    int d=Extended_Euclidean(b,a%b,y,x);
    y=y-(a/b)*x;
    return d;
}
int Inverse_Modulo(int a,int m){
    int x,y,d;
    d=Extended_Euclidean(a,m,x,y);
    if(d==1) return (x+m)%m;
    return -1; //No Solution
}

```

**Big Mod, Fact:**

```

ll bigmod(ll e,ll x){
    if(!x)return 1;
    ll p=bigmod(e,x/2);
    p=(p*p)%mod;
    if(x%2)p=(p*e)%mod;
    return p;
}
void fact_cal(){
    fact[0]=1,inv[0]=1;
    for(int i=1;i<=mx-3;i++){
        fact[i]=(fact[i-1]*i)%mod;
    }
    inv[mx-3]=bigmod(fact[mx-3],mod-2);
    for(int i=mx-4;i>=1;i--)inv[i]=(inv[i+1]*(i+1))%mod;
}

```

**Stirling Number of 2<sup>nd</sup> kind:**

```

ll dp[mx][mx];
ll func(int nn,int kk){

```



```

    if(kk==1)return 1;
    if(nn==kk)return 1;
    if(kk==0)return 0;
    ll &val=dp[nn][kk];
    if(val!=-1)return val;
    val=func(nn-1,kk-1)+1LL*kk*func(nn-1,kk);
    return val;
}

```

#### Pollard RHO:

```

#define pii pair<ll,int>
ll Mul(ll a,ll b,ll Mod){
    ll Ans=0;
    while(b){
        if(b&1) {Ans+=a; if(Ans>=Mod) Ans-=Mod;}
        a+=a; if(a>=Mod) a-=Mod;
        b>>=1;
    }
    return Ans;
}
ll bigMod(ll n,ll r,ll Mod){
    if(r==0) return 1LL;
    ll ret=bigMod(n,r/2,Mod);
    ret=Mul(ret,ret,Mod);
    if(r%2==1) ret=Mul(ret,n,Mod);
    return ret;
}
//Miller-Rabin
bool witness(ll wit,ll n){
    if(wit>=n) return false;
    int s=0; ll t=n-1;
    while(t%2==0) s++,t/=2;
    wit=bigMod(wit,t,n);
    if(wit==1 || wit==n-1) return false;
    for(int i=1;i<s;i++){
        wit=Mul(wit,wit,n);
        if(wit==1) return true;
        if(wit==n-1) return false;
    }
    return true;
}
//Is n prime?
bool miller(ll n){
    if(n==2) return true;
    if(n%2==0 || n<2) return false;
    if(witness(2,n) || witness(7,n) || witness(61,n)) return false;
    return true;
}
// Pollard's Rho
// a must not equal 0 or -2.
// returns a divisor, a proper one when succeeded, equal to n if
failed
// in case of failure, change a
ll rho(ll n,ll a) {
    auto f=[&](ll x){return (Mul(x,x,n)+a)%n; };
    ll x=2,y=2;

```

```

    for(int i=1;;i++){
        x=f(x); y=f(f(y));
        ll d=__gcd(n,abs(x-y));
        if(d!=1) return d;
    }
    return n;
}
ll get_factor(ll n){
    if(n%2==0) return 2;
    if(n%3==0) return 3;
    if(n%5==0) return 5;
    while(true){
        ll a=2+rand()%100;
        ll d=rho(n,a);
        if(d!=n) return d;
    }
    return n;
}
void factorize(ll n,vector<ll> &x) {
    if(n==1) return;
    else if(miller(n)) x.push_back(n);
    else{
        ll d=get_factor(n);
        factorize(d,x);
        factorize(n/d,x);
    }
}
vector<ll>factorize(ll n) {vector<ll>x; factorize(n, x); return x;}
vector<pii>Factors; // store factor
vector<ll>Divisors; //store divisors
void findDiv(int pos,ll val){
    if(pos<0) {Divisors.push_back(val); return;}
    ll Now=1;
    for(int i=0;i<=Factors[pos].second;i++){
        findDiv(pos-1,val*Now);
        Now=Now*Factors[pos].first;
    }
}
void findAllDiv(ll n){
    vector<ll>now=factorize(n);
    sort(now.begin(),now.end());
    Factors.clear();
    Divisors.clear();
    int Count=1;
    for(int i=1;i<now.size();i++){
        if(now[i]==now[i-1]) Count++;
        else {Factors.push_back({now[i-1],Count}); Count=1;}
    }
    Factors.push_back({now.back(),Count});
    findDiv(Factors.size()-1,1);
}
2D Geometry:
typedef double Tf;
typedef Tf Ti;    /// use long long for exactness
const Tf PI = acos(-1), EPS = 1e-9;

```

## JnU-The\_Last\_Phase

```

int dcmp(Tf x) { return abs(x) < EPS ? 0 : (x < 0 ? -1 : 1); }

struct Point {
    Tf x, y;
    Point(Tf x = 0, Tf y = 0) : x(x), y(y) {}
    Point operator + (const Point& u) const { return Point(x + u.x, y + u.y); }
    Point operator - (const Point& u) const { return Point(x - u.x, y - u.y); }
    Point operator * (const long long u) const { return Point(x * u, y * u); }
    Point operator * (const Tf u) const { return Point(x * u, y * u); }
    Point operator / (const Tf u) const { return Point(x / u, y / u); }
    bool operator == (const Point& u) const { return dcmp(x - u.x) == 0 && dcmp(y - u.y) == 0; }
    bool operator != (const Point& u) const { return !(*this == u); }
    bool operator < (const Point& u) const { return dcmp(x - u.x) < 0 || (dcmp(x - u.x) == 0 && dcmp(y - u.y) < 0); }
    friend istream &operator >> (istream &is, Point &p) { return is >> p.x >> p.y; }
    friend ostream &operator << (ostream &os, const Point &p) { return os << p.x << " " << p.y; }
};

Tf dot(Point a, Point b) { return a.x * b.x + a.y * b.y; }
Tf cross(Point a, Point b) { return a.x * b.y - a.y * b.x; }
Tf length(Point a) { return sqrt(dot(a, a)); }
Tf sqLength(Point a) { return dot(a, a); }
Tf distance(Point a, Point b) { return length(a-b); }
Tf angle(Point u) { return atan2(u.y, u.x); }
// returns angle between oa, ob in (-PI, PI]
Tf angleBetween(Point a, Point b) {
    double ans = angle(b) - angle(a);
    return ans <= -PI ? ans + 2*PI : (ans > PI ? ans - 2*PI : ans);
}

// Rotate a ccw by rad radians
Point rotate(Point a, Tf rad) {
    static_assert(is_same<Tf, Tf>::value);
    return Point(a.x * cos(rad) - a.y * sin(rad), a.x * sin(rad) + a.y * cos(rad));
}

// rotate a ccw by angle th with cos(th) = co && sin(th) = si
Point rotatePrecise(Point a, Tf co, Tf si) {
    static_assert(is_same<Tf, Tf>::value);
    return Point(a.x * co - a.y * si, a.y * co + a.x * si);
}

Point rotate90(Point a) { return Point(-a.y, a.x); }
// scales vector a by s such that length of a becomes s
Point scale(Point a, Tf s) {
    static_assert(is_same<Tf, Tf>::value);
    return a / length(a) * s;
}

// returns an unit vector perpendicular to vector a
Point normal(Point a) {
    static_assert(is_same<Tf, Tf>::value);
    Tf l = length(a);
    return Point(-a.y / l, a.x / l);
}

```

```

}
// returns 1 if c is left of ab, 0 if on ab && -1 if right of ab
int orient(Point a, Point b, Point c) {
    return dcmp(cross(b - a, c - a));
}

bool half(Point p) { // returns true for point above x axis or on negative x axis
    return p.y > 0 || (p.y == 0 && p.x < 0);
}

bool polarComp(Point p, Point q) { //to be used in sort()
    function
    return make_tuple(half(p), 0) < make_tuple(half(q), cross(p, q));
}

struct Segment {
    Point a, b;
    Segment(Point aa, Point bb) : a(aa), b(bb) {}
};

typedef Segment Line;

struct Circle {
    Point o;
    Tf r;
    Circle(Point o = Point(0, 0), Tf r = 0) : o(o), r(r) {}
    // returns true if point p is in || on the circle
    bool contains(Point p) {
        return dcmp(sqLength(p - o) - r * r) <= 0;
    }
    // returns a point on the circle rad radians away from +X CCW
    Point point(Tf rad) {
        static_assert(is_same<Tf, Tf>::value);
        return Point(o.x + cos(rad) * r, o.y + sin(rad) * r);
    }
    // area of a circular sector with central angle rad
    Tf area(Tf rad = PI + PI) { return rad * r * r / 2; }
    // area of the circular sector cut by a chord with central angle alpha
    Tf sector(Tf alpha) { return r * r * 0.5 * (alpha - sin(alpha)); }
};

namespace Linear {
    // returns true if point p is on segment s
    bool onSegment(Point p, Segment s) {
        return dcmp(cross(s.a - p, s.b - p)) == 0 && dcmp(dot(s.a - p, s.b - p)) <= 0;
    }
    // returns true if segment p && q touch or intersect
    bool segmentsIntersect(Segment p, Segment q) {
        if(onSegment(p.a, q) || onSegment(p.b, q)) return true;
        if(onSegment(q.a, p) || onSegment(q.b, p)) return true;
    }

    Tf c1 = cross(p.b - p.a, q.a - p.a);
    Tf c2 = cross(p.b - p.a, q.b - p.a);
    Tf c3 = cross(q.b - q.a, p.a - q.a);
    Tf c4 = cross(q.b - q.a, p.b - q.a);
    return dcmp(c1) * dcmp(c2) < 0 && dcmp(c3) * dcmp(c4) < 0;
}

bool linesParallel(Line p, Line q) {
}

```

```

    return dcmp(cross(p.b - p.a, q.b - q.a)) == 0;
}
// lines are represented as a ray from a point: (point, vector)
// returns false if two lines (p, v) && (q, w) are parallel or
collinear
// true otherwise, intersection point is stored at o via
reference
bool lineLineIntersection(Point p, Point v, Point q, Point w,
Point& o) {
    static_assert(is_same<Tf, Ti>::value);
    if(dcmp(cross(v, w)) == 0) return false;
    Point u = p - q;
    o = p + v * (cross(w, u) / cross(v, w));
    return true;
}
// returns false if two lines p && q are parallel or collinear
// true otherwise, intersection point is stored at o via
reference
bool lineLineIntersection(Line p, Line q, Point& o) {
    return lineLineIntersection(p.a, p.b - p.a, q.a, q.b - q.a, o);
}
// returns the distance from point a to line l
Tf distancePointLine(Point p, Line l) {
    return abs(cross(l.b - l.a, p - l.a) / length(l.b - l.a));
}
// returns the shortest distance from point a to segment s
Tf distancePointSegment(Point p, Segment s) {
    if(s.a == s.b) return length(p - s.a);
    Point v1 = s.b - s.a, v2 = p - s.a, v3 = p - s.b;
    if(dcmp(dot(v1, v2)) < 0) return length(v2);
    else if(dcmp(dot(v1, v3)) > 0) return length(v3);
    else return abs(cross(v1, v2) / length(v1));
}
// returns the shortest distance from segment p to segment q
Tf distanceSegmentSegment(Segment p, Segment q) {
    if(segmentsIntersect(p, q)) return 0;
    Tf ans = distancePointSegment(p.a, q);
    ans = min(ans, distancePointSegment(p.b, q));
    ans = min(ans, distancePointSegment(q.a, p));
    ans = min(ans, distancePointSegment(q.b, p));
    return ans;
}
// returns the projection of point p on line l
Point projectPointLine(Point p, Line l) {
    static_assert(is_same<Tf, Ti>::value);
    Point v = l.b - l.a;
    return l.a + v * ((Tf) dot(v, p - l.a) / dot(v, v));
}
} // namespace Linear
typedef vector<Point> Polygon;
namespace Polygonal {
    // returns the signed area of polygon p of n vertices
    Tf signedPolygonArea(Polygon p) {
        Tf ret = 0;
        for(int i = 0; i < (int) p.size() - 1; i++)

```

```

        ret += cross(p[i] - p[0], p[i+1] - p[0]);
    return ret / 2;
}
// given a polygon p of n vertices, generates the convex hull in
ch
// in CCW && returns the number of vertices in the convex hull
int convexHull(Polygon p, Polygon &ch) {
    sort(p.begin(), p.end());
    int n = p.size();
    ch.resize(n + n);
    int m = 0; // preparing lower hull
    for(int i = 0; i < n; i++) {
        while(m > 1 && dcmp(cross(ch[m - 1] - ch[m - 2], p[i] - ch[m -
1])) <= 0) m--;
        ch[m++] = p[i];
    }
    int k = m; // preparing upper hull
    for(int i = n - 2; i >= 0; i--) {
        while(m > k && dcmp(cross(ch[m - 1] - ch[m - 2], p[i] - ch[m -
2])) <= 0) m--;
        ch[m++] = p[i];
    }
    if(n > 1) m--;
    ch.resize(m);
    return m;
}
// for a point o and polygon p returns:
// -1 if o is strictly inside p
// 0 if o is on a segment of p
// 1 if o is strictly outside p
// computes via winding numbers
int pointInPolygon(Point o, Polygon p) {
    using Linear::onSegment;
    int wn = 0, n = p.size();
    for(int i = 0; i < n; i++) {
        int j = (i + 1) % n;
        if(onSegment(o, Segment(p[i], p[j]))) || o == p[i]) return 0;
        int k = dcmp(cross(p[j] - p[i], o - p[i]));
        int d1 = dcmp(p[i].y - o.y);
        int d2 = dcmp(p[j].y - o.y);
        if(k > 0 && d1 <= 0 && d2 > 0) wn++;
        if(k < 0 && d2 <= 0 && d1 > 0) wn--;
    }
    return wn ? -1 : 1;
}
// returns the longest line segment of l that is inside or on the
// simply polygon p. O(n lg n). TESTED: TIMUS 1955
Tf longestSegInPoly(Line l, const Polygon &p) {
    using Linear::lineLineIntersection;
    int n = p.size();
    vector<pair<Tf, int>> ev;
    for(int i=0; i<n; ++i) {
        Point a = p[i], b = p[(i + 1) % n], z = p[(i - 1 + n) % n];
        int ora = orient(l.a, l.b, a), orb = orient(l.a, l.b, b), orz =
orient(l.a, l.b, z);

```

```

if(!ora) {
    Tf d = dot(a - l.a, l.b - l.a);
    if(orz && orb) {
        if(orz != orb) ev.emplace_back(d, 0);
    }
    else if(orz) ev.emplace_back(d, orz);
    else if(orb) ev.emplace_back(d, orb);
}
else if(ora == -orb) {
    Point ins;
    lineLineIntersection(l, Line(a, b), ins);
    ev.emplace_back(dot(ins - l.a, l.b - l.a), 0);
} }
sort(ev.begin(), ev.end());
Tf ret = 0, cur = 0, pre = 0;
bool active = false;
int sign = 0;
for(auto &qq : ev) {
    int tp = qq.second;
    Tf d = qq.first;
    if(sign) {
        cur += d - pre;
        ret = max(ret, cur);
        if(tp != sign) active = !active;
        sign = 0;
    }
    else {
        if(active) cur += d - pre, ret = max(ret, cur);
        if(tp == 0) active = !active;
        else sign = tp;
    }
    pre = d;
    if(!active) cur = 0;
}
ret /= length(l.b - l.a);
return ret;
} // namespace Polygonal
namespace Convex {
    ///Tested on Kattis:fenceortho
    void rotatingCalipersGetRectangle(Point* p, int n, Tf& area,
    Tf& perimeter) {
        using Linear::distancePointLine;
        static_assert(is_same<Tf, Ti>::value);
        p[n] = p[0];
        int l = 1, r = 1, j = 1;
        area = perimeter = 1e100;

        for(int i = 0; i < n; i++) {
            Point v = (p[i + 1] - p[i]) / length(p[i + 1] - p[i]);
            while(dcmp(dot(v, p[r % n] - p[i]) - dot(v, p[(r + 1) % n] - p[i]))
            < 0) r++;
            while(j < r || dcmp(cross(v, p[j % n] - p[i]) - cross(v, p[(j + 1)
            % n] - p[i])) < 0) j++;

```

```

        while(l < j || dcmp(dot(v, p[l % n] - p[i]) - dot(v, p[(l + 1) % n]
        - p[i])) > 0) l++;
        Tf w = dot(v, p[r % n] - p[i]) - dot(v, p[l % n] - p[i]);
        Tf h = distancePointLine(p[j % n], Line(p[i], p[i + 1]));
        area = min(area, w * h);
        perimeter = min(perimeter, 2 * w + 2 * h);
    } }
// returns the left side of polygon u after cutting it by ray a->b
Polygon cutPolygon(Polygon u, Point a, Point b) {
    using Linear::lineLineIntersection, Linear::onSegment;
    Polygon ret;
    int n = u.size();
    for(int i = 0; i < n; i++) {
        Point c = u[i], d = u[(i + 1) % n];
        if(dcmp(cross(b - a, c - a)) >= 0) ret.push_back(c);
        if(dcmp(cross(b - a, d - c)) != 0) {
            Point t;
            lineLineIntersection(a, b - a, c, d - c, t);
            if(onSegment(t, Segment(c, d))) ret.push_back(t);
        }
    }
    return ret;
}
// returns true if point p is in or on triangle abc
bool pointInTriangle(Point a, Point b, Point c, Point p) {
    return dcmp(cross(b - a, p - a)) >= 0
        && dcmp(cross(c - b, p - b)) >= 0
        && dcmp(cross(a - c, p - c)) >= 0;
}
// pt must be in ccw order with no three collinear points
// returns inside = -1, on = 0, outside = 1
int pointInConvexPolygon(const Polygon &pt, Point p) {
    int n = pt.size();
    assert(n >= 3);
    int lo = 1, hi = n - 1;
    while(hi - lo > 1) {
        int mid = (lo + hi) / 2;
        if(dcmp(cross(pt[mid] - pt[0], p - pt[0])) > 0) lo = mid;
        else hi = mid;
    }
    bool in = pointInTriangle(pt[0], pt[lo], pt[hi], p);
    if(!in) return 1;
    if(dcmp(cross(pt[lo] - pt[lo - 1], p - pt[lo - 1])) == 0) return 0;
    if(dcmp(cross(pt[hi] - pt[lo], p - pt[lo])) == 0) return 0;
    if(dcmp(cross(pt[hi] - pt[(hi + 1) % n], p - pt[(hi + 1) % n])) ==
    0) return 0;
    return -1;
}
// Extreme Point for a direction is the farthest point in that
direction
// poly is a convex polygon, sorted in CCW, doesn't contain
redundant points
// u is the direction for extremeness
int extremePoint(const Polygon &poly, Point u = Point(0, 1)) {
    int n = (int) poly.size();
    int a = 0, b = n;

```

```

while(b - a > 1) {
    int c = (a + b) / 2;
    if(dcmp(dot(poly[c] - poly[(c + 1) % n], u)) >= 0 &&
dcmp(dot(poly[c] - poly[(c - 1 + n) % n], u)) >= 0) {
        return c;
    }
    bool a_up = dcmp(dot(poly[(a + 1) % n] - poly[a], u)) >= 0;
    bool c_up = dcmp(dot(poly[(c + 1) % n] - poly[c], u)) >= 0;
    bool a_above_c = dcmp(dot(poly[a] - poly[c], u)) > 0;
    if(a_up && !c_up) b = c;
    else if(!a_up && c_up) a = c;
    else if(a_up && c_up) {
        if(a_above_c) b = c;
        else a = c;
    }
    else {
        if(!a_above_c) b = c;
        else a = c;
    }
}
if(dcmp(dot(poly[a] - poly[(a + 1) % n], u)) > 0 &&
dcmp(dot(poly[a] - poly[(a - 1 + n) % n], u)) > 0)
    return a;
return b % n;
}
// For a convex polygon p and a line l, returns a list of
segments
// of p that are touch or intersect line l.
// the i'th segment is considered (p[i], p[(i + 1) modulo |p|])
// #1 If a segment is collinear with the line, only that is
returned
// #2 Else if l goes through i'th point, the i'th segment is added
// If there are 2 or more such collinear segments for #1,
// any of them (only one, not all) should be returned (not
tested)
// Complexity: O(lg |p|)
vector<int> lineConvexPolyIntersection(const Polygon &p, Line
l) {
    assert((int) p.size() >= 3);
    assert(l.a != l.b);
    int n = p.size();
    vector<int> ret;
    Point v = l.b - l.a;
    int lf = extremePoint(p, rotate90(v));
    int rt = extremePoint(p, rotate90(v) * Ti(-1));
    int olf = orient(l.a, l.b, p[lf]);
    int ort = orient(l.a, l.b, p[rt]);
    if(!olf || !ort) {
        int idx = (!olf ? lf : rt);
        if(orient(l.a, l.b, p[(idx - 1 + n) % n]) == 0)
            ret.push_back((idx - 1 + n) % n);
        else ret.push_back(idx);
        return ret;
    }
    if(olf == ort) return ret;
    for(int i=0; i<2; ++i) {

```

```

int lo = i ? rt : lf;
int hi = i ? lf : rt;
int olo = i ? ort : olf;
while(true) {
    int gap = (hi - lo + n) % n;
    if(gap < 2) break;
    int mid = (lo + gap / 2) % n;
    int omid = orient(l.a, l.b, p[mid]);
    if(!omid) {
        lo = mid;
        break;
    }
    if(omid == olo) lo = mid;
    else hi = mid;
}
ret.push_back(lo);
}
return ret;
}
// Calculate [ACW, CW] tangent pair from an external point
constexpr int CW = -1, ACW = 1;
bool isGood(Point u, Point v, Point Q, int dir) { return orient(Q,
u, v) != -dir; }
Point better(Point u, Point v, Point Q, int dir) { return orient(Q,
u, v) == dir ? u : v; }
Point pointPolyTangent(const Polygon &pt, Point Q, int dir, int
lo, int hi) {
    while(hi - lo > 1) {
        int mid = (lo + hi) / 2;
        bool pvs = isGood(pt[mid], pt[mid - 1], Q, dir);
        bool nxt = isGood(pt[mid], pt[mid + 1], Q, dir);
        if(pvs && nxt) return pt[mid];
        if(!(pvs || nxt)) {
            Point p1 = pointPolyTangent(pt, Q, dir, mid + 1, hi);
            Point p2 = pointPolyTangent(pt, Q, dir, lo, mid - 1);
            return better(p1, p2, Q, dir);
        }
        if(!pvs) {
            if(orient(Q, pt[mid], pt[lo]) == dir) hi = mid - 1;
            else if(better(pt[lo], pt[hi], Q, dir) == pt[lo]) hi = mid - 1;
            else lo = mid + 1;
        }
        if(!nxt) {
            if(orient(Q, pt[mid], pt[lo]) == dir) lo = mid + 1;
            else if(better(pt[lo], pt[hi], Q, dir) == pt[lo]) hi = mid - 1;
            else lo = mid + 1;
        }
    }
    Point ret = pt[lo];
    for(int i = lo + 1; i <= hi; i++) ret = better(ret, pt[i], Q, dir);
    return ret;
}
// [ACW, CW] Tangent
pair<Point, Point> pointPolyTangents(const Polygon &pt, Point
Q) {

```

```

int n = pt.size();
Point acw_tan = pointPolyTangent(pt, Q, ACW, 0, n - 1);
Point cw_tan = pointPolyTangent(pt, Q, CW, 0, n - 1);
return make_pair(acw_tan, cw_tan);
}
}

namespace Circular {
// Extremely inaccurate for finding near touches
// compute intersection of line l with circle c
// The intersections are given in order of the ray (l.a, l.b)
vector<Point> circleLineIntersection(Circle c, Line l) {
    static_assert(is_same<Tf, Ti>::value);
    vector<Point> ret;
    Point b = l.b - l.a, a = l.a - c.o;
    Tf A = dot(b, b), B = dot(a, b);
    Tf C = dot(a, a) - c.r * c.r, D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(l.a + b * (-B - sqrt(D + EPS)) / A);
    if (D > EPS)
        ret.push_back(l.a + b * (-B + sqrt(D)) / A);
    return ret;
}
// signed area of intersection of circle(c.o, c.r) &&
// triangle(c.o, s.a, s.b) [cross(a-o, b-o)/2]
Tf circleTriangleIntersectionArea(Circle c, Segment s) {
    using Linear::distancePointSegment;
    Tf OA = length(c.o - s.a);
    Tf OB = length(c.o - s.b);
    // sector
    if(dcmp(distancePointSegment(c.o, s) - c.r) >= 0)
        return angleBetween(s.a-c.o, s.b-c.o) * (c.r * c.r) / 2.0;
    // triangle
    if(dcmp(OA - c.r) <= 0 && dcmp(OB - c.r) <= 0)
        return cross(c.o - s.b, s.a - s.b) / 2.0;
    // three part: (A, a) (a, b) (b, B)
    vector<Point> Sect = circleLineIntersection(c, s);
    return circleTriangleIntersectionArea(c, Segment(s.a, Sect[0]))
        + circleTriangleIntersectionArea(c, Segment(Sect[0], Sect[1]))
        + circleTriangleIntersectionArea(c, Segment(Sect[1], s.b));
}
// area of intersecion of circle(c.o, c.r) && simple polyson(p[])
// Tested : https://codeforces.com/gym/100204/problem/F -
// Little Mammoth
Tf circlePolyIntersectionArea(Circle c, Polygon p) {
    Tf res = 0;
    int n = p.size();
    for(int i = 0; i < n; ++i)
        res += circleTriangleIntersectionArea(c, Segment(p[i], p[(i +
1) % n]));
    return abs(res);
}
// locates circle c2 relative to c1
// interior (d < R - r) ----> -2
// interior tangents (d = R - r) ----> -1
// concentric (d = 0)

```

```

// secants (R - r < d < R + r) ----> 0
// exterior tangents (d = R + r) ----> 1
// exterior (d > R + r) ----> 2
int circleCirclePosition(Circle c1, Circle c2) {
    Tf d = length(c1.o - c2.o);
    int in = dcmp(d - abs(c1.r - c2.r)), ex = dcmp(d - (c1.r + c2.r));
    return in < 0 ? -2 : in == 0 ? -1 : ex == 0 ? 1 : ex > 0 ? 2 : 0;
}
// compute the intersection points between two circles c1 &&
c2
vector<Point> circleCircleIntersection(Circle c1, Circle c2) {
    static_assert(is_same<Tf, Ti>::value);
    vector<Point> ret;
    Tf d = length(c1.o - c2.o);
    if(dcmp(d) == 0) return ret;
    if(dcmp(c1.r + c2.r - d) < 0) return ret;
    if(dcmp(abs(c1.r - c2.r) - d) > 0) return ret;
    Point v = c2.o - c1.o;
    Tf co = (c1.r * c1.r + sqLength(v) - c2.r * c2.r) / (2 * c1.r *
length(v));
    Tf si = sqrt(abs(1.0 - co * co));
    Point p1 = scale(rotatePrecise(v, co, -si), c1.r) + c1.o;
    Point p2 = scale(rotatePrecise(v, co, si), c1.r) + c1.o;
    ret.push_back(p1);
    if(p1 != p2) ret.push_back(p2);
    return ret;
}
// intersection area between two circles c1, c2
Tf circleCircleIntersectionArea(Circle c1, Circle c2) {
    Point AB = c2.o - c1.o;
    Tf d = length(AB);
    if(d >= c1.r + c2.r) return 0;
    if(d + c1.r <= c2.r) return PI * c1.r * c1.r;
    if(d + c2.r <= c1.r) return PI * c2.r * c2.r;
    Tf alpha1 = acos((c1.r * c1.r + d * d - c2.r * c2.r) / (2.0 * c1.r *
d));
    Tf alpha2 = acos((c2.r * c2.r + d * d - c1.r * c1.r) / (2.0 * c2.r *
d));
    return c1.sector(2 * alpha1) + c2.sector(2 * alpha2);
}
// returns tangents from a point p to circle c
vector<Point> pointCircleTangents(Point p, Circle c) {
    static_assert(is_same<Tf, Ti>::value);
    vector<Point> ret;
    Point u = c.o - p;
    Tf d = length(u);
    if(d < c.r) ;
    else if(dcmp(d - c.r) == 0) {
        ret = { rotate(u, PI / 2) };
    }
    else {
        Tf ang = asin(c.r / d);
        ret = { rotate(u, -ang), rotate(u, ang) };
    }
    return ret;
}
// returns the points on tangents that touches the circle

```

```
vector<Point> pointCircleTangencyPoints(Point p, Circle c) {
    static_assert(is_same<Tf, Ti>::value);
    Point u = p - c.o;
    Tf d = length(u);
    if(d < c.r) return {};
    else if(dcmp(d - c.r) == 0) return {c.o + u};
    else {
        Tf ang = acos(c.r / d);
        u = u / length(u) * c.r;
        return { c.o + rotate(u, -ang), c.o + rotate(u, ang) };
    }
}
// for two circles c1 && c2, returns two list of points a && b
// such that a[i] is on c1 && b[i] is c2 && for every i
// Line(a[i], b[i]) is a tangent to both circles
// CAUTION: a[i] = b[i] in case they touch | -1 for c1 = c2
int circleCircleTangencyPoints(Circle c1, Circle c2,
vector<Point> &a, vector<Point> &b) {
    a.clear(), b.clear();
    int cnt = 0;
    if(dcmp(c1.r - c2.r) < 0) {
        swap(c1, c2); swap(a, b);
    }
    Tf d2 = sqLength(c1.o - c2.o);
    Tf rdif = c1.r - c2.r, rsum = c1.r + c2.r;
    if(dcmp(d2 - rdif * rdif) < 0) return 0;
    if(dcmp(d2) == 0 && dcmp(c1.r - c2.r) == 0) return -1;
    Tf base = angle(c2.o - c1.o);
    if(dcmp(d2 - rdif * rdif) == 0) {
        a.push_back(c1.point(base));
        b.push_back(c2.point(base));
        cnt++;
        return cnt; }
    Tf ang = acos((c1.r - c2.r) / sqrt(d2));
    a.push_back(c1.point(base + ang));
    b.push_back(c2.point(base + ang));
    cnt++;
    a.push_back(c1.point(base - ang));
    b.push_back(c2.point(base - ang));
    cnt++;
    if(dcmp(d2 - rsum * rsum) == 0) {
        a.push_back(c1.point(base));
        b.push_back(c2.point(PI + base));
        cnt++;
    }
    else if(dcmp(d2 - rsum * rsum) > 0) {
        Tf ang = acos((c1.r + c2.r) / sqrt(d2));
        a.push_back(c1.point(base + ang));
        b.push_back(c2.point(PI + base + ang));
        cnt++;
        a.push_back(c1.point(base - ang));
        b.push_back(c2.point(PI + base - ang));
        cnt++;
    }
    return cnt;
}
}}
```

```
// Given a bunch of segments. Check if any two intersect.
// Sweep Line. O(n lg n). TESTED: CF 1359F
namespace IntersectingSegments {
    Tf yvalSegment(const Line &s, Tf x) {
        if(dcmp(s.a.x - s.b.x) == 0) return s.a.y;
        return s.a.y + (s.b.y - s.a.y) * (x - s.a.x) / (s.b.x - s.a.x);
    }
    struct SegCompare {
        bool operator () (const Segment &p, const Segment &q) const
    {
        Tf x = max(min(p.a.x, p.b.x), min(q.a.x, q.b.x));
        return dcmp(yvalSegment(p, x) - yvalSegment(q, x)) < 0;
    } };
    multiset<Segment, SegCompare> st;
    typedef multiset<Segment, SegCompare>::iterator iter;
    iter prev(iter it) {
        return it == st.begin() ? st.end() : --it;
    }
    iter next(iter it) {
        return it == st.end() ? st.end() : ++it;
    }
    struct Event {
        Tf x; int tp, id;
        Event(Ti x, int tp, int id) : x(x), tp(tp), id(id) {}
        bool operator < (const Event &p) const {
            if(dcmp(x - p.x)) return x < p.x;
            return tp > p.tp;
        };
    };
    bool anyIntersection(const vector<Segment> &v) {
        using Linear::segmentsIntersect;
        vector<Event> ev;
        for(int i=0; i<(int) v.size(); ++i) {
            ev.push_back(Event(min(v[i].a.x, v[i].b.x), +1, i));
            ev.push_back(Event(max(v[i].a.x, v[i].b.x), -1, i));
        }
        sort(ev.begin(), ev.end());
        st.clear();
        vector<iter> where(v.size());
        for(auto &cur : ev) {
            int id = cur.id;
            if(cur.tp == 1) {
                iter nxt = st.lower_bound(v[id]);
                iter pre = prev(nxt);
                if(pre != st.end() && segmentsIntersect(*pre, v[id])) return true;
                if(nxt != st.end() && segmentsIntersect(*nxt, v[id])) return true;
                where[id] = st.insert(nxt, v[id]);
            }
            else {
                iter nxt = next(where[id]);
                iter pre = prev(where[id]);
                if(pre != st.end() && nxt != st.end() && segmentsIntersect(*pre, *nxt))
                    return true;
                st.erase(where[id]);
            }
        }
        return false;
    }
}
```

## JnU-The\_Last\_Phase

## // Ashik's extra formula

inner circle radius,  $r = \text{area} * s$ outer circle area,  $A = (\text{abc})/4R$ **BitMask:**ll Set(ll N, ll pos) return  $N = N | (1LL \ll \text{pos})$ ;ll Reset(ll N, ll pos) return  $N = N \& \sim(1LL \ll \text{pos})$ ;bool chk(ll N, ll pos) return (bool)( $N \& (1LL \ll \text{pos})$ );

/// int id= \_\_builtin\_ctz(mask); its give the position of the first one from the left

/// int tot= \_\_builtin\_popcount(mask); number of one bit .

**Digit Dp All digit sum:**

ll dp[15][2][400][2];

const ll mpos=11;

char ch[40];

void convert(ll n){

for(ll i=0; i&lt;mpos; i++){

ch[i]=(n%10)+'0';

n/=10;

}

reverse(ch, ch+mpos);

ch[mpos]=0;

}

ll func(ll pos, ll smallornot, ll digitvalcnt, ll startornot){

if(pos==mpos)

return digitvalcnt;

if(dp[pos][smallornot][digitvalcnt][startornot]!=-1)

return dp[pos][smallornot][digitvalcnt][startornot];

ll be=0, en=9, re=0;

if(!smallornot)

en=ch[pos]-'0';

for(ll i=be; i&lt;=en; i++){

{

ll ismallornot= smallornot | (i&lt;en);

ll idigitvalcnt=digitvalcnt+ i;

ll istartornot= startornot | (i!=0);

re+=func(pos+1, ismallornot, idigitvalcnt, istartornot);

}

return dp[pos][smallornot][digitvalcnt][startornot]=re;

}

func(0,0,0,0);

**SOS DP:**

memset(dp, -1, sizeof(dp));

for(int i=1; i&lt;=n; i++){

scanf("%d", &amp;ar[i]);

dp[ar[i]]=ar[i];

}

for(int i=0; i&lt;22; i++){

for(int mask=0; mask&lt;(1&lt;&lt;22); mask++){

if(chk(mask, i)) dp[mask]=max(dp[mask], dp[mask^(1&lt;&lt;i)]);

}

}

int boro=(1&lt;&lt;22)-1;

for(int i=1; i&lt;=n; i++){

printf("%d ", dp[boro^ar[i]]);

}

/\* iterate all the submask of a mask

for(int mask=1; mask&lt;(1&lt;&lt;sz); mask++){

int tmask=mask&amp;(mask-1);

while(tmask) {

cout&lt;&lt;tmask&lt;&lt;endl;

// dp[mask]=min(dp[mask], dp[tmask]+dp[mask^tmask]);

tmask=(tmask-1)&amp;mask;

}

}\*/

**Combinatorics Notes:**///  $0 * nC0 + 1 * nC1 + 2 * nC2 + 3 * nC3 + \dots + n * nCn = n * 2^{(n-1)}$ .///  $0Cr + 1Cr + 2Cr + 3Cr + 4Cr + 5Cr + 6Cr + \dots + nCr = (n+1)C(r+1)$ ///  $(nC0)^2 + (nC1)^2 + (nC2)^2 + \dots + (nCn)^2 = (2^n)Cn$ 

/// how many ways you can go to (0,0) to (n,m) coordinate (you can only up and right).

like  $n=2, m=3$ , so  $= 5! / (2! * 3!)$ 

if there are more than two dimensions you will do just total moves time! / (x axis moves times! \* y axis moves time! \* .....)

/// you have n balls k bucket # of ways insert the ball into bucket such that every bucket has more than 0 balls

total ways is  $(n-1)C(k-1)$ .modification , any numbers of ball then answer is,  $(n+k-1)C(k-1)$ modification , per bucket condition  $0 \leq k_i < x_i$ for  $0 \leq k_i$ ,  $RESULT1 = (n+k-1)C(k-1)$ for  $k_i \geq x_i$ ,  $val_i = kCi * (n-i * x_i + k-1)C(k-1)$  $RESULT2 = ((-k)^1 * val_1 + ((-k)^2 * val_2 + ((-k)^1 * val_3 + \dots + ((-k)^k * val_k$ 

[But some time we have not calculated overall val1 to valk

, Because  $(n - (x * kth) + k - 1)$  will be  $< 0$ ]Final result =  $RESULT1 - RESULT2$ /// catalan number  $Cn = (1/(n+1)) * ((2^n)Cn)$ In other form  $Cn = ((2^n)C(n)) - ((2^n)C(n+1))$ **Bitset:**

bitset&lt;mx&gt;bt;

bt.set() /// all bit 1

bt.reset() /// all bit 0

bt.count() // total number of 1 bit

bt.\_Find\_first() // palce of the first 1 bit

bt.\_Find\_next() // next one bit

for(int i=bt.\_Find\_first(); i&lt;mx; i=bt.\_Find\_next()) // for traversing all 1 node

**Iterative Stack:**

template&lt;typename T, typename Container = std::deque&lt;T&gt;&gt;

class iterable\_stack

: public std::stack&lt;T, Container&gt;

{

using std::stack&lt;T, Container&gt;::c;

public:

auto begin() { return std::begin(c); }

auto end() { return std::end(c); }

auto begin() const { return std::begin(c); }

auto end() const { return std::end(c); }

};

iterable\_stack&lt;int&gt; st;



```
st.push(2);
for(auto i: st)
    std::cout << i << ' ';
```

#### PBDS:

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
solve(){
    ordered_set os;
    // 10. how many numbers are smaller than a given value(7)
    cout << os.order_of_key(7);
    // 12. how many numbers are greater than a given value(7)
    cout << os.size() - os.order_of_key(8) << "\n";
    // 14. if the given numbers are sorted in ascending order,
    what is the k'th number
    cout << *os.find_by_order(2) << "\n";
    // 16. delete the k'th smallest number
    os.erase(os.find_by_order(k));
    // 22. what is the smallest number which is greater than or
    equal to a given number(7)
    cout << *os.lower_bound(7) << "\n";
    // 23. what is the smallest number which is greater than to a
    given number(7)
    cout << *os.upper_bound(7) << "\n";
}
```

#### Ashrafu's Template:

```
s.sh:
for((i=1;i<100;i++));do
    ./gen $i>int
    ./a<int>out1
    ./brute<int>out2
    diff out1 out2 || break
Done
gen.cpp:
mt19937_64
rng(chrono::steady_clock::now().time_since_epoch().count());
ll my_rand(ll l, ll r) {
    return uniform_int_distribution<ll>(l, r) (rng);
}
```

#### Ashik's Fast I/O:

```
#define faster_io
ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0);
#define watch2(x,y) cout<< _LINE_ << " says: " <<#x<<" =
"<<x<<" <<#y<<" = "<<y <<endl
freopen("input.txt","r",stdin); ("output.txt","w",stdout);
```

#### BIG Integer:

```
import static java.lang.System.in;
import java.util.Scanner;
import java.math.BigInteger;
public class Main {
    public static void main(String[] args) {
        Scanner in = new Scanner(System.in);
        int n;
```

```
n = in.nextInt();
BigInteger ci;
ci = new BigInteger("1");
//BigInteger b = in.nextBigInteger();
//BigInteger carry;
//carry = (a.multiply(b)).divide(a.gcd(b));
for(int i=1; i<=n+1; i++) {
    int temp = 4 * (i+1) - 6;
    BigInteger tem = BigInteger.valueOf(temp);
    ci = ci.multiply(tem);
    ci = ci.divide(BigInteger.valueOf(i+1));
}
System.out.println(ci);
}
```

#### De-arrangement:

$d(n) = (n-1) \cdot (d(n-1) + d(n-2))$  where  $d(0)=1, d(1)=0$

#### Fibonacci:

$F_n = \frac{1}{\sqrt{5}} \left( \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right)$

#### GCD:

159.  $\gcd(a, b) \cdot \text{lcm}(a, b) = |a \cdot b|$
160.  $\gcd(a, \text{lcm}(b, c)) = \text{lcm}(\gcd(a, b), \gcd(a, c))$ .
161.  $\text{lcm}(a, \gcd(b, c)) = \gcd(\text{lcm}(a, b), \text{lcm}(a, c))$ .
162. For non-negative integers  $a$  and  $b$ , where  $a$  and  $b$  are not both zero,
$$\gcd(n^a - 1, n^b - 1) = n^{\gcd(a, b)} - 1$$
163.  $\gcd(a, b) = \sum_{k|a \text{ and } k|b} \phi(k)$
164.  $\sum_{i=1}^n [\gcd(i, n) = k] = \phi\left(\frac{n}{k}\right)$
165.  $\sum_{k=1}^n \gcd(k, n) = \sum_{d|n} d \cdot \phi\left(\frac{n}{d}\right)$
166.  $\sum_{k=1}^n x^{\gcd(k, n)} = \sum_{d|n} x^d \cdot \phi\left(\frac{n}{d}\right)$
167.  $\sum_{k=1}^n \frac{1}{\gcd(k, n)} = \sum_{d|n} \frac{1}{d} \cdot \phi\left(\frac{n}{d}\right) = \frac{1}{n} \sum_{d|n} d \cdot \phi(d)$
168.  $\sum_{k=1}^n \frac{k}{\gcd(k, n)} = \frac{n}{2} \cdot \sum_{d|n} \frac{1}{d} \cdot \phi\left(\frac{n}{d}\right) = \frac{n}{2} \cdot \frac{1}{n} \cdot \sum_{d|n} d \cdot \phi(d)$
169.  $\sum_{k=1}^n \frac{n}{\gcd(k, n)} = 2 * \sum_{k=1}^n \frac{k}{\gcd(k, n)} - 1$ , for  $n > 1$
170.  $\sum_{i=1}^n \sum_{j=1}^n [\gcd(i, j) = 1] = \sum_{d=1}^n \mu(d) \left\lfloor \frac{n}{d} \right\rfloor^2$
171.  $\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{d=1}^n \phi(d) \left\lfloor \frac{n}{d} \right\rfloor^2$
172.  $\sum_{i=1}^n \sum_{j=1}^n i \cdot j [\gcd(i, j) = 1] = \sum_{i=1}^n \phi(i) i^2$
173.  $F(n) = \sum_{i=1}^n \sum_{j=1}^n \text{lcm}(i, j) = \sum_{l=1}^n \left( \frac{(1 + \lfloor \frac{n}{l} \rfloor) (\lfloor \frac{n}{l} \rfloor)}{2} \right)^2 \sum_{d|l} \mu(d) l d$
174.  $\gcd(\text{lcm}(a, b), \text{lcm}(b, c), \text{lcm}(a, c)) = \text{lcm}(\gcd(a, b), \gcd(b, c), \gcd(a, c))$
175.  $\gcd(A_L, A_{L+1}, \dots, A_R) = \gcd(A_L, A_{L+1} - A_L, \dots, A_R - A_{R-1})$ .
176. Given  $n$ , If  $SUM = LCM(1, n) + LCM(2, n) + \dots + LCM(n, n)$ 
then  $SUM = \frac{n}{2} \left( \sum_{d|n} (\phi(d) \times d) + 1 \right)$

/\* Good Luck Ashik, Selim, Ashrafu! \*/