$$Volt = Ampere * Ohm$$

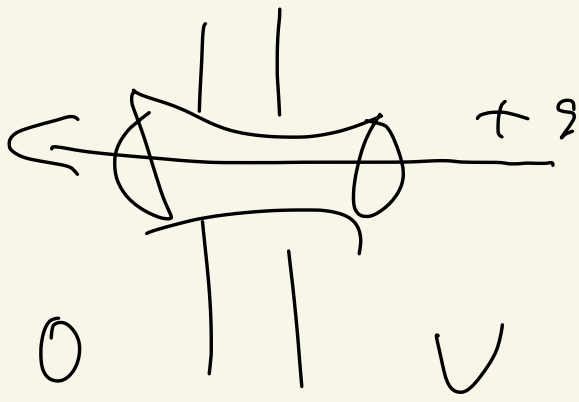
$$Ampere * second = Coulomb$$

$$Farad * Volt = Coulomb$$

$$Farad * Volt/second = Ampere$$

$$Farad * Ohm = second$$

1)

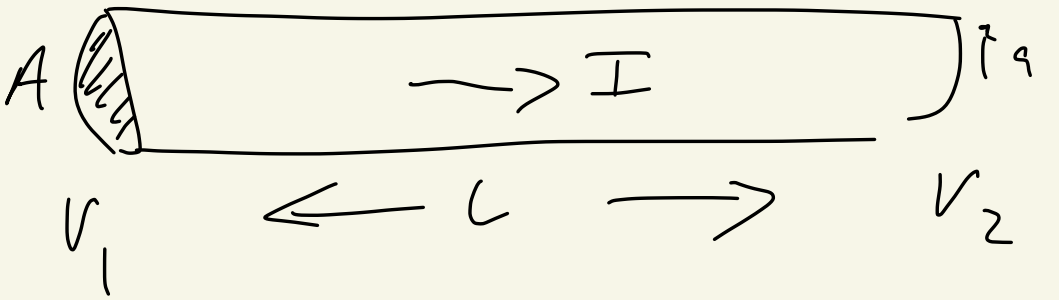


K^+ for example

$$|E| = q|V| \sim k_B T$$

$$|V| = \frac{k_B T}{q} = V_T \simeq 25\,mV$$

2)



$$V_1 - V_2 = I R_L$$

$$R_L \sim \frac{L}{A} = \frac{r_L L}{A}$$

$$r_L = 1\,K\Omega\text{-}mm$$

example $L = 100\,\mu m = 0.1\,mm$

$a = 2\,\mu m = 2 \times 10^{-3}\,mm$

$$A = \pi a^2$$

$$R_L = \frac{(10^3\,\Omega\text{-mm})(0.1\,\text{mm})}{\pi \cdot 4 \times 10^{-6}\,\text{mm}^2}$$

$$= \frac{10^2\,\Omega}{1.2 \times 10^{-5}} = \frac{10}{1.2} \times 10^6\,\Omega$$

$$\sim 8\,\text{M}\Omega$$

example 2

$$G_L = \frac{1}{R_L} = \frac{A}{\rho_L L}$$

channel

$$L = 6\,\mu\text{M} = 6 \times 10^{-6}\,\text{mm}$$

$$A = 0.15\,\text{nm}^2 = 15 \times 10^{-14}\,\text{mm}^2$$

$$G_L = \frac{15 \times 10^{-14}\,\text{mm}^2}{6 \times 10^{-6}\,\text{mm} \cdot 10^3\,\Omega\,\text{mm}} = 2.5 \times 10^{-11}\,S$$

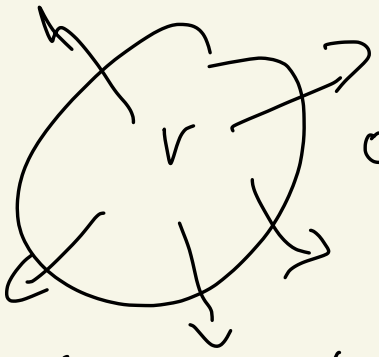
$$= 25 \times 10^{-12}\,S = 25\,\text{pS}$$

$$V_1 - V_2 = 8\,mV$$

$$I = \frac{8\,mV}{8\,M\Omega} = 10^{-9}A = 1\,nA$$

3)

a)



$$I_m = g_m (V - E)$$

E = resting potential $\simeq -70\,mV$

$$\Delta V = V - E \qquad I_m = g_m \Delta V$$

$$g_m \sim A_m \qquad R_m = \frac{1}{g_m} = \frac{r_m}{A}$$

$$r_m = 1\,M\Omega\,mm^2$$

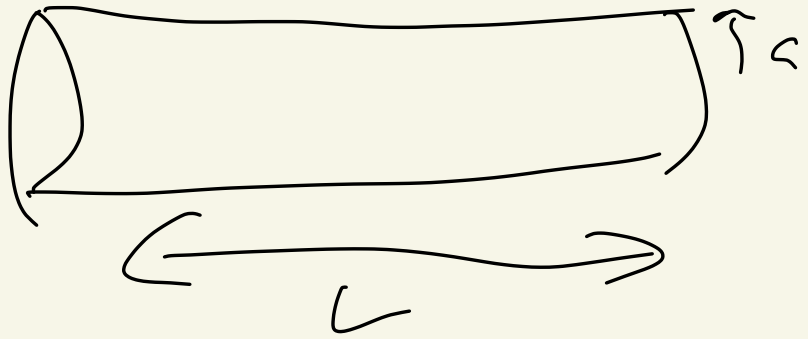
$$A_m = 0.1\,mm^2$$

$$R_m = 10\,M\Omega$$

for $\Delta V = 10\,mV$

$$I = \frac{10\,mV}{10\,M\Omega} = 1\,nA$$

b)



$$R_L = \frac{r_L L}{\pi a^2} \qquad R_m = \frac{r_m}{2\pi a L}$$

what L makes

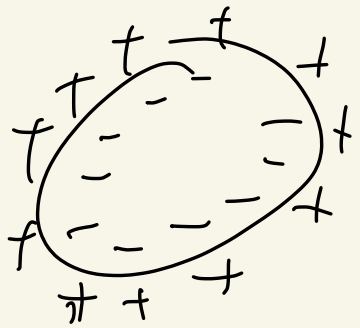
$$R_L = R_m$$

$$\frac{r_L L}{\pi a^2} = \frac{r_m}{2\pi a L}$$

$$L^2 = \frac{r_m}{r_L} \frac{\pi a^2}{2\pi a} = \frac{a r_m}{2 r_L}$$

$$L = \sqrt{\frac{a r_m}{2 r_L}} \Omega \text{ / mm}$$

4)



$Q = CV$

$C = c_m A \quad c_m = 10\,nF/mm^2$

$C = 10\,nF \times 0.1 = 1\,nF$

$Q = 70\,mV \cdot 1\,nF = 7 \times 10^{-11}\,C$

$\approx 10^9$ charges

$= 10^{-5}$ of charges in cell

$Q = 0.7\,nA \cdot 100\,ns = 7 \times 10^{-11}\,C$

$$\frac{dQ}{dt} = I = C\frac{dV}{dt}$$

I = positive inward

$I = g_m (V - E)$ is positive outward

so

$$C \frac{dV}{dt} = -g_m (V - E) + I_{ext}$$

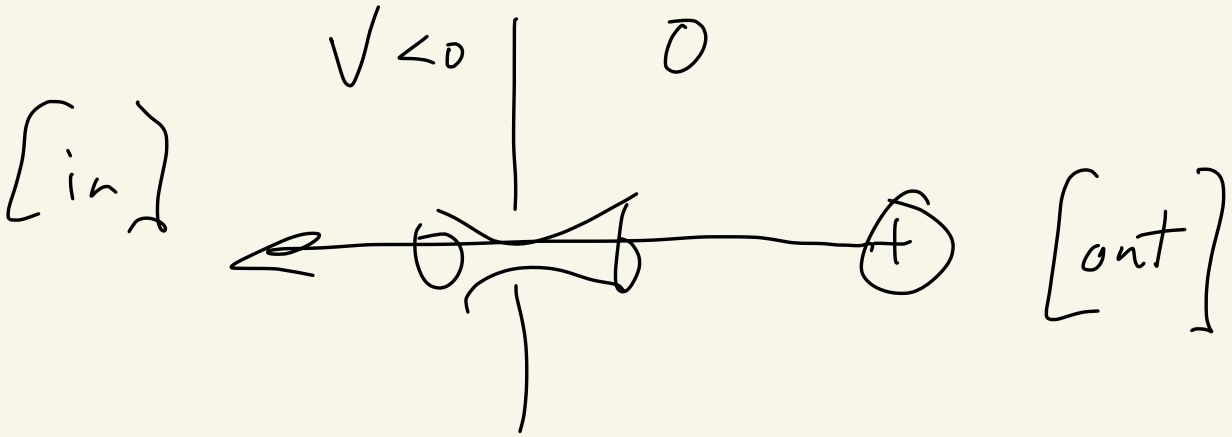
$$10^{-9} F \times \frac{1mV}{ms} = 10^{-9} amps = 1nA$$

5) E?

$$I_m = g_K (V - E_K) + g_{Na} (V - E_{Na}) + \cdots$$

$$g_m = g_K + g_{Na} + \cdots$$

$$E = \frac{g_K E_K + g_{Na} E_{Na} + \cdots}{g_K + g_{Na} + \cdots}$$



$$I_{inward} = [out]\, \rho_{cross}$$

$$I_{outward} = [in]\, \rho_{cross}\, e^{-\xi |V| / k_B T}$$

$$= [in]\, \rho_{cross}\, e^{\frac{V}{V_T}}$$

$V = E$ at zero net current

$$[out]\, \rho_{cross} = [in]\, \rho_{cross}\, e^{\frac{E}{V_T}}$$

$$e^{qE/V_T} = \frac{[out]}{[in]}$$

$$E = V_T \ln\left(\frac{[out]}{[in]}\right)$$ Nernst Potential

for multiply charged ions

$$E = \frac{V_T}{z} \ln\left(\frac{[out]}{[in]}\right)$$

	ion	E	
outward	K^+	$-80mV$	$[in] > [out]$
inward	Na^+	$+50mV$	$[in] < [out]$
inward	Ca^{++}	$+200mV$	$[in] << [out]$
~	Cl^-	$-40mV$	
	exc.	$0mV$	
	inh.	E_{K^+} or E_{Cl^-}	

also ion pumps - Na-K exchanger

6) Passive neuron model

$$C \frac{dU}{dt} = -g_m (V - E) + I_{ext}$$

$$C = c_m A \qquad g_m = \frac{A}{r_m}$$

$$c_m r_m \frac{dU}{dt} = E - V + R_m I_{ext}$$

$$c_m r_m = \tau = 10 \times 10^{-9} F \cdot 1 \times 10^{6} \Omega$$

$$= 10 \text{ ms}$$

$$\tau \frac{dV}{dt} = E - V + R_m I_{ext}$$

$$R_m I_{ext} = I$$

$$\tau \frac{dV}{dt} = E + I - V$$

a) I = constant

$$E + I = V_\infty$$

$$\tau \frac{dV}{dt} = V_\infty - V$$

$$V = V_\infty + X$$

$$\tau \frac{dX}{dt} = -X$$

$$x(t) = x(0)e^{-t/\tau}$$

or $$x(t_2) = x(t_1)e^{-(t_2-t_1)/\tau}$$

or $$x(t+\Delta t) = x(t)e^{-\frac{\Delta t}{\tau}}$$

$$x = V - V_\infty$$

So

$$V(t) = V_\infty + \left(V(0) - V_\infty\right) e^{-t/\tau}$$

or

$$V(t_2) = V_\infty + \left(V(t_1) - V_\infty\right) e^{-(t_2 - t_1)/\tau}$$

or

$$V(t + \Delta t) = V_\infty + \left(V(t) - V_\infty\right) e^{-\frac{\Delta t}{\tau}}$$

b) $\underline{I}(t)$

$$V = E + x$$

$$\tau \frac{dx}{dt} = -x + \underline{I}(t)$$

$$x = e^{-t/\tau} * y$$

$$\tau \frac{dx}{dt} = -x + e^{-t/\tau} \frac{dy}{dt} = -x + I$$

$$\tau \frac{dy}{dt} = e^{t/\tau} I(t)$$

$$y(t) = \frac{1}{\tau} \int_0^t dt' e^{t'/\tau} I(t')$$

$$x(t) = \frac{1}{\tau} e^{-t/\tau} \int_0^t dt' e^{t'/\tau} I(t')$$

$$= \frac{1}{\tau} \int_0^t dt' \, e^{-(t-t')/\tau} I(t')$$

$$= \int_0^t dt' K(t-t') I(t')$$

$$K(t-t') = \frac{1}{\tau} e^{-(t-t')/\tau}$$

2) Integrate & Fire Model

$$\tau \frac{dV}{dt} = E - V + \bar{I}$$

when $V = V_{th}$ spike

and $V \rightarrow V_{reset}$

c) Firing rate for constant $\bar{I}$

$$E + \bar{I} \geq V_{th}$$

at $t = 0$

$$V = V_{reset}$$

when is

$$V(t) = V_{th}$$

$$V_{00} = E + \bar{I}$$

$$V_{th} = V_\infty + (V_{reset} - V_\infty) e^{-t/\tau}$$

$$e^{t/\tau} = \frac{V_{reset} - V_\infty}{V_{th} - V_\infty} = \frac{V_\infty - V_{reset}}{V_\infty - V_{th}}$$

$$t = \tau \ln \left(\frac{V_\infty - V_{reset}}{V_\infty - V_{th}} \right)$$

$$r = \frac{1}{t}$$

8) Other conductances

$$C \frac{dV}{dt} = -g_L (V - E_L) - g_1 (V - E_1) - \cdots$$

$$- g_s (V - E_s) + \cdots$$

$$g \rightarrow g(t)$$