1) Synaptic conductance/current

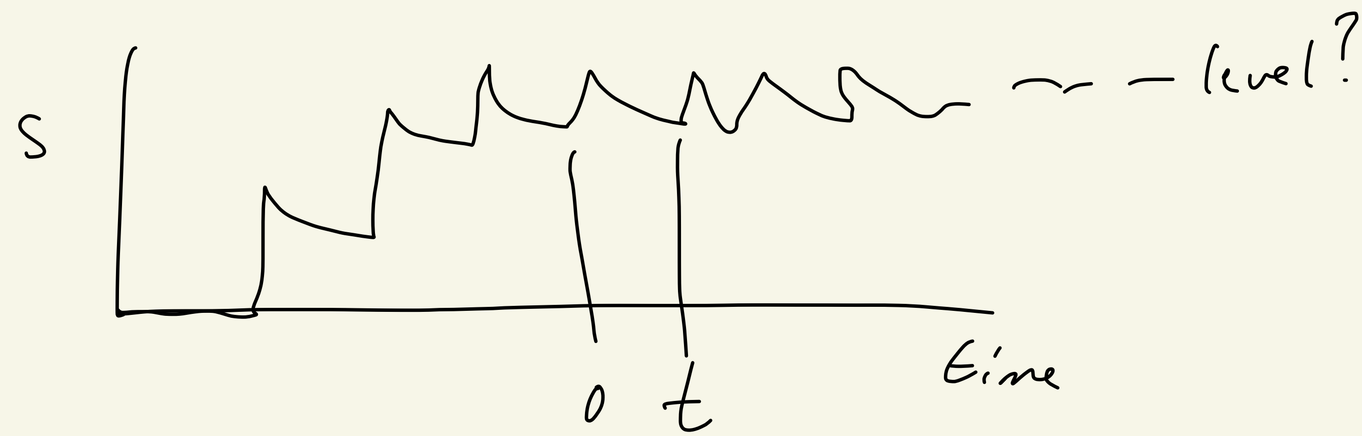
$$I_s = \bar{g}_s \, s \, (V - E_s)$$

$E_s = 0$ (excitatory)

$E_s = -80$ (GABA-B) or -65 (GABA-A)

$$\tau_s \frac{ds}{dt} = -s \qquad \tau_s = \begin{matrix} \text{few ms} \\ \sim 100 \text{ ms} \end{matrix}$$

$s \to s + 1$ on pre-spike



$$\frac{1}{t} = r_{pre}$$

$$s(t) = s(0) = (s(0) + 1)\, e^{-1/r_{pre}\tau_s}$$

$$s(0)\left(1-e^{-1/r_{pre}\tau_s}\right) = e^{-1/r_{pre}\tau_s}$$

$$s(0) = \frac{e^{-1/r_{pre}\tau_s}}{1-e^{-1/r_{pre}\tau_s}}$$

$$\left(s(0)+1\right) = \frac{1}{1-e^{-1/r_{pre}\tau_s}}$$

$$\langle s \rangle = \frac{1}{1-e^{-1/r_{pre}\tau_s}}\frac{1}{t}\int_0^t dt' e^{-t'/\tau_s} \quad t=\frac{1}{r_{pre}}$$

$$= \frac{1}{1-e^{-1/r_{pre}\tau_s}}\frac{\tau_s}{t}\left(1-e^{-1/r_{pre}\tau_s}\right) = r\tau_s$$

$$\langle s \rangle = r_{pre}\tau_s$$

2) Synaptic conductances or currents

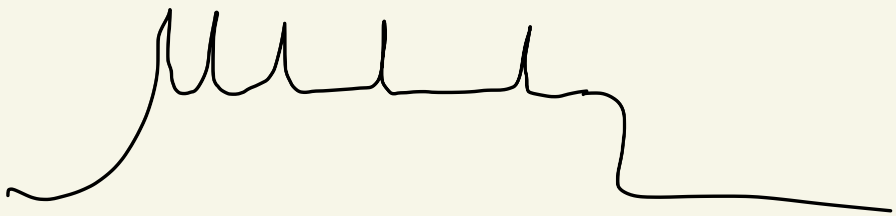
$$I_s = \bar{g}_s s (V-E_s)$$

or $\quad I_s = \bar{g}_s' s \qquad \langle I_s \rangle = \bar{g}_s' r_{pre}\tau_s$

$$\bar{g}_s' = \bar{g}_s (E - E_s)$$

$$\text{or} \quad = \bar{g}_s (V_{th} - E_s)$$

3) Adatation



$$\tau \frac{dV}{dt} = V - E + R_m I_e - g_a A (V - E_a)$$

$$\tau_A \frac{dA}{dt} = -A \qquad \tau_A \sim 100\,\text{ms}$$

$$A \to A + 1 \quad \text{on spike}$$

4) Nonlinear I+F Models

$$\tau_{\mathrm{m}} \frac{dV}{dt} = E_{\mathrm{L}} - V + F(V) + R_{\mathrm{m}} I_{\mathrm{e}} \,.$$

$$F(V) = \Delta_V \exp\left(\frac{V - V_{\mathrm{th}}}{\Delta_V}\right)$$

$F' = \exp(V - V_{th})$

$$F(V) = \frac{(V - E_{\mathrm{L}})^2}{2(V_{\mathrm{th}} - E_{\mathrm{L}})} \,.$$

$F' = \frac{V - E_L}{V_{th} - E_L}$

$$\tau_{\mathrm{m}} \frac{dV}{dt} = E_{\mathrm{L}} - V + F(V) - U + R_{\mathrm{m}} I_{\mathrm{e}} \,.$$

$$\tau_U \frac{dU}{dt} = b(V - E_{\mathrm{L}}) - U \,,$$

$$U \to U + d$$

5) The problem of the mean

$$x_i = \bar{x} + \delta_i$$

$$\frac{1}{N} \sum_i x_i = \bar{x} + \eta$$

$$\eta = \frac{1}{N} \sum_i \delta_i$$

$$\langle \eta \rangle = 0$$

$$\langle \delta_i^2 \rangle = \sigma_i^2$$

$$\langle \eta^2 \rangle = \frac{1}{N^2} \sum_i \langle \delta_i \rangle^2 = \frac{1}{N^2} \sum_i \sigma_i^2$$

$$\bar{\sigma}^2 = \frac{1}{N} \sum \sigma_i^2 \quad \text{so} \quad \langle \eta^2 \rangle = \frac{\bar{\sigma}^2}{N}$$

$$\frac{1}{N} \sum_i x_i = \bar{x} \pm \frac{1}{\sqrt{N}} \bar{\sigma}$$

This is both good news + bad news!

6) Spiking networks

$$\tau \frac{dV_i}{dt} = E - V_i + \sum_j \bar{g}_{ij} S_j$$

$\bar{g}_{ij} \rightarrow \bar{J}_{ij}$ connectivity matrix