Assignment 5

1. In class, we derived the maximum likelihood estimate θ_{ML} for the parameter θ of a model in which the probability of observing x spikes in response to the presentation of a stimulus s is given by a linear-Poisson model:

$$P(x) \sim \mathrm{Pois}(\lambda),\ \ \lambda = \theta s$$

   given a set of datapoints $\{x_i, s_i\}$. Use the same approach to derive an expression that θ_{ML} must satisfy for the linear-nonlinear-Poisson model:

$$P(x) \sim \mathrm{Pois}(\lambda),\ \ \lambda = f(\theta s),$$

   where f is some known function. The solution will be an implicit expression for θ_{ML} involving f, f', x_i, and s_i.

2. From Dayan & Abbott: Simulate the responses of four interneurons in the cercal system of the cricket and check the accuracy of a simple population decoding scheme. In this system, there are four interneurons whose firing rate responses are dependent on the wind direction θ.

   For a true wind direction θ the average firing rates of the four interneurons should be generated as $\mathrm{E}[r_i] = 50\ \mathrm{Hz} \cdot f(\cos(\theta - \theta_i))$, where f is rectified-linear ($f(x) = x$ if $x > 0$, $f(x) = 0$ otherwise), and $\theta_i = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ for $i = 1, 2, 3, 4$. The actual rates, r_i, are then obtained by adding to these mean rates a random number chosen from a Gaussian distribution with zero mean and a standard deviation of 5 Hz (set any rates the come out negative to zero).

   From these rates, construct the x and y components of the population vector:

$$x = \sum_{i=1}^{4} r_i \cos(\theta_i),\ \ y = \sum_{i=1}^{4} r_i \sin(\theta_i) \tag{1}$$

   and, from the direction of this vector, compute an estimate θ_{est} of the wind direction. Average the squared difference $(\theta - \theta_{\mathrm{est}})^2$ over 1000 trials. The square root of of this quantity is the error. Plot the error as a function of θ over the range $[-90°, 90°]$.

3. Compute the Fisher information $I_F(\theta)$ of the above neural population. Use the Cramér-Rao bound to plot a lower bound on the error of the optimal unbiased estimator over the same range as above.