ITN Assignment-4

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① Linearizing a nonlinear feedforward network

input: $\vec{x} \in \mathbb{R}^N$
weights: $W \in \mathbb{R}^{M \times N}$
output: $\vec{r} \in \mathbb{R}^M$

$$\vec{r} = f(W\vec{x})$$
$$r_i = f\left(\sum_{j=1}^{N} w_{ij} x_j\right)$$

by taking the partial derivatives of r with respect to x:

Jacobian $J(\vec{x}) \in \mathbb{R}^{m \times n}$

$$J(\vec{x}) = \begin{bmatrix} \partial r_1/\partial x_1 & \cdots & \partial r_1/\partial x_N \\ \vdots & \ddots & \vdots \\ \partial r_M/\partial x_1 & \cdots & \partial r_M/\partial x_N \end{bmatrix}$$

$$J_{ij} = \frac{\partial r_i}{\partial x_j} = \frac{\partial}{\partial x_j} f\left(\sum_{j=1}^{N} w_{ij} x_j\right) =$$
$$= w_{ij} f'\left(\sum_{j=1}^{N} w_{ij} x_j\right)$$

let $g_i(x) = f'\left(\sum_{j=1}^{N} w_{ij} x_j\right)$

so $J(\vec{x}) = \begin{bmatrix} g_1(\vec{x}) W_1 \\ \vdots \\ g_M(\vec{x}) W_M \end{bmatrix}$

Define G as the $M \times M$ diagonal matrix of $g_i(x)$ values $J(\vec{x}) = G(\vec{x}) W$

so if we linearize r about r_0

$\Delta r = r - r_0 \Leftrightarrow r = r_0 + \Delta r \quad r_0 \simeq f(Wx_0)$

$\Delta x J(x_0) = r - f(Wx_0)$

$\Rightarrow \Delta r \simeq \Delta x J(x_0) = G(x_0) W \Delta x$

the elements of D are the values of $G(x_0)$

ITN_HW4

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1 Intro to Theoretical Neuroscience Homework 4

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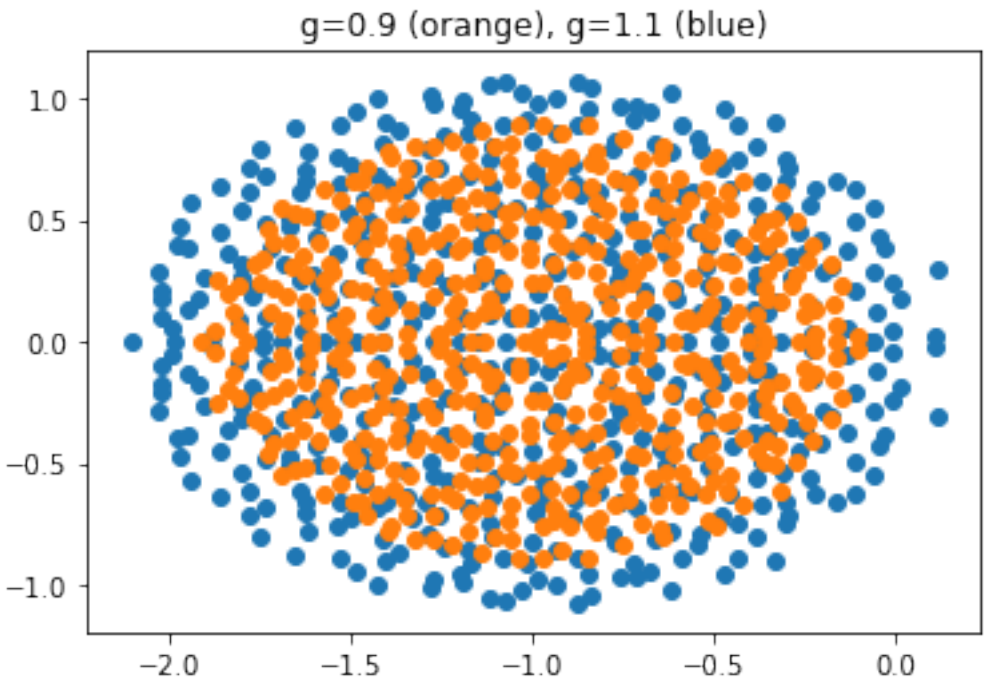
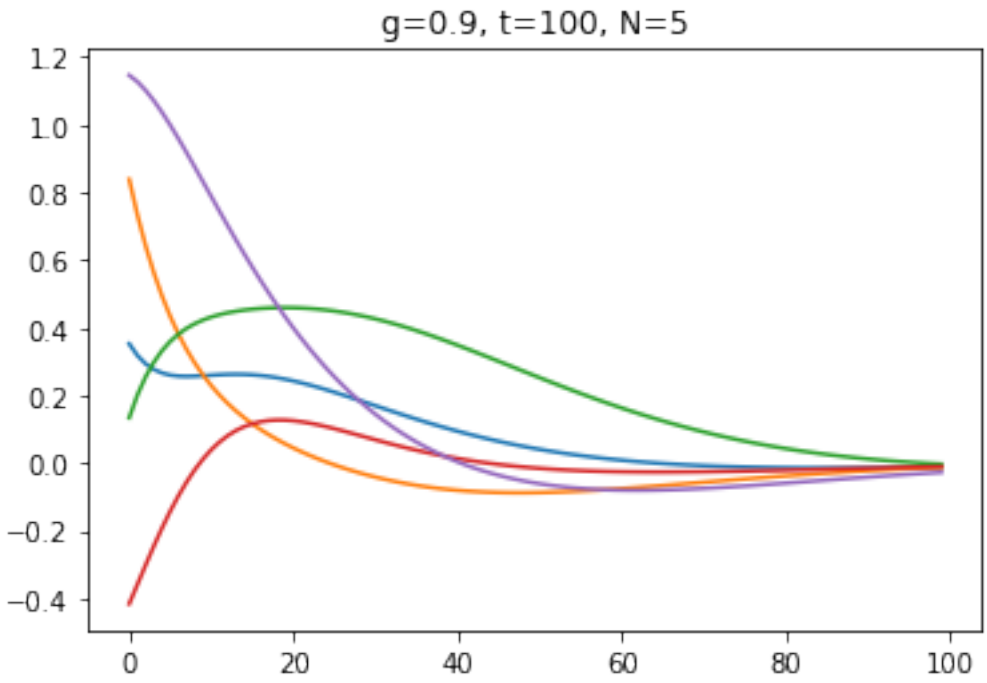
1.1 Question 2

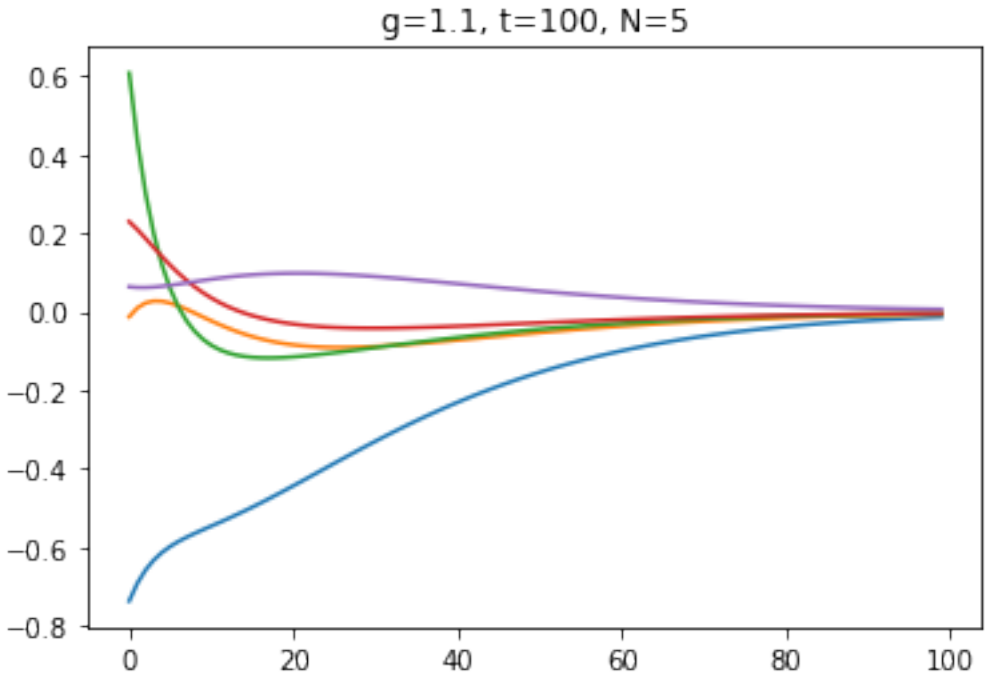
1.1.1 1

In [13]:
```
import numpy as np
import matplotlib.pyplot as plt

# dx(t) = -x(t) + W*f(x(t))

def calc_W(N=500, mean=0, g=0.9):
  W = np.random.normal(mean, g/np.sqrt(N), (N,N))
  return W

def get_evals(M):
  evals, _ = np.linalg.eig(M)
  revals = np.array([e.real for e in evals])
  ievals = np.array([e.imag for e in evals])
  return revals, ievals

W = calc_W(g=1.1)
revals, ievals = get_evals(W-np.identity(W.shape[0]))
plt.scatter(revals,ievals)
W = calc_W(g=0.9)
revals, ievals = get_evals(W-np.identity(W.shape[0]))
plt.scatter(revals,ievals)
plt.title('g=0.9 (orange), g=1.1 (blue)')
plt.show()
```



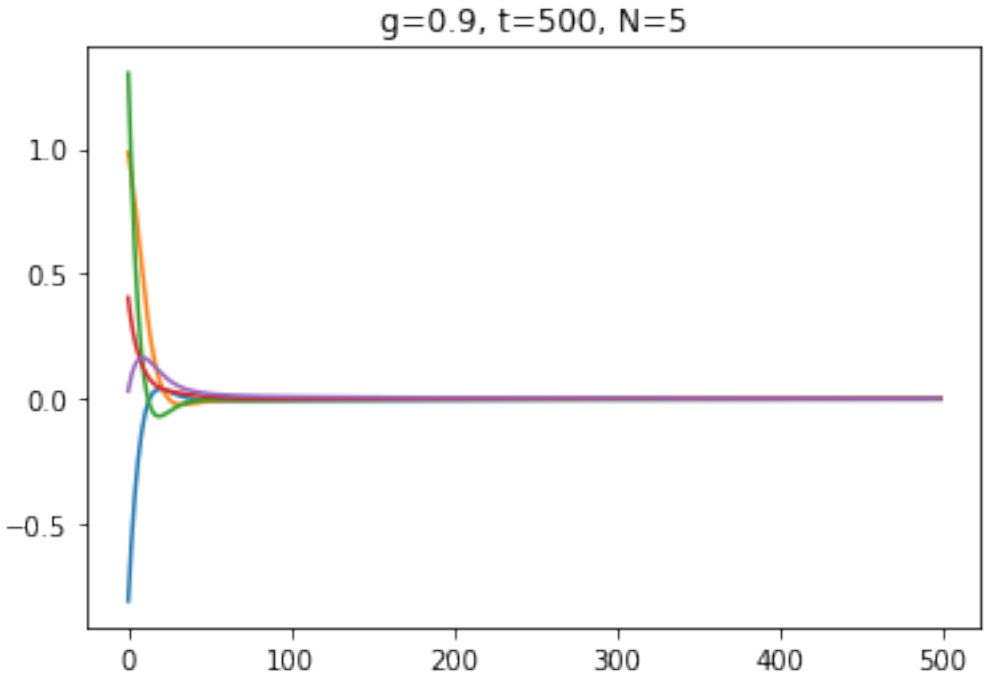

```
In [43]: def simulate_linear_network(g, N=5, time_units=100):
             dt = 1e-1
             M = calc_W(g=g, N=N)-np.identity(N)
             res = np.zeros((time_units, N))
             res[0] = np.random.normal(size=N)
             for i in range(1,time_units):
               dx = M.dot(res[i-1])
               res[i] = res[i-1] + dx*dt
             return res

           plt.plot(simulate_linear_network(0.9))
           plt.title('g=0.9, t=100, N=5')
           plt.show()

           plt.plot(simulate_linear_network(1.1))
           plt.title('g=1.1, t=100, N=5')
           plt.show()
```

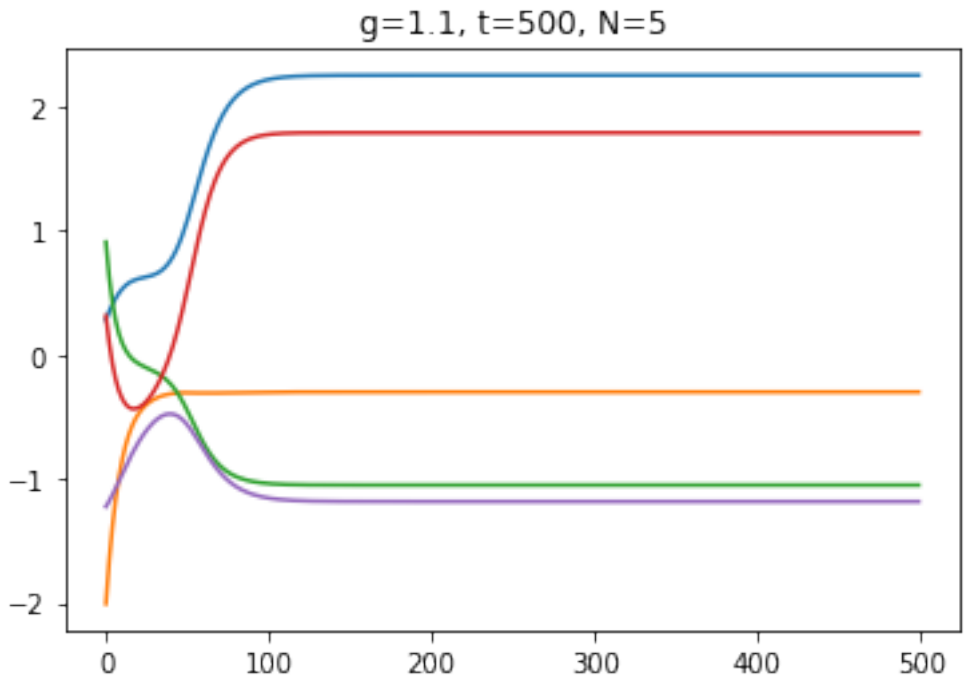


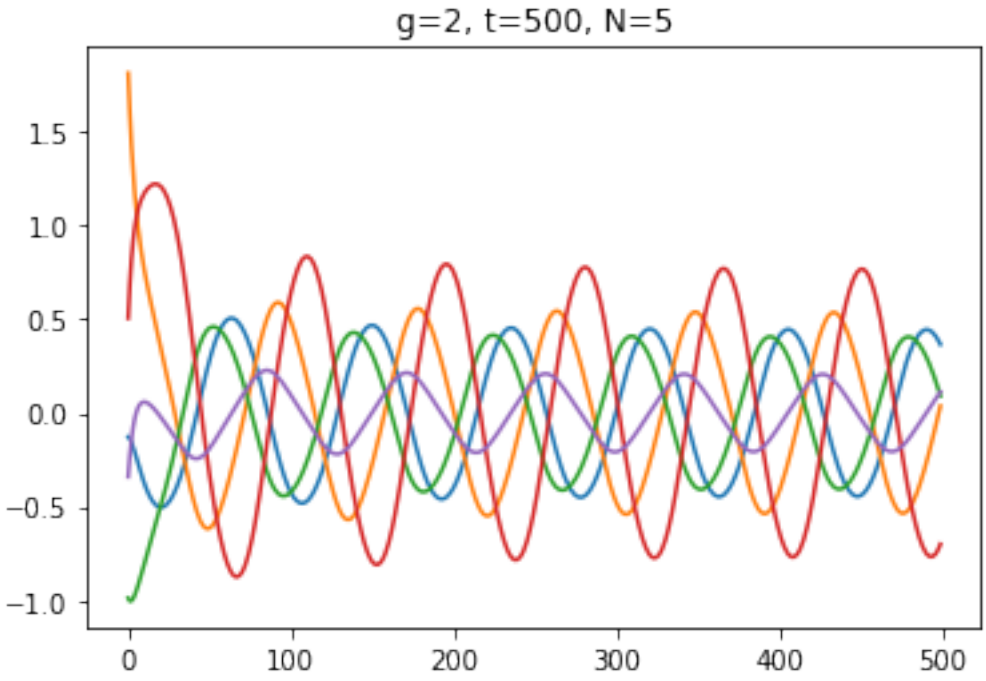



```
In [59]: def simulate_nonlinear_network(g, N=5, time_units=500):
             dt = 1e-1
```

```
  W = calc_W(g=g, N=N)
  res = np.zeros((time_units, N))
  res[0] = np.random.normal(size=N)
  for i in range(1,time_units):
    dx = -res[i-1] + W.dot(np.tanh(res[i-1]))
    res[i] = res[i-1] + dx*dt
  return res

plt.plot(simulate_nonlinear_network(0.9))
plt.title('g=0.9, t=500, N=5')
plt.show()

plt.plot(simulate_nonlinear_network(1.1))
plt.title('g=1.1, t=500, N=5')
plt.show()

plt.plot(simulate_nonlinear_network(2.0, N=5))
plt.title('g=2, t=500, N=5')
plt.show()
```



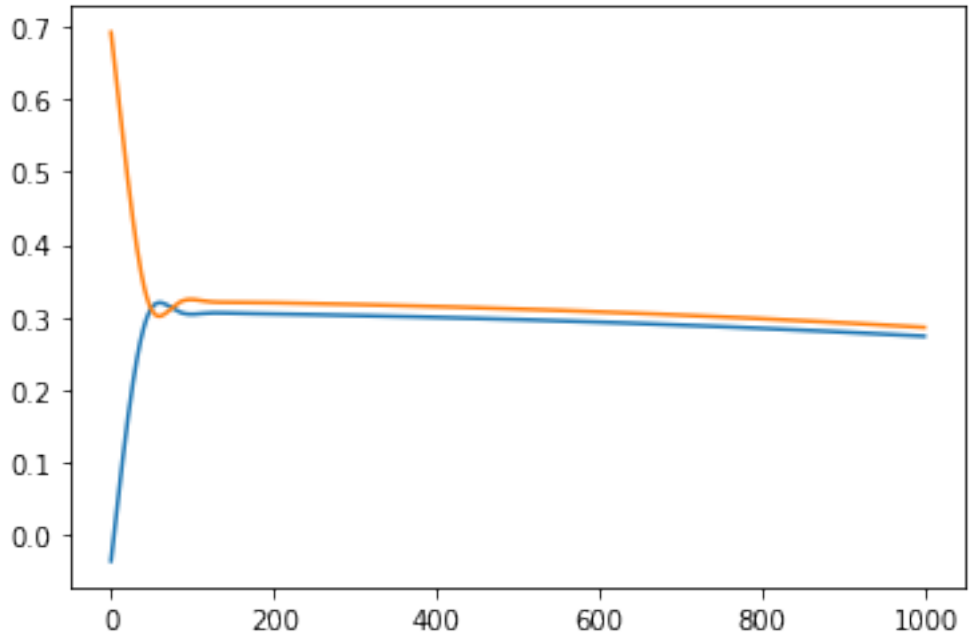




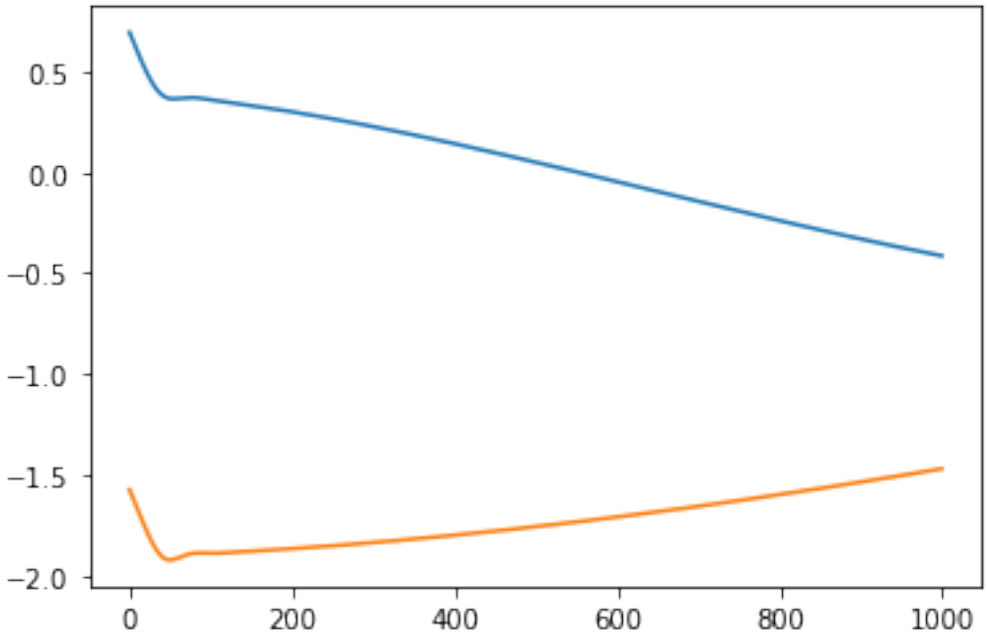

```
In [0]: import torch
        import torch.nn as nn
```

```
        import torch.nn.functional as F
        import torch.optim as optim
        from torch.nn.parameter import Parameter

        class FixedPoint(nn.Module):
          def __init__(self, tau=12.5, a=0.7, b=0.8, I=0):
            super(FixedPoint, self).__init__()
            self.tau = tau
            self.a = a
            self.b = b
            self.I = I
            self.v = Parameter(torch.Tensor(1))
            self.w = Parameter(torch.Tensor(1))
            self.reset_parameters()

          def reset_parameters(self):
            nn.init.uniform_(self.v, a=-2, b=2)
            nn.init.uniform_(self.w, a=-2, b=2)

          def forward(self):
            res = torch.Tensor([0, 0])
            v_term = torch.add(torch.add(torch.add(self.v, torch.div(self.w.pow(3),3.0)),torch
            w_term = torch.div(torch.add(torch.add(self.v,self.a),torch.neg(torch.mul(self.b,s
            return torch.cat([v_term, w_term])

In [0]: def run():
          net = FixedPoint().float()
          optimizer =  optim.Adam(net.parameters(), lr=0.01) # play around with that
          criterion = nn.MSELoss()
          losses = []
          optimal_results = []
          for epoch in range(1000):
            optimizer.zero_grad()   # zero the gradient buffers
            output = net()
            loss = criterion(output, torch.Tensor([0,0]).float())
            losses.append(float(loss))
            loss.backward()
            optimizer.step()
            optimal_results.append([net.v.detach().numpy()[0], net.w.detach().numpy()[0]])
          return losses, optimal_results

In [190]: for i in range(10):
            losses, optimal_results = run()
            print('Run {}, loss {}, fixed point {}'.format(i, losses[-1], optimal_results[-1]))
            plt.plot(np.array(optimal_results))
            plt.show()

Loss: 0.001785819884389639
```

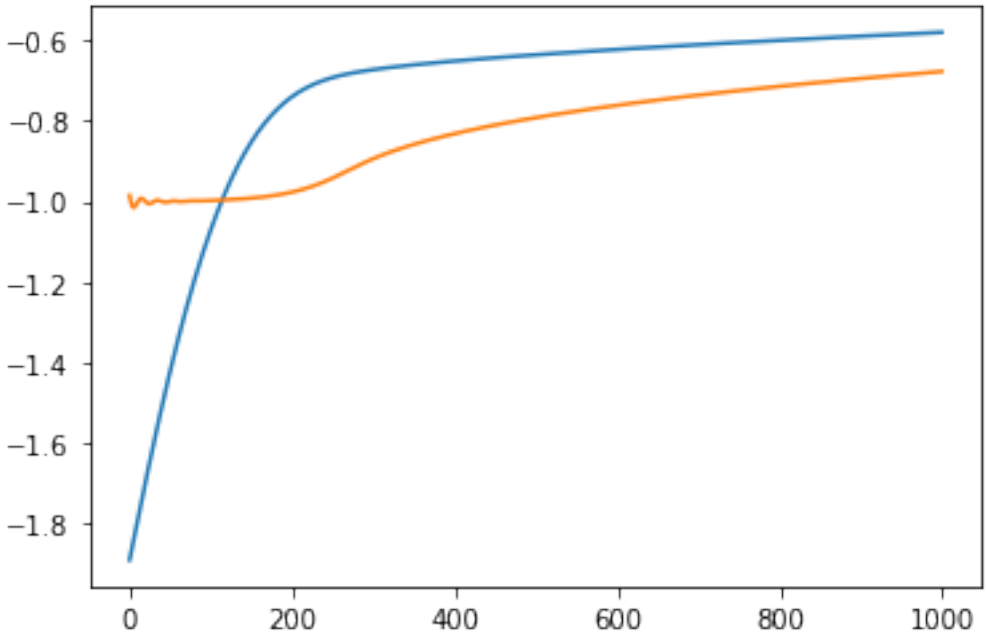
```
Run 0, loss 0.001785819884389639, fixed point [0.27381504, 0.2860338]
```



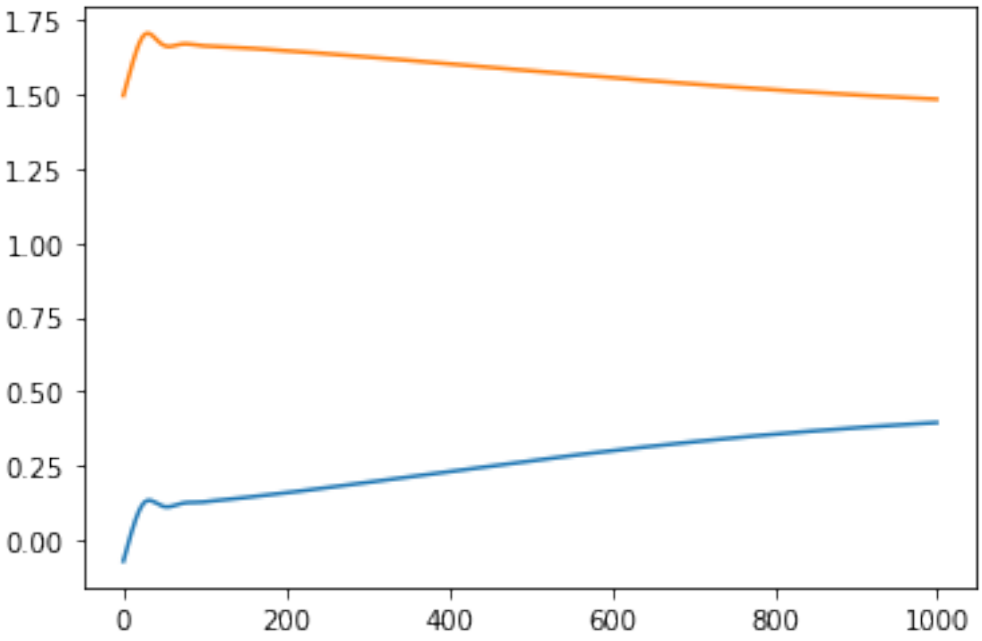

```
Loss: 0.006856880616396666
Run 1, loss 0.006856880616396666, fixed point [-0.41369483, -1.46986]
```



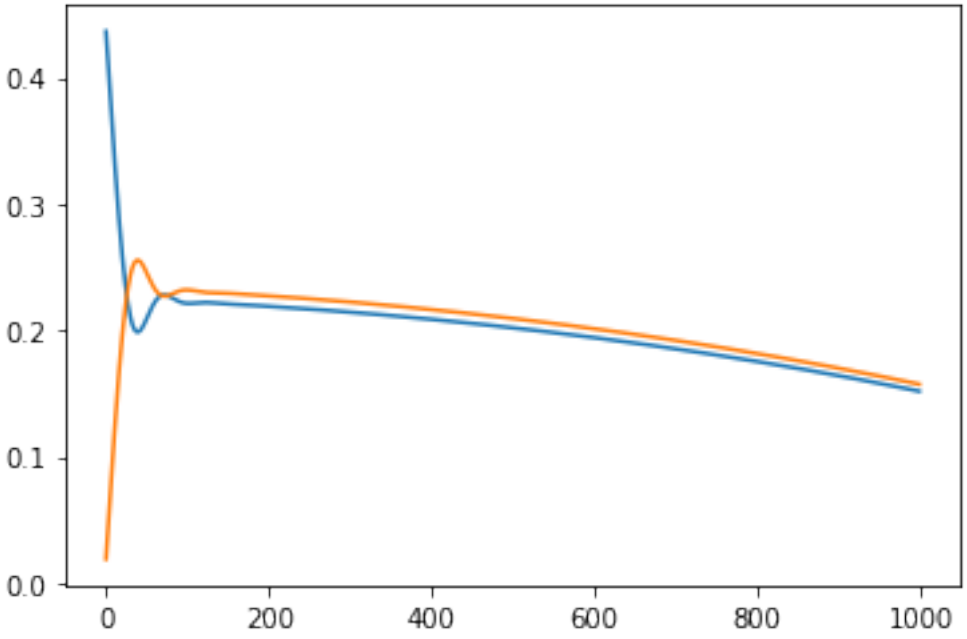
```
Loss: 0.001420895685441792
Run 2, loss 0.001420895685441792, fixed point [-0.5796846, -0.676894]
```



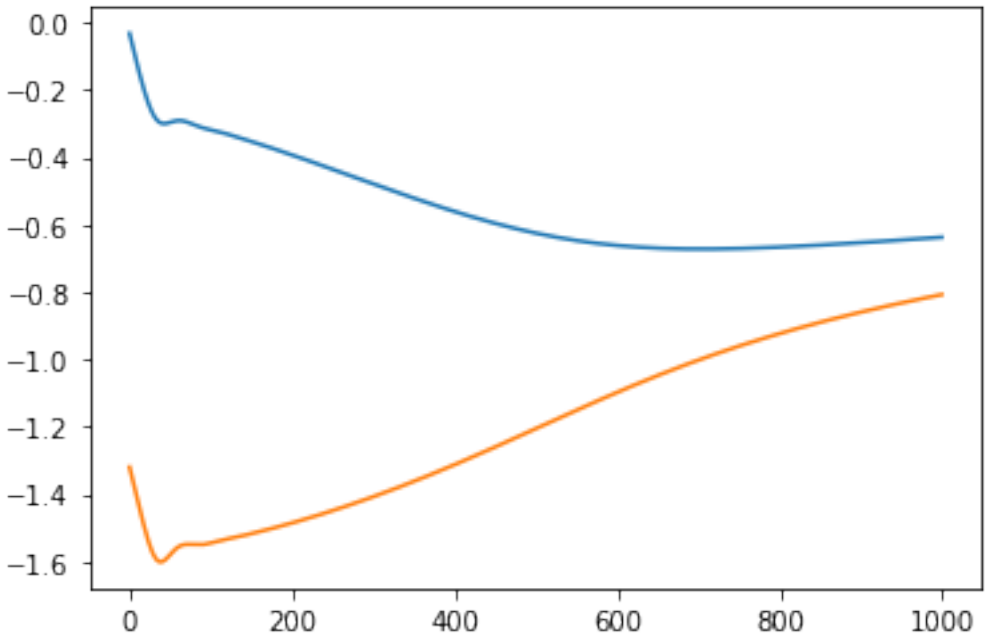

```
Loss: 2.5988047127611935e-05
Run 3, loss 2.5988047127611935e-05, fixed point [0.39629143, 1.4826757]
```



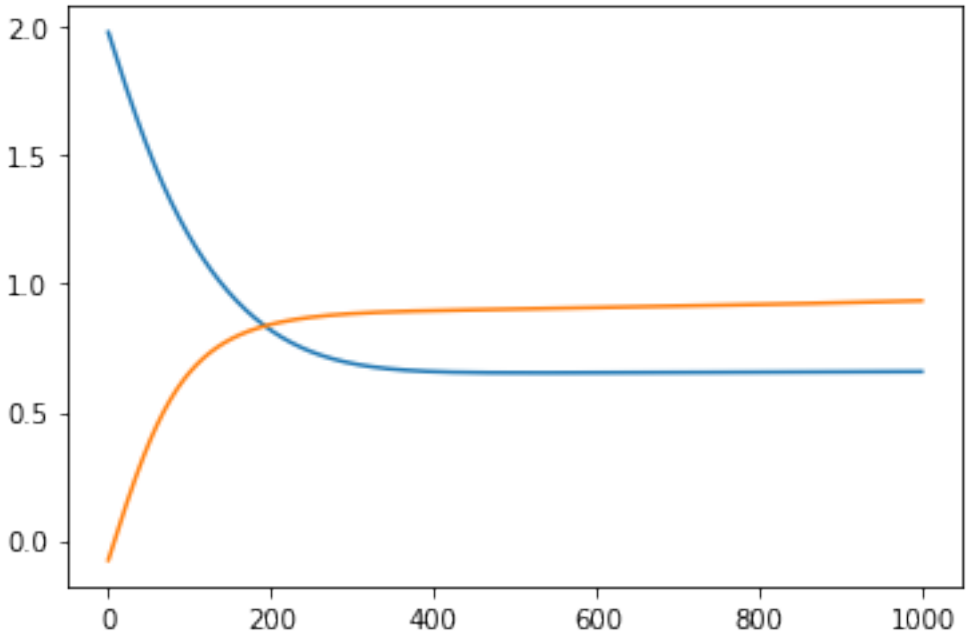

```
Loss: 0.0016953732119873166
Run 4, loss 0.0016953732119873166, fixed point [0.15175432, 0.15728827]
```



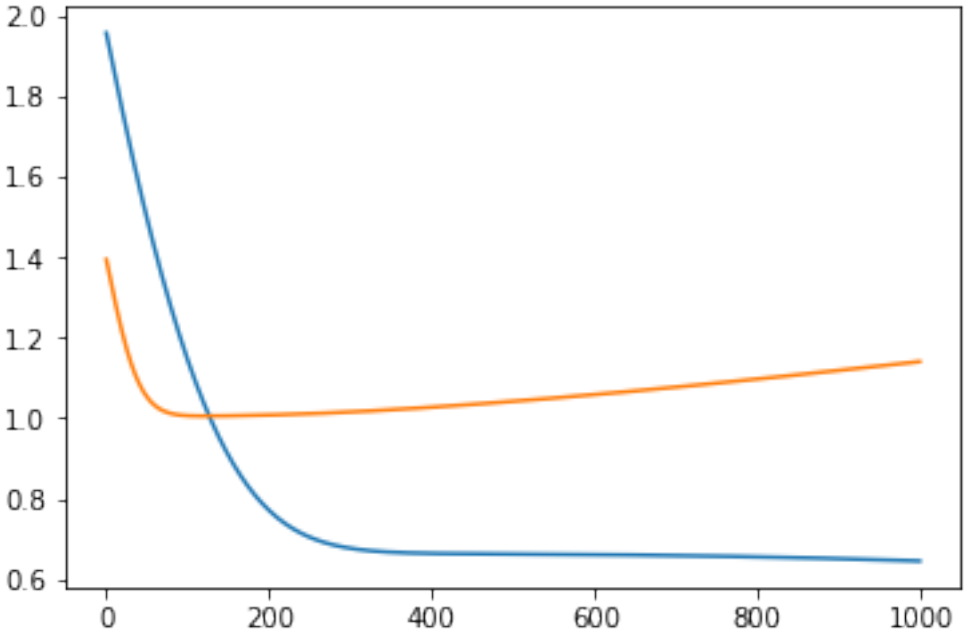
```
Loss: 0.001621963456273079
Run 5, loss 0.001621963456273079, fixed point [-0.63719815, -0.807352]
```



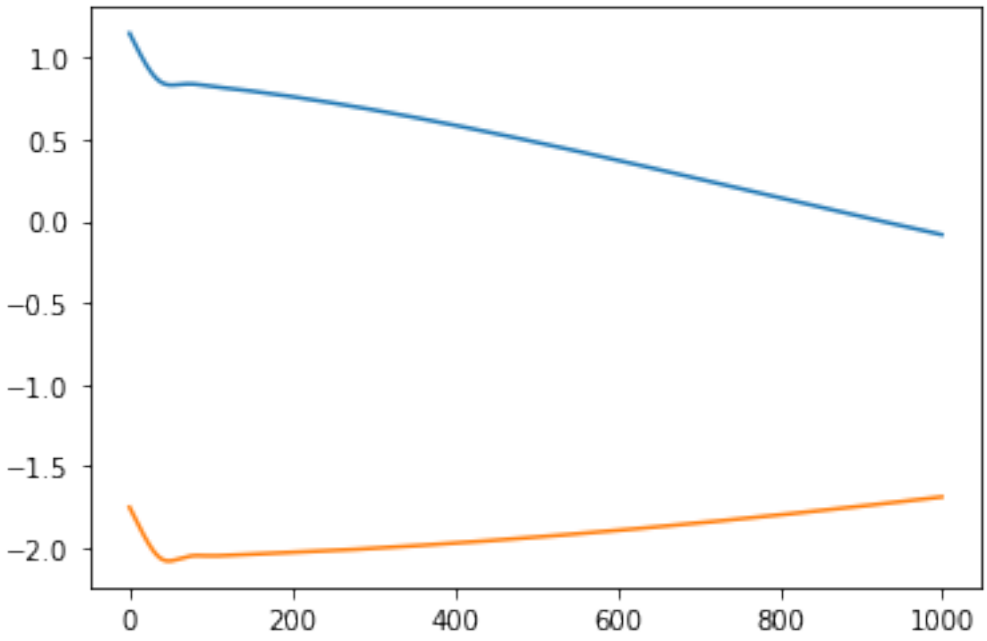

```
Loss: 0.0012021660804748535
Run 6, loss 0.0012021660804748535, fixed point [0.6581857, 0.93459415]
```



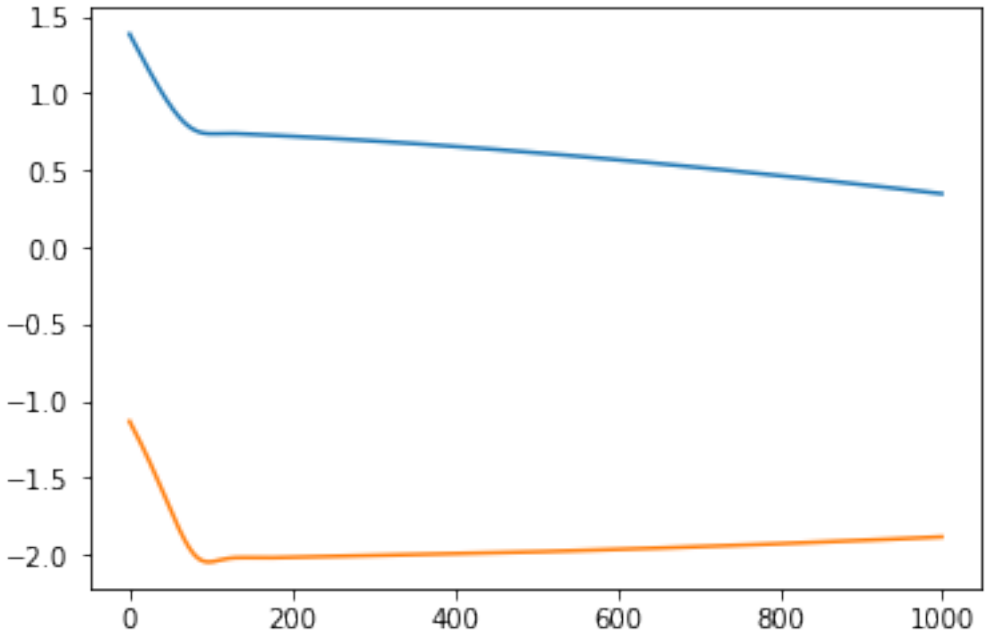
```
Loss: 0.0005991072976030409
Run 7, loss 0.0005991072976030409, fixed point [0.6447493, 1.1407006]
```




```
Loss: 0.012432793155312538
Run 8, loss 0.012432793155312538, fixed point [-0.08264803, -1.6900852]
```



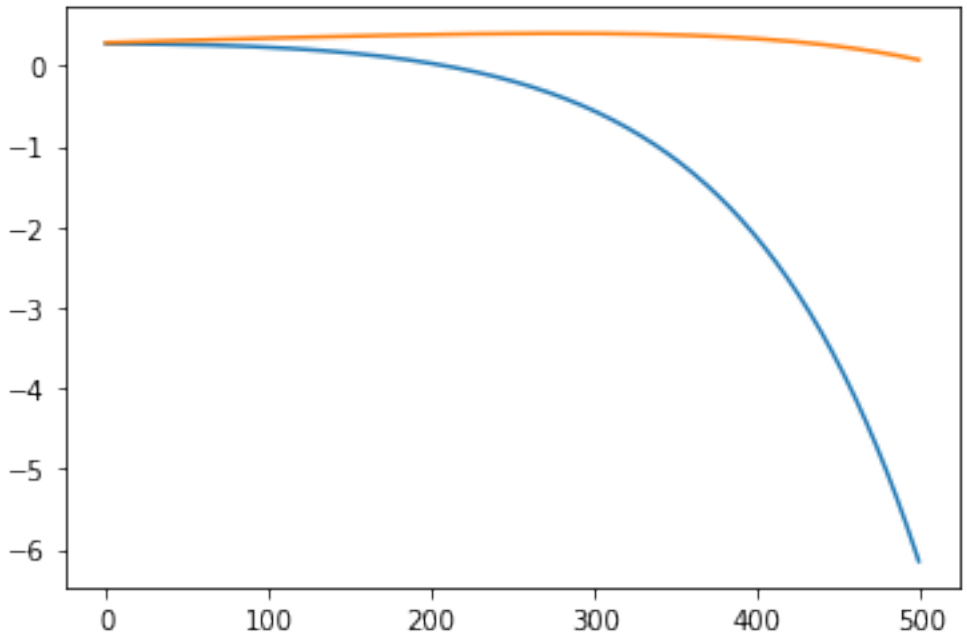

```
Loss: 0.02089967392385006
Run 9, loss 0.02089967392385006, fixed point [0.34739658, -1.8840748]
```



In [206]:
```
def simulate_FHN(init_v, init_w, time_units=500, tau=12.5, a=0.7, b=0.8, I=0):
    dt= 1e-2
    v = np.zeros(time_units)
    w = np.zeros(time_units)
    v[0] = init_v
    w[0] = init_w
    for i in range(1,time_units):
      dv = v[i-1] + w[i-1]**3/3.0 - w[i-1] + I
      dw = (v[i-1] + a - b*w[i-1])/tau
      v[i] = v[i-1] + dv*dt
      w[i] = w[i-1] + dw*dt
    return v,w

v, w = simulate_FHN(0.27381504, 0.2860338)
print(v.shape)
plt.plot(v)
plt.plot(w)
```

```
(500,)
```

Out[206]:
```
[<matplotlib.lines.Line2D at 0x7f954791cc18>]
```



In [0]: