1) Parameters, Labels, Variables

$$I = g(V - E)$$

I: variable being computed

V: input variable

g, E: Parameter

<u>But</u>

$$g = \frac{I}{V - E}$$

g: now variable being computed!

<u>You</u> have to keep track – or ask.

$\boxed{\text{Labels:}}$ V_i or $V(x)$

Often arbitrary: $\int dx\, f(x) \to \int dx'\, f(x')$

Ambiguous: t – all time

t – one time

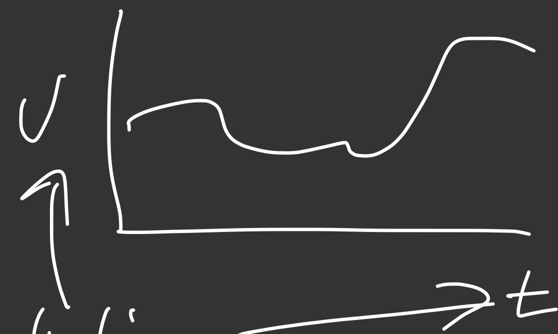
$\boxed{\text{Variables}}$ are also ambiguous

Physical, Holistic, Specific

Physical

Holitic

V = voltage



holistic

Specific

$V(t=1s) = 3$ Volts

You must keep track or ask.

2) More ambiguities

a) $f = ax + b$ (linear)

b) $f(x+y) = f(x) + f(y)$ (linear)

but $(ax+b) + (ay+b) \neq a(x+y) + b$

c) If $f(t)$ & $g(t)$ are solutions

$af(t) + bg(t)$ is solution

$$C\frac{dV}{dt} = -g(V-E) \leftarrow !$$

3) Function as vectors

$$\bar{f} = \frac{1}{n} \sum_{i=1}^{n} f_i$$

$i = 1 \quad \overset{\wedge}{1.5} \quad 2 \quad \overset{\wedge}{2.5} \quad 3 \quad \ldots$

no $\quad \tilde{n} = 2(n-1) + 1$

$$\Delta i = \frac{1}{2} \text{ so } \tilde{n} = \frac{n-1}{\Delta i} + 1$$

$$= \frac{n-1+\Delta i}{\Delta i}$$

and

$$\bar{f} = \frac{\Delta i}{n-1+\Delta i} \text{“}\sum_{i=1}^{n}\text{”} f_i$$

Now let $\Delta i \to 0$

$$\bar{f} = \frac{\Delta i}{n-1} \text{“}\sum_{i=1}^{n}\text{”} f_i \to \frac{1}{n-1} \int_1^n dx\, f(x)$$

continuous label

Dot product

$$\sum_i f_i g_i \rightarrow \int dx\, f(x) g(x)$$

Matrix - Vector Product

$$\sum_j M_{ij} f_j \rightarrow \int dy\, M(x,y) f(y)$$

Toeplitz case

4) Labels in Integrals

$$\int_a^b dx\, f(x) = F(a,b) \text{ } \underline{\underline{\text{not}}} \text{ } x$$

Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_0^x dy\, f(y) = f(x) \qquad \frac{d}{dx}\int_x^{\infty} dy\, f(y) = -f(x)$$

5) $f_i \rightarrow f(x)$ but when we use a computer $f(x) \rightarrow f_i$

A bit archaic!

$\boxed{\text{Derivatives}}$

$x(t) \sim a(t' \leq t)$ - messy expression

- $t' \to -\infty$?

want $\quad x(t) \sim x(t_0)$ and $a(t)$

therefore change - take derivative

no guarantee that it works (it doesn't)

but approximate

$$\left.\frac{dx}{dt}\right|_t \quad \text{or} \quad \frac{dx(t)}{dt} \qquad \text{same ambiguity}$$

if $\left.\frac{dx}{dt}\right|_t \sim a(t' \leq t)$ then $\frac{dx}{dt} = c(t) + y(t)$

where $y(t) \sim a(t' \leq t)$

then

$$\frac{dy}{dt} = g(a) \qquad \text{etc.}$$

$\frac{\Delta x}{\Delta t}$ in limit, just a fraction

$x = c \qquad \frac{dx}{dt} = 0$

$x = at \qquad \Delta x = a\Delta t \qquad \frac{dx}{dt} = a$

$x = at^2 \qquad \Delta x = a(t+\Delta t)^2 - at^2$

$= 2at\Delta t + a\Delta t^2$

$\frac{\Delta x}{\Delta t} \to \frac{dx}{dt} = 2at$

$x = at^n \qquad \frac{dx}{dt} = ant^{n-1}$

$$\frac{d f(g(x))}{dt} = \frac{dg}{dx}\frac{df}{dg} = \frac{\Delta g}{\Delta x}\frac{\Delta f}{\Delta g}$$

Functions:

$$x^a x^b = x^{a+b} \qquad (x^a)^b = x^{ab}$$

so $$(x^a)^{1/a} = x$$

so $$e^x e^y = e^{x+y}, \quad e^{ax} = (e^x)^a$$

$$e^{x+\Delta x} = e^x (e^{\Delta x})$$

$$\frac{\Delta f}{\Delta x} = \frac{e^x (e^{\Delta x} - 1)}{\Delta x} = e^x$$

e defined by $e^{\Delta x} - 1 \to \Delta x$

or $$e^{\Delta x} \to 1 + \Delta x$$

$$e = (1 + \Delta x)^{1/\Delta x}$$

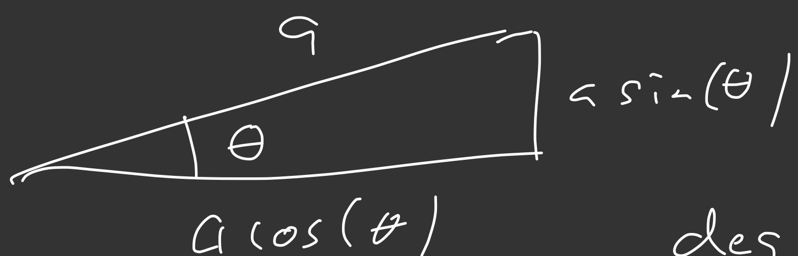
$$\ln(e^x) = x \qquad e^{\ln(x)} = x$$

$$e^{x+y} = e^x e^y$$

$$\ln(xy) = \ln(x) + \ln(y)$$

$$e^{cx} = (e^x)^c$$

$$\ln(x^a) = a\ln(x)$$



$$\text{deg} = \frac{180}{\pi}\theta$$

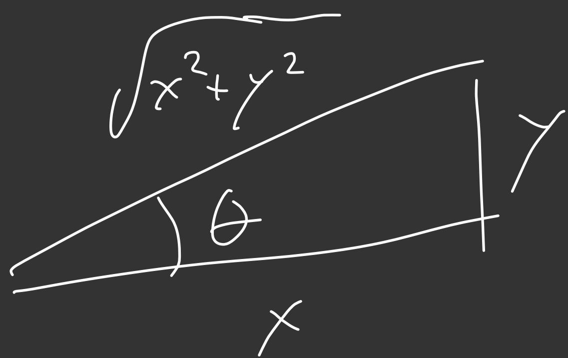
$$\cos(\theta) = \cos(-\theta) \qquad \sin(\theta) = -\sin(-\theta)$$

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$\frac{d\sin(x)}{dx} = \cos(x) \qquad \frac{d\cos(x)}{dx} = -\sin(x)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$



$$\theta = \arctan(y/x)$$
$$= \operatorname{atan2}(y, x)$$

$$a = R\cos(\phi) \qquad b = R\sin(\phi)$$

with $R = \sqrt{a^2+b^2}$

$$a\cos(\theta) + b\sin(\theta) =$$

$$R\left(\cos(\phi)\cos(\theta) + b\sin(\phi)\sin(\theta)\right)$$

$$= R\cos(\theta - \phi)$$

$$e^{ix} = \cos(x) + i\sin(x)$$

$$e^{ix}\,e^{iy} = e^{i(x+y)}$$

$$\cos(x+y) + i\sin(x+y) = \big(\cos(x) + i\sin(x)\big)$$

$$\big(\cos(y) + i\sin(y)\big)$$

$$= \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$+ i\big(\sin(x)\cos(y) + \cos(x)\sin(y)\big)$$

$$\frac{d}{dx} e^{ix} = i e^{ix} = i\cos(x) - \sin(x)$$

Taylor Series

$$f(x) \approx f(x_0) + a_1(x - x_0)$$

$$+ a_2(x - x_0)^2 +$$

$$a_3(x - x_0)^3 + \cdots$$

$$\frac{df}{dx} = a_1 + 2a_2(x-x_0) + 3a_3(x-x_0)^2 + \cdots$$

$$\left.\frac{df}{dx}\right|_{x=x_0} = a_1$$

$$\frac{d^2f}{dx^2} = 2a_2 + 6a_3(x-x_0) + \cdots$$

$$\left.\frac{d^2f}{dx^2}\right|_{x=x_0} = 2a_2 \qquad \left.\frac{d^3f}{dx^3}\right|_{x=x_0} = 6a_3$$

$$f(x) = f(x_0) + \left.\frac{df}{dx}\right|_{x_0}(x-x_0) + \frac{1}{2}\left.\frac{d^2f}{dx^2}\right|_{x_0}(x-x_0)^2 + \frac{1}{6}\left.\frac{d^3f}{dx^3}\right|_{x_0}(x-x_0)^3 + \cdots + \frac{1}{n!}\left.\frac{d^nf}{dx^n}\right|_{x_0}(x-x_0)^n$$

1st order

$$e^x = 1 + x$$

$$\ln(1+x) = x$$

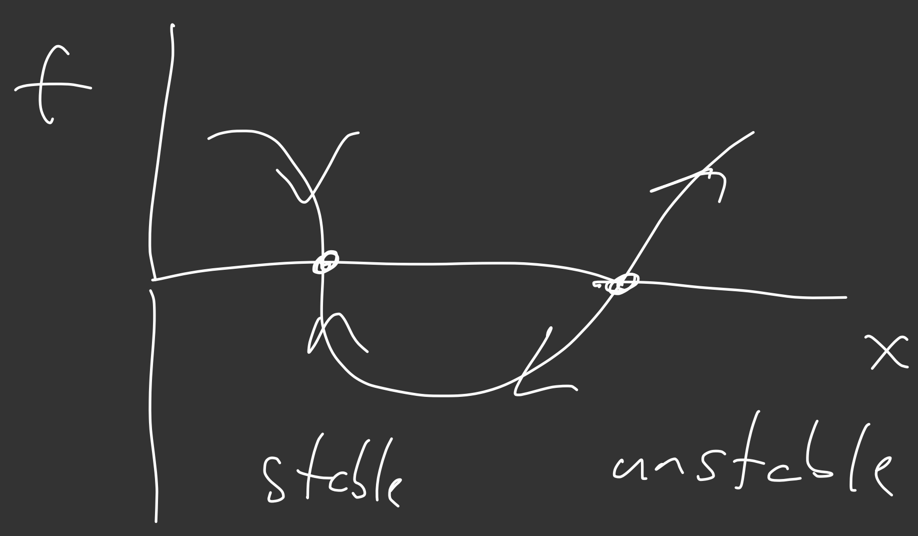
follows

$$\frac{1}{1+x} = 1 - x$$

$\boxed{\text{Differential Equations}}$

$$\frac{dx}{dt} = f(x)$$

$f(\bar{x}) = 0$ is fixed pt



linear 1st order

$$\frac{dx}{dt} = ax \qquad x = x(t) \text{ unknown}$$

so can't integral

but separation of variables

$$\frac{dx}{x} = a\,dt \quad \underline{\text{now}} \text{ integrate}$$

$$\int_{x_0}^{x(t)} \frac{dx'}{x'} = a\int_{t_0}^{t} dt' = a(t-t_0)$$

$$\ln\left(\frac{x(t)}{x_0}\right) = a(t-t_0)$$

$$x(t) = x_0 e^{a(t-t_0)}$$

$a < 0$ stable!

$$a = a_{Re} + i a_{Im}$$

$$x(t) = X_0 e^{a_{Re} t}\left(\cos(a_{Im} t) + i \sin(a_{Im} t)\right)$$

$a_{Re} < 0$ stable oscillation

a better form:

$$\frac{1}{a}\frac{dx}{dt} = -x = \tau \frac{dx}{dt}$$

then
$$x(t) = x_0 e^{-t/\tau}$$

Now

$$\tau \frac{dx}{dt} = -x + b$$

$\bar{x} = b$ f.p. $\quad x' = x - \bar{x}$

$$= x - b$$

$$\tau \frac{dx'}{dt} = -x'$$

$$x(t) = (x_0 - b)e^{-\frac{(t-t_0)}{\tau}} + b$$

$$b = x_\infty$$

$$\tau \frac{dx}{dt} = -x + b(t) \qquad t_0 = 0$$

$$x = \alpha e^{-t/\tau} + x_p(t)$$

$$\tau \frac{dx_p}{dt} = -x_p + b(t)$$

$$x_p(t) = e^{-t/\tau} y(t)$$

$$\tau \frac{dy}{dt} = e^{t/\tau} b(t)$$

$$y(t) = y(0) + \frac{1}{\tau}\int_0^t dt'\, e^{t'/\tau}\, b(t')$$

$$x_p(t) = x_0 e^{-t/\tau} + \frac{e^{-t/\tau}}{\tau}\int_0^t dt'\, e^{t'/\tau}\, b(t')$$

$$= x_0 e^{-t/\tau} + \int_0^t dt'\, K(t-t')\, b(t')$$

$$K(t-t') = \frac{e^{-(t-t')/\tau}}{\tau}$$

Convolutions + Filters

Dimensionless Variables

Taking limits - thinking physically

Stable / Unstable