Bayesian Approaches To Mapping
Tutorial SLAM Summer School

Sebastian Thrun
Carnegie Mellon University
http://www.cs.cmu.edu/~thrun

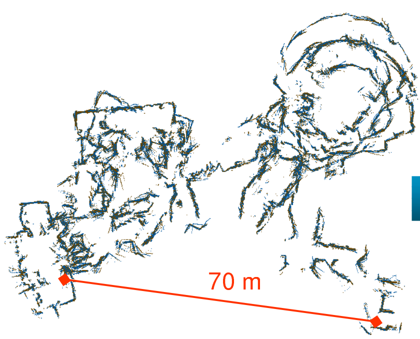
Vocabulary Quiz

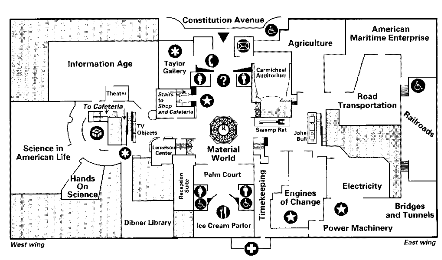
- MCL = Monte Carlo Localization
- SLAM = Simultaneous Localization and Mapping
- EKF = Extended Kalman Filter
- EIF = Extended Information Filter
- ML = Maximum Likelihood
- EM = Expectation Maximization
- BIC = Bayesian Information Criterion
- MCMC = Markov Chain Monte Carlo

Tutorial Outline: Mapping

- The Mapping Problem
- Bayesian Solutions
  - Classical Solution: Kalman Filters
  - Efficient Solution: Sparse Information Filters
  - Nonlinear Solution: Particle Filters (FastSLAM)
- Mapping with Known Poses
  - Occupancy Grid Maps
  - Object Maps
- Summary

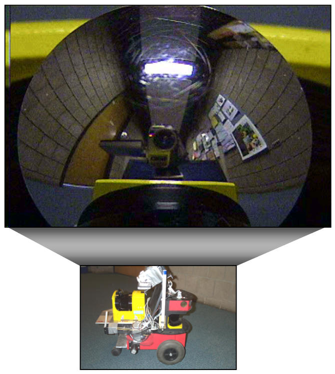
Mobile Robot Mapping

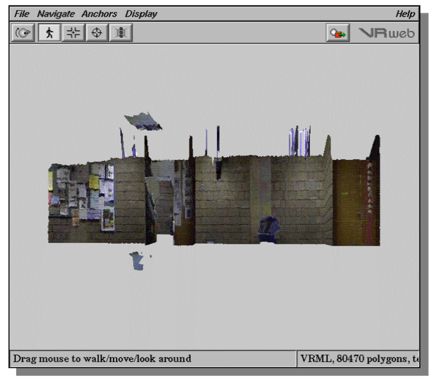




Mobile Robot Mapping

aka Simultaneous Localization and Mapping (SLAM)



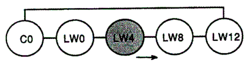


The Mapping Problem

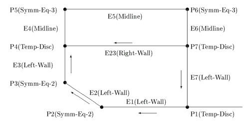
- High dimensional (10^6+)
- Unknown number of dimensions
- Data association problems
- Real-time requirements (sometimes)

…just HORRIBLE!

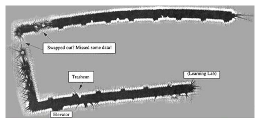
Mapping in AI / Computer Vision



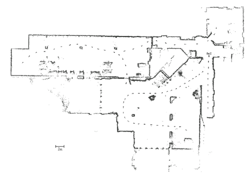
Mataric 1990



Kuipers et al 1991



Elfes/Moravec 1986



Lu/Milios/Gutmann 1997

Konolige et al, 2001

Teller et al, 2000

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- The Mapping Problem
- Bayesian Solutions
  - Classical Solution: Kalman Filters
  - Efficient Solution: Sparse Information Filters
  - Nonlinear Solution: Particle Filters (FastSLAM)
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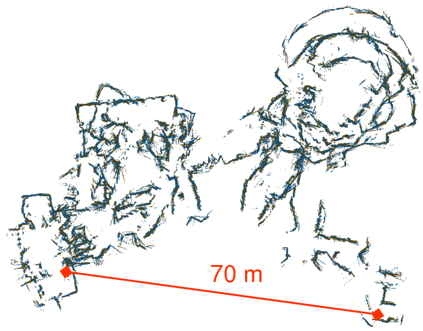
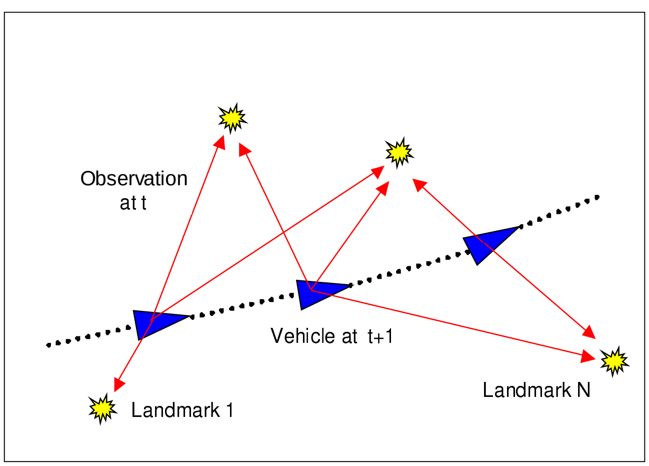
Key Insight:



- Mapping is a Chicken-and-egg problem
  - Mapping with unknown locations
  - Localization with unknown map
- Simultaneous Localization and Mapping (SLAM)

Key Insight: Errors are Correlated



[Smith, Self, Cheeseman, 1990]

Probabilistic Mapping

Localization

$$p(x_t \mid z_{1..t}, u_{1..t}) = \eta\, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1})\, p(x_{t-1} \mid z_{1..t-1}, u_{1..t-1})\, dx_{t-1}$$

$$p(m_t, s_t \mid z_{1..t}, u_{1..t}) = \eta\, p(z_t \mid m_t, s_t) \iint p(m_t, s_t \mid u_t, m_{t-1}, s_{t-1})\, p(m_{t-1}, s_{t-1} \mid z_{1..t-1}, u_{1..t-1})\, dm_{t-1}\, ds_t$$

Mapping

$$p(m, s_t \mid z_{1..t}, u_{1..t}) = \eta\, p(z_t \mid m, s_t) \int p(s_t \mid u_t, s_{t-1})\, p(m, s_{t-1} \mid z_{1..t-1}, u_{1..t-1})\, ds_t$$

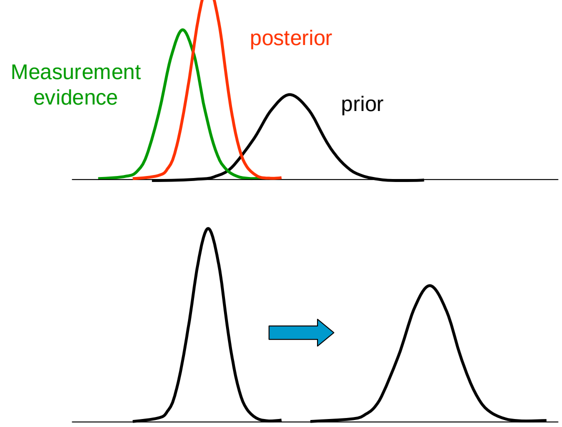
Kalman Filters

$$p(m, x_t \mid z_{1..t}, u_{1..t}) = \left\langle \underset{\mu}{\begin{pmatrix} l_1 \\ l_2 \\ \vdots \\ l_N \\ x \\ y \\ \theta \end{pmatrix}},\ \underset{\Sigma}{\begin{pmatrix} \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} & \sigma_{l_1 x} & \sigma_{l_1 y} & \sigma_{l_1 \theta} \\ \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} & \sigma_{l_2 x} & \sigma_{l_2 y} & \sigma_{l_2 \theta} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 & \sigma_{l_N x} & \sigma_{l_N y} & \sigma_{l_N \theta} \\ \sigma_{l_1 x} & \sigma_{l_2 x} & \cdots & \sigma_{l_N x} & \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} \\ \sigma_{l_1 y} & \sigma_{l_2 y} & \cdots & \sigma_{l_N y} & \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} \\ \sigma_{l_1 \theta} & \sigma_{l_2 \theta} & \cdots & \sigma_{l_N \theta} & \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 \end{pmatrix}} \right\rangle$$

[Smith, Self, Cheeseman, 1990]

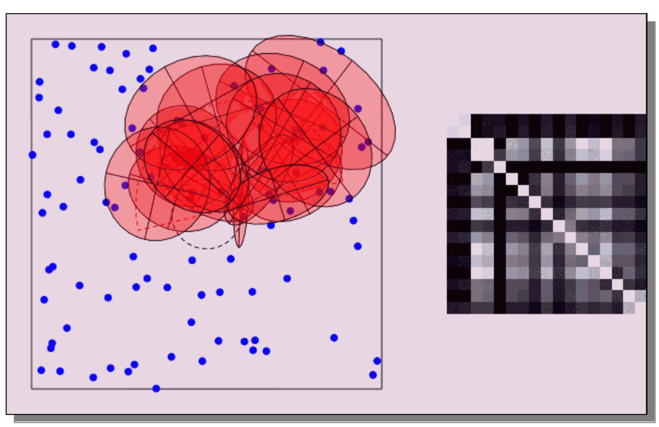
Kalman Filters

$$p(x_t | z_{1..t}, u_{1..t}) = \eta\, p(z_t | x_t)\, p(x_t | z_{1..t-1}, u_{1..t})$$



$$p(x_t | z_{1..t-1}, u_{1..t}) = \int p(x_t | u_t, x_{t-1})\, p(x_{t-1} | z_{1..t-1}, u_{1..t-1})$$

Kalman Filter Mapping



Kalman Filters

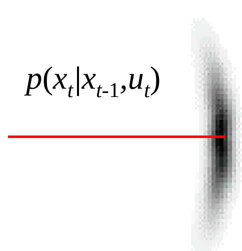
$$p(x_t \mid z_{1..t}, u_{1..t}) \;=\; \eta\, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1})\, p(x_{t-1} \mid z_{1..t-1}, u_{1..t-1})\, dx_{t-1}$$

$$\begin{aligned}
\mu_t &= A\mu_{t-1} + Bu_t + K(z_t - H(A\mu_{t-1} + Bu_t)) \\
\Sigma_t &= (I - K)(A\Sigma_{t-1}A^T + \Sigma_x) \\
K &= (A\Sigma_{t-1}A^T + \Sigma_x)H^T(H(A\Sigma_{t-1}A^T + \Sigma_x) + \Sigma_z)^{-1}
\end{aligned}$$

Extended Kalman Filters (EKFs)

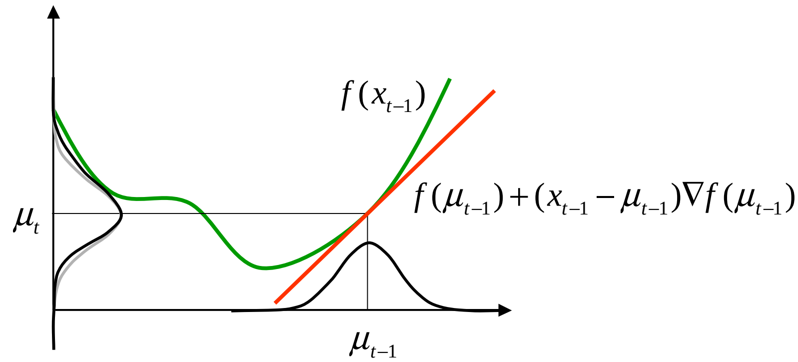
- Assumption:
  - All distributions Gaussian, all models linear

$p(x_t|x_{t-1},u_t)$

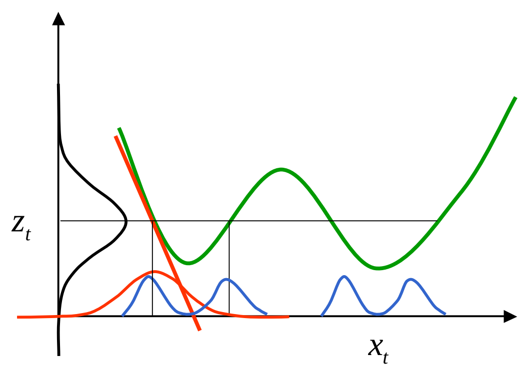


$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = f\left[\begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix}, \begin{pmatrix} v_t \\ \omega_t \end{pmatrix}, \begin{pmatrix} \varepsilon_t \\ \delta_t \end{pmatrix}\right] = \begin{pmatrix} x_{t-1} + (v_t + \varepsilon_t)\Delta t \cos\theta_t \\ y_{t-1} + (v_t + \varepsilon_t)\Delta t \sin\theta_t \\ \theta_t + \Delta t(\omega_t + \delta_t) \end{pmatrix}$$

Linearization via Taylor Expansion



The Fallacy of Linearization



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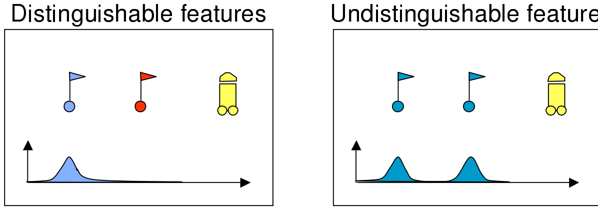
The Key Assumption

- Inverse sensor model $p(x_t|o_t,m)$ must be Gaussian.
- Main problem: Data association

Distinguishable features　　Undistinguishable features

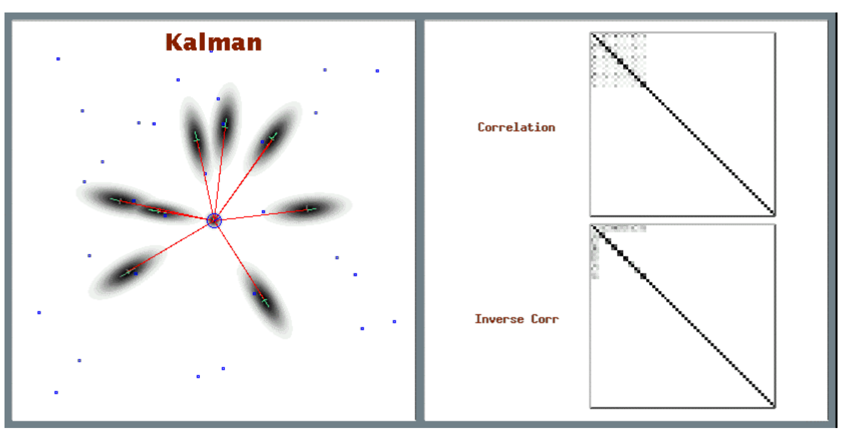


Posterior uni-modal J　　Posterior multi-modal L

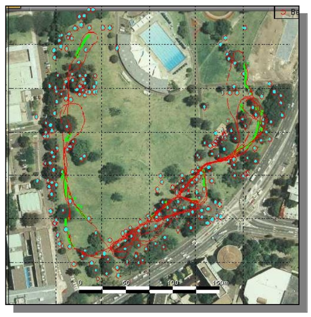
§ In practice:

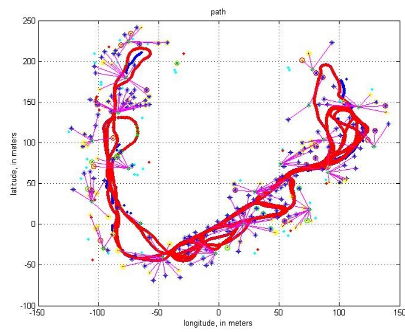
- Extract small set of highly distinguishable features from sensor data
- Discard all other data
- If ambiguous, take best guess for landmark identity

Extended Kalman Filters



Outdoor Mapping

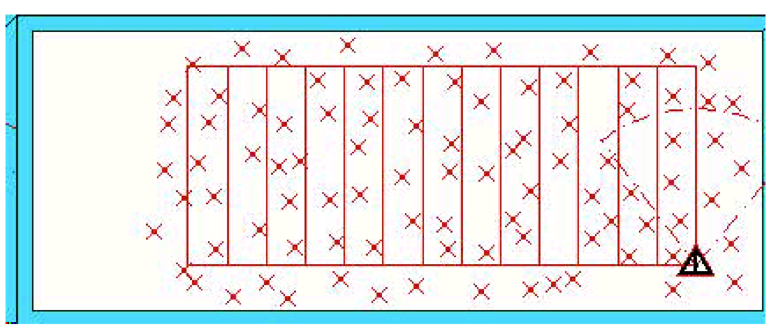




With: Juan Nieto, Jose Guivant, Eduardo Nebot

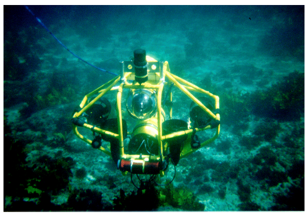
Kalman Filter Mapping

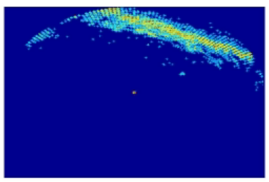
Courtesy of John Leonard, MIT

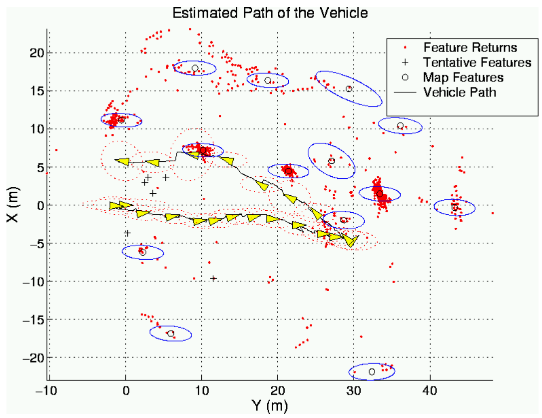


Underwater Mapping

Courtesy of Hugh Durrant-Whyte, Univ of Sydney







EKF Solution

- Advantage
  - Full Bayesian solution
  - Maintains all dependencies
- Disadvantage
  - Does not address data association problem
  - Linear(ized)
  - Slow: $O(N^2)$

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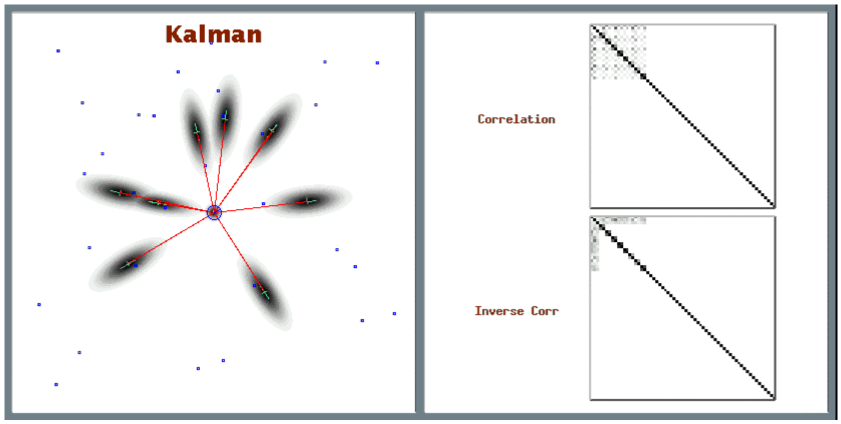
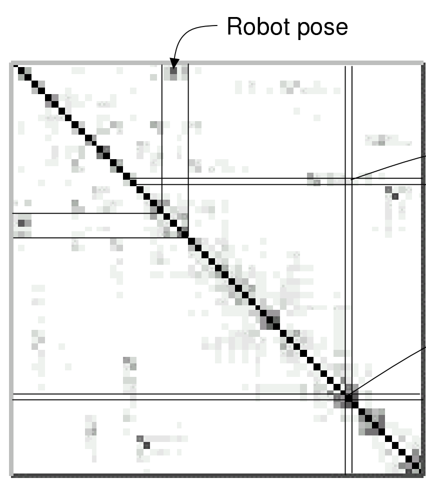
Mapping with SEIFs
Sparse Extended Information Filters

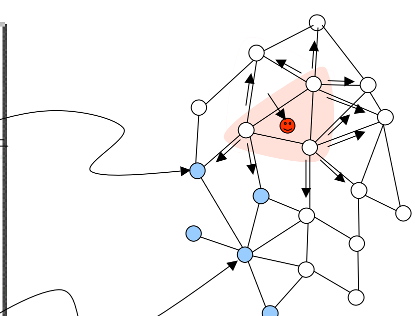
$$O(N^2) \quad \rightarrow \quad \sim O(1)$$

Extended Kalman Filters



Insight #2: Approximate Cond'l Independence

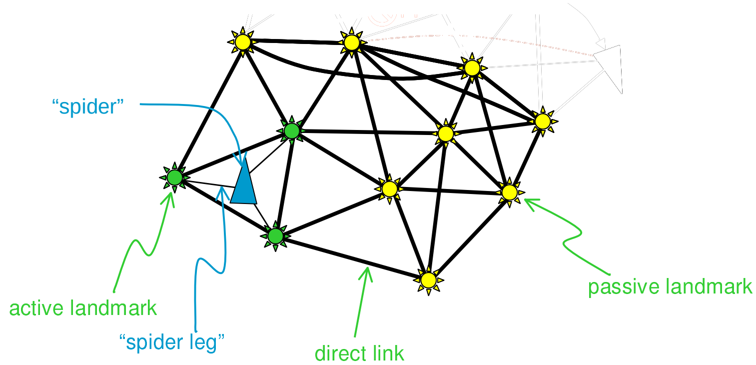




Information matrix $H = \Sigma^{-1}$ (normalized)

SLAM with Sparse Extended Information Filters

The Web of Landmarks



Basic Representation

$$\Sigma, \mu \quad \rightarrow \quad \Sigma^{-1},\ \mu^T \Sigma^{-1}$$

Extended Kalman filter Extended Information filter

See also [Newman, Durrant-Whyte, Nettleton, Tardos…]

Sparse Extended Information Filters

- Representation: Extended information filter
  - Information matrix $H = \Sigma^{-1}$
  - Information state $b = \mu^T H$
- Measurement update

  $b = b + (z - z^* + \mu C)^T Z^{-1} C$ $O(1)$

  $H = H + (C^T Z^{-1} C)$
- Motion update

  $H = H - L$ $\cancel{O(N^2)}$

  $b = b - bL + (H X_s A u)^T$ $O(1)$

  $L = X_\mu (B^{-1} + X_s{}^T H X_s)^{-1} X_\mu{}^T H$
- Enforcing Sparseness

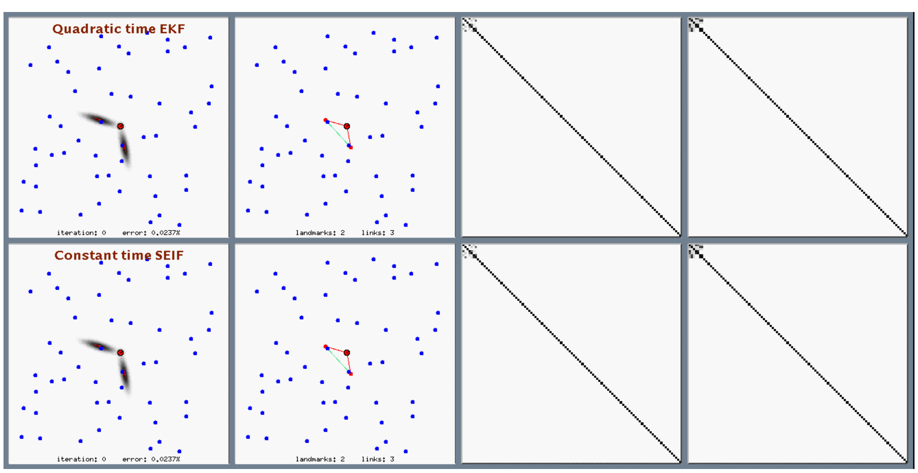
  $H = H - X_{\mu+\text{active}}{}^T H X_{\mu+\text{active}} M$

  $b = b - X_{\mu+\text{active}} M$ $O(1)$

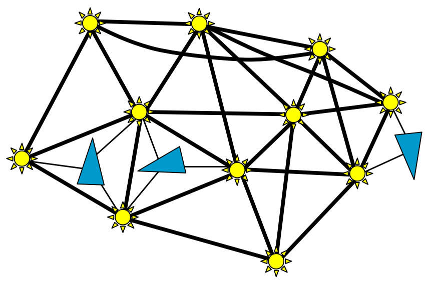
  $M = [\, X_\mu (X_\mu{}^T H X_\mu)^{-1} X_s{}^T + X_{\text{deact}} (X_{\text{deact}}{}^T H X_{\text{deact}})^{-1} X_{\text{deact}}{}^T - X_{\text{active}} (X_{\text{active}}{}^T H X_{\text{active}})^{-1} X_{\text{active}}{}^T \,] X_{\mu+\text{active}}{}^T H X_{\mu+\text{active}}$
- State Recovery (iterative coordinate ascent)

  $\mu_i = H_{i,i}^{-1} [\, \Sigma_{j \neq i} H_{i,j} \mu_j - b_i{}^T]$ $O(1)$

Sparse Extended Information Filters



Implications for Multi-Vehicle SLAM



- Constant message sizes
- Asynchronous

[see a very new paper by Nettleton et al]

ANSER System
University of Sydney

Autonomous Navigation and Sensing Experimental Research.

- 20kg Payload Capacity
- 45min flight duration
- 100 knots cruising speed
- Manual take-off/landing

- Flight Control Computer
- GPS/IMU Filter
- Vision System
- Radar System

ANSER System
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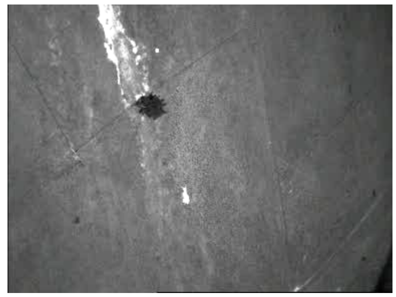
Autonomous Navigation and Sensing Experimental Research.



- 20kg Payload Capacity
- 45min flight duration
- 100 knots cruising speed
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- Flight Control Computer
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- Radar System

5

SEIF Solution

- Advantage
  - Full Bayesian solution
  - Maintains all dependencies
- Disadvantage
  - Does not address data association problem
  - Linear(ized)
  - ~~Slow: $O(N^2)$~~

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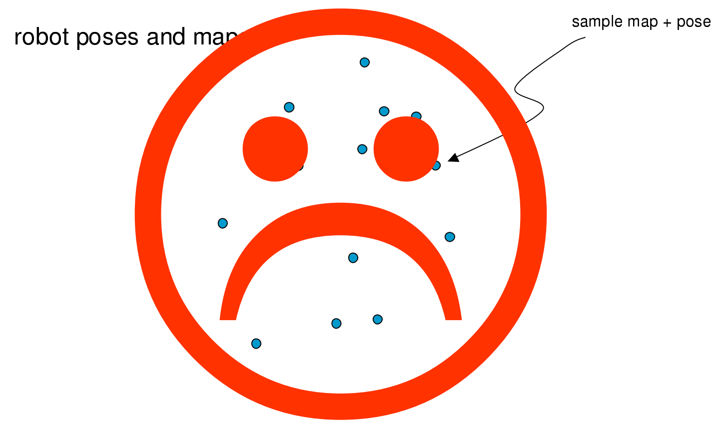
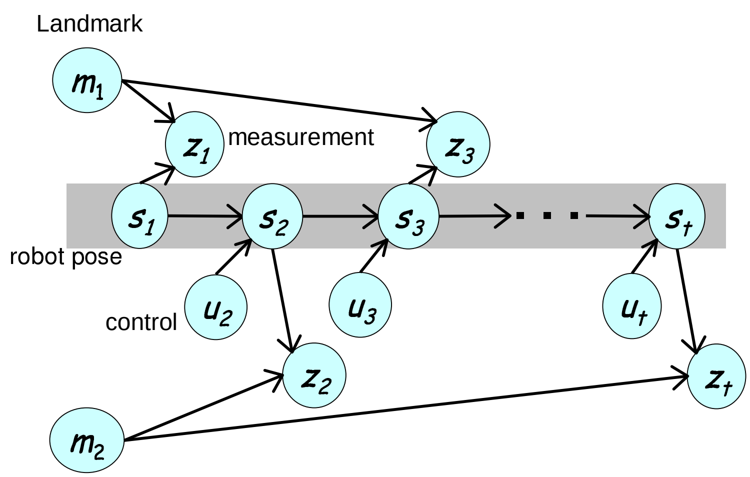
Can We Map With Particle Filters?



$$p(x_t, m \mid z_{0..t}, u_{0..t})$$

Mapping: Structured Generative Model



$$p(m, s_{0...t} \mid z_{0...t}, u_{0...t}) = p(s_{0...t} \mid z_{0...t}, u_{0...t}) \prod_{n=1}^{N} p(m_n \mid s_{0...t}, z_{0...t}, u_{0...t})$$

With K. Murphy, B. Wegbreit and D. Koller

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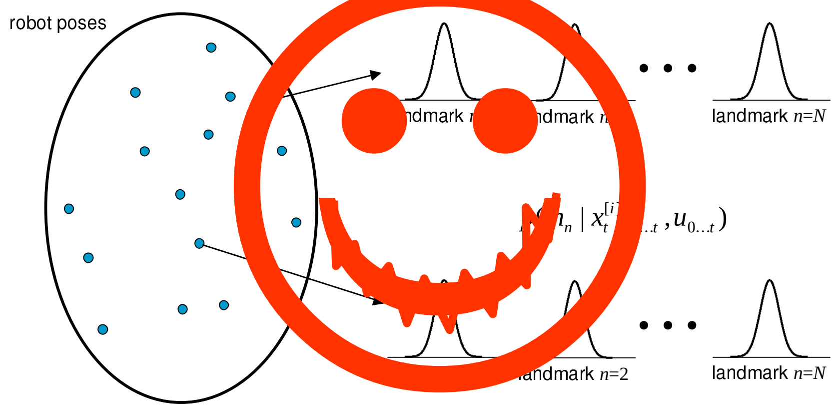
Rao-Blackwellized Particle Filters

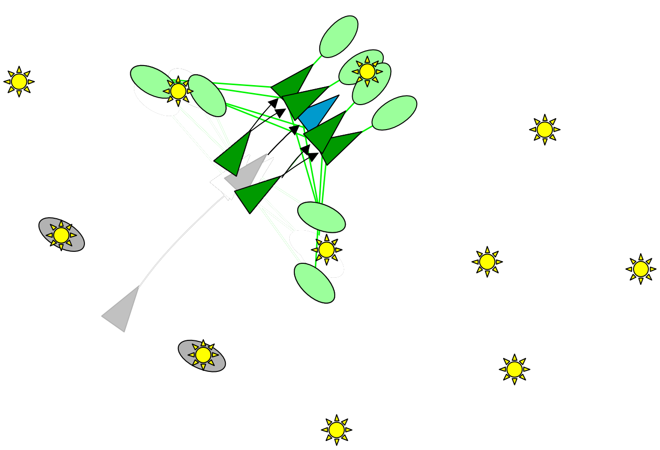
$$p(m, s_{0...t} \mid z_{0...t}, u_{0...t}) \;=\; p(s_{0...t} \mid z_{0...t}, u_{0...t}) \prod_{n=1}^{N} p(m_n \mid s_{0...t}, z_{0...t}, u_{0...t})$$



$$x_t^{[i]} \;\sim\; p(x_t \mid z_{0...t}, u_{0...t})$$

[Murphy 99, Montemerlo 02]

The FastSLAM Algorithm



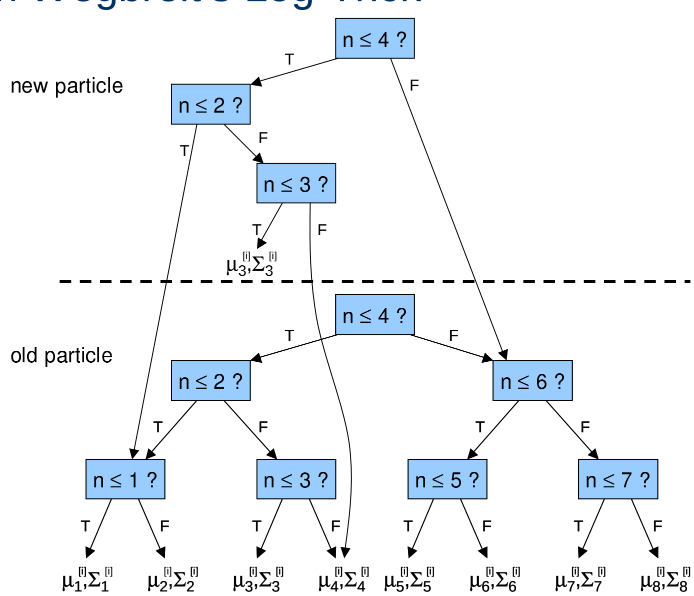
$$p(m, s^t \mid z^t, u^t) \;=\; p(s^t \mid z^t, u^t) \prod_{n=1}^{N} p(m_n \mid s^t, z^t, u^t)$$

The FastSLAM Algorithm

- Take i-th sample $s^{[i],t-1} \in S_{t-1}$ — $O(1)$
- "Guess" next pose $s_t^{[i]} \sim p(s_t \mid u_t s_{t-1}^{[i]})$ — $O(1)$
- Update landmark estimates $\mu_{n,t}^{[i]}, \Sigma_{n,t}^{[i]} \xleftarrow{\text{Kalman}} \mu_{n,t-1}^{[i]}, \Sigma_{n,t-1}^{[i]}$ — $O(1)$
- Calculate Importance Weights $w_t^{[i]} = \ldots$ — $O(1)$
- Resample $p(s^{[i],t}) \in S_t \;\propto\; w_t^{[i]}$ — ~~$O(tN)$~~ ~~$O(N)$~~ $O(\log N)$

See also [Murphy 99, Andrieu et al 99, Doucet et al 00]

Ben Wegbreit's Log-Trick



Advantage of Structured PF Solution

Kalman: $O(N^2)$ ⟶ Rao-B' PFs: $O(M \log N)$

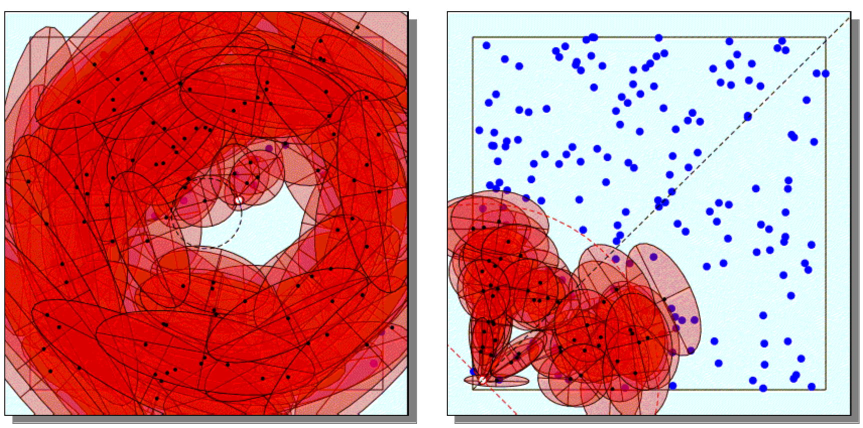
↓ 500 features

↓ Moore's Theorem: $\log N \leq 30$
Experimental: M=250

↓ 100,000+ features

+ global uncertainty, multimodal
+ non-linear systems
+ sampling over data associations

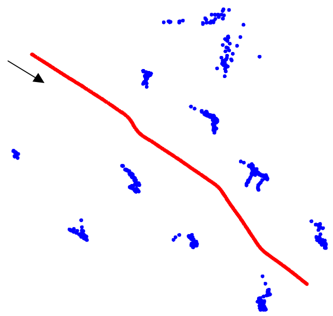
FastSLAM in Simulation



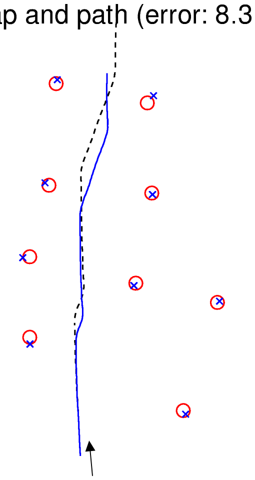
Experiments with up to 100,003 dimensions

Physical Robot Results

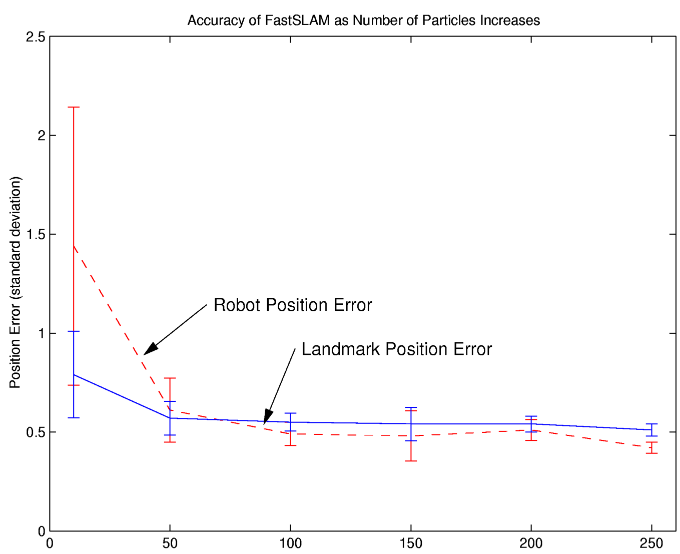
Raw data



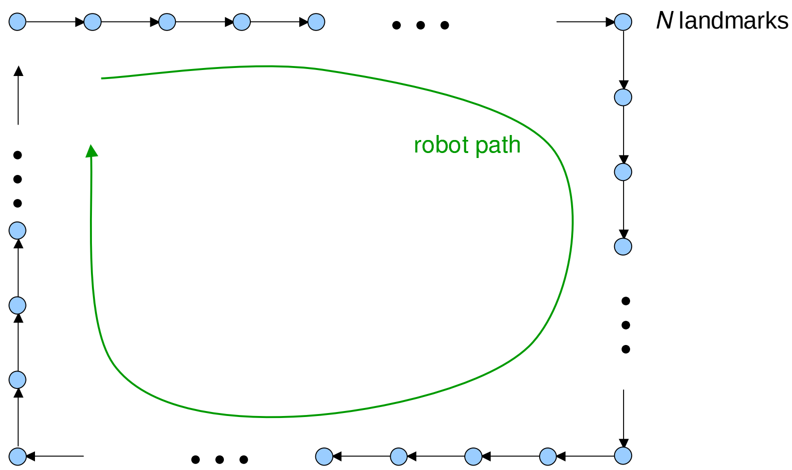
Map and path (error: 8.3 cm)



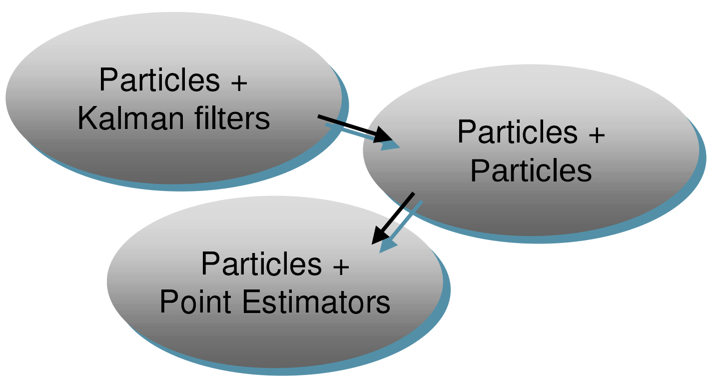
Varying Number of Particles M



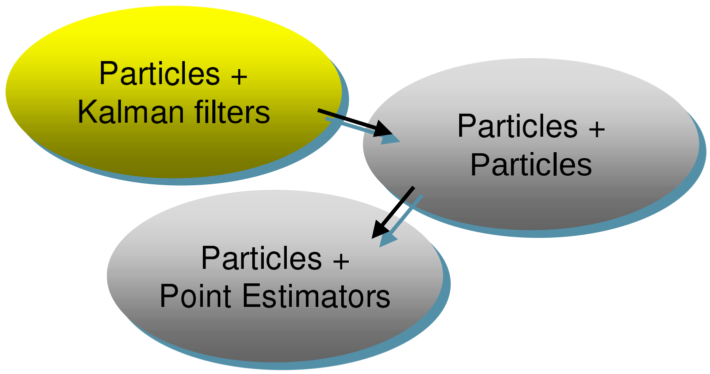
Worst-Case Environment: How many samples do we need?



3 Examples



3 Examples

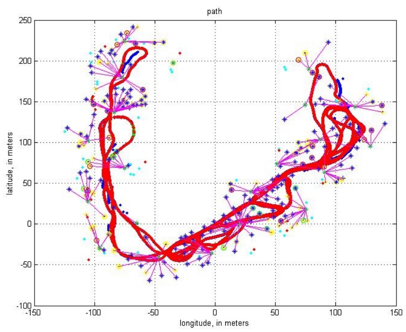


Outdoor Mapping (no GPS)

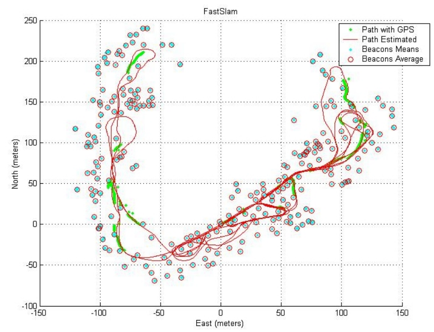
With Juan Nieto, Jose Guivant, Eduardo Nebot, Univ of Sydney

EFK vs FastSLAM

EKF



FastSLAM

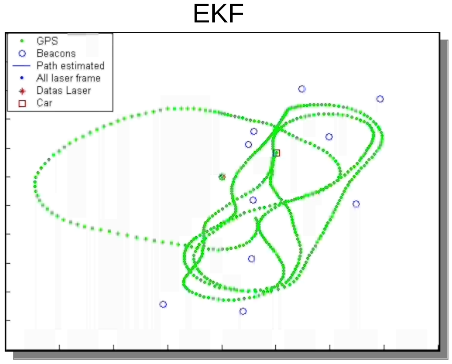


By: Juan Nieto, Jose Guivant, Eduardo Nebot

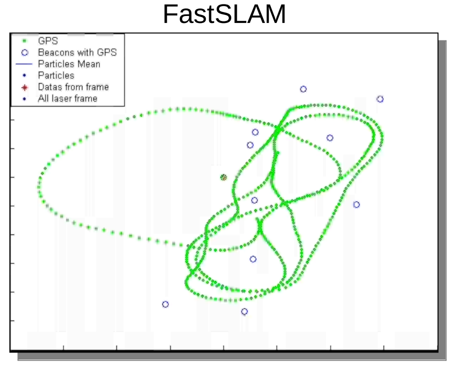
EFK vs FastSLAM

By: Juan Nieto, Jose Guivant, Eduardo Nebot

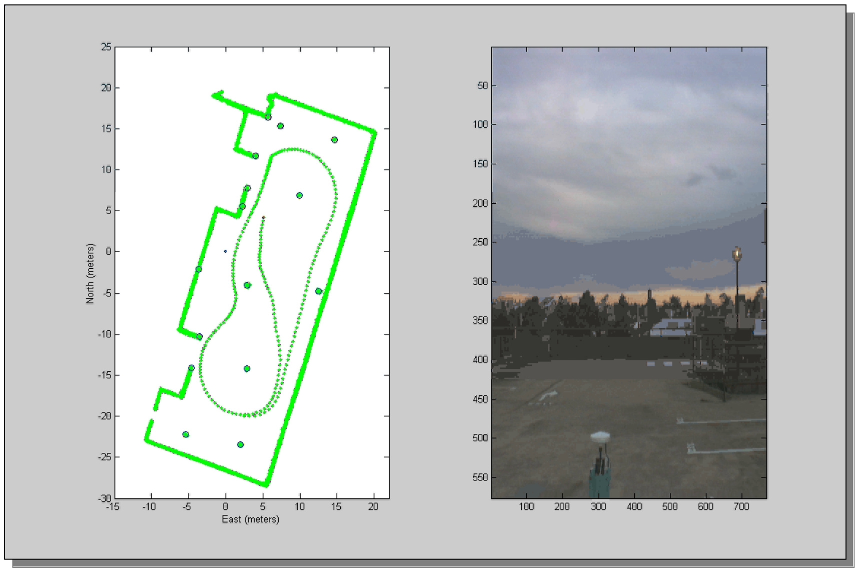
EKF



FastSLAM

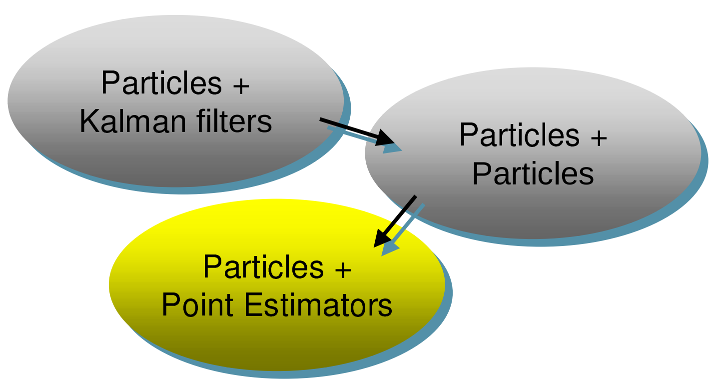


Assumes known data association



With Juan Nieto, Jose Guivant, Eduardo Nebot, Univ of Sydney

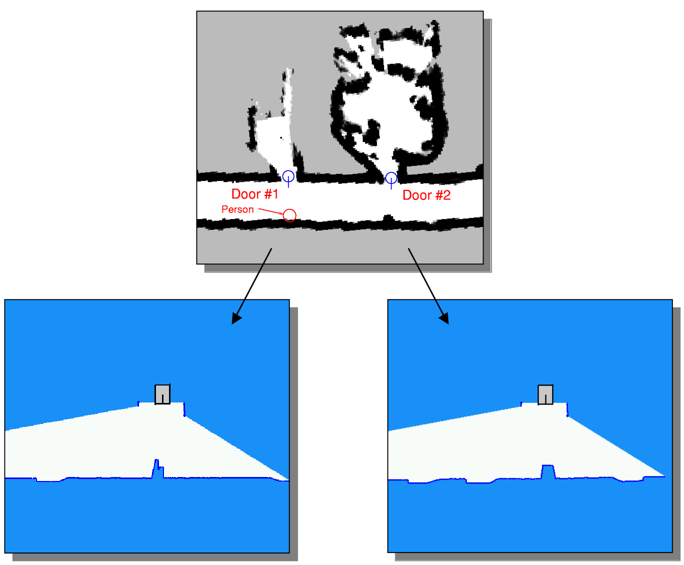
3 Examples



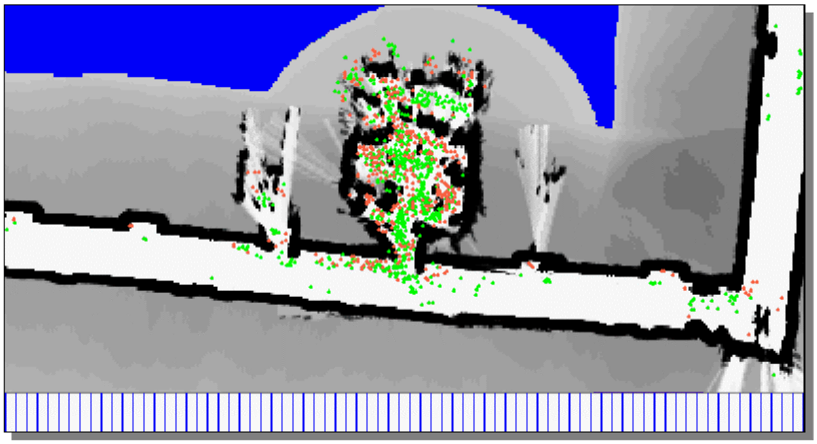
Tracking Moving Features

With: Michael Montemerlo

Tracking Moving Entities Through Map Differencing

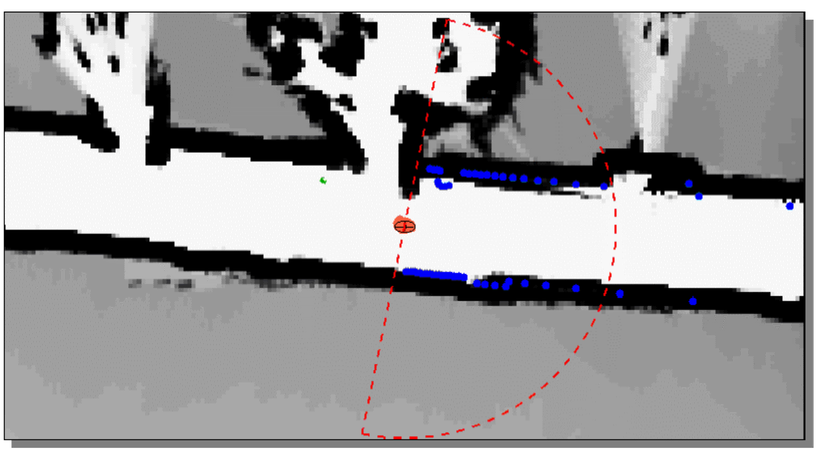


Map-Based People Tracking



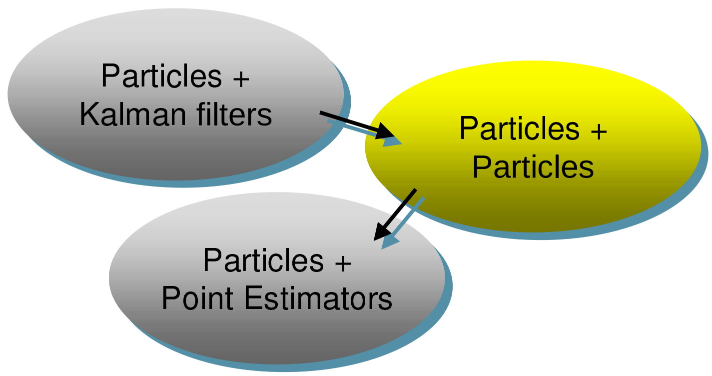
With: Michael Montemerlo

Autonomous People Following

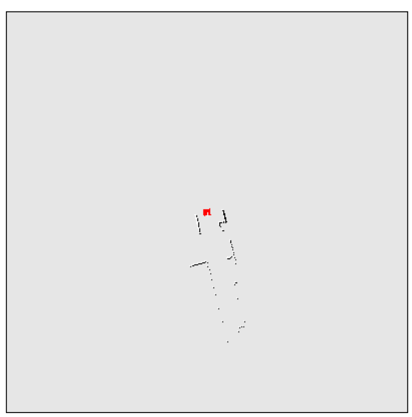


With: Michael Montemerlo

3 Examples

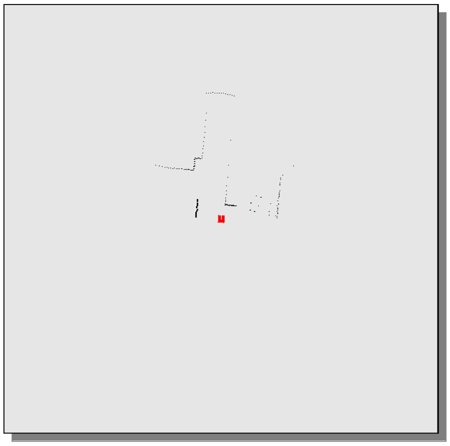


Indoor Mapping

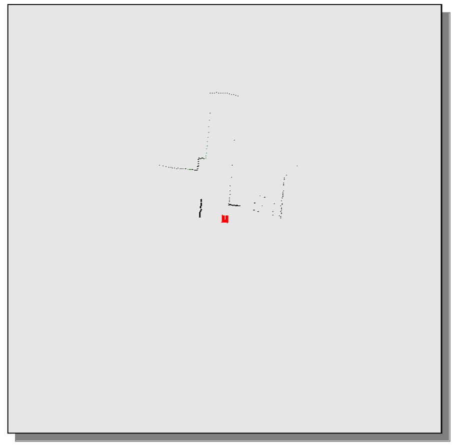


- Map: point estimators (no uncertainty)
- Lazy

Importance of Probabilistic Component

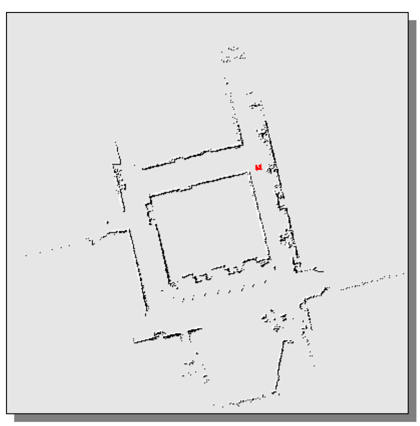


Non-probabilistic

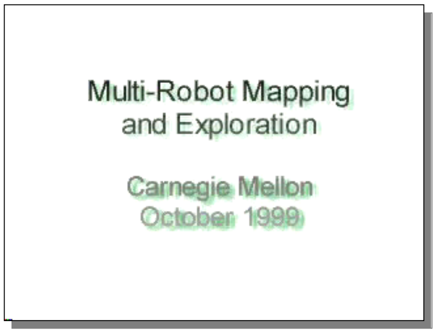


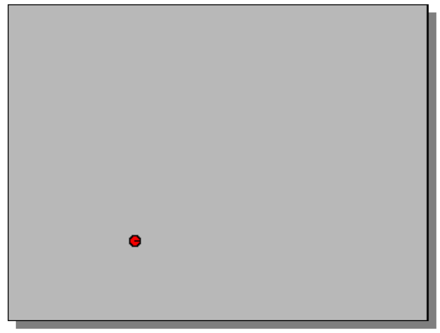
Probabilistic, with samples

Multi-Robot Mapping



Multi-Robot Mapping





DARPA TMR Texas

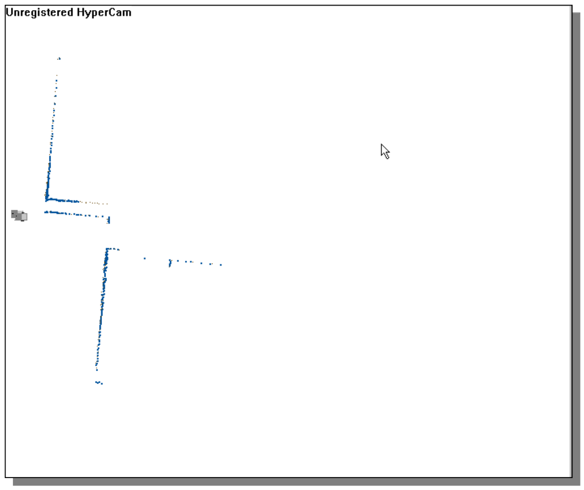
DARPA TMR Maryland

With: Reid Simmons and Dieter Fox

Courtesy of Kurt Konolige, SRI

[Gutmann & Konolige, 00]

Courtesy of Kurt Konolige, SRI



FastSLAM

- Advantage
  - Full Bayesian solution
  - Maintains all dependencies
- Disadvantage
  - ~~Does not address data association problem~~
  - ~~Linear(ized)~~
  - ~~Slow: $O(N^2)$~~

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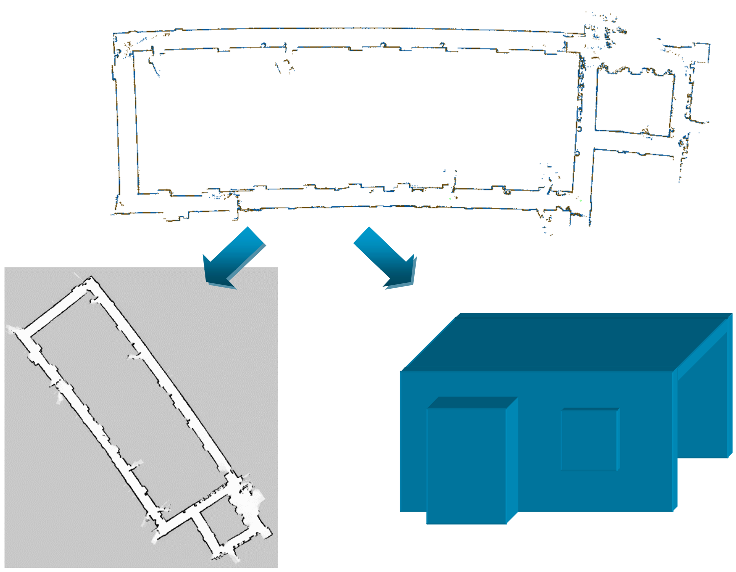
Tutorial Outline: Mapping

- The Mapping Problem
- Bayesian Solutions
  - Classical Solution: Kalman Filters
  - Efficient Solution: Sparse Information Filters
  - Nonlinear Solution: Particle Filters (FastSLAM)
- Mapping with Known Poses
  - Occupancy Grid Maps
  - Object Maps
- Summary

Mapping With Known Poses



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Occupancy Grid Maps: Many Binary Bayes filters

$$p(m, s_t \mid z_{1..t}, u_{1..t}) = \eta\, p(z_t \mid m, s_t) \int p(s_t \mid u_t, s_{t-1})\, p(m, s_{t-1} \mid z_{1..t-1}, u_{1..t-1})\, ds_t$$

$$p(m \mid z_{1..t}, s_{1..t}) = \eta\, p(z_t \mid m, s_t)\, p(m \mid s_{1..t-1}, z_{1..t-1})$$

Poses known

$$p(m_{xy} \mid z_{1..t}, s_{1..t}) = \eta\, p(z_t \mid m_{xy}, s_t)\, p(m_{xy} \mid s_{1..t-1}, z_{1..t-1})$$

Grid cell independence ???

$$p(m_{xy} \mid z_{1..t}, s_{1..t}) = \eta\, p(m_{xy} \mid z_t, s_t) p(z_t \mid s_t) p(m_{xy})^{-1}\, p(m_{xy} \mid s_{1..t-1}, z_{1..t-1})$$

$$p(\bar{m}_{xy} \mid z_{1..t}, s_{1..t}) = \eta\, p(\bar{m}_{xy} \mid z_t, s_t) p(z_t \mid s_t) p(\bar{m}_{xy})^{-1}\, p(\bar{m}_{xy} \mid s_{1..t-1}, z_{1..t-1})$$

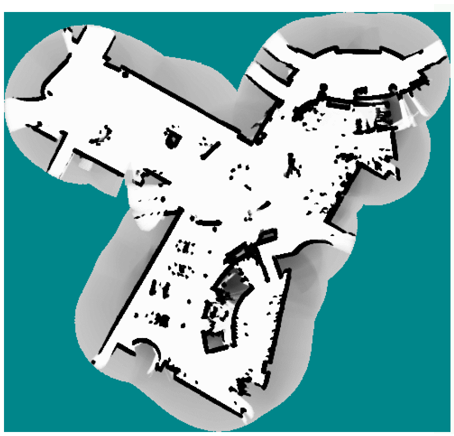
$$\log \frac{p(m_{xy} \mid z_{1..t}, s_{1..t})}{1 - p(m_{xy} \mid z_{1..t}, s_{1..t})} = \log \frac{p(m_{xy} \mid z_t, s_t)}{1 - p(m_{xy} \mid z_t, s_t)} + \log \frac{1 - p(m_{xy})}{p(m_{xy})} + \log \frac{p(m_{xy} \mid s_{1..t-1}, z_{1..t-1})}{1 - p(m_{xy} \mid s_{1..t-1}, z_{1..t-1})}$$

$$p(m_{xy} \mid z_{1..t}, s_{1..t}) = 1 - \left[1 + e^{\log \frac{p(m_{xy} \mid z_{1..t}, s_{1..t})}{1 - p(m_{xy} \mid z_{1..t}, s_{1..t})}} \right]^{-1}$$

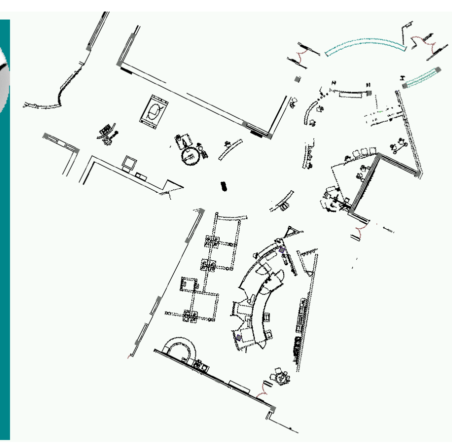
[Elfes/Moravec 88]

Map Generated from Sonar Range Data

Map Generated from Laser Range Data



2D Map, learned



CAD map

3D Occupancy Grid Maps

Courtesy of Hans Moravec

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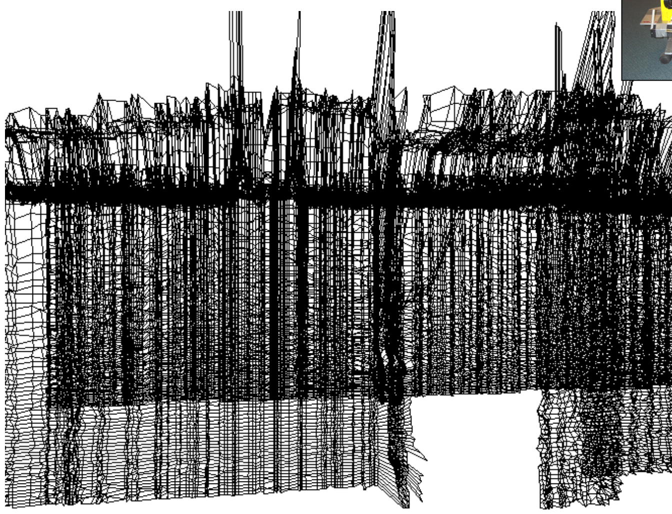
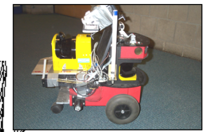
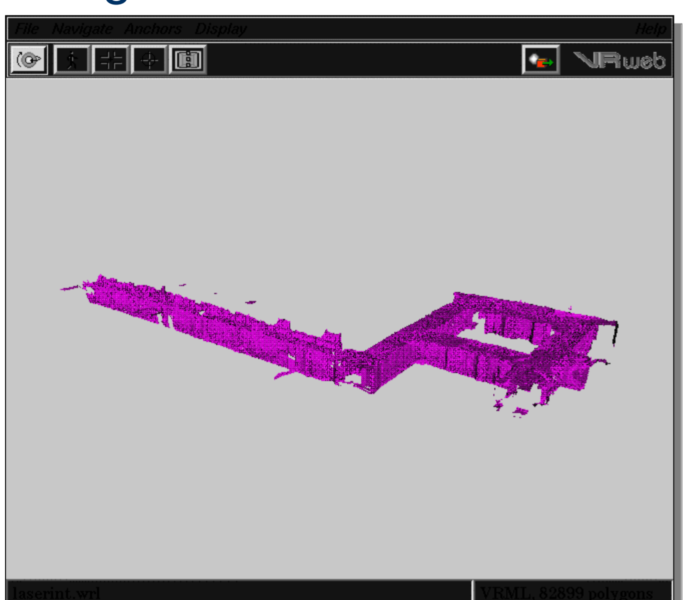
3D Range Data

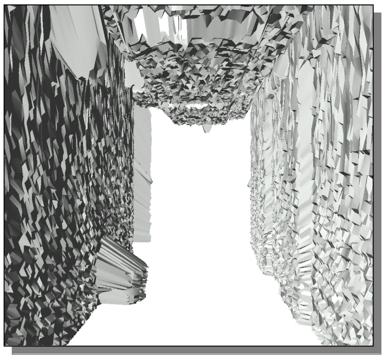


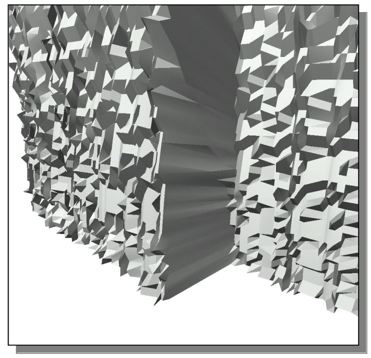


3D Range Data

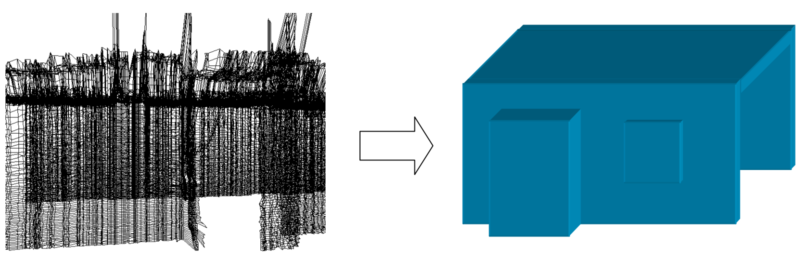


Fine-Grained Structure: Can we do better?





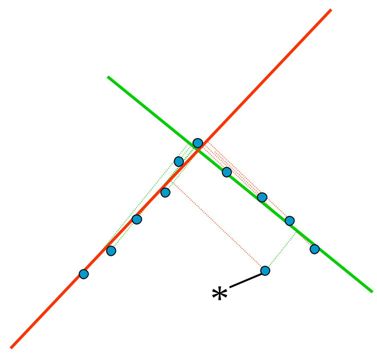
3D Planar Surface Mapping



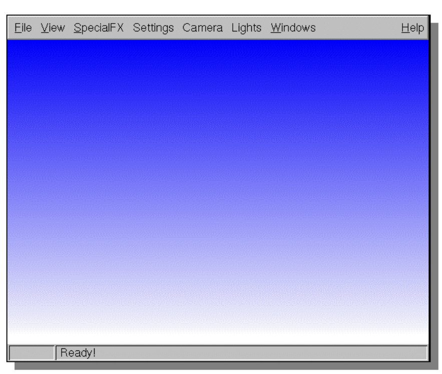
Data

Mostly multi-planar model

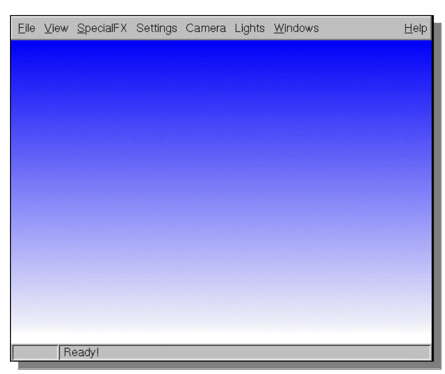
Fitting Planar Surfaces (with EM)



Online Mapping with EM

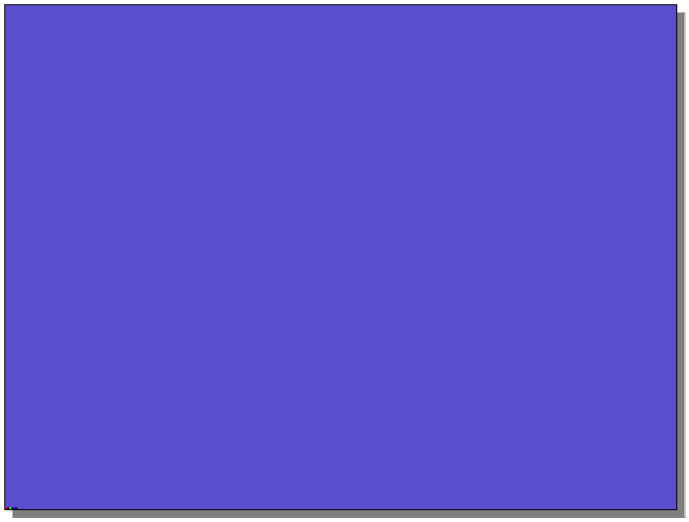


raw data

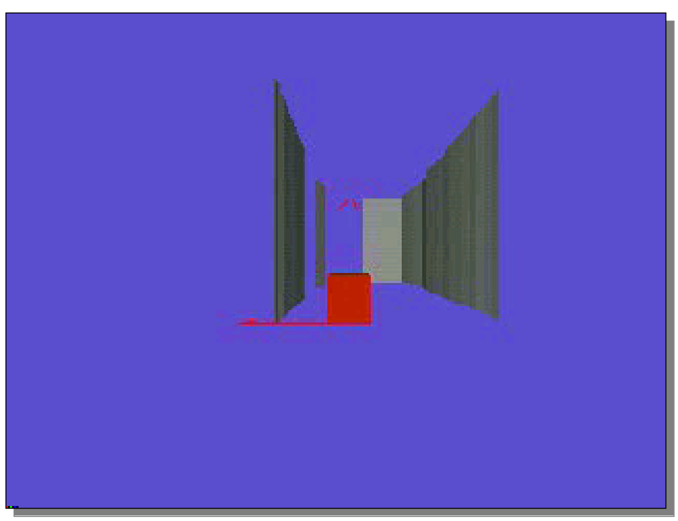


mostly planar map with Online EM

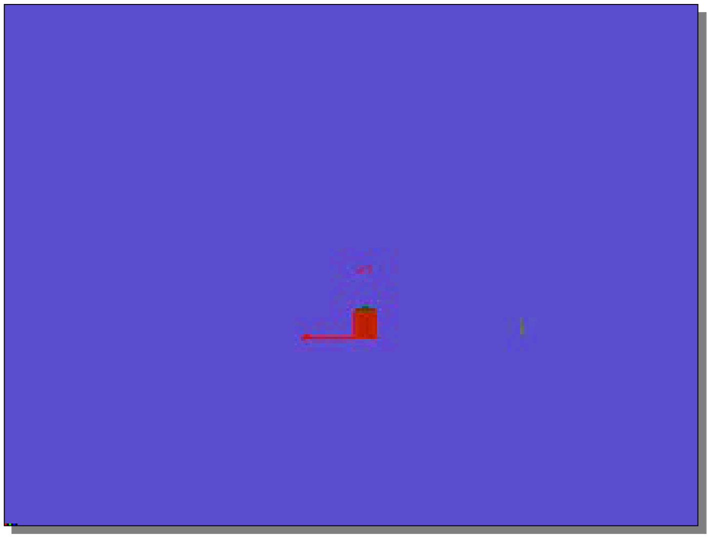
Online Mapping: Carnegie Mellon Univ. Hall



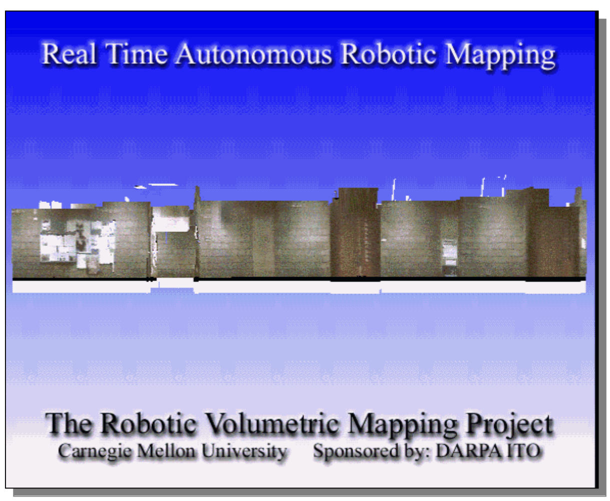
Online Mapping: Stanford Gates Hall



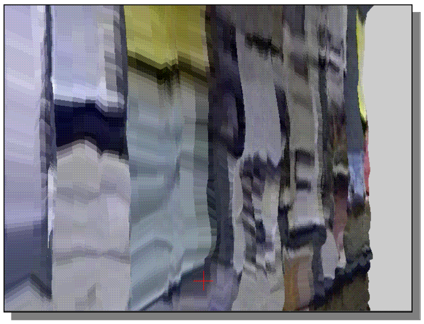
MARS DARPA PI Meeting (2/02 in LA)



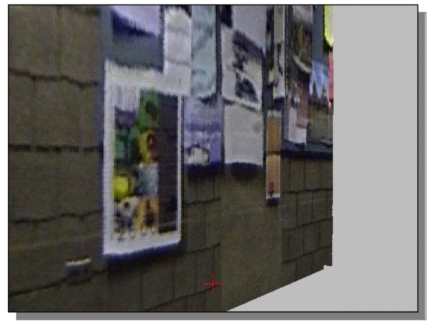
Volumetric Maps: Fly-Through



Quality of 3D Model

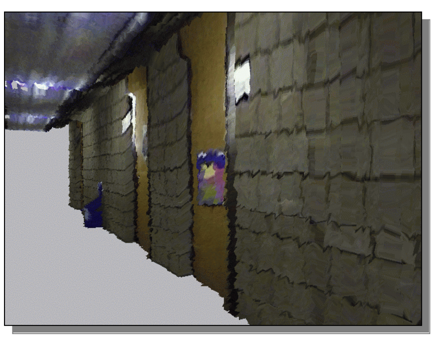


Without EM



With EM

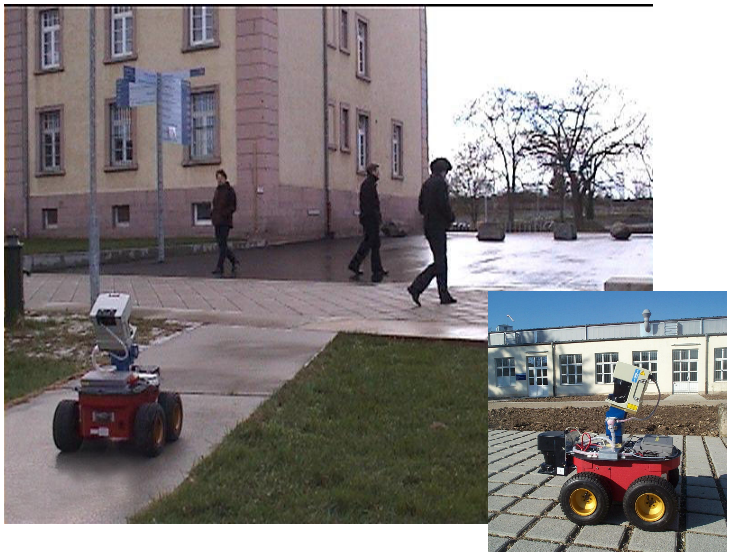
Quality of 3D Model



Without EM

With EM

3D Outdoor Mapping
by Dirk Hähnel, Dirk Schulz and Wolfram Burgard



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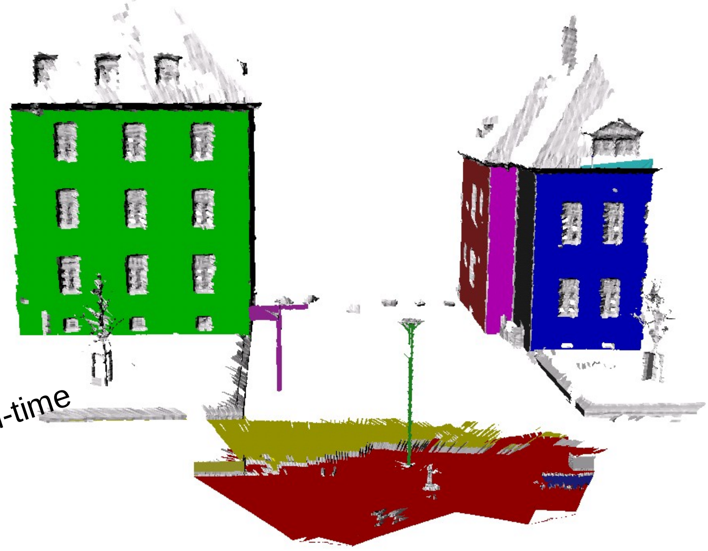

Not real-time

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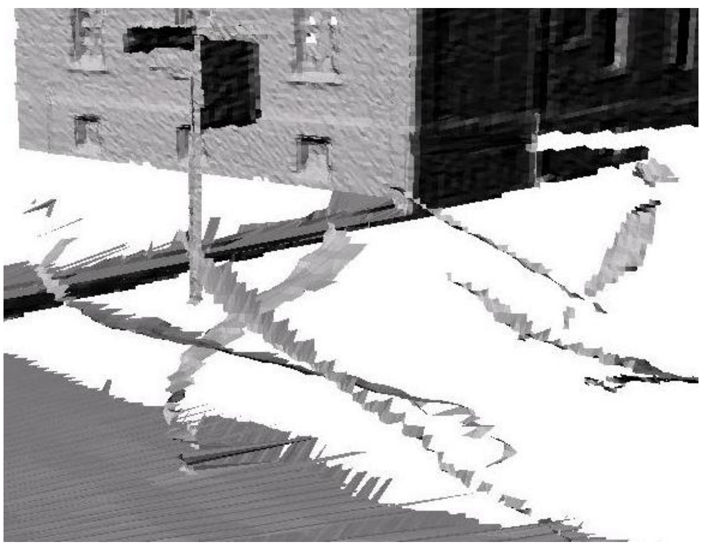

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Mapping: Lessons Learned

- Concurrent mapping and localization: hard robotics problem
- Best known algorithms are probabilistic
  1. EKF/SLAM: Full posterior estimation, but restrictive assumptions (data association)
  2. SEIF: Same, but constant time, multi-robot
  3. FastSLAM: Full posterior estimation, sampling over data association
- Mapping: Even tricky with known poses
  1. Occupancy grid maps: Factorized posteriors
  2. Advanced techniques: Object identification in 3D