Simpson 1/3

Error de tercer orden

$$Error_{(3)} = \int_{x_0}^{x_2} \frac{(s-x_0)(s-x_1)(s-x_2)}{3!} f^{(3)}(\xi)\, ds$$

$$= \frac{f^{(3)}(\xi)}{6} \int_{x_0}^{x_2} (s-x_0)(s-x_1)(s-x_2)\, ds$$

Calculemos

$$\int_{x_0}^{x_2} (s-x_0)(s-x_1)(s-x_2)\, ds$$

$u = s - x_1$ $\quad du = ds$

$s = x_0 \Rightarrow u = x_0 - x_1 = -h$

$s = x_2 \Rightarrow u = x_2 - x_1 = h$

$$= \int_{-h}^{h} (u + x_1 - x_0)(u + x_1 - x_1)(u + x_1 - x_2)\, du$$

$$= \int_{-h}^{h} (u+h)(u)(u-h)\, du = \int_{-h}^{h} u(u^2 - h^2)\, du$$

$g(u) = u(u^2 - h^2)$

$g(-u) = -u(u^2 - h^2) = -g(u)$

g es una función impar

$$\Rightarrow \int_{-h}^{h} g(u)\, du = 0$$

$$\therefore \quad Error_{(3)} = \frac{f^{(3)}(\xi)}{6} \int_{x_0}^{x_2} (s-x_0)(s-x_1)(s-x_2)\, ds = 0 \quad \not\downarrow$$

Simpson 3/8

$$\text{Error} = \int_{x_0}^{x_3} \frac{(s-x_0)(s-x_1)(s-x_2)(s-x_3)}{4!} f^{(4)}(\xi)\, ds$$

$$= \frac{f^{(4)}(\xi)}{24} \int_{x_0}^{x_3} (s-x_0)(s-x_1)(s-x_2)(s-x_3)\, ds$$

$$\int_{x_0}^{x_3} (s-x_0)(s-x_1)(s-x_2)(s-x_3)\, ds \qquad u = s - x_1 \qquad du = ds$$

$s = x_0 \Rightarrow u = x_0 - x_1 = -h$

$s = x_3 \Rightarrow u = x_3 - x_1 = 2h$

$$= \int_{-h}^{2h} (u + x_1 - x_0)(u + x_1 - x_1)(u + x_1 - x_2)(u + x_1 - x_3)\, du$$

$x_1 - x_0 = h$, $x_1 - x_2 = -h$, $x_1 - x_3 = -2h$

$$= \int_{-h}^{2h} (u+h)(u)(u-h)(u-2h)\, du$$

$$= \int_{-h}^{2h} (2h^3 u - h^2 u^2 - 2hu^3 + u^4)\, du$$

$$= 2h^3 \frac{u^2}{2}\Big]_{-h}^{2h} - h^2 \frac{u^3}{3}\Big]_{-h}^{2h} - 2h \frac{u^4}{4}\Big]_{-h}^{2h} + \frac{u^5}{5}\Big]_{-h}^{2h}$$

$$= h^3(4h^2 - h^2) - \frac{h^2}{3}(8h^3 + h^3) - \frac{h}{2}(16h^4 - h^4) + \frac{1}{5}(32h^5 - h^5)$$

$$= 3h^5 - \frac{9}{3}h^5 - \frac{15}{2}h^5 + \frac{33}{5}h^5 = -\frac{75}{10}h^5 + \frac{66}{10}h^5 = -\frac{9}{10}h^5$$

$$\Rightarrow \text{Error} = \frac{1}{24}\left(-\frac{9}{10}h^5\right) f^{(4)}(\xi)$$

$$= -\frac{3}{80} h^5 f^{(4)}(\xi)$$