Efficient Encoding and Decoding of data for Quantum Systems

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IT437 - Quantum Computing

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1 Abstract

Quantum systems hold immense potential for revolutionizing information processing and communication. Efficient data encoding and decoding techniques play a crucial role in harnessing the power of quantum systems. In this paper, we present novel approaches for data encoding and decoding in quantum systems that aim to optimize efficiency and enhance the reliability of quantum information transmission. Classical to Quantum data encoding presents a primary hurdle, as it requires specialized circuits for quantum state initialization. We present decoherence-optimized circuits for Classical to Quantum Encoding using the Flexible Representation of Quantum Images (FRQI) method. For Quantum to Classical Decoding, we utilize the Quantum Wavelet Transform particularly the Quantum Haar Transform through one-stage parallel and pyramidal QHT circuits for more efficient sampling of the output state.

Keywords— Quantum Circuits, Quantum Haar Transforms, FRQI, Hadamard Gates, Ancilla Qubits, Encoding, Decoding

2 Introduction

Quantum computing has emerged with time as the most promising computational paradigm that holds the potential to revolutionize information processing and computation. Noisy Intermediate-Scale Quantum (NISQ) devices, characterized by limited qubit coherence times and high error rates, currently form a significant focus of quantum computing research. Overcoming the challenges inherent in NISQ devices is crucial to harnessing their power for practical applications.

Quantum encoding and decoding are important in quantum computing as they enable efficient representation and processing of classical information in quantum states, harnessing the power of quantum systems. Encoding techniques allow for error correction, enhancing the robustness of quantum algorithms, while decoding methods extract useful classical information from quantum states. These processes are crucial for realizing the potential of quantum computing and overcoming the challenges posed by noise and errors in quantum systems.

One of the primary challenges in utilizing NISQ devices is the classical-to-quantum (C2Q) data encoding process. Efficiently preparing the quantum states required for information processing tasks, demands specialized encoding circuits. However, the design and implementation of these circuits are often complex, and they can exceed the constraints imposed by decoherence. This necessitates the development of innovative encoding techniques that optimize both efficiency and robustness. Similarly, Quantum-to-classical (Q2C) data decoding poses another critical challenge in quantum computing. The conventional approach involves the repeated sampling of the quantum circuit to obtain the classical output. However, this repetitive sampling introduces substantial overhead, leading to increased execution times. Therefore, exploring alternative methods for efficient Q2C data decoding is essential to expedite quantum algorithm execution.

In this paper, our primary focus is on tackling the challenges associated with classical-to-quantum (C2Q) data encoding and quantum-to-classical (Q2C) data decoding for Noisy Intermediate-Scale Quantum (NISQ) devices. To address the C2Q data encoding challenge, we propose the utilization of the Flexible Representation of Quantum Images (FRQI) encoding scheme. This approach provides an efficient and robust method for encoding classical data into quantum states, optimizing the delicate balance between fidelity and circuit complexity. By leveraging FRQI, we achieve the design of encoding circuits that are optimized for decoherence while remaining compatible with the constraints of NISQ devices.

In order to address the challenge of quantum-to-classical (Q2C) data decoding, we propose an innovative approach based on the One-Stage Pyramidal Parallel Quantum Haar Transform (OSPP-QHT). The OSPP-QHT method capitalizes on the inherent properties of the quantum Haar transform to efficiently sample the output state, utilizing the parallelism and hierarchical structure of the pyramidal Haar transform. This approach allows us to minimize the number of measurements needed for precise decoding, resulting in a significant reduction in overhead and, consequently, improving the overall execution time efficiency of NISQ devices. By leveraging the advantages of the OSPP-QHT approach, we aim to optimize the Q2C data decoding process for practical applications in quantum computing.

3 Literature Review

3.1 Background and Related Work

Naveed Mahmud et al.[1] proposed decoherence optimized circuits for Quantum wavelet transforms (QWT), namely the Quantum Haar Transform. QHT has been proven to be more effective than the Classical Discrete Wavelet transforms and is considered effective in the dimensionality reduction of multispectral data. These QHT circuits have been mitigated for minimal depth and decoherence effects and two circuit variants have been proposed: Sequential (d-stage) QHT and parallel one-stage QHT where a d- dimensional QHT operation is applied in just one stage both of these variants have been proposed for pyramidal as well as packet decomposition architectures of QHT.

M. Chaudhary et al.[2] proposed minimal depth circuits for efficient encoding and decoding of data for Quantum systems. They proposed novel approaches for classical to quantum data encoding using optimized amplitude encoding techniques capable of reducing the circuit depths by a factor of 2 and quantum to classical decoding using Quantum haar transform circuits as mentioned in [?] using the one stage parallel pyramidal QHT circuit (OSPP-QHT) and achieved a maximum of 89% reduction in circuit sampling time. H. Ohnishi et al.[?] introduce the factorization of two wavelet transform matrices, namely Pyramid and Packet, through the use of the tensor product and direct sum operations. The authors demonstrate the process of factorizing these matrices and subsequently implementing them as quantum circuits. By employing quantum circuit representations, they leveraged the benefits of quantum computation in wavelet transform computations, potentially leading to improved efficiency and performance in various applications.

Hirota et al.[3] introduce a concept called Flexible Representation of Quantum Images (FRQI), which serves as a means to represent images on quantum computers. This representation takes the form of a normalized state, incorporating information about colors and their respective positions within the images. To facilitate the FRQI state preparation, the researchers propose a constructive polynomial preparation method, starting from an initial state. Furthermore, the study provides us with an algorithm for complex quantum image compression (QIC), enabling efficient storage and retrieval of quantum images. Additionally, various processing operations tailored specifically for quantum images are introduced, allowing for image manipulation on quantum computing platforms. By combining these elements, the authors establish a comprehensive framework for quantum image processing, encompassing image representation, compression, and manipulation.

Fijany et al.[4] proposed the earliest and the most highly effective and comprehensive quantum circuits for two prominent quantum wavelet transforms, namely the quantum Haar and quantum Daubechies D(4) transforms. The methodology employed involves decomposing the classical operators associated with these transforms into combinations of direct sums, direct products, and dot products involving unitary matrices. Naveed et al.[**?**] introduce a novel approach that combines dimension reduction techniques with quantum information processing, aiming to address the challenges posed by domains that generate substantial volumes of data. Specifically, they propose the utilization of the quantum wavelet transform (QWT) as a means to reduce the dimensionality of high spatial resolution data. They develop simplified and optimized emulation architectures for performing the quantum wavelet transform on high-resolution data, surpassing previously reported methods. Additionally, they implement the inverse quantum wavelet transform (IQWT) to ensure accurate data reconstruction without any loss.

3.2 Outcome of Literature Survey

Based on the literature survey conducted, several significant contributions have been made in the field of quantum wavelet transforms and quantum image processing. Naveed Mahmud et al. proposed decoherence optimized circuits for the Quantum Haar Transform (QHT), which demonstrated its superiority over classical discrete wavelet transform. M. Chaudhary et al. introduced minimal depth circuits for efficient data encoding and decoding in quantum systems, achieving significant reductions in circuit sampling time. Hirota et al. introduced the concept of Flexible Representation of Quantum Images (FRQI) and developed a polynomial preparation method for FRQI state generation, along with a quantum image compression algorithm. Fijany et al. presented comprehensive quantum circuits for the quantum Haar and quantum Daubechies D(4) transforms. Naveed et al. proposed the utilization of quantum wavelet transforms for dimensionality reduction of high-resolution data, surpassing previous methods. These findings contribute to advancing the field of quantum image processing and provide insights into the application of quantum wavelet transforms for various data processing tasks.

Table 3.1: Summary of Literature Survey

Authors	Methodology	Merits
Naveed Mahmud et al.[1]	Proposed optimized circuits for Quantum Haar Transform (QHT) with variants including pyramidal and packet forms	Improved effectiveness over classical wavelet transforms and usefulness in decoding of quantum data
Naveed Mahmud et al.[2]	Proposed C2Q and D2Q techniques with 89% reduction in depth for decoding circuits and 2x reduction in depth for encoding	Minimal depth and efficient encoding and decoding for quantum images and better than existing methods
Le et al.[3]	Proposed the FRQI representation of Quantum Images which stores pixel value in ancilla qubits and indexes in seperate registers	Efficient storage and retrieval of quantum images with maximum fidelity and reduced depths
Fijany et al.[4]	Introduced the Quantum Wavelet transforms and its superiority over classical transforms	Improved dimensionality reduction using QWT and simplification of computational states.

3.3 Problem Statement

To create a system capable of efficiently encoding and decoding images on quantum systems with reduced circuit depths to minimize decoherence.

3.4 Objectives

1. To efficiently implement Classical to Quantum data encoding for images with minimal circuit depths and circuit complexity using the Flexible Representation of Quantum Images (FRQI) technique.

2. To efficiently implement Quantum to Classical data decoding technique for images using the Quantum Haar wavelet transform with a one-stage parallel pyramidal QHT circuit.

3. To analyze the efficiency of our algorithms and compare it with other available techniques.

4 Methodology

4.1 Dataset

For our analysis, we are using 2×2, 4×4, 8×8, 16×16, and 32×32 RGB images. As a part of pre-processing, we first convert the images into greyscale so as to perform our calculations only on one channel, then we obtain the array for the image and divide each value by 255 and then multiply each value of intensity with $\pi/2$ to convert the pixel values in the grid to Θ values required in FRQI encoding as per Eq.(2). The multiplication by $\pi/2$ is done due to FRQI rotation gates rotating the qubits by 2Θ leading to a maximum rotation of 180 degrees.

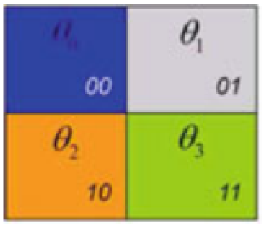
4.2 Implementation Details

1. **C2Q Encoding**: For C2Q encoding, we make use of the FRQI technique. The Flexible Representation of Quantum Images (FRQI) allows for the encoding of image coordinates and color information into quantum states. FRQI focuses on the processing and computations of images, aiming to achieve efficient operations and compression. The complexity of FRQI lies in the number of simple operations involved in its implementation. Using FRQI, we represent the image as a quantum state with the formula given in (1)

$$|I(\Theta)>= 1/2^n \sum_{i=0}^{2^{2n}-1} (cos\Theta_i|0 + sin\Theta_i|1) \otimes |i> \tag{1}$$

$$\Theta_i \in [0, \Pi/2]\,, i = 0, 1, 2, ..., 2^{2n} - 1 \tag{2}$$

It captures all the information about the colors and the corresponding positions or indexes of these colors or intensities. The part of the state i.e. $cos\Theta_i|0 > +sin\Theta_i|1 >$ encodes the color or the intensity values whereas the |i> part encodes the position or the index of the color in the image grid. An example of a 2*2 image is shown in Fig. 4.1. Getting from an initial state $|0 > \otimes(2n + 1)$ to the FRQI state requires the use of a unitary operation, which we will call P. First, Hadamard gates are applied to each of the qubits in the initial



$$|I\rangle = \frac{1}{2}\Big[\big(\cos\theta_0|0\rangle + \sin\theta_0|1\rangle\big)\otimes|00\rangle + \big(\cos\theta_1|0\rangle + \sin\theta_1|1\rangle\big)\otimes|01\rangle + \\ + \big(\cos\theta_2|0\rangle + \sin\theta_2|1\rangle\big)\otimes|10\rangle + \big(\cos\theta_3|0\rangle + \sin\theta_3|1\rangle\big)\otimes|11\rangle\Big]$$

Figure 4.1: FRQI state of a 2*2 image [3]

state. Using controlled rotation gates on the current state transforms it into the FRQI state. Referring to the controlled rotations as R and the Hadamard gates as H, we can say that P = RH. This process uses 2n Hadamard gates and 2^{2n} controlled rotations. These controlled rotations can be implemented by $C^{2n}Ry(2\Theta)$ and NOT operations. It has also been shown that $C^{2n}Ry(2\Theta)$ can be broken down into 2^{2n} simple operations. Therefore, the total number of simple operations to get from the initial state to the FRQI state is $2n+2^{2n}(2^{2n1}+2^{2n}-4)$ which is quadratic in 2^{2n}. The number of gates involved in achieving the Flexible Representation of Quantum Images (FRQI) state can be substantial due to the depth and pixel count of these images. However, to address this challenge and reduce the number of simple gates required, an approach known as Quantum Image Compression can be employed. This process aims to efficiently compress quantum image data, thereby minimizing the number of gates needed for encoding and improving overall circuit complexity. The construction of $C^{2n}Ry(2\Theta)$ gates is given below in Fig 4.2.

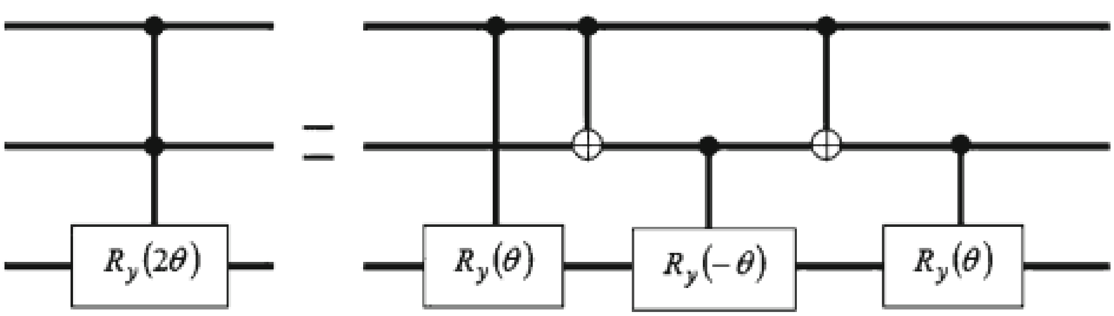


Figure 4.2: $C^{2n}Ry(2\Theta)$ Construction [3]

In summary, FRQI suggests a new way to represent images on quantum computers. It captures information about the colors and positions of pixels in a quantum state. Just like how computers use pixels to display images, this method uses quantum states to store color and position data. Colors are represented by angles, while positions are represented by computational basis states. Different techniques can be used to show how colors appear in the image, like drawing lines or using blocks. When using quantum computers, we need to prepare them properly before working with images. This involves transforming the initial state of the computer to the desired state for the image. We can do this using specific mathematical operations called unitary transforms. These transforms make sure the information is preserved and can be efficiently used. The method described in the paper uses these transforms in a clever way to prepare the quantum computer for image processing.

2. **Q2C Decoding**: In the context of quantum-to-classical (Q2C) decoding, we exploit the Quantum Haar Transform (QHT), a specific type of wavelet transform. The QHT is capable of preserving the local characteristics of data in both space and time and can be decomposed into multiple levels. By applying a multi-level QHT decomposition, we can transform data represented by n qubits into data represented by k = (n - l * d) qubits, where k is smaller than n. Here, l represents the number of decomposition levels, and d denotes the dimensionality of the data (e.g., 1, 2, or 3 for 1-D, 2-D, or 3-D data, respectively). The goal of dimensionality reduction using QHT is to utilize fewer qubits for data representation, thereby reducing the measurement time required. Figure 4.2 provides a visual representation of our proposed methodology for QHT-based Q2C data decoding.

   To implement our Quantum-to-Classical (Q2C) method effectively, we will employ the optimized and efficient one-stage parallel Quantum Haar Transform (QHT) circuit, denoted as Ud-D-QHTpar,opt, which is designed to minimize circuit depth. The QHT can be decomposed into multiple levels using either packet or pyramidal forms. In the packet decomposition approach, the QHT operation is iteratively applied to all qubits at each level, involving all qubits

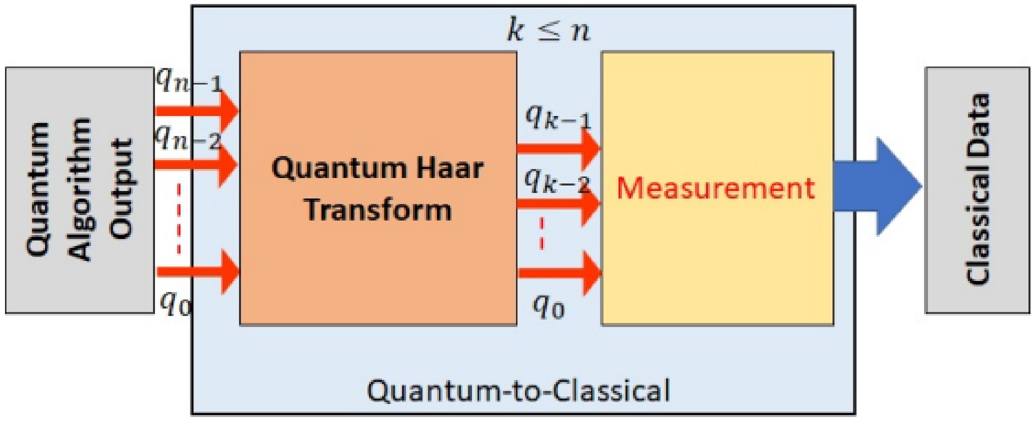


Figure 4.3: Overview of QHT methodology [2]

throughout the entire process. On the other hand, the pyramidal decomposition approach applies the d-dimensional QHT to a reduced number of data qubits at each decomposition level. For our proposed Q2C method, we will adopt the pyramidal decomposition approach. This choice offers the advantage of reducing the number of required qubits and decreasing the depth of the QHT circuit after each decomposition level.

The one-stage parallel Quantum Haar Transform (QHT) method allows for the simultaneous application of the Haar operation, represented by H gates, to all dimensions rather than performing them sequentially on each dimension. By executing the operation all at once, we can organize the Right-rotation (RoR) and Left-rotation (RoL) operations into sets of permutations that occur before and after the Haar operation. You can refer to Figure 4.3 for the circuit diagram illustrating this approach.

By adopting the pyramidal decomposition strategy and leveraging the one-stage parallel QHT method, we aim to enhance the efficiency and effectiveness of our Q2C method. This approach not only reduces the number of qubits required but also decreases the depth of the QHT circuit after each decomposition level, making our method more optimized and suitable for practical quantum computing applications.

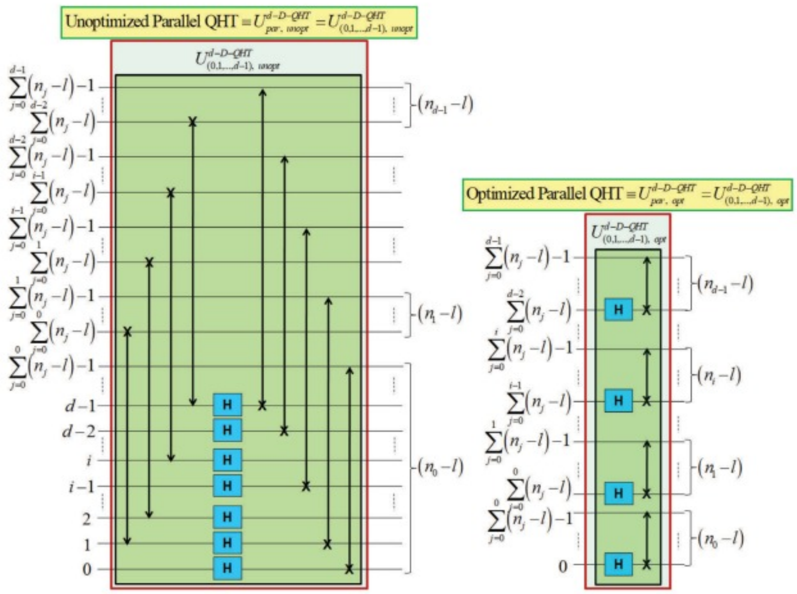


Figure 4.4: Unoptimized and Optimized Parallel QHT circuits [1]

5 Result and Analysis

Our experiments were majorly conducted on penny lane, for varying image sizes as mentioned in 4.1. For an input image with dimensions 32×32, the input state and the output state are given in Fig.4 and Fig.5 below.

Figure 5.1: Original Input image

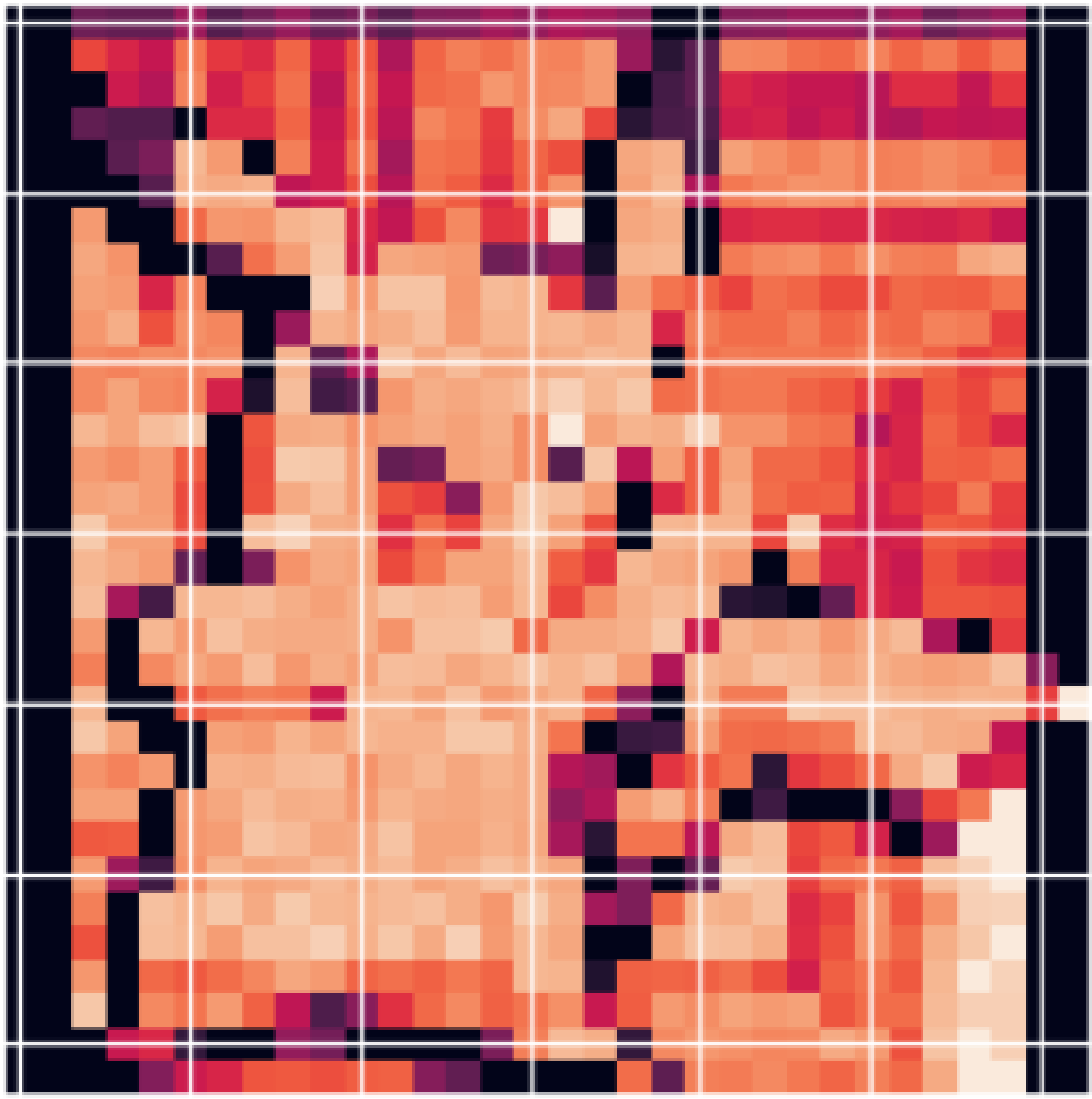


Figure 5.2: Final image after decoding

The results obtained from our analysis of varying image sizes for FRQI encoding are given in Table 5.2 below.

Table 5.1: Experiment Results

Image Size	Qubits	Shots	Runtime	Depth	Ry Gates	X Gates	H Gates
2×2	3	30000	0.20 sec	18	8	6	2
2×2	3	60000	0.35 sec	18	8	6	2
2×2	3	90000	0.50 sec	18	8	6	2
4×4	5	30000	0.24 sec	66	32	30	8
4×4	5	60000	0.51 sec	66	32	30	8
4×4	5	90000	0.82 sec	66	32	30	8
16×16	9	30000	1.3 sec	1026	512	510	8
16×16	9	60000	2.0 sec	1026	512	510	8
16×16	9	90000	2.3 sec	1026	512	510	8
32×32	11	30000	80.2 sec	4098	2048	2046	10
32×32	11	60000	81.5 sec	4098	2048	2046	10
32×32	11	90000	84.3 sec	4098	2048	2046	10

6 Conclusion and Future Work

6.1 Conclusion

In this study, we have introduced methods for achieving depth-optimized and time-efficient classical-to-quantum (C2Q) data encoding using FRQI encoding and quantum-to-classical (Q2C) data decoding using the Quantum Haar Transforms with one-stage parallel circuit methodology. We presented the corresponding quantum circuits for each method and conducted experimental evaluations using IBM Quantum and PennyLanes Lightning Qubit system. Additionally, we implemented related methods for quantitative analysis and analyzed the variation of runtimes and circuit depths for varying sizes of the images. The experimental results demonstrated that our proposed methods aligned with theoretical expectations and significantly enhanced the time efficiency of the C2Q and Q2C processes.

6.2 Future Work

Our future research will focus on exploring further optimizations for the proposed C2Q and Q2C methods including distributed amplitude and angle encoding, NEQR encoding, and many other encoding and decoding techniques, to explore more about the use of Quantum Haar Transforms in other applications, such as well as investigating quantum error correction techniques to improve the fidelity of the output images.

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