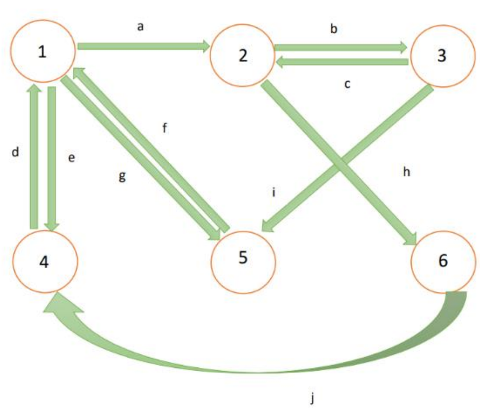
Traffic Allocation Modeling Using Markov Chains

With the increase in vehicles, traffic congestion in many road networks has become more severe and costly. The effects of traffic congestion extend beyond the usual economic and social impacts, reaching areas like the environment and quality of life for urban residents. This situation can put significant pressure on local and national governments to develop transportation infrastructure to keep pace with the growing demand for the movement of people and goods, a trend compounded by the rising number of vehicles. Transportation authorities typically use a combination of network evaluation and design techniques to analyze and propose potential solutions for this complex issue. These approaches are generally divided into two categories: computer traffic simulations, which simulate traffic patterns on road networks with varying levels of detail, and traffic allocation models, which are primarily based on the road network itself.

The use of Markov chains in modeling transportation systems is quite common. However, their application to traffic allocation and network design problems, due to their suitability for modeling complex, dynamic, and large-scale networks, has recently garnered attention. Computer traffic simulation allows us to simulate and analyze traffic under various conditions, helping identify the best strategies to reduce congestion and improve traffic flow. This method, considering various factors like current traffic, road structures, route maps, travel times, and transportation needs, enables precise modeling and comprehensive analysis.

In general, managing traffic as a complex challenge in the transportation sector requires precise and continuous analysis and the provision of suitable solutions. By employing advanced analysis and modeling methods, we hope to achieve greater efficiency in improving citizens' quality of life, reducing environmental pollution, and enhancing transportation performance. This comprehensive approach to traffic management aims to balance the growing demand for mobility with the need for sustainable and efficient transportation networks, ultimately contributing to more livable urban environments and more resilient infrastructure.

Selecting an Urban Area for Traffic Allocation Problem and Creating a Route Graph



a to b	0.7
a to h	0.3
b to c	0.55
b to i	0.45
c to h	0.50
c to b	0.50
d to a	0.30
d to e	0.20
d to g	0.50
d to g	0.50
e to d	1
f to a	0.10
f to e	0.30
f to g	0.60
g to f	1
h to j	1
i to f	1
g to d	1

$$
\begin{array}{c c}
 & \begin{array}{cccccccccc} a & b & c & d & e & f & g & h & i & j \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \\ j \end{array} &
\begin{bmatrix}
0.00 & 0.70 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.30 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.55 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.45 & 0.00 \\
0.00 & 0.50 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.00 & 0.00 \\
0.30 & 0.00 & 0.00 & 0.00 & 0.20 & 0.00 & 0.50 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.10 & 0.00 & 0.00 & 0.00 & 0.30 & 0.00 & 0.60 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00
\end{bmatrix}
\end{array}
$$

based on the above inputs, we consider the following matrix.

The average speed of cars is considered to be 50 km/h. And the one-sided and two-sided nature of the problem is shown in the graph above. And we also see the possible length below:

$$\pi P = \pi$$

$$[\pi1\ \pi2\ \pi3\ \pi4\ \pi5\ \pi6\ \pi7\ \pi8\ \pi9\ \pi10\ ] \times$$

$$
\begin{bmatrix}
0.00 & 0.70 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.30 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.55 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.45 & 0.00 \\
0.00 & 0.50 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.00 & 0.00 \\
0.30 & 0.00 & 0.00 & 0.00 & 0.20 & 0.00 & 0.50 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.10 & 0.00 & 0.00 & 0.00 & 0.30 & 0.00 & 0.60 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00
\end{bmatrix}
$$

$$= [\pi1\ \pi2\ \pi3\ \pi4\ \pi5\ \pi6\ \pi7\ \pi8\ \pi9\ \pi10\ ]$$

$$[0.3\pi4 + 0.1\pi6\ , 0.7\pi1 + 0.5\pi3\ , 0.55\pi2\ , \pi5 + \pi10\ , 0.2\pi4 + 0.3\pi6\ , \pi7 + \pi9\ , 0.5\pi4 + 0.\pi6\ , 0.3\pi1 + 0.5\pi3\ , 0.45\pi2\ , \pi8]$$

$$[\pi1\ \pi2\ \pi3\ \pi4\ \pi5\ \pi6\ \pi7\ \pi8\ \pi9\ \pi10\ ]$$

$0.3\pi4 + 0.1\pi6 = \pi1$ $\quad$ $0.7\pi1 + 0.5\pi3 = \pi2$ $\quad$ $\pi5 + \pi10 = \pi4$ $\quad$ $0.2\pi4 + 0.3\pi6 = \pi5$ $\quad$ $\pi7 + \pi9 = \pi6$

$0.5\pi4 + 0.\pi6 = \pi7$ $\quad$ $0.3\pi1 + 0.5\pi3 = \pi8$ $\quad$ $0.45\pi2 = \pi9$ $\quad$ $\pi8 = \pi10$

I have implemented these steps (stationary) in Python to get the final value. (The desired code is included in the attached file). Based on this implementation, the obtained values were as follows.

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$$[\pi 1\ \pi 2\ \pi 3\ \pi 4\ \pi 5\ \pi 6\ \pi 7\ \pi 8\ \pi 9\ \pi 10\ ] =$$

[0.06815417 0.06580403 0.03619221 0.14294404 0.10440168 0.25270958
0.22309777 0.03854236 0.02961181 0.03854236]

I obtained the value of tt, which is the elapsed time from the beginning to the end of a street, based on physics relations and normalized it. In the following, I have provided you with the obtained values.

$$P_{ii} = \frac{tt_{i-1}}{tt_i}$$

tt_a= 0.2 (2) , tt_b= 0.1 (1) , tt_c= 0.1(1) , tt_d= 0.2(2) , tt_e= 0.2(2) , tt_f=0.14(1.4) , tt_g=0.14(1.4) , tt_h=0.18(1.8) , tt_i= 0.16(1.6) , tt_j=0.4(4)

$$P_{ij} = (1 - P_{ii}) \times tp_{ij}$$

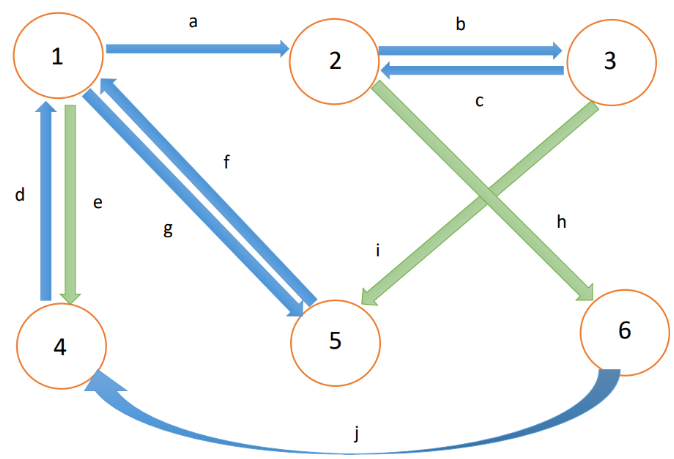
And then, based on the above relationship, I have obtained new values for the desired matrix:

$$\begin{bmatrix}
0.50 & 0.35 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.15 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.55 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.45 & 0.00 \\
0.00 & 0.50 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.00 & 0.00 \\
0.15 & 0.00 & 0.00 & 0.50 & 0.25 & 0.00 & 0.10 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.50 & 0.50 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.07 & 0.00 & 0.00 & 0.00 & 0.21 & 0.29 & 0.43 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.72 & 0.28 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.44 & 0.00 & 0.56 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.63 & 0.00 & 0.00 & 0.37 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.25 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.75
\end{bmatrix}$$

Choosing a utility function reflecting distance, quality, density and accident risk and calculating density

$$\boldsymbol{D_i = \frac{V\pi_i}{L_i N_i}}$$

V is the length of the road as well as L_i which in the above relationship is the total number of cars in the network and N_i is the number of lanes in each street. Therefore, in the figure below, we assume that the blue lines represent 2-lane streets and the green lines represent single-lane streets.


1
2
3
4
5
6
a
b
c
d
e
f
g
h
i
j