Circular Motion

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Constant Angular Velocity

Question 1 (4)

A small object moves in a horizontal circle on a smooth table while attached to a string that passes through a small hole in the centre of the table. The other end of the string is pulled downward so that the radius of the circular path decreases from r_1 to r_2, while the angular momentum of the mass about the hole remains constant.

1. Show that the final angular velocity ω_2 is related to the initial value ω_1 by $\omega_2 = \omega_1 \left(\frac{r_1}{r_2}\right)^2$.

2. Hence determine the ratio of the final kinetic energy to the initial kinetic energy.

Question 2 (5)

A bead of mass m is free to slide on a smooth circular wire of radius R in a vertical plane. The wire rotates with constant angular velocity ω about a vertical diameter.

1. Derive an expression for the equilibrium angle θ (measured from the lowest point) at which the bead can remain stationary relative to the wire.

2. Determine the condition on ω for a stable equilibrium to exist.

Question 3 (4)

A car of mass m travels over the top of a circular hill of radius R at constant speed v.

1. Derive an expression for the normal reaction of the road on the car.
2. Find the maximum speed at which the car can travel without losing contact with the road.

Question 4 (5)

A particle of mass m moves in a horizontal circle on a rough surface of radius r with angular speed ω. The coefficient of friction between the particle and the surface is μ.

1. Determine the magnitude and direction of the frictional force acting on the particle.
2. Find the range of possible values of ω for which circular motion is possible.

Question 5 (4)

A pendulum bob of mass m is attached to a light string of length L and moves in a horizontal circle such that the string makes an angle θ with the vertical.

1. Derive an expression for the angular speed ω in terms of g, L, and θ.
2. Hence, calculate θ if $L = 1.2\,\text{m}$ and $\omega = 3.0\,\text{rad/s}$.

Question 6 (5)

A particle of mass m moves in a circular orbit under the influence of a central attractive force $F = \dfrac{k}{r^2}$.

1. Show that the angular velocity of the particle is $\omega = \sqrt{\dfrac{k}{mr^3}}$.
2. Hence derive an expression for the total energy of the particle in terms of k, m, and r.

Question 7 (5)

A small mass m slides inside a smooth, fixed vertical circular track of radius R. If it starts from rest at an angle θ above the lowest point and moves under gravity only:

1. Derive an expression for its speed when it reaches the bottom.
2. Determine the minimum starting height (in terms of R) required for the mass to complete a full circle without losing contact.

Question 8 (4)

A uniform rod of length L and mass m rotates in a horizontal plane with constant angular velocity ω about one end. Find expressions for:

1. The kinetic energy of the rod,
2. The tension at the mid-point of the rod.

Question 9 (5)

A small mass is attached to a rotating turntable by a string passing through a smooth hole at the centre. The mass moves in a circle of radius r on the table. The string is pulled downward so that the radius decreases slowly and the angular momentum about the centre is conserved.

1. Show that the kinetic energy of the mass increases as r decreases.
2. Determine the work done by the tension in the string as the radius changes from r_1 to r_2.

Question 10 (4)

A Ferris wheel of radius 12 m rotates with constant angular speed such that passengers experience a weight that varies between $0.8W$ and $1.2W$, where W is the true weight. Find:

1. The angular velocity of the wheel.
2. The period of one revolution.