

Roots Of Unity

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Question 1 (3)

Find all the cube roots of unity ($z^3 = 1$) and represent them in the form $a + bi$.

Question 2 (3)

Find all the fourth roots of unity ($z^4 = 1$) and plot them on the complex plane.

Question 3 (4)

Find all the sixth roots of unity ($z^6 = 1$) and express them in exponential form $e^{i\theta}$.

Question 4 (4)

Show that the sum of all the fifth roots of unity ($z^5 = 1$) is zero.

Question 5 (5)

If z is a seventh root of unity ($z^7 = 1$), find the value of

$$1 + z + z^2 + \cdots + z^6$$

Question 6 (4)

If z is a sixth root of unity ($z^6 = 1$), compute the value of

$$z + z^2 + z^4$$

Question 7 (4)

If z is a seventh root of unity ($z^7 = 1$), compute

$$\prod_{k=1}^6 (1 - z^k)$$

Question 8 (5)

Let z_1, z_2, z_3, z_4 be the fourth roots of unity. Compute

$$z_1^2 + z_2^2 + z_3^2 + z_4^2$$

Question 9 (5)

If z is a fifth root of unity ($z^5 = 1$), show that

$$1 + z^2 + z^4 + z^6 + z^8 = 0$$

Question 10 (5)

If z is an eighth root of unity ($z^8 = 1$), compute the sum

$$S = 1 + z + z^3 + z^5 + z^7$$

Question 11 (3)

Point $P(\sqrt{3}, 1)$ is one vertex of an equilateral triangle centered at $(0, 0)$. Find the coordinates of the other two vertices.

Question 12 (3)

A square is centered at the origin with one vertex at $(2, 0)$. Find the coordinates of the other three vertices.

Question 13 (4)

Point $A(1, 0)$ is on a circle centered at the origin. Rotate A 120° counterclockwise around the origin. Find the coordinates of the rotated point.

Question 14 (4)

Vertices $A(1, 2)$ and $B(3, 4)$ are two vertices of a square. Find the coordinates of the remaining two vertices.

Question 15 (4)

Point $P(1, \sqrt{3})$ is one vertex of a regular hexagon centered at the origin. Find the coordinates of the remaining five vertices.

Question 16 (5)

Triangle ABC has centroid at the origin. Vertex $A(3, 1)$ and $B(-1, 2)$ are given. Find the coordinates of vertex C .

Question 17 (5)

Point $P(2, 0)$ is one vertex of a regular pentagon centered at the origin. Find the coordinates of the remaining vertices.

Question 18 (5)

Vertices $A(0, 0)$, $B(2, 0)$, and $C(1, \sqrt{3})$ form an equilateral triangle. Rotate the triangle 60° counterclockwise about the origin. Find the new coordinates of all three vertices.

Question 19 (5)

Vertices $A(1, 0)$, $B(0, 1)$, and $C(-1, 0)$ are three vertices of a square. Find the coordinates of the fourth vertex.

Question 20 (5)

A regular octagon is centered at the origin. One vertex is at $(1, 0)$. Find the coordinates of all the remaining seven vertices.