

# **Proof By Induction**

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# Summation Results

## **Question 1 (3)**

Prove by induction that

$$\sum_{r=1}^n r = \frac{n(n+1)}{2}$$

## **Question 2 (3)**

Prove by induction that

$$\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$$

### **Question 3 (4)**

Prove by induction that

$$\sum_{r=1}^n r^3 = \left( \frac{n(n+1)}{2} \right)^2$$

### **Question 4 (4)**

Prove by induction that

$$\sum_{r=1}^n r(r+1) = \frac{n(n+1)(n+2)}{3}$$

## Question 5 (4)

Prove by induction that

$$\sum_{r=1}^n r \cdot r! = (n+1)! - 1$$

## Question 6 (5)

Prove by induction that

$$\sum_{r=1}^n r^4 = \frac{n(n+1)(2n+1)(3n^2 + 3n - 1)}{30}$$

## **Question 7 (5)**

Prove by induction that

$$\sum_{r=1}^n (2r - 1) = n^2$$

## **Question 8 (5)**

Prove by induction that

$$\sum_{r=1}^n r(r+1)(r+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

## **Question 9 (5)**

Prove by induction that

$$\sum_{r=1}^n r \cdot 2^r = (n-1)2^{n+1} + 2$$

## **Question 10 (5)**

Prove by induction that

$$\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1}$$

# Divisibility Results

## **Question 1 (3)**

Let  $f(n) = 7^n - 1$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 6 for all  $n \in \mathbb{N}$ .

## **Question 2 (3)**

Let  $f(n) = 3^{2n} - 4^n$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 5 for all  $n \in \mathbb{N}$ .

## **Question 3 (3)**

Let  $f(n) = 4^{n+1} - 1$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 3 for all  $n \in \mathbb{N}$ .

### **Question 4 (3)**

Let  $f(n) = 5^n - 1$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 4 for all  $n \in \mathbb{N}$ .

### **Question 5 (4)**

Let  $f(n) = 2^{2n} - 1$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 3 for all  $n \in \mathbb{N}$ .

### **Question 6 (3)**

Let  $f(n) = 3^{2n+1} + 1$ ,  $n \in \mathbb{N}_0$ . Prove by induction that  $f(n)$  is divisible by 4 for all  $n \in \mathbb{N}_0$ .

### **Question 7 (3)**

Let  $f(n) = 2^{2n} + 1$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 5 for all  $n \in \mathbb{N}$ .

### **Question 8 (3)**

Let  $f(n) = 7^n - 2^n$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 5 for all  $n \in \mathbb{N}$ .

### **Question 9 (3)**

Let  $f(n) = 3^{2n} - 2^{2n}$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 5 for all  $n \in \mathbb{N}$ .

## Question 10 (4)

Let  $f(n) = 2^{2n+1} + 3 \cdot 3^{n-1}$ ,  $n \in \mathbb{N}$ . Prove by induction that  $f(n)$  is divisible by 5 for all  $n \in \mathbb{N}$ .