report

November 22, 2024

1 Statistical Information Theory Lab_4_Hamming_code CW

1.1 Section 1

1.1.1 Accuracy of the Hamming Decoder for p = 0.5

The accuracy is defined as:

$$accuracy = P(decode\_hamming(r, m) = s) \quad \text{where} \quad r = n + s \mod 2$$

- r: received codeword after passing through the channel - n: transmitted codeword - s: noise

Derivation

- r has a probability p of each bit being flipped.
- `decode-hamming` only returns the correct codeword if 0 or 1 bits are flipped.
- For m parity bits, the encoded codeword has length $n = 2^m - 1$.
- The number of flipped bits in the encoded codeword can be represented as a **binomial random variable**:

$$F \sim B(n, p)$$

- The probability of correct decoding is:

$$P(decode\_hamming(r, m) = s) = P(F \leq 1) = \binom{n}{0} p^0 (1-p)^n + \binom{n}{1} p^1 (1-p)^{n-1}$$

- For p = 0.5:

$$P(F \leq 1) = \left(\frac{1}{2}\right)^n + \frac{n}{2}\left(1 - \frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n (1 + n)$$

- Substituting $n = 2^m - 1$:

$$= \frac{2^m - n + 1}{2^n} = \frac{2^{m-n}}{2^n} = 2^{-k}$$

  where: $k = 2^m - 1 - m$

1. For $m = 2:$ $\quad n = 3,\ k = 1,\ accuracy = 2^{-1} = 0.5$
2. For $m = 3:$ $\quad n = 7,\ k = 4,\ accuracy = 2^{-4} = 0.0625$
3. For $m = 4:$ $\quad n = 15,\ k = 11,\ accuracy = 2^{-11} \approx 0.000488$

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