9.5.3 Inverse Z-Transform with MATLAB

Symbolic MATLAB can be used to compute the inverse one-sided Z-transform. The function iztrans provides the sequence that corresponds to its argument. The following script illustrates the use of this function.

```
%%%%%%%%%%%%%%%%%%%
% Inverse Z-transform
%%%%%%%%%%%%%%%%%%%
syms n z
        x1 = iztrans((z * (z + 1))/((z + 0.5) * (z - 0.5)))
        x2 = iztrans((2 - z)/(2 * (z - 0.5)))
        x3 = iztrans((8 - 4 * z ^ (-1))/(z ^ (-2) + 6 * z ^ (-1) + 8))
```

The above gives the following results:

```
x1 = 3/2 * (1/2) ^ n - 1/2 * (-1/2) ^ n
x2 = -2 * charfcn[0](n) + 3/2 * (1/2) ^ n
x3 = -3 * (-1/4) ^ n + 4 * (-1/2) ^ n
```

Notice that the Z-transform can be given in positive or negative powers of z, and that when it is nonproper the function charfcn[0] corresponds to $\delta[n]$.

Partial Fraction Expansion with MATLAB

Several numerical functions are available in MATLAB to perform partial fraction expansion of a Z-transform and to obtain the corresponding inverse. In the following we consider the cases of single and multiple poles.

(1) Simple Poles

Consider finding the inverse Z-transform of

$$X(z) = \frac{z(z+1)}{(z-0.5)(z+0.5)} = \frac{(1+z^{-1})}{(1-0.5z^{-1})(1+0.5z^{-1})} \qquad |z| > 0.5$$

The MATLAB function residuez provides the partial fraction expansion coefficients or residues $r[k]$, the poles $p[k]$, and the gain k corresponding to $X(z)$ when the coefficients of its denominator and of its numerator are inputted. If the numerator or the denominator is given in a factored form (as is the case of the denominator above) we need to multiply the terms to obtain the denominator polynomial. Recall that multiplication of polynomials corresponds to convolution of the polynomial coefficients. Thus, to perform the multiplication of the terms in the denominator, we use the MATLAB function conv to obtain the coefficients of the product. The convolution of the coefficients $[1 - 0.5]$ of $p_1(z) = 1 - 0.5z^{-1}$ and $[1 \;\; 0.5]$ of $p_2(z) = 1 + 0.5z^{-1}$ gives the denominator coefficients. By means of the MATLAB function poly we can obtain the polynomials in the numerator and denominator from the zeros and poles. These polynomials are then multiplied as indicated before to obtain the numerator with coefficients $\{b[k]\}$, and the denominator with coefficients $\{a[k]\}$.