Course 4190.101.001
Discrete Mathematics
Homework 1: Logic and Proofs

March 8, 2016

Due date: March 22, 2016, 23:59

Most of the problems of this homework are selected from the textbook, *Discrete Mathematics and its Applications*. These questions can be used as the purpose of your homework ONLY. The submission file is expected to be in `pdf` format and its name should be `hw1_STUDENT-ID.pdf`. Please send an e-mail to `dshan@bi.snu.ac.kr` with the submission file before the homework deadline. The proper subject of the e-mail is `[DM HW1] YOUR_NAME, STUDENT-ID`.

1. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

   a) $p \wedge q$ b) $p \rightarrow \neg q$ c) $\neg q \rightarrow p$

   d) $\neg p \rightarrow \neg q$ e) $p \leftrightarrow \neg q$ f) $\neg p \wedge (p \vee \neg q)$

2. State the converse, contrapositive, and inverse of each of these conditional statements.

   a) If it snows tonight, the I will stay at home.

   b) I go to the beach whenever it is a sunny summer day.

   c) When I stay up late, it is necessary that I sleep until noon.

3. What is the value of x after each of these statements is encountered in a computer program, if $x = 1$ before the statement is reached?

   a) **if** $1 + 1 = 2$ **then** $x := x + 1$

   b) **if** $(2 + 3 = 5)$ *AND* $(3 + 4 = 7)$ **then** $x := x + 1$

   c) **if** $(1 + 1 = 2)$ *XOR* $(1 + 2 = 3)$ **then** $x := x + 1$

4. Each inhabitant of a remote village always tells the truth or always lies. A villager will only give a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit, the other branch leads deeper into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?

5. Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said "Carlos did it." John said "I did not do it." Carlos said "Diana did it." Diana siad "Carlos lied when he said that I did it."

   a) If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.

   b) If the authorities also know that exactly one is lying, who did it? Explain your reasoning.

6. Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

7. Show that $\{|\}$ (a set of logical operator which contains only $NAND$) is a complete collection of logcal operators.

8. Show that $p \mid (q \mid r)$ and $(p \mid q) \mid r$ not equivalent, so that the logical operator $\mid$ is not associative.

9. Let $P(x)$ be the statement "$x + 1 > 2x$." If the domain consisits of all integers, what are these truth values?

   a) $P(1)$

   b) $\exists x P(x)$

   c) $\forall x P(x)$

   d) $\exists x \neg P(x)$

10. Express the negation of these propositions using quantifiers, and then express the negation in English.

    a) Some driviers do not obey the speed limit.

    b) There is somoeone in this class who dows not have a good attitude.

11. Find a common domain for the variable x, y, z, and w for which the statement $\forall x \forall y \forall z \exists w ((w \neq x) \wedge (w \neq y) \wedge (w \neq z))$ is true and another common domain for these variables for which it is false.

12. Use quantifiers and logical connectives to express the fact that a quadratic polynomial with real number coefficients has at most two real roots.

13. Identify the error or errors in this arguemnt that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true that $\forall x P(x) \vee \forall x Q(x))$ is true.

1. $\forall x(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal instantiation from (1)
3. $P(c)$	Simplification from (2)
4. $\forall x(P(x))$	Universal generalization from (3)
5. $Q(c)$	Simplification from (2)
6. $\forall x(Q(x))$	Universal generalization from (5)
7. $\forall x P(x) \vee \forall x Q(x))$	Conjunction from (4) and (6)

14. Use rules of inference to show that if $\forall x(P(x) \to (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

15. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

    a) a proof by contraposition.

    b) a proof by contradiction.

16. Prove that $\sqrt[3]{n}$ is irrational.

17. Prove or disprove that you can use dominoes to tile a 5×5 checkerboard with three corners removed.

$$\text{One domino} = \square\blacksquare$$