CS 5000 Homework 4

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Problem 1

Is there a regular language over an alphabet of 3 symbols (e.g., $\Sigma = \{a, b, c\}$ or $\Sigma = \{0, 1, 2\}$) that has infinitely many non-regular proper subsets? If yes, state such a language and some of its proper non-regular subsets. If not, show why such a language cannot exist.

Solution

Yes, there are many regular languages over an alphabet of three symbols that have infinitely many non regular proper subsets.
An example of a regular language over an alphabet of 3 symbols ($\Sigma = \{a, b, c\}$) that has an infinite number of non regular proper subsets is :
$L = \{a^i b^j c^k \mid i, j, k \geq 0\}$
L is a regular language because an FA can be easily constructed from it.
Language L contains infinitely many non regular proper subsets shown below.

$L_1 = \{a^i b^j c^1 \mid i, j \geq 0 \text{ and } i = j\}$
$L_2 = \{a^i b^j c^2 \mid i, j \geq 0 \text{ and } i = j\}$
$L_3 = \{a^i b^j c^3 \mid i, j \geq 0 \text{ and } i = j\}$
...
$L_k = \{a^i b^j c^k \mid i, j \geq 0 \text{ and } i = j\}$

We can see that for every value of $k \in \mathbb{N}$, there is a non-regular language which is a proper subset of the regular language, $L = \{a^i b^j c^k \mid i, j, k \geq 0\}$).
The natural numbers are infinite and there exists a non regular language which is a proper subset of L for every natural number. Thus, the regular language L has infinitely many non regular languages which are proper subsets of L.

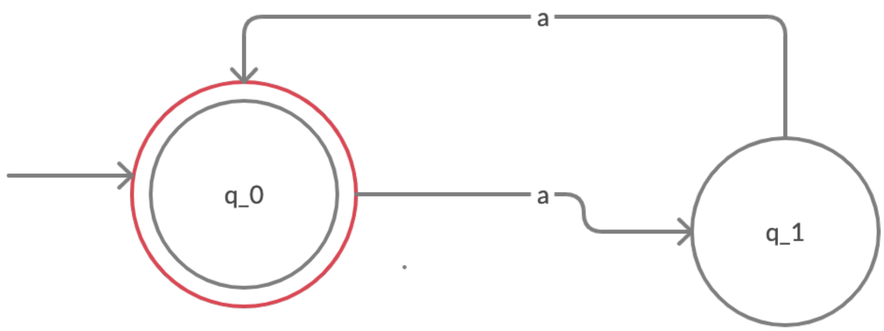
Problem 2

Review the Pumping Lemma and its proof in lecture 6. For each language, state if it's regular or not and sketch a proof of your statement.
1. $L = \{a^n b^n c^n \mid n \geq 0\}$
2. $L = \{xx^R \mid x \in \Sigma^*\}$, where x^R is the reversal of x and $\Sigma = \{a\}$
3. $L = \{xcx^R \mid x \in \Sigma^*\}$, where $\Sigma = \{a, b\}$
4. $L = \{a^n c^m b^p \mid n + m = p\}$
5. $L = \{0^n 1^m \mid n \geq m\}$

Solution

1. No, not regular.
Assume L is regular and let m be the number of states (from the pumping lemma).
Take $w = \{a^m b^m c^m\}$.
Since $w \in L$ and $\mid w \mid \geq m$, the pumping lemma must apply. Specifically,
$w = xyz$ where y $\neq \Lambda$ and $\mid xy \mid \leq m$
$\mid xy \mid$ cannot be less than or equal to m, because this would require there to be an uneven number or a's, b's, and c's, defying $w = \{a^m b^m c^m\}$. Thus we have a contradiction. The language is not regular.

2. Yes, regular.
$L = \{xx^R \mid x \in \Sigma^*\}$ must have an even number of a's. We can represent this language as a finite automata.



Therefore, the given language is regular.

3. No, not regular.
Assuming that the language is regular- Take the example, $x = abb$. Then $x^R = bba$, and we have abbcbba, a string in the language L.
By the pumping lemma, let $x = ab$, $y = bc$, and $z = bba$.
Take the string $xy^k z$. For $k = 2$, $xy^k z = abbcbcbba$.
This string does not belong to the language L as we cannot have more than one c. It does not satisfy the Pumping Lemma. Therefore, the language is not regular.

4. No, not regular.
Assuming that the language is regular- Take the example, $n = 3$, $m = 3$, $p = 6$. Then, *aaacccbbbbbb* is a string in the language.
By the pumping lemma, let $x = a$, $y = a$ and z=*acccbbbbbb*.
Take the string, $xy^k z$. For $k = 2$, $xy^k z = aaaacccbbbbbb$.
This string is not in the language as $4 + 3 \neq 6$. It does not satisfy the Pumping Lemma. Therefore, the language is not regular.

5. No, not regular.
Assuming that the language is regular- Take the example, $n = 4$, $m = 3$. Then, 0000111 is a string in the language.

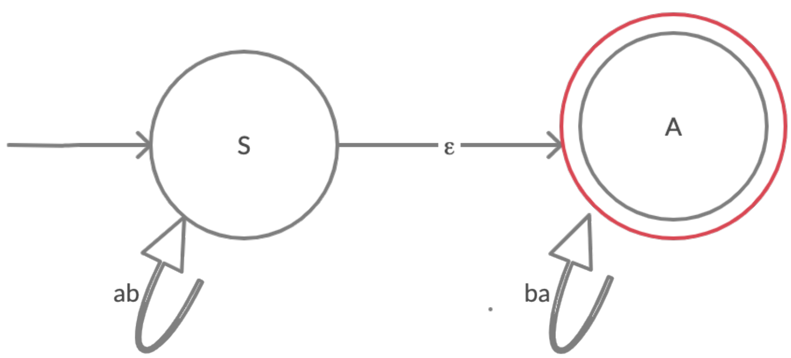
By the pumping lemma, let $x = 000$, $y = 01$ and $z = 11$.
Now take the string xy^kz. For $k = 2$, $xy^kz = 000010111$
This string is not in the language as it does not have all 1's then all 0's. It does not satisfy the Pumping Lemma. Therefore, the language is not regular.

Problem 3

3.1
Consider a grammar $G = (\{S, A\}, \{a, b\}, S, P)$, where the set of productions P includes the following productions: $S \to abS \mid A$ and $A \to baA \mid \epsilon$. Show that $L(G)$ is regular. Hint: Use the theorem on slide 19 in Lecture 7 and its proof.
3.2
Consider a grammar $G = (\{S,\}, \{a, c, d\}, S, P)$, where the set of productions P includes the following productions: $S \to Scd \mid a \mid \epsilon$. Show that $L(G)$ is regular. Hint: Review slide 33 in Lecture 7 and use the theorem on slide 34 and its proof.

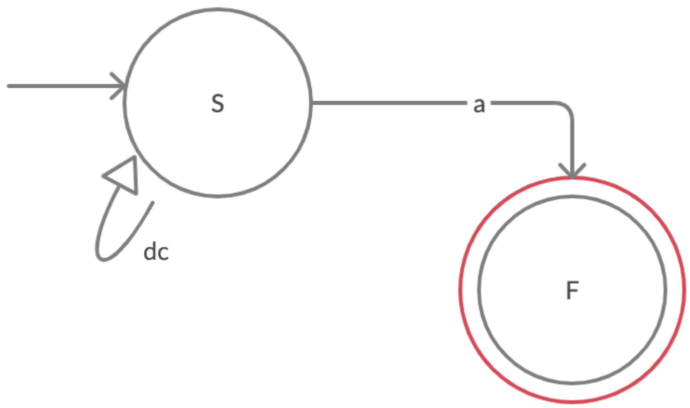
Solution

3.1
If G is a right linear grammar, L(G) is regular, so we will construct an NFA, M, such that L(M) = L(G) by adding a corresponding path to the FA for every type of production. The FA is as follows:

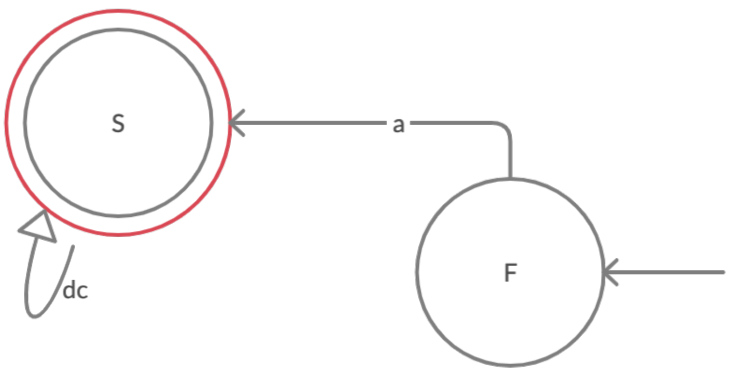


3.2
If G is a left linear grammar, then L(G) is regular. We will convert G to a right linear grammar G_R such that $L(G) = [L(G_R)]^R$ then construct an NFA and reverse the directions of the arcs. The reversal of the given grammar is:
$S \to dcS \mid a \mid \epsilon$

The resulting FA is as follows:



We now must reverse the directions of the arrows and swap the start and final states. This produces the following FA:

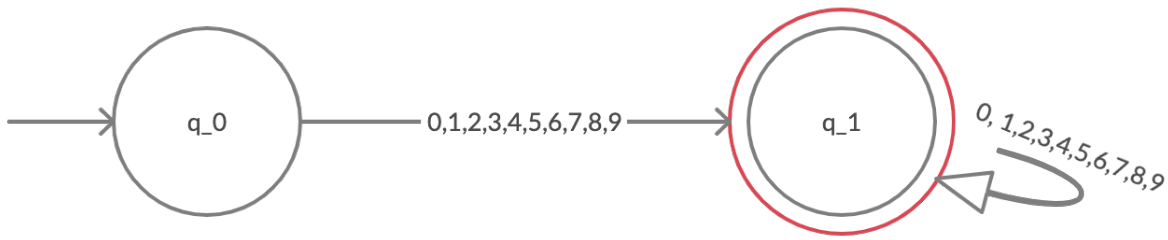


Problem 4

Let L be the set of all strings consisting of one or more digits, where each digit is one of the symbols in $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Show that L is regular by constructing a right linear grammar for it. See slide 19 in Lecture 7 for the definition of a right linear grammar.

Solution

The FA for L is as follows:



The right linear grammar for L is as follows.
$S \to 0S \mid 1S \mid 2S \mid 3S \mid 4S \mid 5S \mid 6S \mid 7S \mid 8S \mid 9S \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$
A right linear grammar was constructed for L, so L is regular.

Problem 5

Recall that a non-regular grammar is a grammar that is neither right linear nor left linear. See slides 19 and 32 in lecture 7 for the definitions of right linear and left linear grammars. Show that a regular language can be generated by a non-regular grammar. Hint: take a simple regular language and construct a non-regular grammar for it.

Solution

Take the example language, $R(L) = \{a, b\}$
We can construct the non regular grammar for that language.
$S \to AB$
$A \to a$
$B \to b$
From that grammar, we can derive a regular language.
$S \to AB$
$S \to aB$ (by above $\{A \to a\}$
$S \to ab$ (by above $\{B \to b\}$

$S \to ab$ is a regular language.

We have shown that a non-regular grammar can generate a regular language.