CS 5000 Homework 3

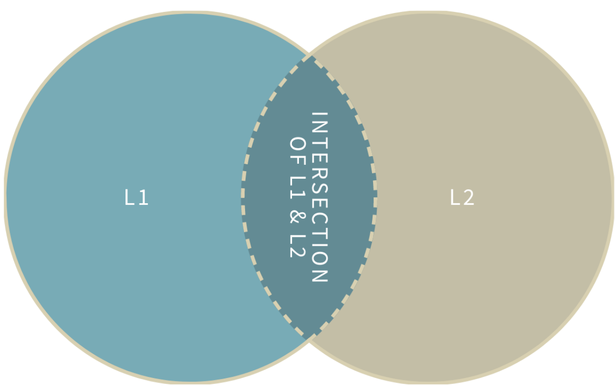
Sally Devitry (A01980316)

Problem 1

Let Σ be an alphabet. A finite language over Σ is a finite set of strings over Σ, i.e., a finite subset of Σ^* . Let L_1 be a regular language over Σ and L_2 be a finite language over Σ. Show that $L_1 \cap L_2$ is regular.

Solution

If L_1 is a regular language, then $L_1 \cap L_2$ is regular even if L_2 is not regular because the intersection of both languages must be completely contained in L_1. The diagram below makes it extra clear that any language accepted by $L_1 \cap L_2$ is always contained in L_1.



Therefore, if L_1 is a regular language, $L_1 \cap L_2$ is also a regular language.

Problem 2

Let Σ be an alphabet. Let $L_1 = \Sigma^*$. Let L_2 be a regular language over Σ. Show that $L_2 - L_1$ is regular.

Solution

Let M_1 and M_2 be the DFA's that accept the regular languages, L_1 and L_2, respectively.
$M_2 = (Q_1, \Sigma_1, \delta_1, q_0^1, F_1)$
$M_2 = (Q_2, \Sigma_2, \delta_1, q_0^2, F_2)$
To prove that $L_1 - L_2$ is a regular language we must construct a DFA, $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes the language $L_1 - L_2$

Q = pair of states, one from M_1 and one from M_1 (a.k.a. $Q_1 \times Q_2$)
$Q = \{(q_1, q_2) \mid q_1 \in Q_1$ and $q_2 \in Q_2$
$q_0 = (q_0^1, q_0^2)$
$\delta((q_a^1, q_b^2), x) = (\delta(q_a^1, x), \delta(q_b^2, x))$
$F = \{(q_1, q_2) \mid q_1 \in F_1$ and $q_2 \notin F_2\}$

The DFA for $L_1 - L_2$ accepts when M_1 accepts and M_2 rejects. A DFA can be constructed for $L_1 - L_2$, so $L_1 - L_2$ is indeed a regular language.

Problem 3

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA, where
1. $Q = \{q_0, q_1, q_2, q_3, q_4\}$;
2. $\Sigma = \{0, 1\}$;
3. $\delta(q_0, 0) = q_1; \delta(q_0, 1) = q_3; \delta(q_1, 0) = q_2; \delta(q_1, 1) = q_4; \delta(q_2, 0) = q_1; \delta(q_2, 1) = q_4; \delta(q_3, 0) = q_2; \delta(q_3, 1) = q_4; \delta(q_4, 0) = q_4; \delta(q_4, 1) = q_4$.
4. q_0 is the start state;
5. $F = \{q_2, q_4\}$.
Minimize M with the algorithm of the Myhill-Nerode Theorem (Lecture 5) and draw your minimal DFA.

Solution

To begin minimizing the DFA using Myhill-Nerode Theorem, we will create a table of all states. Half of the table is ignored (denoted with black squares) because the pairs are repeated.

A checkmark in the below table means that in that pair, one is a final state and the other is a nonfinal state.

	q_0	q_1	q_2	q_3	q_4
q_0	■	■	■	■	■
q_1		■	■	■	■
q_2	✓	✓	■	■	■
q_3			✓	■	■
q_4	✓	✓		✓	■

Next, we must check all unmarked pairs (P, Q) to see if $[\delta(P, x), \delta(Q, x)]$ is marked (x is any transition). If so, (P, Q) should also be marked.

For the unmarked pair (q_0, q_1), on 0 from both states we get the pair (q_1, q_2) which is marked. So we will mark (q_0, q_1)

For the unmarked pair (q_0, q_3), on 0 from both states we get the pair (q_1, q_2) which is marked. So we will mark (q_0, q_1)

For the unmarked pair (q_1, q_3), on 0 or 1 from both states we get the pair (q_2, q_2) or (q_4, q_4) which is not a valid pair, so we need not add a mark.

For the unmarked pair (q_2, q_4), on 0 from both states we get the pair (q_1, q_4) which is marked. So we will mark (q_2, q_4)

The new table looks as follows:

	q_0	q_1	q_2	q_3	q_4
q_0	■	■	■	■	■
q_1	✓	■	■	■	■
q_2	✓	✓	■	■	■
q_3	✓		✓	■	■
q_4	✓	✓	✓	✓	■

The pair (q_1, q_3) is unmarked, meaning we can combine these states into an individual state. The resulting minimized DFA is as follows: