CS 5000 Homework 1

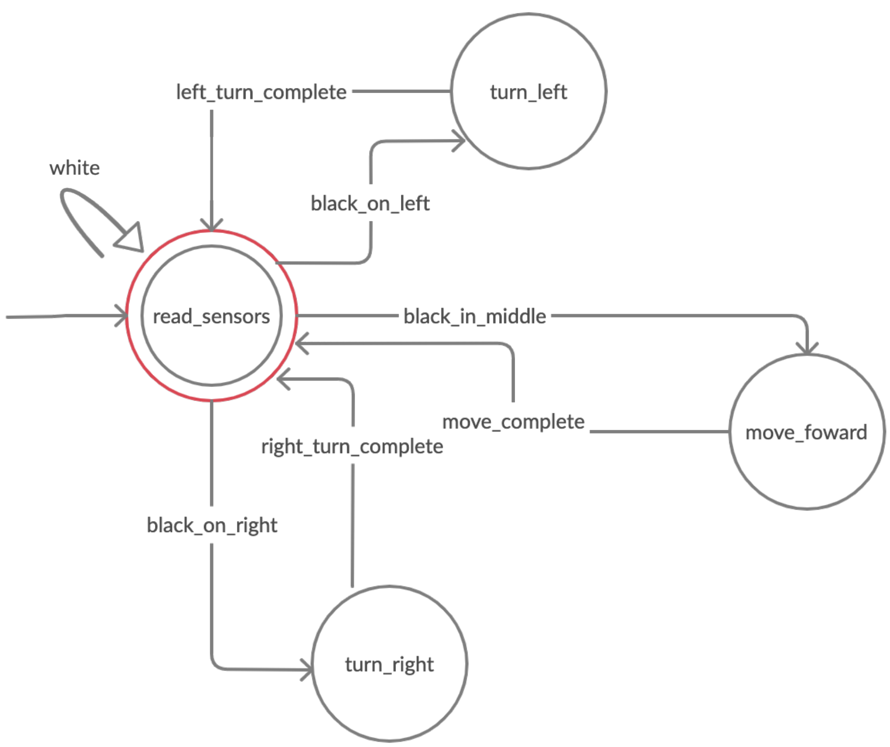
Sally Devitry (A01980316)

Problem 1

Design an FA to have a Junun robot follow a black line on the white background. You can design either a DFA or an NFA. Specify your FA in a graph that clearly identifies the start state, each transition, and the final states (if there are any).

Solution

Q = {read_sensors, turn_left, turn_right, move_forward}
Σ = {black_on_left, black_on_right, black_on_middle, left_turn_complete, right_turn_complete, move_complete, white}

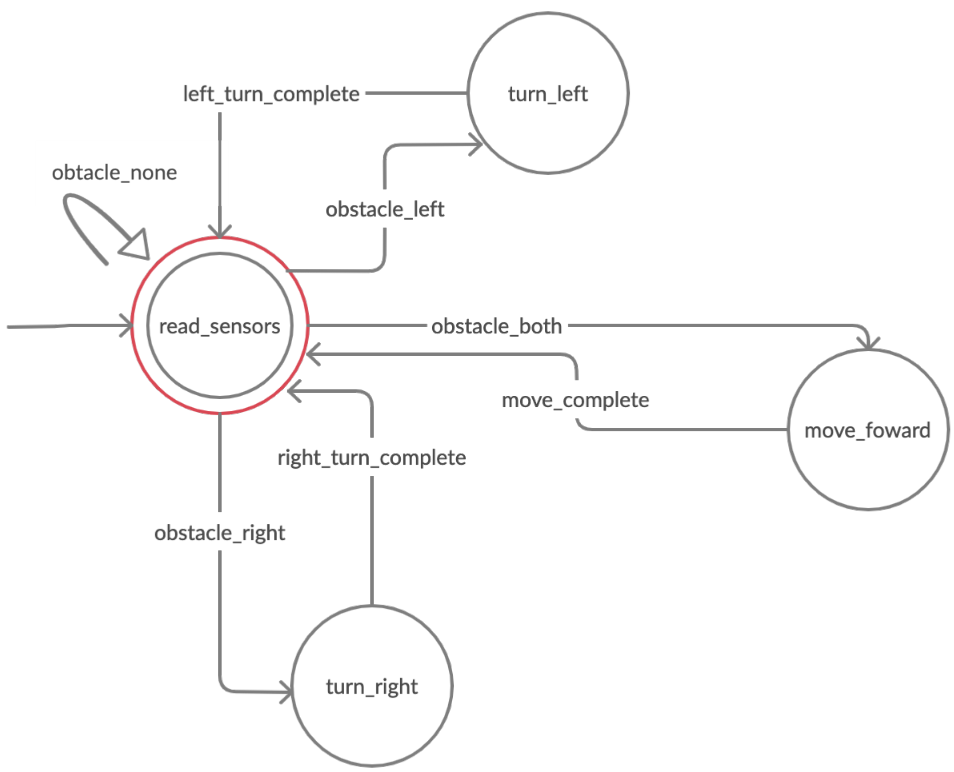


Problem 2

Design an FA to have a Junun robot follow a blob with its two front sonars. You can design either a DFA or an NFA. Specify your FA in a graph that clearly identifies the start state, each transition, and the final states (if there are any).

Solution

Q = {read_sensors, turn_left, turn_right, move_forward}
Σ = {obstacle_left, obstacle_right, obstacle_both, obstacle_none, left_turn_complete, right_turn_complete, move_complete }

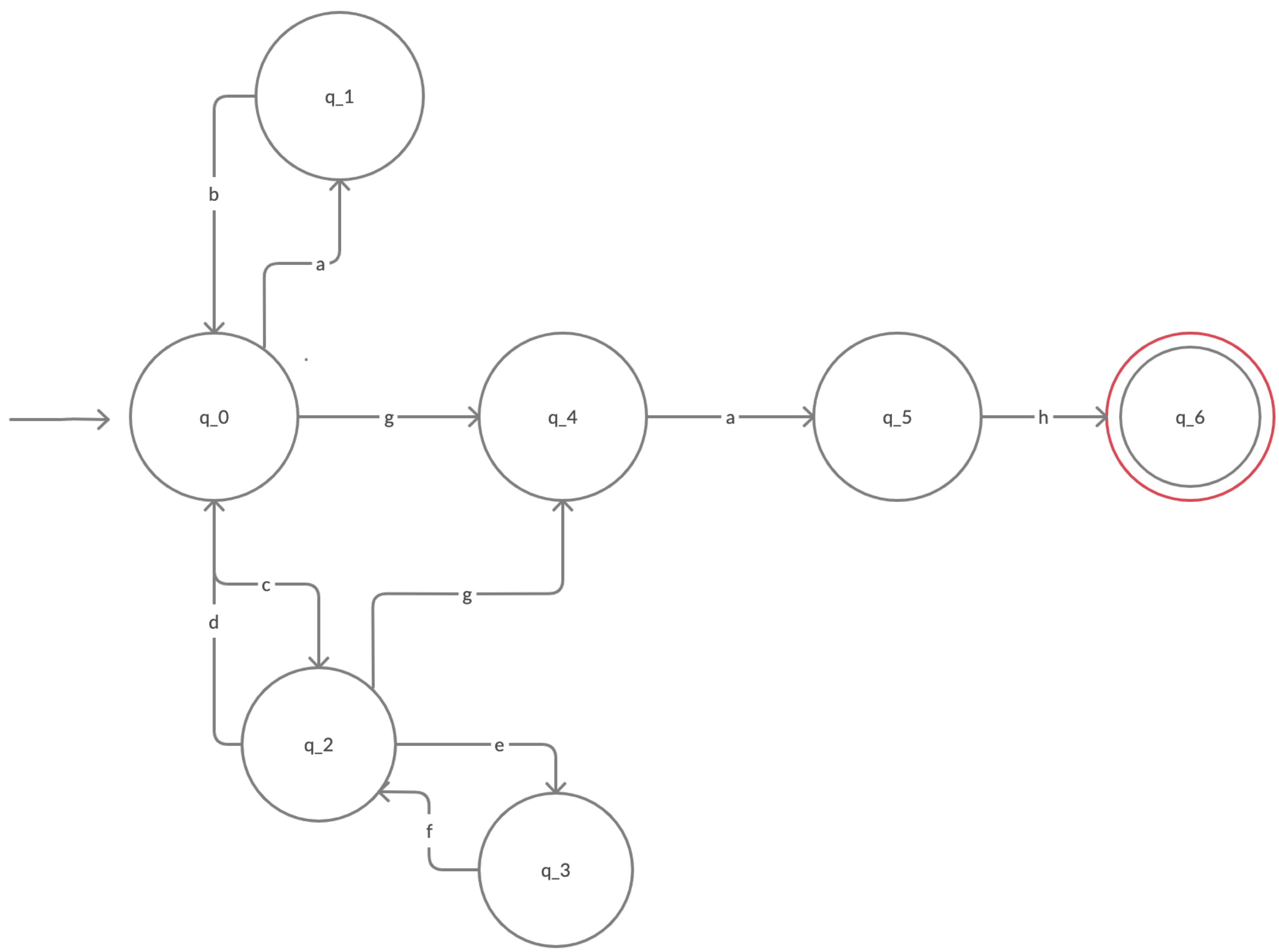


Problem 3

Design a DFA for the camera-arm unit that solves the Sussman anomaly. You do not have to be too formal when you specify your DFA. A clear graph drawing where all states and transitions are specified is perfectly acceptable. You may assume that your camera-arm unit has the same physical and perceptual capabilities as the camera-arm unit discussed in Lecture 2.

$q_0 = \{$ on(B, T), on(A, T) on(C, A) clear(B), clear(C) $\}$
$q_1 = \{$ on(A, T), on(C, A), on(B, C), clear(B) $\}$
$q_2 = \{$ on(A, T), on(B, T), on(C, B), clear(A) clear(C) $\}$
$q_3 = \{$ on(B, T), on(C, B), on(A, C), clear(A) $\}$
$q_4 = \{$ on(A, T), on(B, T), on(C, T), clear(A), clear(B), clear(C) $\}$
$q_5 = \{$ on(A, T), on(C, T), on(B, C), clear(A), clear(B) $\}$
$q_6 = \{$ on(C, T), on(B, C), on(A, B), clear(A) $\}$
$F = \{q_6\}$

$a =$ puton(B, C)
$b =$ puton(B, T)
$c =$ puton(C, B)
$d =$ puton(C, A)
$e =$ puton(A, C)
$f =$ puton(A, T)
$g =$ puton(C, T)
$h =$ puton(A, B)



Solution

Problem 4

Use the ϵ-closure to construct an NFA M that has no ϵ-transitions and is equivalent to M (i.e., $L(M) = L(M)$). Specify M' both with a graph similar to the one in Figure 1 and the table for its transition function δ' similar to Table 1.

Solution

The NFA, M, without ϵ-transitions is as follows:

Let
$r_0 = \{q_0, q_1, q_2\}$
$r_1 = \{q_1, q_2\}$
$r_2 = \{q_2\}$

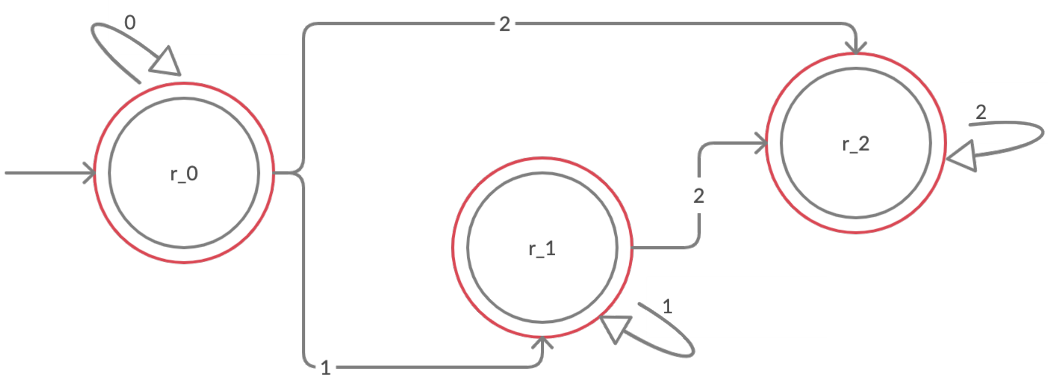


Table 1: Transition function δ for the NFA immediately above. The symbol $\emptyset$ denotes the empty set.

states/inputs	0	1	2	ϵ
r_0	$\{r_0\}$	$\{r_1\}$	$\{r_2\}$	$\{r_0\}$
r_1	$\{\emptyset\}$	$\{r_1\}$	$\{r_2\}$	$\{r_1\}$
r_2	$\{\emptyset\}$	$\{\emptyset\}$	$\{r_2\}$	$\{r_2\}$

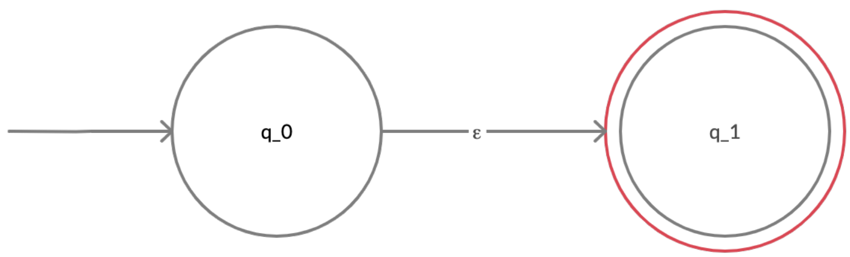
Problem 5

Let M be an NFA with ϵ-transitions and let M' be an equivalent NFA without ϵ-transitions obtained with the ϵ-closure. Show that $\epsilon \in L(M)$ if and only if $\epsilon \in L(M')$.

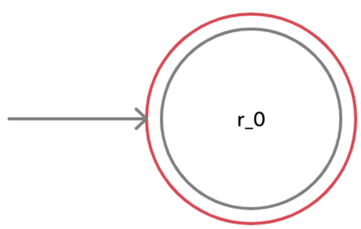
Solution

For ϵ to be in the language of M, the ϵ-closure of the start state must include the end state. For ϵ to be in the language of M, all edges on the path from the start state to to the accepting state must have label ϵ. This translates into an intersection of the accepting states on all states in M'.
A simple example demonstrates this point.

The following is an NFA with ϵ transitions.



After converting the above NFA into an equivalent NFA with no ϵ transitions, we have the following, where $r_0 = \{q_0, q_1\}$



Given F as the set of accepting states, the following is true and shows that $\epsilon \in L(M)$ if and only if $\epsilon \in L(M')$.

if $\epsilon \in L(M')$ then $q_0 \in F$
if $q_0 \in F$ then $\epsilon \in L(M)$

if $\epsilon \notin L(M')$ then $q_0 \notin F$
if $q_0 \notin F$ then $\epsilon \notin L(M)$