Revision Notes by Sally Yang

MICROECONOMIC PRINCIPLES

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CONSTRAINED OPTIMISATION

PRODUCTION FUNCTION $\phi(\underline{z})$ ← input vector $(z_1, z_2, \ldots z_m)$

* also called "techniques"

Property : $\phi(\underline{0}) = 0$

↳ No need to consider -ve π bc firm can always stop production and make $\pi = 0$

① Cobb-Douglas
- $\prod z_i^{\alpha_i}$ ⇒ Convert to $\phi(\underline{z})$ (monotonic transformation) $= \sum \alpha_i \ln z_i$

* Will same MRTS and same isoquant shape
* But don't forget to convert q back!

② Perfect Complements (Leontief)
- $\min\{\alpha_1 z_1, \ldots, \alpha_m z_m\}$

③ Perfect Substitutes

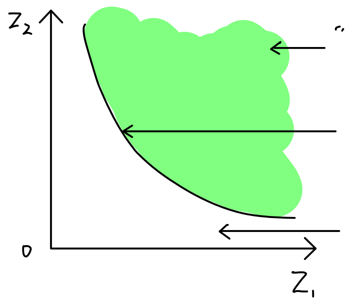
④ Constant Elasticity of Substitution (CES)

⑤ Non-Convex to the Origin

⑥ Quasilinear

INPUT REQUIREMENT SET $Z(q) = \{\underline{z} : \phi(\underline{z}) \geq q\}$, for some arbitrary q

The set of all feasible $\underline{z}$ (input sets / techniques)



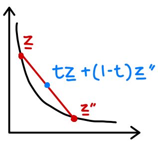
"Inside" : Feasible but inefficient

Boundary : Feasible and technically efficient $(\phi(\underline{z}) = q)$. Contour of q

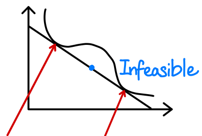
"Outside" : Infeasible

CONVEXITY Implies divisibility

- Any two $\underline{z}$ and $\underline{z}''$ in $Z(q)$, draw a straight line connecting them, and any point on that line $t\underline{z} + (1-t)\underline{z}''$, $0 < t < 1$, also lie in $Z(q)$



- Implies divisibility — if $\underline{z}$ and $\underline{z}''$ are feasible, so is a mixture of them
- Combination of two techniques may produce more output



- Non-convexity implies some degree of indivisibility — may have to choose between techniques
- Not strictly convex ⇒ interval of z-values

Q-ISOQUANT $\{\underline{z} : \phi(\underline{z}) = q\}$.

The boundary of $Z(q)$.

- Can also rearrange to have a particular z_i on LHS for substitution purposes.
- Can touch the axis if ≥ 1 output is not essential.
- Flat segments — locally perfect substitutes in production

MARGINAL PRODUCT

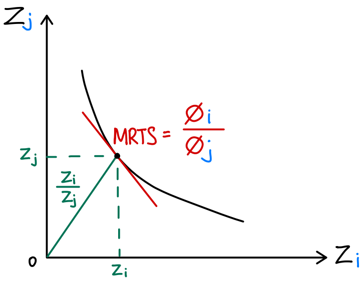
MARGINAL PRODUCT of z_i is $MP_i = \phi_i(\underline{z}) = \frac{\partial \phi(\underline{z})}{\partial z_i}$

- Keep the other zs constant (can lump them into some constant A) first

MARGINAL RATE OF TECHNICAL SUBSTITUTION

If ϕ is differentiable at $\underline{z}$, MRTS of input j for $i = \frac{\phi_i(\underline{z})}{\phi_j(\underline{z})}$ or $-\frac{dz_j}{dz_i}$ ← rewrite ϕ in terms of z_j

- Rate at which you trade off one input against the other along the isoquant, given constant q. If you $\uparrow z_j$ 1 unit, how much $\downarrow z_i$?
- Relative value / implicit price of z_j in terms of z_i. Higher MRTS $\Rightarrow$ smaller relative use of z_j
- Slope of boundary at $\underline{z}$



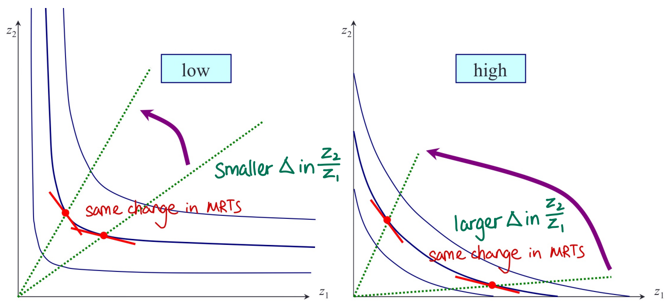
INPUT RATIO

INPUT RATIO $\frac{z_2}{z_1}$

- Slope of the ray through $\underline{z}$

ELASTICITY

$$\sigma_{ij} = \sigma_{ji} = -\frac{\partial \log\left(\frac{z_j}{z_i}\right)}{\partial \log\left(\frac{\phi_j(\underline{z})}{\phi_i(\underline{z})}\right)} = \frac{\left(\frac{\phi_j(\underline{z})}{\phi_i(\underline{z})}\right)}{\left(\frac{z_j}{z_i}\right)}\left[\frac{\partial\left(\frac{z_j}{z_i}\right)}{\partial\left(\frac{\phi_j(\underline{z})}{\phi_i(\underline{z})}\right)}\right] = \frac{\%\Delta \text{ in input ratio}}{\%\Delta \text{ in MRTS}} \geq 0$$

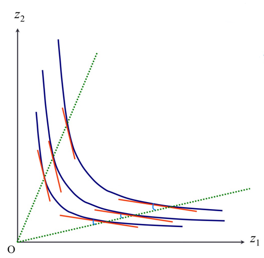


HOMOTHETIC / HOMOGENEOUS

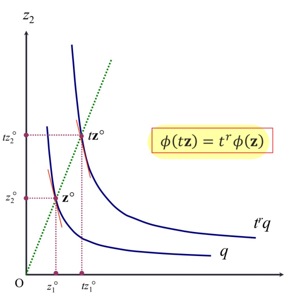
A homothetic function $g := \varphi(f)$, where f is homogeneous and $\varphi: \mathbb{R} \to \mathbb{R}$.

- For any (i,j), g has same slope (MRTS) for all vectors in a given direction (ray)

$$\frac{g_i(\underline{z})}{g_j(\underline{z})} = \frac{g_i(t\underline{z})}{g_j(t\underline{z})}$$



Any ray through the origin (input ratio), all tangents have the same MRTS



Homogeneous of degree r

↱ e.g. doubling input more than doubles output!

Increasing returns to scale (IRTS): $\phi(t\underline{z}) > t\phi(\underline{z})$ for any $t>1$ $r>1$

Decreasing returns to scale (DRTS): $\phi(t\underline{z}) < t\phi(\underline{z})$ for any $t>1$ $r<1$

Constant returns to scale (CRTS): $\phi(t\underline{z}) = t\phi(\underline{z})$ for any $t>0$ $r=1$

↳ (LR)MC is constant!!

LRAC = LRMC

LRTC linear

- Specify localised IRTS by limiting t
- Typically, IRTS when q is small; DRTS when q is large
- If ϕ is concave, $Z(q)$ is concave $\forall q$

TYPES OF U & x^*

Lagrangian
λ, x_i^*
Total expenditure
Min Cost f^n
Conditional (firm) / Hicksian (consumer) dd

Budget Constraint: $\sum_i p_i x_i \leq y$

"preference parameter" make $\sum_i \alpha_i = 1$ first!

COBB-DOUGLAS

$\prod_i x_i^{\alpha_i}$

Monotonic transformation to $\alpha_1 \log x_1 + \alpha_2 \log x_2 + \ldots$

new, transformed y

U-max: $\mathcal{L} = \sum_i \alpha_i \log x_i + \lambda(y - \sum_i p_i x_i)$

$$\begin{cases} \frac{\alpha_i}{x_i} = \lambda p_i \\ \sum_i p_i x_i = y \end{cases} \Rightarrow \begin{cases} \lambda = \frac{\sum \alpha_i}{y} = \frac{1}{y} \\ x_i^* = \frac{\alpha_i y}{p_i \sum \alpha_i} = \frac{\alpha_i y}{p_i} \end{cases}$$

Always spends α_i of income on x_i^*

transformed alr

C-min: $\mathcal{L} = \sum_i p_i x_i + \lambda(\nu - \sum_i \alpha_i \log x_i)$

$C(\underline{p}, \nu) = Ae^{\nu} p_1^{\alpha_1} \cdots p_n^{\alpha_n}$, $A = \alpha_1^{-\alpha_1} \cdots \alpha_n^{-\alpha_n}$

$h_i^* = \frac{C(\underline{p}, \nu)\alpha_i}{p_i}$

Subsistence minimum

Must have β_i of x_i first before new units of x_i bring Joy™

COBB-DOUGLAS WITH MINIMUM CONSUMPTION

$\prod (x_i - \beta_i)^{\alpha_i}$ make $\sum_i \alpha_i = 1$ first!

$\mathcal{L} = \sum_i \alpha_i \log(x_i - \beta_i) + \lambda(y - \sum_i p_i x_i)$

$x_i^* = \beta_i + \frac{\alpha_i(y - \sum p_i \beta_i)}{p_i}$ ← Discretionary income - share of income after spending on min. consumption

minimum

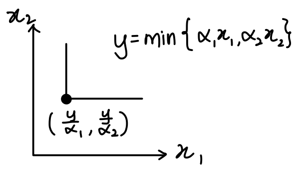
$C(\underline{p}, \nu) = Ae^{\nu} p_1^{\alpha_1} \cdots p_n^{\alpha_n} + \sum_{i=1}^{n} p_n \beta_n$, $A = \alpha_1^{-\alpha_1} \cdots \alpha_n^{-\alpha_n}$

$h_i^* = \beta_i + \frac{\alpha_i (Ae^{\nu} p_1^{\alpha_1} \cdots p_n^{\alpha_n})}{p_i}$

PERFECT COMPLEMENTS

$\min\{\alpha_1 x_1, \ldots, \alpha_m x_m\}$

No need to make $\Sigma = 1$



$\alpha_1 x_1 = \alpha_2 x_2 = \ldots = y$

$x_i^* = \frac{y}{\alpha_i}$

Total expenditure is $y \sum \frac{p_i}{\alpha_i}$

$C(\underline{p}, \nu) = \nu \sum \frac{p_i}{\alpha_i}$

$h_i^* = \frac{\nu}{\alpha_i}$

PERFECT SUBSTITUTES

$\sum_i \alpha_i x_i$

If $\frac{\alpha_1}{\alpha_2} > \frac{p_1}{p_2}$ $\quad x_1^* = \frac{y}{\alpha_1}$, $x_2^* = 0$

$\frac{\alpha_1}{\alpha_2} < \frac{p_1}{p_2}$ $\quad x_1^* = 0$, $x_2^* = \frac{y}{\alpha_2}$

$\frac{\alpha_1}{\alpha_2} = \frac{p_1}{p_2}$ $\quad x_1^* \in [0, \frac{y}{\alpha_1}]$, $x_2^* = \frac{y - p_1 x_1^*}{\alpha_2} \in [0, \frac{y}{\alpha_2}]$

Total expenditure is $\min\{\frac{p_1 y}{\alpha_1}, \ldots, \frac{p_m y}{\alpha_m}\}$

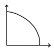
$C(\underline{p}, \nu) = \min\{\frac{p_1 \nu}{\alpha_1}, \ldots, \frac{p_m \nu}{\alpha_m}\}$

h_i^* = same as x_i^* but replace y with ν

CONVERT TO PRODUCTION

$y \to q$
$x \to z$
$w \to p$
MRS → MRTS
expenditure → cost C
$\nu \to q$

NON-CONVEX TO ORIGIN

 $\sum_i \alpha_i x_i^2$

$$(x_1^*, x_2^*) = \begin{cases} \left(\sqrt{\frac{y}{\alpha_1}}, 0\right) & \text{if } \frac{p_1}{p_2} < \sqrt{\frac{\alpha_1}{\alpha_2}} \\ \left(0, \sqrt{\frac{y}{\alpha_2}}\right) & \text{if } \frac{p_1}{p_2} > \sqrt{\frac{\alpha_1}{\alpha_2}} \\ \left(\sqrt{\frac{y}{\alpha_1}}, 0\right) \text{ or } \left(0, \sqrt{\frac{y}{\alpha_2}}\right) & \text{if } \frac{p_1}{p_2} = \sqrt{\frac{\alpha_1}{\alpha_2}} \end{cases}$$

Total expenditure is $\min\left\{p_1\sqrt{\frac{y}{\alpha_1}}, \dots, p_m\sqrt{\frac{y}{\alpha_m}}\right\}$

$C(p,v) = \min\left\{p_1\sqrt{\frac{v}{\alpha_1}}, \dots, p_m\sqrt{\frac{v}{\alpha_m}}\right\}$

h_i^* = same as x_i^* but replace y with v

CONSTANT ELASTICITY OF SUBSTITUTION

This general case links non-convex, perfect substitutes, perfect complements and Cobb-Douglas

$y = (\alpha_1 x_1^\beta + \alpha_2 x_2^\beta + \dots)^{\frac{1}{\beta}} = \left(\sum \alpha_i x_i^\beta\right)^{\frac{1}{\beta}}$

$\frac{\phi_i}{\phi_j} = \left(\frac{z_i}{z_j}\right)^{1-\beta}$

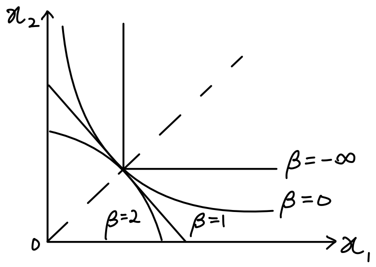
MRTS m — input ratio r

Elasticity as $\%\Delta$ in input ratio given a $\%\Delta$ in input prices (MRTS)

$\sigma = \frac{dr}{r} \Big/ \frac{dm}{m} = \frac{1}{1-\beta}$ is constant!

$-\frac{\partial \log(\frac{z_2}{z_1})}{\partial \log(\frac{\phi_1}{\phi_2})}$

Non-convex	$\beta = 2$	$\sigma = -1$
Perfect substitute	$\beta \to 1$	$\nexists\, \sigma$
Cobb-Douglas	$\beta \to 0$	$\sigma = 1$
Perfect complement	$\beta \to -\infty$	$\sigma = 0$



$x_i^* = \frac{y\left(\frac{p_i}{\alpha_i}\right)^{-\sigma}}{\sum p_i\left(\frac{p_i}{\alpha_i}\right)^{-\sigma}}$ Interior and tangency. works only for $\beta \in (-\infty, 0) \cup (0, 1)$

$h_1^* = y\left[\alpha_1 + \alpha_2\left(\frac{\alpha_1}{\alpha_2}\frac{p_2}{p_1}\right)^{1-\sigma}\right]^{-\frac{1}{\beta}}$ Decreasing in p_1 if $\beta < 1$
Homogeneous of degree 0

STAGE I OPTIMISATION - COST MINIMISATION

PROFIT $\Pi = pq - \sum_{i=1}^{m} w_i z_i$

- Assume perfect competition so all prices (p and $\underline{w}$) are exogenously given (fixed)
  - Take a given q — so pq is constant. Just minimise cost

ISOCOST LINE $C = \sum_{i=1}^{m} w_i z_i$, for some given cost C

1. Identify Objective function
2. Identify Constraints
3. Draw isoquants $\xrightarrow{\text{or}}$ check that ϕ is convex: rewrite ϕ to be $z_j = \dots$ check $\frac{\partial z_j}{\partial z_i} < 0$, $\frac{\partial^2 z_j}{\partial z_i^2} > 0$ and differentiable
4. Identify type of solution (corner/interior/unconventional?)
5. Identify relevant method (e.g. Lagrangian)
6. Solve

LAGRANGIAN $\mathcal{L}(\underline{z}, \lambda; \underline{w}, q) = \sum_{i=1}^{m} w_i z_i + \lambda(q - \phi(\underline{z}))$ for some q and $\underline{w}$, $z_i \geq 0 \ \forall i$.

- Get First Order Conditions (i.e. set partial derivatives $= 0$)
- Use if Z is strictly convex ($\Leftrightarrow \phi$ is strictly concave) and tangent is defined (i.e. ϕ differentiable)
  - will get tangency condition $\underbrace{\frac{\phi_i(\underline{z}^*)}{\phi_j(\underline{z}^*)}}_{\text{MRTS}} = \underbrace{\frac{w_i}{w_j}}_{\text{Price ratio}}$ ($\leq$, if z_j is not used)
  - Can also try if linear (convex, but not strictly) $\rightarrow$ but may get corner solution

COST-MINIMISING INPUT $z_i^* = H^i(\underline{w}, q)$

- H^i is the conditional input demand function — demand for z_i depending on the level q (See later)

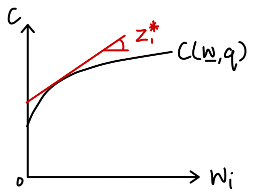
MINIMUM COST FUNCTION $C(\underline{w}, q) = \min_{\{\underline{z} \geq 0,\ \phi(\underline{z}) \geq q\}} \sum_{i=1}^{m} w_i z_i = \sum_{i=1}^{m} w_i H^i(\underline{w}, q)$, $q \geq 0$

$$= \sum_{i=1}^{m} w_i z_i^* + \lambda^* \big(q - \underbrace{\phi(\underline{z}^*)}_{\text{diff}}\big)$$

COST-MINIMISING VALUE FOR LAGRANGE MULTIPLIER $\lambda^* = \lambda^*(\underline{w}, q) = C_q(\underline{w}, q)$ = marginal cost of output at q

PROPERTIES

Just draw
- Non-decreasing and continuous in $\underline{w}$ (may not $\uparrow$ with w_i if w_i is non-essential and too expensive)
- Concave in $\underline{w}$
- Homogeneous of degree one in $\underline{w}$. $C(t\underline{w}, q) = t\,C(\underline{w}, q)$
- Strictly increasing in at least one w_i
- Strictly increasing in q (if continuous)
- Shephard's Lemma: $\frac{\partial C(\underline{w}, q)}{\partial w_i} = z_i^* = H^i(\underline{w}, q)$



STAGE 2 OPTIMISATION - OUTPUT OPTIMISATION

Once the appropriate input policy conditional upon an arbitrary output level is known $(C(\underline{w},q))$ choose the appropriate output level (q)

Sub min cost f^n to get the problem:

$$\max_{\{q\geq 0\}} pq - C(\underline{w},q)$$

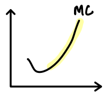
1. MARGINAL COST

$\boxed{\text{DIFFERENTIATE}}$ wrt q, If $q^* > 0$, $p = C_q(\underline{w},q^*)$; If $q^* = 0$, $p \leq C_q(\underline{w},q^*)$ so price $\leq$ MC

Need to check 2nd derivative $\frac{\partial^2 C(\underline{w},q)}{\partial q^2} = \frac{\partial}{\partial q}\left(p - C_q(\underline{w},q)\right) = -C_{qq}(\underline{w},q) \leq 0$ (i.e. maximum)

$$\Rightarrow C_{qq}(\underline{w},q) \geq 0$$

★ Since $C_q(\underline{w},q) = \lambda^* = $ MC curve, derivative ≥ 0 $\Rightarrow$ MC must be rising/constant

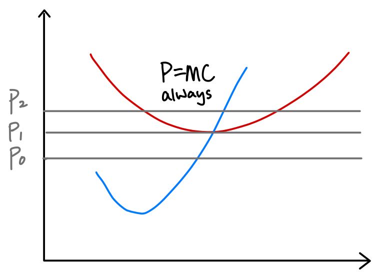


2. AVERAGE COST

$\Pi = pq - C(\underline{w},q) \geq 0$ Assume no $-$ve Π

$$\Downarrow$$

$$AC = \frac{C(\underline{w},q)}{q} \leq p \quad \text{if } q > 0$$



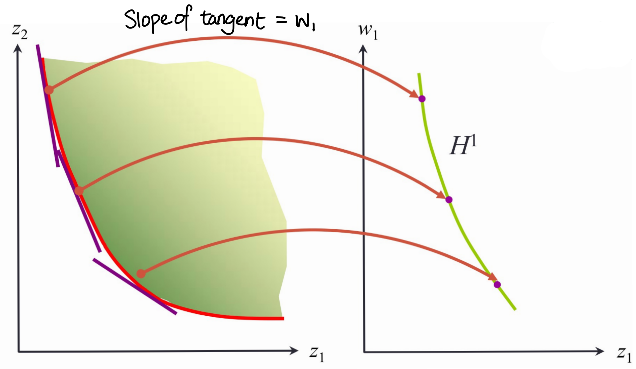
$p > AC$. $\Pi > 0$

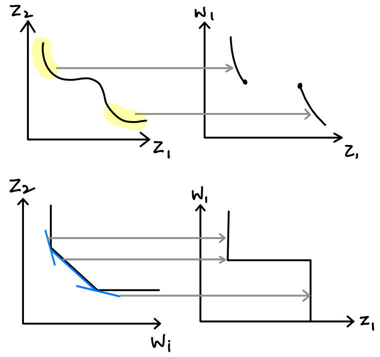
$p = AC$. $\Pi = 0$

$p < AC$. Then $q^* = 0$, $\Pi = 0$.

FIRMS' CONDITIONAL INPUT DEMAND & SUPPLY

- Have convex Z, smooth boundary
- Keep q and w_2 constant; vary w_1 (hence slope of isocost changes)





CONDITIONAL INPUT DEMAND $z_i^* = H^i(\underline{w}, q)$, where $(z_1^*, \ldots, z_m^*)$ are the cost-min inputs for $\underline{w}$ and q

- Demand for input z_i given $\underline{w}$ and q (i.e. conditional upon output q)
- $C_{ji} = C_{ij}$ so $H^i_j = H^j_i$ → Effect of w_i on conditional dd for z_j = Effect of w_j on conditional dd for z_i
- "Downward-sloping" (or constant): Shephard's Lemma: $C_i(\underline{w}, q) = H^i(\underline{w}, q)$

$C_{ii}(\underline{w}, q) = H^i_i(\underline{w}, q)$

Since C is concave, $C_{ii} \leq 0$

so $H^i_i(\underline{w}, q) \leq 0$

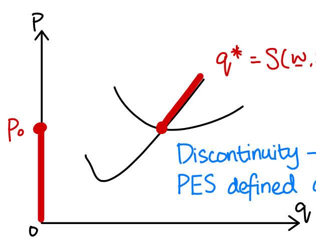
SUPPLY The locus of quantities q that satisfies the problem $\max_q (pq - C(q))$ for any given price p

$q^* = S(\underline{w}, p)$, where q^* is the Π-max o/p for $\underline{w}$ and p

$$S(p) = \begin{cases} q & \text{satisfying } p = MC \quad \text{if } p > \min AC(q) \\ 0 \text{ or } q & \text{satisfying } p = MC \quad \text{if } p = \min AC(q) \\ 0 & \text{if } p < \min AC(q) \end{cases}$$

RISES/FALLS WITH MC

- Sub this into FOC: $C_q(\underline{w}, S(\underline{w}, p)) = p$
- Differentiate wrt p: $C_{qq}(\underline{w}, S(\underline{w}, p)) S_p(\underline{w}, p) = 1 \Rightarrow S_p(\underline{w}, p) = \frac{1}{C_{qq}(\underline{w}, S(\underline{w}, p))}$
- Supply slope is positive when MC is rising



$$q^* = S(\underline{w}, p) = \begin{cases} 0 & \text{if } p < p_0 \\ 0 \text{ or } q & \text{if } p = p_0 \\ \boxed{} & \text{if } p > p_0 \end{cases}$$

Discontinuity – if P<AC, just stop pdⁿ!
PES defined only for $p > p_0$

RISES/FALLS WITH INPUT PRICE

- FOC: $C_q(\underline{w}, S(\underline{w}, p)) = p$
- Differentiate wrt w_j: $C_{qj}(\underline{w}, S(\underline{w}, p)) + C_{qq}(\underline{w}, S(\underline{w}, p)) S_j(\underline{w}, p) = 0$

→ How does MC react to Δw_j

→ Slope of MC (How MC responds to Δq)

$$S_j(\underline{w}, p) = -\frac{C_{qj}(\underline{w}, S(\underline{w}, p))}{C_{qq}(\underline{w}, S(\underline{w}, p))}$$

→ Δw_j directly affects MC. Typically > 0.
→ Δw_j indirectly affects MC through Δq. Can be < 0 or > 0

UNCONDITIONAL DEMAND

$z_i^* = D^i(\underline{w}, p) = H^i(\underline{w}, S(\underline{w}, p))$ — sub supply into q; D^i depends on p now

- H^i conditional on q, but D^i conditional on p
- Diff wrt w_j: $D^i_j(\underline{w}, p) = H^i_j(\underline{w}, q^*) + H^i_q(\underline{w}, q^*) S_j(\underline{w}, p)$

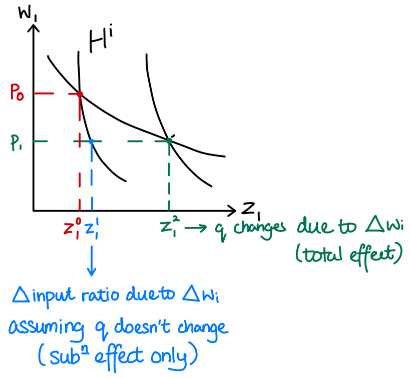
  Shephard $= H^i_j(\underline{w}, q^*) + C_{iq}(\underline{w}, q^*) S_j(\underline{w}, p)$

$$D^i_j(\underline{w}, p) = H^i_j(\underline{w}, q^*) - \frac{C_{iq}(\underline{w}, q^*)\, C_{jq}(\underline{w}, S(\underline{w}, p))}{C_{qq}(\underline{w}, S(\underline{w}, p))}$$

Total effect — Substitution effect (−ve) — Output effect (all terms ≥ 0)

☆ Symmetric: $D^i_j = D^j_i$. Effect of w_i on dd_{z_j} = Effect of w_j on dd_{z_i}

☆ This is dd for inputs only, not goods (dd need not be symmetrical)



$$D^i_i(\underline{w}, p) = H^i_i(\underline{w}, q^*) - \frac{(C_{iq}(\underline{w}, q^*))^2}{C_{qq}(\underline{w}, S(\underline{w}, p))}$$

INPUT PRICES AND DEMAND (THEOREM)

- $H^i_j = H^j_i$. Effect of $\uparrow w_j$ on H^i = Effect of $\uparrow w_i$ on H^j (symmetry)
- $D^i_j = D^j_i$
- $H^i_i \leq 0$, $D^i_i \leq 0$ downward-sloping
- $D^i_i \leq H^i_i$

  $|D^i_i| \geq |H^i_i|$

SHORT-RUN PROBLEM

amt of z_m in SR is fixed

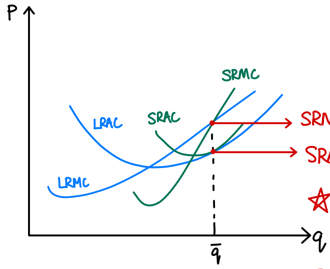
Choose (find optimal) q and $\underline{z}$ to max $\Pi := pq - \sum_{i=1}^{m} w_i z_i$ $\quad (q \geq 0, z_i \geq 0, \phi(\underline{z}) \geq q, \ z_m = \bar{z}_m$

$$\tilde{C}(\underline{w}, q, \bar{z}_m) := \min_{z_m = \bar{z}_m} \sum_{i=1}^{m} w_i z_i$$

$$\underset{\text{SRTC}}{\tilde{C}(\underline{w}, q, \bar{z}_m)} \geq \underset{\text{LRTC}}{C(\underline{w}, q)}$$

$$\Downarrow$$

$$\underset{\text{SRAC}}{\frac{\tilde{C}(\underline{w}, q, \bar{z}_m)}{q}} \geq \underset{\text{LRAC}}{\frac{C(\underline{w}, q)}{q}}$$



SRMC cuts LRMC at $\bar{q}$

SRAC touches LRAC at $\bar{q}$

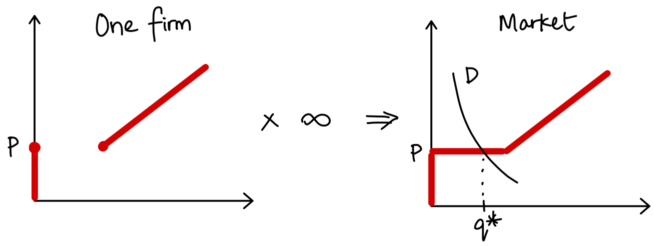
☆ Supply less elastic in SR

o/p for which $\bar{z}_m$ would be optimal

THEOREM: SHORT-RUN DD/SS

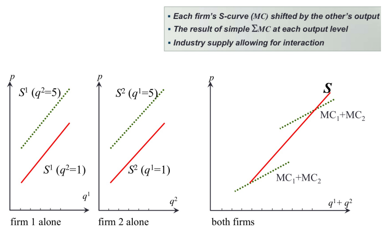
- At $\bar{q}$, SRTC = LRTC (so SRAC = LRAC)
- At $\bar{q}$, SRMC = LRMC
- SRMC at least as steep as LRMC
- LR dd at least as elastic as SR dd
- At $\bar{q}$, SR and LR input demands are equal

☆ Monopolist does not have supply curve
it uses market demand to set q



If total no. of firms = N,
At q^*

NEGATIVE EXTERNALITY



Supply less responsive than ΣMC

NONCONCAVE ϕ

May result in discontinuous supply

- But if there are many firms, average behaviour may still look conventional

MARKET EQUILIBRIUM – PRICE-TAKING

Determining the equilibrium no. of firms in an industry

1. Assume first firm makes $\Pi > 0$
2. Is $pq - C \leq$ Set-up costs of a new firm? ★ Recall: Π-max firm enters if $P \geq AC = \frac{C}{q}$
   - YES : Stop. Arrived at eqm no. of firms
   - NO : Continue
3. Number of firms goes up by 1.
4. $\uparrow q \cdot \downarrow p$. Firms adjust o/p

★ In the limit (as $n_{firms} \longrightarrow N_{eqm}$) $\Pi \downarrow\downarrow \cdots = 0$

MARKET EQUILIBRIUM – PRICE-SETTING

★ monopolist has no SS-curve since price isn't exogenous

$\boxed{\text{INVERSE DEMAND}}$ $p = p(q)$

- Makes price a determinate function of o/p. The market takes this price
- Not when firm is competing with rival — need to introduce other vars (e.g. info on rival)

$\boxed{\text{PRODUCT DEMAND ELASTICITY}}$ $\eta = \frac{dq}{q} \div \frac{dp(q)}{p(q)} = \frac{p(q)}{q} \div \frac{dp(q)}{dq} = \frac{p(q)}{q p_q(q)}$

Δq in response to Δp

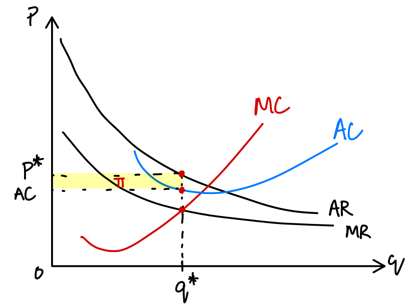
- Typically, dd is downward-sloping, so $p_q(q) < 0$
- η is thus negative

$\boxed{\text{AVERAGE REVENUE}}$ $p(q)$ same as dd

$\boxed{\text{TOTAL REVENUE}}$ $p(q)q$

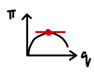
$\boxed{\text{MARGINAL REVENUE}}$ $p(q) + p_q(q)q$

- Revenue monopolist gets by unloading extra q on the market
- If dd is downward-sloping ($p_q(q) < 0$) then $\boxed{MR < AR}$



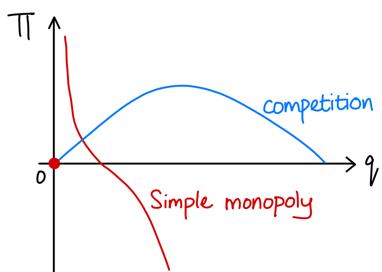
$\boxed{\text{OPTIMISE : MR=MC}}$ $\max\limits_{q \geq 0} \Pi(q)$, $\Pi(q) = p(q)q - C(q)$

- Conditions for interior maximum : $\exists q$ s.t. $\Pi_q(q) = 0$ $\Pi_{qq}(q) < 0$



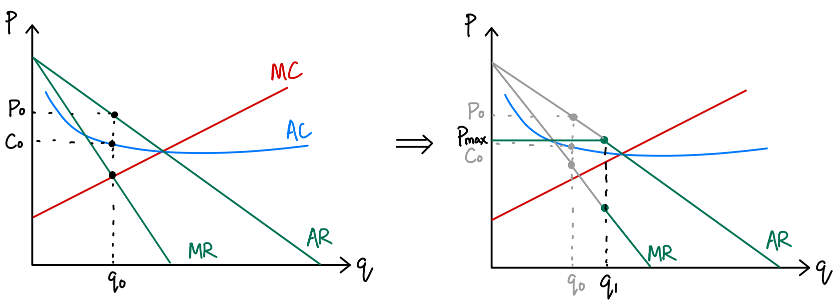
- FOC : $p(q) + p_q(q)q = C_q(q) \iff$ MR = MC
- Get optimal o/p q^*, sub in to get optimal price $p(q^*)$
- P > MC : $p(q) > C_q(q)$ if $|\eta| < \infty$. $\uparrow|\eta| \Rightarrow \downarrow P - AC$

$p(q) + p_q(q)q = C_q(q) \Leftrightarrow p(q)\left(1 + \frac{1}{\eta}\right) = C_q(q) \Leftrightarrow p(q) = \frac{C_q(q)}{\left(1 + \frac{1}{\eta}\right)}$



if dd inelastic $\forall q$, $\nexists$ Π-max q^*
monopolist selects q^* s.t. $\frac{d\Pi}{dq} = 0$
can $\uparrow\Pi$ as long as $\downarrow q$, but $\Pi(0) = 0$

$\boxed{\text{MONOPOLY REGULATION-PRICE CAP}}$



CONSUMER

A consumption is a list of commodities $\underline{x} = (x_1, \dots, x_n)$
Set of feasible consumption bundles $X = \mathbb{R}^n_+$
Assume all prices $\underline{p} = (p_1, \dots, p_n)$ are known

BUDGET CONSTRAINT
$\sum_{i=1}^{n} p_i x_i$

AXIOM OF RATIONAL CHOICE

Consumer always makes a choice, and chooses the most preferred bundle available (affordable)

WEAK AXIOM OF REVEALED PREFERENCE
If $\underline{x} \triangleright \underline{x}'$, then $\underline{x}' \ntriangleright \underline{x}$.

- $\underline{x}$ is revealed $\triangleright$ preferred to $\underline{x}'$ if $\underline{x}$ is actually selected when $\underline{x}'$ is also available
- If $\underline{x}'$ is selected when prices $= \underline{p}$, then $\underline{x}$ must be unavailable. ($\underline{p}^T\underline{x} > \underline{p}^T\underline{x}'$)
- Preferences can be cyclical under WARP (cannot draw indif. curves)

WEAK PREFERENCE RELATION
$\forall \underline{x}, \underline{x}' \in X$, $\underline{x} \succcurlyeq \underline{x}' \Leftrightarrow$ "$\underline{x}$ is at least as good as $\underline{x}'$"

- Made operational through a set of axioms:

1. Completeness — Either $\underline{x} \succcurlyeq \underline{x}'$, or $\underline{x}' \succcurlyeq \underline{x}$, or both are true.
2. Transivity — If $\underline{x} \succcurlyeq \underline{x}'$ and $\underline{x}' \succcurlyeq \underline{x}''$, then $\underline{x} \succcurlyeq \underline{x}''$. ☆ consistency — if you always strictly prefer a bundle to another, no indifference curve
3. Continuity — $\forall \underline{x} \in X$, the "not-worse-than-$\underline{x}$" and "not-better-than $\underline{x}$" set are both closed in X.
4. Greed — $\underline{x} > \underline{x}' \Rightarrow U(\underline{x}) > U(\underline{x}')$
5. Strict Quasiconcavity — $U(\underline{x}) = U(\underline{x}') \Rightarrow \forall 0<t<1,\ U(t\underline{x} + (1-t)\underline{x}') > U(\underline{x})$ — Prefers diversity. No bumps or flat segments
6. Smoothness — U is twice-differentiable everywhere and $U_{ij}(\underline{x}) = U_{ji}(\underline{x})$ (no kinks)

- Indifference relation $\underline{x} \sim \underline{x}'$: $\underline{x} \succcurlyeq \underline{x}'$ and $\underline{x}' \succcurlyeq \underline{x}$
- Strict preference relation $\underline{x} \succ \underline{x}'$: $\underline{x} \succcurlyeq \underline{x}'$ and not $\underline{x}' \succcurlyeq \underline{x}$

PREFERENCE REPRESENTATION THEOREM

Given axioms 1-3 hold, $\forall \underline{x}, \underline{x}' \in X$, $\exists$ a continuous utility function U s.t. $\underline{x} \succcurlyeq \underline{x}' \Leftrightarrow U(\underline{x}) \geq U(\underline{x}')$

GREED AXIOM
$\underline{x} > \underline{x}' \Rightarrow U(\underline{x}) > U(\underline{x}')$

☆ $\underline{x} > \underline{x}'$: $x_i \geq x_i'$ for all i and $x_i > x_i'$ for at least one i

- Indifference curve can't be vertical/horizontal. No bliss point.
- Gives a clear northeast direction of increasing U

MARGINAL RATE OF SUBSTITUTION
$MRS_{ij} = \frac{U_j(\underline{x})}{U_i(\underline{x})}$ (of x_i for x_j)

- Marginal willingness to pay for x_j measured in terms of x_i
- Independent of cardinal representation (monotonic transformation) of U

START

Utility maximisation

$$\mathcal{L} = U(\underline{x}) + \mu\left(y - \sum_{i=1}^{n} p_i x_i\right)$$

At optimum $U_i(\underline{x}^*) = \mu^* p_i$

$= V_y p_i$

for SWF

$\downarrow$ FOCs

Marshallian demands

$$x_i^*(\underline{p}, y) = D^i(\underline{p}, y)$$

At equilibrium, $x_i^* = h_i^*$

Slutsky Equation

$$\underbrace{D^i_j(\underline{p},y)}_{\text{Total effect}} = \underbrace{H^i_j(\underline{p},v)}_{\text{Sub effect}} - \underbrace{x_j^* D^i_y(\underline{p},y)}_{\text{Income effect}}$$

* $i=j$ for own-price effect

Roy's identity

$$x_i^* = -\frac{V_i(\underline{p},y)}{V_y(\underline{p},y)}$$

$V_y = \mu^*$ (from max U)

Indirect utility

$$V(\underline{p}, y) = U(\underline{x}^*)$$

☆ $V_y(\underline{p},y) = \frac{1}{C_v(\underline{p},v)}$

$\lambda^* = \frac{1}{\mu^*}$ ← MU of money

↑ MC of utility

Same v

Rearrange to get y wrt v →

← $y = C(\underline{p}, v)$

Rearrange to get v wrt y

START

Cost minimisation

$$\mathcal{L} = \sum_{i=1}^{n} p_i x_i + \lambda\left(v - U(\underline{x})\right)$$

$\downarrow$ FOCs

Hicksian demands

$$h_i^*(\underline{p}, v) = H^i(\underline{p}, v)$$

Shephard's lemma

$$h_i^*(\underline{p}, v) = C_i(\underline{p}, v)$$

Cost function

$$C(\underline{p}, v) = \sum p_i h_i^*$$

Compensating variation

$$CV(\underline{p} \to \underline{p}') = C(\underline{p}, v) - C(\underline{p}', v)$$

Shephard, if only p_1 changed $= \int_{p_1'}^{p_1} H^1(p, p_2, \dots, p_n, v)\,dp$

Or solve $V(\underline{p}, y) = V(\underline{p}', y - CV)$

Equivalent variation

$$EV(\underline{p} \to \underline{p}') = C(\underline{p}, v') - C(\underline{p}', v')$$

Shephard, if only p_1 changed $= \int_{p_1'}^{p_1} H^{new}(p_1, p_2, \dots, v')\,dp$

Or solve $V(\underline{p}, y + EV) = V(\underline{p}', y)$

Consumer surplus

$$\Delta CS = \int_{p_1'}^{p_1} D^1(p, p_2, \dots, p_n, y)\,dp$$

PRIMAL : UTILITY - MAXIMISATION

$$\max \quad U(\underline{x}) + \mu\left(y - \sum_{i=1}^{n} p_i x_i\right)$$

⋮ FOCs

$$\underset{\text{implicit price}}{MRS} = \frac{U_i(\underline{x}^*)}{U_j(\underline{x}^*)} = \frac{p_i}{p_j} = \underset{\text{mkt price}}{\text{Price ratio}}$$

U-maxxing commodity dd for $\underline{p}$ and y

ORDINARY DEMAND FUNCTIONS $x_i^* = D^i(\underline{p}, y)$

Must satisfy properties

- Adding-up property : $\sum_{i=1}^{n} p_i D(\underline{p}, y) = y$ i.e. Σ expenditure on each good = income
- D^i is homogeneous of deg 0 in $\underline{p}, y$: $D^i(t\underline{p}, ty) = D^i(\underline{p}, y)$
  - As D^i is derived from FOC, scaling both $\underline{p}$ and y by t doesn't affect $x_1^*, x_2^*, \ldots, x_n^*$

DUAL : COST - MINIMISATION

$$\min_{x_i > 0} \sum_{i=1}^{n} p_i x_i + \lambda\left(v - U(\underline{x})\right)$$

utility index (specified utility level)

EXPENDITURE FUNCTION $C(\underline{p}, v) = \min_{\{x_i \geq 0,\, U(\underline{x}) \geq v\}} \sum_{i=1}^{n} p_i x_i = \sum_{i=1}^{n} p_i \underset{\text{Hicksian}}{x_i^*} = y$ minimised cost/constraint income

e.g. 2018 LTQ3

Properties

1. Non-decreasing and continuous in $\underline{p}$
2. Homogeneous of degree 1 in $\underline{p}$
3. Concave in $\underline{p}$
4. Strictly increasing in v and at least one p_i
5. Differential $\frac{\partial C(\underline{p}, v)}{\partial p_i} = x_i^*$ is defined at every point. It is the optimal demand for x_i.

HICKSIAN / COMPENSATED DEMAND FUNCTIONS $x_i^* = C_i(\underline{p}, v) = H^i(\underline{p}, v)$

Demand of a consumer over a bundle of goods that minimises ther expenditure at a fixed level of utility v.

- Well-defined and continuous if U is strictly concave - contoured
- $H^i_i \leq 0$ (downward - sloping) and < 0 if U is smooth
- Less price-elastic than D^i for normal good ∵ D^i reflects both income and substition effects. H^i reflects sub^n eff only
- Homogeneous of degree 0 in $\underline{p}$
- $H^i_j = H^j_i$

SLUTSKY EQUATION

At equilibrium,

$H^i(\underline{p}, v) = D^i(\underline{p}, y)$

$H^i(\underline{p}, v) = D^i(\underline{p}, C(\underline{p}, v))$

$H^i_j(\underline{p}, v) = D^i_j(\underline{p}, y) + D^i_y(\underline{p}, y) C_j(\underline{p}, v)$

$$\underset{\text{Total effect}}{D^i_j(\underline{p}, y)} = \underset{\text{Sub effect}}{H^i_j(\underline{p}, v)} - \underset{\text{Income effect}}{x_j^* D^i_y(\underline{p}, y)}$$

If $H^i_j > 0$, x_i and x_j are net substitutes

If $H^i_j < 0$, x_i and x_j are net complements

$H^i_j = H^j_i$ but $D^i_j \neq D^j_i$.

Sub^n eff symmetric but income eff (hence total eff) asymmetric

SUBSTITUTION EFFECT

Effect on demand as a result of a fall in the relative price of good j, While the budget was adjusted to keep the person on the same indifference curve.

INCOME EFFECT

Change in consumption of each good due to an increase in consumer's real spending power alone

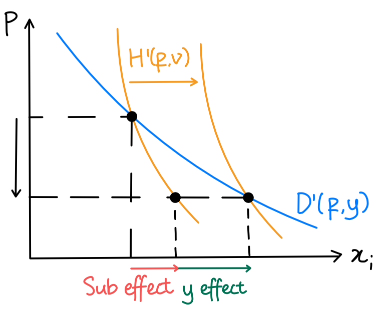
GIFFEN GOOD

$$D^i_i(\underline{p},y) = \underbrace{H^i_i(\underline{p},v)}_{\text{always } \leq 0} - x_i^* \underbrace{D^i_y(\underline{p},y)}_{\text{can } \geq 0 \text{ or } \leq 0!}$$

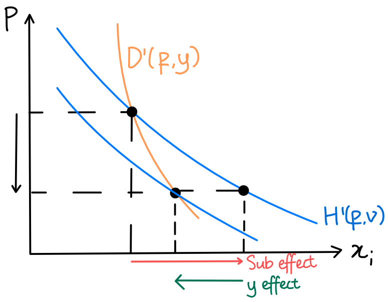
If $D^i_y < 0$, increase in income will cause demand for x_i to fall (inferior good)
Possible for D^i to slope upwards if $D^i_i > 0$.

OWN-PRICE EFFECT (FOR NORMAL GOODS)

If a consumer's demand for a good never decreases when income alone increases (i.e. normal good, $D^i_j \geq 0$), then his demand for that good must always decrease when its price (alone) increases.



Normal Good



Inferior Good

INDIRECT UTILITY FUNCTION

$$V(\underline{p},y) = \max_{\{x_i \geq 0,\ \sum_{i=1}^{n} p_i x_i \leq y\}} U(\underline{x}) = U(\underline{x}^*) = v$$

constraint utility / maximal utility

Max utility consumer gets maximising utility subject to budget constraint given $\underline{p}, y$

Properties

- $V_i(\underline{p},y) \leq 0$
  $V_i < 0$ for at least 1: $V_i = 0$ if you're not consuming x_i (utility not hurt by $\uparrow p_i$), but you must consume something
- $V_y(\underline{p},y) = \mu^*$. It's the marginal $\uparrow$ in maximal utility due to $\uparrow y$
- V is homogeneous of deg 0 in all prices and income: $V(t\underline{p},ty) = V(\underline{p},y)$ and quasiconvex in prices
- and ...

ROY'S IDENTITY

$$x_i^* = -\frac{V_i(\underline{p},y)}{V_y(\underline{p},y)}$$

$V(\underline{p}, C(\underline{p},v)) = v$

$V_i(\underline{p},y) + V_y(\underline{p},y)\, C_i(\underline{p},v) = 0$

$$C_i(\underline{p},v) = -\frac{V_i(\underline{p},y)}{V_y(\underline{p},y)}$$

MARGINAL CHANGES

We have $V(\underline{p},y) = v$ and $C(\underline{p},v) = y$ at optimum

Thus, $V(\underline{p}, C(\underline{p},y)) = v$

$V_y(\underline{p},y)\, C_v(\underline{p},v) = 1$

$$C_v(\underline{p},v) = \frac{1}{V_y(\underline{p},y)}$$

Sub in $V_y(\underline{p},y) = \mu^*$ and $C_v(\underline{p},v) = \lambda^*$

$$\underbrace{\lambda^*}_{\text{MC of utility}} = \frac{1}{\underbrace{\mu^*}_{\text{MU of money}}}$$

CONSUMER SURPLUS

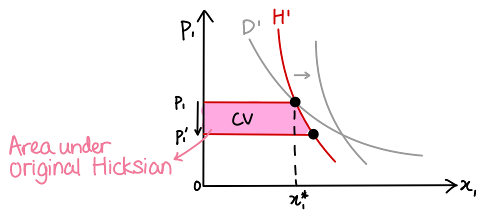
Given indirect utility function $v = V(\underline{p}, y)$, if $\underline{p}$ changes to $\underline{p}'$, utility changes to $v' = V(\underline{p}', y)$

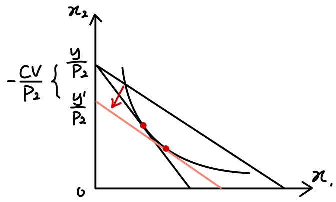
COMPENSATING VARIATION $CV(\underline{p} \to \underline{p}') = C(\underline{p}, v) - C(\underline{p}', v) \overset{\text{Shephard}}{=} \int_{p_1'}^{p_1} H'(p, p_2, \dots, p_n, v)\, dp$

CV is the change in income (or equivalent expenditure function C) needed to keep V on the same utility v as before (i.e. undo effect of $\underline{p}$ change)

- Takes original utility level v as reference point
- $CV = -\Delta$cost of hitting v. Sign of CV, EV same as Δv (opposite of Δp_i)
- Can also solve $V(\underline{p}, y) = V(\underline{p}', y - CV)$

- If $\underline{p} = (p_1, \dots, p_n) \to \underline{p}' = (p_1', \dots, p_n)$, $CV(\underline{p} \to \underline{p}') = \int_{p_1'}^{p_1} \frac{\partial C(\underline{p}, v)}{\partial p_1}\, dp_1 \overset{\text{Shephard}}{=} \int_{p_1'}^{p_1} H'(p, p_2, \dots, p_n, v)\, dp$





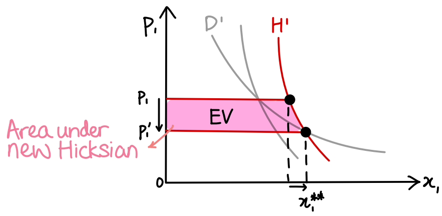
Using U and B

EQUIVALENT VARIATION $EV(\underline{p} \to \underline{p}') = C(\underline{p}, v') - C(\underline{p}', v') = \int_{p_1'}^{p_1} H^{new}(p_1, p_2, \dots, v')\, dp$

EV is the change in income (or equivalent expenditure function C) needed to bring V to v' without changing $\underline{p}$ (mimic effect of going from $\underline{p}$ to $\underline{p}'$)

- Takes terminal utility level v' as reference point
- $EV = -\Delta$cost of hitting v'. Sign of CV, EV same as Δv (opposite of Δp_i)
- Can also solve $V(\underline{p}, y + EV) = V(\underline{p}', y)$

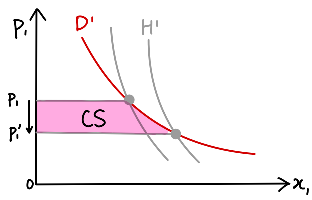
CV and EV always have the same sign as welfare change. We use EV and CV to measure utility in \$\$ terms



$$EV(\underline{p}' \to \underline{p}) = C(\underline{p}', v) - C(\underline{p}, v) = -CV(\underline{p} \to \underline{p}')$$

EV and CV look at same welfare change $|v - v'|$ in opposite directions.

CONSUMER'S SURPLUS $\Delta CS = \int_{p_1'}^{p_1} D'(p, p_2, \dots, p_n, y)\, dp$



For normal goods: $CV \leq \Delta CS \leq EV$
For inferior goods: $CV > \Delta CS > EV$
(equality if income effect = 0.)

CONSUMER'S PRICE INDEX

Given base-year prices $\underline{p}$ and utility v,
and current-year prices $\underline{p}'$ and utility v',

$$I_{CV} = \frac{C(\underline{p}', v)}{C(\underline{p}, v)}$$ (%Δ in cost of hitting the "target" utility level v)

$$I_{EV} = \frac{C(\underline{p}', v')}{C(\underline{p}, v')}$$ (%Δ in cost of hitting the "target" utility level v')

These are exact price indices and require no estimation

$I_{EV} = I_{CV}$ if utility function is homothetic 4.13

$\boxed{\text{LASPEYRES INDEX}}$ $I_L = \frac{\sum_{i=1}^{n} p_i' x_i}{\sum_{i=1}^{n} p_i x_i} \begin{matrix} \geq C(\underline{p}', v) \\ = C(\underline{p}, v) \end{matrix}$ $I_L \geq I_{CV}$ (overestimate with base-year basket)

$\boxed{\text{PAASCHE INDEX}}$ $I_P = \frac{\sum_{i=1}^{n} p_i' x_i'}{\sum_{i=1}^{n} p_i x_i'} \begin{matrix} = C(\underline{p}', v') \\ \geq C(\underline{p}, v) \end{matrix}$ $I_P \leq I_{EV}$ (underestimate with base-year basket)

CONSUMER & MARKET

An individual owns $R_1, \dots, R_n$ of commodities $1, \dots, n$
Given market prices $p_1, \dots, p_n$

$\boxed{\text{INCOME}}$ $y = \sum_{i=1}^{n} p_i R_i$ i.e. selling your stuff

$\boxed{\text{BUDGET CONSTRAINT}}$ $\sum_{i=1}^{n} p_i x_i \leq \sum_{i=1}^{n} p_i R_i$

$\boxed{\text{MODIFIED SLUTSKY}}$ Substituting $y = \sum_{i=1}^{n} p_i R_i$ into $x_i^* = D^i(\underline{p}, y)$,

$$x_i^* = D^i(\underline{p}, \sum_{i=1}^{n} p_i R_i)$$

$$\frac{dx_i^*}{dp_j} = D_j^i(\underline{p}, \sum_{i=1}^{n} p_i R_i) + D_y^i(\underline{p}, \sum_{i=1}^{n} p_i R_i)\, R_j$$

Sub in $D_j^i(\underline{p}, y) = H_j^i(\underline{p}, v) - x_j^* D_y^i(\underline{p}, y)$

Get $\dfrac{dx_i^*}{dp_j} = H_j^i(\underline{p}, v) + (R_j - x_j^*)\, D_y^i(\underline{p}, y)$

>0 if net supplier (happy if $\uparrow p$)

<0 if net demander (unhappy if $\downarrow p$)

SIMPLE ECONOMY

NET OUTPUT VECTOR $q = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix}$

List of all potential inputs/outputs of a pdn process

- >0 : output
- <0 : input
- $=0$: intermediate good

TECHNOLOGY / PRODUCTION SET $Q = \mathbb{R}^n$

- Set of all technically feasible processes. Exogenously given.

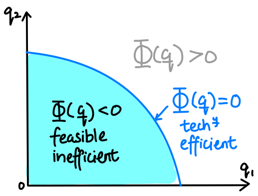
Axioms

- Possibility of inaction : $0 \in Q$
- No free lunch : $Q \cap \mathbb{R}^n_+ = \{0\}$ (i.e. ≥ 1 must be -ve)
- Irreversibility : $Q \cap (-Q) = \{0\}$
- Free disposal : If $q^\circ \in Q$ and $q \leq q^\circ$, $q \in Q$
- Additivity : If $q, q' \in Q$, $q + q' \in Q$ → rules out DRTS! ⇒ CRTS
- Divisibility : If $q \in Q$, $0<t<1$, $tq \in Q$. → rules out IRTS!
  - Can combine and divide techniques

COMBINATION OF SETS

- If no externalities then combination of Q^1 and Q^2 is just $Q^1 + Q^2$
  - If Q^1 and Q^2 are also convex, $Q^1 + Q^2$ are convex.

MULTI-GOOD PRODUCTION FUNCTION $\Phi(q) \leq 0$



- Φ non-decreasing in each q_i
- Boundary is transformation curve
- Slope : marginal rate of transformation $MRT_{ij} = \dfrac{\Phi_j(q)}{\Phi_i(q)}$

Given single-good production function $q \leq \phi(z_1, z_2)$

Rewrite as $\Phi(q_1, q_2, q_3) = -\phi(-q_1, -q_2) + q_3 \leq 0$, $\begin{cases} q_1 = -z_1 \\ q_2 = -z_2 \\ q_3 = q \end{cases}$

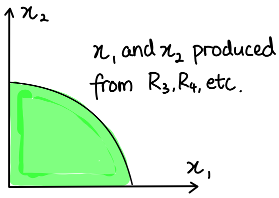
ROBINSON CRUSOE ECONOMY

no trade
one agent
all commodities produced from existing stocks

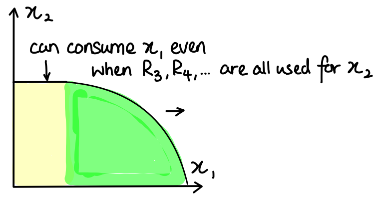
maximise $U(\underline{x})$ by choosing $\underline{q}$ and $\underline{x}$

Subject to
- $\underline{x} \in X$ — logically feasible C^n
- $\Phi(\underline{q}) \leq 0$ — technically feasible
- $\underline{x} \leq \underline{q} + \underline{R}$ — materials balance – can't consume more than net o/p + resource stock

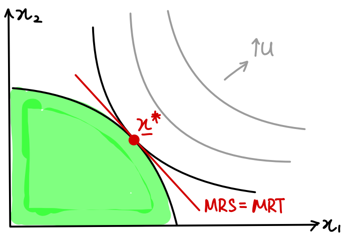
ATTAINABLE SET FOR CONSUMPTION $A(\underline{x}) = \{\underline{x} \mid \underline{x} \in X, \underline{x} \leq \underline{q} + \underline{R};\ \Phi(\underline{q}) \leq 0\}$ (Or production-possibility set)



$R_1 = R_2 = 0$



$R_1 > 0,\ R_2 = 0$



At optimum,

$$MRS = \frac{U_i(\underline{x})}{U_j(\underline{x})} = \frac{\Phi_i(\underline{q})}{\Phi_j(\underline{q})} = MRT$$

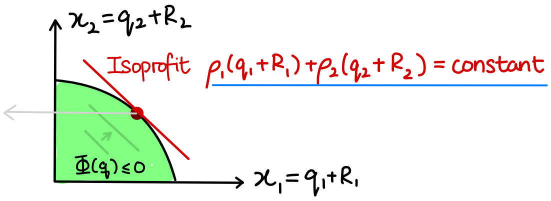
$$\Phi(q, z) = q + z - \frac{1}{4}$$

DECENTRALISATION THEOREM

If attainable set is convex, utility function is concave-contoured and satisfies greed axiom
Then there exist shadow prices $p_1, \ldots, p_n$ s.t. maximisation problem above is equivalent to the two-stage problem
(bc no mkt in single agent eco^y)

① **INCOME MAXIMISATION** $\max\limits_{\underline{q}} \sum_{i=1}^{n} p_i(q_i + R_i)$ subject to $\Phi(\underline{q}) \leq 0$

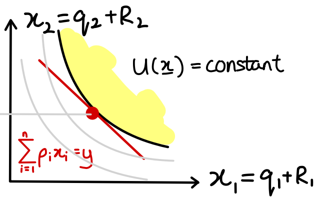
$$\frac{\Phi_i(\underline{x})}{\Phi_j(\underline{x})} = \frac{p_i}{p_j}$$

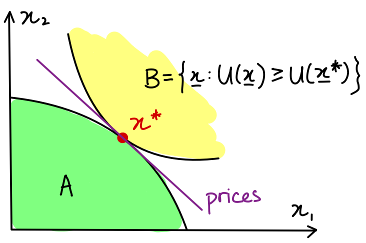


→ "National income"/isoprofit line
Value of resource stocks $= \sum_{i=1}^{n} p_i R_i$
Profits $= \sum_{i=1}^{n} p_i q_i$

② **UTILITY MAXIMISATION** $\max\limits_{\underline{x}} U(\underline{x})$ subject to $\underline{x} \in X$, $\sum_{i=1}^{n} p_i x_i \leq y$ ← = the max from ①

$$\frac{p_i}{p_j} = \frac{U_i(\underline{x})}{U_j(\underline{x})}$$





x^* maximises income on A
x^* minimises expenditure on B ("better-than-x" set)
Production/Consumption decentralised!
☆ fails when non-convex, but next page (Trade) can address

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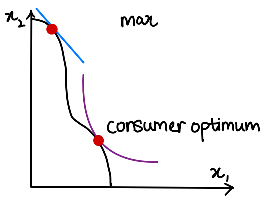
TRADE

NONCONVEXITY ISSUE

Decentralisation fails!

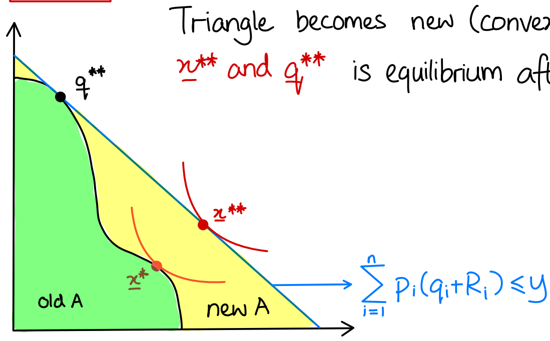


TRADE

$\underline{x}^*$ is the autarkic equilibrium : $x_1^* = q_1^*$, $x_2^* = q_2^*$

Triangle becomes new (convex) attainable set with trade!

$\underline{x}^{**}$ and $\underline{q}^{**}$ is equilibrium after trade



GENERAL EQUILIBRIUM

ECONOMY

Resource stocks $R_1, R_2, \ldots$

Households with $U_1, U^2, \ldots$

Firms with $\Phi^1, \Phi^2, \ldots$

ALLOCATION or "state of the economy"

Collection of goods vectors $[\underline{x}] = [\underline{x}^1, \underline{x}^2, \ldots]$ (For hh1, hh2)

Collection of net-output vectors $[\underline{q}] = [\underline{q}_1, \underline{q}_2, \ldots]$ (For firm 1)

COMPETITIVE ALLOCATION $\underline{a} = ([\underline{x}], [\underline{q}], \underline{p})$ where $\underline{x}^h$ maximises U^h (i.e. lies on offer curve of hh)

$\underline{q}^f$ maximises Φ^f

MATERIALS BALANCE (ECONOMY) $\sum_{h=1}^{n_h} x_i^h \leq \sum_{f=1}^{n_f} q_i^f + \sum_{h=1}^{n_h} R^h$ or $\underline{x} \leq \underline{q} + \underline{R}$

This simple aggregation property holds if

- Rivalness – All private goods, no joint consumption
- No externalities in production (if $q_1, q_2 \in Q$, then $q_1 + q_2 \in Q$)

INCOME $y^h = \underbrace{\sum_{i=1}^{n} p_i R_i^h}_{\text{rents}} + \sum_{i=1}^{n} \underbrace{S_f^h}_{\text{share in firm } f\text{'s profits hh } h \text{ gets}} \Pi^f = \sum_{i=1}^{n} p_i \left(R_i^h + \sum_{f=1}^{n_f=1} S_f^h q_i^f\right)$ in Robinson Crusoe economy (homo-deg1 in $\underline{p}$)

- So income is a function of prices!

DISTRIBUTION $\underline{d} = ([\underline{R}], [\underline{S}])$. Exogenous

COMPETITIVE EQUILIBRIUM A competitive allocation $\underline{a}$ in which the material balance holds

IN MANY-AGENT ECONMY The separation result carries over from Crusoe! decentralise to get

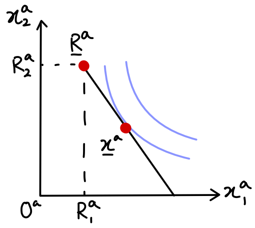
Attainable set $A = \{ \underbrace{\underline{x}}_{\sum_h x^h} : \underline{x} \leq \underbrace{\underline{q}}_{\sum_f q^f} + \underline{R} \, , \, \underbrace{\Phi(\underline{q})}_{\sum_f \Phi^f(q^f)} \leq 0 \}$. works assuming no ext^y

"Better-than" set $B = \{ \sum_h \underline{x}^h : U^h(\underline{x}^h) \geq U^h(\underline{x}^{*h}) \}$ ← $x_i = \sum_h x_i^h$ if private good

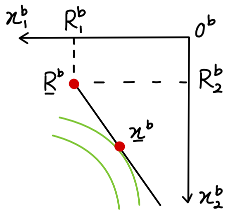
If A and B both convex, we can find prices that form a supporting hyperplane for A and B. Under those prices, each firm maximises profits and households maximise utility subject to budget constraint.

This means we can decentralise to get $\underline{x}^*$ that maximises aggregate income over A and minimises aggregate expenditure over B.

EDGEWORTH'S BOX

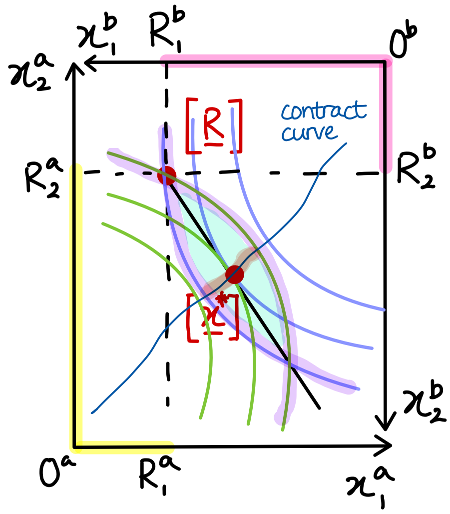


Person A



Person B

→



Reservation utilities of A & B (pass through [R]. Can guarantee just by not trading)

Set of all mutually beneficial trades

Core of the exchange ecoʸ (Any CE must be here)

$[R] = \left[(R_1^a, R_2^a), (R_1^b, R_2^b) \right]$ is property distribution / resource endowment

⇒ changing it may change eqm prices, allocation, incomes (inequality!)

line through $[R]$ and $[x^*]$ has slope $\frac{P_1}{P_2}$

$\boxed{\text{CONTRACT CURVE}}$ set of all feasible and Pareto efficient (possible, mutually beneficial) allocations

- Locus of points of tangency of MRS (and intersections of offer curves)
- Passes thru the CE
- Solve $\begin{cases} MRS^A = MRS^B \\ x_i^A + x_i^B = R_i^A + R_i^B \quad \text{for } i = 1,2 \end{cases}$
- Write as $x_1^A = f(x_2^A)$ or $x_1^B = f(x_2^B)$

$\boxed{\text{EG CALCULATION}}$

① Normalise prices (e.g. $\rho = \frac{P_1}{P_2}$)

② Get x^{*a}, x^{*b} in ρ-terms by maximising $U^{a/b}$ subject to resource constraint $\rho x_1^a + x_2^a \leq \rho R_1^a + R_2^a$

③ Sub into materials balance of each good $x_1^a + x_1^b = R_1^a + R_1^b$ to find ρ, the equilibrium price ratio

or excess demand (they're equivalent)

④ Use ρ to find $\underline{a}$ and incomes

COALITION/PRICE-TAKING

$\boxed{\text{COALITION}}$

In a two-person exchange economy, coalitions {Alf} {Bill} block allocations below their reservation utilities and coalition {Alf, Bill} blocks points outside of contract curve.

If we replicate the economy N-fold (N Alf-tribe, N Bill-tribe, identical pref / $\underline{R}$)
More allocations get blocked. $N \to \infty$ then core shrinks to set of CEs only

☆ can have multiple CE

EXCESS DEMAND FUNCTION

$$E_i(p) = \underbrace{x_i(p)}_{\sum \text{individual dd for good } i} - \underbrace{q_i(p)}_{\text{net o/p}} - R_i$$ ⇒ Dynamically writing materials balance

In equilibrium, for all goods i $\quad E_i(p^*) \le 0$ and $E_i(p^*) = 0$ if $p_i > 0$

PROPERTIES

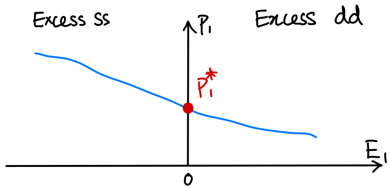
Walras' Law : $\sum_{i=1}^{n} p_i E_i(p) = 0 \quad \forall p$ ☆ since either $E_i = 0$ or $p_i = 0$!

➜ You just need to solve n-1 equations in n-1 unknowns — can deduce the last one $E_n(p) = -\frac{1}{p_n}\sum_{i=1}^{n-1} p_i E_i(p)$

Homogeneity of degree 0 : $\forall p, \forall t > 0: E_i(tp) = E_i(p)$

➜ Only relative prices matter. Normalise by any positive number!

- Follows from the same property in individual consumers' dd and firms' ss fns



FINDING EQUILIBRIUM

☆ normalise prices!

1. Compute q and Π using technologies Φ
2. Compute y_h and x^h using property rights
3. Aggregate the xs and qs and get $E(p) = \underline{x} + \underline{q} - \underline{R}$
4. Let $E_1 = E_2 ... = 0$ to find equilibrium p.

PRICE NORMALISATION

Conversion of n prices to n-1 relative prices

- Divide by price of a numéraire good (i.e. p_1 and $p_2 \Rightarrow p = \frac{p_1}{p_2}$ and 1)
- Divide by a composite (basket, or $\sum p_i x_i$, or $\sum p_i$)

↓

$J = \{p \mid p \ge 0, \sum_{i=1}^{n} p_i = 1\}$ is convex and compact

EXISTENCE

If $\forall i \; E_i : J \to \mathbb{R}$ is bounded below and continuous, $p^* \in J$ exists

- If each Φ is concave and each U^h quasiconcave, E continuous

To check no. of equilibria: Let $E(p) = 0$ and solve for p. See how many solutions

TÂTONNEMENT

A mechanism in which you adjust prices according to sign of E_i

- If $E_i > 0$, increase p_i
- If $E_i < 0$ and $p_i > 0$, decrease p_i
- Is linear (i.e. no $\approx$) if $\frac{dp_i(t)}{dt} = \alpha_i E_i(p(t))$ if $p_i(t) \ge 0$ and 0 otherwise
- Given WARP, distance between p^* and $p(t)$ falls under tâtonnement

⇒ Uniqueness and stability!

CHECK LOCAL STABILITY

$\frac{dE_i}{dp} < 0$ in vicinity of solution

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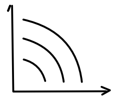
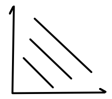
RISK & UNCERTAINTY

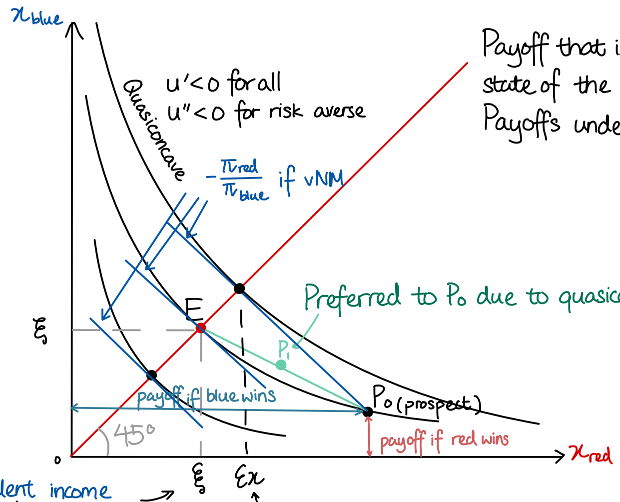
States of the world	$\omega \in \Omega$
Payoffs (depends on ω)	$x_\omega \in X$
Prospects	$\{x_\omega : \omega \in \Omega\}$
Ex ante	Before the realisation
Ex post	After the realisation



Risk-loving
$u''<0$



Risk-neutral
$u''=0$



Payoff that is independent of the state of the world that will occur
Payoffs under complete certainty

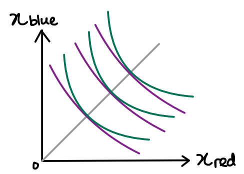
Preferred to P_0 due to quasiconcave indiff curve (risk-averse) ↓risk ↑utility

Will take risk if ξ > initial wealth

Certainty-equivalent income
Amt of $x_{red/blue}$ that, if received with complete certainty, will give same utility as P_0
(indiv will accept ξ as straight swap for P_0)

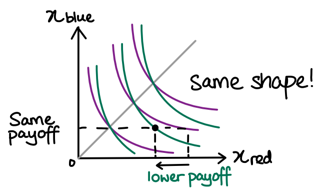
expected payoff

Same π, different u



More risk averse (sharply concave)

Same u, different π



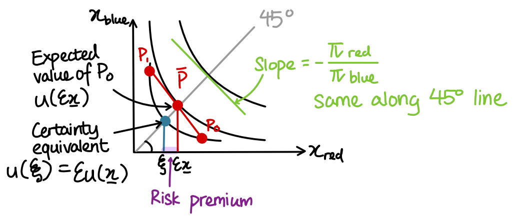
Thinks P(red wins) is higher

EXPECTED UTILITY THEOREM

If von-Neumann-Morgenstern axioms hold, preferences/utility can be represented as

Subjective Probabilities (normalise to $\sum = 1$)

$$\sum_{\omega \in \Omega} \pi_\omega u(x_\omega) = \mathcal{E}u(x)$$



If risk averse, $U(\bar{P}) \geq U(P_0)$

$\Rightarrow u(\mathcal{E}x) \geq \pi_{red} u(x_{red}) + \pi_{blue} u(x_{blue})$

$\Rightarrow u(\pi_{red} x_{red} + \pi_{blue} x_{blue}) \geq \pi_{red} u(x_{red}) + \pi_{blue} u(x_{blue})$

To find certainty equivalent ξ given prospect $P = (x_{red}, x_{blue})$, solve $u(\xi) = \sum \pi_\omega u(x_\omega)$

Note: If risk-averse, $U(\mathcal{E}x) \geq U(\xi)$
MU $u'(x) > 0$

VNM FROM AXIOMS

AXIOMS

Transivity, Continuity, Completeness, Greed

State Irrelevance

- State that is realised has no intrinsic value to the person
- Only payoffs $\underline{x}$ matter. People enjoy $\underline{x}$ as much on blue/red day

Independence

- If $U(P(\underline{z})) \geq U(P'(\underline{z}))$ for one $\underline{z}$, then it does for all $\underline{z}$.
- $\underline{z}$ is the payoff if one particular state of the world ϖ
- No disappointment/regret.
- e.g. If $P(0,0,z) \succsim P(1,2,z)$, then $P(0,0,x) \succsim P(1,2,x)$

Revealed Likelihood

- $\underline{x}$ and $\underline{x}^*$ be payoffs. $\underline{x}^* \succsim \underline{x}$.
- $\Omega_0, \Omega_1 \subseteq \Omega$
- $P_0 = [\underline{x}^* \text{ if } \omega \in \Omega_0, \underline{x} \text{ if } \omega \notin \Omega_0]$ $\quad P_1 = [\underline{x} \text{ if } \omega \in \Omega_1, \underline{x}^* \text{ if } \omega \notin \Omega_1]$
- If $P_0 \succsim P_1$ for some $\underline{x}, \underline{x}^*$, it does for all $\underline{x}, \underline{x}^*$

So expected utility:

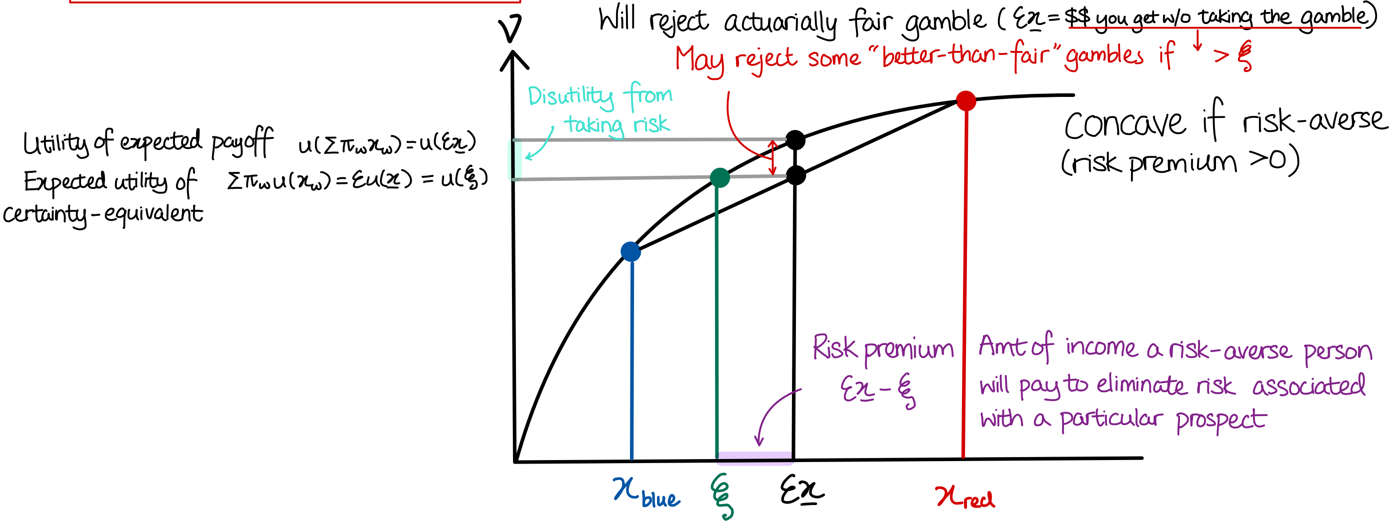
$$\mathcal{E}u(\underline{x}) = \pi_{\text{red wins}} U_{\text{of the world where red wins}}(x_{red}, x_{blue}) + \pi_{\text{blue wins}} U_{\text{of the world where blue wins}}(x_{red}, x_{blue})$$

$$= \pi_{\text{red wins}} U_{\text{of the world where red wins}}(x_{red}) + \pi_{\text{blue wins}} U_{\text{of the world where blue wins}}(x_{blue}) \quad \text{by state irrelevance}$$

$$= \underbrace{\pi_{\text{red wins}} U(x_{red}) + \pi_{\text{blue wins}} U(x_{blue})}_{\text{additively separable}} \quad \text{by independence}$$

$$\sum_{\omega \in \Omega} \pi_\omega u(\underline{x}_\omega) = \mathcal{E}u(\underline{x})$$

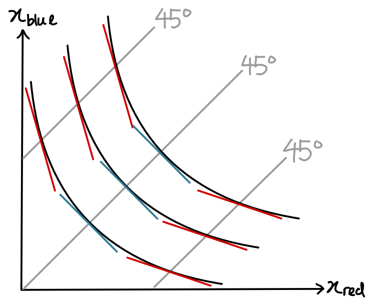
FELICITY/CARDINAL UTILITY FUNCTION



ABSOLUTE RISK AVERSION INDEX

$$\alpha(x) = -\frac{U_{xx}(x)^{<0 \text{ if concave}}}{U_x(x)_{>0}} \quad \begin{cases} >0 \text{ if risk-averse} \\ =0 \text{ if risk-neutral} \\ <0 \text{ if risk-loving} \end{cases}$$

- independent of scale and origin
- normalised rate of decrease of marginal felicity
- If $\hat{u}$ is a concave transformation of (more concave/"sharply-curved" than) u, then $\hat{\alpha}(x) > \alpha(x)$, $\hat{\rho}(x) > \rho(x)$
- If constant $\alpha(x) = \alpha$, then $u(x) = -\frac{1}{\alpha} e^{-\alpha x}$ and $\uparrow x \Rightarrow \uparrow \rho$



Slope of IC same along <u>any 45° line</u>

RISK PREMIUM

For small risks, risk premium $(\mathcal{E}x - \xi) \approx \frac{1}{2}\alpha(x)\,\mathrm{var}(x)$

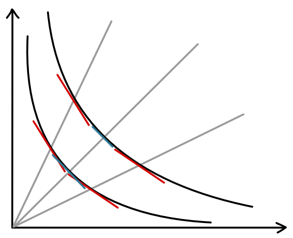
RELATIVE RISK AVERSION INDEX

$$\rho(x) = -x\frac{U_{xx}(x)}{U_x(x)} = x\alpha(x)$$

- independent of scale and origin
- elasticity of marginal felicity
- If risk-averse/neutral, $\uparrow \rho(x) \Rightarrow \uparrow \alpha(x)$. Converse false.
- If constant $\rho(x) = \rho$, $u(x) = \frac{1}{1-\rho} x^{1-\rho}$

☆ $\frac{d\rho(x)}{dx} = \alpha(x) + x\frac{d\alpha(x)}{dx}$

So $\frac{d\rho(x)}{dx}$ and $\frac{d\alpha(x)}{dx}$ have same sign if $\alpha \geq 0$ (x must >0)



Homothetic - Slope same along any ray through origin

LOTTERY

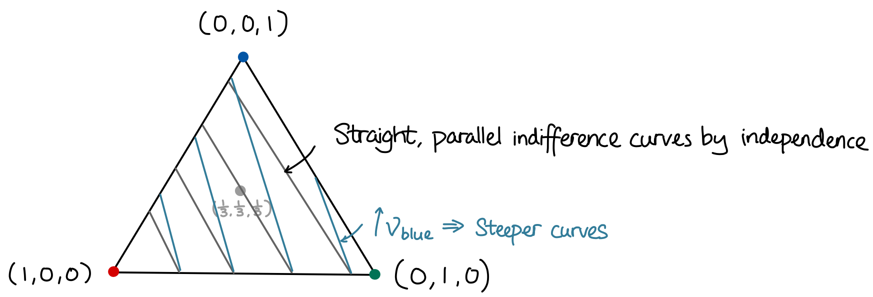
Fix payoffs, choose between probability distributions $\underline{\pi}^0, \underline{\pi}', \underline{\pi}''$

Assume transivity, independence, continuity

Then preferences can be von Neumann - Morgenstern

$\sum_{\omega \in \Omega} \pi_\omega u(x_\omega) = \sum_{\omega \in \Omega} \pi_\omega v_\omega$ where $v_\omega = u(x_\omega)$

- Linear in probabilities $U(\alpha\pi + (1-\alpha)\pi') = \alpha U(\pi) + (1-\alpha) U(\pi')$

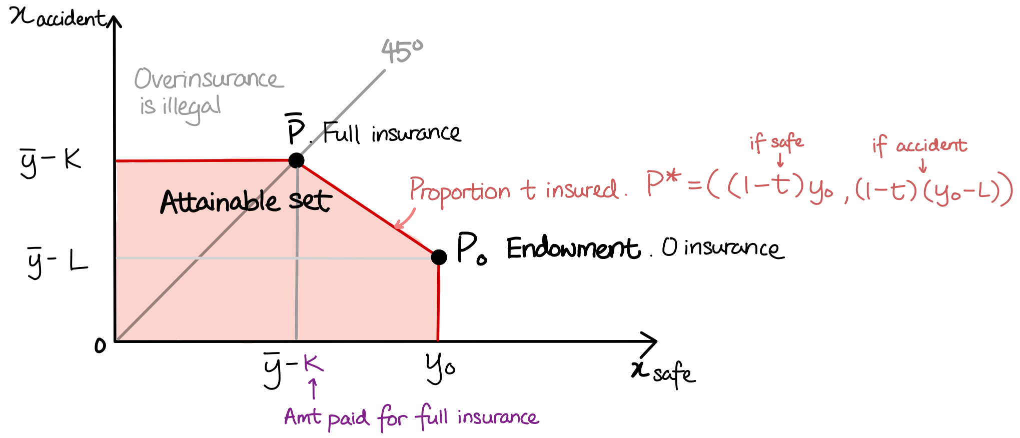


$\pi_{red} + \pi_{blue} + \pi_{green} = 1$

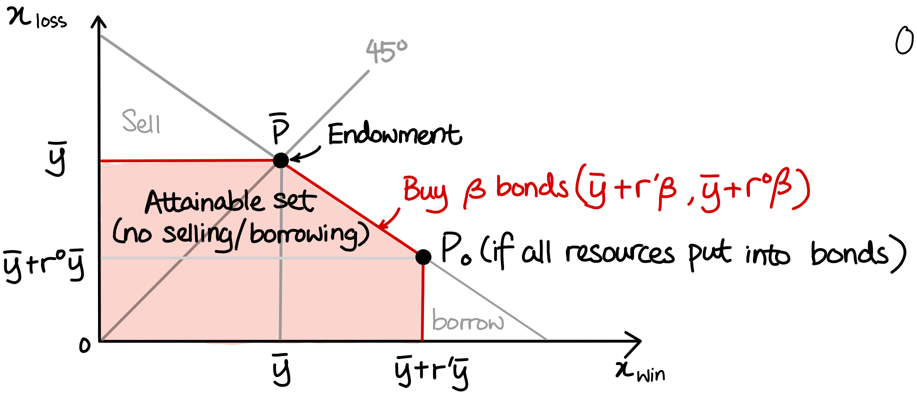
If $v_{red} > v_{green} > v_{blue}$, slope $\frac{d\pi_{blue}}{d\pi_{red}} > 0$

UNCERTAINTY & TRADE

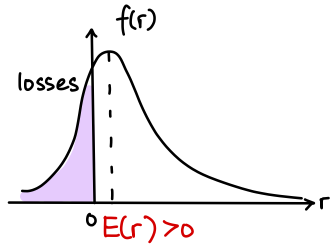
INSURANCE



PORTFOLIO CHOICE



Only two possible outcomes for r
Favourable: $r = r' > 0$
Unfavourable: $r = r^o < 0$



Choose β to maximise expected utility

$\mathcal{E}u(\bar{y}+r\beta) = \pi_{safe}u(\bar{y}+r'\beta) + \pi_{accident}u(\bar{y}+r^o\beta)$

FOC at interior solution, marginal utility of $\uparrow\beta$, $\mathcal{E}(ru_y(\bar{y}+r\beta)) = 0$ ✯ $u_y(\cdot)>0$

$\boxed{\text{WILL ALWAYS BUY IF EXPECTED RETURN} > 0}$

If $\beta=0$, $\mathcal{E}(ru_y(\bar{y}+r\beta)) = \mathcal{E}(ru_y(\bar{y})) = \overset{>0}{u_y(\bar{y})}\,\mathcal{E}r > 0$ if $\mathcal{E}r>0$

Can $\uparrow$ utility by $\uparrow\beta$. Will always take risk if
- vNM
- nonsatiated
- $\mathcal{E}r>0$

$\boxed{\text{EFFECT OF WEALTH ON } \beta^*}$

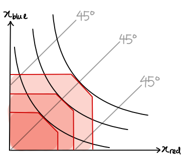
Differentiate $\mathcal{E}(ru_y(\bar{y}+r\beta^*)) = 0$ wrt y

$$\mathcal{E}\left[ru_{yy}(\bar{y}+r\beta^*)\left(1+r\frac{\partial\beta^*}{\partial\bar{y}}\right)\right] = 0$$

$$\frac{\partial\beta^*}{\partial\bar{y}} = -\frac{\mathcal{E}(ru_{yy}(\bar{y}+r\beta^*))}{\mathcal{E}(r^2u_{yy}(\bar{y}+r\beta^*))}$$
>0 if risky asset profits. <0 if risky asset loses
<0. $u_{yy}<0$ since risk averse

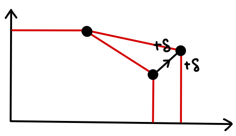
If $x\uparrow \overset{\text{DARA}}{\Rightarrow} \alpha(x)\downarrow$, $\frac{\partial\beta^*}{\partial\bar{y}} > 0$ so $\uparrow\bar{y}$ (endowment) $\Longrightarrow \uparrow\beta^*$ bonds bought

Note: CARA (constant α) implies $\beta^* \perp\!\!\!\perp \bar{y}$



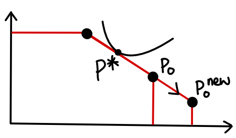
$\boxed{\text{EFFECT OF } \Delta\mathcal{E}r \text{ ON } \beta^*}$

Returns shift from (r', r^o) to $(r'+\delta, r^o+\delta)$. $\delta>0$
$\uparrow\beta$ if DARA



$\boxed{\text{EFFECT OF VAR}(r) \text{ ON } \beta^*}$

Returns shift from (r', r^o) to (tr', tr^o) $t>0$
Curve extended, P^* unchanged, so $\downarrow\beta^*$



8.13

$\boxed{\text{PROPORTIONAL TAX}}$ Buy β bonds, ROR $= r \in \mathbb{R}$

	On income with full loss offset	On wealth
Expected post-tax wealth, y	$\bar{y}+(1-t)r\beta$	$(\bar{y}+r\beta)(1-t)$
Optimal portfolio β^* (Choose β to max $\mathcal{E}u(y)$)	Differentiate wrt β FOC: $\mathcal{E}(u_y(y)r)=0$	FOC: $\mathcal{E}(u_y(y)r)=0$
$\frac{\partial\beta^*}{\partial t}$	Differentiate FOC wrt t $\mathcal{E}(u_{yy}(y)r^2(\underbrace{-\beta^*+(1-t)\frac{\partial\beta^*}{\partial t}}_{=0}))=0$ $\frac{\partial\beta^*}{\partial t}=\frac{\beta^*}{1-t}>0$ $\quad \uparrow t \longrightarrow \uparrow\beta^*$	Differentiate FOC wrt t $\frac{\partial\beta^*}{\partial t}=$

WELFARE

Social states $\theta \in \Theta$

$\theta \succsim^h \theta'$: individual h thinks state θ is at least as good as θ'.

Social ordering $\succsim = \underbrace{\Sigma}_{\text{constitution}}\underbrace{(\succsim^1, \succsim^2, \ldots, \succsim^h)}_{\text{profile of preferences}}$

☆ $\succsim$ invariant if utility (ordinal) is transformed monotonically

☆ Assume society is individualistic

AXIOMS

Universality : Σ is defined for all logically possible profiles of preferences

Pareto unanimity : $\forall \theta, \theta' \in \Theta, (\theta \succ^h \theta' \; \forall h \Rightarrow \theta \succ \theta')$ think majority voting

Independence of irrelevant alternatives : If two profiles are identical for some $\hat{\Theta} \subset \Theta$, then the derived social orderings should be identical over $\hat{\Theta}$.

Non-dictatorship : $\nexists h$ s.t. $\forall \theta, \theta' \in \Theta$, $\theta \succ^h \theta' \Rightarrow \theta \succ \theta'$

ARROW'S IMPOSSIBILITY THEOREM

No Σ satisfies above axioms if there are >2 social states

PARETO

A social state θ is Pareto superior to θ' if $\forall h, \; \underbrace{v^h(\theta)}_{\text{Utility person } h \text{ gets under } \theta} \geq v^h(\theta')$
$\exists h$ s.t. $v^h(\theta) > v^h(\theta')$

A social state θ is Pareto efficient if it is feasible$^{\in \Theta}$ and $\nexists \theta'$ Pareto superior to θ

EFFICIENT ALLOCATION

Assume private goods

$$\mathcal{L} = U^1(x^1) + \sum_h \lambda_h \underbrace{\left(U^h(x^h) - v^h\right)}_{\text{utility constraint of every other hh}} - \sum_f \mu_f \underbrace{\Phi^f(q^f)}_{\text{technical feasibility}} + \sum_i K_i \underbrace{(\overset{\sum_f q_i^f}{q_i} + R_i - \overset{\sum_h x_i^h}{x_i})}_{\text{materials balance}}$$

FOCs : (d wrt x_i^h, q_i^f)

$$\begin{cases} \lambda_h \underbrace{U_i^h}_{\text{MU of hh } h \text{ wrt } \Delta \text{ in } x_i^h}(x^h) = K_i \\ \mu_f \Phi_i^f(q^f) = K_i \end{cases}$$

for each good i, hh h, firm f

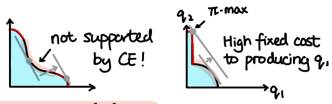
$$\frac{U_i^1(x^1)}{U_j^1(x^1)} = \ldots = \frac{U_i^h(x^h)}{U_j^h(x^h)} = \frac{K_i}{K_j}$$

MRS = Shadow price ratio

$$\frac{\Phi_i^1(q^1)}{\Phi_j^1(q^1)} = \ldots = \frac{\Phi_i^f(q^f)}{\Phi_j^f(q^f)} = \frac{K_i}{K_j}$$

MRT = Shadow price ratio

Welfare Theorem 1
If all consumers are greedy and there are no externalities, private ownership and private goods, then any competitive equilibrium is Pareto efficient



Welfare Theorem 2
If, in addition to conditions for Theorem 1, there are no non-convexities then any Pareto efficient allocation can be supported by a competitive equilibrium

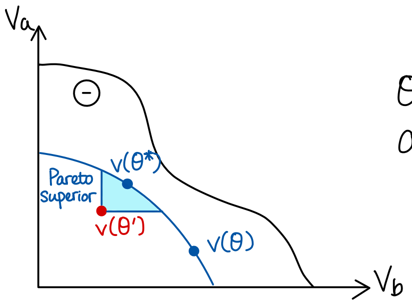
i.e. there is a property distribution and a price system such that people actually choose the Pareto efficient allocation

ACCESSIBLE θ is accessible from θ' if $\sum_{h=1}^{n_h} CV^h(\theta' \to \theta) > 0$

monetary value of a change from θ' to θ state for person h
>0 means welfare gain

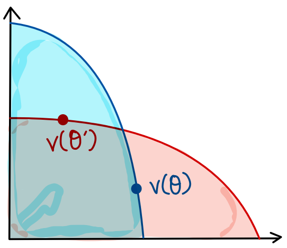
$\sum > 0$ means we gain enough from move to compensate individuals who lose out

POTENTIALLY SUPERIOR



θ is potentially superior to θ' if θ is accessible from θ' and $\exists \theta^* \in \hat{\Theta}(\theta)$ s.t. θ^* is Pareto superior to θ'.

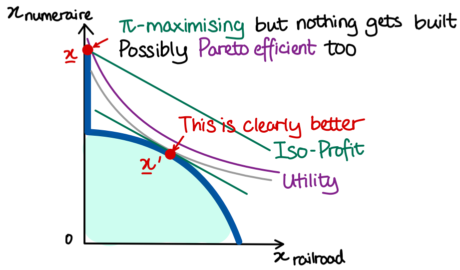
subset of Θ accessible from θ



But θ can be potentially superior to θ' and θ' to θ

MARKET FAILURE

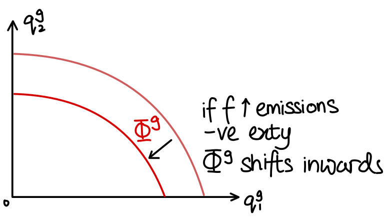
$\boxed{\text{IRTS}}$ High one-time/upfront fixed costs.



EXTERNALITY

- A type of "involuntary" transaction
- Agents can't be excluded from transaction using conventional price mechanism

PRODUCTION EXTERNALITY



$\dfrac{\partial \Phi^g}{\partial q_i^f} > 0$ if negative production externality

If it used to be that $\Phi^g(q^g) = 0$,
a $\uparrow q_i^f$ will cause q^g to now be infeasible ($\Phi(q^g) > 0$)

Maximise $U^1(x^1)$ subject to

$$\begin{cases} U^h(x^h) \geq v^h, & h = 2, \dots, n_h \\ \Phi^f(q^f, (q_\cdot)^{-f}) \leq 0, & f = 1, \dots, n_f \\ x_i \leq q_i + R_i, & i = 1, \dots, n \end{cases}$$

Consider $q_\cdot$ production by all other firms ($-f = 1, \dots, f-1, f+1, \dots, n_f$)

FOCs (for all h, f):

$$\begin{cases} \lambda_h U_i^h(x^h) = K_i, & i = 1, \dots, n \\ \mu_f \Phi_1^f(q^f) + \sum_{g=1}^{n_f} \mu_g \dfrac{\partial \Phi^g(\cdot)}{\partial q_1^f} = K_1 & \\ \mu_f \Phi_i^f(q^f) = K_i, & i = 2, \dots, n \end{cases}$$

☆ Term equals 0 when $g = f$, the polluting firm

MARGINAL EXTERNALITY

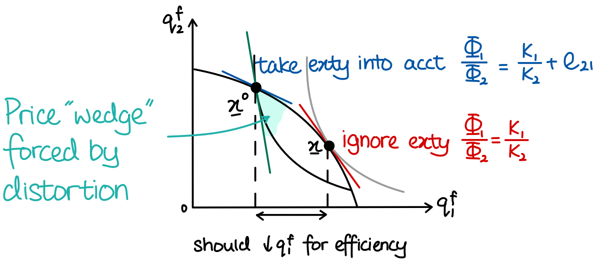
$$e_{21}^f = \sum_{g=1}^{n_f} \frac{1}{\Phi_2^g} \left(-\frac{\partial \Phi^g}{\partial q_1^f} \right)$$

<0 if $-$ve exty
>0 if $+$ve exty

Marginal externality caused by firm f producing good 1 q_1^f on all firms measured in terms of good 2 (numeraire good)

Thus, from the FOCs, $\dfrac{\Phi_1^f(q^f)}{\Phi_2^f(q^f)} - e_{21}^f = \dfrac{K_1}{K_2}$

MRT adjusted by exty = Shadow price ratio



x° inefficient! Consumers' on lower U

Producer prices ≠ Consumer prices ! $\dfrac{\Phi_1^f(q^f)}{\Phi_2^f(q^f)} = \dfrac{U_1^h(x^h)}{U_2^h(x^h)} + e_{21}^f$

POLICIES FOR EXTERNALITY

Correcting the distorted FOC $\frac{\Phi_1^f(q^f)}{\Phi_2^f(q^f)} = \frac{P_1}{P_2} + e_{21}^f$

* mkt prices introduced

- (Pigouvian) Tax : $t = -e_{21}^f$ >0 if $-$ve exty, <0 if $+$ve exty (subsidy)
- Merge polluting firm with victim to internalise exty
- Bribe (!)

BRIBE

ASSUMPTION

- Just 2 firms, firm 1's output of q_2 imposes costs on firm 2
- Just 2 goods (unimpt)
- Full info on Φ, exty, activity
- Costless communication

FIRM 2

Firm 2 pays firm 1 a bribe $\beta(q_i^1)$, decreasing in q_i^1 as a disincentive to pollute

So firm 2 chooses β, q^2 to max profits $\sum_{i=1}^n p_i q_i^2 - \beta - \mu_2 \Phi^2(q^2; q_i^1)$

FOCs : $\begin{cases} p_i - \mu_2 \Phi_i^2(q^2, q_i^1) = 0 \\ -1 + \mu_2 \frac{d\Phi^2(q^2, q_i^1)}{dq_i^1} \frac{dq_i^1}{d\beta} = 0 \end{cases}$

$$\Downarrow$$

$$-1 - \mu_2 \Phi_2^2(q^2; q_i^1) e_{21}^1 \frac{dq_i^1}{d\beta} = 0$$

$$\Downarrow$$

$\frac{d\beta}{dq_i^1} = p_2 e_{21}^1$ <0 if $-$ve exty

FIRM 1

Firm 1 knows bribe is conditional on q_i^1, under its control, and responds rationally

So firm 1 chooses q^2 to max profits $\sum_{i=1}^n p_i q_i^1 + \beta(q_i^1) - \mu_1 \Phi^1(q^1)$

FOCs: $\begin{cases} p_1 + \frac{d\beta(q_i^1)}{dq_i^1} - \mu_1 \Phi_1^1(q^1) = 0 \\ p_i - \mu_1 \Phi_i^1(q^1) = 0, \quad i = 2, \dots, n \end{cases}$

Combining:

$$\frac{\Phi_1^1(q^1)}{\Phi_2^1(q^1)} = \frac{p_1}{p_2} + e_{21}^1$$

- Bribe has internalised the exty
- Works the same as Pigovian tax, but without external guidance
- Require full info and both firms to work rationally (Coase). Stringent assumptions!
- If firm 1 can sell rights to pollute (i.e. produce x_i^1) indefinitely, firm 2 can't afford the bribe

WASTE MEASUREMENT

Total waste in the economy is Λ = Consumer welfare loss (measured as $|EV|$) − change in profits

SOCIAL WELFARE FUNCTION

$W(v^a, v^b, \ldots)$ for households $a, b, \ldots$

VEIL OF IGNORANCE

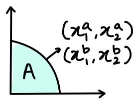
To determine shape of W, we use veil of ignorance approach — each individual born into a social state randomly ("lottery on identity"), so utility function is vN-M: $\sum_{\omega \in \Omega} \pi_\omega u(\underline{x}_\omega) = \sum_{\omega \in \Omega} \pi_\omega v_\omega$

Replace Ω with a set of identities $\{1, 2, \ldots, n_h\}$, assume each identity is equally probable $\pi_h = \frac{1}{n_h}$ *questionable!

Then $W(v^a, v^b, \ldots) = \frac{1}{n_h} \sum_{h \in \{1, \ldots, n_h\}} v^h$

RELATING TO ALLOCATION OF GOODS

* $y = f(x_1, \ldots, x_n)$, then the total differential is $dy = \frac{\partial f(\underline{x})}{\partial x_1} dx_1 + \ldots + \frac{\partial f(\underline{x})}{\partial x_n} dx_n$



$\rightarrow$ $v^a = U^a(x_1^a, x_2^a)$, $v^b = U^b(x_1^b, x_2^b)$ $\rightarrow$ $W(v^a, v^b) = W(U^a(\underline{x}^a), U^b(\underline{x}^b))$

*Assumes everyone is selfish

↳ Individualistic welfare model

Effect on a change in x_i^h on v^h, $\quad \frac{\partial v^h}{\partial x_i^h} = U_i^h(\underline{x}^h)$ U must be cardinal to be aggregated

Effect on a change in $x_1^h, \ldots, x_n^h$ on v^h, $\quad dv^h = \sum_{i=1}^{n} U_i^h(\underline{x}^h)\, dx_i$

Effect on a change in $v^1, \ldots, v^{n_h}$ on W, $\quad dW = W_1 dv^1 + W_2 dv^2 + \ldots$ Sub^n!

Effect on a change in $\underline{x}^1, \underline{x}^2 \ldots$ on W is $\quad dW = W_1 \sum_{i=1}^{n} U_i^1(\underline{x}^1) dx_i^1 + \ldots + W_{n_h} \sum_{i=1}^{n} U_i^{n_h}(\underline{x}^{n_h}) dx_i^{n_h}$

$$= \sum_{i=1}^{n_h} \left[W_h \sum_{i=1}^{n} U_i^h(\underline{x}^h)\, dx_i^h \right]$$

Person h's "weight" depends on importance of his utility to W and MU of x_i^h to him

SWF MAXIMUM

The gov chooses allocation $[\underline{x}^h]$ to Max $W(U^1(\underline{x}^1), \ldots)$ subject to $\Phi(\underline{x}) \leq 0$ ← tech feasibility, materials balance

No pdn, just gov redistributing goods

$\mathcal{L} = W(U^1(\underline{x}^1), \ldots) - \lambda \Phi(\underline{x})$ bc $0 - \Phi(\underline{x}) \geq 0$

FOCs: $W_h(\ldots) U_i^h(\underline{x}^h) - \Phi_i(\underline{x})$ for all h, i

Thus $\frac{U_i^h(\underline{x}^h)}{U_j^h(\underline{x}^h)} = \frac{U_i^\ell(\underline{x}^\ell)}{U_j^\ell(\underline{x}^\ell)}$ MRS equated across all agents. Pareto efficiency! (9.3)

and $W_h U_i^h(\underline{x}^h) = W_\ell U_i^\ell(\underline{x}^\ell)$ Social marginal utility of good i equal across all agents * $W_h U_i^h = \frac{dW}{dx_i^h}$, effe

W is maximised only when transferring goods (e.g. good i) from h to ℓ/ℓ to h doesn't ↑W

SWF MAXIMUM - MARKET ECONOMY

Each hh max U^h subject to $\sum p_i x_i^h \leq y^h$. Gain indirect utility function $V^h(\underline{p}, y^h)$

Then W becomes $W(V^1(\underline{p}, y^1), \ldots)$

and at optimum, $U_i^h(\underline{x}^h) = V_y^h p_i$ So $W_h V_y^h = W_\ell V_y^\ell = M$. Welfare value of gaining \$1 equated across all agents

Effect of Δy on W

ΔY ΔP ON W

ΔY: $\quad dW = \sum_{h=1}^{n_h} W_h dU^h = \sum_{h=1}^{n_h} W_h V_y^h dy^h \overset{\text{near the optimum}}{=} M \sum_{h=1}^{n_h} dy^h$ Proportional to change in national income $(y^1 + \ldots + y^{n_h})$

Roy's identity: $V_i = -V_y x_i$ $\quad$ Private gd: $x_i = \sum_h x_i^h$

Δp (y^h unchanged): $\quad dW = \sum_{i=1}^{n} \sum_{h=1}^{n_h} W_h V_i^h dp_i = -\sum_{i=1}^{n} \sum_{h=1}^{n_h} W_h V_y^h x_i^h dp_i = -M \sum_{i=1}^{n} x_i dp_i$ near an optimum

Δexpenditure by all hh on all goods

Thus, near an optimum, ΔW can be measured by Δnational expenditure / national income 9.12

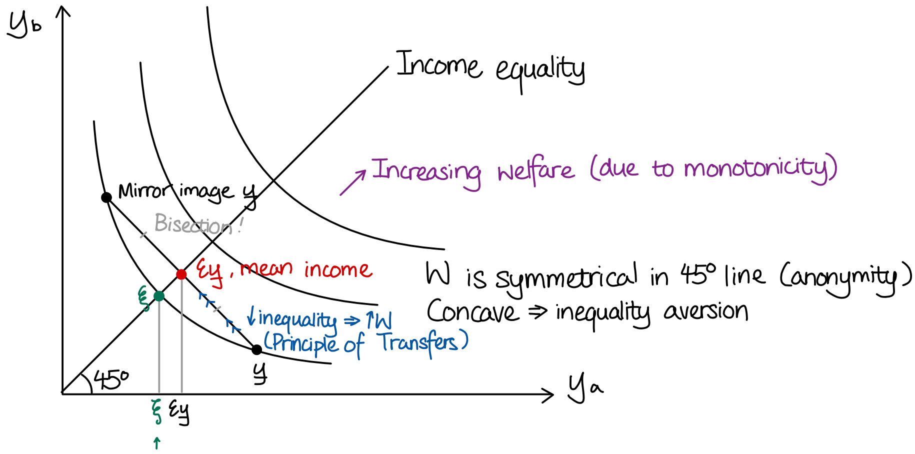
same in mkt eco^y

Assumptions: Comparability btwn ppl, Well-informed agents, private goods, complete markets. ignores inequality

INEQUALITY

ASSUMPTIONS

Incomes can be compared across people
Anonymity: labelling of individual incomes doesn't matter
Monotonicity: increase any person's income $\Rightarrow \uparrow W$
Population principle: $W(y) > W(y') \Rightarrow W(y,y,y,\ldots) > W(y',y',y',\ldots)$
Principle of Transfers: Transfer from a poorer to a richer person
Decomposability: $W(y) > W(y') \Rightarrow W(y,z) > W(y',z)$
Scale irrelevance: $W(y) > W(y') \Rightarrow W(\lambda y) > W(\lambda y')$, $\lambda > 0$



Welfare-equivalent income: Income that gives same W as y if given to all

$I = 1 - \frac{\xi}{\mathcal{E}y}$ is % of income society would give up to eliminate inequality

$\Rightarrow \xi = \mathcal{E}y(1-I)$ welfare = mean income × (1 − inequality)

Total differential $\frac{d\xi}{\xi} = \frac{d\mathcal{E}y}{\mathcal{E}y} - \frac{dI}{1-I}$

SOCIAL EVALUATION FUNCTION

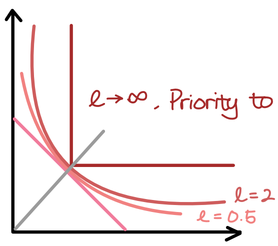
Social evaluation function, $\zeta(y) = \frac{y^{1-\ell}}{1-\ell}$. ℓ is index of inequality aversion

$\uparrow \ell \Rightarrow \uparrow I$

Assume W is decomposable/ additively separable,

$$W = \frac{1}{n_h}\sum_{h=1}^{n_h} \zeta(y^h) = \mathcal{E}\zeta(y^h) = \frac{1}{n_h}\sum_{h=1}^{n_h} \frac{(y^h)^{1-\ell}}{1-\ell}$$

- We can then solve $\zeta(\xi) = W$ to find ξ as a function of $\mathcal{E}y$, then determine $I = 1 - \frac{\xi}{\mathcal{E}y}$.
- Using the above specifications makes ζ concave and $I \in [0,1]$ $\forall \ell \geq 0$.
- Using $\zeta(y) = \frac{y^{1-\ell}-1}{1-\ell}$ gives the same end result.



$\ell \to \infty$, Priority to the poorest (Rawlsian) $W = \min_h y^h$

$\ell = 0$, indifferent to inequality (Benthamite) $W = \sum_{h=1}^{n_h} y^h$. Maximises total income

SACRIFICE

will be negative

Government has to take a maximum of $\left|\frac{dy^a}{dy^b}\right|$ from a to give 1 unit of income to b w/o changing welfare.

when W is maximised

The sacrifice, $\frac{dy^a}{dy^b}$, is calculated using $\sum_{h=1}^{n_h} W_h V_y^h dy^h = dW = 0$

$W_h = \frac{\partial W}{\partial V_h}$, $V_y^h = \frac{\partial V^h}{\partial y^h}$

If there are just a and b,
a has A times the income of b ($y^a = Ay^b$)
Welfare is $W = \sum_{h=a,b} \frac{1}{1-e}(y^h)^{1-e}$

then $\left|\frac{dy^a}{dy^b}\right| = A^e$

↑ inequality aversion e ⇒ ↑ max sacrifice of a