Revision Notes by Sally Yang

MACROECONOMIC PRINCIPLES

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GDP

Value of all G&S produced in an economy in a period of time
- Use market value where possible
- Final goods only
- Flow, not stock

PRODUCTION APPROACH GDP = sum of all value added over all industries producing G&S

Value added = $\underset{P\times Q}{\text{Value}}$ of o/p produced − $\underset{P\times Q}{\text{value}}$ of intm goods used in production

use cost of pdn (public services)/impute if no mkt value

EXPENDITURE APPROACH GDP = hhs'C + I + gov's C + net exports

hhs'C	Purchases of final G&S by hhs
I	Purchases of new capital goods by firms (fixed I) + Purchases of new residential structures + Net Δin inventories of goods
gov's C	Purchases of goods to provide public services (incl. own pdn)
net exports	Exports − Imports

- Capital goods used for future pdn included
- Intermediate goods excluded

INCOME APPROACH GDP = Sum of all income derived from production

= Wages + rents + net interest (on debt) paid by firms + profits (inc. Δinventories and I) + indirect taxes

- Incomes are paid by firms (or gov in public service)
- As indirect taxes are deduced from profits, must add them back for consistency

possible sources

ALTERNATIVES

GNP = GDP + net international income

Net National Product = GNP − Depreciation of capital − cost of maintaining existing K stock

NNP at basic prices = NNP − Indirect taxes − impact of indirect taxes on mkt prices

ISSUES WITH GDP

GDP would be a good measure of welfare or living standards if market prices reflect value of goods and services to consumers. But need to adjust for price differences (time and cross country), divide by population. Still silent on inequality.

ISSUES

- No market prices for all goods (public services valued at cost, impute rents for owner occupied housing)
- Total benefits derived from goods greater than price (internet services with no charges)
- Cost of production not reflected in price (pollution)
- Increase in GDP may be due to spending to deal with 'bads'
- Excludes non market activities
- Labour and capital treated as 'free'
- Earning wages uses up time so better to divide income by hours of work for a productivity measure
- Should subtract depreciation since capital stock must be maintained

REAL GDP Account for inflation

$$\text{Real GDP}_{2|1} = \sum_i p_1^i x_2^i$$

in year 2 at year 1 prices

$$\text{Real GDP growth at year 1 prices} = \frac{\sum_i p_1^i x_2^i - \sum_i p_1^i x_1^i}{\sum_i p_1^i x_1^i} \quad = NGDP_1$$

- Can overestimate real growth due to consumers substituting away from goods that become relatively more expensive (substitution bias)

$$\text{Real GDP growth at year 2 prices} = \frac{\overbrace{\sum_i p_2^i x_2^i}^{=NGDP_2} - \sum_i p_2^i x_1^i}{\sum_i p_2^i x_1^i}$$

- Can underestimate real growth due to substitution bias, again. Should establish there there was is a difference in growth rates of individual quantities and relative prices.

CHAIN-WEIGHTED REAL GDP Fisher Ideal Index gives geometric average of GDP scaling factors

Take geometric average of growth calculated using year 1 prices and growth calculated using year 2 prices

$$\sqrt{\frac{RGDP_{2|1}}{RGDP_{1|1}} + \frac{RGDP_{2|2}}{RGDP_{1|1}}} \overset{e.g.}{=} \sqrt{3\times 1.36} = 2.02$$

We can then use year 1 RGDP using year 1 or year 2 prices (the choice doesn't affect growth rate, just a choice of units) to calculate year 2 chain weighted RGDP

Better since it uses an average instead of two extremes, and uses both years' price data instead of just one year.

CPI

Cost of buying a basket of goods based on past consumption patterns

$$\text{Inflation as } \%\Delta CPI \text{ (year 1 basket)} = \frac{\sum_i p_2^i x_1^i - \sum_i p_1^i x_1^i}{\sum_i p_1^i x_1^i}$$

Can overestimate real inflation due to consumers substituting away from goods that become relatively more expensive (substitution bias)

$$\text{Inflation as } \%\Delta CPI \text{ (year 2 basket)} = \frac{\sum_i p_2^i x_2^i - \sum_i p_1^i x_2^i}{\sum_i p_1^i x_2^i}$$

Can underestimate real inflation due to consumers substituting away from goods that become relatively more expensive (substitution bias)

Note: calculating real GDP growth changes quantity x, calculating inflation changes prices
Note: sometimes CPI is normalised (with base year CPI=1. Doesn't matter.)

CHAIN-WEIGHTED INFLATION

Take geometric average of inflation calculated using year 1 basket and inflation calculated using year 2 basket

e.g. $\sqrt{1.62 \times 0.73} = 1.09$

GDP DEFLATOR

Ratio between nominal and real GDP

GDPDEF = NGDP/Real GDP

$$= \frac{DEF_2 - DEF_1}{DEF_1}, \quad DEF_1 = \frac{NGDP_1}{RGDP_{1|1}} = 1, \quad DEF_2 = \frac{NGDP_2}{RGDP_{2|1}}$$

% change in GDP Deflator between years 1 and 2 calculated using year 1 prices equals inflation calculated using year 2 basket (when the baskets equal production, not always) The GDP deflator tends to overstate real growth, which is the flip side of the year 2 basket inflation understating inflation

GDP Deflator covers all goods and services, including non-consumer goods, but CPI only covers a hypothetical basket of consumer goods for the average domestic household.

PPP EXCHANGE RATE

If we use country A's basket,

\$1 is equivalent in purchasing power to $\dfrac{\text{Cost of country A basket of goods in country A in \$}}{\text{Cost of country A basket of goods in country B in £}} \times £1$

Country A currency

Country B currency

If we use country B's basket,

\$1 is equivalent in purchasing power to $\dfrac{\text{Cost of country B basket of goods in country A in \$}}{\text{Cost of country B basket of goods in country B in £}} \times £1$

Country A currency

Country B currency

In practice, we use Fisher ideal index $\sqrt{1.57 \times 1.85} = 1.7$

PPP exchange rates using A/B-basket

We can then (re)calculate the GDP of country B in \$ using the PPP exchange rate. If country B is initially poorer than A then the PPP adjusted GDP will usually higher than before, reducing the income gap between A and B (goods are more affordable in developing countries)

MICROECONOMIC FOUNDATIONS

HOUSEHOLDS finding optimal labour supply N^s

☆ assume a representative hh

☆ Static model – No borrowing/saving. No future.

CONSTRAINT
All vars in real

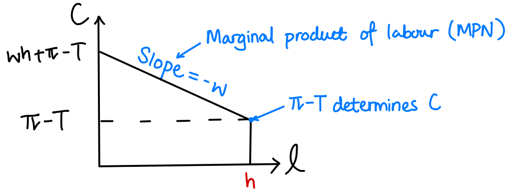
Budget : Consumption $C = wN^s + \pi - T$

(net) wage rate; labour supply (hours); non-wage income; total taxes

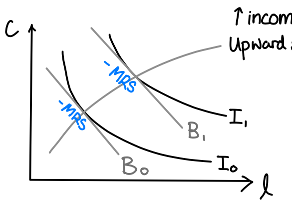
Time : Leisure $\ell = h - N^s$

h ↳ total no. of hrs available (fixed)

$$\boxed{C + w\ell = wh + \pi - T}$$



INDIFFERENCE CURVES



- Convex to O
- ↑ℓ ⇒ ↓MRS

OPTIMISE

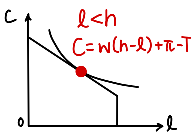
Participation decision

Interior Solution

Tangency condition: Net wage rate = MRS

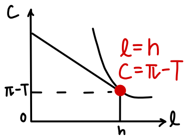
w, or $(1-t)w$ with prop. tax

Slope of constraint = Slope of indifference curve



Corner solution

- Not working. $\ell = h$. MRS > Net wage rate
  - ↑π − T ☆ T can be −ve if received net transfer
  - ↓w flatter constraint
  - Steeper indifference curve strong pref for leisure



MARGINAL RATE OF SUBSTITUTION

- −Slope of indifference curve
- ↑ℓ —— ↓MRS
- Amt of C-goods needed to compensate a hh for loss of 1 hr of leisure

EFFECT OF ↑w

SUBSTITUTION EFFECT

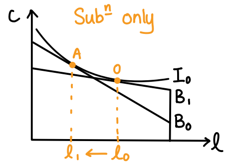
↑cost of leisure ⇒ ↓ℓ ↑N^s

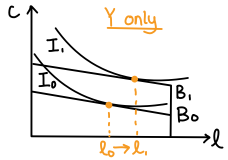
- Isolated effect: Steeper gradient, same indifference curve

INCOME EFFECT

↑$wN^s = Y$ ⇒ ↗C ↗ℓ

- Isolated effect: Parallel ↑ shift of constraint





Overall: B pivoted upwards. ↗C but ?N^s

LABOUR SUPPLY
MRS = w or net wage rate $(1-t)w$

- Vary w and see how ℓ^* ($N^s = h - \ell$) responds
- If Sub effect > Y effect (esp SR), N^s is upward-sloping. may be inelastic if Yeff = subeff in LR, possibly downward-sloping
- ↑π − T ⇒ ↑I (income effect) ⇒ ←N^s

PROPORTIONAL INCOME TAX

$T = twN^s$

fixed % of wage; tax rate; hrs of labour; Net wage rate

- Causes slope of hh budget constraint $= -(1-t)w$
- $MRS = (1-t)w$ at optimum

FIRMS choosing optimal labour demand N^d

OPTIMISE PROFITS
☆ No maintenance cost of capital, all profits distributed as dividends

$\pi = Y - wN^d = zF(K, N^d) - wN^d$

pdn of o/p; wage bill; TFP (given); capital input; pdn fn

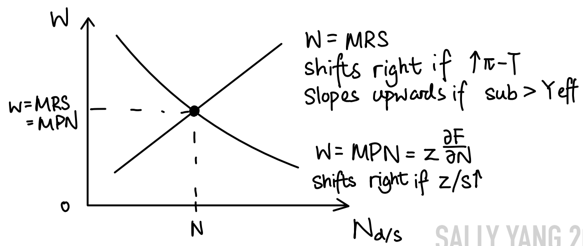
FOC $MP_N = z\frac{\partial F}{\partial N} = w$

MARGINAL PRODUCT OF LABOUR
$MP_N = z\frac{\partial F}{\partial N}$

- Units of C created by 1 additional hr of labour
- Different from MRS, which reflects preferences
- ↑N ⟹ ↓MP_N

LABOUR DEMAND
$MP_N = z\frac{\partial F}{\partial N} = w$

- Downward-sloping since ↑N ⟹ ↓MP_N
- Horizontal if $Y = zN$
- Shifts upwards with ↑z



MICROECONOMIC FOUNDATIONS

EFFICIENCY

COMPETITIVE EQUILIBRIUM

- Take w as given
- hhs optimise N^s w B constraint
- firms optimise N^d w pdn fn
- Balanced budget $G = T$ (no debt - one-period model)
- Real wages adjust so that markets clear

$$N^d = N^s$$

which implies $MP_N = w^* = MRS$

also implies the goods market clears too:

$$C + G = Y$$ pdn of o/p

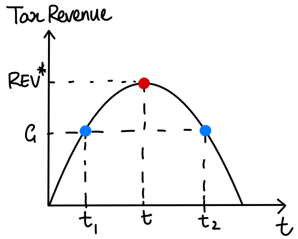
PARETO EFFICIENCY

$MP_N = MRT = MRS$

Impossible for one agent to be strictly better off w/o making at least one other strictly worse off

- Maximises utility given the physical / technological capability
- If $MP_N = MRT = MRS$ at competitive equilibrium, then it is also efficient
- MRT is marginal rate of tranformation : units of a good that must be forgone to create 1 unit of another in this case, it's time into goods $(= MP_N)$
- Cannot make the rep. hh better off thru any gov intervention that affects N^S or C
- Proportional income tax drives wedge between MRS and MP_N

LAFFER CURVE



Sample equation
$REV = t\underbrace{w(h-\ell)}_{\text{tax base}}$

Mechanical effect : $\uparrow t \Rightarrow \uparrow REV$

Negative incentive effect : $\uparrow t \Rightarrow \downarrow (h-\ell) \Rightarrow \downarrow REV$

Two different tax rates can give same revenue but different unemployment

t_1 : Mechanical eff > −ve incentive

- if $\uparrow t$ can $\uparrow REV$

t_2 ("wrong side") : −ve incentive > Mechanical eff.

- $\uparrow$ Unemployment
- Should $\downarrow t$ to $\uparrow REV$

NEOCLASSICAL PRODUCTION FN

Link between factor inputs and output of goods

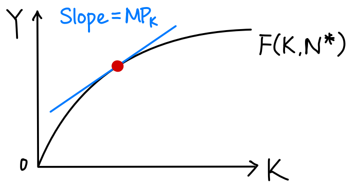
Upper case for aggregates

Y: Agg. o/p
K: Agg. capital
L: Labour
I: Agg. Investment
N: Total no. of workers
C: Agg. Consumption

Lower case for per worker

y: o/p per worker
k: capital per worker
ℓ: land per worker
c: consumption per worker

$$Y = zF(K,N)$$



1. Constant returns to scale

$$\lambda Y = zF(\lambda K, \lambda N) \quad \lambda > 0$$

2. Positive but diminishing MPK and MP_N

$$\frac{\partial Y}{\partial K} = zF_K(K,N) > 0 \qquad \frac{\partial Y}{\partial N} = zF_{N}(K,N) > 0$$

$$\frac{\partial^2 Y}{\partial K^2} = zF_{KK}(K,N) < 0 \qquad \frac{\partial^2 Y}{\partial N^2} = zF_{NN}(K,N) < 0$$

3. Inada conditions

$K \to 0 \Rightarrow MP_K \ \& \ MP_N \to \infty$

$K \to \infty \Rightarrow MP_K \ \& \ MP_N \to 0$

Guarantees a unique steady state

COBB-DOUGLAS $Y = zK^{\alpha}N^{1-\alpha}$

MALTHUSIAN MODEL OF GROWTH

- Population is endogenous
- Assumption: Ignore gov, closed eco^{y}, no I or S (all pdn consumed)

Production function $Y = zF(L, N)$ land (fixed)

$$y \equiv \frac{Y}{N} = \frac{zF(L,N)}{N} \overset{CRTS}{=} zF\left(\frac{L}{N}, 1\right) \equiv zf(\ell)$$

per worker pdn fn; land/worker

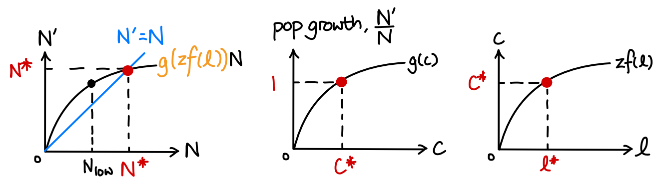
- $\uparrow N \Rightarrow \downarrow \frac{L}{N} = \ell \overset{f \text{ is increasing}}{\Rightarrow} \downarrow f(\ell) = y$

Market clearing condition $c = y = zf(\ell)$

Population growth $\frac{N'}{N} = g(c)$ *g is an increasing function

- Consumption/worker affects (fertility) population growth
- When market clears, $N' = Ng(y) = Ng(zf(\ell))$

STEADY STATE $N^* = N' = N$



If pop drops from $N^* \to N_{low}$, $\uparrow\ell \to \uparrow y \to \uparrow c \to \uparrow g(c) \to \uparrow N \to \downarrow\ell \to \downarrow y \to \downarrow c$

Thus, $g(c^*) = 1 \Rightarrow c^* = g^{-1}(1)$

$c^* = zf(\ell^*) \Rightarrow \ell^* = f^{-1}\left(\frac{c^*}{z}\right)$

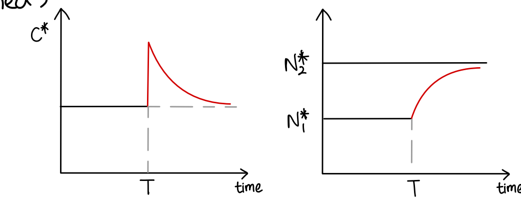
$N^* = \frac{L}{\ell^*}$

Or, in one line: solve $g\left(zF\left(\frac{L}{N^*}, 1\right)\right) = 1$

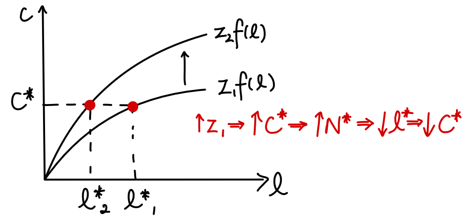
☆ LRSOL(c) determined solely by g.
Improvements in pdn tech or $\uparrow L$ doesn't affect SOL in LR!

- Pre-IR, GDP stagnant
- Malthus did not predict effects of tech adv on fertility – opp cost of raising a large family
- Didn't consider capital accumulation (L is limited, K is not)

$\uparrow z$ (TRANSITION) Malthusian trap



1. $\uparrow y = zf(\ell)$ (pop^n yet to rise)
2. $\uparrow c = y$ (market clearing)
3. $\uparrow \frac{N'}{N} = g(c) > 1$
4. $N' \longrightarrow N_2^*$ (higher steady state)
5. $\downarrow c$ ($\uparrow N$ offsets SOL change)

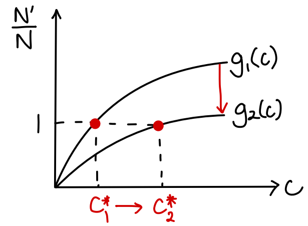


$\uparrow z_1 \Rightarrow \uparrow c^* \to \uparrow N^* \Rightarrow \downarrow \ell^* \Rightarrow \downarrow c^*$

No LR change in living standards!

POPULATION CONTROL Escaping the trap

$$\downarrow g(c) \overset{\frac{N'}{N}=1}{\Longrightarrow} \uparrow c^* \text{ (and } \uparrow \ell^*)$$



Living standards can rise to a higher level before pop^n starts to increase

SOLOW MODEL OF GROWTH

PDN FN

$Y = zF(K,N)$

POP GROWTH

$N' = (1+n)N$

★ exogenous (diff from Malthus!)

rate of pop growth constant

CAPITAL ACCUM^L

$K' = (1-d)K + I$

capital stock ★ physical goods only

depreciation

$I = S = sY$ ← hh's income

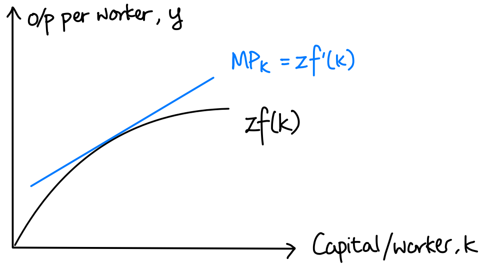
savings rate (constant)

★ closed economy. No G/T.

PER-WORKER

$$y = \frac{Y}{N} = \frac{zF(K,N)}{N} \overset{CRTS}{=} zF\left(\frac{K}{N},1\right) = zf(k)$$

$$f'(k) > 0,\quad f''(k) < 0$$



Concave: Diminishing marginal product of capital

COBB-DOUGLAS

$Y = zK^{\alpha}N^{1-\alpha}$ $\quad 0<\alpha<1$

- α is capital share (% of income capital receives)
- $MPN = z(1-\alpha)K^{\alpha}N^{-\alpha} = (1-\alpha)Y$ = workers' net wages
- Assume firms are competitive, price takers

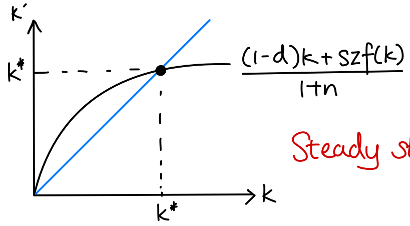
EQUILIBRIUM

- $Y = C + I$ (income-expenditure identity)
- $Y = C + S$ (consumers' budget constraint)
- Thus $I = S = sY$
- $K' = (1-d)K + sY$

$$\Rightarrow \frac{K'}{N} = (1-d)\frac{K}{N} + s\frac{Y}{N}$$

$$\Rightarrow \frac{K'}{N'}\left(\frac{N'}{N}\right) = (1-d)k + sy$$

$$\Rightarrow (1+n)k' = (1-d)k + szf(k)$$



Steady state is $k^* = k' = k$

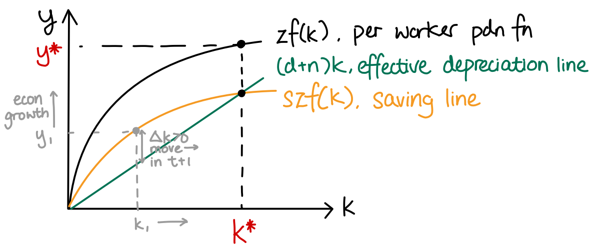
Solve $(1+n)k^* = (1-d)k^* + szf(k^*)$ to find k^*

CHANGE IN K

Dynamic Equation

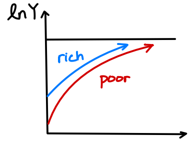
$$k' - k = \frac{szf(k) - (d+n)k}{1+n}$$

- Δk equal to saving (and investment) per worker minus investment per worker needed to maintain existing stock of capital per worker
- $k < k^*$: capital replaced faster than it depreciates
- $k > k^*$: capital replaced slower than it depreciates
- Economy then converges to unique (concavity and Inada) k^*



★ when z increases, k grows faster ($\frac{k'-k}{k}$ higher) when s is high

CONDITIONAL CONVERGENCE



- If countries share same f(k), z, s, n, d, they approach the same steady state
  - Poorer country (starting with less k) grows faster

STEADY STATE

k and y not growing, N, K, Y growing at rate n

- Pop^n can grow without harming living standards (more benign)

Exists if saving line crosses depreciation line (concavity, falling MP_K)

Unique if crosses only once (ignore 0)

Converge to it if saving > dep when $k > k^*$, < when $k < k^*$

Neoclassical F(K,N) satisfies all of the above

GOLDEN RULE

Steady state with optimal s

$\uparrow s$ → $\uparrow$growth + $\uparrow y^* k^*$ (but falling MP_K) $\uparrow C$ / → $\downarrow$% of output for con^n $\downarrow C$ → $\exists$ s that maximises C

Steady state of consumption/person $c^* = (1-s)y = (1-s)zf(k)$

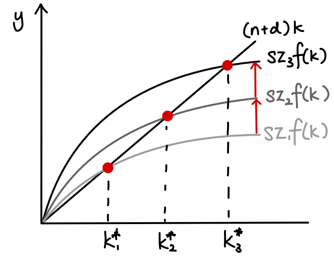
- Find k^* that maximises $c = zf(k^*) - (d+n)k^*$

$\Rightarrow$ where MP_k = slope of $zf(k) = d+n$

TECHNOLOGICAL PROGRESS

$\uparrow z$

$\uparrow z \Rightarrow \uparrow k^* \uparrow y^* \Rightarrow \uparrow$growth of y (temporary)



Cannot explain LR growth through capital accumulation

- k-accum is the growth mechanism endogenous to Solow
- But $\downarrow MP_K$ = harder to accum k as $k \uparrow$
- LR $\uparrow y$ thus exogenous in Solow model

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SOLOW MODEL WITH TECHNOLOGICAL PROGRESS

New production function $Y = F(K, BN)$
where B is labour-augmenting technology growing at exogenous rate g. $B' = (1+g)B$

GROWTH OF EFFECTIVE LABOUR

$B'N' = BN(1+g)(1+n) = BN(g+n)$ if g and n are small

CAPITAL / EFFECTIVE WORKER

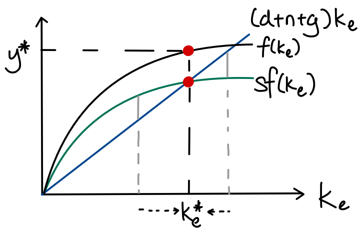
$k_e = \frac{K}{BN}$

$$y = \frac{Y}{BN} = \frac{F(K,BN)}{BN} \overset{CRTS}{=} F\left(\frac{K}{BN}, 1\right) = f(k_e)$$

$$k_e' - k_e = \frac{sf(k_e) - (d+n+g)k_e}{1+n+g}$$

$k_e \uparrow$ if $sf(k_e) > (d+n+g)k_e$

$k_e \downarrow$ if $sf(k_e) < (d+n+g)k_e$



STEADY STATE

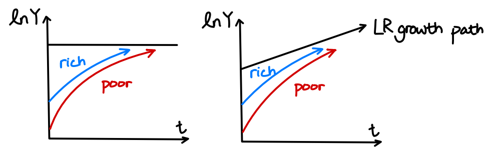
Growth rates of

k_e	y_e	k	y	N	K	Y
0	0	g	g	n	$g+n$	$g+n$

Solve $sf(k_e^*) = (d+n+g)k_e^*$

CONDITIONAL CONVERGENCE

- If countries share same $f(k)$, z, s, n, d, g they approach the same steady state
- With new B, will converge to the LR growth path!



Model w/o B   Model w B

TFP DIFFERENCE

If we let TFP = level of tech, poor countries should be skipping R&D and borrowing rich countries' tech

- But institutions affect efficiency with which they utilise FOP
- Corruption, risk of expropriation
- Taxes, regulation (IP law)

GROWTH ACCOUNTING

Q: How much of observed Y differences can differences in

$Y = K^\alpha (BN)^{1-\alpha} = zK^\alpha N^{1-\alpha}$ (i.e. $z = B^{1-\alpha}$)

$$\frac{\Delta Y}{Y} = \alpha \frac{\Delta K}{K} + (1-\alpha)\underbrace{\frac{\Delta N}{N}}_{=n} + \underbrace{\frac{\Delta z}{z}}_{=g}$$

Solow residual / Growth rate of TFP,

$$\frac{\Delta z}{z} = \frac{\Delta Y}{Y} - \left[\alpha \frac{\Delta K}{K} + \underbrace{(1-\alpha)}_{\text{capital share of income}} \frac{\Delta N}{N}\right]$$

In LR, $\frac{\Delta K}{K} = n+g$ so $\frac{\Delta Y}{Y} = n+g$, $\frac{\Delta y}{y} = g$

- If Country A has higher $\frac{\Delta K}{K}$ than B but lower $(n+g)$, A and B's output will grow at same rate $\left(\frac{\Delta Y}{Y}\right)$ in SR but B grows faster than A in LR (where $\frac{\Delta Y}{Y} = n+g$)
- Country with fastest tech growth (g) has highest growth in living standards (y) in LR ($\frac{\Delta y}{y} = g$)

COBB-DOUGLAS WITH TECH PROGRESS

In steady state, $sf(k_e^*) = (n+g+d)k_e^*$

With Cobb-Douglas, $f(k_e^*) = (k_e^*)^\alpha$

Then $k_e^* = \left(\frac{s}{d+n+g}\right)^{\frac{1}{1-\alpha}}$, $y^* = B\left(\frac{s}{d+n+g}\right)^{\frac{\alpha}{1-\alpha}}$

- e.g. if $s_A = 5s_B$, $y_A^* = 5^{\frac{\alpha}{1-\alpha}} y_B^*$

ENDOGENOUS GROWTH MODEL (1)

AK-MODEL

$Y = A^{\text{TFP}} K^{\alpha} N^{1-\alpha}$, $0<\alpha<1$

If A is constant, $\alpha = 1$, not neoclassical

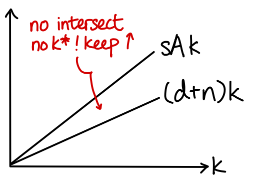
Then
$$\begin{cases} Y = AK \\ y = Ak \\ MP_K = A \text{ (not diminishing!)} \end{cases}$$

Key condition for endo growth
If diminishing

LONG-RUN GROWTH

Dynamic equation, $k'-k = \dfrac{sAk-(d+n)k}{1+n}$

If $sA > d+n$, $k \uparrow$ at constant rate
$y \uparrow$ at constant rate A



LEARNING-BY-DOING

- Skills/knowledge gained as free by-product during production (non-rivalrous, nonexcludable)
- Rationale: Positive externality in R&D, so MP_K not diminishing $\Rightarrow$ AK model

so $Y = nY_i$
drop n for simplicity

P^d f^n of rep^v firm i: $Y_i = K_i^{\alpha}(BN_i)^{1-\alpha}$

- $\alpha < 1$ (diminishing MP_K at firm level)
- Economy-wide knowledge, $B = \lambda K$
  economy-wide capital stock
  - Unlikely in reality — IP laws, poor countries can imitate others' tech so B not just dep on own K
  - Can be true on world-economy level
- $\lambda > 0$ $\Rightarrow$ positive externality

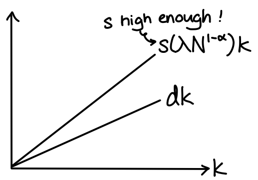
$$Y = \lambda^{1-\alpha} N^{1-\alpha} K$$

LONG-RUN GROWTH

Take N constant ($n=0$)

Dynamic equation, $k'-k = sy - dk$
$= s(\lambda N)^{1-\alpha}k - dk$

Growth of k is $\dfrac{k'-k}{k} = s(\lambda N)^{1-\alpha} - d$ ☆constant!



endo growth since stock of knowledge determined by endo level of k through learning-by-doing

☆ No Convergence!

SCALE EFFECTS

$N\uparrow \underset{\text{knowledge spillovers}}{\Rightarrow} \dfrac{k'-k}{k}\uparrow \Rightarrow \uparrow y$ growth rate

Set $B = Ak$ to remove effect

HUMAN CAPITAL

- Predicts endo LR growth (AK-model) w/o change in tech

HUMAN CAPITAL

$H^{s\prime} = b(1-u)H^s$ ☆linear - no dim returns in pdn of human cap

b = efficiency of human capital accum
u = % of time working
H^s = stock of human capital

- If $b(1-u) > 1$, endo growth in H, y

HH CONSUMPTION

$C = wuH^s$

w = real wage for each eff unit of lab

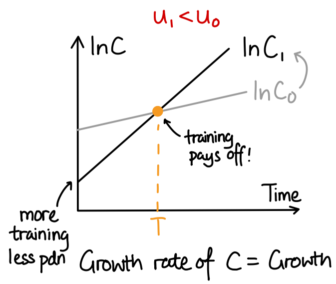
FIRM PDN

$Y = \underbrace{zuH^d}_{\text{efficiency units of lab}} = C$ in eqm

PROFITS

$\pi = Y - wuH^d = (z-w)uH^d$

$w<z$: hire more
$w>z$: don't hire more
$w=z$: indifferent



Growth rate of C = Growth of $H = b(1-u)-1$
$\downarrow u \Rightarrow \uparrow$ slope of $\ln C$

No convergence. Richer country starts off H

ENDOGENOUS GROWTH MODEL (2)

R & D

Ideas, unlike K, are non-rivalrous tho excludable (incentive to R&D)

$L = L_Y + L_A$ (in pdn; in R&D (not in pdn))

$L_A = \gamma_A L$ (γ_A: % of workers in R&D)

$y =$

$Y = AL_Y$ ignores K/H

$y = A\frac{L_Y}{L} = A(1-\gamma_A)$ y grows at rate $\hat{A}$

GROWTH RATE OF A

$\hat{A} = \frac{A'-A}{A} = \frac{\gamma_A L}{\mu}$

μ: cost of new inventions (lab input in R&D needed to double A)

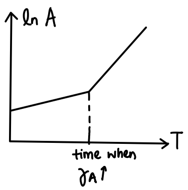
- Independent of A
  - ☆ No diminishing returns to R&D assumed!
    - ↳ not really true, looking at real life evidence
    - ↳ if MC of inventing (μ) is increasing, $\hat{A} \to 0$ as A increases
      - ↳ Cannot sustain endogeneous long run growth. A will tend towards (but never reach) some value

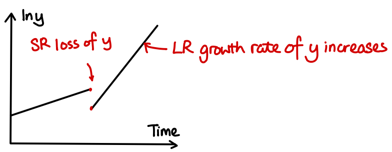
RETURNS

- Increasing? Past discoveries support further discoveries
- Decreasing? More advanced ideas are harder to get
- Assume constant — cancel out, maybe

SHIFTING LABOUR INTO R&D

$\uparrow\gamma_A$ → $\downarrow y$ (one-time); → $\uparrow\hat{A}$ → faster $\uparrow y$





R & D (TWO COUNTRIES)

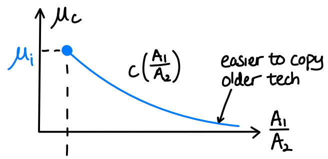
$Y_j = A_j(1-\gamma_{A_j})L_j \;,\; j=1,2$

Assume
- $L_1 = L_2 = L$ ignore scale effects
- $\gamma_{A_1} > \gamma_{A_2}$
- $\mu_c < \mu_i$ (μ_c: cost of imitation; μ_i: cost of invention)

Country 1 is technology leader
gets new tech by invention
Country 2 is the follower
gets new tech by invention and imitation

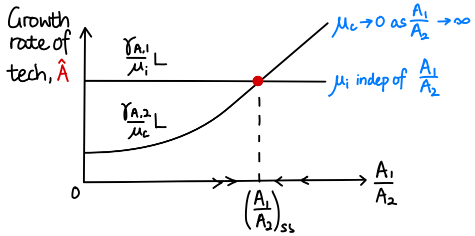
COST OF IMITATION

$\mu_c = c\left(\frac{A_1}{A_2}\right)$ How far country 1 is ahead



Assume
- c is downward sloping
- $c \to 0$ as $\frac{A_1}{A_2} \to \infty$
- $c(1) = \mu_i$

STEADY STATE OF $\hat{A}$

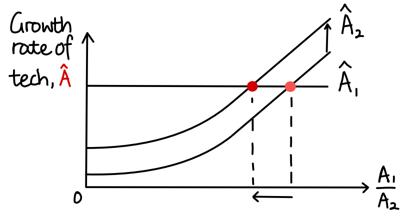


In steady state,

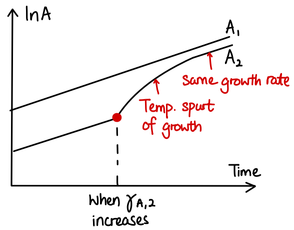
$\frac{\gamma_{A,1}}{\mu_i}L = \frac{\gamma_{A,2}}{\mu_c}L \Rightarrow \mu_c = \frac{\gamma_{A,2}}{\gamma_{A,1}}L$

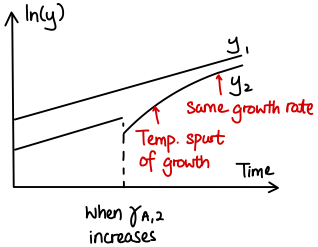
$\frac{A_1}{A_2} = c^{-1}(\mu_c) = c^{-1}\left(\frac{\gamma_{A,2}}{\gamma_{A,1}}\mu_i\right)$

↑R&D IN FOLLOWER



$\uparrow\gamma_{A,2} \Rightarrow$ Same $\hat{A}_2$ in LR. $\downarrow\left(\frac{A_1}{A_2}\right)_{ss}$

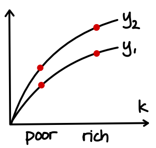


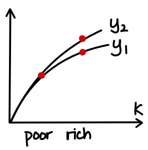


- Copying has diminishing returns
- The closer you get, less you can copy
- To overtake, must invest in invention

CONVERGENCE

Model doesn't explain large inequalities
- Tech developed in leader may not suit follower
  - Tech change is "neutral" if $\uparrow A \Rightarrow$ proportional upward shift in $f(k)/y$. e.g. $y = Ak^\alpha$
  - But it can be "capital-biased". e.g. $y = Ak(k^\alpha)$





CONSUMPTION PUZZLE

KEYNESIAN

$C = a + bY$ (C: aggregate con^n; Y: current agg. Y) $a>0,\ 0<b<1$

$C = a$ if $Y=0$ (paid for from borrowing or savings)

$b = MPC$: ΔC for every 1 unit ΔY

If $a=0$, $C \propto Y$

AVERAGE PROPENSITY TO CONSUME

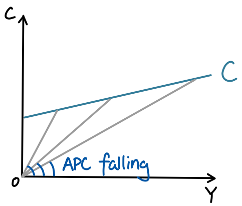
$APC = \frac{C}{Y} = \frac{a}{Y} + b$

AVERAGE PROPENSITY TO SAVE

$APS = 1 - APC$

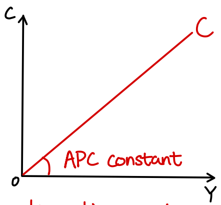
- Since $a>0$, $Y\uparrow \Rightarrow APC\downarrow \Rightarrow APS\uparrow$

KUZNETS PUZZLE



household cross-section

Across hh at fixed time t,
$Y\uparrow \Rightarrow APC\downarrow$
$a>0$

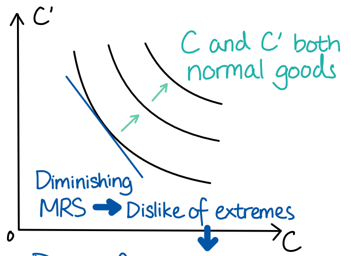


long time-series

Within country over time,
$Y\uparrow \Rightarrow APC$ —
$a=0$

TWO-PERIOD MODEL

t_1 = present $\quad t_2$ = future



Desire for consumption Smoothing

☆ non-durables & services only (durables like cars actly volatile)

LIFETIME BUDGET CONSTRAINT

Current: $C + S = y - t$

Future: $C' = y' - t' + (1+r)S$

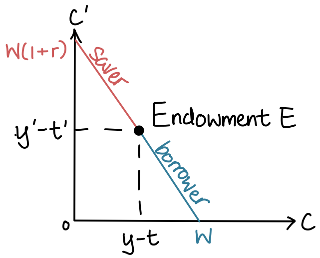
Lifetime: $C + \frac{C'}{1+r} = W$

$W = y - t + \frac{y'-t'}{1+r}$

Lifetime wealth

☆ Hh is i/r taker

- If no initial assets, W is present discounted value of income after tax



MAXIMISE LIFETIME UTILITY

$U(c,c') = u(c) + \beta u(c')$

$0<\beta<1$ due to impatience

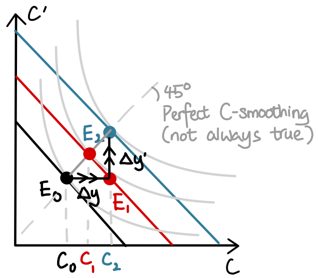
Subject to $C + \frac{C'}{1+r} = W$

At optimal (c, c'), utility max, tangency :

$MRS = \frac{MU_c}{MU_{c'}} = \frac{u'(c)}{\beta u'(c')} = 1+r$

rel. price

ΔINCOME



TEMPORARY

Δy but y' unchanged $\Rightarrow$ horizontal shift

- $c_1 - c_0 < y_1 - y_0$
- some Δy is saved to smooth out C

PERMANENT

horizontal (Δy) + vertical shift ($\Delta y'$)

- $c_2 - c_0 = y_2 - y_0$
- $c_2' - c_0' = y_2' - y_0'$
- $\Delta y \propto \Delta c$ and $\Delta y' \propto \Delta c'$

LIFE CYCLE THEORY OF CONSUMPTION

Life-cycle pattern in labour income

Older $\Rightarrow$ high y but expects low y'
$\Rightarrow$ optimal to save. $c<y$

Younger $\Rightarrow$ low y but expects high y'
$\Rightarrow$ optimal to borrow. $c>y$

- Cross-section predicts similar consumption levels for a wide range of income
- C is a function of wealth

PERMANENT INCOME THEORY

Permanent income : Constant level of C that can be sustained over time given path of income expected ($c=c'$ given y, y')

Transitory income : Deviation of current income from perm. income

In cross-section, income variation mostly transitory (different points in life; temp shocks)

- ΔC less than proportionate to ΔY
- Falling APC with Y

In time series, income variation mostly permanent (e.g. LR ↑GDP) (transitory factors on invs cancel out on agg)

- $\Delta C \propto \Delta Y$
- Constant APC

SALLY YANG 2021

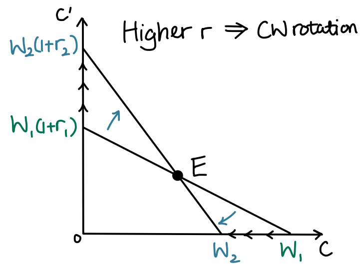
CONSUMPTION SMOOTHING

Aggregate C theoretically smoother than income ... BUT

Durables C more volatile than GDP
- Houses, cars not bought regularly; no "smoothing"
- Behaves like I

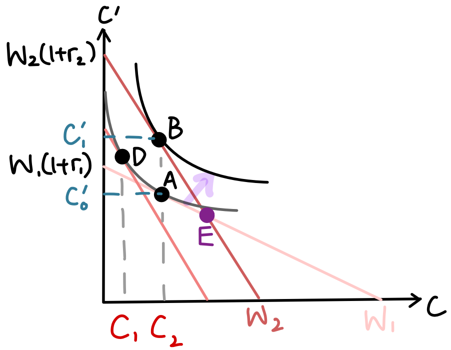
Nondurables C half as volatile as GDP
- Still more volatile than expected! Why?

INTEREST RATES & C

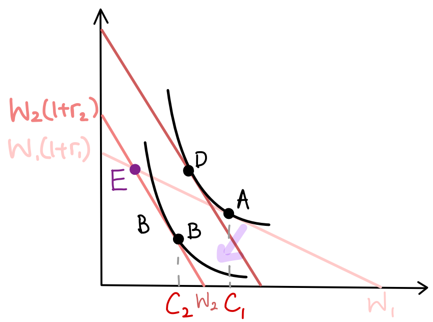


Higher r ⇒ CW rotation about E!

SAVER ↑r



BORROWER ↑r



Substitution effect: A → D
↓C ↑C' same indifference curve

Income effect: D → B
↑C ↑C' higher indifference curve

cancel

Overall: D → B
?ΔC ↑↑C' ?ΔS

Substitution effect: A → D
↓C ↑C' same indifference curve

cancel

Income effect: D → B
↓C ↓C' lower indifference curve

Overall: D → B
↓↓C ?ΔC' ↑S
borrow less

CONCLUSION

↓r incentivise spending for borrowers. income and substitution effects reinforce
↓r may not incentivise spending for savers if income effect ⩾ substitution effect

★ If $y'=0$ (saver), $C \perp\!\!\!\perp r$. Two effects cancel out

FISCAL POLICY ☆ all in real ☆ 2-period

GOV PRESENT-VALUE BUDGET CONSTRAINT

Current: $G = T + B$ ← issue bonds

Future: $G' + (1+r)B = T'$ repay debt (no default)

Present-value: $G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$

RICARDIAN EQUIVALENCE

If $T = Nt$ (lump-sum on N hhs)

$$G + \frac{G'}{1+r} = N\left(t + \frac{t'}{1+r}\right)$$

Consumer's $W = y + \frac{y'}{1+r} - \left(t + \frac{t'}{1+r}\right)$

$$= y + \frac{y'}{1+r} - \frac{1}{N}\left(G + \frac{G'}{1+r}\right)$$

a share of present-value gov exp

$W \perp\!\!\!\perp t, t'$!!

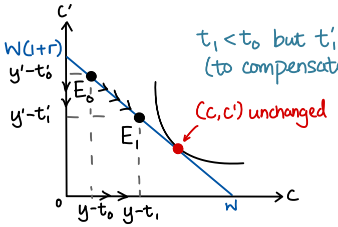
Timing of taxes (whether ↓t↑B or ↑t'↓B doesn't affect lifetime wealth.)

Effect on C is the same.

Limitations: Credit mkt imperfect, distortionary taxes

DEFICIT-FINANCED ↓T

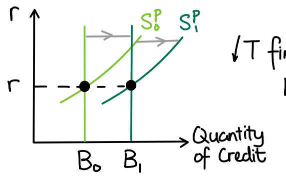
B↑ so that G—



$t_1 < t_0$ but $t_1' > t'$ (to compensate)

Income, $Y = C + G$ is unchanged

Interest rate r unaffected



↓T financed by ↑B is matched by ↑private saving

PENSIONS

FULLY FUNDED

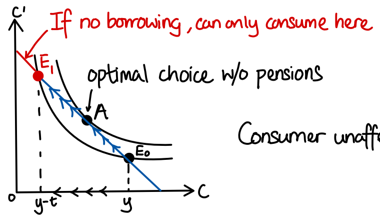
Mandated savings programme

- Assets acquired when young, sold in retirement

pensions when old ↓ $b' = (1+r)t$ ↙ mandated savings

$$W = (y-t) + \frac{y'-b'}{1+r}$$

$$= y + \frac{y'}{1+r} \text{ unaffected!}$$



Consumer unaffected!

- Worse-off only if mandates higher saving than consumer'd choose and cannot borrow in anticipation of benefits (i.e. wanted to consume on blue but can only consume on red)
- Special case of Ricardian equivalence (PS7)
- Rationale: prevent moral hazard
  if gov forced to support old who didn't save enough, even less incentive to save

LIMITATIONS

- Δt may affect distribution
  - Winners↑, Losers↓, agg cancel
- Consumers live shorter than gov
  - Intergenerational redistribution of debt
- Assumes income (willingness to work) is exogenous

So distortionary (Y/C) tax has same effect as lump-sum

But PS2 model shows ↑t can affect income-leisure tradeoff

PAY-AS-YOU-GO

$$N' = (1+n)N$$

pop growth (n); young alive currently (N'); old alive currently (N)

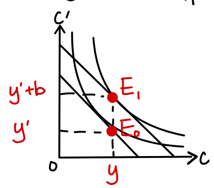
Total benefits old get $Nb = N't$ Total contributions young pay

$$\Rightarrow t = \frac{b}{1+n}$$

If pension introduced in time T

OLD IN T

Always better off

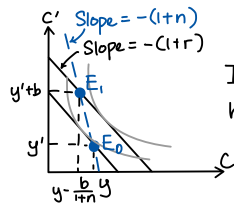


* Diagram assumes they know pension will be introduced when young

YOUNG IN T

$$W = y - \frac{b}{1+n} + \frac{y'+b}{1+r}$$

$$= y + \frac{y'}{1+r} + b\left(\frac{1}{1+r} - \frac{1}{1+n}\right)$$



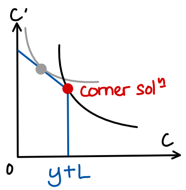
If $n > r$, W↑ budget↑ welfare↑

- Benefits current and future generations only if pop^n growth rate (implicit return from social security) exceeds interest rate (return from private credit markets)
- If incomes are growing, GDP growth > real r
- Remedies the mkt failure
  - current gen can't trade with future unborn gens
- Popular during baby booms and when r is low but aging society...
- hard to reform since one gen will have to t but no b
- If gov wants to ↓t without ↓b by raising debt, there's only a steady state level of debt if $n > r$ PS7

CREDIT-MARKET IMPERFECTIONS

Empirically, current y effect on current c is larger than expected if people where smoothing c.

BORROWING CONSTRAINT



$c \leq y + L$ max size of loan (assume $L \leq \frac{y'}{1+r}$)

If $c^* \leq y+L$, constraint doesn't matter

If $c^* > y+L$, a 1-unit ↑y ⇒ 1-unit ↑c

⇒ MPC = 1

⇒ Ricardian equivalence fails

LIMITED COMMITMENT

Introduce collateral to borrow against

Consumer starts off owning housing H at price p

H illiquid; cannot be sold in t_1

LIFETIME BUDGET CONSTRAINT

$$c + \frac{c'}{1+r} = y - t + \frac{y' - t' + pH}{1+r}$$

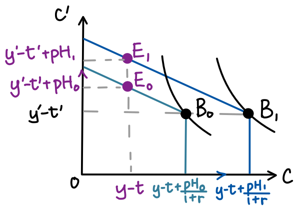
COLLATERAL CONSTRAINT

repayment of principal + interest $-s(1+r) \leq pH$ housing value

⇓ $s = y - t - c$

$$c \leq y - t + \frac{pH}{1+r}$$

↑H (WEALTH EFFECT)



★ with collateral, banks can offer $r_1 = r_2$ (no kinks at E)

★ Ability to use collateral opens access to credit and helps smooth c

⇒ Consuming at B > at E

⇒ But p fluctuate ⇒ c greatly affected

INTEREST-RATE SPREADS

saver borrower

$r_1 < r_2$

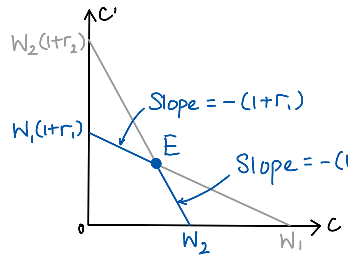
Gov can borrow at r_1

Lender's constraint:

$$c + \frac{c'}{1+r_1} = y + \frac{y'}{1+r_1} - \left(t + \frac{t'}{1+r_1}\right) = W_1$$

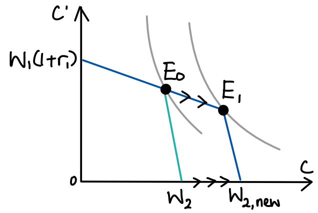
Borrower's constraint:

$$c + \frac{c'}{1+r_2} = y + \frac{y'}{1+r_2} - \left(t + \frac{t'}{1+r_2}\right) = W_2$$



CURRENT TAX CUT ↓t, ↑t' by $\frac{\Delta t}{1+r_1}$

2020 Q7 bc gov borrows at r_1



ASYMMETRIC INFORMATION

2020 Q7 for elab

Deposit (saving) rate $= r_1$

Loan (borrowing) rate $= r_2$

All borrow L.

a of borrowers never defaults

$1-a$ always defaults. Bank can't tell.

For $\pi = aL(1+r_2) - L(1+r_1) = 0$

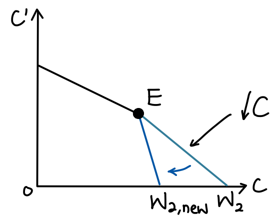
$\Rightarrow 1 + r_2 = \frac{1+r_1}{a}$ $(a<1)$

⇒ Default premium : $r_2 > r_1$

• ↓a ⇒ ↑$(r_2 - r_1)$

If ≥0, this is a diff source of credit mkt imperfection — imperfect competition causing markup $r_2 > r_1$

FINANCIAL CRISIS ↑a ↑r_2 ↓c



Changes in severity of asym. info causes variation in int. rate spreads

SALLY YANG 2021

INVESTMENT

★ Largest, most volatile % of total I

Business fixed I : Business' spending on eqp for pdn
Residential I : Buy new houses
Inventory I : Change in inventory stock

TWO-PERIOD MODEL

$Y = zF(K,N)$ ← neoclassical
$Y' = z'F(K',N')$

$K' = I + (1-d)K$

↑ current I only ready for pdn use in future

Current dividend $\pi = \underbrace{Y - wN}_{\text{current profits}} - I$

if <0, firm issues new shares/ borrows

Expected future dividends $\pi' = Y' - w'N' + \underbrace{(1-d)K'}_{\text{undepreciated K sold off}}$

Present value of dividends

$V = \pi + \frac{\pi'}{1+r} = Y - wN - (K' - (1-d)K) + \frac{Y' - w'N' + (1-d)K'}{1+r}$

OPTIMAL DECISION

Optimal employment N^* at $MP_N = w$
Optimal investment I^* at $MB_I = MC_I$

$I \to \uparrow K \to \pi$ in future

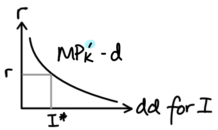
$MB_I = \frac{MP_K' + 1 - d}{1+r}$ ← resale value; $1+r$: converted to present value

$MC_I = 1$ — 1 unit of K costs 1

Choose K' (same as choosing I) to max
$V = Y - wN - (K' - (1-d)K) + \frac{Y' - w'N' + (1-d)K'}{1+r}$

FOCs:
$MP_N = w$
$MP_N' = w'$
$MP_K' = r + d$ ← opp cost for not investing in financial mkts (r); maintenance cost (d)

user cost of K



expected future TFP: $\uparrow z'$ or $\downarrow K \Rightarrow \uparrow MP_K'$ (future MP of capital) at each r $\Rightarrow \uparrow dd_I$ (→ shift)

BORROW TO INVEST

I^* is same no matter how financed (retained π, borrowing)
in the absence of financing constraints

If borrow I now and repay $(1+r)I$ later,
$\pi = Y - wN$
$\pi' = Y' - w'N' + (1-d)K' - (1+r)I$
Substituting $I = K' - (1-d)K$ to max $\pi + \frac{\pi'}{1+r}$
FOC for K' is still $MP_K' - d = r$
Same amt of investment!

EFFICIENT MARKETS HYPOTHESIS

$V = \frac{\pi'}{1+r}$ ← present value of π'

Share price in current period (a claim to π')

share price = present value of future expected dividends

TOBIN'S Q

Should buy more capital if $q > 1$ Also see PS8

$q = \frac{\text{Expected value of a firm's shares / market value of installed capital}}{\text{Replacement cost of installed capital}}$

$= \frac{V}{K'} \overset{EMH}{=} \frac{\left(\frac{\pi'}{1+r}\right)}{K'} = \dots = \frac{MP_K' - 1 + d}{1+r}$

Neoclassical investment theory consistent w q theory
if share price = present value of future expected dividends (EMH)

$q = 1 \Rightarrow MP_K' - d = r$! optimum

$MP_K' > r + d \Rightarrow$ profit rate $\uparrow \Rightarrow$ stock mkt value $\uparrow \Rightarrow q \uparrow$

Limitations PS8
- q is hard to measure
- doesn't measure human capital, brand value, etc
- If not CRTS... ???

Model suggests I depends primarily on π', but in reality I depends on π too

FINANCING CONSTRAINTS

ASYMMETRIC INFORMATION

Deposit (saving) rate $= r_1$
Loan (borrowing) rate $= r_2$
Default premium, $x = r_2 - r_1$

Then for firms borrowing to invest,
$MP_K' - d - x = r_2$ ($\uparrow x \Rightarrow \downarrow I$) negative relationship

But if using π, $MP_K' - d = r_2$

Thus, when $x > 0$, $\uparrow \pi \Rightarrow \uparrow I$

UNEMPLOYMENT - EFFICIENCY WAGE THEORY

EFFICIENCY WAGES

Worker productivity increases with wage

- Firms don't perfectly know workers' skill and
- commitment
- Wage paid affects effort put in and who applies
  Firms set wages to maximise returns from workforce relative to pay
- Hence stickiness

REASON - MORAL HAZARD

High wages discourage shirking

- Firm cannot directly control worker effort, but can imperfectly monitor workers
- Workers don't like putting in effort, but fear being caught shirking and dismissed
- Current wage relative to other jobs is cost of losing a job
- Higher wage provides greater incentive not to shirk

Assumptions: workers cannot commit to effort when employed, no reputational concerns, dismissal is worst punishment available for those caught shirking

REASON - ADVERSE SELECTION

- Firms don't know which workers are good and who will apply
- Offer high wages to attract and retain (save training costs) better workers

Labour force $L = E + U$

Participation rate $= \frac{L}{P}$ ← Working-age pop^n
<100% (school, childbearing, hysteresis)

Unemployment rate, $u = \frac{U}{L}$

Employment rate $= \frac{E}{P}$ ☆Not L!

Under EWT,

Effort per worker is $e(w)$. $e'(w) > 0$ increasing fn

Effective units of labour $E = e(w)N$

Then $Y = F(K, E)$

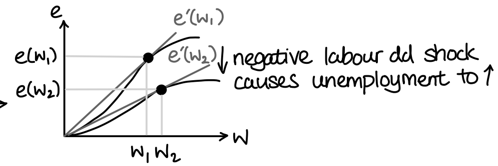
The firm chooses w and N to max

$$\pi = F(K,E) - wN$$

FOCs: (Diff wrt N): $e(w)\frac{\partial F(E,N)}{\partial E} = w$

(Diff wrt w): $e'(w)\frac{\partial F(E,N)}{\partial E} = 1$

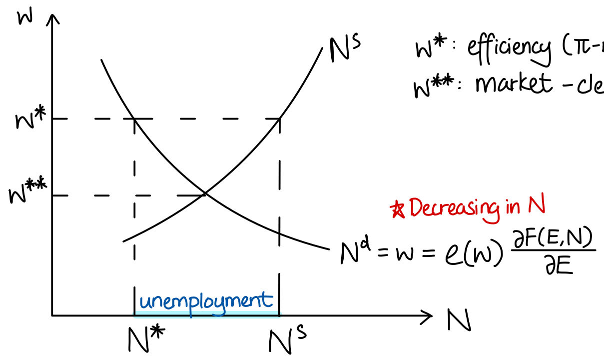
$$\Rightarrow \underbrace{e'(w)}_{\text{marginal effect of wage on effort}} = \underbrace{\frac{e(w)}{w}}_{\text{mean effort/\$}}$$



i.e maximise effort/unit wage $\frac{e(w)}{w}$

or minimise cost of inducing effort from each worker $\frac{w}{e(w)}$ ☆ $\pi = F(K,E) - \frac{w}{e(w)}E$

w^* depends solely on $e(w)$ — sticky wrt other factors



w^*: efficiency (π-max) wage
w^{**}: market-clearing real wage

We should expect w*>w** and hence positive unemployment at w*. If there is no unemployment (surplus of labour), w* will provide no incentive to exert effort: you can always join another firm (also offering w*) if your boss fires you. Effort should depend positively on wage relative to other jobs and on unemployment, because it means you may not be able to get another job

Even if labour demand falls, firms have no incentive to change w* so unemployment increases

But model's state of excess supply does not explain positive vacancy rate (jobs openings infilled), and why people go in and out of unemployment

UNEMPLOYMENT - SEARCH THEORY

Δ Unemployment, $U_t - U_{t-1} = s(L - U_t) - f U_t$

job separation rate — s; job finding rate — f; Size of labour force (constant) — L

Inflow: $s(L-U_t)$ Outflow: fU_t

Unemployment rate $u_t = \frac{U_t}{L}$

$u_{t+1} - u_t = s(1-u_t) - fu_t$

E(time for unemployed to find job) $= \frac{1}{f}$

E(duration of a job) $= \frac{1}{s}$

STEADY STATE $u_{t+1} = u_t = u^*$ ★ adjusts quickly (< 1 year)

Solve $\boxed{s(1-u^*) = fu^*} \Rightarrow \boxed{u^* = \frac{s}{s+f} = \frac{1}{1+\frac{f}{s}}}$

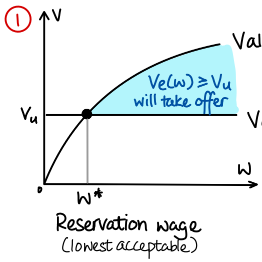
$\uparrow f / \downarrow \frac{1}{f} / \downarrow s / \uparrow \frac{1}{s} \Rightarrow u^* \downarrow$

Empirically more impt

"ONE-SIDED" SEARCH MODEL

s, w exogenous. $s > 0$

e.g. housing, marriage

① 

Value of being employed, $V_e(w)$ — Each offer's wage — will shift down if income tax increases

Value of being unemployed, $V_u > 0$. Includes non-wage income, value of leisure, unemployment benefits, expected future wage when employed

= Value of continuing search

JOB FINDING RATE

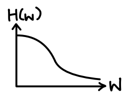
P(an unemployed worker gets offer) / Rate of getting job offers / % of unemployed getting offer in duration t. Exogenous

$$f = pH(w^*) = p[1 - F(w^*)]$$

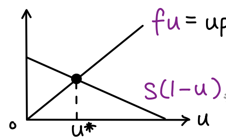
$F(x) = P(\text{an offer with } w < x)$ ★ a CDF! Non-decreasing in w^*

so $H(w^*) = P(\text{accepting an offer} \mid \text{receiving an offer}) = P(\text{wage offered} \geq w^* \mid \text{received offer})$

- $\uparrow w^* \Rightarrow \downarrow f$
- If income tax $\uparrow$, then $V_e \downarrow$



$H(w)$ is the fraction of firms offering wages above w^*



Determined in graph ①

★ empirically quite flat, effect of s is small

Under this model, unemployment more benign and even efficient because of voluntary aspect and ensuring good matches

Unemployment benefit theoretically inefficient as it increases unemployment, but may still be needed (efficient) to give poor people time to find good match

To find u^*:

① Solve for w^* by letting $V_e(w) = V_u$

② Solve for $f = pH(w^*)$

③ Solve for $u^* = \frac{1}{1+\frac{f}{s}} = \frac{1}{1+\frac{pH(w^*)}{s}}$

EQUILIBRIUM SEARCH MODEL

Determine f with 3 key elements
1. Matching function (ease of matching vacancies with potential workers)
2. Nash bargaining over wages
3. Job creation condition (determines number of vacancies)

MATCHING FUNCTION

no. of matches, efficiency parameter, no. unemployed, vacancies

$m < u$
$m < v$

$$m = \mu M(u, v) \quad \text{Matching fn}$$

Neoclassical
- CRTS
- Marginal products positive and diminishing

Tightness of labour market $\theta = \frac{v}{u}$ Ratio of "buyers" to "sellers"

$$f = \frac{m}{u} = \frac{\mu M(u,v)}{u} \overset{CRS}{=} \mu M(1, \theta) = f(\theta)$$ increasing in θ

BEVERIDGE CURVE

$$\boxed{u = \frac{s}{s + f(\theta)}}$$ (s exogenous)

v inversely related to u $\uparrow v \Rightarrow \uparrow \frac{v}{u} \Rightarrow \uparrow f \Rightarrow \downarrow u^*$

. Also, $v = \frac{\theta s}{s + f(\theta)}$, $\frac{dv}{d\theta} > 0$, so $\downarrow u \Rightarrow \uparrow v \uparrow \theta$

e.g. if $f(\theta) = \mu \theta^{1-\eta}$, $\mu v^{1-\eta} = \frac{s(1-u)}{u^\eta}$

WAGE BARGAINING

Surplus = gains from a deal compared to no deal

Worker's surplus = w - b ← unemployment benefit

Firm's surplus = $\underbrace{y}_{\uparrow \pi} - w - (\underbrace{-cy\theta}_{\text{cost/unit time to keep searching}})$

$\uparrow$ o/p (worker's pdy); cost of worker walking away from offer & finding a replacement (derivation not needed)

Worker and firm will only agree on a wage deal if both have non negative surplus

Total surplus = y - b + cyθ Wage cancels out as it is a transfer between two parties

Given exogenous bargaining power γ (workers) and 1-γ (firms), the worker will get γ of the total surplus

$$w - b = \gamma(y - b + cy\theta)$$

WAGE CURVE

$$\boxed{w = (1-\gamma)b + \gamma(y + cy\theta)}$$

$\uparrow \theta \Rightarrow \uparrow w$

$\uparrow \gamma \Rightarrow$ steeper curve

- Wage is a weighted average between worker's outside option b and what the worker is worth to firm $y + cy\theta$
- Positive relationship between θ and w

JOB CREATION DECISION

Firm's decision to open vacancies

Present value of π after vacancy is filled : $\frac{y-w}{r+s}$
- If profits y-w were received forever, then present value = $\frac{y-w}{r}$
- But must discount profits at s in addition to r

Expected cost of filling a vacancy : $\frac{cy}{q(\theta)}$
- cy : cost of keeping vacancy open per unit time
- $q(\theta) = \frac{m}{v} = \frac{\mu M(u,v)}{v} = \mu M(\frac{1}{\theta}, 1)$: rate at which vacancies are filled

Job creation condition requires them to be equivalent.

JOB-CREATION CURVE

MB of posting vacancy
$$\boxed{\frac{y-w}{r+s} = \frac{cy}{q(\theta)}}$$
prob-adjusted MC of posting vacancy

☆☆ $\uparrow \theta \Rightarrow \downarrow q(\theta)$

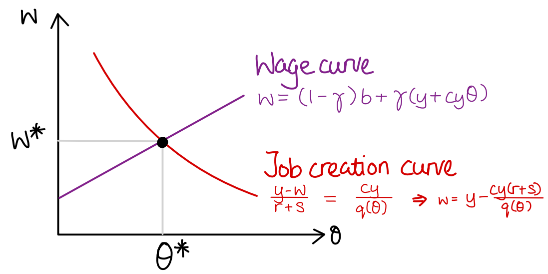
$\frac{1}{q}$ = E(time vacancy is open till worker found)

$\uparrow w \Rightarrow \downarrow v \Rightarrow \downarrow \theta \Rightarrow \downarrow \frac{cy}{q(\theta)}$

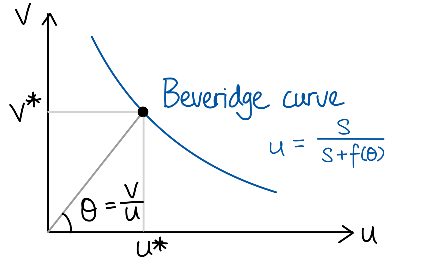
- Higher wages discourages job creation ; $\frac{cy}{q(\theta)}$ will fall until firms willing to create jobs again
- Negative relationship between θ and w

(Slopes of all 3 curves intuitively explained in PS9)

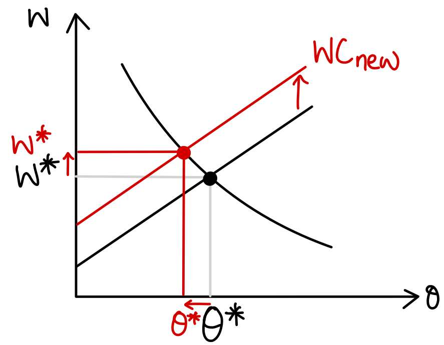
① Plot and find θ^*



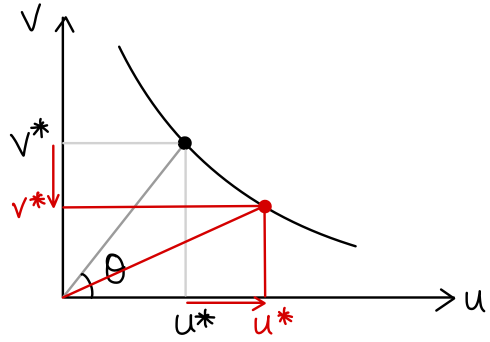
② Sub $v = u\theta^*$ into BC to find u^*



$\boxed{\uparrow b}$ Results in $\uparrow u^*$



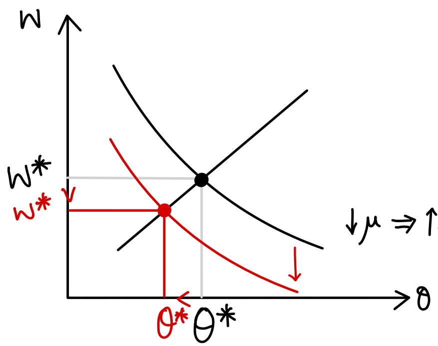
Workers negotiate higher wages as outside option is improved



Firms create higher jobs, increasing unemployment

$\boxed{\downarrow \mu}$

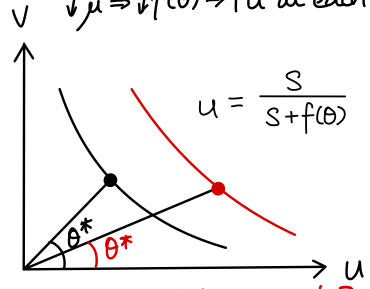
E.g. less migration, less efficient matching



$\downarrow \mu \Rightarrow \uparrow q(\theta) \Rightarrow \downarrow \frac{1}{q(\theta)} \Rightarrow \downarrow w = y + \frac{cy(r+s)}{q(\theta)}$

More “mismatch” raises recruiting costs and causes fewer vacancies to be created. Lower tightness reduces bargained wages.

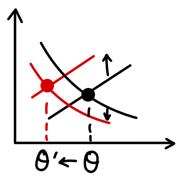
$\downarrow \mu \Rightarrow \downarrow f(\theta) \Rightarrow \uparrow u$ at each v

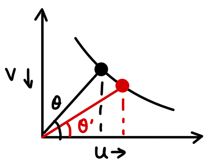


$u^*\uparrow$ but v^*?

More “mismatch” lowers job finding rate for a given number of vacancies which increases the unemployment rate

$\boxed{\downarrow y}$





☆ Extent of $\downarrow u$ rises with worker's bargaining power γ

GENERAL EQUILIBRIUM
dynamic ver of W2

Two-period model of a closed economy
3 markets: Labour, goods, financial

AGENTS

Rep. hh: Decides on C vs ℓ, C vs s

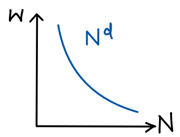
Rep. firm: Decides on N, I

Gov: Consumes resources, taxes, issues debt

W8 FIRM
★ nothing new: review of "Investment"

Assume CRTS, diminishing MP_K

LABOUR DEMAND



+1 worker ⇒ output $+MP_N$
⇒ output $-w$ (cost)

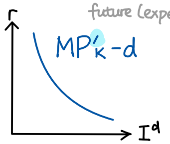
$MP_N = w$

Diminishing MP_N means lower w is needed for additional N (-ve slope)

$\uparrow MP_N$, e.g. ↑TFP, ↑current capital stock K will $\uparrow N^D$

INVESTMENT DEMAND

future (expected)



$MP'_K - d = r$

Diminishing MP_K: additional I will only be procured at lower opp cost (r)

Changes to d or MP'_K, e.g

$\uparrow z'$ or $\downarrow K$ ⇒ $\uparrow MP'_K$ at each r ⇒ $\uparrow dd_I$ (→ shift)
natural disaster

↑Interest rate spread will $\downarrow dd_I$ ($MP_K - x - d = r$)

GOVERNMENT
W6

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

Bonds $B = G - T$ are sold to finance deposit

If $G > T$, then $B > 0$, $G' < T'$.

If ↑r endogenously, ↑T.

HOUSEHOLD
W6, but labour endogenous

Hh lifetime budget constraint

$$C + \frac{C'}{1+r} = w(h-\ell) + \pi - T + \frac{w'(h-\ell') + \pi' - T'}{1+r}$$

Firm's profits (dividends) (π); lump sum tax (T)

Taking FOCs...

OPTIMALITY CONDITIONS

C^a vs ℓ: $MRS_{\ell,c} = w$

Timing of C^a: $MRS_{c,c'} = 1+r$

Timing of ℓ: $MRS_{\ell,\ell'} = \frac{(1+r)w}{w'}$

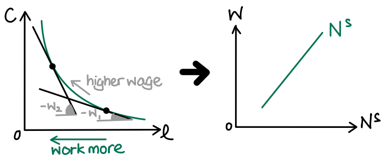
Working 1h more today gives extra income w worth $w(1+r)$ tmr, so you can $\uparrow \ell'$ by $\frac{(1+r)w}{w'}$ while still affording the same C bundle

NO INCOME EFFECT

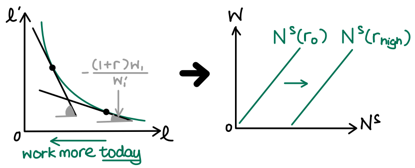
- Income effect of $\Delta w / \Delta r$ ignored in this model as hh owns firms and pays taxes; faces offsetting gains/losses e.g. better wages $\uparrow \ell$ but $\downarrow \pi$; higher interest rate $\uparrow C$ but $\uparrow t$ (gov repays at ↑r)
- Only substitution effect considered
  - $\Delta w/r$ causes budget to pivot around the same indifference curve

LABOUR SUPPLY

Upward sloping: when $w\uparrow$, $\ell\downarrow$.



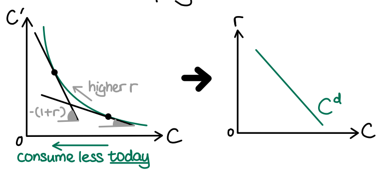
Higher interest rate shifts labour supply



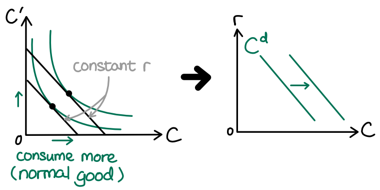
Higher G/T also $\overrightarrow{N^s}$
- taxes reduce wealth impact is shared by $\downarrow \ell$ and $\downarrow C$ (smoothing)
- Though if t isn't lump sum...

CONSUMPTION

Downward sloping



$\uparrow w$ (wealth effect) shifts C^d



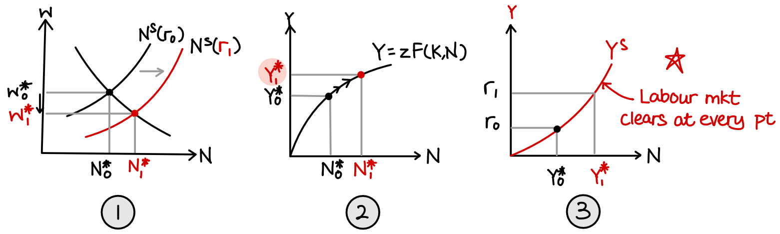
GENERAL EQUILIBRIUM

AGGREGATE SUPPLY OF GOODS

1. Given r, find w^*, N^* in labour market
2. Put N^* into pdn fn $Y = zF(K,N)$ to find Y^*
3. Plot r against Y^* to get output supply curve

★ Y^s will shift right if ↑pdy (i.e. $z\uparrow$ or $K\uparrow$: pdn fn scales up)

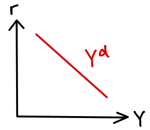
★ Everything that shifts N^s will shift Y^s .. except r



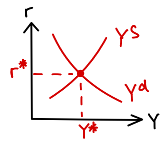
AGGREGATE DEMAND OF GOODS

$Y^d = C^d + I^d + G$ ← exogenous

both decrease with r



GOODS MARKET



r adjusts to clear the goods mkt

By construction of Y^s, labour mkt clears too

By Walras' Law, the financial mkt (last mkt in ecy) clears too

Thus, no need to look at financial mkt

- All 3 mkts clear. General eqm!

- Used mostly for LR analysis
- In SR mkts don't clear, so dd/ss don't exist and r indeterminate...
- That's when Central Banks come in!

GENERAL EQUILIBRIUM : EXAMPLES

☆ N must go in the same direction as Y and opposite to w, if they're ambiguous

↑GOV SPENDING

Temporary ↑G (G' unchanged)
Mainly a demand shock

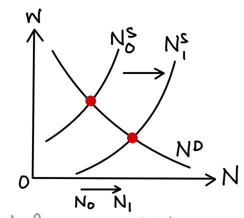
↑ tax burden (assume lump sum) but per-period ↑T less than ↑G

DIRECT ↑G ↑Y^d

INDIRECT Negative wealth effect: ↓C^d ↓Y^d, ↑N^s ↑Y^s

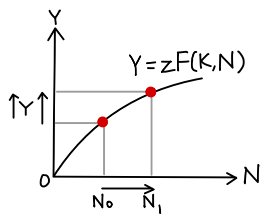
Since $\Delta Y^s + \Delta C^d < \Delta G$, $\Delta C^d < \Delta G$, so Y^d shifts right overall
↳ because c' and ℓ' will also fall! Smoothing!

Labour

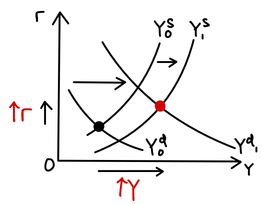


☆ if gov exp went into pdy enhancement, N^D may ↑

Pdn Fn



Goods



WHERE DOES G GO

If gov expenditure went into productivity enhancement ↑z' then ↑N^d and ↑I^d

CROWDING OUT

$\Delta Y < \Delta G$, implying C and I fell (displaced)

- ↓C^d: Negative wealth effect, sub^n effect of ↑r
- ↓I^d: ↑r increases cost of capital

This effect is larger if tax is corporate

WEALTH EFFECT

Caused by Δ in z, z', K, G, G' (through taxes), d PS10 Q3
Changes in timing of T (without ΔG) doesn't matter

- Makes hhs better off, demand more normal goods (C, ℓ, C', ℓ')
- Shifts C^d and hence Y^d to the right
- Shifts N^s and hence Y^s to the left

↑CURRENT TFP

Mainly a supply shock
eg: temp. fall in energy prices
☆ z' unchanged, so I_{dd} unchanged

DIRECT ↑MP_N ↑N^d ↑N + Pdn f^n scales up ⇒ ↑Y^s

WEALTH Positive wealth effect: ↑C^d ↑Y^d, ↓N^s ↓Y^s (by less than original shift)
e.g. lower energy costs boosts income temporarily

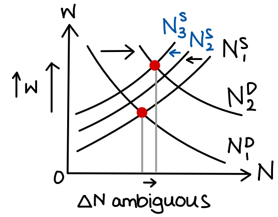
SECOND INDIRECT

- Shift in Y^s likely larger than Y^D, so ↓r
- N^s may shift further left, leaving ΔN ambiguous

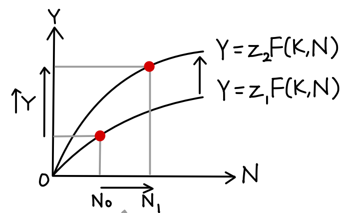
OUTCOME

- Definite ↑Y, ↑w
- Possible ↓r, ↑N

Labour

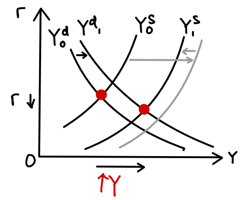


Pdn Fn



even if ΔN = 0, ↑z is enough to ↑Y

Goods



GENERAL EQUILIBRIUM : EXAMPLES

↑ FUTURE TFP

e.g. discovery of new tech that's not immediately available for use

INDIRECT $\uparrow z'$ $\uparrow MP_K'$ $\uparrow I^d$ $\uparrow Y^d$

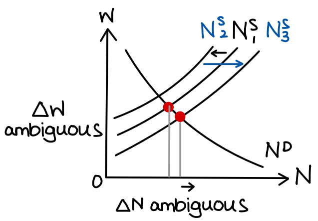
- No direct effect on Y^s since tech isn't available yet

WEALTH Positive wealth effect : $\uparrow C^d$ $\uparrow Y^d$, $\downarrow N^s$ $\downarrow Y^s$

SECOND INDIRECT

- Huge $\uparrow r$
- N^s may shift further left, leaving ΔN ambiguous

Labour



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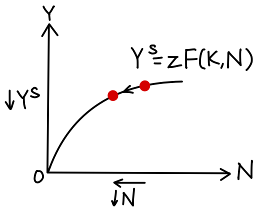
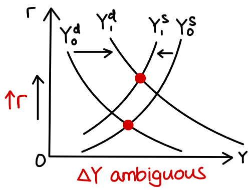


Goods



OUTCOME

- Definite $\uparrow r$
- Possible $\Delta Y, \Delta N, \Delta W$

Both effects indirect. Cannot compare extent of Y^d/Y^s shift

↓K e.g. natural disaster/war

DIRECT $\downarrow Y^s$

$\downarrow MP_N$ $\downarrow N^d$ $\downarrow Y^s$

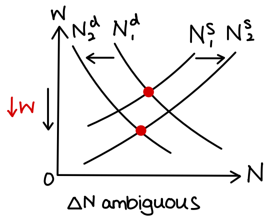
$\uparrow MP_K'$ $\uparrow I^D$ $\uparrow Y^d$

WEALTH

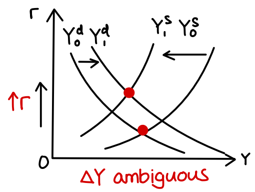
Negative wealth effect : $\downarrow C^d$ $\downarrow Y^d$, $\uparrow N^s$ $\uparrow Y^s$

$\Delta Y^d + \Delta Y^s$ due to wealth effect smaller than $\Delta Y^d + \Delta Y^s$ from direct effect : $\uparrow Y^d$ $\downarrow Y^s$ overall

Labour



Goods



OUTCOME

- Definite $\uparrow r$ $\downarrow W$
- Possible $\Delta Y, \Delta N$

ASYMMETRIC INFO

- Asym. info btwn borrowers/lenders worsen
  Interest-rate spread increases

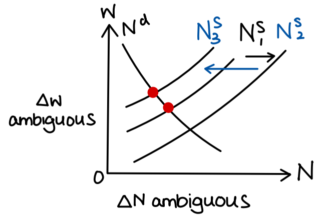
Firms are borrowers, so $\downarrow I^d$ $\downarrow Y^d$

Some hhs borrow : $\downarrow C^d$ $\downarrow Y^d$ $\uparrow N^s$ $\uparrow Y^s$

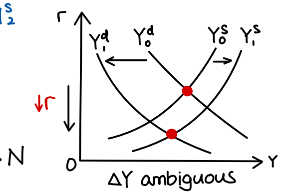
SECOND INDIRECT

$\downarrow r$ may cause $N^s \downarrow$

Labour



Goods



OUTCOME

- Definite $\downarrow r$
- Possible $\downarrow Y$, $\downarrow N$, ΔW

likely (dd-side effect > supply)

EXAMPLES SUMMARY

- Gov spending
- Current/ Future TFP
- Destruction of capital stock
- Credit-market risk
- Uncertainty about future income/propensity to save PS10
- Depreciation rate PS10
- Payroll tax/ tax on profits PS10