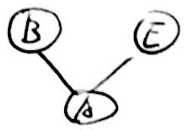
Problem 2.

a). MRE:



$$P(A,E,B) = \frac{\phi_1(A,B)\cdot\phi_2(A,E)}{Z}.$$

Z - normalization constant.

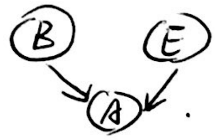
b). $$P(B,E|A) = \frac{P(A,E,B)}{P(A)} = \frac{\frac{1}{Z}\phi_1(A,B)\phi_2(A,E)}{\frac{1}{Z}\sum_{E\in\{0,1\}}\phi_2(A,E)\sum_{B\in\{0,1\}}\phi_1(A,B)}$$

$$P(B=0,E=1|A=1] = \frac{\phi_1(A=1,B=0)\,\phi_2(A=1,E=1)}{\sum_{E\in\{0,1\}}\phi_2(A,E)\sum_{B\in\{0,1\}}\phi_1(A,B)} \neq 1$$

In principal, when the alarm is triggered, one of the two events must happen, but the prob $\neq 1$, therefore, it looks unreasonable.

c). BN

$$P(A,E,B) = P(B)P(E)P(A|B,E)$$



d). $$P(A|E,B) = \frac{P(A,E,B)}{P(E,B)}$$

$P(A=0|E=1,B=0) = 0$ $P(A=1|E=0,B=1) = 1$

$P(A=1|E=1,B=0) = 1$ $P(A=0|E=0,B=1) = 0$

E	B	$P(A=0\|E,B)$	$P(A=1\|E,B)$
0	0	1	0
0	1	0	1
1	0	0	1
1	1	0	1

$\Rightarrow$

A \ E,B	00	01	10	11
0	1	0	0	0
1	0	1	1	1

Note: In this senario, when $E,B=0$, alarm won't be triggered
when $E=B=1$, alarm will be triggered.

e). $\because$ $P[B=1] = \delta_B \quad P[E=1] = \delta_E$.

$P(A=0, E=0, B=0] = P(A=0 \mid E=0, B=0)\, P(E=0, B=0)$

$= 1 \cdot (1-\delta_B)(1-\delta_E)$

(Based on the given morginal probability and the ~~previous~~ conditional probability calculatted in (d))

$P(A=1, E=0, B=1) = P(A=1 \mid E=0, B=1)\, P(E=0, B=1)$

$= (1-\delta_E)\delta_B$

$P(A=1, E=1, B=0) = P(A=1 \mid E=1, B=0)\, P(E=1, B=0)$

$= (1-\delta_B)\delta_E$

$P(A=1, E=1, B=1) = P(A=1 \mid E=1, B=1)\, P(E=1, B=1)$

$= \delta_B \delta_E$

$P(\text{others}) = 0$ (Because the conditional prob $= 0$)

A \ (E,B)	00	01	10	11
0	$(1-\delta_B)(1-\delta_E)$	0	0	0
1	0	$(1-\delta_E)\delta_B$	$(1-\delta_B)\delta_E$	$\delta_B\delta_E$

f)
$$P[B=1|E=1,A=1] = \frac{P(B=1,E=1,A=1)}{P(E=1,A=1)}$$

When the alarm is triggered and earthquake happens, burglary rarely happens.

$$= \frac{\delta_B \delta_E}{\sum_{B\in\{0,1\}} P(B,E=1,A=1)} = \frac{\delta_B \delta_E}{(1-\delta_B)\delta_E + \delta_B \delta_E}$$

$$= \delta_B$$

$$P[B=1|E=0,A=1] = \frac{P(B=1,E=0,A=1)}{P(E=0,A=1)} = \frac{(1-\delta_E)\delta_B}{\sum_{B=\{0,1\}} P(B,E=0,A=1)}$$

when the alarm is triggered and earthquake doesn't happen, burglary must happen.

$$= \frac{(1-\delta_E)\delta_B}{(1-\delta_E)\delta_B + \underbrace{P(B=0,E=0,A=1)}_{0}} = 1$$

Based on this scenario, B should be independent of E (no necessary relationship between them. But given A, B and E will be dependent as mentioned in BN. Therefore, compared with MRF, BN sounds more reasonable.

① For BN, $B \perp E$, but given A, B and E are dependent.
② For MRF, B and E are dependant. Give A, they are independent.