Module Five - Multiple Integration

MAT325: Calculus III: Multivariable Calculus

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Introduction:

Module Five builds on one-dimensional integration by introducing double and triple integrals for functions of two and three variables, respectively. Double and triple integrals in rectangular, cylindrical, and spherical coordinate system are studied, and general approaches to change of variables in multiple integrals is presented.

In this MATLAB assignment, you'll learn how to evaluate double and triple integrals in various coordinate systems.

Review the code and comments provided in the "Examples" section below, and then use this information to complete the problems listed in the "Problems" section.

Make sure to run your code so all relevant computations/results are displayed, delete the "Introduction" and "Examples" sections, and then export your work as a PDF file for submission (your submission only needs to contain the "Problems" section that you completed).

Examples:

Example 1 - Double Integral in Rectangular Coordinates

In this example, we'll compute the integral $\int_0^5 \int_0^4 xy \, dx \, dy$.

```
% Example 1 Code - Double Integral in Rectangular Coordinates

% Clear the workspace
clear all;

% To perform double-integration over a rectangular region, it is usually
% helpful to define x and y as symbolic variables
syms x y;

% Define the function the function handle for f(x,y)
f = @(x,y) x.*y;

% The integral2() function can be used to evaluate double integrals
```

```
% For the rectangular region be [0,4] X [0,5] we have:
format long;
xMin = 0;
xMax = 4;
yMin = 0;
yMax = 5;
integral2(f,0,4,0,5)
```

```
ans =
   100
```

We can verify the integral2() results manually be computing:

$$\int_0^5 \int_0^4 xy\, dx\, dy = \int_0^5 \frac{x^2}{2}\Big|_0^4 y\, dy = 8 \int_0^5 y\, dy = 8\,\frac{y^2}{2}\Big|_0^5 = 8 \cdot \frac{25}{2} = 100.$$

Example 2 - Double Integral in Polar Coordinates

Consider the integral $\int\int_R (1 + r^2)\, dA$ where R is a circle of radius 2 in the xy-plane. For this circular region, we can describe R as the set of points $0 \le r \le 2$, $0 \le \theta \le 2\pi$ where $dA = r\, dr\, d\theta$. Thus, the integral to evaluate is $\int_0^{2\pi} \int_0^2 (1 + r^2) r\, dr\, d\theta$.

For calling the integral2() function in MATLAB, we must define an integral using variables x and y. We can re-write the integral above by making the substitutions $\theta = x$, $r = y$, resulting in the integral:

$$\int_0^{2\pi} \int_0^2 (1 + y^2) y\, dy\, dx$$

We are now ready to use integral2() to compute the numerical value of this integral.

```
% Example 2 Code - Double Integral in Polar Coordinates

% Clear the workspace
clear all;

%setup the function
clear all;
f = @(x,y) (1+y.^2).*y;

%setup limits of integration
xmin = 0;
xmax = 2*pi;
ymin = 0;
ymax = 2;

%compute the integral
format long;
integral2(f,xmin,xmax,ymin,ymax,'Method','tiled')
```

```
ans =
  37.699111843085738
```

Example 3 - Triple Integral in Spherical Coordinates

Consider the integral in spherical coordinates $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\cos(\phi)} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$.

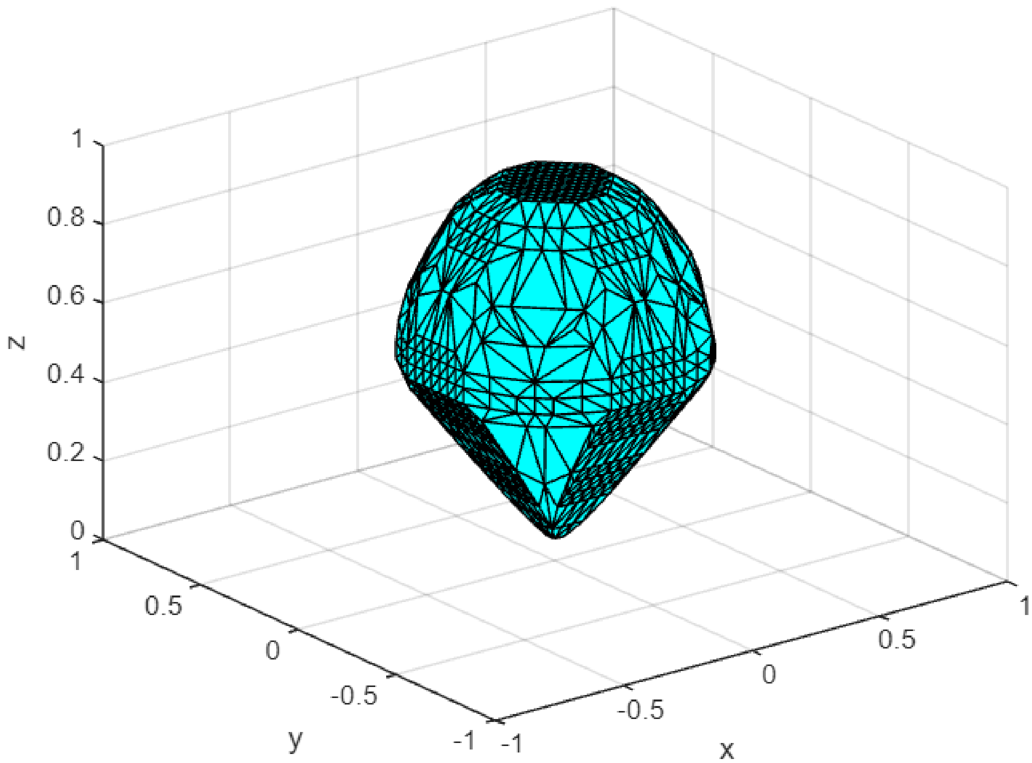
The region of integration can be visualized numerically using meshgrid() and the trisurf() function.

```
%setup a rectangular grid
x       = linspace(-1,1,50);
y       = linspace(-1,1,50);
z       = linspace(-1,1,50);
[X,Y,Z] = meshgrid(x,y,z);

%convert to a spherical grid
rho   = sqrt(X.^2 + Y.^2 + Z.^2);
theta = atan(Y./X);
phi   = atan(sqrt(X.^2 + Y.^2)./Z);

%find the indices that satisfy the integral limits
plotInd = (phi>=0 & phi<=pi/4) & (rho>=0 & rho<=cos(phi));

%plot the surface using the trisurf function()
figure;
K1 = convhull(X(plotInd),Y(plotInd),Z(plotInd));
trisurf(K1,X(plotInd),Y(plotInd),Z(plotInd),'Facecolor','cyan');
xlim([-1 1]);
ylim([-1 1]);
xlabel('x');
ylabel('y');
zlabel('z');
```



The integral can be evaluated using the integral3() function. Note that the limits on ρ are a function of ϕ, so an additional function handle is needed to define those integral limits.

For calling the integral3() function in MATLAB, we must define an integral using variables x, y, and z. We can re-write the integral above by making the substitutions $\rho = z$, $\phi = y$ and $\theta = x$. The resulting integral is:

$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\cos(y)} z^2 \sin(y)\, dz\, dy\, dx$$

We are now ready to use integral3() to compute the numerical value of this integral.

```
%setup the function
clear all;
f = @(x,y,z) (z.^2.*sin(y));

%setup limits of integration
xmin = 0;
xmax = 2*pi;
ymin = 0;
ymax = pi/4;
zmin = 0;
zmax = @(x,y) cos(y);  % Note: This function handle define the upper limits
on z as a function of variable y

%compute the integral
```

```
format long;
integral3(f,xmin,xmax,ymin,ymax,zmin,zmax,'Method','tiled')
```

```
ans =
   0.392699081696143
```

Problems:

Problem 1: Use MATLAB and the integral2() function to evaluate the integral of the function $f(x, y) = x^3 - 2y^3$ over the rectangular region $R = [1, 2] \times [2, 4]$.

```
% Problem 1 Code Here
clear all;

syms x y;

f = @(x, y) x.^3 -2*.^3;

x_min = 1;
x_max = 2;
y_min = 2;
y_max = 4;

I = integral2(f, x_min, x_max, y_min, y_max)
```

Problem 2: Use MATLAB and the integral2() function to evaluate the integral $\int\int_R (x + y)\, dA$ over the region $R = \{(x, y)|2 \le x^2 + y^2 \le 4, y \ge 0\}$.

Put your math/explanation here...

```
% Problem 2 Code Here
clear all;

clear all;

f = @(x, y) x + y;

x_min = -2;
x_max = 2;
y_min = @(x) sqrt(2- x.^2);
y_max = @(x) sqrt(4- x.^2);

format long;
I = integral2(f, x_min, x_max, y_min, y_max)
```

Problem 3: Use MATLAB and the integral3() function to evaluate the integral

$$\int_0^{2\pi}\int_0^2\int_0^{\sqrt{16-r^2}} r^2\,dz\,dr\,d\theta.$$

Put your math/explanation here...

```
% Problem 3 Code Here

x         = linspace(-2, 2, 30);
y         = linspace(-2, 2, 30);
z         = linspace(0, 4, 30);
[X, Y, Z] = meshgrid(x, y, z);

rho = sqrt(X.^2 + Y.^2 + Z.^2);
theta = atan(Y, X);
phi = acos(Z ./ rho);

R = sqrt(X.^2 + Y.^2);

plotInd = (R <= 2) & (Z <= sqrt(16 - R.^2)) & (rho <= 4);

figure;
K1 = convhull(X(plotInd), Y(plotInd), Z(plotInd));
trisurf(K1, X(plotInd), Y(plotInd), Z(plotInd), 'FaceColor', 'cyan',
'EdgeColor', 'black');
xlim([-2, 2]);
ylim([-2, 2]);
xlabel('x');
ylabel('y');
zlabel('z');
title('Plot of Triple Integral');
grid on;

clear all;

f = @(r, theta, z) r.^2;

x_min = -2;
x_max = 2;
y_min = -2;
y_max = 2;
z_min = 0;
z_max = @(x, y) sqrt(16 -x.^2 -y.^2);

format long;
I = integral3(f, x_min, x_max, y_min, y_max, z_min, z_max, 'Method',
'tiled');
```