Project Two Template

MAT325: Calculus III: Multivariable Calculus

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Problem 1: Consider the iterated integral in cylindrical coordinates $\int_0^{\pi/3} \int_0^1 \int_{r^3}^{r} r \, dz \, dr \, d\theta.$

a) Use MATLAB and the trisurf() function to plot the solid region indicated by the integral limits. Choose an appropriate view to best visualize the region and label axes appropriately.

Solution:

```
%code
theta = linspace(0, pi/3, 30);
r = linspace(0, 1, 20);

[R, Theta] = meshgrid(r, theta);

Z1 = R;
Z2 = R.^3;

X1 = R .*cos(Theta);
Y1 = R .*sin(Theta);
X2 = X1;
Y2 = Y1;

V1 = [X1(:), Y1(:), Z1(:)];
V2 = [X2(:), Y2(:), Z2(:)];
V = [V1; V2];

F = delaunay(V(:,1), V(:,2));
```

```
Warning: Duplicate data points have been detected and removed.
 Some point indices will not be referenced by the triangulation.
```

```
trisurf(F, V(:,1), V(:,2), V(:,3))

xlabel('x');
ylabel('y');
zlabel('z');
title('Solid Region Defind by the Integral in Cylindrical Coordinates');
```

```
view(30, 45);
grid on;
```

b) Solve the integral to find a value for the volume V of the solid.

Solution:

Put your math/explanation here...

$$V = \frac{-2\pi}{45}$$

c) Use MATLAB and the integral3() function to numerical solve the iterated integral. Verify that your numerical value computed below matches the value computed in part (b).

Solution:

Put your math/explanation here...

```
%code
clc;
clear;
close all;

f = @(r, z, theta) r;

theta_lower = 0;
theta_upper = pi/3;
r_lower = 0;
r_upper = 1;
z_lower = @(r) r;
z_upper = @(r) r.^3;

V = integral3(@(r, z, theta) f(r, z, theta),...
                r_lower, r_upper, ...
                z_lower, z_upper, ...
                theta_lower, theta_upper)
```

```
V =
-0.1396
```

Problem 2: Consider the integral

$$\int_{-2\sqrt{2}}^{2\sqrt{2}} \int_{-\sqrt{8-x^2}}^{\sqrt{8-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$$

a) Use MATLAB and the trisurf() function to plot the solid region given by the integral limits. Choose an appropriate view to best visualize the region and label axes appropriately.

Solution:

```
%code
clc;
clear;
close all;

R = (4);
[x, y, z] = sphere(50);
x = R * x;
y = R * y;
z = R * z;

[x_base, y_base] = meshgrid(linspace(-R, R, 50), linspace(-R, R, 50));
mask = x_base.^2 + y_base.^2 <= R^2;
x_base = x_base(mask);
y_base = y_base(mask);

z_base = sqrt(16 - x_base.^2 - y_base.^2);

cyl_radius = sqrt(8);
inside_cylinder = (x.^2 + y.^2) <= cyl_radius^2;

x = x(inside_cylinder);
y = y(inside_cylinder);
z = z(inside_cylinder);

K = convhull(x, y, z);

figure;
trisurf(K, x, y, z, 'FaceColor', 'cyan', 'EdgeColor', 'none');

xlabel('x');
ylabel('y');
zlabel('z');
title('The Plot of The Solid Over The Region Given by The Integral');
view(45, 25);
grid on;
axis equal;
```

b) Convert the integral into an integral in spherical coordinates.

Put your math/explanation here...

$\int_0^{2\pi} \mathrm{d}\theta \int_0^{\frac{\pi}{2}} \sin(\phi * d\phi \int_0^4 \rho^3 \, d\rho$

c) Solve the integral from part (b).

Put your math/explanation here...

I = 2*π(2.747)

I=17.28

d) Use MATLAB and the integral3() function to numerically solve the integral. Verify that your numerical value computed below matches the value computed in part (c).

Put your math/explanation here...

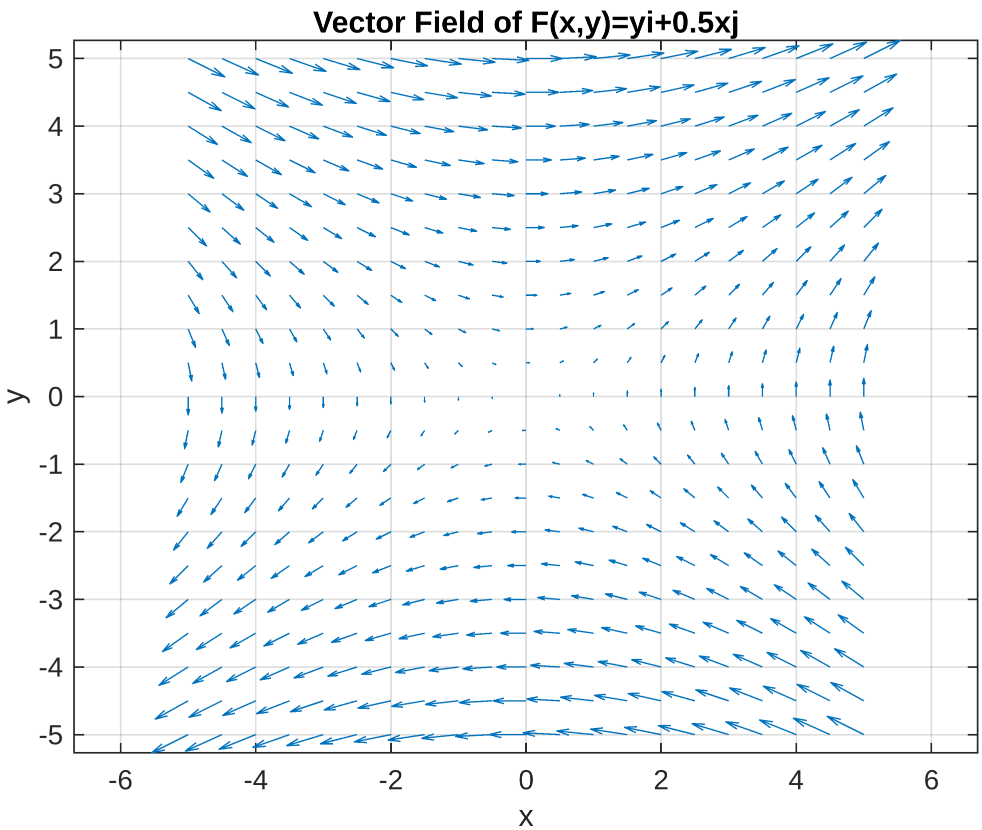
```
%code
clc;
close;
clear all;

f = @(x, y, z) sqrt(x.^2 + y.^2) .* sqrt(x.^2 + y.^2 + z.^2);

x_min = -2*sqrt(2);
x_max = 2*sqrt(2);

y_min = @(x) -sqrt(8 - x.^2);
y_max = @(x) sqrt(8 - x.^2);

z_min = @(x, y) -sqrt(16 - x.^2 -y.^2);
z_max = @(x, y) sqrt(16 - x.^2 -y.^2);

integral3(f, x_min, x_max, y_min, y_max, z_min, z_max);
```

Problem 3: Consider the vector field $\mathbf{F}(x, y) = y\mathbf{i} + 0.5x\mathbf{j}$.

a) Use MATLAB and the quiver() function plot the vector field.

```
%code
clc;
clear;
close all;

x = -5:0.5:5;
y = -5:0.5:5;

[X, Y] = meshgrid(x, y);
```

```
U = Y;
V= 0.5 *X;

figure;
quiver(X, Y, U, V);
xlabel('x');
ylabel('y');
title('Vector Field of F(x,y)=yi+0.5xj');
axis equal;
grid on;
```



b) Consider moving an object along a path C, with starting point $(1, 0)$ and end point $(0, 1)$. Find a parameteriziation $\mathbf{r}(t)$ for the curve.

Put your math/explanation here...

r(t) = (1, t), 0< t <1

c) Compute the work done by the vector field in moving the object along the path C.

Put your math/explanation here...

W = 0.5 units

Problem 4: Consider the line integral $\int_C (6x - 5y)dx + (2x - 4y)dy$ where C is the ellipse given by $\frac{x^2}{4} + y^2 = 1$ in the counterclockwise direction.

a) Find a parameterization $\mathbf{r}(t)$ for the curve.

Put your math/explanation here...

r(t) = <cos(t), sin(t)>

b) Use Green's Theorem to evaluate the integral.

Put your math/explanation here...

$7 \int \int_D \mathrm{dA} = 7(6\pi) = 42\pi$

Problem 5: Consider the vector field $\mathbf{F}(x, y, z) = 4y\mathbf{i} + z\mathbf{j} + 2y\mathbf{k}$ and let the surface S be part of the sphere $x^2 + y^2 + z^2 = 9$ that is above the plane $z = 0$.

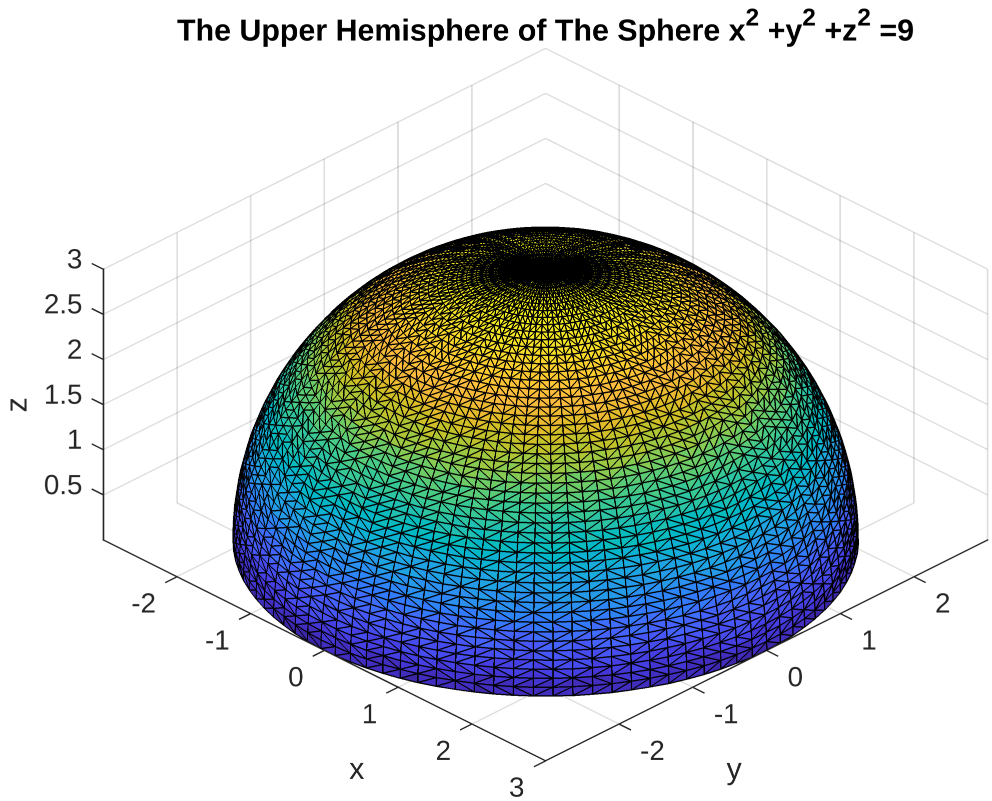
a) Use MATLAB and the trisurf() function to plot S. Choose an appropriate view to best visualize the region and label axes appropriately.

Solution:

```
%code
R = 3;

theta = linspace(0, 2*pi, 100);
phi = linspace(0, pi/2, 50);

[Theta, Phi] = meshgrid(theta, phi);

X = R *cos(Theta) .* sin(Phi);
Y = R *sin(Theta) .* sin(Phi);
Z = R *cos(Phi);

figure;

tri = delaunay(X(:), Y(:));
```

```
Warning: Duplicate data points have been detected and removed.
 Some point indices will not be referenced by the triangulation.
```

```
trisurf(tri, X(:), Y(:), Z(:))

xlabel('x');
ylabel('y');
zlabel('z');
title('The Upper Hemisphere of The Sphere x^2 +y^2 +z^2 =9');
view(45, 30);
axis equal;
grid on;
```



b) Use Stoke's Theorem to evalute the integral $\int\int_S (\text{curl}\,\mathbf{F}\cdot\mathbf{N}\,dS)$ for the given vector field $\mathbf{F}(x, y, z)$ and surface S.

Solution:

Put your math/explanation here...

$W = -36\pi$

Problem 6: Consider the vector field $\mathbf{F}(x, y, z) = (x^2 + 2y^2)\mathbf{i} + (x^2y - z)\mathbf{j} + (2y + 3z)\mathbf{k}$ and let

the surface S be the surface of the cube $0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$, excluding the $z = 0$ face.

a) Compute $\operatorname{div} \mathbf{F}$.

Solution:

Put your math/explanation here...

$$\operatorname{div} F = x^2 + 2x + 3$$

b) Use the Divergence theorem to compute the surface integral $\int\int_S \mathbf{F} \cdot \mathbf{N}\, ds$ for the given vector field $\mathbf{F}$ and surface S.

Solution:

Put your math/explanation here...

$$\int\int_S F \cdot N \,\mathrm{dS} = \frac{152}{3}$$

c) Use MATLAB and the integral3() function to numerically solve the integral. Verify that your numerical value computed below matches the value computed in part (b).

```
%code
f = @(x, y, z) x.^2 + 2*x + 3;

x_min = 0;
x_max = 2;

y_min = 0;
y_max = 2;

z_min = 0;
z_max = 2;

integral3(f, x_min, x_max, y_min, y_max, z_min, z_max);
```