Module Two - Vector-Valued Functions

MAT325: Calculus III: Multivariable Calculus

Student Name JoAnn Olney

Date 1/19/25

Introduction:

Vector-valued functions, calculus of vector-valued functions, and properties of curves such as arc length and curvature are some of the key topics studied in Module Two.

In this MATLAB assignment, you'll learn how to plot vector-valued functions and compute quantities such as arc length and curvature.

Review the code and comments provided in the "Examples" section below, and then use this information to complete the problems listed in the "Problems" section.

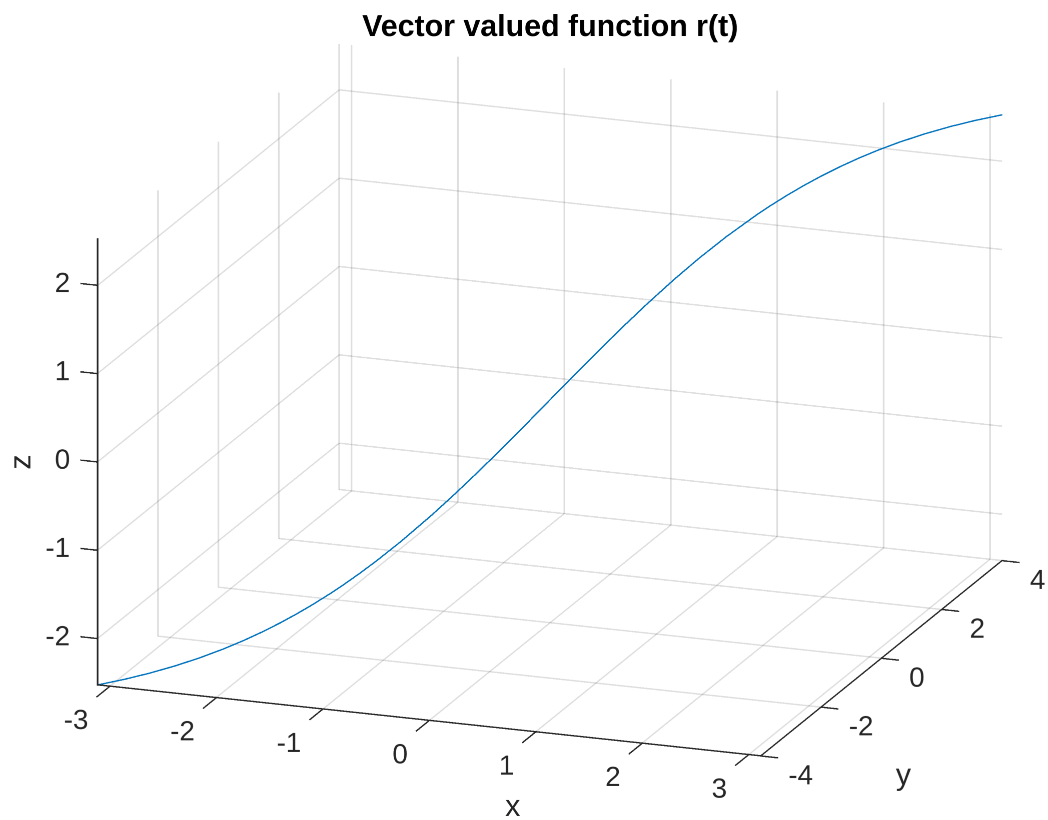
Make sure to run your code so all relevant computations/results are displayed, delete the "Introduction" and "Examples" sections, and then export your work as a PDF file for submission (your submission only needs to contain the "Problems" section that you completed).

Examples:

Example 1 - Plotting Vector-Valued Functions

```
% Example 1 Code - Plotting vector-valued functions

% Clear the workspace
clear all;

% To plot a vector-valued function parameterize by variable t, it is
% usually helpful to define t as a symbolic variable
syms t;

% Consider the vector-valued function
% r(t) = 2tan(t)*i + 4t*j + 3sin(t)*k

% We plot this vector-valued function in a similar manner to how
% parameterized curves were plotted in Module One
x(t) = 2*tan(t);
y(t) = 4*t;
```

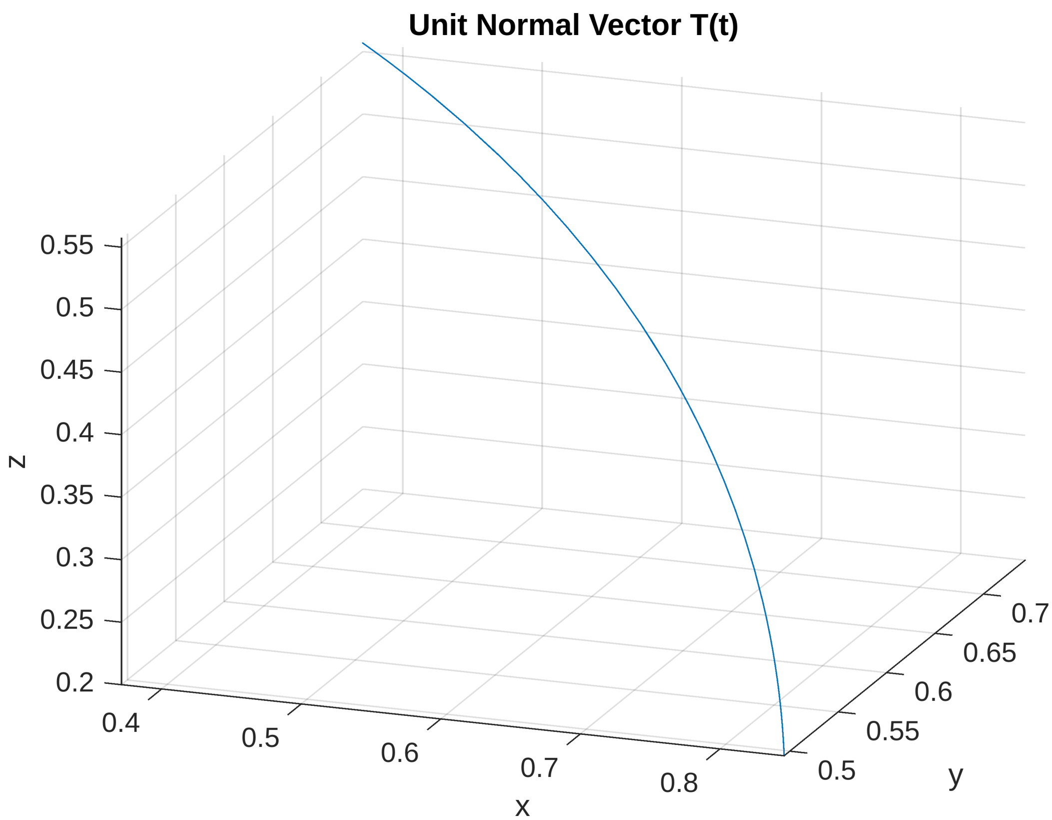
```
z(t) = 3*sin(t);

% The "fplot3" function plots a symbolic function over some interval. The
% second argument [-1,1] tells fplot3 to plot from t = -1 to t = 1
figure;
fplot3(x(t),y(t),z(t),[-1,1]);
xlabel('x');
ylabel('y');
zlabel('z');
title('Vector valued function r(t)');
view([20 25]);
```



Example 2 - Unit Tangent Vector

```
% Example 2 Code - Unit Tangent Vector

% Clear the workspace
clear all;

% To plot a vector-valued function parameterize by variable t, it is
% usually helpful to define t as a symbolic variable
syms t;

% Consider the same vector-valued function from Example 1
% r(t) = 2tan(t)*i + 4t*j + 3sin(t)*k
```

```
% To compute the unit tangent vector T(t), we need to commpute r'(t)/||r(t)||
% We can easily define each component of r'(t) be taking derivates as
rPrimeX(t) = 2*(sec(t)).^2;
rPrimeY(t) = 4 + 0*t;  %note, we include 0*t so still a function of t
rPrimeZ(t) = 3*cos(t);

% We can compute ||r(t)|| as
rNorm(t) = sqrt(rPrimeX(t).^2 + rPrimeY(t).^2 + rPrimeZ(t).^2);

% And finally compute each component of T(t)
TX(t) = rPrimeX(t)./rNorm(t);
TY(t) = rPrimeY(t)./rNorm(t);
TZ(t) = rPrimeZ(t)./rNorm(t);

% We can now plot T(t) over [-1,1]
figure;
fplot3(TX(t),TY(t),TZ(t),[-1,1]);
xlabel('x');
ylabel('y');
zlabel('z');
title('Unit Normal Vector T(t)');
view([20 25]);
```



Example 3 - Arc Length

```
% The MATLAB function int() can be used to evalute indefinite and definite
% integrals. This can be useful when performing arc length computations.
%
% Consider the vector-valued function
% r(t) = 3cos(t)i + 3sin(t)j + 2t^2k
%
% First, we compute ||r'(t)|| as
```

$\mathbf{r}^{'}(t) = -3\sin(t)\mathbf{i} + 3\cos(t)\mathbf{j} + 4t\mathbf{k}$

$||\mathbf{r}^{'}(t)|| = \sqrt{9\sin^2(t) + 9\cos^2(t) + 16t^2} = \sqrt{9 + 16t^2}$

```
% We compute the arc length of r(t) over one full rotation in the xy plane
% (i.e. t = 0 to 2pi) using int() as follows:
syms t;
rNorm(t) = sqrt(9 + 16*t^2);
symbolicAnswer = int(rNorm,0,2*pi)
```

symbolicAnswer =

$$\frac{9\,\mathrm{asinh}\left(\frac{8\pi}{3}\right)}{8} + \pi\,\sqrt{64\pi^2 + 9}$$

```
%Note: While it's nice that MATLAB was able to provide an exact symbolic
%answer in this case, sometimes it's nice to get a simplified numerical
%answer instead of a complicated symbolic expression. The function vpa()
%can used to evalute "symbolicAnswer" into a numerical value
numericalAnswer = vpa(symbolicAnswer)
```

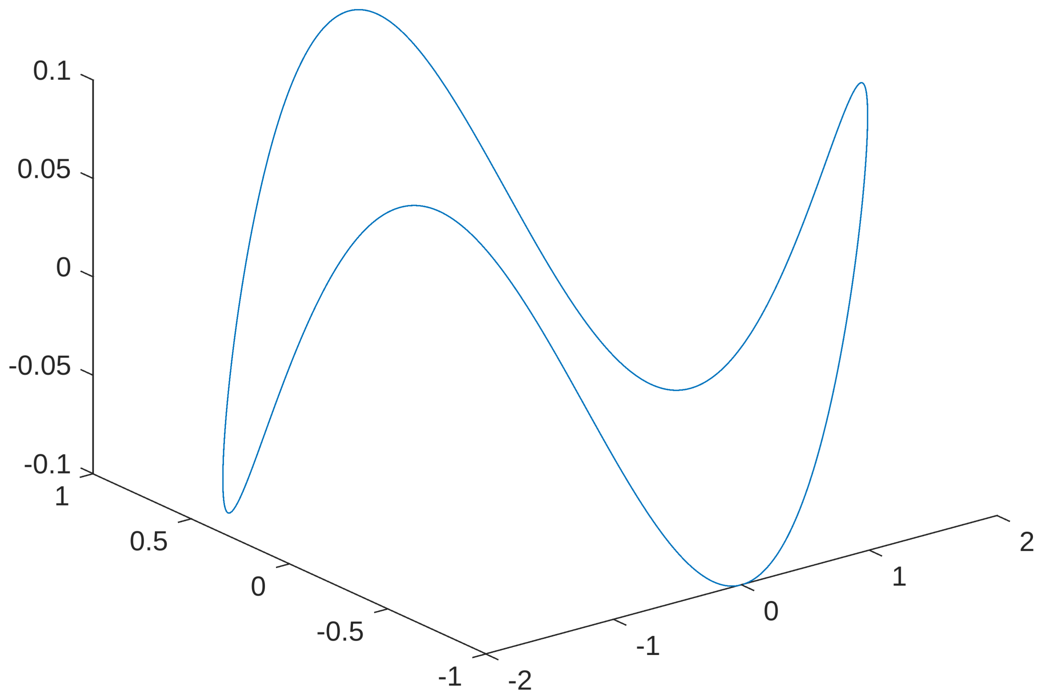
numericalAnswer = $82.692376388902254057823291684207$

Problems:

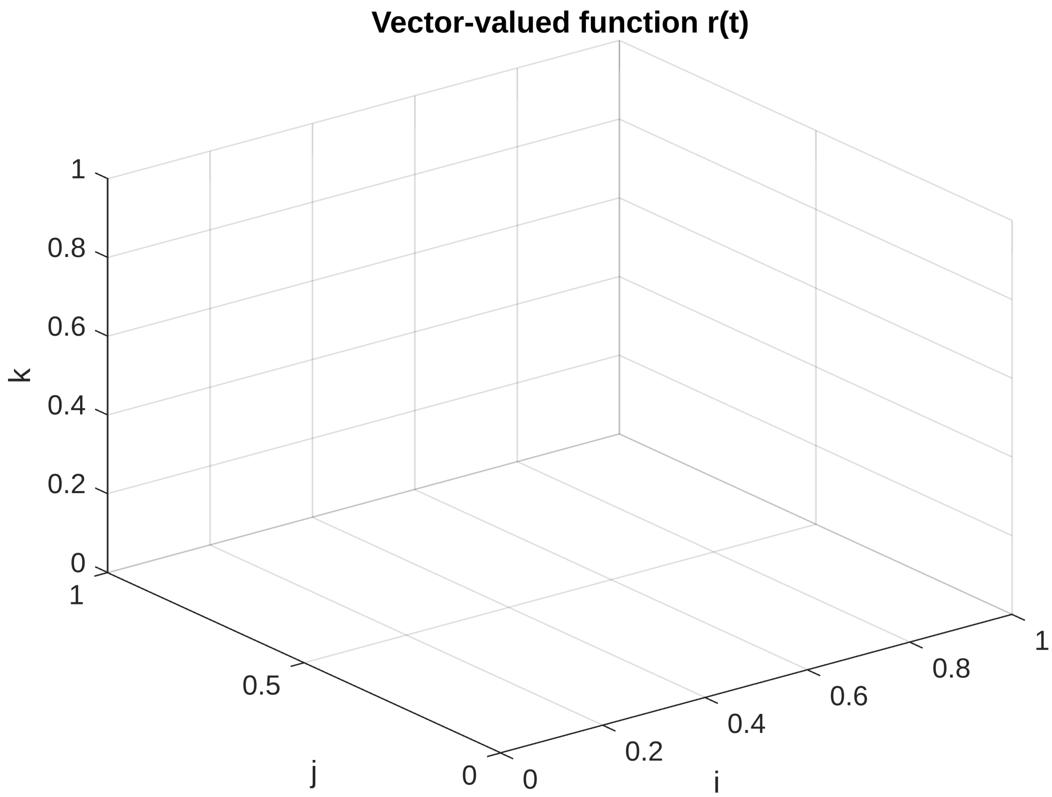
Problem 1: Use MATLAB to plot the vector-valued function $\mathbf{r}(t) = 2\cos(t)\mathbf{i} - \sin(t)\mathbf{j} + 0.1\sin(3t)\mathbf{k}$ on the interval $0 \leq t \leq 2\pi$. Choose an appropriate view to best visualize the curve, label axes appropriately, and title the figure.

```
% Problem 1 Code Here
t = linspace(0, 2*pi, 1000);
x = 2*cos(t);
y = -sin(t);
```

```
z = 0.1*sin(3*t);

plot3(x, y, z);
```



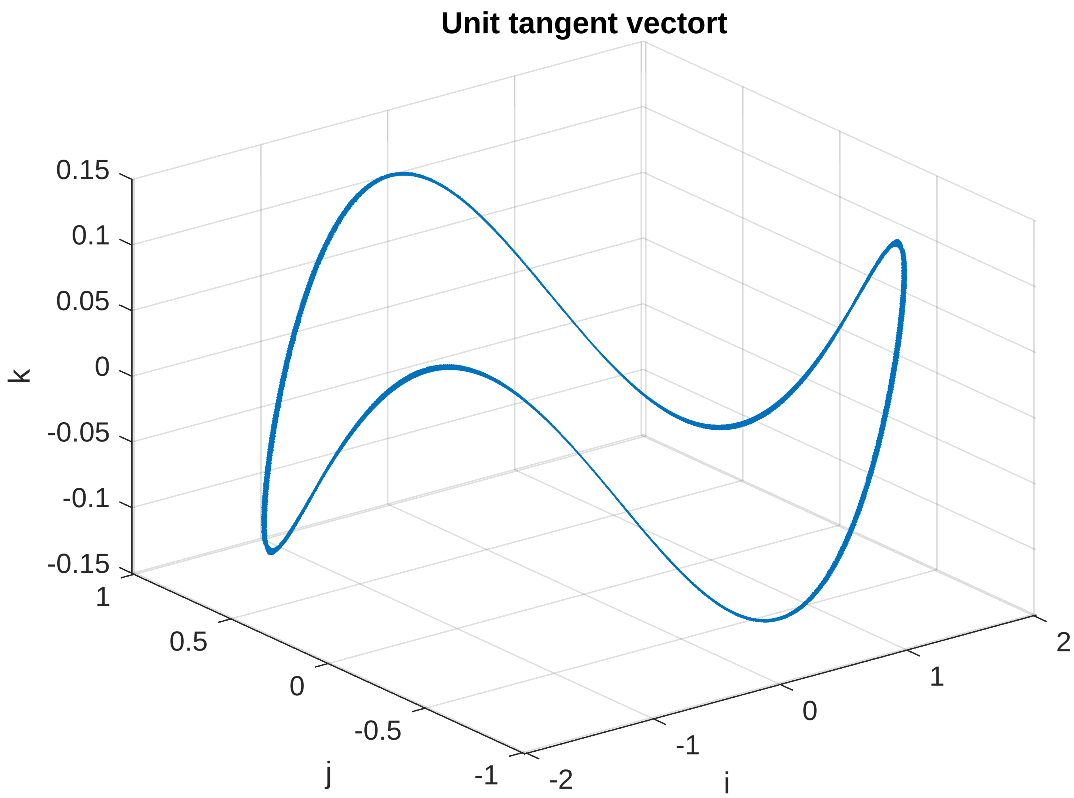

```
figure;
xlabel('i');
ylabel('j');
zlabel('k');
title('Vector-valued function r(t)')

grid on;
view(-37.5, 30);
```



Problem 2: Consider the again the vector-valued function $\mathbf{r}(t) = 2\cos(t)\mathbf{i} - \sin(t)\mathbf{j} + 0.1\sin(3t)\mathbf{k}$ on the interval $0 \leq t \leq 2\pi$. Compute the unit tangent vector $\mathbf{T}(t)$ and plot for $0 \leq t \leq 2\pi$. Choose an appropriate view to best visualize the curve, label axes appropriately, and title the figure.

```
% Problem 2 Code Here
clear all;

syms t;

t = linspace(0, 2*pi, 1000);
r = [2*cos(t); -sin(t); 0.1*sin(3*t)];

dr = diff(r, 1, 2);

mag_dr = sqrt(sum(dr.^2, 1));

T = dr ./ mag_dr;

figure;
```

```
quiver3(r(1, 1: end-1), r(2,1:end-1), r(3,1:end-1), T(1, :), T(2, :),
T(3, :));
xlabel('i');
ylabel('j');
zlabel('k');
title('Unit tangent vectort')

grid on;
view(-37.5, 30);
```



Problem 3: Consider the again the vector-valued function $\mathbf{r}(t) = 2\cos(t)\mathbf{i} - \sin(t)\mathbf{j} + 0.1\sin(3t)\mathbf{k}$ on the interval $0 \le t \le 2\pi$. Compute an expression for $||\mathbf{r}'(t)||$, then use the MATLAB functions int() and vpa() to compute its arc length on the interval $0 \le t \le 2\pi$.

Put your math/explanation here...

```
% Problem 3 Code Here
syms t;

r_prime = [-2*sin(t), -cos(t), 0.3*cos(3*t)];

r_prime_norm = norm(r_prime);
```

```
arc_length = int(r_prime_norm, t, 0, 2*pi);
arc_length_value = vpa(arc_length);
```