Module Three - Multivariable Functions and Limits

MAT325: Calculus III: Multivariable Calculus

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Introduction:

Module Three expands on concepts you are familiar with for single-variable functions to multi-variable functions. Being able to visualize or plot multi-variable functions, computing the limit of a multi-variable function, determining if a multi-variable function is continuous, and computing partial derivatives of multi-variable functions are some of the key topics studied in Module Three.

In this MATLAB assignment, you'll learn how to plot multi-variable functions and compute partial derivatives of multi-variable functions.

Review the code and comments provided in the "Examples" section below, and then use this information to complete the problems listed in the "Problems" section.

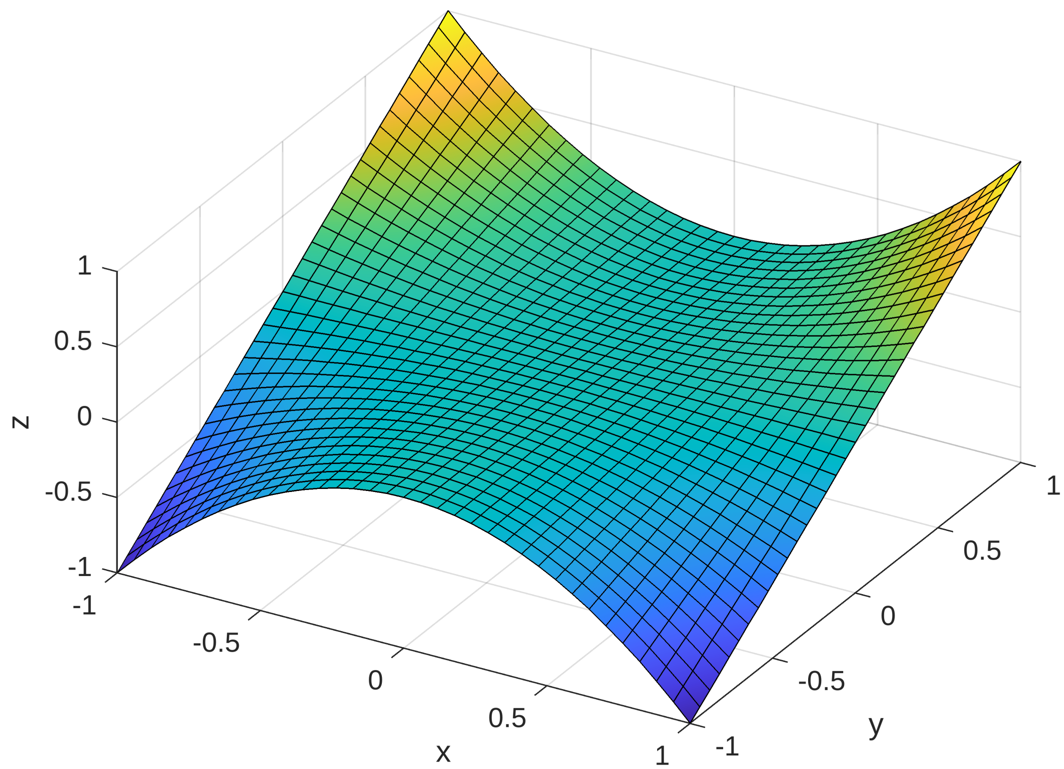
Make sure to run your code so all relevant computations/results are displayed, delete the "Introduction" and "Examples" sections, and then export your work as a PDF file for submission (your submission only needs to contain the "Problems" section that you completed).

Examples:

Example 1 - Plotting Multi-Variable Functions

```
% Example 1 Code - Plotting multi-variable functions

% Clear the workspace
clear all;

% To plot a multi-variable function parameterized by variables x and y, it
% is usually helpful to define x and y as symbolic variables
syms x y;

% Compute the function z(x,y)
z(x,y) = x^2*y;

% Plot the surface
% The second argument sets bounds [xMin, xMax, yMin, yMax]
```

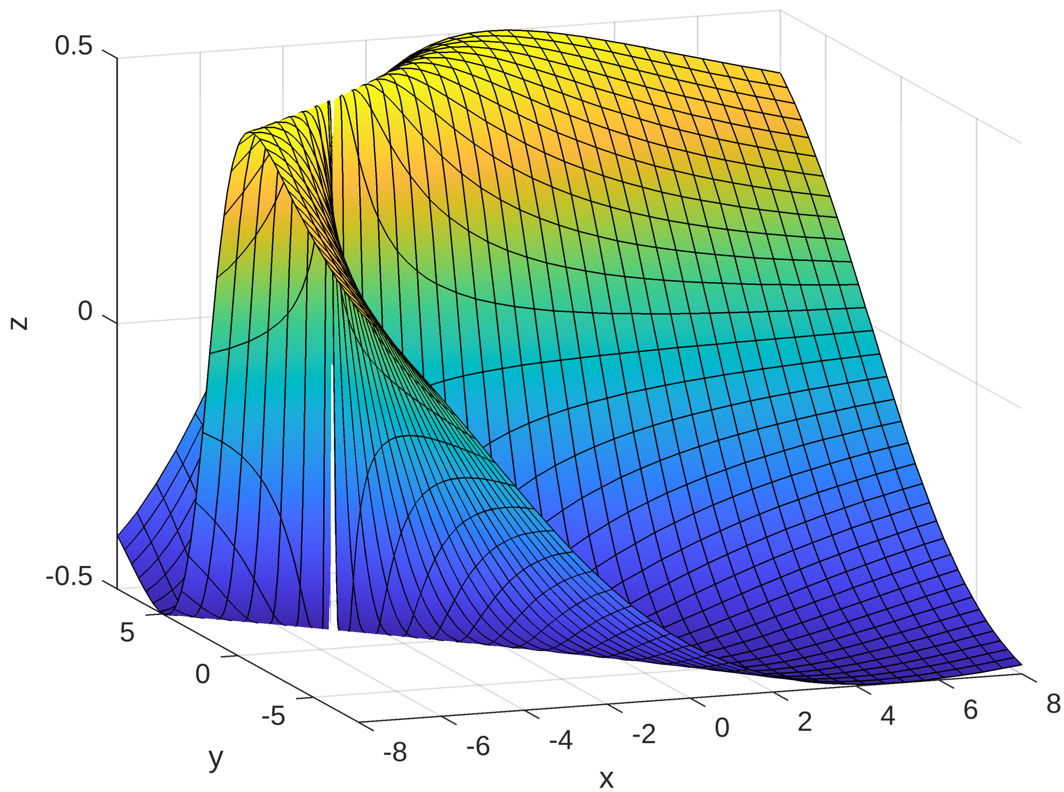
```
figure;
fsurf(z(x,y),[-1,1,-1,1]);
xlabel('x');
ylabel('y');
zlabel('z');
view([30 45]);
```



Example 2 - Limits of Multi-Variable Functions

```
% Example 2 Code - Limits of Multi-Variable Functions

% Clear the workspace
clear all;

% To plot a multi-variable function parameterized by variables x and y, it
% is usually helpful to define x and y as symbolic variables
syms x y;

% Compute the function z(x,y) = (x+5)*(y-2)./( (x+5).^2 + (y-2).^2 );
z(x,y) = (x+5)*(y-2)./( (x+5).^2 + (y-2).^2 );

% Plot the surface
figure;
fsurf(z(x,y),[-8,8,-8,8]);
xlabel('x');
ylabel('y');
```

```
zlabel('z');
view([-380 15]);
```




```
% From visually examining the surface, we see that approaching the point
% (-5,2) from some paths results in a value of 0.5, while approaching along
% other paths yields a value of -0.5. Thus, it appears this function does
% not have a limit at the point (-5,2).
% PLEASE NOTE: Examining a visualization/plot of a function is not a
% rigorous way of determining if a limit does (or does not) exist. However,
% plots such as these can often provide insight/guidance into establishing
% a more formal or rigorous proof of the limit existing (or not).
```

Example 3 - Partial Derivatives

```
% The MATLAB function diff() can be used to compute derivatives of symbolic
% expressions. For example, consider the function
%
% f(x,y) = x^2*y + 5*x*y + 12*y^2 - 5*x + 17*y + 4
%
% First, define the symbols needed and the function
%
syms x y;
f(x,y) = x^2*y + 5*x*y + 12*y^2 - 5*x + 17*y + 4;
```

```
% Compute the partial derivative with respect to x
diff(f(x,y),x)
```

ans = $5y + 2xy - 5$

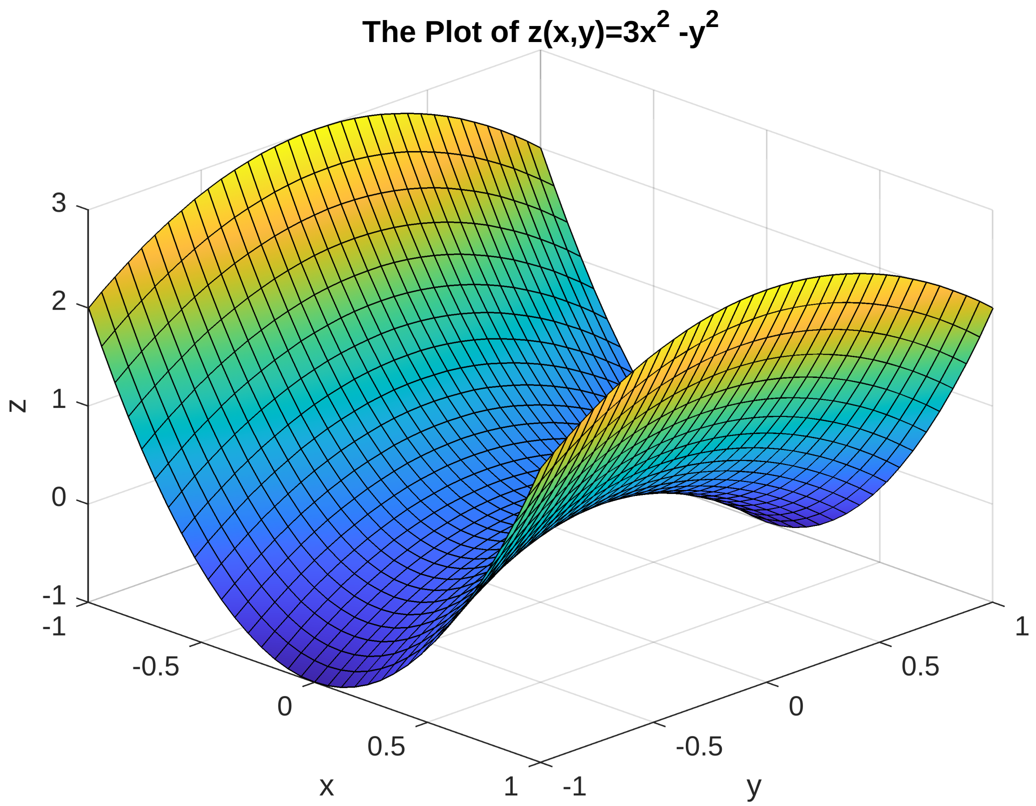
```
% Compute the partial derivative with respect to y
diff(f(x,y),y)
```

ans = $x^2 + 5x + 24y + 17$

Problems:

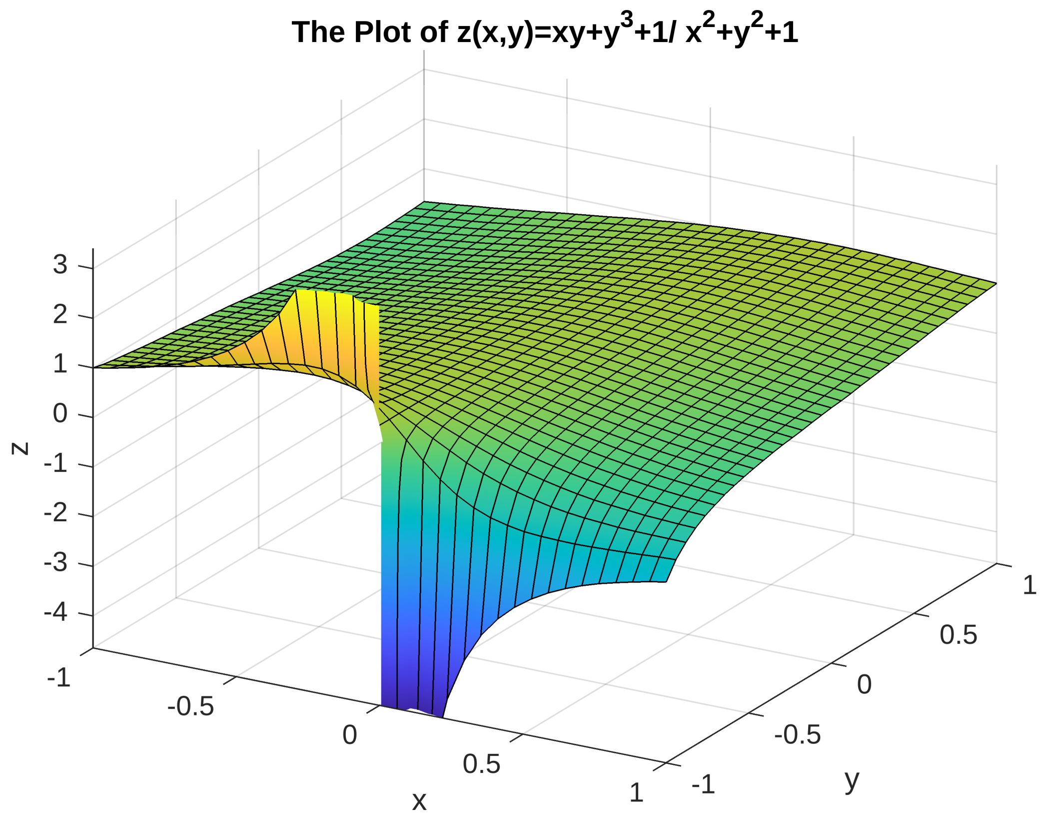
Problem 1: Use MATLAB to plot the multi-variable function $z(x, y) = 3x^2 - y^2$ over the region $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. Choose an appropriate view to best visualize the surface, label axes appropriately, and title the figure.

```
% Problem 1 Code Here
clear all;
syms x y;

z(x,y) = 3*x.^2 -y.^2;

figure;
fsurf(z(x,y), [-1, 1, -1, 1]);
xlabel('x');
ylabel('y');
zlabel('z');
title('The Plot of z(x,y)=3x^2 -y^2');
grid on;
view([45, 30]);
```



Problem 2: Use MATLAB to plot the multi-variable function $z(x, y) = \dfrac{xy + y^3 + 1}{x^2 + y^2 + 1}$ **over the region** $-1 \le x \le 1$, $-1 \le y \le 1$. **Choose an appropriate view to best visualize the surface, label axes appropriately, and title the figure. From your examination of the figure does the limit** $(x, y) \to (0, 0)$ **of the function exist? Yes or no? Explain.**

```
% Problem 2 Code Here
clear all;
syms x y;

z(x,y) = (x .*y + y.^3 +1)./ (x.^2 + y.^3 +1);

figure;
fsurf(z(x,y), [-1, 1, -1, 1]);
xlabel('x');
ylabel('y');
zlabel('z');
title('The Plot of z(x,y)=xy+y^3+1/ x^2+y^2+1');
grid on;
view([30, 30]);
```



%Yes the limit of the function does exisit. When you evaluate the limit at (0,0) you get 1 for the y %and x. I solved mathmatically first for the limit.

%So, for y when I plugged in 0 it quickly becomes 1/ x^2 +1

%And as x approaches zero this becomes 1/0^2 +1 =1.

%For x I also sub in zero #getting y^3+1/y^3+1 =1.

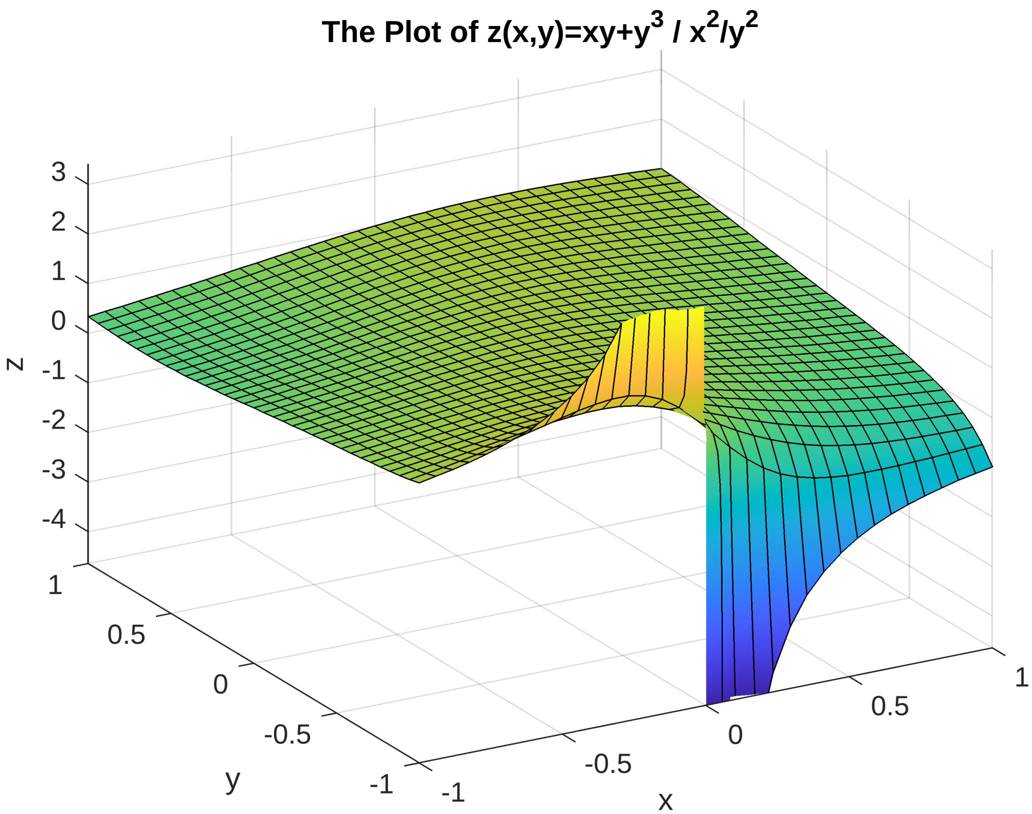
%As y approaches 0 this gets one as well, z(0,0) =1.

%Then I subbed y=x and also recieved one, z(x,x) = 1. (-x^2 -x^3 +1)/ (x^2 -x^3 +1) = 1.

%The limit lim(x,y) ->(0,0) z(x,y) does exist and is 1.

Problem 3: Use MATLAB to plot the multi-variable function $z(x, y) = \dfrac{xy + y^3}{x^2 + y^2}$ **over the region** $-1 \le x \le 1$, $-1 \le y \le 1$. **Choose an appropriate view to best visualize the surface, label axes appropriately, and title the figure. From your examination of the figure does the limit** $(x, y) \to (0, 0)$ **of the function exist? Yes or no? Explain.**

```
% Problem 3 Code Here
clear all;
syms x y;

z(x,y) = (x .*y +y.^3 +1) ./ (x.^2 +y.^3 +1);

fsurf(z(x,y), [-1, 1, -1, 1]);

xlabel('x');
ylabel('y');
zlabel('z');
title('The Plot of z(x,y)=xy+y^3 / x^2/y^2');
grid on;
view([-30,30]);
```



Problem 4: Consider the multi-variable function $f(x, y) = \cos(xy + 3x)$. Use MATLAB and the diff() function to compute the partial derivatives $\frac{\partial}{\partial x} f(x, y)$ and $\frac{\partial}{\partial y} f(x, y)$.

```
% Problem 4 Code Here
clear all;
syms x y;

f = cos(x*y + 3*x);

df_dy = diff(f, y);
df_dx = diff(f, x);
```