Project One Template

MAT325: Calculus III: Multivariable Calculus

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Problem 1: Consider the plane defined by the equation $2x - 5y + 10z + 60 = 0$.

a) Use MATLAB and the surf() function to plot the plane over the interval $x \in [-2, 2]$, $y \in [-2, 2]$. Choose an appropriate view to best visualize the plane and label axes appropriately.

Solution:

```
%code
clear all;

x = linspace(-2,2,20);
y = linspace(-2,2,20);

[X,Y] = meshgrid(x,y);

Z = -2*X+0.5*Y-6;

figure;
surf(X,Y,Z);

xlabel('x');
ylabel('y');
zlabel('z');
title('Plane of 2x-5x+10z+60=0');
view(30,45);
grid on;
```

b) Where does the plane intersect the x-axis, y-axis, and z-axis? Provide a coordinate (x, y, z) for each intersection.

Solution:

Put your math/explanation here...

It intersects the x-axis at (-30,0,0)

it intersects the y-axis at (0,12,0)

It intersects the z-axis at (0,0,-6)

c) Compute the normal vector $\mathbf{n}$ to the plane. Express $\mathbf{n}$ in terms of the standard unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

Solution:

Put your math/explanation here...

A=2

B= -5

C=10,

Thus, n = 2i-5j+10k

d) Use MATLAB to plot the plane (as in part (a)) and a normal vector to the plane using the surfnorm() function at the point $(0, 0, -6)$. Normalize and scale the surfnorm()-computed vector to show it equals the value found in part (c). Make sure to use the axis equal command so the plane and vector look orthogonal as desired.

Solution:

```
%code
clear all;

[x,y] = meshgrid(-10:1:10, -10:1:10);

z = -0.2*x+0.5*y-6;

hold on;

n_theroy = [2, -5, 10];
n_theroy = n_theroy / norm(n_theroy);

[~,px] = [min(abs(x(1, :)))];
[~,py] = [min(abs(y(:, 1)))];

n_surf = [nx(py, px), ny(py, px), nz(py,px) ];
n_surf = n_surf / norm(n_surf);

surf(x, y, z, 'FaceAlpha', 0.5);
hold on;

figure;
xlabel('x');
ylabel('y');
```

```
zlabel('z');
title('Plane of surfnorm() function');
grid on;

quiver3(0, 0, -6, n_theroy(1), n_theroy(2), n_theroy(3), 'LineWidth', 2, 5);

legend('Plane', 'Normal Vector at (0, 0, -6)');
axis equal;
hold off;
```

Problem 2: Consider a moving projectile with position vector given as $\mathbf{r}(t) = 2\cos(t)\mathbf{i} - 2\sin(t)\mathbf{j} + \sqrt{t}\,\mathbf{k}$.

a) Use MATLAB and the plot3() function to plot the projectile location for time $t \in [0, 10]$. Choose an appropriate view to best visualize the curve and label axes appropriately.

Solution:

```
%code
clear all;

t = linspace(0, 10, 100);

x = 2*cos(t);
y = -2*sin(t);
z = sqrt(t);

figure;
grid on;
plot3(x, y, z, 'LineWidth', 2);
hold on;

xlabel('x');
ylabel('y');
zlabel('z');
title('The Curve of r(t) = 2cos(t)i -2sinj +sqrt(t)k');

view(45, 30);

legend('Trajectory', 'Start (t=0)', 'End (t=10)');

axis equal;
hold off;
```

b) Compute the velocity vector $\mathbf{v}(t)$ and acceleration vector $\mathbf{a}(t)$.

Solution:

Put your math/explanation here...

The velocity vector is v(t)= (-2sint)i + (-2cost)j + (1/2sqrt(t))k

The acceleraction vector is a(t) = (-2cost)i + (2sint)j + (-1/4t^3/2)k

c) Use MATLAB to replicate your plot from part (a) and then use the quiver3() function to plot the velocity vector of the particle at times $t = 1$, $t = 2$, and $t = 5$. Choose an appropriate view to best visualize the curve and label axes appropriately.

Solution:

```
%code
clear all;

t = linspace(0, 10, 100);

x = 2*cos(t);
y = -2*sin(t);
z = sqrt(t);

vx = -2*sin(t);
vy = -2*cos(t);
vz = 1./(2*sqrt(t));

t

figure;
grid on;
plot3(x, y, z, 'LineWidth', 2);

xlabel('x');
ylabel('y');
zlabel('z');
title('Velocity Vectors of r(t) = 2cos(t)i - 2sin(t)j + sqrt(t)k');

hold on;

t_values = [1, 2, 5];
x_points = 2*cos(t_values);
y_points = -2*sin(t_values);
z_points = sqrt(t_values);
vx_points = -2*sin(t_values);
vy_points = -2*cos(t_values);
vz_points = 1./(2*sqrt(t_values));
```

```
quiver3(x_points, y_points, z_points, vx_points, vy_points, vz_points, 0.5,
'r', 'LineWidth', 1.5);

axis equal;
view(45, 30);
legend('Trajectory', 'Velocity at t=1', 'Velocity at t=2', 'Velocity at t=5')
hold off;
```

d) Compute unit normal vector $\mathbf{N}(t)$.

Solution:

Put your math/explanation here...

$$N(t) = \frac{-2\cos(t)i + 2\sin(t)j - \dfrac{1}{4t^{\frac{3}{2}}}k}{\operatorname{sqrt}\left(4 + \dfrac{1}{16t^3}\right)}$$

e) Use MATLAB to replicate your plot from part (c) and then use the quiver3() function to plot the unit normal vector of the particle at times $t = 1$, $t = 2$, and $t = 5$. Your figure should now contain the original curve, the velocity vectors from part (c), and the unit normal vectors from part (d). Choose an appropriate view to best visualize the curve and label axes appropriately.

Solution:

```
%code
t_values = [1, 2, 5];

r = @(t) [2*cos(t); -2*sin(t); sqrt(t);];
v = @(t) [2*cos(t); -2*sin(t); sqrt(t);];

figure;
hold on;

t = linspace(0, 5, 100);
plot3(2*cos(t), -2*sin(t), sqrt(t), 'b', 'LineWidth', 1.5 );

grid on;

for t_val = t_values
    pos = r(t_val);
    vel = v(t_val);

    unit_normal = vel / norm(vel);
```

```
    quiver3(pos(1), pos(2), pos(3), vel(1), vel(2), vel(3), 0, r,
'LineWidth', 1.5);
    quiver3(pos(1), pos(2), pos(3), unit_normal(1), unit_normal(2),
unit_normal(3), 0, 'LineWidth', 1.5);


xlabel('x');
ylabel('y');
zlabel('z');
title('Normal Vector, Velocity Vector, Original Curve');
view(45, 30);
hold off;
```

f) Compute an expression for the dot product between the velocity vector $\mathbf{v}(t)$ and unit normal vector $\mathbf{N}(t)$, i.e. compute $\mathbf{v}(t) \cdot \mathbf{N}(t)$. For values of t computed for the figure, does your expression for the dot product seem reasonable based on your plots from part (e)? Yes or no? Explain.

Solution:

Put your math/explanation here...

$$v(t) * N(t) = -\frac{\frac{1}{8t^2}}{\mathrm{sqrt}\left(4 + \frac{1}{16t^3}\right)}$$

The dot product of the velocity vector and the unit normal vector based on the calculation I did, seems to be reasonable. The dot product treds towards zero execpt at some very small numbers and is excepted it is nearly orthogonal especially at large (t) values.

Problem 3: Consider the surface defined by the equation $z = \ln(5x^2 + y + 3)$.

a) Use MATLAB and the surf() function to plot the plane over the interval $x \in [-2, 2]$, $y \in [-2, 2]$. Choose an appropriate view to best visualize the plane and label axes appropriately.

Solution:

```
%code
clear all;

x= -2:0.1:2;
y = -2:0.1:2;
```

```
[X, Y] = meshgrid(x,y);

Z = log(5*X.^2 +Y +3);

surf(X, Y, Z);

figure;

xlabel('x');
ylabel('y');
zlabel('z');
title('Surface of z = ln(5x^2 + y + 3)')

grid on;
view(45, 30);
```

b) Find an equation for a tangent plane to the surface at the point $P = (1, 1)$.

Solution:

Put your math/explanation here...

$$z = \ln(9) + \frac{10}{9}(x - 1) + \frac{1}{9}(y - 1)$$

c) Use MATLAB to replicate your plot from part (a) but now also plot the tangent surface computed in part (b). Also, use the plot3() function to plot the point where the tangent plane was computed at. Set the view so it's clear the tangent plane is indeed tangent to the surface at the given point.

Solution:

```
%code
clear all;

x = -2:0.1:2;
y = -2:0.1:2;

[X, Y] = meshgrid(x, y);

Z = log(5*X.^2 +Y +3);

Z_tangent = log(9) + (10/9)*(X-1) + (1/9)*(Y-1);

surf(X, Y, Z_tangent, 'FaceAlpha', .6, 'EdgeColor', 'none');
hold on;

figure;
plot3(1, 1, log(9), 'ro', 'Color', 'r', 'MarkerSize', 8);
```

```
xlabel('x');
ylabel('y');
zlabel('z');
title('Adding Tangent Srface to Org. Plot');
view(45, 30);
grid on;
hold off;
```

Problem 4: Consider the function $f(x, y) = x^3 - 5x + 4xy - y^2$ for $x \in [-3, 3]$, $y \in [-3, 3]$.

a) Use MATLAB and the surf() function to plot the surface over the given closed and bounded region. Choose an appropriate view to best visualize the plane and label axes appropriately.

Solution:

```
%code
clear all;

x = -3:0.1:3;
y = -3:0.1:3;

[X, Y] = meshgrid(x, y);

Z = X.^3 -5*X +4*X.*Y -Y.^2;

figure;
surf(X, Y, Z);

xlabel('x');
ylabel('y');
zlabel('z');
title('The Surface of f(x,y)= x^3 -5x +4xy -y^2 over closed region');

view(45, 30);
grid on;
```

b) Find the critical points of the function on the bounded region and use the MATLAB plot3() function to plot the critical points on the surface from part (a).

Solution:

Put your math/explanation here...

```
%code
clear all;

x = -3:0.1:3;
y = -3:0.1:3;

[X, Y] = meshgrid(x, y);

Z = X.^3 -5*X +4*X.*Y -Y.^2;

x1 = (-8 + 2*sqrt(31))/6;
x2 = (-8 - 2*sqrt(31))/6;

y1 = 2*x1;
y2 = 2*x2;

z1 = x1^3 -5*x1 +4*x1*y1 -y1^2;
z2 = x2^3 -5*x2 +4*x2*y2 -y2^2;

figure;
surf(X, Y, Z);
hold on;

plot3(x1, y1, Z1);
plot3(x2, y2, Z2);

xlabel('x');
ylabel('y');
zlabel('z');
title('Org Bound Surface Plot with Critical Points ');

view(45,30);
grid on;
hold off;
```

c) Use the second derivative test to classify each critical point as either a local maximum, local minimum, or saddle point.

Solution:

Put your math/explanation here...

The function has a critical point which is a saddle point.