Module Six - Vector Calculus

MAT325: Calculus III: Multivariable Calculus

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Introduction:

Module Six covers vector fields, which assign vectors to points in space. Line integrals are also introduced and studied. An important result called Green's Theorem is also used, which provides an important relationship between one-dimensional line integrals and two-dimensional integrals over a region.

In this MATLAB assignment, you'll learn how to plot vectors fields, evaluate line integrals, and use Green's Theorem to evaluate integrals.

Review the code and comments provided in the "Examples" section below, and then use this information to complete the problems listed in the "Problems" section.

Make sure to run your code so all relevant computations/results are displayed, delete the "Introduction" and "Examples" sections, and then export your work as a PDF file for submission (your submission only needs to contain the "Problems" section that you completed).

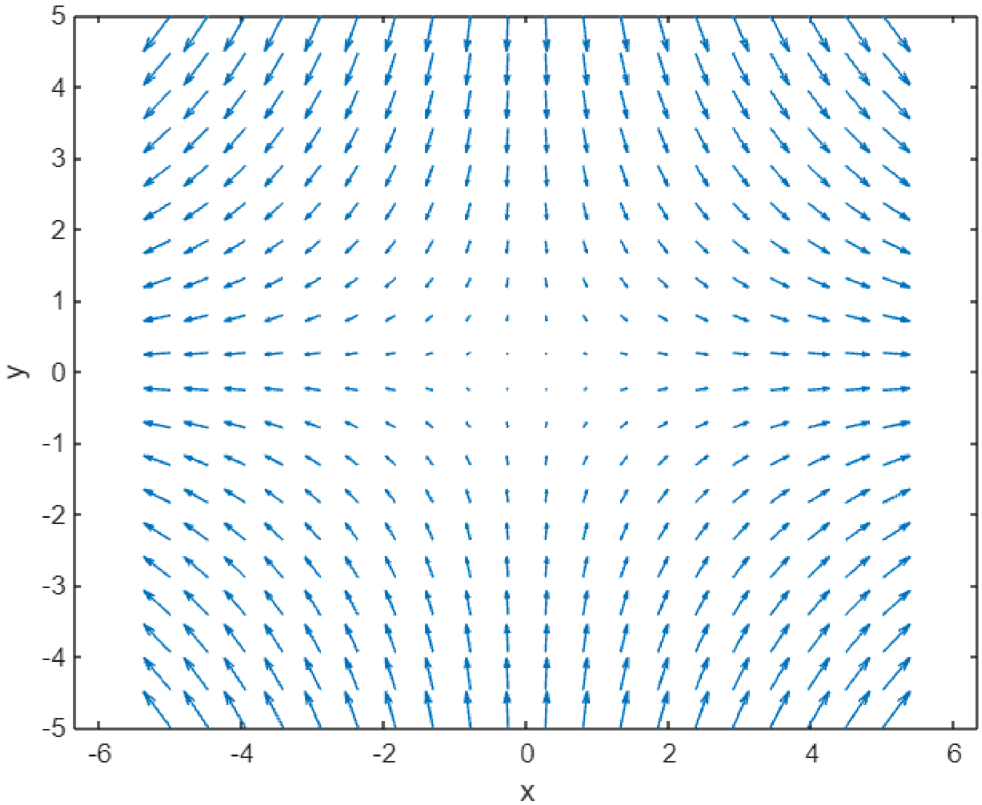
Examples:

Example 1 - Vector Field Plotting

In this example, we'll plot the meshgrid() and quiver() functions to plot the vector field $\mathbf{F}(x, y) = 3x\mathbf{i} - 4y\mathbf{j}$.

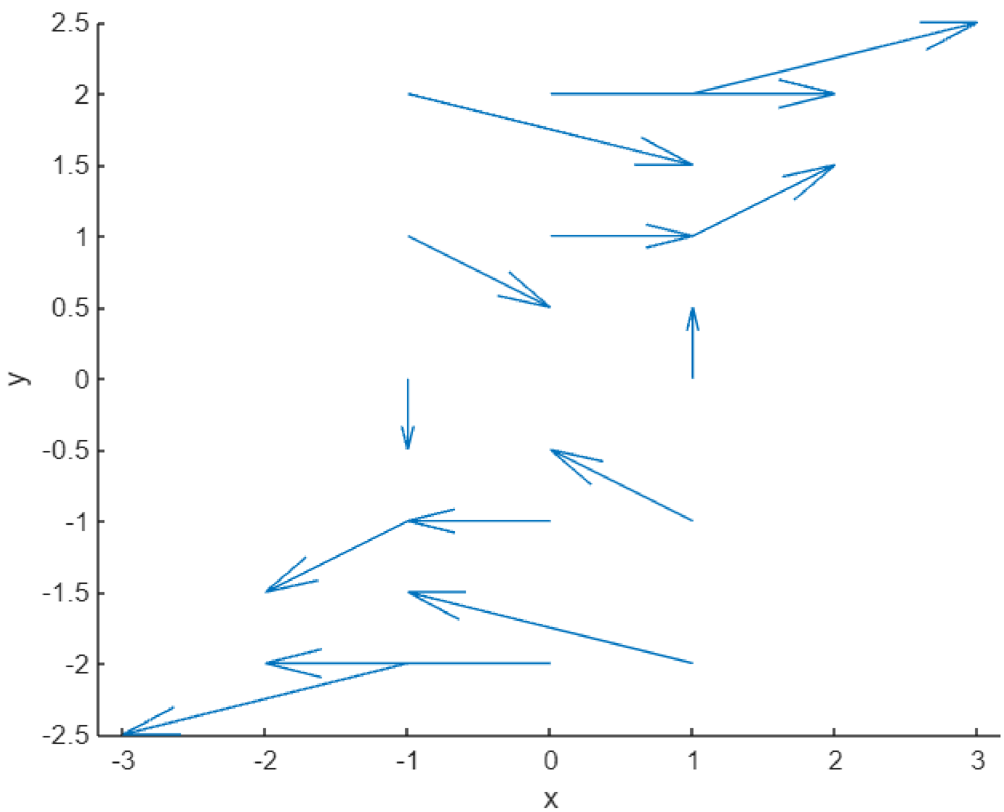
```
% Example 1 Code - Vector Field Plotting

% Clear the workspace
close all;
clear all;
clc;

%setup a dense grid of points for plotting
x = linspace(-5,5,20);
y = linspace(-5,5,20);
[X, Y] = meshgrid(x,y);

%compute the vector field
```

```
Fi = 3*X;
Fj = -4*Y;

%calling the quiver() function with its default values will result in a
%quiver plot that performs automatic scaling of the arrows. This is useful
%when you're more interested in the direction of the vectors, not the
%absolute length.
quiver(X,Y,Fi, Fj);
axis equal;
xlabel('x');
ylabel('y')
```




```
figure;
hold on;

%setup a sparse meshgrid for plotting
%a sparse grid like this is more useful if scaling is set to off and you
%want the vectors plotted to be their actual values, not scaled values
x = [-1 0 1];
y = [-2 -1 0 1 2];
[X, Y] = meshgrid(x,y);

%compute the vector field
Fi = Y;
Fj = 0.5*X;
```

```
%calling the quiver() function with the 'off' argument will disable
%automatic scaling of the arrows. This will show the vectors true length
%at the desired locations, but can often result in arrows that overlap with
%each other (which may make the visualization not as nice to look at).
quiver(X,Y,Fi, Fj,'off');
axis equal;
xlabel('x');
ylabel('y')
```



Example 2 - Line Integral Evaluation

Consider the line integral $\int_0^{3\pi} \sqrt{1+4t}\,dt$. The MATLAB function integral() can be used to easily numerically evaluate the integral.

```
% Example 2 Code - Line Integral Evaluation

%define integration variable t
syms t

%setup the function handle
f = @(t) sqrt(1+4*t);

%compute the integral
```

```
integral(f,0,3*pi)
```

```
ans =
  39.956966068902418
```

Example 3 - Green's Theorem

Consider the line integral $\oint_C x^2y^2\,dx + (xy+2)\,dy$ where the curve C is a rectangular region with vertices $(0,0)$, $(5,0)$, $(5,2)$, $(0,2)$, oriented in the counterclockwise direction.

Using Green's Theorem, we let $\mathbf{F}(x,y) =< x^2y^2, xy+2 >$. Also, $Q_x = 2xy^2$, $P_y = x$, so we can write

$$\oint_C x^2y^2\,dx + (xy+2)\,dy = \int\int_D (Q_x - P_y)\,dA = \int_0^2 \int_0^5 (2xy^2 - x)\,dx\,dy$$

We can numerically evaluate this two-dimensional integral using the code below

```
% Example 3 Code - Green's Theorem

%define integration variables
syms x y;

%setup the function
clear all;
f = @(x,y) (2*x.*y.^2 - x);

%setup limits of integration
xmin = 0;
xmax = 5;
ymin = 0;
ymax = 2;

%compute the integral
format long
integral2(f,xmin,xmax,ymin,ymax,'Method','tiled')
```

```
ans =
  41.666666666677578
```

Problems:

Problem 1: Use MATLAB and the quiver() function to plot the vector field $\mathbf{F}(x,y) = 2x^2\mathbf{i} + xy\mathbf{j}$. Label axes appropriately and title the figure.

```
% Problem 1 Code Here
close all;
clear all;
clc;

x = linspace(-5, 5, 20);
```

```
y = linspace(-5, 5, 20);

[X, Y] = meshgrid(x, y);

Fi = 2 * X.^2;
Fj = X .* Y;

quiver(X, Y, Fi, Fj);
axis equal;
xlabel('x');
ylabel('y');
title('Vector Field of F(x,y)=2x^2i + xyj');
grid on;

figure;
hold on;

x = [-3 1.5 3];
y = [-3 1.5 3];

[X, Y] = meshgrid(x, y);

Fi = 2 * X.^2;
Fj = X .* Y

quiver(X, Y, Fi, Fj, 'off');
axis equal;
xlabel('x');
ylabel('y');
```

Problem 2: Consider the line integral $\int_C (2x + 4y^2)\, ds$ where C is the curve parameterized by $x = t$ and $y = 3t$ for $0 \le t \le 2$. Find a simplified expression for the line integral and then use the MATLAB integral() function to compute its value.

Put your math/explanation here...

```
% Problem 2 Code Here
syms t

f = @(t) (2*t + 36*t.^2) * sqrt(10);

integral(f, 0, 2);
```

Problem 3: Consider the line integral $\oint_C (2x^2 + y)\,dx + (xy^2 - 4)\,dy$ where C is the rectangular region with vertices $(-1, 0)$, $(3, 0)$, $(3, 6)$, $(-1, 6)$, oriented in the counterclockwise direction. Use Green's Theorem to find a simplified expression for the line integral and then use the MATLAB integral2() function to compute its value.

Put your math/explanation here...

```
% Problem 3 Code Here
syms x y;

clear all;

f = @(y, x) (y-1);

xmin = -1;
xmax = 3;
ymin = 0;
ymax = 6;

format long;
integral2(f, ymin, ymax, xmin, xmax, 'Method', 'tiled');
```