# MODULE THREE PROBLEM SET



JoAnn Olney

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Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

## Problem 1

A 125-page document is being printed by five printers. Each page will be printed exactly once.

(a) Suppose that there are no restrictions on how many pages a printer can print. How many ways are there for the 125 pages to be assigned to the five printers?

(b) One possible combination: printer A prints out pages 2-50, printer B prints out pages 1 and 51-60, printer C prints out 61-80 and 86-90, printer D prints out pages 81-85 and 91-100, and printer E prints out pages 101-125.

The printers are represented by the letters $A, B, C, D, E$.
For every page printed there are 5 printer options.
We can let the 5 printers be base number, and the exponent will be the number of pages.
There is a total number of ways that the 125 pages can be assigned to the
5 printers which can be shown as $5^1 25$ .

item Suppose the first and the last page of the document must be printed in color, and only two printers are able to print in color. The two color printers can also print black and white. How many ways are there for the 125 pages to be assigned to the five printers?

Now the first and last pages need to be color, and there are 2 color printers.
So we will print the 123 regular pages since color printers can also do black and white.
So for those pages there are 5 printers avalible.
The total ways will be $5^1 23$.
For the first and last page, there are 2 printers,
$2^2 = 4$.
The total ways will be $= (5^1 23) * 4$

item Suppose that all the pages are black and white, but each group of 25 consecutive pages (1-25, 26-50, 51-75, 76-100, 101-125) must be assigned to the same printer. Each printer can be assigned 0, 25, 50, 75, 100, or 125 pages to print. How many ways are there for the 125 pages to be assigned to the five printers?

We divide 125 by 25 to determine how many groups of consecutive pages there are. Then we will multiple that by 5 times since there are 5 printers.
We get 3125

PROBLEM 2

Ten kids line up for recess. The names of the kids are:

$$\{\text{Alex, Bobby, Cathy, Dave, Emy, Frank, George, Homa, Ian, Jim}\}.$$

Let $S$ be the set of all possible ways to line up the kids. For example, one order might be:

(Frank, George, Homa, Jim, Alex, Dave, Cathy, Emy, Ian, Bobby)

The names are listed in order from left to right, so Frank is at the front of the line and Bobby is at the end of the line.

Let $T$ be the set of all possible ways to line up the kids in which George is ahead of Dave in the line. Note that George does not have to be immediately ahead of Dave. For example, the ordering shown above is an element in $T$.

Now define a function $f$ whose domain is $S$ and whose target is $T$. Let $x$ be an element of $S$, so $x$ is one possible way to order the kids. If George is ahead of Dave in the ordering $x$, then $f(x) = x$. If Dave is ahead of George in $x$, then $f(x)$ is the ordering that is the same as $x$, except that Dave and George have swapped places.

(a) What is the output of $f$ on the following input?
(Frank, George, Homa, Jim, Alex, Dave, Cathy, Emy, Ian, Bobby)

The output of f(x) on the given sequence is:
(Frank, Ian, Dave, Homa, Jim, Alex, Bobby, Frank, George, Cathy)

(b) What is the output of $f$ on the following input?
(Emy, Ian, Dave, Homa, Jim, Alex, Bobby, Frank, George, Cathy)

The output of f(x) on the given sequence is:
(Emy, Ian, George, Homa, Jim, Alex, Bobby, Frank, Dave, Cathy)

(c) Is the function $f$ a $k$-to-1 correspondence for some positive integer $k$? If so, for what value of $k$? Justify your answer.

$k$ to 1 is identical $k = 1$ for bijection $k$ to 1.
The ratio of $f$: x-y for any y that there is one $k$ from such that
$f(x) = y$ 1to1 corresponds to another bijection.
Two values will give the same output which are interchangeable places for George and Dave,
this suggests that for every output from $f$ is obtained by exactly two different inputs.
So $k = 2$

(d) There are 3628800 ways to line up the 10 kids with no restrictions on who comes before whom. That is, $|S| = 3628800$. Use this fact and the answer to the previous question to determine $|T|$.

Let $S$ be the set of all possible ways to line up the kids where:
$|S| = 3628800$
Let $T$ be the set of all possible ways to line up the kids where George is ahead of Dave.
The function $f$ is defined by:
$|T| = |S|\ 2$
$|T| = 1814400$

## Problem 3

Consider the following definitions for sets of characters:

- Digits $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Letters $= \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$
- Special characters $= \{*, \&, \$, \#\}$

Compute the number of passwords that satisfy the given constraints.

(i) Strings of length 7. Characters can be special characters, digits, or letters, with no repeated characters.

There are 10 digits, 26 letters and 4 special chars.
That means there are 40 total characters used for this password.
The equation would then turn out to be $(40 * 39 * 38 * 37 * 36 * 35 * 34)$.
This is because we start with 40 characters and then say the char a is taken out, that leaves 39, and so on.
That leave 39 choices of character number 2 and so on.
The total number passwords is: 93963542400.

(ii) Strings of length 6. Characters can be special characters, digits, or letters, with no repeated characters. The first character can not be a special character.

Since the first character cant be a special char,
there are 36 options for the first character.
After choosing the first character there are 39 options,
and so on, since we can not repeat.
The total number of characters can be represented by:
$36 * 39 * 38 * 37 * 36 * 35$
The final value comes to $= 2487270240$

How many different passwords are there that contain only digits and lower-case letters and satisfy the given restrictions?

(iii) Length is 7 and the password must contain at least one digit.

If there are 10 digits and 26 letters,
there are 36 possible characters to use for the password.
$26^7$ is the number of passwords that contain just letters and
$36^7$ is the number of passwords that contain letters and digits.
Therefore, $36^7 - 26^7$ is the total number of passwords that contain at least one digit.
$36^7 - 26^7 = 70332353920$

(iv) Length is 7 and the password must contain at least one digit and at least one letter.

There are 36 possible characters to use.
The total number of passwords of length 7 using these characters is $36^7$.
Passwords with only letters of length 7 is $26^7$
and passwords of length 7 using digits is $10^7$.
Using complementary counting, we subtract the unwanted cases,
only the letters or only the digits, from the total number.
Thus, our equation becomes $36^7 - 26^7 - 10^7$
So, the number of passwords of length 7 that contain at least 1 letter and 1 digit is 70322353920.

## Problem 4

A group of four friends goes to a restaurant for dinner. The restaurant offers 12 different main dishes.

(i) Suppose that the group collectively orders four different dishes to share. The waiter just needs to place all four dishes in the center of the table. How many different possible orders are there for the group?

To find how many dif possible orders there are for the group to select 4 dif dishes from a list of 12,
we consider the problem of selecting w/o regard to the order of selection.
For this case $n = 12$ the number of dishes and $k = 4$ the number of dishes to choose.
When we plug in these values we get $11880/24 = 495$
Therefore, there is 495 different ways to select 4 different dishes from a list of 12,
considering that the order in which they are selected doesnt matter.

(ii) Suppose that each individual orders a main course. The waiter must remember who ordered which dish as part of the order. It's possible for more than one person to order the same dish. How many different possible orders are there for the group?

Since each choice is independent of the others you multiply the number of options for each person.
Thus, the calculation becomes $12^4$.
$12^4 = 20736$
This means there are 20736 possible combinations of dishes that the group of 4 can choose,
accounting for repetitions.

Problem 5

A university offers a Calculus class, a Sociology class, and a Spanish class. You are given data below about two groups of students.

(i) Group 1 contains 170 students, all of whom have taken at least one of the three courses listed above. Of these, 61 students have taken Calculus, 78 have taken Sociology, and 72 have taken Spanish. 15 have taken both Calculus and Sociology, 20 have taken both Calculus and Spanish, and 13 have taken both Sociology and Spanish. How many students have taken all three classes?

$|A| = 61$ (students who took calculus)
$|B| = 78$ (students who took sociology)
$|C| = 72$ (students who took spanish)
$|A \cap B|$ (students who took both calculus and sociology)
$|A \cap C|$ (students who took both calculus and spanish)
$|B \cap C|$ (students who took both sociology and spanish)
We want to find the number of students who have taken all 3, which is shown by, $|A \cap B \cap C|$.
Using the inclusion-exclusion principle, we have:
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
Substituting the know values:
$170 = 61 + 78 + 72 - 15 - 20 - 13 + |A \cap B \cap C|$
Calculating the rt side:
$170 = 211 - 48 + |A \cap B \cap C|$
$170 = 163 + |A \cap B \cap C|$
$|A \cap B \cap C| = 170 - 163 = 7$,
Thus the number of students who have taken all 3 classes is 7.

(ii) You are given the following data about Group 2. 32 students have taken Calculus, 22 have taken Sociology, and 16 have taken Spanish. 10 have taken both Calculus and Sociology, 8 have taken both Calculus and Spanish, and 11 have taken both Sociology and Spanish. 5 students have taken all three courses while 15 students have taken none of the courses. How many students are in Group 2?

$A = 32$ (students took calculus)
$B = 22$ (students took sociology)
$C = 16$ (students took spanish)
$D = 15$ (students took no courses)
We will use the inclusion-exclusion principle to find the total number of students in group 2
The principle for 3 sets states:
$|A \cap B \cap C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
To begin substituting the values:
$|A \cup B \cup C| = 32 + 22 + 16 - 10 - 8 - 11 + 5$

$= 61$
To find the total num of students in group 2 we add those who took no courses:
$|A \cup B \cup C| + D = 61 + 15 = 76$
So, the total number of students in group 2 is 76.

Problem 6

A coin is flipped five times. For each of the events described below, express the event as a set in roster notation. Each outcome is written as a string of length 5 from $\{H, T\}$, such as $HHHTH$. Assuming the coin is a fair coin, give the probability of each event.

(a) The first and last flips come up heads.

The set contains 8 specific sequences of coin flip.
Each sequence consists of 5 flips.
When flipping a coin there are 2 possible outcomes, heads or tails.
For the 5 flips, the total number of outcomes is $2^5 = 32$
To find the probability $p(E)$ that the first and last flip come up heads we note:
that 8 favorable outcomes in set $S$.
So the probability can be calculated by:
$p(E) = \frac{|S|}{x} = \frac{8}{32} = \frac{1}{4}$

(b) There are at least two consecutive flips that come up heads.

Let $S$ be the sample space of all possible outcomes when you flip a coin 5 times,
The number of out comes is given by:
$|S| = 2^5 = 32$
We let $E$ be the event that there are at least two consecutive heads flipped.
The total number of favorable outcomes is $|E| = 14$
The probability $p(E)$ of the event $E$ is given by
the fraction of the number of outcomes to the total number of outcomes:
$p(E) = \frac{|E|}{|S|} = \frac{14}{32} = \frac{7}{16}$
Therefore, the probability that there are at least two consecutive heads is 7/16.

(c) The first flip comes up tails and there are at least two consecutive flips that come up heads.

The total number of outcomes is given by: $|S| = 2^5 = 32$
We find the outcomes where the first flip is tails and there is at least two consecutive heads:
$\{THHHH, THHTH, THTHH, TTHHH, THHHT, TTTHH, TTHHT, THHTT\}$
Counting these outcomes we find there are 8 valid outcomes.
The probability $p(E)$ of the event $E$ is given by:
$p(E) = \frac{8}{32} = \frac{1}{4} = 0.25$

PROBLEM 7

An editor has a stack of $k$ documents to review. The order in which the documents are reviewed is random with each ordering being equally likely. Of the $k$ documents to review, two are named "Relaxation Through Mathematics" and "The Joy of Calculus." Give an expression for each of the probabilities below as a function of k. Simplify your final expression as much as possible so that your answer does not include any expressions in the form
$\left( \begin{array}{c} a \\ b \end{array} \right)$.

(a) What is the probability that "Relaxation Through Mathematics" is first to review?

The total number of ways to review $k$ docs is given by $k!$ of $k$.
The number of ways to arrange the remaining $k-1$ docs is $(k-1)!$
Thus, the probability of $P$ that "Relaxation through Mathematics" is the first to review is given by: $P$

(b) What is the probability that "Relaxation Through Mathematics" and "The Joy of Calculus" are next to each other in the stack?

We consider both docs as a single entity , we can treat them as one doc.
So this will reduce the problem to $k-1$ docs:
the block of the two docs together and $k-2$ other docs.
The number of ways to arrange these $k-1$ entities is $(k-1)!$.
But, the two docs that are within the block can be arranged among themselves in $2! = 2$ ways.
Thus, the total number of arrangements where these 2 docs are next to each other is:
$2 \cdot (k-1)!$
As we have already established, the total number of all $k$ docs is $k!$.
Therefore the probability $P$ they will be next to each other is:
$P = \frac{2 \cdot (k-1)!}{k!} = \frac{2 \cdot (k-1)!}{k \cdot (k-1)!} = \frac{2}{k}$