MODULE FIVE PROBLEM SET

JoAnn Olney

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

Indicate whether the two functions are equal. If the two functions are not equal, then give an element of the domain on which the two functions have different values.

(a)

$$f : \mathbb{Z} \to \mathbb{Z}, \text{ where } f(x) = x^2.$$
$$g : \mathbb{Z} \to \mathbb{Z}, \text{ where } g(x) = |x|^2.$$

Both of the functions are the same because
x^2 and $|x|^2$ both give a positive value.

(b)

$$f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, \text{ where } f(x, y) = |x + y|.$$
$$g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, \text{ where } g(x, y) = |x| + |y|.$$

Both of the functions arent equal because
of either x or y are of opposite signs.

PROBLEM 2

The domain and target set of functions f and g is $\mathbb{R}$. The functions are defined as:

- $f(x) = 2x + 3$
- $g(x) = 5x + 7$

(a) $f \circ g$?

$2(5x + 7) + 3 = 10x + 17$

(b) $g \circ f$?

$5(2x + 3) + 7 = 10x + 22$

(c) $(f \circ g)^{-1}$?

$\frac{x-17}{10}$

(d) $f^{-1} \circ g^{-1}$?

$\frac{x-22}{10}$

(e) $g^{-1} \circ f^{-1}$?

$\frac{x-17}{10}$

Are any of the above equal?

Yes c and e both have the same solution of:
$\frac{x-17}{10}$

Problem 3

(a) Give the matrix representation for the relation depicted in the arrow diagram. Then, express the relation as a set of ordered pairs.

The arrow diagram below represents a relation.

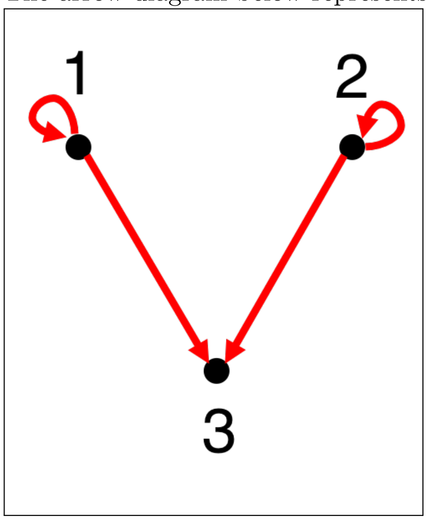

Figure 1: *An arrow diagram shows three vertices, 1, 2, and 3. An arrow from vertex 1 points to vertex 3, and another arrow from vertex 2 points to vertex 3. Two self loops are formed, one at vertex 1 and another at vertex 2.*

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{matrix}$$

(b) Draw the arrow diagram for the relation.
The domain for the relation A is the set $\{2, 5, 7, 8, 11\}$. For x, y in the domain, xAy if $|x - y|$ is less than 2.

Problem 4

For each relation, indicate whether the relation is:

- Reflexive, anti-reflexive, or neither
- Symmetric, anti-symmetric, or neither
- Transitive or not transitive

Justify your answer.

(a) The domain of the relation L is the set of all real numbers. For x, $y \in \mathbb{R}$, xLy if $x < y$.

A relation is reflexive if $\forall x, (x, x) \in L$
Since $x < x$ is never true, L, is not reflexive
A relation is antireflxive if $\forall x, (x, x) \notin L$
which is true here, so L is antireflixive. If $(x, y) \in L$,
then (x, y) must also be in L. For EX: if $x < y$ it isnt true
that $y < x$. Thus, L is not symmestric. For L to be antisymmestric
$(x, y) \in L$ and $(y, x) \in L$ must imply $x = y$ Since L
cant contain both it is antisymmextric. For transitivity, if
$(x, y) \in L$ and $(y, z) \in L$, then (x, z) must also
be in L. Since if $x < y$ and $y < z$ it follows that $x < Z$
thus L is transitive.

(b) The domain of the relation A is the set of all real numbers. xAy if $|x-y| \leq 2$

For any x, $|x - x| = 0 \leq 2$. Hence, A is reflexive.
Since $(x, x) \in A$ it is not antireflexive.
If $(x, y) \in A$, then $|x - y| \leq 2$, which implies
$|y - x| \leq 2$, hence A is symmectric. Since both
$(x, y) \in A$ and $(y, x) \in A$ can exist even if
$x \neq y$, A is not antisymmetric. Consider $(x, y) \in A$
and $(y, z) \in A$. It is not gauranteed that $|x - z| \leq 2$
Therefore A is not transitive.

(c) The domain of the relation Z is the set of all real numbers. xZy if $y = 2x$

For any x, $x \neq 2x$ unless $x = 0$, thus Z is not reflexive.
Since $(x, x) \notin Z$, it is antireflexive.
If $(x, y) \in Z$ then $y = 2x$. The reverse,
$x = 2y$, is not guaranteed, therefore Z is not symmestric.
If $(x, y) \in Z$ and $(y, z) \in Z$, implies $y = 2x$ and
$z = 2y = 4x$. Thus Z is not transitive.

PROBLEM 5

The number of watermelons in a truck are all weighed on a scale. The scale rounds the weight of every watermelon to the nearest pound. The number of pounds read off the scale for each watermelon is called its measured weight. The domain for each of the following relations below is the set of watermelons on the truck. For each relation, indicate whether the relation is:

- Reflexive, anti-reflexive, or neither
- Symmetric, anti-symmetric, or neither
- Transitive or not transitive

Justify your answer.

(a) Watermelon x is related to watermelon y if the measured weight of watermelon x is at least the measured weight of watermelon y. No two watermelons have the same measured weight.

The relation is reflexive because for any watermelon
x the measured weight of x is at least the measured
weight of x itself. Thus x related x holds true.
The relation is not symmetric, for EX: if $x = 20$ lbs and
$y = 21$ lbs then y is related to x, since $21 > 20$
but x is not related y since $20 \not> 21$.
The relation is antisymmetric because if x is related to y
and y is related to x it must be true that $x = y$, however
since no two watermelon have the same weight this condition holds.
The relation is transitive. If x is related to y, i.e.,
weight of $x >$ weight of y and y is related to z, i.e.
weight of $y >$ weight of z, then it follows that
$weight(x) > weight(y) > weight(z)$ implying x is related to z

(b) Watermelon x is related to watermelon y if the measured weight of watermelon x is at least the measured weight of watermelon y. All watermelons have exactly the same measured weight.

The relation is reflexive since x will always
be related to itself $weight(x) > weight(x)$.
The relation is symmetric, if x is related to y
since they have the same weight, then y is related to x.
The relation is transitive because if x is related to y
and y is related z, then x is related to z since
they all have the same weights.

Problem 6

Part 1. Give the adjacency matrix for the graph G as pictured below:

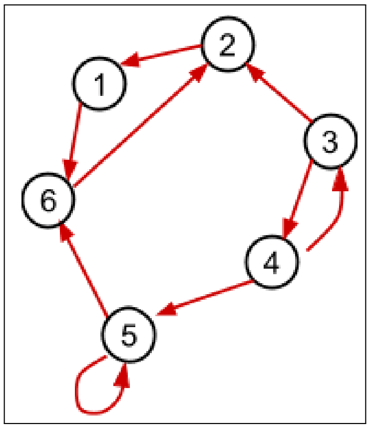

Figure 2: *A graph shows 6 vertices and 9 edges. The vertices are 1, 2, 3, 4, 5, and 6, represented by circles. The edges between the vertices are represented by arrows, as follows: 4 to 3; 3 to 2; 2 to 1; 1 to 6; 6 to 2; 3 to 4; 4 to 5; 5 to 6; and a self loop on vertex 5.*

0	0	0	0	0	1
1	0	0	0	0	0
0	1	0	1	0	0
0	0	1	0	1	0
0	0	0	0	1	1
0	1	0	0	0	0

Part 2. A directed graph G has 5 vertices, numbered 1 through 5. The 5×5 matrix A is the adjacency matrix for G. The matrices A^2 and A^3 are given below.

$$A^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Use the information given to answer the questions about the graph G.

(a) Which vertices can reach vertex 2 by a walk of length 3?

The vertex that can be reached from vertex 2 by a walk of length 3 is vertex 2.

(b) Is there a walk of length 4 from vertex 4 to vertex 5 in G? (Hint: $A^4 = A^2 \cdot A^2$.)

No the walk doesnt exist of length 4
from vertex 4 to 5.

Problem 7

Part 1. The drawing below shows a Hasse diagram for a partial order on the set $\{A, B, C, D, E, F, G, H, I, J\}$

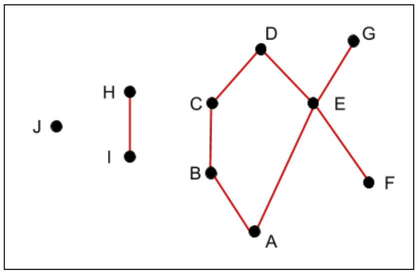

Figure 3: *A Hasse diagram shows 10 vertices and 8 edges. The vertices, represented by dots, are as follows: vertex J; vertices H and I are aligned vertically to the right of vertex J; vertices A, B, C, D, and E forms a closed loop, which is to the right of vertices H and I; vertex G is inclined upward to the right of vertex E; and vertex F is inclined downward to the right of vertex E. The edges, represented by line segments, between the vertices are as follows: Vertex J is connected to no vertex; a vertical edge connects vertices H and I; a vertical edge connects vertices B and C; and 6 inclined edges connect the following vertices, A and B, C and D, D and E, A and E, E and G, and E and F.*

(a) What are the minimal elements of the partial order?

The elements that satisfy the minimalist conditions
are J, I, A, F

(b) What are the maximal elements of the partial order?

The elements that satisfy the maximality conditions
are J, H, G, D

(c) Which of the following pairs are comparable?

(A, D), (J, F), (B, E), (G, F), (D, B), (C, F), (H, I), (C, E)

$(B, E), (C, F), (C, E), (J, F)$ are not comparable
$(A, D), (G, F), (D, B), (H, I)$ are comparable

Part 2. Each relation given below is a partial order. Draw the Hasse diagram for the partial order.

(a) The domain is $\{3, 5, 6, 7, 10, 14, 20, 30, 60\}$. $x \leq y$ if x evenly divides y.

(b) The domain is $\{a, b, c, d, e, f\}$. The relation is the set:

$\{(b, e), (b, d), (c, a), (c, f), (a, f), (a, a), (b, b), (c, c), (d, d), (e, e), (f, f)\}$

PROBLEM 8

Determine whether each relation is an equivalence relation. Justify your answer. If the relation is an equivalence relation, then describe the partition defined by the equivalence classes.

(a) The domain is a group of people. Person x is related to person y under relation M if x and y have the same favorite color. You can assume that there is at least one pair in the group, x and y, such that xMy.

Consider the given relation M.A person has the same biological mother or father as herself or himself. Thus, the relation M is is reflexive. If a person x has the same biological mother or father as person y, then y has the same biological mother or father as x. Thus, the relation M is symmetric. If person x has the same biological mother or father as person y and y has the same biological mother or father as z then not necessarily the person x has the same biological mother or father as z. Thus M is not transitive. Therefore, the given relation M is not necessarily an equivalence relation.

(b) The domain is the set of all integers. xEy if $x + y$ is even. An integer z is even if $z = 2k$ for some integer k.

If x is an ineger, then $x + x = 2x$ is also an integer. Thus, the relation E is reflexive. If x, y are integers and $x + y$ is even, then $y + x$ is also even. Thus the relation E is symmetric. Consider x, y, z are integers. Assume $x + y$ is even, that is $y + z = 2k$ for some integer k. Then, $x + z = 2j - y + 2k - y = 2(j + k - y)$. Since $(j + k - y)$ is an integer, $x + z$ is also an integer. Therefore, the given relation is an equivalence relation.