MODULE ONE PROBLEM SET

JoAnn Olney

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

In the following question, the domain of **discourse** is a set of male patients in a clinical study. Define the following predicates:

- $P(x):$ x was given the placebo

- $D(x):$ x was given the medication

- $M(x):$ x had migraines

Translate each of the following statements into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

Sample question: Some patient was given the placebo and the medication.

- $\exists x\ (P(x)\ \wedge\ D(x))$
- *Negation:* $\neg\exists x\ (P(x)\ \wedge\ D(x))$
- *Applying De Morgan's law:* $\forall x\ (\neg P(x)\ \vee\ \neg D(x))$
- *English: Every patient was either not given the placebo or not given the medication (or both).*

(a) Every patient was given the medication or the placebo or both.

(b) $\forall x\ (D(x); \vee\ P(x))$

(c) Negation: $\neg\forall x\ (D(x)\ \vee\ P(x))$

(d) Applying De Morgans Law: $\exists x\ (\neg D(x)\ \wedge\ \neg D(x))$

(e) English: Some patients not given the mediation and were not given the placebo

- Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \to q \equiv \neg p \vee q$.)

- Logic: $\forall x\ (P(x)) \longrightarrow M(x))$
- Applying conditional identity: $\forall x\ (\neg P(x) \vee M(x))$
- Negate the expression: $\neg\forall x\ (\neg P(x) \vee M(x))$
- De Morgans Law: $\exists x\ (\neg\neg P(x) \wedge \neg M(x))$
- Double negation: $\exists x\ (P(x) \wedge \neg M(x))$
- English: some patients were given the placebo and did not get migraines.

(1) There is a patient who had migraines and was given the placebo.

(2) Logic: $\exists x\ (M(x) \wedge P(x))$

(3) Negation: $\neg\exists x\ (M(x) \wedge P(x))$

(4) Apply de morgans law: $\forall x\ (\neg M(x) \vee \neg P(x))$

(5) English: Every patient did not have a migraine or was not given a placebo.

PROBLEM 2

Use De Morgan's law for quantified statements and the laws of propositional logic to show the following equivalences:

(a) $\neg\forall x\ (P(x) \wedge \neg Q(x))\ \equiv\ \exists x\ (\neg P(x) \vee Q(x))$

$\neg\forall x\ (P(x) \wedge \neg Q(x)) \equiv \exists x\ (\neg P(x) \vee Q(x))$
Apply De morgans law: $\exists x\ (\neg P(x) \vee \neg\neg Q(x)) \equiv \exists x\ (\neg P(x) \vee Q(x)$
Apply double negation: $\exists x\ (\neg P(x) \vee Q(x)) \equiv \exists x\ (\neg P(x) \vee Q(x))$

(b) $\neg\forall x\ (\neg P(x) \to Q(x))\ \equiv\ \exists x\ (\neg P(x) \wedge \neg Q(x))$

$\neg\forall x\ \ (\neg P(x) \to Q(x)) \equiv \exists x\ \ (\neg P(x) \wedge \neg Q(x))$
Apply conditional identities law: $\neg\forall x\ \ (\neg(\neg(P(x) \vee Q(x)) \equiv \exists x\ \ (\neg P(x) \wedge \neg Q(x))$
Apply the double negation rule: $\neg\forall x\ \ (P(x) \vee Q(x)) \equiv \exists x\ \ (\neg P(x) \wedge \neg Q(x))$
Apply De morgans law: $\exists x\ \ (\neg P(x) \wedge \neg Q(x)) \equiv \exists x\ \ (\neg P(x) \wedge \neg Q(x))$

(c) $\neg\exists x\ \big(\neg P(x) \vee (Q(x) \wedge \neg R(x))\big)\ \equiv\ \forall x\ \big(P(x) \wedge (\neg Q(x) \vee R(x))\big)$

$\neg\exists x\ \neg P(x) \vee (Q(x) \wedge \neg R(x)) \equiv \forall x\ P(x) \wedge (\neg Q(x) \vee R(x))$
Apply De morgans law: $\forall x\ \ (\neg\neg P(x) \wedge (\neg Q(x) \vee \neg\neg R(x))) \equiv \forall x\ \ (P(x) \wedge (\neg Q(x) \vee R(x)))$
Apply the double negation law: $\forall x\ \ (P(x) \wedge (\neg Q(x) \vee R(x))) \equiv \forall x\ \ (P(x) \wedge (\neg Q(x) \vee R(x)))$

Problem 3

The domain of **discourse** for this problem is a group of three people who are working on a project. To make notation easier, the people are numbered 1, 2, 3. The predicate $M(x,\ y)$ indicates whether x has sent an email to y, so $M(2,\ 3)$ is read "Person 2 has sent an email to person 3." The table below shows the value of the predicate $M(x,\ y)$ for each $(x,\ y)$ pair. The truth value in row x and column y gives the truth value for $M(x,\ y)$.

M	1	2	3
1	T	T	T
2	T	F	T
3	T	T	F

Determine if the quantified statement is true or false. Justify your answer.

(a) $\forall x\, \forall y\, (x \neq y) \ \rightarrow\ M(x,\ y))$

This statement is False: This statement requires each individual to send an email to each person except them self. Person 1 sent an email to "everyone", which means themselves included which fails the given statement.

(b) $\forall x\, \exists y\ \ \neg M(x,\ y)$

This statement is False: As the statement says every person sent an email to at least on individual. The negation of the statement says not every person sent an email to at least one individual. And using the chart, this is false because everyone person sent an email.

(c) $\exists x\, \forall y\ \ M(x,\ y)$

This statement is True: This statement is true based on the given context that one person emails everyone(person 1)

PROBLEM 4

Translate each of the following English statements into logical expressions. The domain of **discourse** is the set of all real numbers.

(a) The reciprocal of every positive number less than one is greater than one.

$\forall x\ \forall y\ (0 < x < 1) \rightarrow (\frac{1}{x} > 1)$

(b) There is no smallest number.

$\forall x\ \forall y\ (x\ ^{\neg\infty}) \rightarrow (y < x)$

(c) Every number other than 0 has a multiplicative inverse.

$\forall x\ (x \neq 0) \rightarrow (\frac{1}{x})$

Problem 5

The sets A, B, and C are defined as follows:

$$A = tall, grande, venti$$
$$B = foam, no - foam$$
$$C = non - fat, whole$$

Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

(a) Write an element from the set $A \times B \times C$.

(tall, foam, non-fat)

(b) Write an element from the set $B \times A \times C$.

(foam, tall, non-fat)

(c) Write the set $B \times C$ using roster notation.

B x C = (foam,non-fat), (foam,whole), (no-foam,non-fat), (no-foam,whole)