

# The story of the Hitachi experiment

Thinking about the physical large building unit different from its small building characteristics in the nanostructure as obeying the laws of quantum mechanics.

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by describing an experiment on a big scale one that involves beams of electrons in an electron microscope. This remarkable experiment was carried out at Hitachi Labs in Japan.

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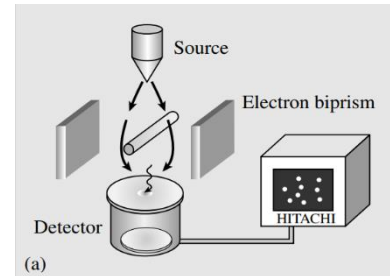


Fig 1.1(a)The Hitachi electron diffraction

A key feature of the experiment was the use of a very low electron beam current. The current was so low that the probability of even one electron being in the apparatus at any time was tiny. The probability that two were in the device at the same time was therefore vanishingly small. Electrons were observed landing at random on the screen, one at a time, as shown in Fig. 1.1(b).

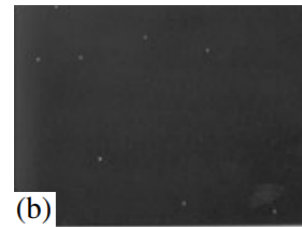


Fig. 1.1(b) First electron distribution

As time passed, the number of detected electrons increased, resulting in a greater density of spots on the screen as shown in Fig 1.1 (c). As only one electron was in the column at any one time, the passage to the left or the right of the biprism was surely a random series of events. If the electrons behave as independent particles, then they must hit the screen randomly with some overall distribution determined by the electron source. But this is not how the electrons behaved.

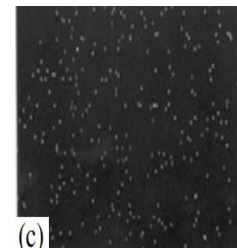
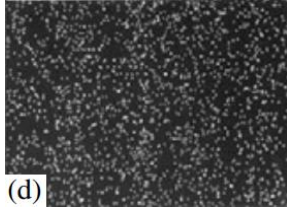
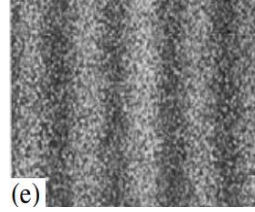


fig 1.1 (c) second electron distribution

As time progressed, some regions were seen to acquire more spots—the pattern is discernable in Fig. 1.1(d). becoming much clearer on longer exposure as shown in Fig. 2.1(e).



(d)  
Fig1.1(d) third distribution



(e)  
fig1.1(e)fourth distribution

### **Particles and waves**

To know the quantum mechanics, be familiar with the particles and waves to know the behavior of the probability amplitude for finding electrons.

Electrons with a momentum of magnitude  $p_e$  (when specifying both magnitude and direction use the vector symbol  $\vec{p}_e$ ) have a “wavelength”  $\lambda$  associated with the probability of finding the particle given by  $\lambda = h/p_e$ .

where  $h$  is Planck’s constant,  $6.63 \times 10^{-34}$  Js, and  $p_e$  is the momentum,  $mv$ , of an electron of mass  $m_e$  ( $m_e = 9.1 \times 10^{-31}$  kg) undergoing linear motion with velocity,  $v$ (m/s). The classical definition of momentum (as mass  $\times$  velocity) is valid at speeds much less than that of light.

### **Uncertainty principle**

**uncertainty principle**, also called **the Heisenberg uncertainty principle** or **indeterminacy principle** statement, by Werner Heisenberg, that the position and the velocity of an object cannot both be measured exactly, at the same time, even in theory.

It states that we cannot know both the position and speed of a particle, such as a photon or an electron, with perfect accuracy; the more we nail down the particle's position, the less we know about its speed and vice versa.

In other words, if there were a probability of shrinking a tortoise down to the size of an electron, it would only be able to precisely calculate its speed *or* its location, not at the same time.

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Heisenberg, arguing in a similar but more mathematical manner, derived his famous “uncertainty principle” relating the product of the uncertainty in position,  $dx$ , and the uncertainty in momentum,  $dp$ , in a measurement made on a quantum system as follows:  $dx dp \geq h/4\pi$  . (2.7)

(this is a relationship between the magnitudes of these vector quantities.) The units of Planck’s constant—Joule-seconds—are equivalent to position times momentum ( $\text{kg (m/s)}^2 \times \text{s} = \text{kg (m/s)} \times \text{m}$ ).

The uncertainty in position in the Hitachi experiment (with a light detector) would be the spacing between fringes on the screen or  $dx = z\delta\theta \sim hz/dp_e$ .

where  $z$  is the distance between the biprism and the screen. The uncertainty in momentum is on the order of the angular difference between the direction of the beams emerging from source 1 or source 2 times the magnitude of the incident momentum or  $dp \approx dp_e/z$ .

The dimensions of Planck’s constant—Joule-seconds—suggest an equivalent statement of Heisenberg’s uncertainty principle in terms of energy and time:  $dE dt \sim h$ .

### **The Hitachi microscope as a quantum system**

The key to the observation of quantum behavior in the Hitachi experiment lies in the fact that the electrons do not interact with any other particles as they transit the possible paths through the

instrument. The only interaction occurs after the biprism when the electrons hit the detection screen. Even though the electron appears to be a particle at that point.

This experiment led to the making of a new type of microscope that helped many fields discover and make life more progressive. This type is an electron microscope.

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They work like other optical microscopes, except that they use electron beams instead of photons for imaging the specimen. They are used to know an object's composition and structure. It can magnify images up to 10,000,000 times and has a resolution of fifty picometers or 0.05 nanometers.

Uses of electron microscopes vary from investigating biological to inorganic specimens.

### **Who Invented the Electron Microscope?**

In the 1930s, scientists seemed to show interest in exploring the interior structure of organic cells' fine details. However, the available instrument, the light microscope, could not give 10,000 times plus the magnification required to watch the interior parts of the cell such as the nucleus, mitochondria, etc. The light microscope can magnify only 1000 times with a resolution of 0.2 micrometers. Thus, a need to develop high magnification and resolution microscopes came to the surface.

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In 1942, the first scanning electron microscope debuted, and its first commercial instrument was developed around 1965.

### **Probability amplitudes and the rules of quantum mechanics**

In quantum mechanics, a probability amplitude is a complex-valued function that describes an uncertain or unknown quantity. For example, each particle has a probability amplitude describing its position. This amplitude is the wave function, expressed as a function of position.

The mechanics of classical particles are straightforward and described by Newton's second law of motion. The acceleration of a particle is given by the ratio of the net force acting on it to its mass. Once its initial position and velocity are given, its subsequent motion is completely determined. However, quantum mechanics tells us that the very idea of a classical particle is an artifact of a complicated world where interactions have smeared out the underlying quantum behavior.

The uncertainty principle tells about both the position and the velocity of a "particle." Because of this, the underlying laws of motion for a quantum particle have to be framed in such a way that lets us make predictions only for quantities that are the average of many individual measurements.

Knowing the position and momentum would be found by averaging many repeated measurements in nominally identical, but repeated experiments. But the outcome of the measurements on any one "particle" is stochastic, with a spread of data given by the uncertainty principle. Therefore, instead of predicting the position and momentum of any one particle, quantum mechanics tells us how to predict the probability that a "particle" has a certain position. To **codify this process**, a new variable is introduced, called the "probability amplitude,"  $\psi$ .

The following list provides some rules for making predictions based on  $\psi$ :

A- Probability amplitudes and wavefunctions: The probability that a particle is at a certain point in space and time is given by the square of a complex number called the probability amplitude,  $\psi$ .

Being a complex number means that it has two components in general: one a real number and the other an imaginary number. These components behave similarly to the orthogonal components of a 2-D vector. A function that encodes all the (real and imaginary) values of  $\psi$  at various points in space and time is called the wavefunction,  $\psi(r, t)$ . It is important to realize that the probability amplitude is not a probability (it cannot be, because it is, in general, a complex number). Rather, it is a tool used to calculate the probability that a measurement will yield a particular value.

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B- Calculating probability amplitudes for one “path”: Even though  $\psi$  is not a probability per se, calculate the overall probability amplitude (i.e., a particle goes from A to C via B) as the product of the probability amplitude for going from A to B with the probability amplitude for going from B to C. This is analogous to the multiplication rule for calculating the probability of classical independent events, but the quantum mechanical rules apply only to a quantum system where the particle is not disturbed significantly as it passes from A to C (via B).

C- Calculating probabilities from probability amplitudes: The probability that a particular outcome is measured is given by the square of the corresponding probability amplitude.

D- More than one possible “path” through the apparatus: Now turn to the rule that will let us understand the interference fringes observed in the Hitachi experiment. To work out the probability amplitude for some outcome when there is more than one possible path, add the probability amplitudes for each possible pathway.

This is like classical probability but applied to the probability amplitude. Once again, this rule only applies if the system remains undisturbed up to the point of measurement of the outcome. The probability of an outcome is the square of this sum of probability amplitudes. It is this addition of two (or more) complex numbers, followed by squares, that gives the quantum interference pattern.

Illustrating the process with the Hitachi experiment, suppose that one possible way through the apparatus in the Hitachi experiment corresponds to a probability amplitude:  $\psi_{ac1} = x + iy$  the probability amplitude for the second possible way is as follows:  $\psi_{ac2} = u + iv$ .

Then according to the rules just stated, the probability to arrive at some point of the screen,  $\psi(r)$ , is given by  $|\psi(r)|^2 = (\psi_{ac1} + \psi_{ac2})^* (\psi_{ac1} + \psi_{ac2}) = [(x + u) - i(y + v)][(x + u) + i(y + v)] = (x + u)^2 + (y + v)^2 = x^2 + u^2 + 2xu + y^2 + v^2 + 2yv$ .

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#### Fig 1.1(a)The Hitachi electron diffraction

A key feature of the experiment was the use of a very low electron beam current. The current was so low that the probability of even one electron being in the apparatus at any time was tiny. The probability that two were in the device at the same time was therefore vanishingly small. Electrons were observed landing at random on the screen, one at a time, as shown in Fig. 1.1(b).

#### Fig. 1.1(b) First electron

distribution

As time passed, the number of detected electrons increased, resulting in a greater density of spots on the screen as shown in Fig 1.1 (c). As only one electron was in the column at any one time, the passage to the left or the right of the biprism was surely a random series of events. If the electrons behave as independent particles, then they must hit the screen randomly with some overall distribution determined by the electron source. But this is not how the electrons behaved.

#### fig 1.1 (c) second electron distribution

As time progressed, some regions were seen to acquire more spots—the pattern is discernable in Fig. 1.1(d). becoming much clearer on longer exposure as shown in Fig. 2.1(e).

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### **Particles and waves**

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### **Uncertainty principle**

**uncertainty principle**, also called the **Heisenberg uncertainty principle** or **indeterminacy principle** statement, by Werner Heisenberg, that the position and the velocity of an object cannot both be measured exactly, at the same time, even in theory.

It states that we cannot know both the position and speed of a particle, such as a photon or an electron, with perfect accuracy; the more we nail down the particle's position, the less we know about its speed and vice versa.

In other words, if there were a probability of shrinking a tortoise down to the size of an electron, it would only be able to precisely calculate its speed *or* its location, not at the same time. measurement of the position of the electron imparted just enough momentum to alter its direction by about the amount of its angle of diffraction before the measurement.

Heisenberg, arguing in a similar but more mathematical manner, derived his famous “uncertainty principle” relating the product of the uncertainty in position,  $dx$ , and the uncertainty in momentum,  $dp$ , in a measurement made on a quantum system as follows:  $dx dp \geq h/4\pi$ . (2.7) (this is a relationship between the magnitudes of these vector quantities.) The units of Planck’s constant—Joule-seconds—are equivalent to position times momentum ( $\text{kg (m/s)}^2 \times \text{s} = \text{kg (m/s)} \times \text{m}$ ).

The uncertainty in position in the Hitachi experiment (with a light detector) would be the spacing between fringes on the screen or  $dx = z\delta\theta \sim hz/dpe$ .

where  $z$  is the distance between the biprism and the screen. The uncertainty in momentum is on the order of the angular difference between the direction of the beams emerging from source 1 or source 2 times the magnitude of the incident momentum or  $dp \approx dpe/z$ .

The dimensions of Planck’s constant—Joule-seconds—suggest an equivalent statement of Heisenberg’s uncertainty principle in terms of energy and time:  $dE dt \sim h$ .

### **The Hitachi microscope as a quantum system**

The key to the observation of quantum behavior in the Hitachi experiment lies in the fact that the electrons do not interact with any other particles as they transit the possible paths through the instrument. The only interaction occurs after the biprism when the electrons hit the detection screen. Even though the electron appears to be a particle at that point.

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### **Probability amplitudes and the rules of quantum mechanics**

In quantum mechanics, a probability amplitude is a complex-valued function that describes an uncertain or unknown quantity. For example, each particle has a probability amplitude describing its position. This amplitude is the wave function, expressed as a function of position.

The mechanics of classical particles are straightforward and described by Newton's second law of motion. The acceleration of a particle is given by the ratio of the net force acting on it to its mass. Once its initial position and velocity are given, its subsequent motion is completely determined. However, quantum mechanics tells us that the very idea of a classical particle is an artifact of a complicated world where interactions have smeared out the underlying quantum behavior.

The uncertainty principle tells about both the position and the velocity of a "particle." Because of this, the underlying laws of motion for a quantum particle have to be framed in such a way that lets us make predictions only for quantities that are the average of many individual measurements.

Knowing the position and momentum would be found by averaging many repeated measurements in nominally identical, but repeated experiments. But the outcome of the measurements on any one "particle" is stochastic, with a spread of data given by the uncertainty principle. Therefore, instead of predicting the position and momentum of any one particle, quantum mechanics tells us how to predict the probability that a "particle" has a certain position. To **codify this process**, a new variable is introduced, called the "probability amplitude,"  $\psi$ . The following list provides some rules for making predictions based on  $\psi$ :

A- Probability amplitudes and wavefunctions: The probability that a particle is at a certain point in space and time is given by the square of a complex number called the probability amplitude,  $\psi$ .

Being a complex number means that it has two components in general: one a real number and the other an imaginary number. These components behave similarly to the orthogonal components of a 2-D vector. A function that encodes all the (real and imaginary) values of  $\psi$  at various points in space and time is called the wavefunction,  $\psi(r, t)$ . It is important to realize that the probability amplitude is not a probability (it cannot be, because it is, in general, a complex number). Rather, it is a tool used to calculate the probability that a measurement will yield a particular value.

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