Week_3 605 assignment

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Problem set_1

(1) What is the rank of the matrix A?

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

For this problem, I decided to implement a function to calculate the row reduced matrix and the rank to practice my coding skills. I implemented a generic function to calculate any '$m \times n$'. I have three cases for ranking a matrix as following:

1. m > n
2. m < n
3. Finally m = n

Those were the edge cases we need to work on to built the function.

```
a_mEn <- matrix(c(1,2,3,4,
                  -1,0,1,3,
                  0,1,-2,1,
                  5,4,-2,-3), 4, byrow=T)
# a <- matrix(c(0,1,2,1,2,7,2,1,8), ncol = 3)
a_mEn
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```
a_mbn <- matrix(c(1,2,3,4,
                  -1,0,1,3), 4, byrow=T)
a_mbn
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    3    4
## [3,]   -1    0
## [4,]    1    3
```

```
a_mln <- matrix(c(1,2,3,4,
                  -1,0,1,3), 2, byrow=T)
a_mln
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
```

```
get_echoln <- function(a) {
  U = a
  n = ncol(a)
  m = nrow(a)
  if(m == n) {
    for (i in 1:n) {
      for (j in 2:m) {
        if(U[j,i] != 0 & j > i) {
          # Add multiples of the pivot row to each of the lower rows,
          # so every element in the pivot column of the lower rows equals 0.
          mplier = U[[j,i]]/U[[i,i]]
          # reduce by reduction and subtitute in the U matrix
          U[j,] = U[j,] - mplier * U[i,]
        } else if (U[j,i] != 0 & j == i) {
          U[j,] = U[j,] / U[[j,i]]
        }# end if
      } # end if
    } # end for
  } else if(m < n) {
    for (i in 1:n) {
      for (j in 2:m) {
        if(U[j,i] != 0 & j > i) {
          U[i,] = U[i,] / U[[i,i]]
          mplier = U[[j,i]]/U[[i,i]]
          U[j,] = U[j,] - mplier * U[i,]

        } else if(U[j,i] != 0 & j == i) {
          U[i,] = U[i,] / U[[i,i]]
        } # end if
      } # end for
    } # end for
  } else if (m > n) {
    for (i in 1:n) {
      for (j in 2:m) {
        if(U[j,i] != 0 & j > i) {
          U[i,] = U[i,] / U[[i,i]]

          mplier = U[[j,i]]/U[[i,i]]

          U[j,] = U[j,] - mplier * U[i,]

        } else if(U[j,i] != 0 & j == i) {
          U[i,] = U[i,] / U[[i,i]]
        } # end if
      } # end for
    } # end for
  } # end if
  return(round(U, digits = 1))
}
```

```
equal = get_echoln(a_mEn)
equal
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3  4.0
## [2,]    0    1    2  3.5
## [3,]    0    0    1  0.6
## [4,]    0    0    0  1.0
```

```
# greater = get_echoln(a_mbn)
# greater
#
# lesser = get_echoln(a_mln)
# lesser
```

```
# rank needs to be modified
ranking = function(cd) {
  rank = 0
  # sol = as.array(colSums(cd))
  # [1]  1  3  6 10
  for (i in 1:nrow(cd)) {
    if(sum(cd[i,]) > 0 & ncol(cd) == nrow(cd)) {
      rank = rank + 1
    } else if (sum(cd[i,]) > 0 & ncol(cd) > nrow(cd)) {
      rank = rank + 1
      # rank = max(nrow(cd), rank)
    } else if (sum(cd[i,]) > 0 & ncol(cd) < nrow(cd)){
      rank = rank + 1
      # rank = max(ncol(cd), rank)
    }
  }
  return(rank)
}

r1 = ranking(equal)
r1
```

```
## [1] 4
```

```
# r2 = ranking(greater)
# r2
#
# r3 = ranking(lesser)
# r3
```

(2) Given an '$m \times n$' matrix where m > n, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

- If m is greater than n, then the maximum rank of the matrix is **n** *(number of columns)*.
- If m is less than n, then the maximum rank of the matrix is **m** *(number of rows)*.

(3) What is the rank of matrix B?

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

```
B <- matrix(c(1,2,1,
              3,6,3,
              2,4,2), 3, byrow = T)

B
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2
```

```
B_echoln = get_echoln(B)
B_echoln
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    0    0    0
## [3,]    0    0    0
```

```
B_rank = ranking(B_echoln)
B_rank
```

```
## [1] 1
```

Problem set_2

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Steps to solution:-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\lambda I_3 = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$det(A - \lambda I_n) = 0$$

$$det \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(4-\lambda)(6-\lambda) = 0$$

$$Eigenvalues\, of\, A:$$

$$\lambda = 1,\ \lambda = 4,\ \lambda = 6$$

Eigenvectors:

$$\lambda = 1$$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$The\, first\, pivot\, is\, 0.\, x_1 = free.\, Let\, the\, value = 1.$$

$$3x_2 + 5x_3 = 0\, and\, 5x_3 = 0$$

$$x_{\lambda=1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 4$$

$$\begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$Second\, pivot\, is\, 0.\, x_2 = free.\, Let\, the\, value = 1.$$

$$-3x_1 + 2x_2 + 3x_3 = 0\, and\, 2x_3 = 0$$

$$x_3 = 0,\ x_2 = 1\, and\, x_1 = 2/3$$

$$x_{\lambda=4} = \begin{bmatrix} 2/3 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = 6$$

$$\begin{bmatrix} -5 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$Third\, pivot\, is\, 0.\, x_3 = free.\, Let\, the\, value = 1.$$

$$-5x_1 + 2x_2 + 3x_3 = 0 and, -2x_1 + 5x_3 = 0$$

$$x_3 = 1,\ x_2 = 5/2,\ and\, x_1 = 8/5$$

$$x_{\lambda=6} = \begin{bmatrix} 8/5 \\ 5/2 \\ 1 \end{bmatrix}$$

Confirm with buit-in function in r

```
A <- matrix(data = c(1,0,0,
                     2,4,0,
                     3,5,6), nrow = 3, ncol = 3, byrow = FALSE)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    0    4    5
## [3,]    0    0    6
```

```
eign_A <- (eigen(A))$values
eign_A
```

```
## [1] 6 4 1
```