HW2

Doc

1. It is trivial to show that if A is not symmetric, then A^TA $\neq$ AA^T as the dimensions differ by the rules of matrix multiplication (e.g., 3 x 4 x 4 x 3 is a 3 x 3 vs. 4 x 3 x 3 x 4 which is a 4 x 4). If symmetric, it is also trivial to show that by the rules of matrix multiplication that for all i $\neq$ j that elements $a_{ji} = a_{ij}$ for A^TA $==$ AA^T. Let C be the product of the two matrices. Then set up equalities for the two proposed matrix products.

2. You provided some great approaches for deriving LU factorization, so I thought I would provide a unique method-nonlinear optimization. Specifically, the below approach minimizes the L2 norm between the original matrix and the variables used to construct the upper / lower matrix. I demonstrate on a 3x3 and 4x4. Note: simply build the matrix from the results of the minimization. There are some advantages and disadvantages to this method, but I thought you might want to see it.

```r
f=function(x){
a=matrix(c(3,4,5,1,4,6,3,7,11), byrow=FALSE, nrow=3)
(x[4]-a[1,1])^2+
(x[5]-a[1,2])^2+
(x[6]-a[1,3])^2+
(x[1]*x[4]-a[2,1])^2 +
(x[1]*x[5]+x[7]-a[2,2])^2+
(x[1]*x[6]+x[8]-a[2,3])^2+
(x[2]*x[4]-a[3,1])^2 +
(x[2]*x[5]+x[3]*x[7]-a[3,2])^2+
(x[2]*x[6]+x[3]*x[8]+x[9]-a[3,3])^2
}
mynlm = nlm(f, p=c(rep(1,9)), gradtol=1e-30, iterlim=1000)

f=function(x){
a=matrix(c(3,4,5,1,4,6,3,7,11,4,4,3,4,14,11,3), byrow=FALSE, nrow=4)
(x[7]-a[1,1])^2+
(x[8]-a[1,2])^2+
(x[9]-a[1,3])^2+
(x[10]-a[1,4])^2 +
(x[1]*x[7]-a[2,1])^2 +
(x[1]*x[8]+x[11]-a[2,2])^2+
(x[1]*x[9]+x[12]-a[2,3])^2+
(x[1]*x[10]+x[13]-a[2,4])^2+
(x[2]*x[7]-a[3,1])^2+
(x[2]*x[8]+x[3]*x[11]-a[3,2])^2+
(x[2]*x[9]+x[3]*x[12]+x[14]-a[3,3])^2+
(x[2]*x[10]+x[3]*x[13]+x[15]-a[3,4])^2+
(x[4]*x[7]-a[4,1])^2+
(x[4]*x[8]+x[5]*x[11]-a[4,2])^2+
(x[4]*x[9]+x[5]*x[12]+x[6]*x[14]-a[4,3])^2+
(x[4]*x[10]+x[5]*x[13]+x[6]*x[15]+x[16]-a[4,4])^2
}

mynlm = nlm(f, p=c(rep(1,16)), gradtol=1e-30, iterlim=1000)
```