SElshahawy_Assig9

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Question_11, page 363, Introduction to probability

The price of one share of stock in the Pilsdorff Beer Company (see Exer- cise 8.2.12) is given by Yn on the nth day of the year. Finn observes that the differences Xn = Yn+1 − Yn appear to be independent random variables with a common distribution having mean = 0 and variance 2 = 1/4. If Y1 = 100, estimate the probability that Y365 is (a) ≥ 100 (b) ≥ 110 (c) ≥ 120

Answer:-

The summation of random variables will be as the following:

$$S_n = \sum_{x=1}^{x=n} X_1 + X_2 + X_3 + ... + X_n$$

Now substitute X by $Y_{n+1} - Y_n$

$$S_n = \sum (Y_2 - Y_1) + (Y_3 - Y_2) + ... + (Y_{n+1} - Y_n)$$

Then we end up with:

$$S_n = Y_{n+1} - Y_n$$

$$S_n = Y_{n+1} - 100 \quad (where \quad Y_n = 100)$$

Now we have to get the mean and standard deviation

$$varS_n = \sigma^2 S_n = \frac{1}{4} \times \sum_{X=0}^{n} \sigma_X^2 = \frac{n}{4}$$

$$SDS_n = \sqrt{\frac{n}{4}} = \frac{\sqrt{n}}{2}$$

Consider n = 364 where $n + 1 = 365$

$$S_{364} = Y_{365} - 100 Y_{365} = S_{364} + 100$$

calculate variance for n = 364

```
n = 364
var_s_364 = n/4
var_s_364
```

```
## [1] 91
```

```r
sd_s_364 = sqrt(n)/2
sd_s_364
```

```
## [1] 9.539392
```

(a) ≥ 100

```r
z = (100-100) / sd_s_364
pnorm(z, lower.tail = FALSE)
```

```
## [1] 0.5
```

(b) ≥ 110

```r
z = (110-100) / sd_s_364
pnorm(z, lower.tail = FALSE)
```

```
## [1] 0.1472537
```

(c) ≥ 120

```r
z = (120-100) / sd_s_364
pnorm(z, lower.tail = FALSE)
```

```
## [1] 0.01801584
```

Question_2

Calculate the expected value and variance of the binomial distribution using the moment generating function.

Answer:-

I will start with the definition of $M_x(t) \quad = \quad E(e^{tx})$ for a descrete variables:

$$M_x(t) \quad = \quad \sum_{x=0}^{n} e^{tx} \cdot \quad p(x) \quad \longrightarrow \quad 1$$

from the binomial theory:

$$p(x) \quad = \quad \binom{n}{x} \cdot \quad p^x \cdot \quad (1-p)^{n-x} \quad \rightarrow \quad 2$$

by substitute equation 2 into equation 1 we get:

$$M_x(t) \quad = \quad \sum_{x=0}^{n} e^{tx} \cdot \quad \binom{n}{x} \cdot \quad p^x \cdot \quad (1-p)^{n-x}$$

take e and p the same brackets to the power of x we get:

$$M_x(t) \quad = \quad \sum_{x=0}^{n} \binom{n}{x} \cdot \quad (e^t \quad p)^x \cdot \quad (1-p)^{n-x} \quad \rightarrow \quad 3$$

and we know from the Binomial theorm that

$$\sum_{x=0}^{n} \binom{n}{x} \quad y^x \qquad z^{n-x} \quad = \quad (y \quad + \quad z)^n \quad for \quad any \quad x, y, z \quad \epsilon \quad R \qquad \rightarrow \quad 4$$

Then we can use the Binomial theorm to get the MGF from equation 3 then we get:

$$M_x(t) \quad = \quad ((e^t \quad p) \quad + \quad (1-p))^n \quad for \quad t \quad \epsilon \quad R \qquad \rightarrow \quad done(the \quad expected \quad value)$$

$$var(x) = secondmoment - first \quad moment^2 \quad at \quad t \quad = \quad 0$$

$$M_x^{'}(t) \quad = \quad n((e^t \quad p) \quad + \quad (1-p))^{n-1} \cdot \quad e^t \quad p \quad \rightarrow \quad 1st \quad moment$$

$$M_x^{'}(0) \quad = \quad np \quad \rightarrow \quad M_x^{'}(0)^2 \quad = \quad n^2p^2 \quad \rightarrow \quad 5$$

$$M_x^{''}(t) \quad = \quad n \cdot (n-1) \cdot ((e^t \quad p) \quad + \quad (1-p))^{n-2} \cdot \quad e^{2t} \quad p^2 \quad + \quad ((e^t \quad p) \quad + \quad (1-p))^{n-1} \quad \cdot \quad n \quad \cdot \quad e^t \quad p \quad \rightarrow \quad 2nd$$

$$M_x^{''}(0) \quad = \quad n \cdot (n-1) \quad p^2 \quad + \quad np \quad \rightarrow \quad 6$$

by subtracting equation 5 from 6 we get variance

$$var(x) \quad = \quad n \cdot (n-1) \quad p^2 \quad + \quad np \quad - \quad n^2p^2$$

$$var(x) \quad = \quad np(1-p) \quad \rightarrow \quad done \quad (the \quad variance)$$

Question_3

Calculate the expected value and variance of the exponential distribution using the moment generating function.

Answer:-

for the exponential distribution:

$$f(x) = \lambda e^{-\lambda x}$$

The MGF should be:

$$M_x(t) = \frac{\lambda}{\lambda - t} \quad where \quad t < \lambda$$

Getting the first moment:

$$M_x^{''}(t) = \frac{\lambda}{(\lambda - t)^2}$$

The second moment:

$$M_x^{''}(t) = \frac{2\lambda}{(\lambda - t)^3}$$

$$E(x) = M_x(t) \quad at \quad t = 0$$

$$E(x) = \frac{1}{\lambda}$$

$$var(x) = M_x^{''}(t) - M_x^{'}(t)^2 \quad at \quad t = 0$$

$$var(x) = \frac{1}{\lambda^2}$$