First we show that the Hamiltonian is constant.

Given: $H(q,p) = \frac{pM^{-1}p}{2} + U(q)$ $\quad$ $f(q) = -\nabla U(q)$

$Mq' = p$

$p' = f(q)$

$$\frac{d(H(q,p))}{dt} = \frac{d}{dt}\left(\frac{p^2}{2M} + U(q)\right)$$

$$= \frac{2p}{2M}\frac{dp}{dt} + \frac{dU(q)}{dq}\cdot\frac{dq}{dt}$$

$$= \frac{p}{M}p' - f(q)\,q' = \frac{p}{M}f(q) - f(q)\times\frac{p}{M} = 0$$

$\Rightarrow$ H is constant (for scalars)

Observations: $\quad 0 \leq t \leq 2000$

RKDP45: For $n=1000$ ($h=2$), the max error in $H(t)-H(0)$ is approx 2×10^{-3}. If we calculate the slope (assume it to be straight line) we have

slope $\approx \frac{2\times10^{-3}}{2\times10^{3}} = 10^{-6}$ which is very small.

For $n=2000$ ($h=1$), the max error in $H(t)-H(0)$ is approx 3×10^{-5}. If we calculate the slope (assume it to be straight line) we have

slope $\approx \frac{3\times10^{-5}}{2\times10^{3}} = 1.5\times10^{-8}$ << slope for $n=1000$

so as h increases (range of t remains same) the slope is decreasing by a lot. So the rate of change of $H(t)-H(0)$ keeps decreasing.

So the value tends to 0 as n increases (h decreases). We know that 0 is the actual solution of $H(t)-H(0)$.