

Note: The actual value of  $H(p, q)$  is constant  $\forall t \geq 0$ .

So  $H(t) - H(0) = 0$  (Actual value)

Also the actual values of  $p, q$  (found using ODE 45) show a symmetric behaviour

Observations:

Forward Euler: For  $h = 2.3684$  and  $h = 2.3685$  the

graph  $H(t) - H(t_0)$  vs  $t$  is almost same. So no

abnormal behaviour there. The graph for  $h = 2$  is

more accurate (closer to 0) with max value

being around 0.5. After that error almost remains

same. But the Hamiltonian in this case has a lot

of error and there is no symmetry in the error

(Actual values of  $p, q$  are found to be symmetric).

This method is not very good for the given equations

Leapfrog: For  $h = 2$  the value  $H(t) - H(t_0)$  is very

close to 0. It is also showing a symmetric

behaviour (which should be there). The

error is of the order  $10^{-5}$ . Even for  $h = 2.3684$

and  $h = 2.3685$  the symmetric behaviour is shown.

But in case of  $h = 2.3684$  there is a sudden spike

in the value (goes to approx  $2 \times 10^{-3}$ ). This

sudden increase in the error is because of

resonance instability. For  $h = 2.3685$  there

is no drastic instability. This is a good method

for the given equations.

Symplectic Euler : It's behaviour is very similar to

leapfrog. It is also showing resonance instability.

But in this case even for  $h = 2.3685$  (along with  $h = 2.3684$ )  
we observe resonance instability (unlike leapfrog).

This is a good method for the given equations.