

Since we don't know the actual values of p or q , so we use the values of $H(t)$. Error in this case is given by $H_{\text{approx}}(t) - H_{\text{actual}}(t)$. Now $H_{\text{actual}}(t) = \text{constant} = H(t_0)$ so we have error $= H(t) - H(t_0) = H(t) - H(0)$. So our error f^n is $H(t) - H(0)$.

RKDP_45: For $n=250$ ($h=8$) the error f^n is of order 10^{38} which is a very large value. So we can convincingly say that for $h=8$ the solution does not converge to the actual solution. Moreover this h does not lie in the absolute stability region.

As the value of n increases, the solution starts to converge. For $n=500$ ($h=4$), there are drastic changes in error though max error is very small (order of 10^{-3}) which is very less than for what it was for $n=250$. So now the solution starts converging.

For $n=1000$ ($h=2$), we get an approximately straight line whose slope is of the order 10^{-6} (calculated in the previous part). So now rate of change of error is very small and no sudden change is observed. Max error is of order 10^{-3} .

For $n=2000$ ($h=1$), we again do not get any drastic changes and rate of change of errors (assume approx straight line) is of the order 10^{-8} . So now rate of change of error is even smaller. So we can observe that error keeps getting smaller. Max error is of order 10^{-5} .

For $n=4000$ ($h=0.5$) the error is even smaller (10^{-6}).
So we can say that we enter the absolute stability region
for $n=500$ ($h=4$) but not for $n=250$ ($h=8$)
and the solution does not converge for $n=250$ ($h=8$)
and later starts converging as n increases.

Classical RK 4 Method :

For $n=250$ ($h=8$). Max error is 0.25 which is not
as large as in RKDP-45 but still the solution is not converging
in this case too. Moreover we can say that this h is outside the
absolute stability region.

For $n=500$ ($h=4$). Max error is of 10^{-4} (-5×10^{-4}).
This is much better than $n=250$ ($h=8$). and now we can
see that the solution is very close to the actual solution.
drastic change is shown initially but it stabilises later.

For $n=1000$ ($h=2$). Max error is of order 10^{-4} (-5×10^{-4}).
The drastic change shown near 0 is lesser in this case.

For $n=2000$ ($h=1$). Max error is of order 10^{-4} (-1×10^{-6})
But the change in error is very linear (no sharp changes)

Approx slope of this line $\approx 10^{-7}$.

For $n=4000$ ($h=0.5$). Max error is of order 10^{-5} (-7×10^{-6})

The rate of change in error is again approx linear.

But now the rate is even smaller (approx $\approx 10^{-8}$).

So we see that as the value of n increases the solution
gets closer to 0 (i.e converges to the actual solution).

But for $n = 250$ ($h = 8$), the values lie outside of absolute stability region and values do not converge to actual solⁿ.

But for $n = 250$ ($h = 8$), the solution given by Classical RK4 method is closer to the actual solution as compared to RKDP_45 (for $n = 250$ ($h = 8$)).