19K-1043

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LA Activity

Q1. c). $9x_1^2 - x_2^2 + 4x_3^2 + 6x_1x_2 - 8x_1x_3 + x_2x_3$

Expressing in quadratic form of eq

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 9 & 3 & -4 \\ 3 & -1 & 1/2 \\ -4 & 1/2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q2). b). $-7x_1x_2$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 0 & -7/2 \\ -7/2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

c). $x_1^2 + x_2^2 - 3x_3^2 - 5x_1x_2 + 9x_1x_3$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 & -5/2 & 9/2 \\ -5/2 & 1 & 0 \\ 9/2 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q4). find eq of matrix

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} -2 & 7/2 & 1 \\ 7/2 & 0 & 6 \\ 1 & 6 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$-2x_1^2 + 3x_3^2 + 7x_1x_2 + 2x_1x_3 + 12x_2x_3$

Q6). Find Orthogonal Change of Variable of Q & express Q in new variable

$$Q = 5x_1^2 + 2x_2^2 + 4x_3^2 + 4x_1x_2$$

In x^TAx form:-

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 5 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

For EignVectors : $\det(\lambda I - A) = 0$

$$\det\left\{\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 5 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\} = \begin{bmatrix} \lambda-5 & -2 & 0 \\ -2 & \lambda-2 & 0 \\ 0 & 0 & \lambda-4 \end{bmatrix}$$

$\det(\lambda I - A) = 0$

$\Rightarrow (\lambda-4)\{(\lambda-5)(\lambda-2) - 4\} = 0 \quad — (A)$

$(\lambda-4)(\lambda^2 - 5\lambda - 2\lambda + 10) - 4\lambda + 16 = 0$

$(\lambda-4)(\lambda^2 - 7\lambda + 10) - 4\lambda + 16 = 0$

$\lambda^3 - 7\lambda^2 + 10\lambda - 4\lambda^2 + 28\lambda - 40 - 4\lambda + 16 = 0$

$\lambda^3 - 11\lambda^2 + 34\lambda - 24 = 0$

For Eign Vector Roots: ~~Put λ = 4~~ By eq (A)

$(\lambda-4)(\lambda^2 - 7\lambda + 10 - 4) = 0$

$(\lambda-4)(\lambda^2 - 7\lambda + 6) = 0$

Either: $\boxed{\lambda_1 = 4}$, $\lambda^2 - 7\lambda + 6 = 0$

$\lambda^2 - 6\lambda - \lambda + 6 = 0$

$\lambda(\lambda - 6) - 1(\lambda - 6) = 0$

$\boxed{\lambda_2 = 1}$, $\boxed{\lambda_3 = 6}$

For $\lambda = 1$:-

$$\left[\begin{array}{ccc|c} -4 & -2 & 0 & 0 \\ -2 & -1 & 0 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} -4 & -2 & 0 & 0 \\ -2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ $x-1$; x_2

$x_1 + 1/2\, x_2 = 0$, $x_3 = 0$

$x_1 = -1/2\, x_2$

Parametric: $x_2 = t$, $x_1 = -1/2\, t$, $x_3 = 0$

$$\therefore P_2 = \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

For $\lambda = 4$:-

$$\left[\begin{array}{ccc|c} -1 & -2 & 0 & 0 \\ -2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore x_1 = 0 \; , \; x_2 = 0$

Parametric: $x_1 = 0, x_2 = 0, x_3 = t$

$$P_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

For $\lambda = 6$:-

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ -2 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x_1 = 2x_2 \; , x_3 = 0$

Parametric:-

$x_2 = t \quad x_1 = 2t \quad x_3 = 0$

$$\therefore \quad P_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Now Apply Gram Schmidt Process:-

$$V_1 = P_2 = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

$$V_2 = X_2 - \frac{(x_2, V_1)}{\|V_1\|^2} V_1$$

$$(x_2, V_1) = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = 0$$

$$\therefore \quad V_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$V_3 = x_3 - \frac{(x_3; V_1)\,V_1}{\|V_1\|^2} - \frac{(x_3, V_2)\,V_2}{\|V_2\|^2}$$

$$(x_3, V_1) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = 0$$

$$(x_3, V_2) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$V_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Orthonormalization:

$$V_1 = \frac{V_1}{\|V_1\|} = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix}$$

$$V_2 = \frac{V_2}{\|V_2\|} = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix}$$

$$V_3 = \frac{V_3}{\|V_3\|} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore P = \begin{bmatrix} 1/\sqrt{5} & 0 & 2/\sqrt{5} \\ -2/\sqrt{5} & 0 & 1/\sqrt{5} \\ 0 & 1 & 0 \end{bmatrix}$ $P^T = \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 0 & 1 \\ 2/\sqrt{5} & 1/\sqrt{5} & 0 \end{bmatrix}$

$D = PAP^T$

$= \begin{bmatrix} 1/\sqrt{5} & 0 & 2/\sqrt{5} \\ -2/\sqrt{5} & 0 & 1/\sqrt{5} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 8 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 0 & 1 \\ 2/\sqrt{5} & 1/\sqrt{5} & 0 \end{bmatrix}$

$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$\therefore Q = y^T (P^T A P) y = y^T D y$

$\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$\therefore \quad y_1^2 + 4y_2^2 + 6y_3^2.$

Q8. $Q = 2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_2x_3$

$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\det(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix} = \begin{bmatrix} \lambda-2 & -2 & +2 \\ -2 & \lambda-5 & 4 \\ 2 & 4 & \lambda-5 \end{bmatrix}$$

$= (\lambda-2)(\lambda^2-10\lambda+9)-8\lambda+8=0$

$= \lambda^3-12\lambda^2+21\lambda-10=0$

For Eigen Values:- Put $\lambda=1$

$(1)^3-12(1)+21(1)-10 = 0=0$

$\therefore \boxed{\lambda=1}$

$$\begin{array}{r|l} & \lambda^2-11\lambda+10 \\ \hline \lambda-1 & \cancel{\lambda^3}-12\lambda^2+21\lambda-10 \\ & \cancel{\lambda^3} \mp \lambda^2 \\ \hline & \cancel{-11\lambda^2}+21\lambda-10 \\ & \cancel{\mp 11\lambda^2} \pm 11\lambda \\ \hline & 10\lambda \cancel{-10} \\ & \cancel{10\lambda} - \cancel{10} \\ \hline & 0 \end{array}$$

$\lambda^2-10\lambda-\lambda+10=0$

$\boxed{\lambda=1}$, $\boxed{\lambda=10}$

For $\lambda=1$:

$$\left[\begin{array}{ccc|c} -1 & -2 & 2 & 0 \\ -2 & -4 & 4 & 0 \\ 2 & 4 & -4 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$P_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ $P_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

For $\lambda = 10$:
$$\left[\begin{array}{ccc|c} 8 & -2 & 2 & 0 \\ -2 & 5 & 4 & 0 \\ 2 & 4 & 5 & 0 \end{array}\right] = \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} -1/2 \\ -1 \\ 0 \end{bmatrix}$$

Using Gram Schmidt Process

$$V_1 = P_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$V_2 = P_2 - \frac{(P_2, V_1)\, V_1}{\|V_1\|^2}$$

$$(P_2, V_1) = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = -4$$

$$\|V_1\| = \sqrt{5}$$

$$V_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + \frac{4}{5} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2/5 \\ 4/5 \\ 1 \end{bmatrix}$$

Orthonormality :-

$$V_1 = \frac{V_1}{\|V_1\|} = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix}, \quad V_2 = \frac{V_2}{\|V_2\|} = \begin{bmatrix} -1/3 \\ -2/3 \\ 2/3 \end{bmatrix}, \quad V_3 = \begin{bmatrix} 2/\sqrt{45} \\ 4/\sqrt{45} \\ 5/\sqrt{45} \end{bmatrix}$$

$$P = \begin{bmatrix} -2/\sqrt{5} & 2/\sqrt{45} & -1/3 \\ 1/\sqrt{5} & 4/\sqrt{45} & -2/3 \\ 0 & 3/\sqrt{45} & 2/3 \end{bmatrix}$$

$$D = P^T A P$$

$$= \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 2/\sqrt{45} & 4/\sqrt{45} & 3/\sqrt{45} \\ -1/3 & -2/\cancel{\sqrt{45}}\,3 & 2/3 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} & 2/\sqrt{45} & -1/3 \\ 1/\sqrt{5} & 4/\sqrt{45} & -2/3 \\ 0 & 3/\sqrt{45} & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\therefore Q = y^T D y$$

$$\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\therefore y_1^2 + y_2^2 + 10y_3^2 .$$