Q. Show that A and B are Similar

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 3 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

For Similarity Transformation : $B = P^{-1}AP$

$\therefore$ det B = det A

$\det |A| = 1(0) + 1(0) = 0$

$\det |B| = 1(2) + (-1)(2) = 2 - 2 = 0.$

$\therefore$ A and B are not similar. $\because$ det A & det B is singular $\therefore$ P is not invertible

Q. Find matrix P that diagonalizes A

$$A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$$

As we know that : $\det(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} = \begin{bmatrix} \lambda - 1 & 0 \\ -6 & \lambda + 1 \end{bmatrix}$$

$\det |A| = (\lambda - 1)(\lambda + 1) + 0$

$= \lambda^2 \cancel{+\lambda} \cancel{-\lambda} - 1 = (\lambda^2 - 1) = (\lambda - 1)(\lambda + 1) = 0$

For $\lambda = 1$:

$$\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using Gauss Jordan Elimination

$$\left[\begin{array}{cc|c} 1 & -2/6 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 - 2/6\, x_2 = 0$

$x_1 = 2/6\, x_2$

Parametric Eq: $x_2 = t$, $x_1 = 2/6\, t$

$\therefore P_1 = \begin{bmatrix} 2/6 \\ 1 \end{bmatrix}$

For $\lambda = -1$:

$$\begin{bmatrix} -2 & 0 \\ -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using Gauss Jordan Elimination

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ -6 & 0 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = 0 \qquad x_2 = 0$

Using Parametric:

~~Put~~ $= 0 \qquad x_2 = t$

$\therefore P_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\therefore P = \begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix}$

$|P| = (1/3)$

$\operatorname{Adj} P = \begin{bmatrix} 1 & 0 \\ -1 & 1/3 \end{bmatrix}$

$P^{-1} = \begin{bmatrix} 1/3 & 0 \\ -1/3 & 1 \end{bmatrix}$

$\therefore P^{-1} A P = \begin{bmatrix} 1/3 & 0 \\ -1/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1/3 & -1 \\ -17/3 & -1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -8/9 \end{bmatrix}$

$$P^{-1} = \frac{\begin{bmatrix} 1 & 0 \\ -1 & 1/3 \end{bmatrix}}{1/3} = \begin{bmatrix} 3 & 0 \\ -3 & 1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 3 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Proved!