"Key Points"

Chap#1

* Ex 1.1

- Types of Sol:-

(1) One Sol. → $x=y$

(2) No Solution → $0=x$

(3) Infinite Sol → Inconsistent; $0=0$

- Augmented Sol :-

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

* Ex 1.2

- Row Echelon (Gaussian Elimination)
- Reduced Row Echelon (Gauss-Jordan-Elimination)
- Trival Sol ($x=y=z==0$); Non-Trival ($x=y=z\neq 0$)
- If RREF have any row which is completely zero then it inconsistent and vice versa.

* Ex 1.4

$$x=\frac{du-bv}{ad-bc} \; ; \; y=\frac{av-cu}{ad-bc} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}$$

* Ex 1.5

- Elementary Matrix → which convert into Identity by applying only 1 operation
- Inverse Algorithm → To find inverse of matrix } convert given matrix into RREF and apply all operation on Identity

* Ex 1.6

- Find inverse by Inverse Algorithm then * by b To find x, y, z in matrix
- If more then one b then double augment the matrix

$$\left[\begin{array}{cc|c|c} a_1 & a_2 & b_1 & b_1 \\ a_3 & a_4 & b_2 & b_2 \end{array}\right]$$

* Ex 1.7

- Upper Triangle → entries below ~~upper~~ main diagonal are 0
- lower Triangle → entries above main diagonal are 0
- Invertible → main diagonal contain all non-zero elements

Chap#2

2.1

- Determinant by RREF → convert A into RREF → All rules of Inter
- $\det(A) = \det(A^T)$

2.3

- $\det(kA) = k^n \det(A)$ ∴ order of matrix $= n$
- $\det = 0$, Invertible vice versa

Chap#4

4.3

- Scalar Multiple → linear Dependent
- linear Combination → $k_1u_1 + k_2u_2 + \cdots$
- Trival → linearly Independant | Non-Trival → Dependant
- $\det = 0$, lie on same plain | $\det \neq 0$, lie not on same plain
- $\det = 0$, linearly Dependent / $\det \neq 0$, linearly Independent
- functions → wronskian

4.4

- Spaning → No. of comp. of vectors = Dimension
  → highest power = Dimension → Polynomial
- linearly Independent & spanning → Basis
- Cordinate Vectors → $(k_1, k_2, \ldots)$
- Basis for $\mathbb{R}^3 \rightarrow (1,0,0), (0,1,0), (0,0,1)$

4.5

- No. of Basis = Dimension

4.7

- RREF → Any row having all zeros → inconsistent
- $AX=0$ → Non-homo ; $AX=b$ → homo
- Entirely → Transpose → Then RREF
- row space → RREF
- column space (basis) → RREF → mark col having 1 → Take original matrix-A

4.8

- Rank + Nullity = No. of col.
- Rank → No. of non-zeros rows
- Nullity → No. of zero rows
- No. of leading variables → Rank
- No. of parameters → Nullity
- Rank = $\min(m * n)$
- Dimension Space = Nullity

Chap #5

5.1

- $Ax = \lambda x$ → λ = eigen value , x = eigen vector
- $\det(\lambda I - A) = 0$

5.2

- $\det A = \det B$ → Similar Matrice
- Main Diagonal → λ → Upper, lower Triangular Diagonal matrices
- No. of Bases (r, s, t) = Size of matrix → diagonalizable
- AM → No. of λ ; GM → Basis produce by each λ
- $GM \leq AM$
- $\lambda I = P^{-1}AP$
- $A^{100} = P^{-1} D^{100} P$ $\therefore D = \lambda I$ [λ obtained by solving $\det(\lambda I - A) = 0$]

1.8

$\mathbb{R}^n \rightarrow \mathbb{R}^m$ $(4 * 5)$ $\mathbb{R}^5 \rightarrow \mathbb{R}^4$

Chap #6

6.1

All operations to be applied on main eq. given in q.s

- $\langle u, v \rangle = u \cdot v = u_1 v_1 + u_2 v_2 + \ldots$
- $\| u \| = \sqrt{u \cdot u}$
- $d(u, v) = \| u - v \| \Rightarrow \sqrt{u - v \cdot u - v}$
- $\begin{bmatrix} \sqrt{w_1} & 0 \\ 0 & \sqrt{w_2} \end{bmatrix}$ → Find matrix q.s → $2u_1v_1 + 3u_2v_2 \Rightarrow w_1 = 2, w_2 = 3$
- $\langle p, q \rangle = a_0 b_0 + a_1 b_1 + \ldots$
- $\| p \| \sqrt{a_0^2 + a_1^2 + \ldots}$

- $\text{Trace}(u, v) = u_1v_1 + u_2v_2 + \cdots$
- $\langle p, q \rangle = p(x_0)q(x_0) + p(x_1)q(x_1) + \cdots$
- $d(p, q) = \sqrt{(p-q)x_0^2 + (p-q)x_1^2 + \cdots}$

Ex #6.2

- $\theta = \cos^{-1}\left(\frac{\langle u, v \rangle}{\|u\| \cdot \|v\|}\right)$
- $\langle u, v \rangle = 0 \rightarrow$ orthogonal
- $\|u\| = 1 \rightarrow$ orthonormal

Ex #6.3

- $u = \frac{v}{\|v\|}$
- linear Combination $\Rightarrow \frac{\langle u, v_1 \rangle}{\|v_1\|^2}\|v_1\| + \frac{\langle u, v_2 \rangle}{\|v_2\|^2}\|v_2\| + \cdots$
- $\text{Proj}_v u = \frac{\langle u, v \rangle}{\|v\|} v$
- $\text{Proj}_{v\perp} u = u - \text{Proj}_v u$
- $v_1 = u_1$; $v_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{\|v_1\|^2} v_1$; $v_3 = u_3 - \frac{\langle u_3, v_1 \rangle}{\|v_1\|^2} v_1 - \frac{\langle u_3, v_2 \rangle}{\|v_2\|} v_2$

$\downarrow$

Gram Schmid Process

Chap#7

7.1

$A^{-1} = A^T \rightarrow$ If

$AA^T = A^TA = I \rightarrow$ Orthogonal

EX #7.2

- $P^T A P = D(\lambda I)$
- Garam Schmid Process
- For P :- $u_1 = \dfrac{x_1}{\|x_1\|}$, $u_2 = \dfrac{x_2}{\|x_2\|}$, $u_3 = \dfrac{x_3}{\|x_3\|}$

  $P = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$

  Then $P^{-1} A P$
- Orthogonically Diagonalized
  - → Basis for each eigenspace of A
  - → Apply Garam Schmid Process
  - → Form matrix $= P$
  - → $P^T A P$

EX #7.3

- $a_1 x_1^2 + a_2 x_2^2 + 2a_3 x_1 x_2 \rightarrow \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a_1 & a_3 \\ a_3 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
- $a_1 x_1^2 + a_2 x_2^2 + a_3 x_3^2 + 2a_4 x_1 x_2 + 2a_5 x_1 x_3 + 2a_6 x_2 x_3$

  ↳ $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} a_1 & a_4 & a_5 \\ a_4 & a_2 & a_6 \\ a_5 & a_6 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

  ↳ Consider A

  ↳ Apply ~~chp#5~~ eigen space and vector