1916-1043

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Q. transform basis $\{u_1, u_2, u_3, u_4\}$ into orthonormal basis using gram schmidt

$u_1 = (0, 2, 1, 0)$ $u_2 = (1, -1, 0, 0)$ $u_3 = (1, 2, 0, -1)$

$u_4 = (1, 0, 0, 1)$

Using Gram Schmidt Process

① $v_1 = u_1 = (0, 2, 1, 0)$

② ~~$v_2 = u_2 - \frac{(u_2, v_1)}{\|v_1\|^2} v_1 = u_2 - \frac{(0,2,1,0)(0,2,1,0)}{\|v_1\|^2} v_1$~~

~~$\|v_1\|^2 = \sqrt{0^2+2^2+1^2+0^2} = \sqrt{5} = 5$~~

~~$v_2 = u_2 - \frac{5(0,2,1,0)}{5} = (0,2,1,0) - (0,2,1,0)$~~

~~$= (0, 0, 0, 0)$~~

② $v_2 = u_2 - \frac{(u_2, v_1)}{\|v_1\|^2} v_1$

$(u_2, v_1) = (1,-1,0,0)(0,2,1,0) = -2$

$\|v_1\|^2 = 5$

$v_2 = (1,-1,0,0) - \frac{(-2)(0,2,1,0)}{5}$

$= (1,-1,0,0) - \left(0, \frac{-4}{5}, \frac{-2}{5}, 0\right)$

$= \left(1, \frac{-1}{5}, \frac{2}{5}, 0\right)$

③ $V_3 = U_3 - \frac{(U_3, V_1)V_1}{||V_1||^2} - \frac{(U_3, V_2)V_2}{||V_2||^2}$

$(U_3, V_1) = (1,2,0,-1)(0,2,1,0) = (0+4+0+0) = 4$

$(U_3, V_2) = (1,2,0,-1)\left(1, \frac{-1}{5}, \frac{2}{5}, 0\right) = 1 - \frac{2}{5} + 0 + 0 = \frac{3}{5}$

$||V_2||^2 = \sqrt{1^2 + 1/5^2 + 2/5^2 + 0} = \sqrt{1 + \frac{1}{25} + \frac{4}{25}} = \sqrt{\frac{25+5}{25}} = \frac{\sqrt{30}}{5} = \frac{326}{255}$

$V_3 = U_3 - \frac{(4)}{5}(0,2,1,0) - \frac{3/8}{26/8}(1, -1/5, 2/5, 0)$

$= U_3 - (0, 8/5, 4/5, 0) - (1/2, -1/10, 2/10, 0)$

$= (1,2,0,-1) - (0, 8/5, 4/5, 0) - (1/2, -1/10, 2/10, 0)$

$= (1, 2/5, -4/5, -1) - (1/2, -1/10, 2/10, 0)$

$V_3 = (1/2, 1/2, -1, -1)$

$V_4 = U_4 - \frac{(U_4, V_1)V_1}{||V_1||^2} - \frac{(U_4, V_2)V_2}{||V_2||^2} - \frac{(U_4, V_3)V_3}{||V_3||^2}$

$(U_4, V_1) = (1,0,0,1)(0,2,1,0) = 0$

$(U_4, V_2) = (1,0,0,1)(1, -1/5, 2/5, 0) = 1$

$(U_4, V_3) = (1,0,0,1)(1/2, 1/2, -1, -1) = 1/2 - 1 = -1/2$

$||V_3||^2 = \sqrt{(1/2)^2 + (1/2)^2 + 1 + 1} = \sqrt{\frac{1}{4} + \frac{1}{4} + 1 + 1} = \sqrt{5/2} = 5/2$

$$V_4 = u_4 - (0,0,0,0) - \frac{1}{6/5}\left(1, -1/5, 2/5, 0\right) - \frac{(-1/\cancel{2})}{5/\cancel{2}}\left(1/2, 1/2, -1, -1\right)$$

$$= u_4 - (5/6, -1/6, 1/3, 0) - (-1/10, -1/10, 1/5, 1/5)$$

$$= u_4 - (14/15, -1/15, 2/15, -1/15)$$

$$= (1, 0, 0, 1) - (14/15, -1/15, 2/15, -1/15)$$

$$V_4 = (1/15, 1/15, -2/15, 16/15)$$

For orthonormal basis :-

$$w_1 = \frac{V_1}{\|V_1\|} = \frac{(0, 2, 1, 0)}{\sqrt{5}} = \left(0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$$

$$w_2 = \frac{V_2}{\|V_2\|} = \frac{(1, -1/5, 2/5, 0)}{\sqrt{30}/5} = \left(\frac{5}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, 0\right)$$

$$w_3 = \frac{V_3}{\|V_3\|} = \frac{(1/2, 1/2, -1, -1)}{\sqrt{5/2}} = \frac{1/2}{\sqrt{5/2}}, \frac{1/2}{\sqrt{5/2}}, \frac{-1}{\sqrt{5/2}}, \frac{-1}{\sqrt{5/2}}$$

$$w_4 = \frac{V_4}{\|V_4\|} = \frac{(1/15, 1/15, -2/15, 16/15)}{\sqrt{262}/15} = \frac{1}{\sqrt{262}}, \frac{1}{\sqrt{262}}, \frac{-2}{\sqrt{262}}, \frac{16}{\sqrt{262}}$$

Q2. R^3 be Euclidean Inner Product. Find orthonormal basis for subspace spanned by $(0,1,2), (-1,0,1)$ $(-1,1,3)$

$V_1 = (0,1,2) \quad V_2 = (-1,0,1) \quad V_3 = (-1,1,3)$

For Orthogonal basis:

$(V_1 \cdot V_2) = (0,1,2)(-1,0,1) = 0+0+2 = 2$

$(V_1 \cdot V_3) = (0,1,2)(-1,1,3) = 0+1+6 = 7$

$(V_2 \cdot V_3) = (-1,0,1)(-1,1,3) = 1+0+3 = 4$

$\|V_1\| = \sqrt{0+1+4} = \sqrt{5} \quad \|V_2\| = \sqrt{1+0+1} = \sqrt{2}$

$\|V_3\| = \sqrt{1+1+9} = \sqrt{11}$

$w_1 = \dfrac{V_1}{\|V_1\|} = \dfrac{0}{\sqrt{5}}, \dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}} = \left(0, \dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}}\right)$

$w_2 = \dfrac{V_2}{\|V_2\|} = \dfrac{-1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}$

$w_3 = \dfrac{V_3}{\|V_3\|} = \dfrac{-1}{\sqrt{11}}, \dfrac{1}{\sqrt{11}}, \dfrac{3}{\sqrt{11}}$