And, $\sum x_i y_i = \left(\sum x_i\right) \beta_0 + \beta_1 \left(\sum x_i^2\right)$

$$\Rightarrow \sum x_i y_i = \sum x_i \left(\bar{y} - \beta_1 \bar{x}\right) + \beta_1 \left(\sum x_i^2\right)$$

$$\Rightarrow \sum x_i y_i = (n\bar{x}) \left(\bar{y} - \beta_1 \bar{x}\right) + \beta_1 \left(\sum x_i^2\right)$$

[Note: $\beta_0 = \bar{y} - \beta_1 \bar{x}$ and $\sum x_i = n\bar{x}$]

$$\Rightarrow \sum x_i y_i = n\bar{x}\bar{y} + \beta_1 \left(\sum x_i^2 - n(\bar{x})^2\right)$$

$$\Rightarrow \sum x_i y_i - n\bar{x}\bar{y} = \beta_1 \left(\sum x_i^2 - n(\bar{x})^2\right)$$

$$\therefore \hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n(\bar{x})^2}$$

where, $\bar{x} = \dfrac{\sum x_i}{n}$

$\bar{y} = \dfrac{\sum y_i}{n}$