Question # 1: P(n) = 1+2+3+ . . . . . + n = $\frac{\mathbf{n(n+1)}}{\mathbf{2}}$ for all integers n ≥ 1.

Solution:

Basis step: n=1

n = $\frac{n(n+1)}{2}$

1 = $\frac{1(1+1)}{2}$

1 = $\frac{1(2)}{2}$ = $\frac{2}{2}$

1 = 1 Hence P(n) is true for n=1.

Inductive step

n = k

P(k) = 1+2+3+..... + k = $\frac{k(k+1)}{2}$

n = k+1

Since P(k) = 1+2+3+..... + k = $\frac{k(k+1)}{2}$

Add (k+1) both side

1+2+3+..... + k +(k+1) = (k+1) + $\frac{k(k+1)}{2}$

= (k+1) $(1 + \frac{k}{2})$

= (k+1) $(\frac{2+k)}{2})$

= (k+1) $(\frac{k+2}{2})$

Hence p(n) is true for n = k +1.

Rough work

n = $\frac{n(n+1)}{2}$

(k+1) = (k+1) $(\frac{k+1+1}{2})$

(k+1) = (k+1) $(\frac{k+2}{2})$

Question #2: 1+3+5+ . . . . . . (2n - 1) = n^2 for all integers n ≥1.

Solution:

Basis step: **n =1**

(2(1) - 1) = 1^2

(2 - 1) = 1

1 = 1 Hence P(n) is true for n=1.

Inductive step

n = k

1+3+5+ . . . . . . (2k - 1) = k^2

n = k+1

1+3+5+ . . . . . . (2k - 1) = k^2

Add (2k+1) both side

1+3+5+ . . . . . . (2k - 1) + (2k + 1) = k^2+(2k + 1)

= k^2+ 2k + 1

$= (k + 1)^2$

Hence p(n) is true for n = k +1.

Rough work
(2(k + 1) - 1) = $(k + 1)^2$
2k +2-1 = $(k + 1)^2$
2k+1 = $(k + 1)^2$

Question # 3: $2^0 + 2^1 + 2^2 + \ldots\ldots 2^n = 2^{n+1} - 1$ for all nonnegative integers.

Solution:

Basis Step   n = 0

$2^n = 2^{n+1} - 1$

$2^0 = 2^{0+1} - 1$

$1 = 2^1 - 1 = 2 - 1$

1 = 1   Hence P(n) is true for n=1.

Inductive Step

n = k

$2^0 + 2^1 + 2^2 + \ldots\ldots 2^k = 2^{k+1} - 1$

n = k +1

Add 2^{k+1} both side

= $2^{k+1} + 2^{k+1} - 1$

=$2^{k+1}\,(1+1) - 1$

=$2^{k+1}\,(2) - 1$

= $2^{k+1}\,(2^1) - 1$

= $2^{k+1+1} - 1$

= $2^{k+2} - 1$

Hence p(n) is true for n = k +1.

Rough work

$2^n = 2^{n+1} - 1$

$2^{k+1} = 2^{k+1+1} - 1$

$2^{k+1} = 2^{k+2} - 1$