National University of Computer & Emerging Sciences, Karachi
Spring-2020 - Department of Computer Science
Bachelor of Science (Computer Science and Software Engineering)
Final Examination
June 30, 2020, 09:00 am – 12:00 pm

Course Code: CS211	**Course Name: Discrete Structures**
Instructor Names: Mr. Shoaib Raza	
Student Roll No:	**Section No:**

Instructions:

- Read each question completely before answering it. There are **6 questions and 4 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Answer all the questions in given sequence of the question paper. Step by step solution is required.
- Expected solution time is mentioned for each question. Try to follow the timings.
- Make sure that your answers are not plagiarized or copied from any other sources. In case of plagiarism, **"F- Grade"** in the course will be awarded.

Total Time: 3 Hours **Maximum Marks**: 100

Question # 1: Propositional Logic and Rules of Inference (Expected time = 20 minutes) **[4x4=16 points]**

(i) Let ***a, b, c*** and ***d*** be the propositions

a: Ali is enrolled in BS Mathematics. ***b:*** Ali is enrolled in BS Computer Science.

c: Ali studies Discrete Structures. ***d:*** Ali is an intelligent student.

Write these propositions using ***a, b, c*** and ***d*** and logical connectives (including negations):

a) Ali is enrolled in BS Mathematics or Computer Science.
b) If Ali doesn't study Discrete Structures, he is not enrolled in BS Computer Science.
c) If Ali studies Discrete Structures then he is intelligent student.
d) Ali is not enrolled in BS Mathematics.

(ii) Using the premises(statements) from **part(i)**, apply rules of inference to obtain conclusion(s) from these premises. Also, explain the rules of inference used to obtain each conclusion from the premises.

(iii) Using the truth table prove or disprove that the contrapositive of statement ***(b)*** in **part (i)** is equivalent to the converse of its inverse. Also write the statements in words.

(iv) Determine using laws of Logic if the following statement is a tautology, contradiction or a contingency.
$((p \vee q) \wedge (p \rightarrow r)) \rightarrow (q \vee r)$

Question # 2: Predicate and Quantifiers (Expected time = 15 minutes) **[3x4=12 points]**

Let ***P(f)*** be the statement "***f*** has an WhatsApp" and ***Q (f, g)*** be the statement "***f*** and ***g*** have chatted over the WhatsApp," where the domain for the variables ***f*** and ***g*** consists of all students in your Discrete Structure class.
(i) Use quantifiers to express each of these statements.

(a) Not everyone in your discrete structure class has WhatsApp.
(b) No one in the discrete structure class has chatted with Bilal.

(ii) Express the following using logical expression with nested quantifiers:
(a) $\exists f \neg P(f)$
(b) $\exists f \forall g\ Q(f, g)$

(iii) Most of the Historical buildings in the Pakistan are in Karachi and Lahore. Table # 1 lists the 10 Historical buildings in the Pakistan and some data about them.
Consider ***H*** to be the set of all the historical buildings, ***K*** to be the set of historical buildings that are in Karachi, and ***L*** to be the set of historical buildings that are in Lahore. Furthermore, consider ***P*** to be the set of historical buildings that are exactly 1546 Sq. ft. tall. Use the information given to find the truth value (and explain) of each statement below.

(a) $\exists x \in L$ such that $\forall y \in K$, ***x*** has greater area than ***y.***
(b) $\forall x \in K$, $\forall y \in P$, ***x*** has more books written than ***y.***

Building Name	Location	Area (Sq. ft.)	Number of books written	Year Built
Lahore Shahi Fort	Lahore	2276	84	1566
Frere Hall	Karachi	1951	88	1865
Badshahi Mosque	Lahore	1896	68	1671
Mohatta Palace Museum	Karachi	1889	78	1927
Minar-e-Pakistan	Lahore	1750	82	1960
Shalimar Bagh	Lahore	1700	35	1641
Wazir Mansion	Karachi	1636	63	1866
St. Patrick's Cathedral	Karachi	1627	80	1881
Haveli of Nau Nihal Singh	Lahore	1546	57	1830
Tomb of Jahangir	Lahore	1546	32	1637

Table # 1: Data of historical buildings in Karahi and Lahore

Question # 3: Set theory and Functions (Expected time = 25 minutes) **[4x4=16 points]**

Ali is enrolled in CS211-Discrete Structures this semester. Due to the pandemic, he missed the lectures for the topics "Set theory and Functions". He asked his friend to help him in understanding the topic and asked the questions given below:

(i) Suppose that $A = \{b, d, f\}$, $B = \{b, f\}$, $C = \{d, f\}$, and $D = \{d, f, h\}$.
(a) Determine which of these sets are subsets of each other.
(b) Determine the Cardinality of each set.

(ii) Among ***5000*** patients admitted to a hospital, ***2500*** are diagnosed with Pneumonia, ***3000*** with Bronchitis, and ***1000*** with both pneumonia and bronchitis. Using Venn diagram determine:
(a) The number of patients diagnosed with pneumonia or bronchitis (or both).
(b) The number of patients not diagnosed with pneumonia or bronchitis.

(iii) Using Set Identities, prove or disprove the following set operations: $P - (Q \cap R) = (P - Q) \cap (P - R)$.

(iv) Suppose $g: P \rightarrow Q$ and $f: Q \rightarrow R$ where $P = \{1, 2, 3, 4\}$, $Q = \{a, b, c\}$, $R = \{2, 7, 10\}$, and ***f*** and ***g*** are defined by $f = \{(a, 10), (b, 7), (c, 2)\}$ and $g = \{(1, b), (2, a), (3, a), (4, b)\}$.
(a) Is Function ***f*** and ***g*** invertible? If yes find f^{-1} and g^{-1} or if not why?
(b) Find $f \circ g$ and $g \circ f$ if exists.

QUESTION # 4: Number Theory, Combinatorics and Discrete Probability (Expected time = 1 Hr.) [6x4=24 points]

(i) Ali is very fond of playing Cricket. He has collected tennis ball in a Basket. He's not sure exactly how many he has today, so when he

aligns them in rows of ***7*** balls, ***2*** balls are left;
aligns them in rows of ***17*** balls, ***3*** balls are left;
aligns them in rows of ***19*** balls, ***5*** balls are left.

He's positive that there are fewer than ***700*** balls but how many does he have?

Hint: In this problem, you are supposed to use the following Theorems:

a. Chinese Remainder Theorem
b. The Euclidean Algorithm Lemma
c. Bézout's Theorem
d. Linear congruences

(ii) After counting the number of balls Ali had, encrypt the number word using Ceaser Cipher.
Hint: you have to convert the number in word counting. i.e. Five hundred and twenty-one.

(iii) Use Fermat's' theorem, what is the value of $a^{302} \pmod 7$, where ***a*** is the sum of all the digits of the number obtained in **part (i).**

(iv) Ali purchases a book entitled "Discrete Mathematics and Its Applications". Suppose that ISBN-10 of the textbook is ***125973128Q***. How Ali can find the check digit to validate the originality of the book?

(v)
(a) Mr. Shoaib Raza is teaching Discrete structures this semester and is preparing a final exam. He has a pool of ***15*** questions on Graph theory, ***12*** questions on Number theory, ***7*** questions on Propositional logic, **5** questions on trees, ***9*** questions on Relations and ***10*** questions on Set theory. He wishes to put ***3*** questions on each topic on the exam. Determine the number of different ways the above things can happen.

(b) Suppose the grades earned by the students for a Discrete Structures class this semester are ***A, A-, B+, B, B-, C+, C, C-, D*** and ***F,*** and that there are ***97*** students in the class. At least how many students will earn the exact same grade?

(vi)
(a) Ali plays for FAST-NUCES cricket team. FAST-NUCES cricket team squad consists of ***15*** players which includes ***2*** wicket-keepers, ***5*** bowlers, ***3*** all-rounders and ***5*** batsmen. In how many ways, can cricket squad choose a captain, a vice-captain and a wicket-keeper from among themselves?

(b) Ali is also serving as the president of Sportics society at FAST-NUCES. He is purchasing footballs for FUTSAL tournament. The probability of purchasing defective footballs is ***15%***. What is the probability of not purchasing defective footballs?

QUESTION # 5: Relations and Graph Theory (Expected time = 40 minutes) [5x4=20 points]

Consider a scenario from World War II where two Germen soldiers are communicating with each other through a secure channel. Head of Platoon want to communicate two different messages to each of them through separate communication channels. Both of them can received the message but can't reply back. After receiving the message from the head of platoon, these two soldiers communicate the received message to one of their subordinates each. These subordinates can't reply to them but have a one-way communication channel with the Platoon Head so that they can confirm the message delivery to the head of platoon.

(i) Transform the given scenario into a digraph. Show the in-degree and out-degree of each vertex.

(ii) Using diagraph in **part (i)**, determine whether the relation is an equivalence relation OR a partial-order relation? Show all of your steps.

(iii) Determine if the following two graphs ***G*** and ***H*** as shown in figure# 1 & 2 are isomorphic. If they are, give function ***f: V (G) → V (H)*** that define the isomorphism. If they are not, give the reason why?

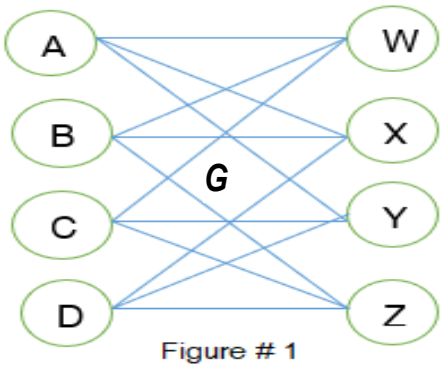

Figure # 1

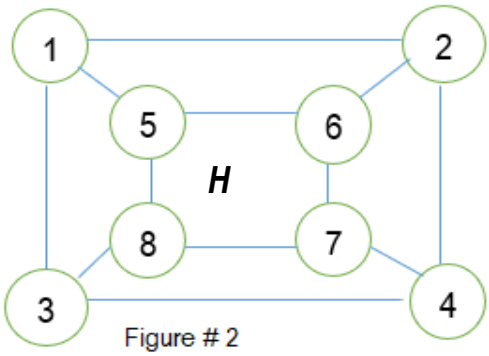

Figure # 2

(iv) Consider the graphs ***G*** and ***H*** from **part (iii)**.
(a) Is Graph ***G*** a complete Bipartite Graph? Explain.
(b) Determine if Euler and Hamilton circuits exist in Graph ***H***. If yes, show the circuit, if not explain why?

(v) Find the minimum spanning tree of the graph shown in figure # 3 where the degree of each vertex in the spanning tree does not exceed **2**.

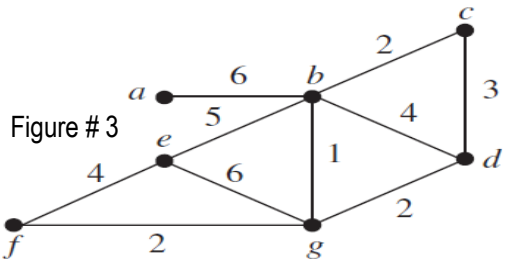

Figure # 3

QUESTION # 6: Binomial theorem, Trees and Proofs (Expected time = 20 minutes) **[3x4=12 points]**

(i)
(a) What is the co-efficient of x^7y^{10} in the expansion of $(3x - 2y)^{17}$.
(b) FAST-NUCES Sportics society is organizing the Chess tournament. Suppose ***1000*** participants have registered for the chess tournament. Now they are using a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one participant has not lost. (Assume there are no ties.)

(ii)
(a) Proof that for all integers ***z***, $4(z^2 + z + 1) - 3z^2$ is a perfect square.
(b) Prove that if ***a*** and ***b*** are positive, even integers, then ***ab*** is divisible by ***4***.

(iii) Prove by mathematical Induction. $P(n) = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all natural numbers ***n***.

ALL THE BEST 😊