Salma 19K-1043 BS SE (A)

Discrete Final Exam

"I pledge on my honor that I will not give or receive any unauthorized assistance on this examination".

Q #1

(i)

a). $a \vee b$

b). $\neg c \rightarrow \neg b$

c). $c \rightarrow d$

d). $\neg a$

(ii)

a). ~~$a \vee b$~~ $a \vee b$

P a ∴ Using ~~Simplification~~

b). $\neg c \rightarrow \neg b$

$\Rightarrow (a \vee b) \wedge (\neg c \rightarrow \neg b) \wedge (c \rightarrow d) \wedge \neg a$

$\Rightarrow (a \vee b) \wedge \neg a \wedge (\neg c \rightarrow \neg b) \wedge (c \rightarrow d)$

$\Rightarrow b \wedge (\neg c \rightarrow \neg b) \wedge (c \rightarrow d)$ ∴ ~~Simpli~~ Disjunctive Syllogism

$\Rightarrow (b \rightarrow c) \wedge b \wedge (c \rightarrow d)$ ∴ Contrapositive

$\Rightarrow c \wedge (c \rightarrow d)$ ∴ Modus Ponen

$\Rightarrow d$ ∴ Modus Ponen

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(2)

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(iii)

Contrapositive : $b \to c$

"If Ali is entrolled in BS Compiles Science, then Ali studies Discrete Structures".

Inverse: $\neg c \to \neg b$ & $\Rightarrow c \to b$

"If Ali studies Discrete Structures, then he is entrolled in BS Computer Science

Converse of its inverse : $b \to c$

"If Ali is entrolled in BS Compiles Science, then Ali studies Discrete Structures"

		contrapositive	inverse	converse
b,	c ,	$b \to c$	$c \to b$	$b \to c$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

∴ Proved that contrapositive is equivalent to converse of its inverse

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(3)

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(iv)

$((p \vee q) \wedge (p \to r)) \longrightarrow (q \vee r)$

$\because p \to q = \neg p \vee q$

$\neg [(p \vee q) \wedge (p \to r)] \vee (q \vee r)$

$\neg (p \vee q) \vee \neg (\neg p \vee r) \vee (q \vee r)$ ∵ Implication

$(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \vee r)$ ∵ De-Morgan, ∵ Negation

$(\neg p \wedge \neg q) \vee [((p \wedge \neg r) \vee q) \vee (p \wedge \neg r) \vee r]$ ∵ Distributive

$(\neg p \wedge \neg q) \vee [(p \vee q) \wedge (q \vee \neg r) \vee (p \vee r) \wedge (r \vee \neg r)]$ ∵ Distrib

$(\neg p \wedge \neg q) \vee [(p \vee q) \wedge (q \vee \neg r) \vee (p \vee r) \wedge T]$ ∵ Negati

$(\neg p \wedge \neg q) \vee (p \vee q) \wedge (q \vee \neg r) \vee (p \vee r)$ ∵ Identity

$(\neg p \wedge \neg q) \vee (p \vee q) \wedge (p \vee q) \vee (r \vee \neg r)$ ∵ ~~Const~~

$(\neg p \wedge \neg q) \vee (p \vee q) \vee T$ ∵ Negatio

$(\neg p \wedge \neg q) \vee T$ ∵ Universal

T ∵ Universal

∴ Proved Hai it is Tautology.

Soln

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Q#2

(i)

(a) ~~$\neg \forall x\, J(x)$~~ $\neg \forall f\, P(f)$

(b) $\neg \exists f\, Q(f, Bilal)$

(ii)

(a) There is a student in your class who has not whatsapp.

(b) There is a student in your class who has chatted with everyone over the whatsapp.

(iii)

(a). There is a building in lahore such that it has greater area then all building in Karachi.

True. Lahore Shaifort has greater area then all building in Karachi

(b) All buildings in Karachis has more books written than All buildings that are exactly 1546 sq. ft.

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False. There are not all books of Karachi that are more written than all books of building exactly 1546 sq ft

Q#3

(i)

(a) B and C are subset of A
C is also subset of D.

(b) Cardenality : A = 3 , B = 2, C = 2, D = 3

(ii)

(a) A = 2500 , B = 3000 , A∩B = 1000

$|A|+|B| - |A \cap B| = 2500 + 3000 - 1000$

$= 4500$

(b) A = 2500, B = 3000 , A∩B = 1000

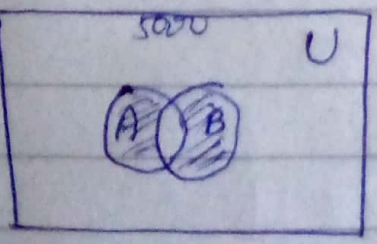
$|A|+|B| - |A \cap B| = 2500 + 3000 - 1000$

$= 4500$

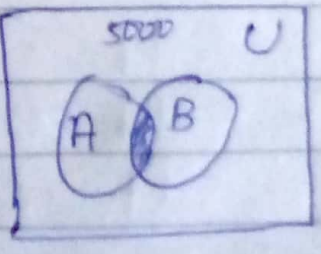
$= 45000 - 4500$ bronchitis

$= 500$ (Not diagnosed with penumia or

(a)



(b)



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(iii)

$P-(Q\cap R) = (P-Q)\cap(P-R)$

$\because A-B = A\cap\bar{B}$

Apply on b.s

$P\cap\overline{(Q\cap R)} = (P\cap\bar{Q})\cap(P\cap\bar{R})$

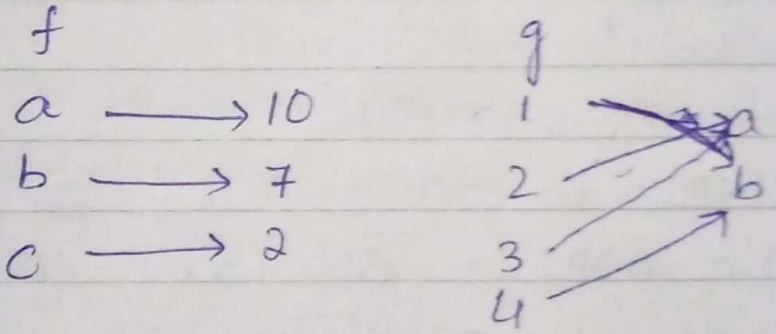
$P\cap(\overline{Q\cap R}) = P\cap(\bar{Q}\cap\bar{R})$ ∵ Comutative

$P\cap(\overline{Q\cap R}) \neq P\cap(\overline{Q\cup R})$

∴ Disproved!

(iv)

(a)



f is invertible but g does not because it is not bijective function

$f^{-1} = \{(10,a),(7,b),(2,c)\}$

Zahra

(7)

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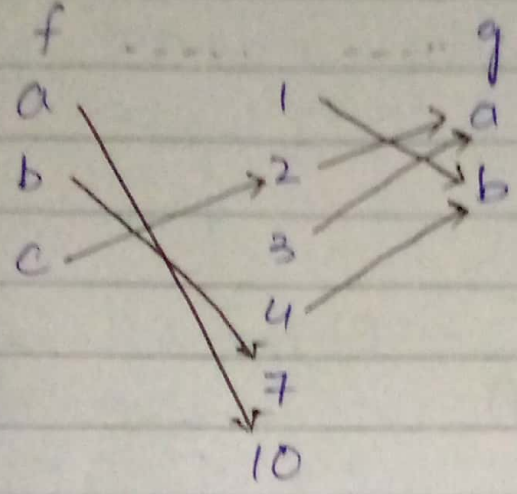
(v)



fog gof

c → a a → c

Q # 4

(i)

~~(a)~~

$a_1 = 2$, $m_1 = 7$, $a_2 = 3$ $m_2 = 17$

$a_3 = 5$, $m_3 = 19$

$m = m_1 \times m_2 \times m_3 = 7 \times 17 \times 19 = 2261$

$M_1 = \frac{m}{m_1} = \frac{2261}{7} = 323$

$M_2 = \frac{m}{m_2} = \frac{2261}{17} = 133$

$M_3 = \frac{m}{m_3} = \frac{2261}{19} = 119$

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$y_k = \overline{M_k} \bmod m_k$

$y_1 = \overline{323} \bmod 7$

$a = qd + r$

$323 = (46)7 + 1$

Backward :-

$1 = 1 \cdot 323 + (-46)(7)$

$y_1 = 1$

$y_2 = \overline{133} \bmod 17$

$a = qd + r$

$133 = (7)17 + 14$

$17 = (1)14 + 3$

$14 = (4)3 + 2$

$3 = (1)2 + 1$

Backward :-

$1 = 1 \cdot 3 - 1 \cdot 2$

$= 1 \cdot 3 - 1 \cdot (1 \cdot 14 - 4 \cdot 3)$

$= 1 \cdot 3 - 1 \cdot 14 + 4 \cdot 3$

$= 5 \cdot 3 - 1 \cdot 14$

$= 5 \cdot (1 \cdot 17 - 1 \cdot 14) - 1 \cdot 14$

$= 5 \cdot 17 - 5 \cdot 14 - 1 \cdot 14$

$= 5 \cdot 17 - 6 \cdot 14$

$= 5 \cdot 17 - 6 \cdot (1 \cdot 133 - 7 \cdot 17)$

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$= 5 \cdot 17 - 6 \cdot 133 + 42 \cdot 17$

$= 47 \cdot 17 + (-6)(133)$

$= -6 + 17 = 11$

$\boxed{y_2 = 11}$

$y_3 = 119 \bmod 19$

$a = qd + r$

$119 = (6)19 + 5$

$19 = (3)5 + 4$

$5 = (1)4 + 1$

Backward :-

$1 = 1 \cdot 5 - 1 \cdot 4$

$= 1 \cdot 5 - 1 \cdot (1 \cdot 19 - 3 \cdot 5)$

$= 1 \cdot 5 - 1 \cdot 19 + 3 \cdot 5$

$= 4 \cdot 5 - 1 \cdot 19$

$= 4 \cdot (1 \cdot 119 - 6 \cdot 19) - 1 \cdot 19$

$= 4 \cdot 119 - 24 \cdot 19 - 1 \cdot 19$

$= (4)(119) + (-25)(19)$

$\boxed{y_3 = 4}$

Using Chinese Remainder theorem:

$x = (a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3) \bmod m$

$= [(2)(323)(1) + (3)(133)(11) + (5)(119)(4)] \bmod m$

$x = 7415 \bmod 2261 = 632$

Soln

(10)

Solve 19K-1043

(ii)

Number of balls = 632

Six hundred and thirty two

18, 8, 23, 7, 20, 13, 3, 17, 4, 3, 0, 13, 3

19, 7, 8, 17, 19, 24, 19, 22, 14

Ceaser Cipher

$f(p) = (p+3) \bmod 26$

~~21 mod 26, 8 m 11 mod 26,~~

21, 11, 26, 10, 23, 16, 6, 20, 7, 6, 3, 16, 6

22, 10, 11, 20, 22, 27, 22, 25, 17

21 mod 26 = 21 , 11 mod 26 = 11

26 mod 26 = 0 , 10 mod 26 = 10

23 mod 26 = 23 , 16 mod 26 = 16

6 mod 26 = 6 , 20 mod 26 = 20

7 mod 26 = 7 , 6 mod 26 = 6

3 mod 26 = 3 , 16 mod 26 = 16

6 mod 26 = 6 , 22 mod 26 = 22

10 mod 26 = 10 , 11 mod 26 = 11

20 mod 26 = 20 , 22 mod 26 = 22

27 mod 26 = 1 , 22 mod 26 = 22

25 mod 26 = 25 , 17 mod 26 = 17

Solve

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Encryption: V L A K X Q G U H G D Q G W
K L U W B W Z R.

(iii)

$a = 6+3+2 = 11$

$11^{302} \bmod 7$

Using Fermat's: $a^{P-1} = 1 \bmod P$

$11^{7-1} = 1 \bmod 7$

$11^{6} = 1 \bmod 7$

$11^{302} \bmod 7$ =

gcd (302, 6) = $a = qd + r$

$302 = (50)\ 6 + 2$

$= (11)^{50 \times 6 + 2} \bmod 7$

$= (11^6)^{50} \cdot 11^2 \bmod 7$

$= (1)\ 11^2 \bmod 7 = 121 \bmod 7 = 2.$

(iv)

125973128 Q

$x_{10}(Q) = \{(1\times1) + (2\times2) + (3\times5) + (4\times9) + (5\times7) + (6\times3) + (7\times1) + (8\times2) + (9\times8)\} \bmod 11$

$= 204 \bmod 11 = 6$

$Q = 6 \times 10 = 60$

$(204 + 60) \bmod 11 = 0.$ Validated.

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(12)

Solve 19K-1043

(v)

(a)

Using Combinations:

$15C3 + 12C3 + 7C3 + 5C3 + 9C3 + 10C3$

$\because C = \frac{n!}{(n-r)!\,r!}$

$455 + 220 + 35 + 10 + 84 + 120$

$= 924$

(b)

Grades = 10 Student - 97

$P(\text{At least}) = 1 - \frac{10}{97} = 0.897$

(vi)

(a)

Captain : $15P1 = \frac{15!}{(15-1)!} = 15$

Vice-Captain: $14P1 = \frac{14!}{(14-1)!} = 14$

Wicke keper : $15P2 = \frac{15!}{(15-2)!} = 210$

Solve

(13)

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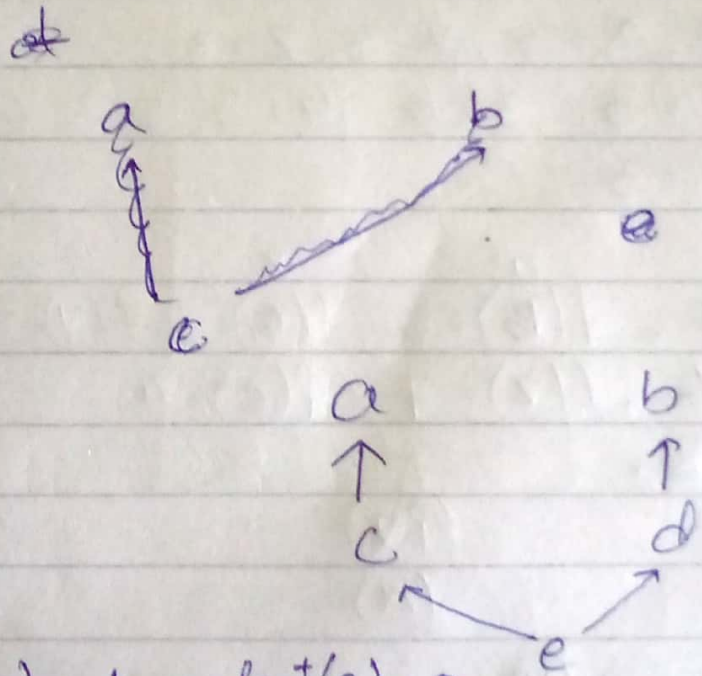
(b)

$P(\text{not defective}) = 1 - P(\text{defective})$

$= 1 - \frac{15}{100} = \frac{17}{20} = 0.85$

Q # 5

(i)



$\deg^-(a) = 1 \quad \deg^+(a) = 0$

$\deg^-(b) = 1 \quad \deg^+(b) = 0$

$\deg^-(c) = 1 \quad \deg^+(c) = 1$

$\deg^-(d) = 1 \quad \deg^+(d) = 1$

$\deg^-(e) = 0 \quad \deg^+(e) = 2$

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(ii)

{(e, c), (e, d), (c, a), (d, b)}

Partial Relation because no symetric is there.

(iii)

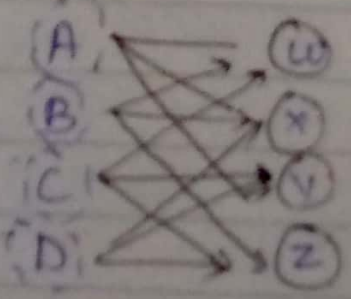
Edges = 8.

degrees = A = 3, B = 3, C = 3, D = 3
W = 3, X = 3, Y = 3, Z = 3
1 = 3, 2 = 3, 3 = 3, 4 = 3, 5 = 3, 6 = 3
7 = 3, 8 = 3

f(A) = 1, f(B) = 2, f(C) = 3 f(Y) = 5
f(B) = 6, f(z) = 7, f(c) = 4
f(D) = 8.

(iv)

(a)



Complete Bipartite Graph because every adjacent node is different.

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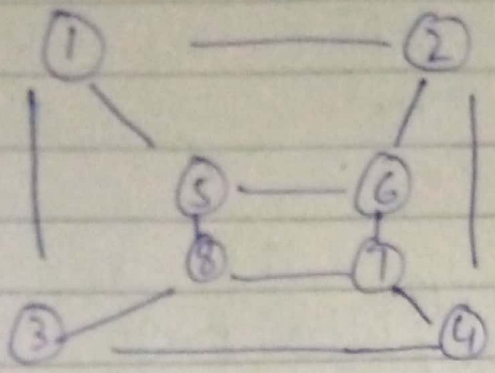
(15)

Sol:- 19K-1043

(b)

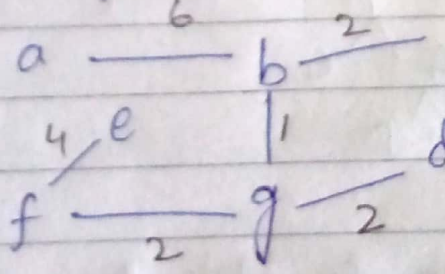


Euler Circuit = No because degrees are not even. All degrees are odd.

Hamilton Circuit: 1, 2, 4, 3, 8, 7, 6, 5, 1
Valid Circuit.

(v)

b.q.- Using Prims Algorithm



Minimum = 17.

(16)

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Q#6

(i)

(a)

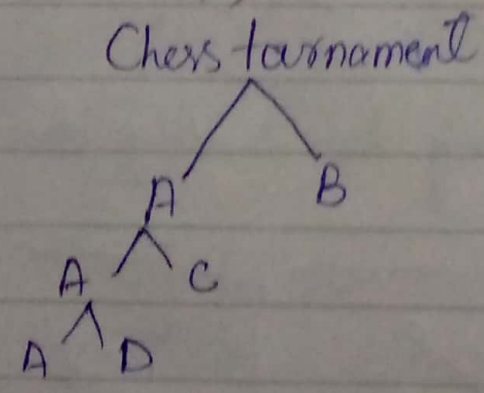
$x^7 y^{10}$ coefficient = ?

$$(3x-2y)^{17} = \sum_{j=0}^{17} \binom{17}{j} (3x)^{17-j} (-2y)^{j}$$

$$= \binom{17}{10} (3x)^{17-10} (-2y)^{10}$$

$$= 19448\,(3x)^{7}(-2y)^{10}.$$

(b)



If two participants play then one must lose $1000 - 1 = 999$.

Sher

(17)

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(Q11)

(a)

$4(z^2+z+1)-3z^2$

~~4(z~~ let $m = z$

$4m^2+4m+4-3m^2$

m^2+4m+4

$(m)^2+2(2(m)+(2)^2$

$(m+2)^2$

∴ z is perfect square.

(b)

$a = 2r$ and $b = 2k$

$\frac{a \times b}{4} = \frac{(2r)(2k)}{4} = \frac{4rk}{4} = rk$

$= \frac{rk}{4} = 4\left(\frac{rk}{16}\right)$

$\frac{rk}{16} = p$

$= 4p$. Proved!

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(18)

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(iii)

Basic Step = $n = 1$

$$(1)^2 = \frac{(1)(1+1)(2(1)+1)}{6}$$

$$\boxed{1 = 1}$$

Inductive Steps-

Put $n = k$

$$1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

$(k+1)^2$ Put :- Add both

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left\{\frac{k(2k+1)}{6} + (k+1)\right\}$$

$$= (k+1)\left\{\frac{2k^2 + k + 6k + 6}{6}\right\}$$

$$= (k+1)\left\{\frac{2k^2 + 7k + 6}{6}\right\}$$

$$= (k+1)\left\{\frac{2k^2 + 4k + 3k + 6}{6}\right\}$$

Rough

$k+1$:-

$$\frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\frac{(k+1)(k+2)(2k+3)}{6}$$

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$$= (k+1)\left\{\frac{2k(k+2)+3(k+2)}{6}\right\}$$

$$= (k+1)\left\{\frac{(2k+3)(k+2)}{6}\right\}$$

$$= \frac{(k+1)(2k+1+1)(k+1+1)}{6}$$. Proved!

Sidra.