# DS ASSIGNMENT #3

Q1). a). Level of n is 3

b). Level of a is 0

c). Height of tree is 5

d). Childrens are u and v

e). Parents of g is d

f). Siblings of j are k and l

g). Descendants are m, s, t, x, y

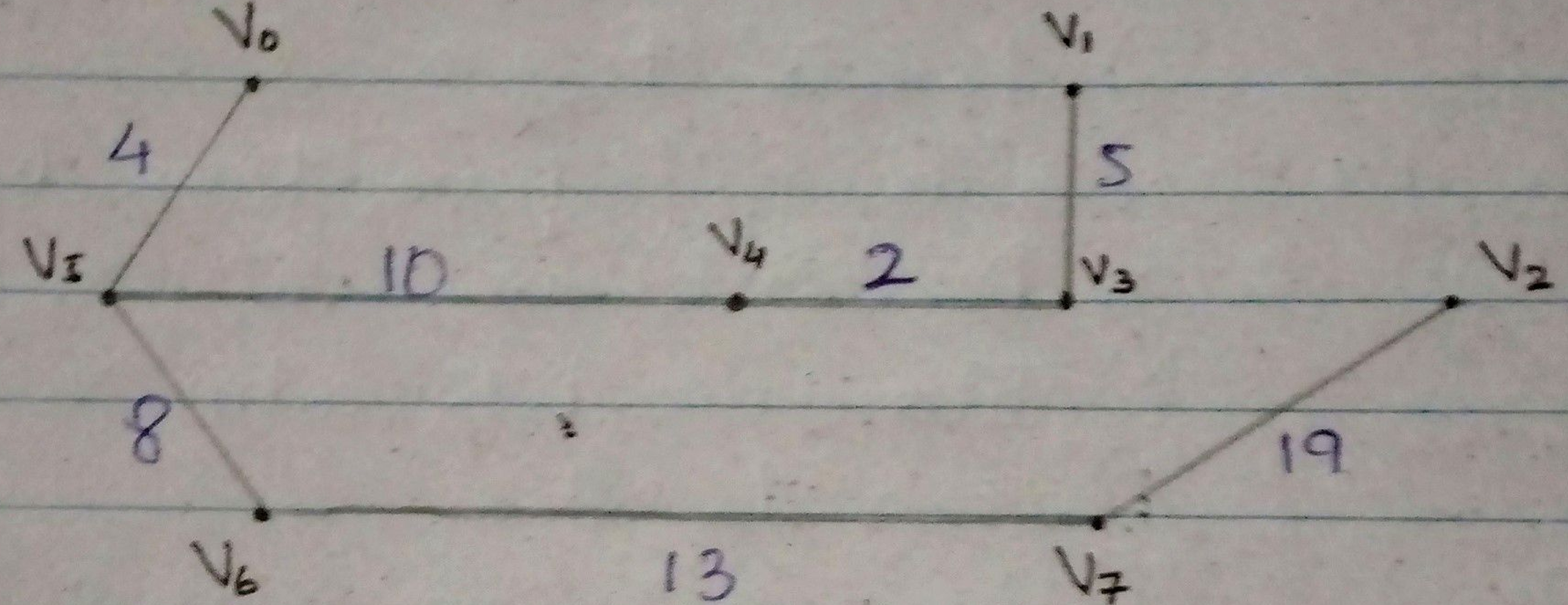
h). Internal node: b, d, i, o, h, n, v, c, f, m, t, e, k.

j). Leaves are: j, q, r, l, s, x, y, g, u, z, w, p

i). Ancestors of z: v, n, h, d, a.

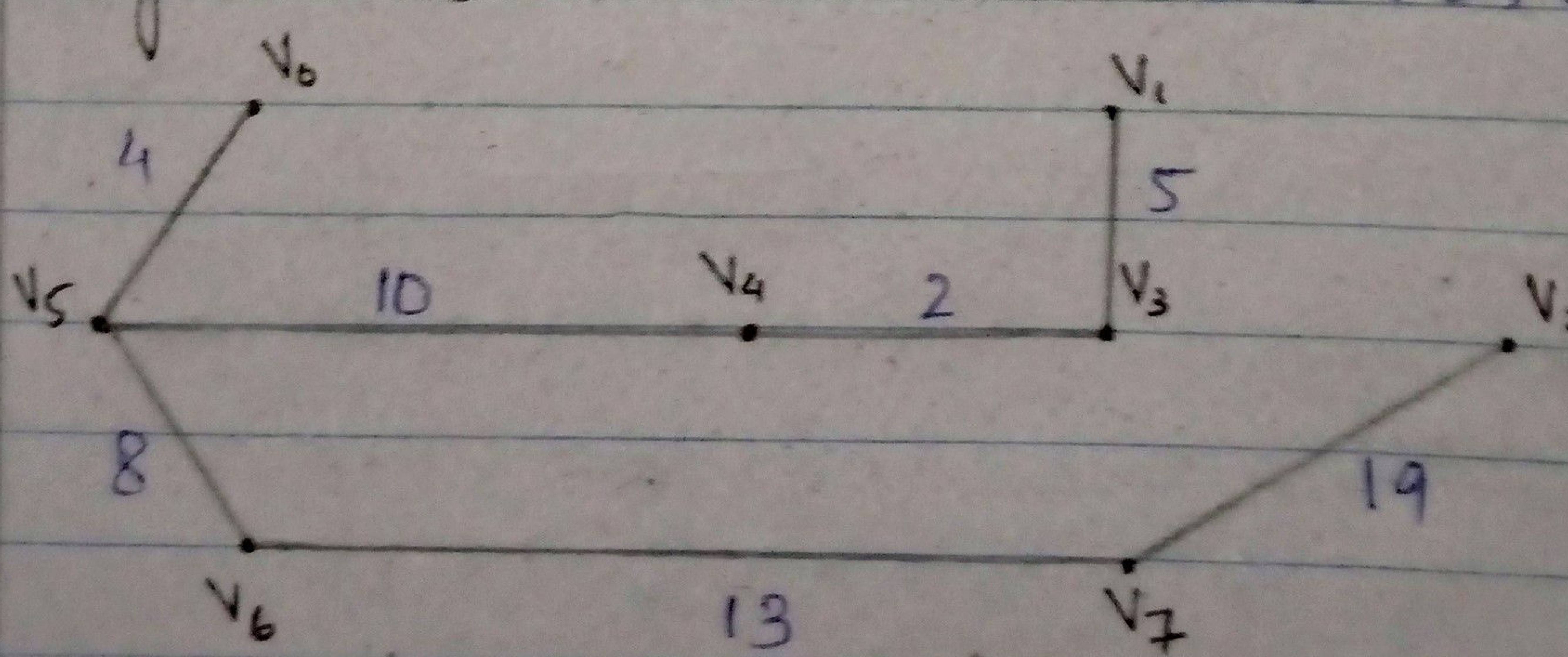
## Q2). a). By Kruskal's :-

$V_3, V_4 = 2$ , $V_0, V_5 = 4$ , $V_1, V_3 = 5$
$V_1, V_4 = 7^{\times}$ , $V_5, V_6 = 8$ , $V_4, V_5 = 10$
$V_0, V_1 = 12^{\times}$ , $V_6, V_7 = 13$ , $V_4, V_7 = 15^{\times}$
$V_3, V_7 = 18^{\times}$ , $V_2, V_7 = 19$ , $V_1, V_2 = 20^{\times}$




Order: $\{V_3, V_4\}$, $\{V_0, V_5\}$, $\{V_1, V_3\}$, $\{V_5, V_6\}$, $\{V_4, V_5\}$, $\{V_6, V_7\}$, $\{V_2, V_7\}$

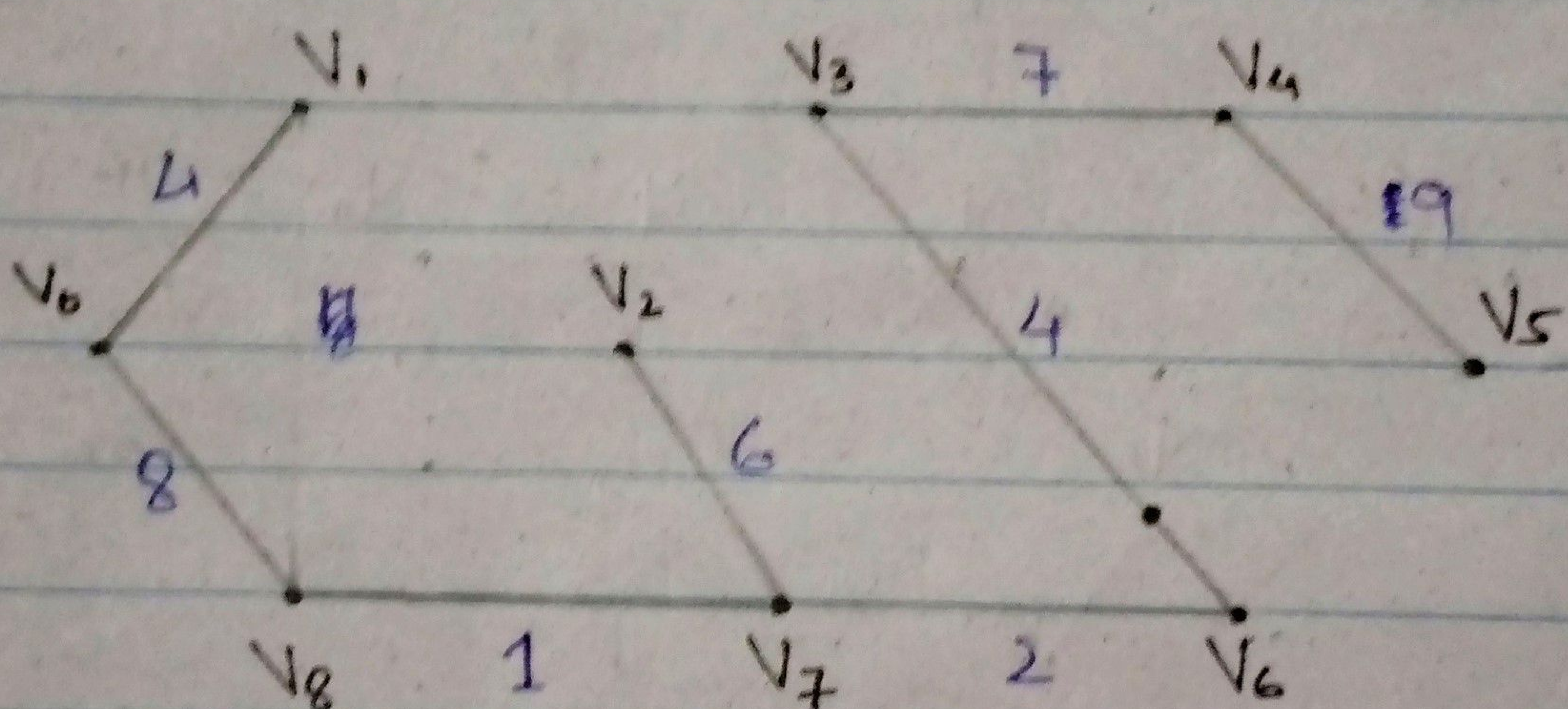
## By Prim's :-




Order: $\{V_3, V_4\}$, $\{V_0, V_5\}$, $\{V_1, V_3\}$, $\{V_5, V_6\}$, $\{V_4, V_5\}$, $\{V_6, V_7\}$, $\{V_2, V_7\}$

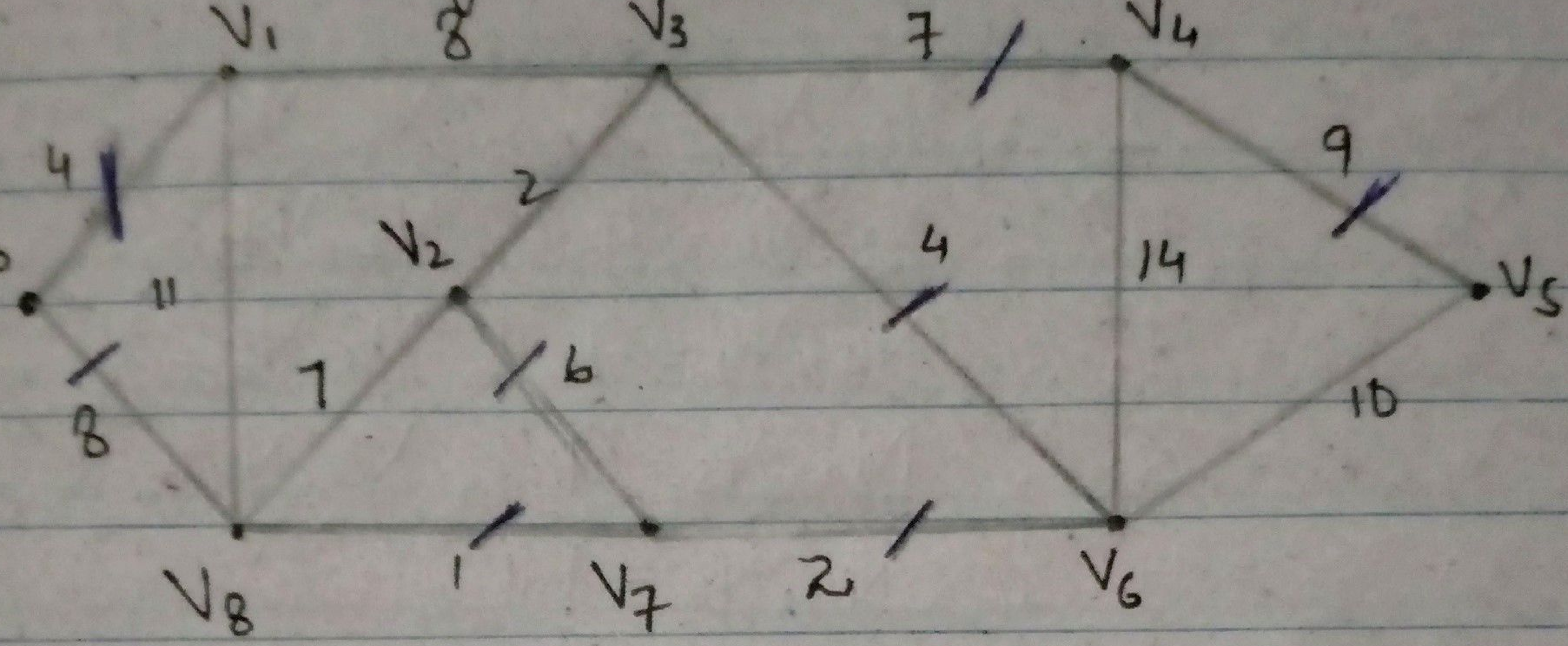
Q2). b). By Kruskal's:

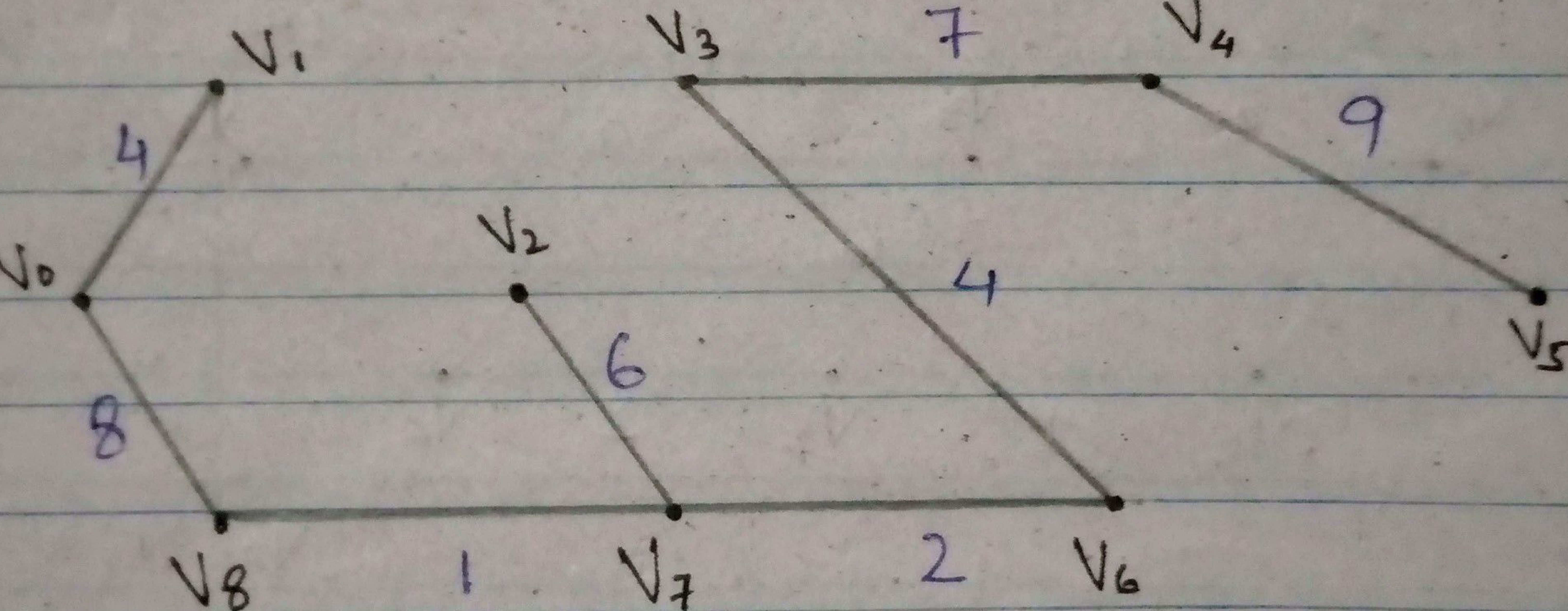
$V_7, V_8 = 1$, $V_2, V_3 = 2$, $V_6, V_7 = 2$,
$V_0, V_1 = 4^*$, $V_3, V_6 = 4$, $V_2, V_7 = 6$,
$V_2, V_8 = 7^{\times}$, $V_3, V_4 = 7$, $V_1, V_3 = 8^{\times}$,
$V_0, V_8 = 8$, $V_4, V_5 = 9$, $V_5, V_6 = 10^{\times}$,
$V_1, V_8 = 11$, $V_4, V_6 = 14^{\times}$




Order: $\{V_7, V_8\}$, $\{V_6, V_7\}$, $\{V_0, V_1\}$, $\{V_3, V_6\}$,
$\{V_2, V_7\}$, $\{V_3, V_4\}$, $\{V_0, V_8\}$,
$\{V_4, V_5\}$, $\cancel{\{V_1, V_8\}}$

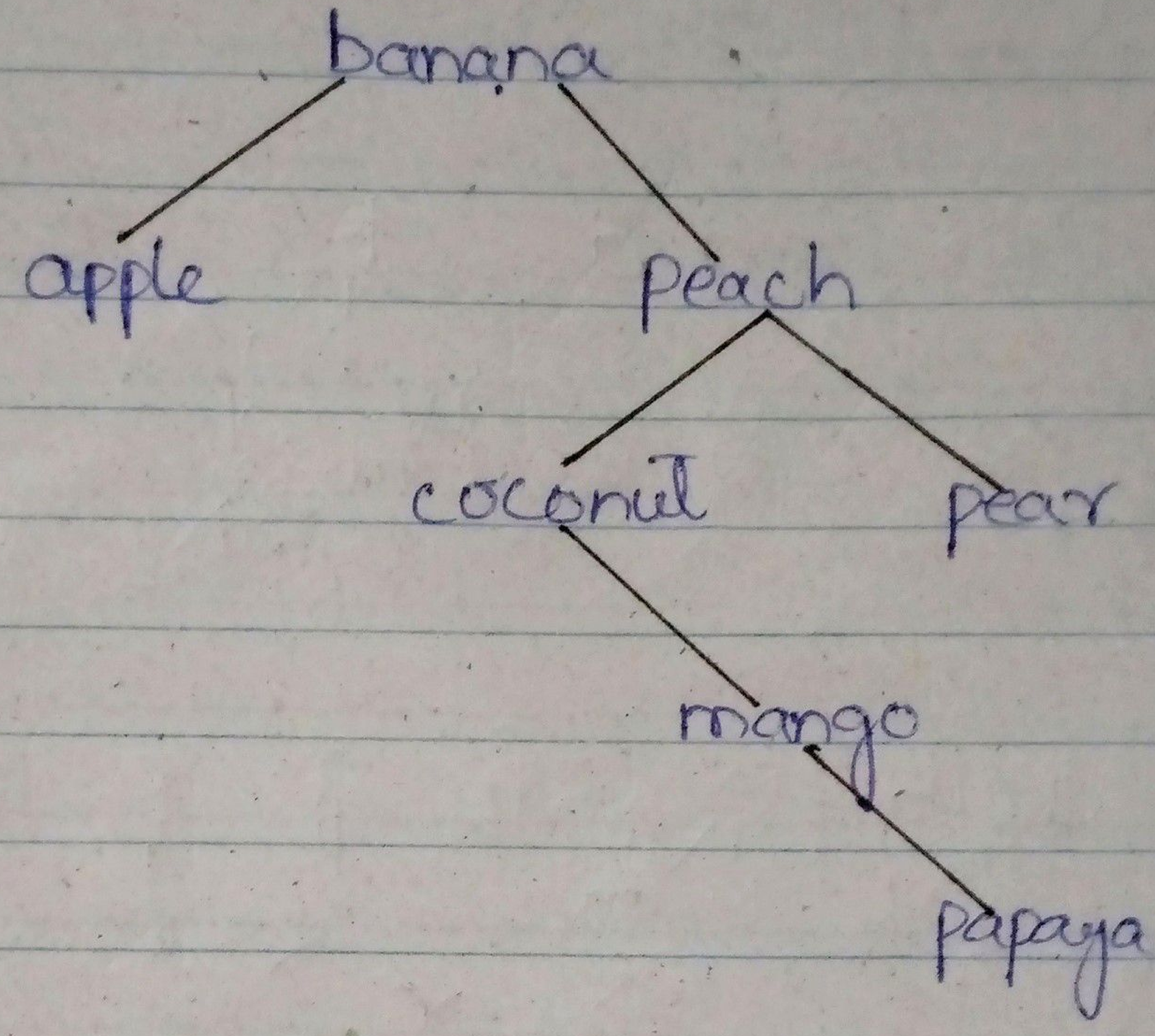
Q2). b). By PRIM'S :-




Order: $\{V_7, V_8\}$, $\{V_6, V_7\}$, $\{V_3, V_6\}$, $\{V_0, V_1\}$, $\{V_2, V_7\}$, $\{V_0, V_8\}$, $\{V_4, V_5\}$

Q3). a).




b). i). Vertices = 10000

Edges = ?

∵ A tree with $n$ vertices has $n-1$ edges

∴ Edges = $10000-1$

Edges = $9999$

ii). Internal Vertices = 1000

$m = 2$

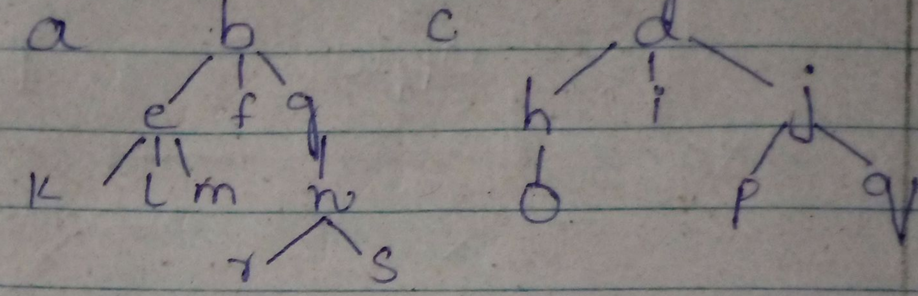
∴ Edges = $m \times$ internal vertices

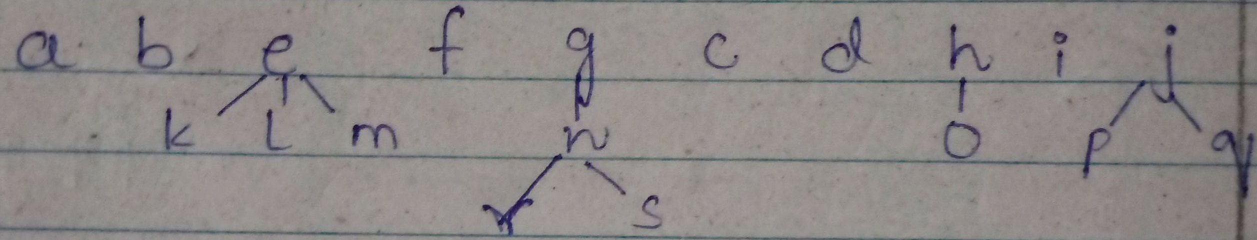
$= 2 \times 1000$

$= 2000$

Q4). a). a).

Pre Order:



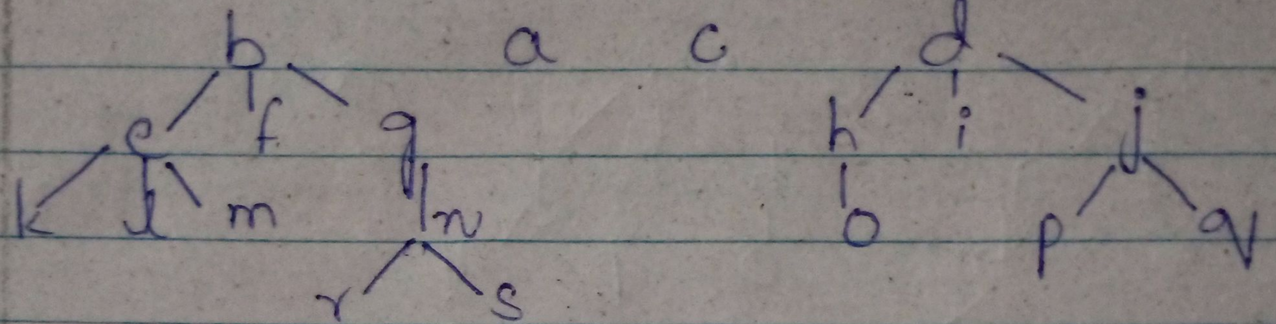



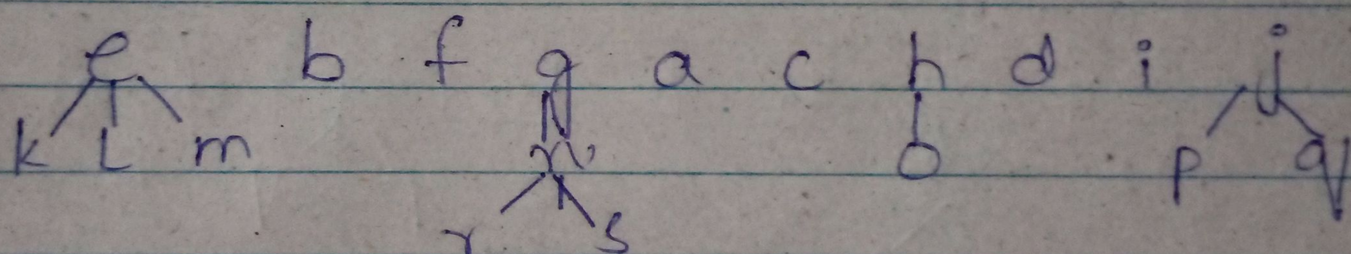

a b e k l m f g n c d h o
r s

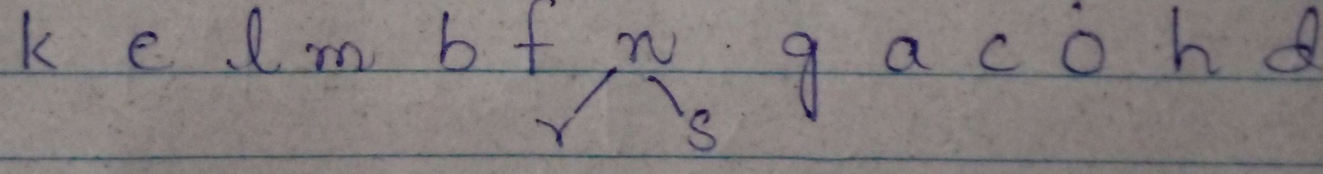
i j p q

a b e k l m f g n r s c d h
o i j p q

# INORDER :-



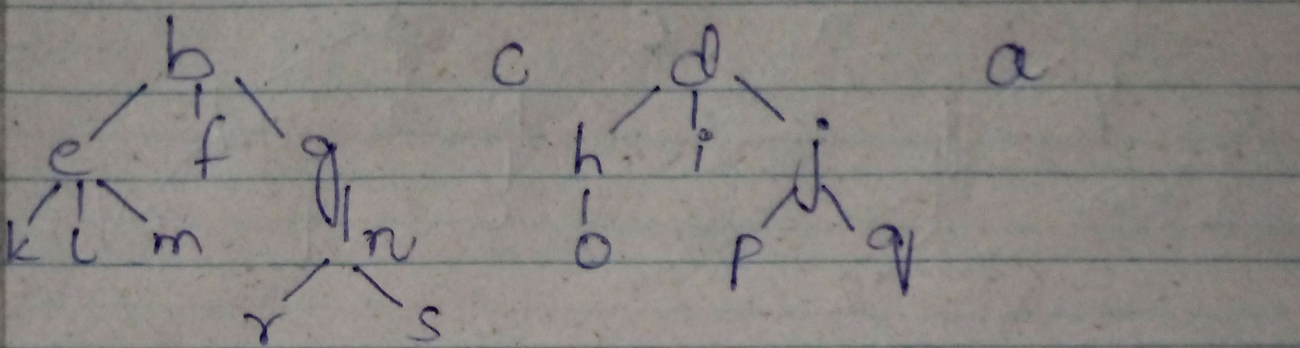



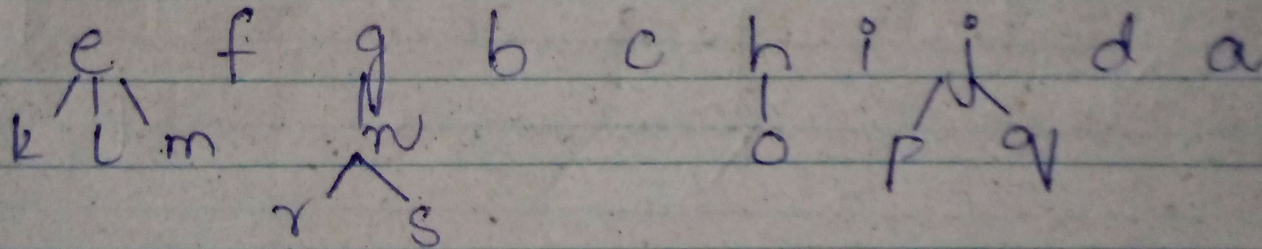



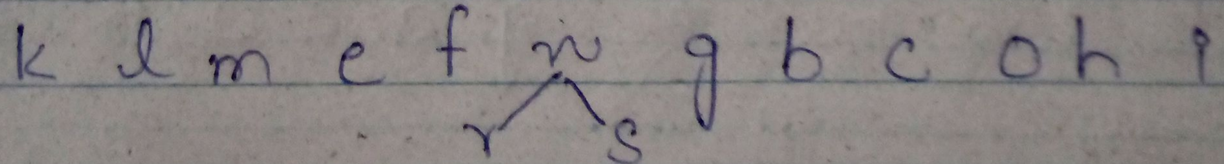

i p j q

k e l m b f r n s g a c o h d
i p j q

# POST ORDER :-




p q j d a

k l m e f r s n g b c o h
i p q j d a

Q4). a). b).

PRE ORDER :-

- a b c
- a b d e c f g h
- a b d e i j c f g h k l
- a b d e i j m n o c f g h k l p

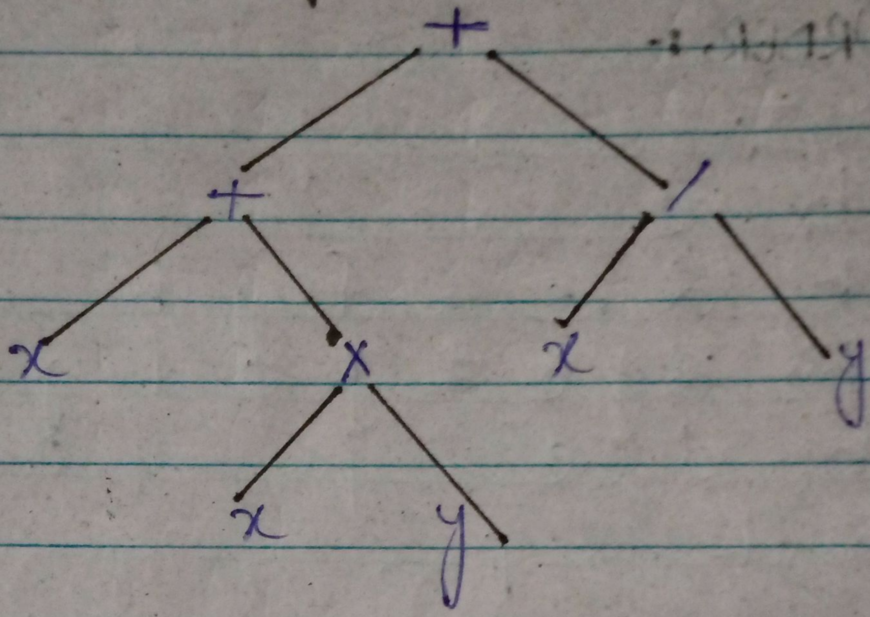
IN ORDER :-

- b a c
- d b e a f c g h
- d b i e j a f c g k h l
- d b i e m j n o a f c g k h p l

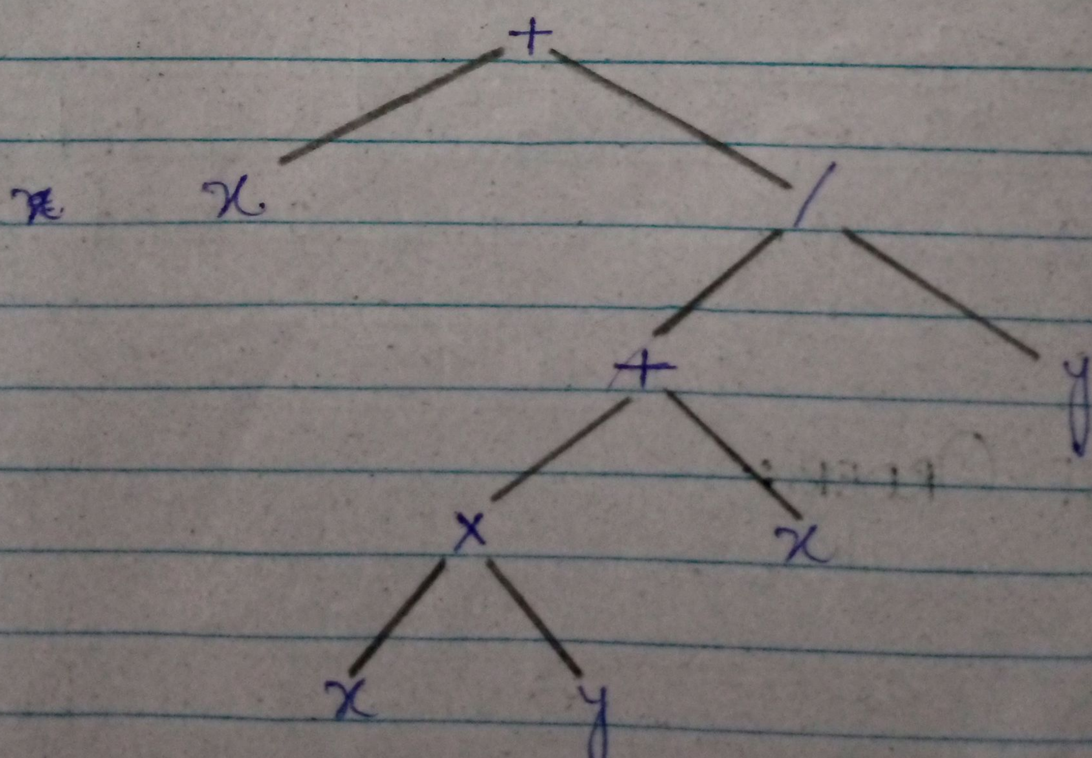
POST ORDER :-
- b c a
- d e b f g h c a
- d i j e b f g k l h c a
- d i m n o j e b f g k p l h c a

Q4). b).

$(x + xy) + (x/y)$.



$x + ((xy + x)/y)$ :-



Q5). a). $(x + xy) + (x/y)$

i). Prefix notation:-

$+ + x \times x y / x y$

ii). Postfix notation:-

$x x y \times + x y / +$

iii). Infix notation:-

$((x + (x \times y)) + (x/y))$

Expression: $x + ((xy + x)/y)$

i). Prefix notation:-

$+ x / + \times x y x y$

ii). Postfix notation:-

$x x y \times x + y / +$

iii). Infix notation:-

$(x + (((x \times y) + x)/y))$

Q5). b). i). $+ - \uparrow 3\ 2 \uparrow 2\ 3 / 6 - 4\ 2$

Prefix: $+ - \uparrow 3\ 2 \uparrow 2\ 3 / 6\ (4-2)$

$+ - \uparrow 3\ 2 \uparrow 2\ 3 / 6\ 2$

$+ - \uparrow 3\ 2 \uparrow 2\ 3\ (6/2)$

$+ - \uparrow 3\ 2 \uparrow 2\ 3\ 3$

$+ - \uparrow 3\ 2\ (2^3)\ 3$

$+ - \uparrow 3\ 2\ 8\ 3$

$+ - (3)^2\ 8\ 3$

$+ - 9\ 8\ 3$

$+ (9-8)\ 3$

$+ 1\ 3$

$1+3 \Rightarrow 4$

ii). $4\ 8 + 6\ 5 - \times 3\ 2 - 2\ 2 + \times /$

Postfix: $(4+8)\ 6\ 5 - \times 3\ 2 - 2\ 2 + \times /$

$12\ (6-5) \times 3\ 2 - 2\ 2 + \times /$

$(12 \times 1)\ 3\ 2 - 2\ 2 + \times /$

$12\ (3-2)\ 2\ 2 + \times /$

$12\ 1\ (2+2) \times /$

$12\ 1\ 4 \times /$

$12\ (1 \times 4)\ /$

$(12/4)$

$3$

Q6). a). Using Product Rule:-

Each Offices = 37  Floor = 27

$\therefore$ Total Office = $37 \times 27$

= 999 offices

b). Colors of shirts = 12

Versions = Male & Female

Sizes = 3

$\therefore$ 12 colors come in each sizes and for two genders separately

Using Product and Sum Rule:-

Total types of shirt = $(12 \times 3) + (12 \times 3)$

= 72

Q7). a). Three letters can people have that is $26 \times 26 \times 26 = 17576$

b). By Using Product Rule and given that it is not repeated.

$\therefore$ $26 \times 25 \times 24 = 15600$

Q 8). a). By using Product and Sum Rule, a hexadecimal digits string can be of 10, 26 or 58

$\therefore (16)^{10} + (16)^{26} + (16)^{58}$

$= 6.9 \times 10^{69}$

b). Using Product and Subtraction Rule

String with $x$ : $26^4 = 456,976$

without $x$ : $25^4 = 390,625$

$\therefore 456976 - 390625 = 66351$

Q9). a). Using counting function by product rule the domain is $m$ where as range is 2, therefore $2^m$ function can be form.

b). The choices will be from domain to codomain by one-to-one will be

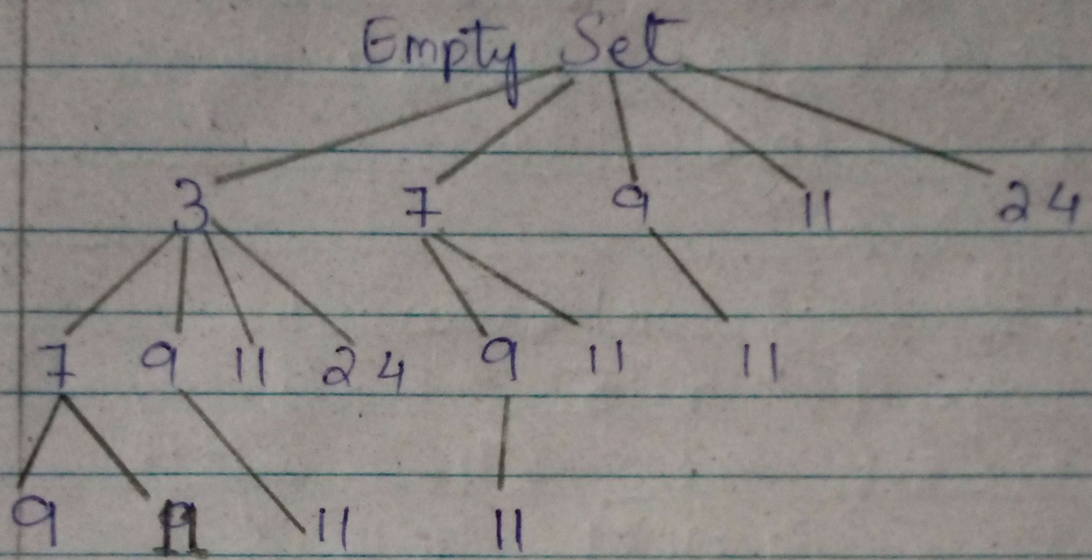
let $m = 5$

$\therefore m(m-1)(m-2)(m-3)(m-4)$

~~(m-5)~~

$5 \times 4 \times 3 \times 2 \times 1 = 120$ ways

Q10). a).




$\therefore$ There are 17 subsets..

b). Using Permutation,

$n = 2$ (A or B)

$r = 2$ (Two games winner win)

$\therefore$ $2P2 = \dfrac{2!}{(2-2)!} = 2$. ways

Q11). a). Using Combination

$n = 8$ $r = 3$

$8C3 = \dfrac{8!}{(8-3)!\,3!} = 56$ ways

Q11). b). Using Combination

$n = 12 \quad r = 6$

$$12C6 = \frac{12!}{(12-6)!\,6!} = 924 \text{ ways}$$

c). ~~#~~ Using Combination

$n = 9 \quad r = 5$

$$9C5 = \frac{9!}{(9-5)!\,5!} = 126 \text{ ways}$$

Q12). a). Using Permutation

$n = 20 \quad r = 5$

$$20P5 = \frac{20!}{(20-5)!} = 1860480 \text{ ways}$$

b). Using Permutation

$n = 16 \quad r = 4$

$$16P4 = \frac{16!}{(16-4)!} = 43680 \text{ ways}$$

c). Using Permutation

$n = 15 \quad r = 2$

$$15P2 = \frac{15!}{(15-2)!} = 210 \text{ ways}$$

Q 13). a). 3 breads, 5 meats, 4 cheese and 6 condiments

Required is 1 meat, 2 bread, 1 cheese and 3 condiments

$= 5C1 \times 3C2 \times 4C1 \times 6C3$

$= 1200$

b). Using Product Rule:-

$(15)(48)(24)(34)(28)(28) = 460615680$ faces

Q14). a). Using Subtraction Rule:-

Bit string begining with three 0's $= 2^7$
$= 128$

Bit string ending with two 0's $= 2^8$
$= 256$

Bit string starting with three 0's and ending with two 0's $= 2^5$
$= 32$

$\therefore$ Bit strings $= 128 + 256 - 32$
$= 352$

Q14). b). Using Subtraction Rule:-

Bit string starting with 0 = $2^4 = 16$

Bit string ending with two 1's = $2^3$
$= 8$

Bit string starting with 0 and ending with two 1's = $2^2 = 4$

$\therefore$ Bit String = $16 + 8 - 4 = 20$

Q15). a). Students = 30 ⇒ N

Total letters = 26 ⇒ K

Result = 2

$$\therefore \left\lceil \frac{N}{K} \right\rceil = 2 \quad \because \text{Pigeon Hole Rule}$$

$$\left\lceil \frac{30}{26} \right\rceil = 2 \quad . \text{ Proved!}$$

b). Population = 8008278 ⇒ N

Hairs = 1000000 ⇒ K

Result = 9

Using Pigeonhole Principle

$$\left\lceil \frac{N}{K} \right\rceil = \left\lceil \frac{8008278}{1000000} \right\rceil = 9. \text{ Proved!}$$

(Q15). c). Time Period = 38 ⇒ K
Classes = 677 ⇒ N

Using Pigeonhole Rule

$$\left\lceil \frac{N}{K} \right\rceil = \left\lceil \frac{677}{38} \right\rceil = 18 \text{ rooms}$$

(Q16). a). Coefficient $x^5 = ?$ in $(1+x)^{11}$

Using Binomial Theorem

$T_{r+1} = {}^{n}C_r \, a^{n-r} \, b^r$

Here $r = 5$ $a = 1$ $b = x$ $n = 11$

$T_{5+1} = {}^{11}C_5 \, (1)^{11-5} \, x^5$

$T_6 = 252 x^5$

∴ Coefficient of $x^5$ is 252.

b). Coefficient of $a^7 b^{17} = ?$ $(2a-b)^{24}$

Using Binomial Theorem

$T_{r+1} = {}^{n}C_r \, a^{n-r} \, b^r$

Here $r = 17$ $a = 2a$ $b = -b$ $n = 24$

$T_{17+1} = {}^{24}C_{17} \, (2a)^{24-17} \, (-b)^{17}$

$T_{18} = (346104) 2 a^7 (-b)^{17}$

∴ Coefficient of $a^7 b^{17} = -(346104)(2)^7$

$= -44301312$

Q17). a). Let $a|b$ : $\frac{b}{a} = k$

$\boxed{a = \frac{b}{k}}$ and $\boxed{b = ak}$

also $b|c$ : $\frac{c}{b} = s$

$\boxed{c = sb}$ —— (I)

Here in eq (I) Put the value of b

$c = s(ak)$

$\frac{c}{a} = sk$

Put the value of 's' and k

$\frac{c}{a} = \frac{c}{\cancel{b}} \cdot \frac{\cancel{b}}{a}$

$\frac{c}{a} = \frac{c}{a}$

$\boxed{1 = 1}$ Hence Proved!

Q17). b). let $\frac{b}{a} = t$ and $\frac{c}{a} = s$

$b = at$ and $c = as$

$\therefore$ $b + c = at + as$

$= a(t+s)$

$\therefore$ Hence Proved $a \mid (b+c)$.

Q18). a). If there is an integer, atleast integer then $2^n - 1$ is prime.

Suppose an integer. $n = 7$ which is greater then 5

$\therefore$ $2^7 - 1 = 128 - 1 = 127$ which is prime.

OR by Condradition that: There is not an integer $n > 5$ then $2^n - 1$ is not prime.

*However in above example it can be seen that there is an integer on which $2^n - 1$ is prime.

$\therefore$ Proved that on $n > 5$, $2^n - 1$ is prime.

Q18). b). By using Contradiction :

If p divides a and a+1 both.

therefore : let $\frac{a}{p} = s$ $\frac{a+1}{p} = t$

$a = ps$ — (i) and $a+1 = pt$ — (ii)

Now subtracting eq (i) & (ii)

$a+1-a = ps - pt$

$1 = p(s-t)$

$\frac{1}{p} = s-t$

which is not possible

∴ Our assumption is wrong. Hence if $p|a$, then $p \nmid (a+1)$.

Q19). a). Squaring both sides

$(\sqrt{a+b})^2 = (\sqrt{a} + \sqrt{b})^2$

$a+b = a + 2\sqrt{a}\sqrt{b} + b$

$\cancel{a}+\cancel{b}-\cancel{a}-\cancel{b} = 2\sqrt{a}\sqrt{b}$

$0 = \sqrt{a}\sqrt{b}$

∴ If any number among them is zero then $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.

Q19).b). By Contrapositive : If $x <= 1$ or $x >= -1$ then $|x| <= 1$

Suppose $x = -2$

(a) $\cancel{x} -2 <= -1$ then $|-2| <= 1$

$2 \not<= 1$

Suppose $x = 2$

(b) $\cancel{x >= -1}$ $2 >= -1$

$|2| <= 1$

$2 \not<= 1$

∴ Our assumption is wrong. Hence Proved!

Q20).a). By Contradiction:-

The product of two rational is irrational

∴ $r = \frac{a}{b}$ $s = \frac{m}{n}$

We suppose $r \times s$ = irrational

Putting values

$rs = \frac{a}{b} \frac{m}{n} = \frac{am}{bn}$

Our assumption is wrong. Therefore proved that the product of two irrational is irrational.

Q20). b). By contradiction :-
The sum of rational and irrational is rational

$r$ = rational $\quad$ $s$ = irrational

$\therefore$ $r+s$ = rational

$$r = \frac{a}{b} \quad \text{and } r+s = \frac{c}{d} \quad — (A)$$

Put value of 'r' in eq (A)

$$\frac{a}{b} + s = \frac{c}{d}$$

$$s = \frac{c}{d} - \frac{a}{b} = \frac{bc-ad}{bd}$$

(rational)

Our assumption is wrong because $s$ is irrational not rational

$\therefore$ Proved that sum of rational and irrational is rational

Q21). a). Suppose $n = 7$

Using Counter example

$$n, n+2 = 7, 7+2$$
$$= 7, 9$$

where 9 is not a prime number.

$\therefore$ $n, n+2$ is not prime for every prime numbers.

b). Let the prime numbers $p_1, p_2, \ldots, p_n$

Suppose $R = p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdots p_n$

let define number $T$ that is

$$T = R + 1$$

Suppose if $T$ is not prime number then there exist a factor that divides into $T$. So that factor is $d$.

Suppose $d$ is not new prime, this implies that $d$ divides $R$ and $T$

$$\because T = R + 1$$

$\therefore$ $d$ must divide $R$ and $R+1$

$$\therefore \frac{(R+1) - R}{d} = \frac{1}{d}$$

which is not possible. Therefore our assumption is wrong. So, set of prime numbers is infinite.

Q22). a). By contradiction:-

If $m$ & $n$ are odd, then $n+m$ is odd.

let $m = 2r+1$ $\quad n = 2k+1$

$m+n = 2r+1 + 2k+1$

$= 2r+2k+2$

$= 2(r+k+1)$

let $(r+k+1) = p$

$= 2p$

Our assumption is wrong. Therefore, if $m$ and $n$ are odd integers then $m+n$ is even.

b). By Contrapositive: If $m$ and $n$ are not both even and $m$ and $n$ are not both odd then ~~n and m~~ $m+n$ is not even.

So when $m$ is even and $n$ is odd then $m+n$ is odd

let $m = 2k$ $\quad n = 2r+1$

$m+n = 2k + 2r+1$

$= 2(k+r)+1$

let $k+r = p$

$= 2p+1$

Hence Proved!

Q23). a). By Contradiction: $6-7\sqrt{2}$ is rational

let $6-7\sqrt{2} = \frac{m}{n}$

Squaring on both sides

$36 - 84\sqrt{2} + 98 = \frac{m^2}{n^2}$

$134 - 84\sqrt{2} = \frac{m^2}{n^2}$

$134n^2 - 84\sqrt{2}\,n^2 = m^2$

Here common factor exist. Our assumption is wrong. Therefore $6-7\sqrt{2}$ is irrational.

b). By Contradiction: $\sqrt{2}+\sqrt{3}$ is rational

let $\sqrt{2}+\sqrt{3} = \frac{m}{n}$

Squaring both sides

$2 + 2\sqrt{2}\sqrt{3} + 3 = m^2/n^2$

$(5 + 2\sqrt{2}\sqrt{3})n^2 = m^2$

$5n^2 + 2\sqrt{2}\sqrt{3}\,n^2 = m^2$

Here common factor exist. Our assumption is wrong. Therefore $\sqrt{2}+\sqrt{3}$ is irrational

Q24). a). i). $(4,2),(2,4),(1,5),(5,1) \Rightarrow \frac{4}{36} = \frac{1}{9}$

ii). $(4,3),(3,4),(6,1),(1,6),(5,2),$
$(2,5) \Rightarrow \frac{6}{36} = \frac{1}{6}$

iii). $(1,4),(4,1),(2,4),(4,2),(3,4),(4,3)$
$= \frac{6}{36} = \frac{1}{6}$

b). Non-distinct digits lessthen 100:
$\{11, 22, 33, 44, 55, 66, 77, 88, 99\} \Rightarrow 9$
$P = \frac{\text{Event}}{\text{Sample Space}} = \frac{9}{99} = \frac{1}{11}$

Q25). a). Basic Step $= n \Rightarrow 1$
$n^2 = \frac{n(n+1)(2n+1)}{6}$

$1 = \frac{1(1+1)(2(1)+1)}{6}$

$\boxed{1 = \frac{6}{6} \Rightarrow 1}$

Hence $P(n)$ is true for $n = 1$

Inductive Step: $n = k$

$$1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Put $n = k+1$

$$1^2 + 2^2 + \dots + (k+1)^2 = \frac{k(k+1)(2k+1)}{6}$$

$$(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= (k^2+3k+2)(2k+3)/6$$

$$= 2k^3+3k^2+6k^2+9k+4k+6/6$$

$$= \frac{2k^3+9k^2+13k+6}{6}$$

Add $(k+1)^2$ on both sides

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2 =$$

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$\frac{k(k+1)(2k+1) + (k^2+2k+1)(6)}{6}$$

$$\frac{(k^2+k)(2k+1) + (6k^2+12k+6)}{6}$$

$$\frac{(2k^3+3k^2+k) + (6k^2+12k+6)}{6}$$

$$= \frac{2k^3+9k^2+13k+6}{6}$$

Hence $P(n)$ is true for $n = k+1$

Q25). b). Basic Step = $n=0$

$2^n = 2^{n+1}-1$

$2^0 = 2^{0+1}-1$

$2^0 = 2-1$

$\boxed{1 = 1}$

Inductive Step: $n=k$

$1+2+2^2+\dots+2^k = 2^{k+1}-1$

Put $n=k+1$

$1+2+\dots+2^{k+1} = 2^{k+1}-1$

Add $2^{k+1}$ on both sides

$1+\dots+2^k+(2^{k+1}) = 2^{k+1}-1+2^{k+1}$

$= 2^{k+2}-1$

$= 2^k \cdot 2^2 - 1$

$2^{k+1} = 2^{k+2}-1$

$= 2^k \cdot 2^2 - 1$

$\neq$