1

Salman 19K-1043 BS SE (A)

Discrete Final Exam

"I pledge on my honor that I will not give or receive any unauthorized assistance on this examination".

Q # 1

(i)

a). $a \vee b$

b). $\neg c \rightarrow \neg b$

c). $c \rightarrow d$

d). $\neg a$

(ii)

a). $\cancel{a \vee b}$ $\cancel{a \vee b}$

~~p a ∴ Using simplification~~

b). $\cancel{\neg c \rightarrow \neg b}$

$\Rightarrow (a \vee b) \wedge (\neg c \rightarrow \neg b) \wedge (c \rightarrow d) \wedge \neg a$

$\Rightarrow (a \vee b) \wedge \neg a \wedge (\neg c \rightarrow \neg b) \wedge (c \rightarrow d)$

$\Rightarrow b \wedge (\neg c \rightarrow \neg b) \wedge (c \rightarrow d)$ ∵ ~~Simpli~~ Disjunctive Syllogism

$\Rightarrow (b \rightarrow c) \wedge b \wedge (c \rightarrow d)$ ∵ Contrapositive

$\Rightarrow c \wedge (c \rightarrow d)$ ∵ Modus Ponen

d ∵ Modus Ponen

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(iii)

Contrapositive : $b \to c$
"If Ali is enrolled in BS Computer Science, then Ali studies Discrete Structures".

Inverse: $\neg c \to \neg b$ => $c \to b$
"If Ali studies Discrete structures, then he is enrolled in BS Computer Science

Converse of its inverse : $b \to c$
"If Ali is enrolled in BS Computer Science, then Ali studies Discrete Structures"

b,	c,	contrapositive $b \to c$	inverse $c \to b$	converse $b \to c$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

∴ Proved that contrapositive is equivalent to converse of its inverse

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(iv)

$((p \vee q) \wedge (p \to r)) \to (q \vee r)$

$\because p \to q = \neg p \vee q$

$\neg[(p \vee q) \wedge (p \to r)] \vee (q \vee r)$

$\neg(p \vee q) \vee \neg(\neg p \vee r) \vee (q \vee r)$ $\because$ Implication

$(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \vee r)$ $\because$ De-Morgan,

$\because$ Negation

$(\neg p \wedge \neg q) \vee [((p \wedge \neg r) \vee q) \vee (p \wedge \neg r) \vee r]$ $\because$ Distributive

$(\neg p \wedge \neg q) \vee [(p \vee q) \wedge (q \vee \neg r) \vee (p \vee r) \wedge (r \vee \neg r)]$

$\because$ Distrib

$(\neg p \wedge \neg q) \vee [(p \vee q) \wedge (q \vee \neg r) \vee (p \vee r) \wedge T]$ $\because$ Negati

$(\neg p \wedge \neg q) \vee (p \vee q) \wedge (q \vee \neg r) \vee (p \vee r)$ $\because$ Identity

$(\neg p \wedge \neg q) \vee (p \vee q) \wedge (p \vee q) \vee (r \vee \neg r)$ ~~$\because$ Comt~~

$(\neg p \wedge \neg q) \vee (p \vee q) \vee T$ $\because$ Negatin

$(\neg p \wedge \neg q) \vee T$ $\because$ Universal

T $\because$ Universal

$\therefore$ Proved that it is Tautology.

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Q#2

(i)

(a) ~~$\neg \forall x\, J(x)$~~ $\neg \forall f\, P(f)$

(b) $\neg \exists f\, Q(f, \text{Bilal})$

(ii)

(a) There is a student in your class who has not whatsapp.

(b) There is a student in your class who has chatted with everyone over the whatsapp.

(iii)

(a). There is a building in lahore such that it has greater area then all building ~~of~~ in Karachi.

True. Lahore Shai Fort has greater area then all building in Karachi.

(b) All buildings in Karachis has more books written than All buildings that are exactly 1546 sq. ft.

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False. There are not all books of Karachi that are ^more^ written then all books of building exactly 1546 sq ft

Q#3

(i)

(a) B and C are subset of A
C is also subset of D.

(b) Cardonality : A = 3 , B = 2, C = 2, D = 3

(ii)

(a) A = 2500 , B = 3000 , $A \cap B = 1000$

$|A| + |B| - |A \cap B| = 2500 + 3000 - 1000$
$= 4500$

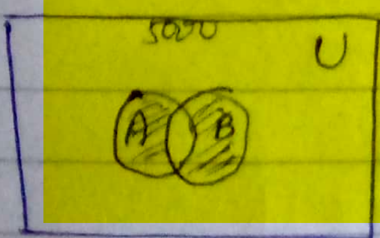
(b) A = 2500, B = 3000 , $A \cap B = 1000$

$|A| + |B| - |A \cap B| = 2500 + 3000 - 1000$
$= 4500$

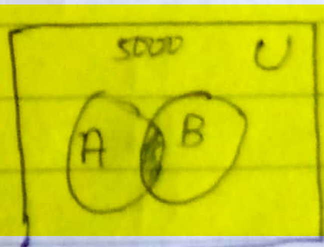
$= 5000 - 4500$

$= 500$ (Not diagnosed with penumia or bronchitis)

(a)



(b)



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(iii)

$P-(Q\cap R) = (P-Q)\cap(P-R)$

$\because$ $A-B = A\cap\bar{B}$

Apply on b.s

$P\cap\overline{(Q\cap R)} = (P\cap\bar{Q})\cap(P\cap\bar{R})$

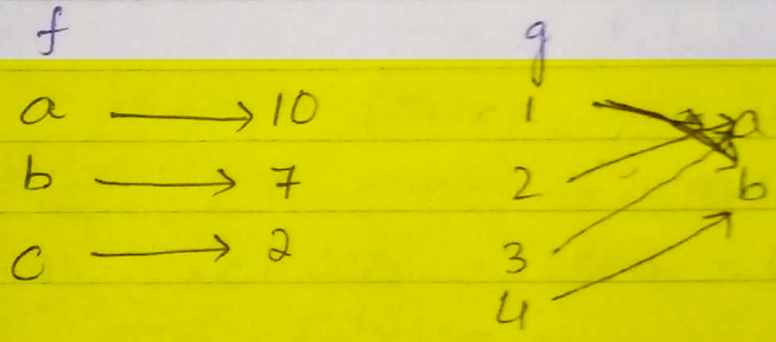
$P\cap\overline{(Q\cap R)} = P\cap(\bar{Q}\cap\bar{R})$ $\because$ Comutative

$P\cap\overline{(Q\cap R)} \neq P\cap(\overline{Q\cup R})$

$\therefore$ Disproved!

(iv)

(a)



f is invertible but g does not because it is not bijective function

$f^{-1} = \{(10,a),(7,b),(2,c)\}$

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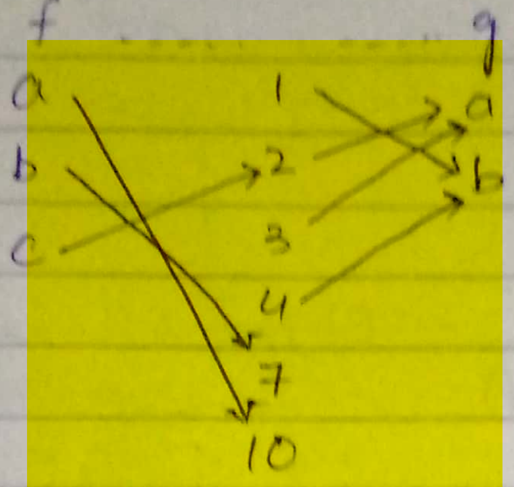
(v)



f o g	g o f
$c \to a$	$a \to c$

Q # 4

(i)

~~(a)~~

$a_1 = 2$, $m_1 = 7$, $a_2 = 3$ $m_2 = 17$

$a_3 = 5$, $m_3 = 19$

$m = m_1 \times m_2 \times m_3 = 7 \times 17 \times 19 = 2261$

$M_1 = \frac{m}{m_1} = \frac{2261}{7} = 323$

$M_2 = \frac{m}{m_2} = \frac{2261}{17} = 133$

$M_3 = \frac{m}{m_3} = \frac{2261}{19} = 119$

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$y_k = \overline{M_k} \bmod m_k$

$y_1 = \overline{323} \bmod 7$

$a = qd + r$

$323 = (46)7 + 1$

Backward :-

$1 = 1 \cdot 323 + (-46)(7)$

$y_1 = 1$

$y_2 = \overline{133} \bmod 17$

$a = qd + r$

$133 = (7)17 + 14$

$17 = (1)14 + 3$

$14 = (4)3 + 2$

$3 = (1)2 + 1$

Backward :-

$1 = 1 \cdot 3 - 1 \cdot 2$

$= 1 \cdot 3 - 1 \cdot (1 \cdot 14 - 4 \cdot 3)$

$= 1 \cdot 3 - 1 \cdot 14 + 4 \cdot 3$

$= 5 \cdot 3 - 1 \cdot 14$

$= 5 \cdot (1 \cdot 17 - 1 \cdot 14) - 1 \cdot 14$

$= 5 \cdot 17 - 5 \cdot 14 - 1 \cdot 14$

$= 5 \cdot 17 - 6 \cdot 14$

$= 5 \cdot 17 - 6 \cdot (1 \cdot 133 - 7 \cdot 17)$

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$= 5 \cdot 17 - 6 \cdot 133 + 42 \cdot 17$

$= 47 \cdot 17 + (-6)(133)$

$\therefore -6 + 17 = 11$

$\boxed{y_2 = 11}$

$y_3 = 119 \bmod 19$

$a = qd + r$

$119 = (6)19 + 5$

$19 = (3)5 + 4$

$5 = (1)4 + 1$

Backword :-

$1 = 1 \cdot 5 - 1 \cdot 4$

$= 1 \cdot 5 - 1 \cdot (1 \cdot 19 - 3 \cdot 5)$

$= 1 \cdot 5 - 1 \cdot 19 + 3 \cdot 5$

$= 4 \cdot 5 - 1 \cdot 19$

$= 4 \cdot (1 \cdot 119 - 6 \cdot 19) - 1 \cdot 19$

$= 4 \cdot 119 - 24 \cdot 19 - 1 \cdot 19$

$= (4)(119) + (-25)(19)$

$\boxed{y_3 = 4}$

Using Chinese Remainder theorem:-

$x = (a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3) \bmod m$

$= [(2)(323)(1) + (3)(133)(11) + (5)(119)(4)] \bmod m$

$x = 7415 \bmod 2261 = 632$

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(ii)

Number of balls = 632

Six hundred and thirty two

18, 8, 23, 7, 20, 13, 3, 17, 4, 3, 0, 13, 3

19, 7, 8, 17, 19, 24, 19, 22, 14

Ceaser Cipher

$f(p) = (p+3) \bmod 26$

~~21 mod 26, 8 m 11 mod 26,~~

21, 11, 26, 10, 23, 16, 6, 20, 7, 6, 3, 16, 6

22, 10, 11, 20, 22, 27, 22, 25, 17

21 mod 26 = 21 , 11 mod 26 = 11

26 mod 26 = 0 , 10 mod 26 = 10

23 mod 26 = 23 , 16 mod 26 = 16

6 mod 26 = 6 , 20 mod 26 = 20

7 mod 26 = 7 , 6 mod 26 = 6

3 mod 26 = 3 , 16 mod 26 = 16

6 mod 26 = 6 , 22 mod 26 = 22

10 mod 26 = 10 , 11 mod 26 = 11

20 mod 26 = 20 , 22 mod 26 = 22

27 mod 26 = 1 , 22 mod 26 = 22

25 mod 26 = 25 , 17 mod 26 = 17

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Encryption: VLAKXQGU~~H~~GDQGW
KLUWBWZR.

(iii)

$a = 6+3+2 = 11$

$11^{302} \bmod 7$

Using Fermat's: $a^{P-1} \equiv 1 \bmod P$

$11^{7-1} = 1 \bmod 7$

$11^{6} = 1 \bmod 7$

$11^{302} \bmod 7 =$

gcd $(302, 6) =$ $a = qd + r$

$302 = (50)\,6 + 2$

$= (11)^{50\times6+2} \bmod 7$

$= (11^{6})^{50} \cdot 11^{2} \bmod 7$

$= (1)\;11^{2} \bmod 7 = 121 \bmod 7 = 2.$

(iv)

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$x_{10}(Q) = \{(1\times1) + (2\times2) + (3\times5) + (4\times9) + (5\times7) + (6\times3) + (7\times1) + (8\times2) + (9\times8)\} \bmod 11$

$= 204 \bmod 11 = 6$

$Q = 6\times10 = 60$

$(204+60) \bmod 11 = 0.$ Validated.

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(v)

(a)

Using Combinations:-

$15C3 + 12C3 + 7C3 + 5C3 + 9C3 + 10C3$

$\because C = \frac{n!}{(n-r)!\,r!}$

$455 + 220 + 35 + 10 + 84 + 120$

$= 924$

(b)

Grades = 10 Student - 97

$P(\text{At Least}) = 1 - \frac{10}{97} = 0.897$

(vi)

(a)

Captain : $15P1 = \frac{15!}{(15-1)!} = 15$

Vice-Captain : $14P1 = \frac{14!}{(14-1)!} = 14$

Wickekeper : $15P2 = \frac{15!}{(15-2)!} = 210$

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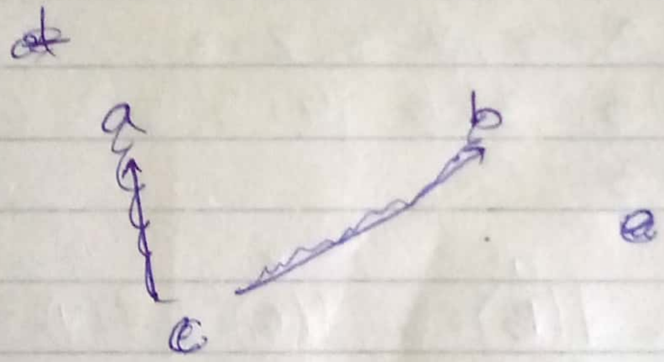
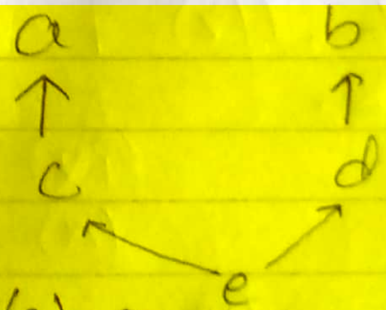
(b)

$$P(\text{not defective}) = 1 - P(\text{defective})$$
$$= 1 - \frac{15}{100} = \frac{17}{20} = 0.85$$

Q # 5

(i)





$deg^-(a) = 1 \quad deg^+(a) = 0$

$deg^-(b) = 1 \quad deg^+(b) = 0$

$deg^-(c) = 1 \quad deg^+(c) = 1$

$deg^-(d) = 1 \quad deg^+(d) = 1$

$deg^-(e) = 0 \quad deg^+(e) = 2$

(14)

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(ii)

{(e, c), (e, d), (c, a), (d, b)}

Partial Relation because no symetric is there.

(iii)

Edges = 8.

degrees = A = 3, B = 3, C = 3, D = 3

W = 3, X = 3, Y = 3, Z = 3

1 = 3, 2 = 3, 3 = 3, 4 = 3, 5 = 3, 6 = 3

7 = 3, 8 = 3

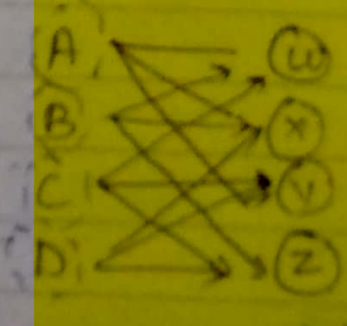
f(A) = 1, f(B) = 2, f(C) = 3 f(Y) = 5

f(B) = 6, f(Z) = 7, f(C) = 4

f(D) = 8.

(iv)

(a)



Complete Bipartite Graph because every adjacent node is different.

15

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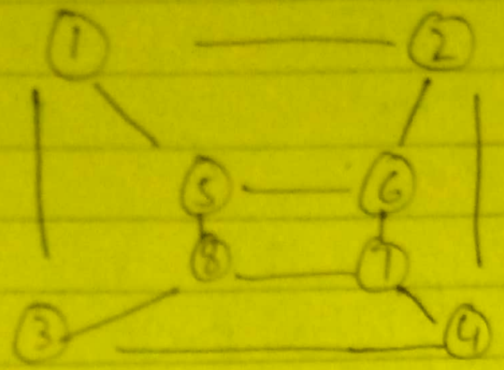
(b)



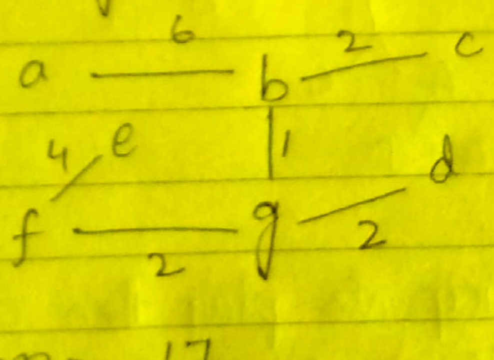
Euler Circuit = No because degrees are not even. All degrees are odd.

Hamilton Circuit: 1, 2, 4, 3, 8, 7, 6, 5, 1

Valid Circuit.

(v)

~~b.g.~~ Using Prims Algorithm



Minimum = 17.

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Q#6

(i)

(a)

$x^7 y^{10}$ Coefficial = ?

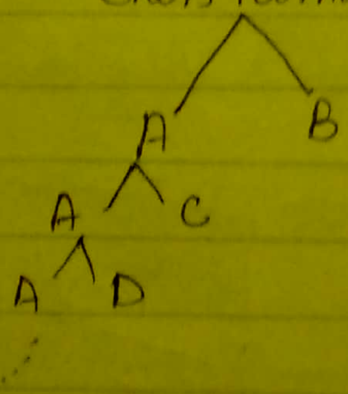
$$(3x-2y)^{17} = \sum_{j=0}^{17} \binom{17}{j} (3x)^{17-j} (-2y)^{j}$$

$$= \binom{17}{10} (3x)^{17-10} (-2y)^{10}$$

$$= 19448\,(3x)^{7} (-2y)^{10}.$$

(b)

Chess tournament



If two participants play then one must lose $1000 - 1 = 999$.

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$\binom{10}{11}$

(a)

$4(z^2+z+1)-3z^2$

~~4(z~~ let $m=z$

$4m^2+4m+4-3m^2$

m^2+4m+4

$(m)^2+2(2)(m)+(2)^2$

$(m+2)^2$

∴ z is perfect square.

(b)

$a=2r$ and $b=2k$

$\frac{a\times b}{4}=\frac{(2r)(2k)}{4}=\frac{4rk}{4}=$

$=\frac{4(rk)}{4}=4\left(\frac{rk}{16}\right)$

∵ $\frac{rk}{16}=p$

$=4p$. Proved!

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18

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(iii)

Basic Step = $n = 1$

$$(1)^2 = \frac{(1)(1+1)(2(1)+1)}{6}$$

$$\boxed{1 = 1}$$

Inductive Steps-

Put $n = k$

$$1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

$(k+1)^2$ Put :- Add both

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left\{\cancel{(k+1)}\,\frac{k(2k+1) + (k+1)}{6}\right\}$$

$$= (k+1)\left\{\frac{2k^2 + k + 6k + 6}{6}\right\}$$

$$= (k+1)\left\{\frac{2k^2 + 7k + 6}{6}\right\}$$

$$= (k+1)\left\{\frac{2k^2 + 4k + 3k + 6}{6}\right\}$$

RHS

$k+1$:-

$$\frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\frac{(k+1)(k+2)(2k+3)}{6}$$

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$$= (k+1)\left\{\frac{2k(k+2)+3(k+2)}{6}\right\}$$

$$= (k+1)\left\{\frac{(2k+3)(k+2)}{6}\right\}$$

$$= \frac{(k+1)(2k+1+1)(k+1+1)}{6}$$. Proved!

Selm.