Introduction

Linear Circuit Analysis

EECE 202

Number of credits: *3 credits*
Prerequisites: *EECE 201*
Contact Hours: *3 hrs Lecture*

Textbook/material required: *R. A. DeCarlo and P-M. Lin, "Linear Circuit Analysis; The Time Domain, Phasor and Laplace Transform Approach", 3rd Edition, 2009.*

Learning Outcomes

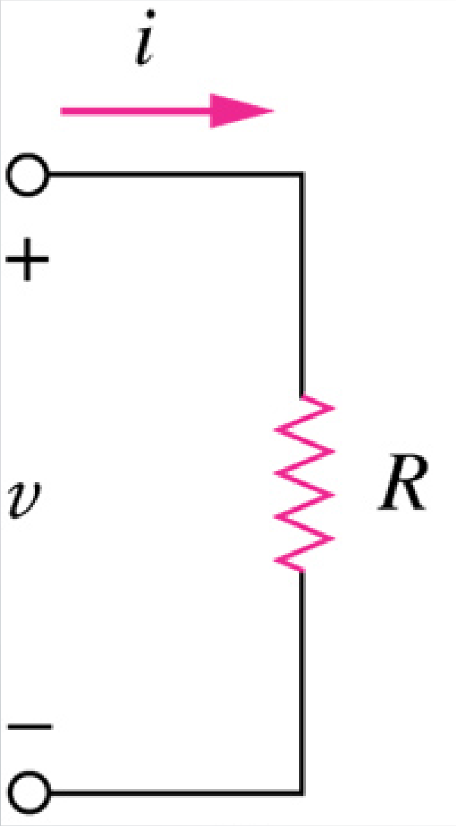
By the end of this course, the student should be able to:

1. *An ability to compute impedances and admittances of components and circuits. SO[1]*
2. *An ability to compute responses of linear circuits with and without initial conditions via one-sided Laplace transform techniques. SO[1]*
3. *An ability to compute responses to linear circuits using transfer function and convolution techniques. SO[1]*
4. *An ability to analyze and compute responses of linear circuits containing mutually coupled inductors and ideal transformers in the s-domain. SO[1]*
5. *An ability to analyze basic two port circuits using the various types of two port parameters and be able to construct such parameters from a given circuit. SO[1]*
6. *An ability to analyze and design basic LP, BP, HP and resonant circuits in the s-domain. SO[1]*
7. *an ability to work within a team, develop hands-on experience, draw conclusion and communicate results through the offered course project. SO[2,3,5]*

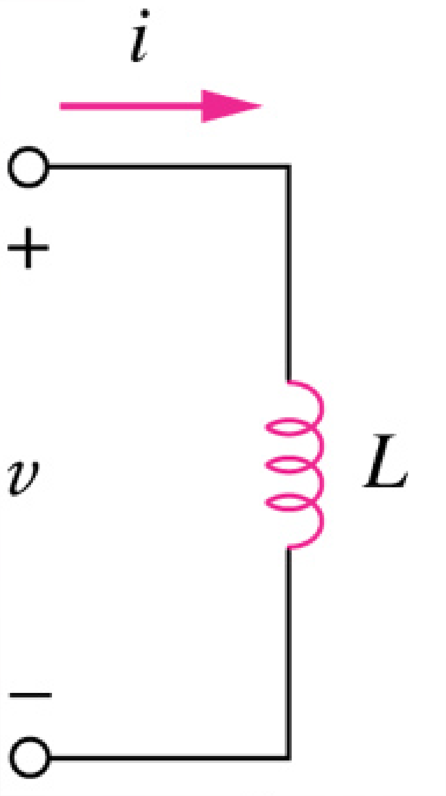
	Assessment		***Weight***
Attendance			5%
Assignments	PD1 – Conceptual Design Presentation (Week 10)		3 x 10%
	PD2 – Technical Report (10%) (Week 12)		
	PD3 – Prototype Demonstration (10%) (Week 14)		
Graded Class Activity (GCA)	GCA – 1 (Week 6, L2)		2 x 10%
	GCA – 2 (Week 11, L2)		
Moodle Quiz	Quiz (Week 4)		5%
Midterm	Week 8		20%
Final exam	Week 16		20%
		Total	**100%**

Revision on EE201

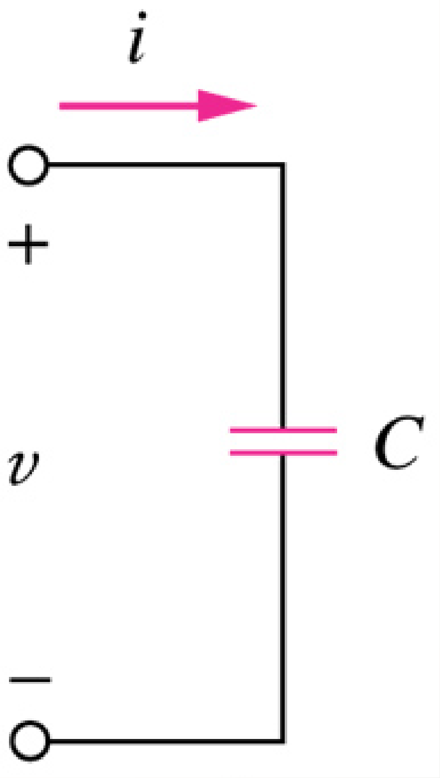
Passive Elements



Resistor



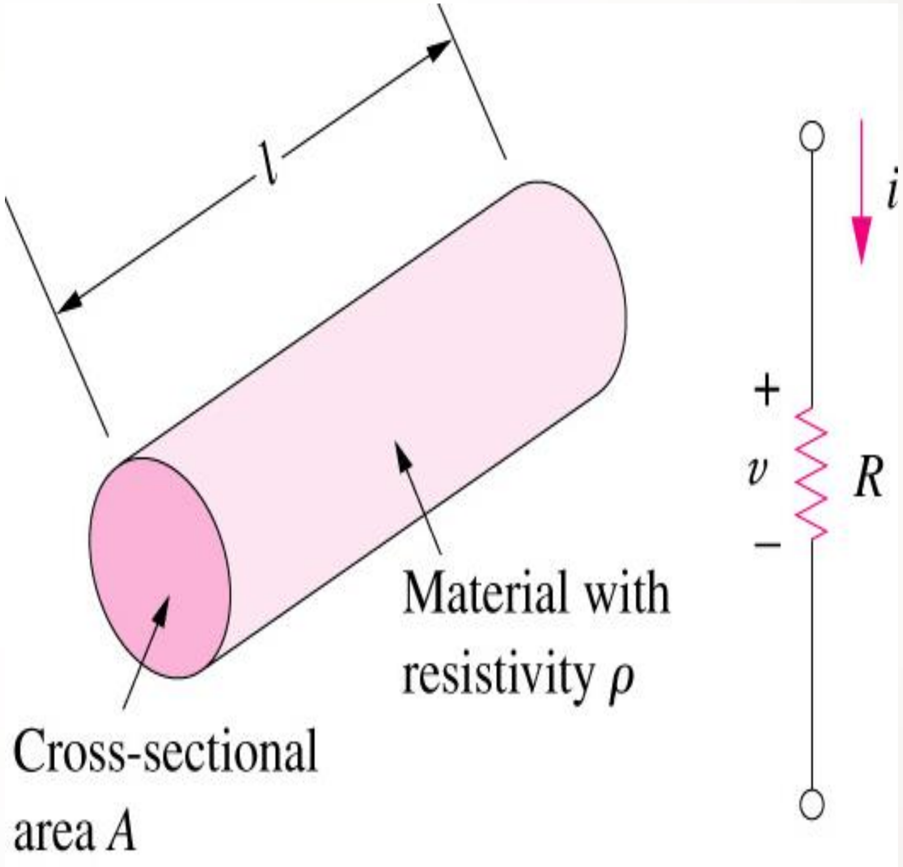
Inductor



Capacitor

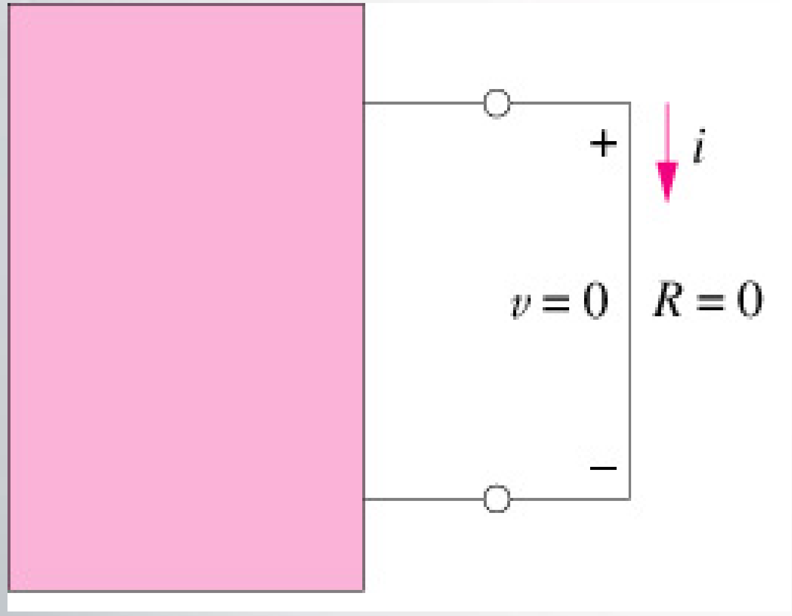
Ohm's Law

Ohm's law states that the voltage "V" across a resistor "R" is directly proportional to the current "I" flowing through the resistor

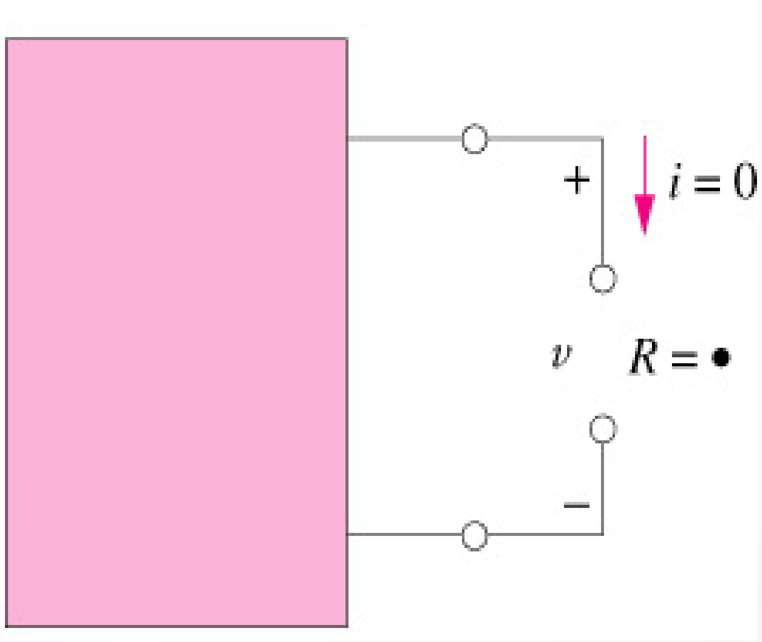


$$V = I \times R$$

As R varies from zero to infinity, it is necessary to study the relation between V and I at the extreme values of R

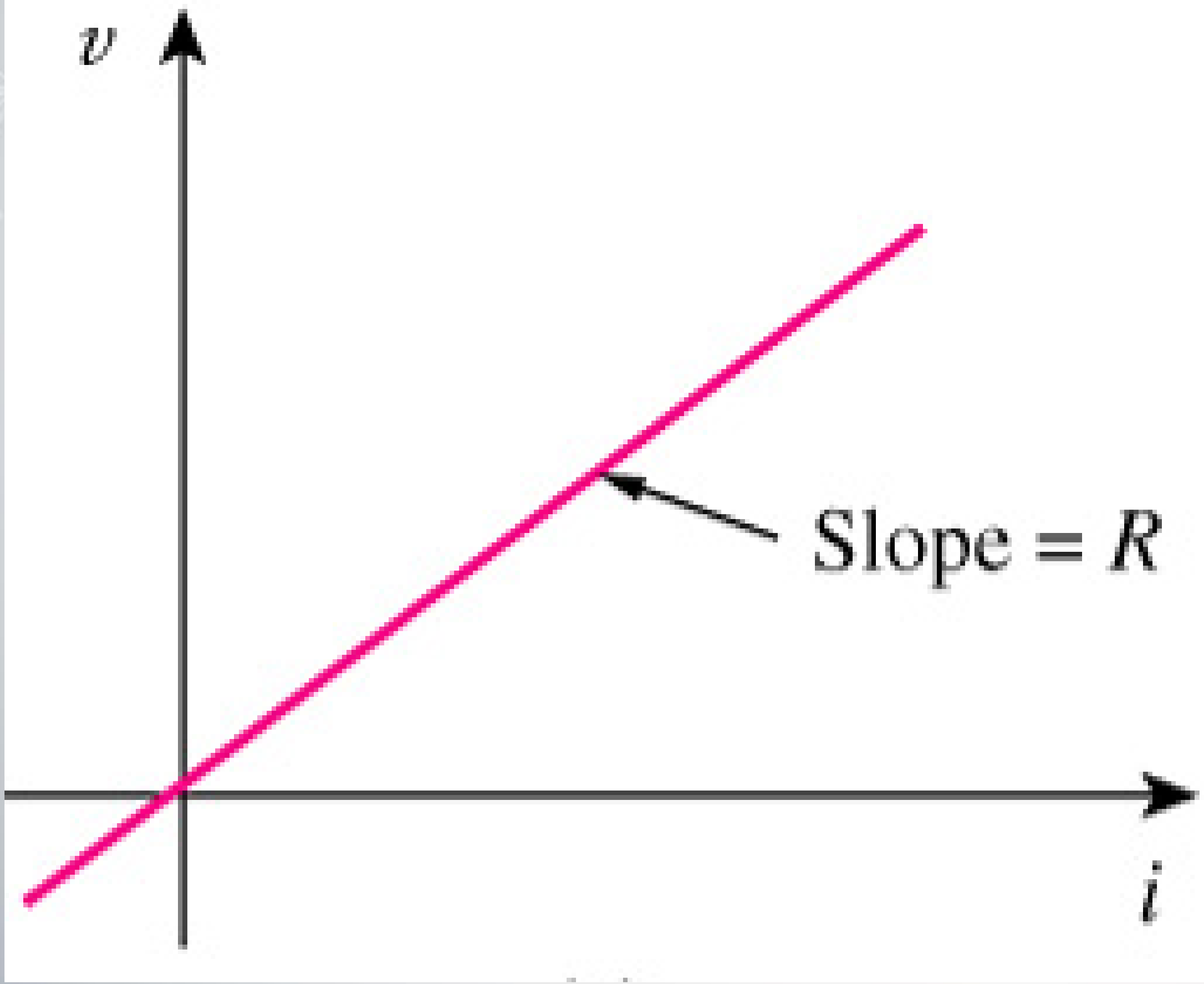


Short Circuit Case
R = zero, V = zero



Open Circuit Case
R = ∞, I = zero

V-I characteristics according to Ohm's law



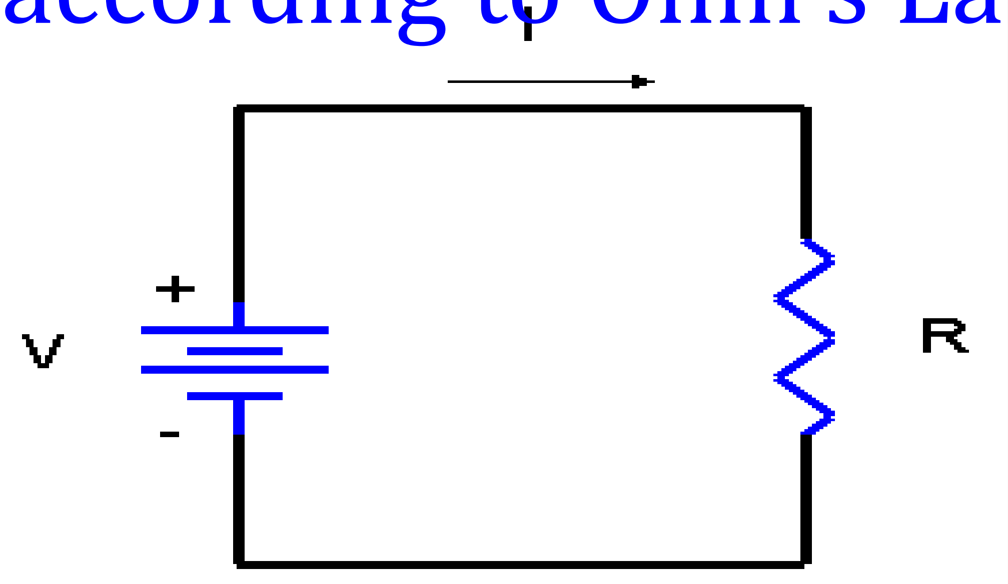
V = I×R

I = V/R

R = V/I

Power calculations according to Ohm’s Law

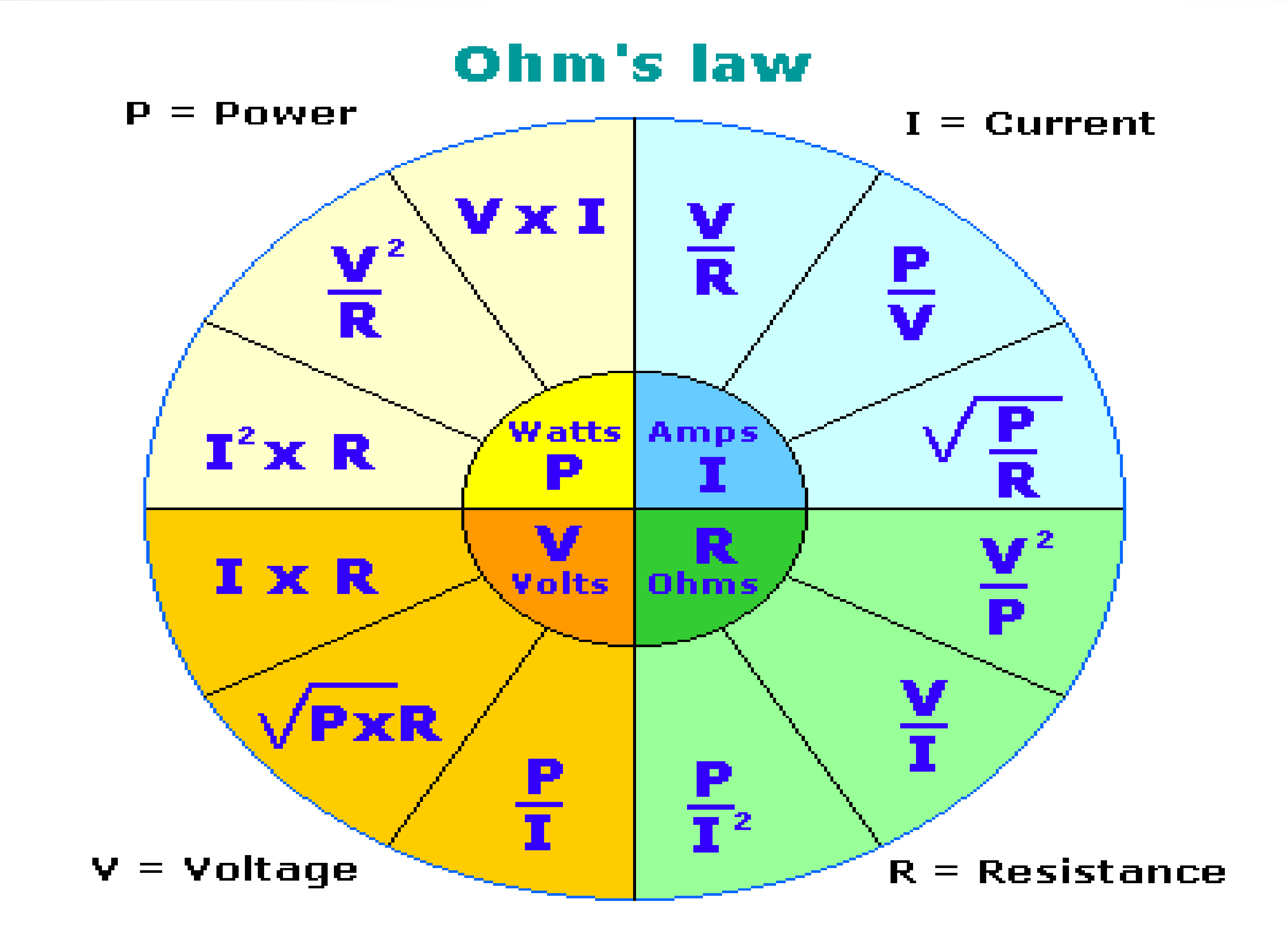
$$V = I \times R$$



$$P = I^2 \times R$$

$$P = V \times I$$

$$P = V^2 / R$$



Ohm's law
P = Power
I = Current
V = Voltage
R = Resistance
Watts
P
Amps
I
V
Volts
R
Ohms
V x I
$\frac{V^2}{R}$
I^2 x R
I x R
$\sqrt{PxR}$
$\frac{P}{I}$
$\frac{V}{R}$
$\frac{P}{V}$
$\sqrt{\frac{P}{R}}$
$\frac{V^2}{P}$
$\frac{V}{I}$
$\frac{P}{I^2}$

Example 7

A resistor formed of a copper conductor of length 0.1 m and cross-sectional area 4×10^{-4} m^2, the resistivity of copper is 1.72×10^{-8} Ω.m find the current flowing through the resistor when a 10 V battery is connected across its terminals.

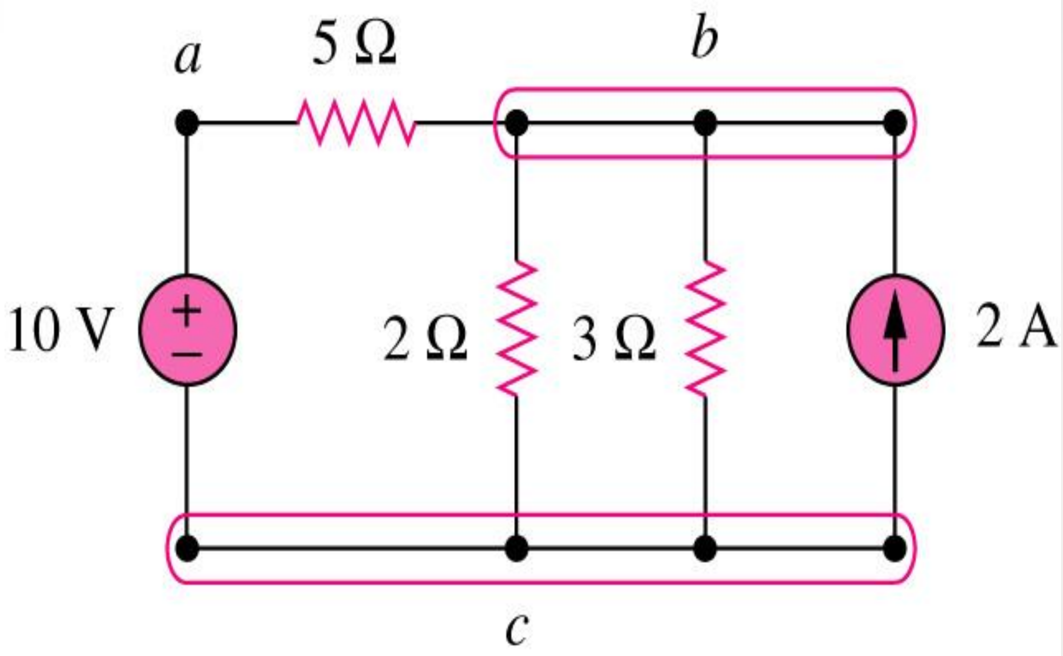
Solution

$$\text{Since } I = V/R$$

$$\text{And } R = \delta L/A = 1.72\times10^{-8} \times 0.1/4\times10^{-4} = 43\times10^{-7}\ \Omega$$

$$\text{Then } I = 10/43\times10^{-7} = 2.3\ \text{MA}$$

Nodes, Branches and Loops



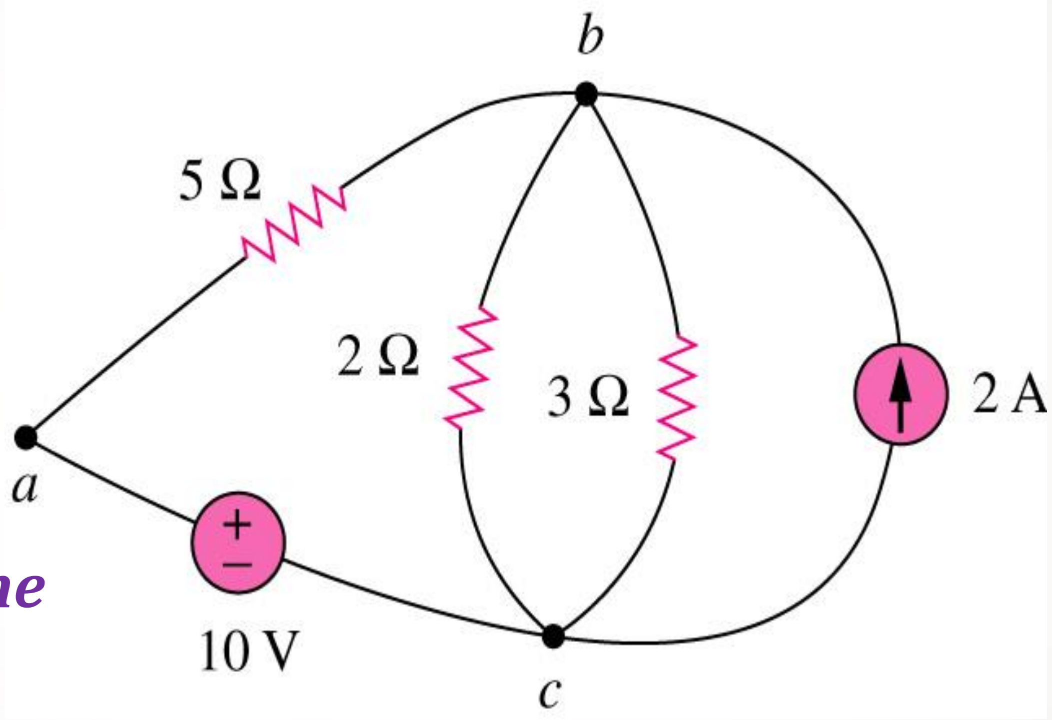
Branch: represents a single element such as a voltage source or a resistor

A branch represents any element with two terminals

(Vb-10)/5 - (Vb/2) - (Vb/3) – 2 = 0

There are five branches in the shown figure, namely the 10 V voltage source, the 2-A current source and the three resistors.

Nodes, Branches and Loops



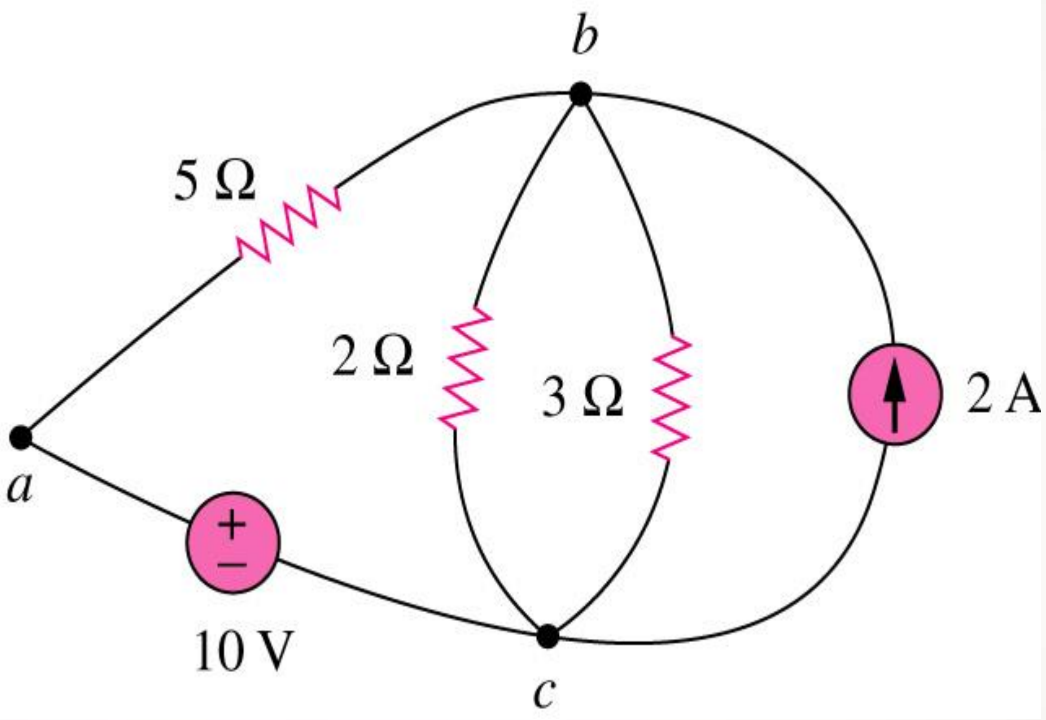
Node: is the point of connection between two or more branches

There are three nodes in the shown circuit, a, b and c

A node is usually indicated by a dot in a circuit, but if a short circuit connects two nodes, the two nodes constitute a single node

Nodes, Branches and Loops

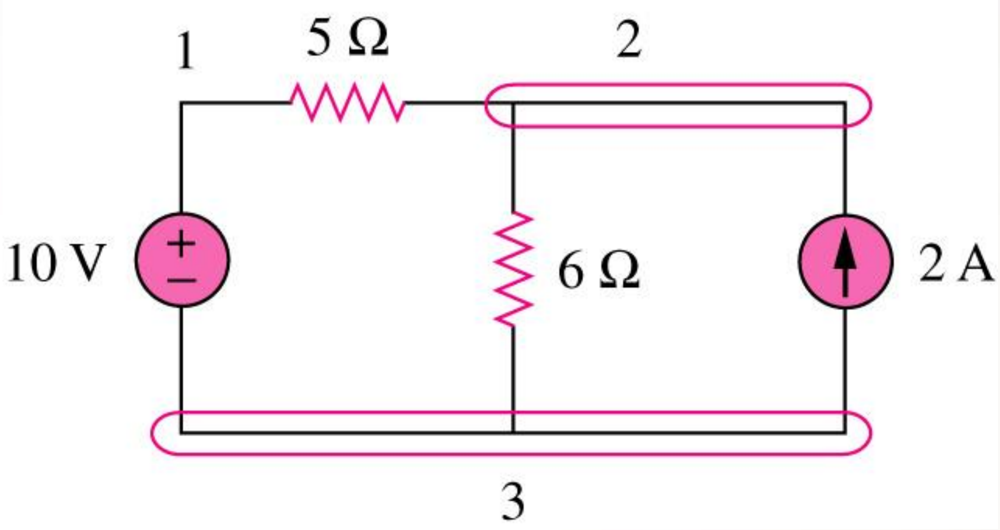
Loop: is a closed path formed by starting at a node, passing through a set of nodes and returning to the starting node without passing through any node more than once



abca is a loop, where the flow from b to c could be done via: the 2 ohm resistor, the 3 ohm resistor, ………………….etc

Example(8)

Determine the number of branches and nodes in the shown circuit



Solution

As the circuit has four elements, then there are *four branches* as follows:

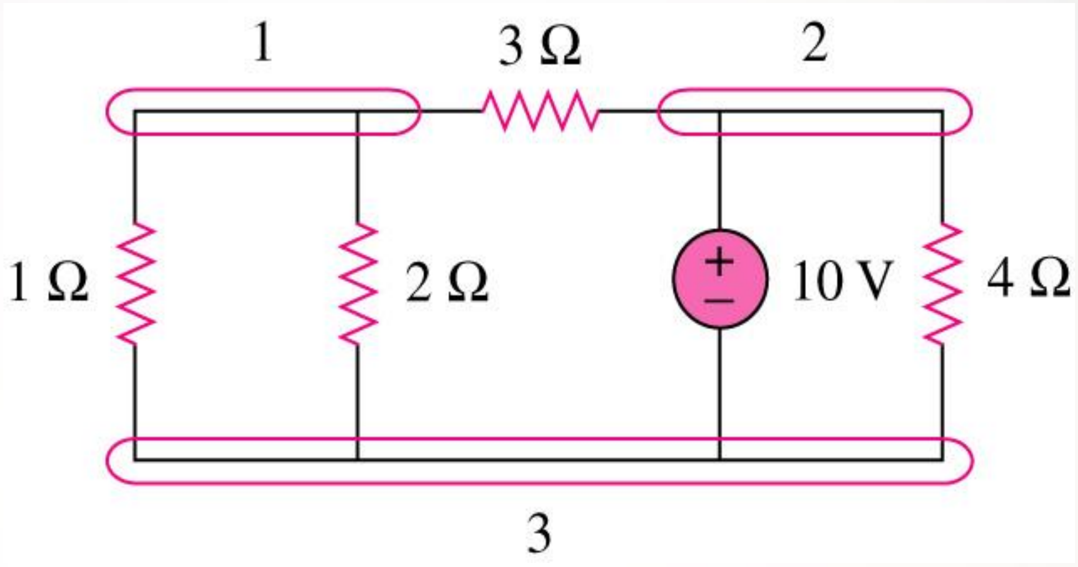
- ❑ the 10 V voltage source
- ❑ the 2A current source
- ❑ the 5 ohm resistor
- ❑ the 6 ohm resistor

There are *three nodes* in the circuit 1, 2 and 3

$$(V2-10/5) + (V2/6) - 2 = 0$$

Example(9)

Determine the number of branches and nodes in the shown circuit



Solution

As the circuit has five elements, then there are *five branches* as follows:

- ❑ the 10 V voltage source
- ❑ the 1ohm resistor
- ❑ the 2 ohm resistor
- ❑ the 3 ohm resistor
- ❑ the 4 ohm resistor

There are *three nodes* in the circuit 1, 2 and 3

Resistor's Arrangements

There are three possible arrangements for resistors in an electrical circuit

Series

Parallel

Series & Parallel

Resistors in Series

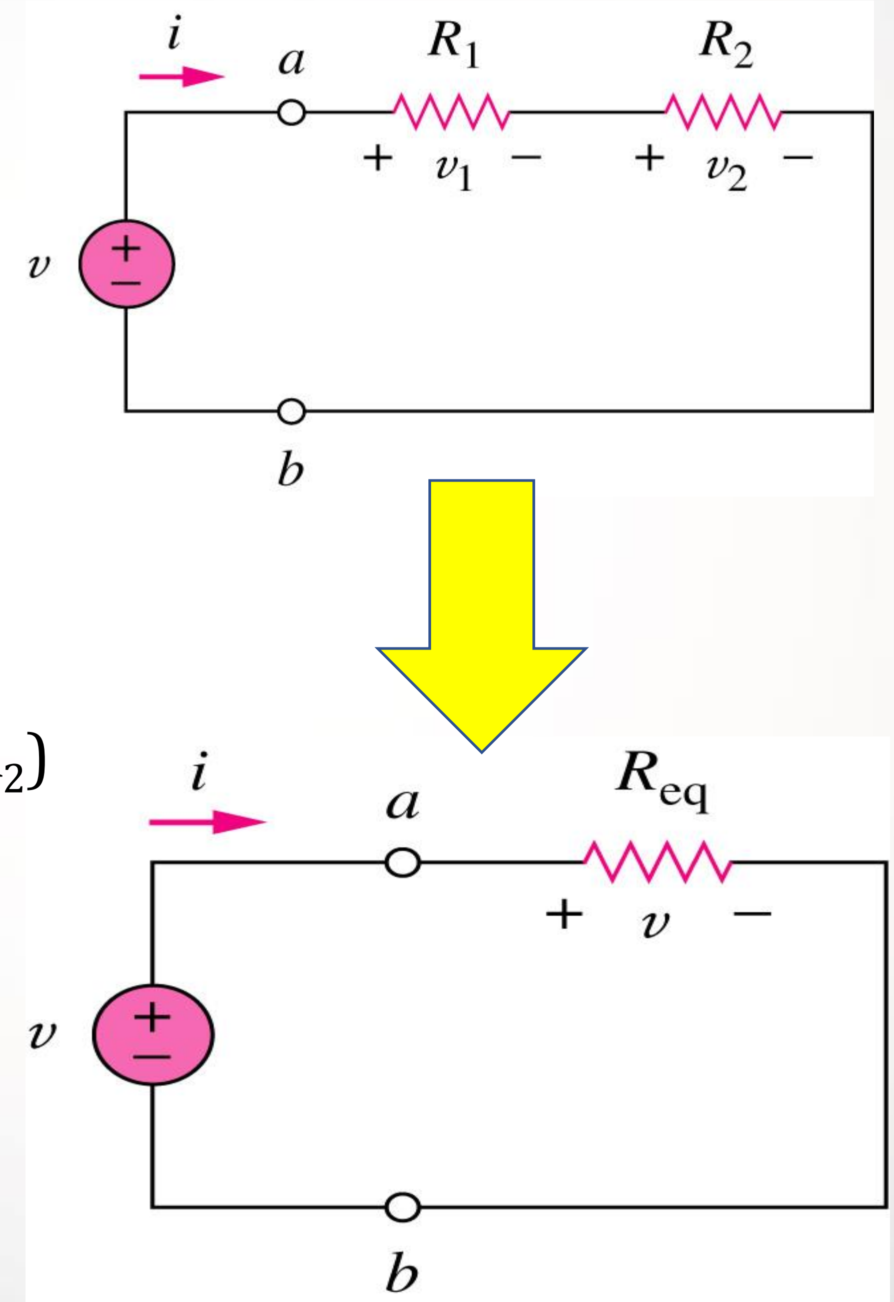
Two or more elements are said to be in *series* if they exclusively share a single node and consequently *carry the same current*

$V = V_1 + V_2 = I\,R_1 + I\,R_2 = I\,(R_1 + R_2)$

$V = I\,R_{eq}$

For N resistors placed in series

$R_{eq} = R_1 + R_2 + \ldots\ldots\ldots\ldots\ldots\ldots R_N$



Resistors in Parallel

Two or more elements are said to be in *parallel* if they are connected to the same two nodes and consequently have the *same voltage across them*

$$V = I_1R_1 = I_2R_2$$

But since $I = I_1 + I_2$, then

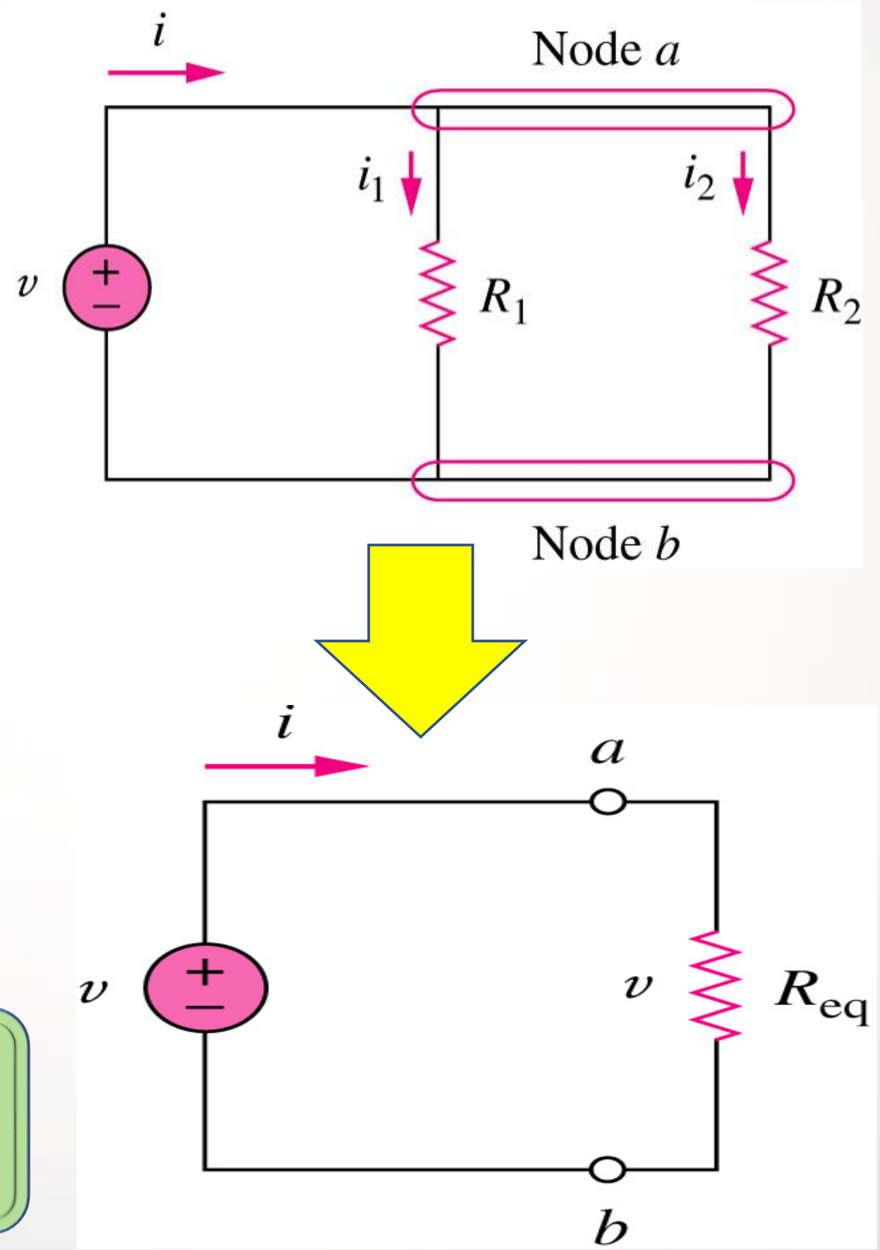
$$I = (V/R_1 + V/R_2) = V\,(R_1+R_2)/R_1R_2$$

and as, $R_{eq} = V/I$…then

$$R_{eq} = R_1R_2/(R_1+R_2)$$

For N resistors placed in parallel

$$1/R_{eq} = 1/R_1 + 1/R_2 + \ldots\ldots\ldots\ldots\ldots 1/R_N$$



Conductance “G”

The conductance is a measure of how well an element will conduct electric current

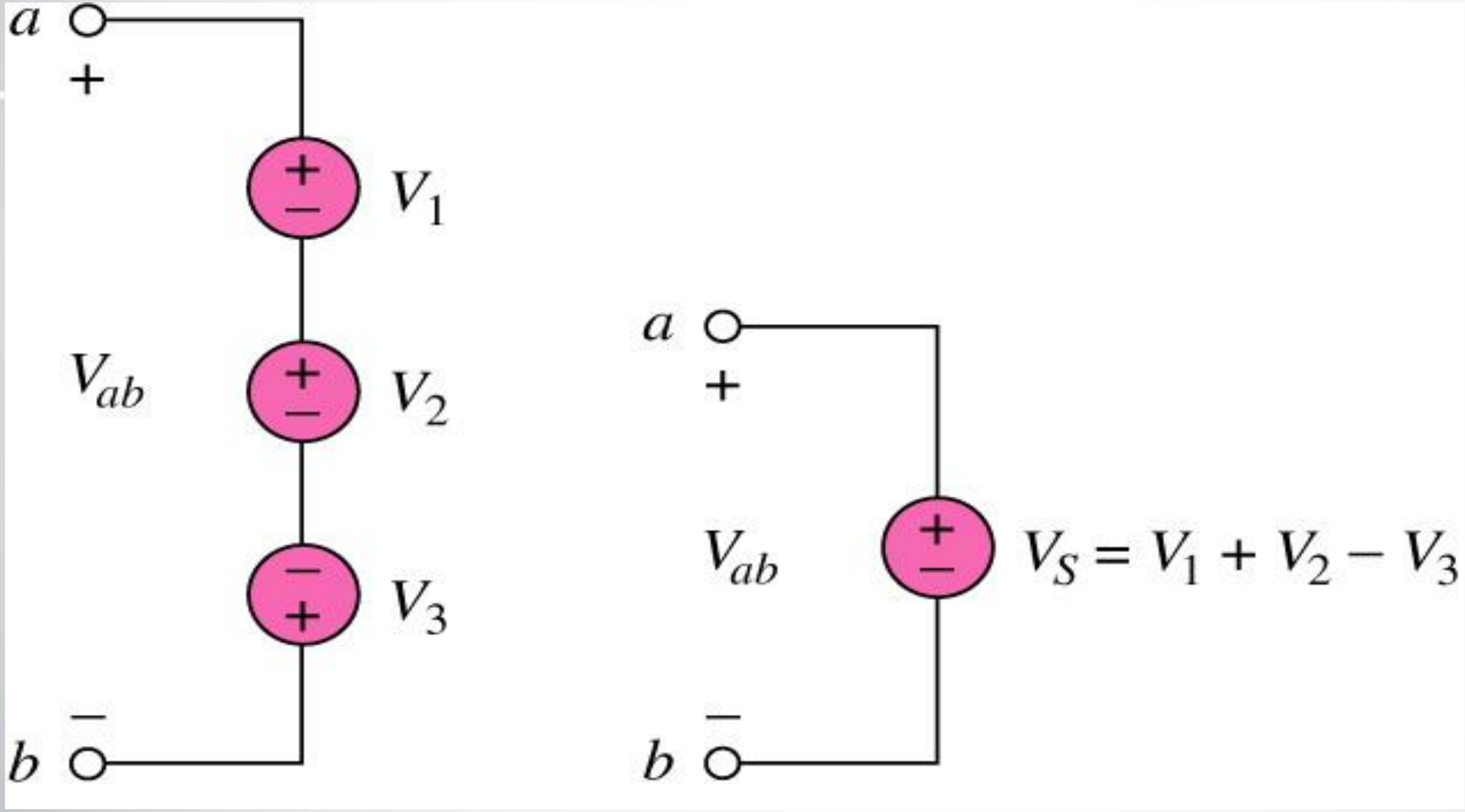
The unit of conductance is “Siemens” and with the symbol of “S”

$$G = 1/R = I/V \text{ (S)}$$

Note: the equivalent of N parallel resistors could be calculated using conductance, where:

$$G_{eq} = G_1 + G_2 + \ldots\ldots\ldots\ldots\ldots G_N$$

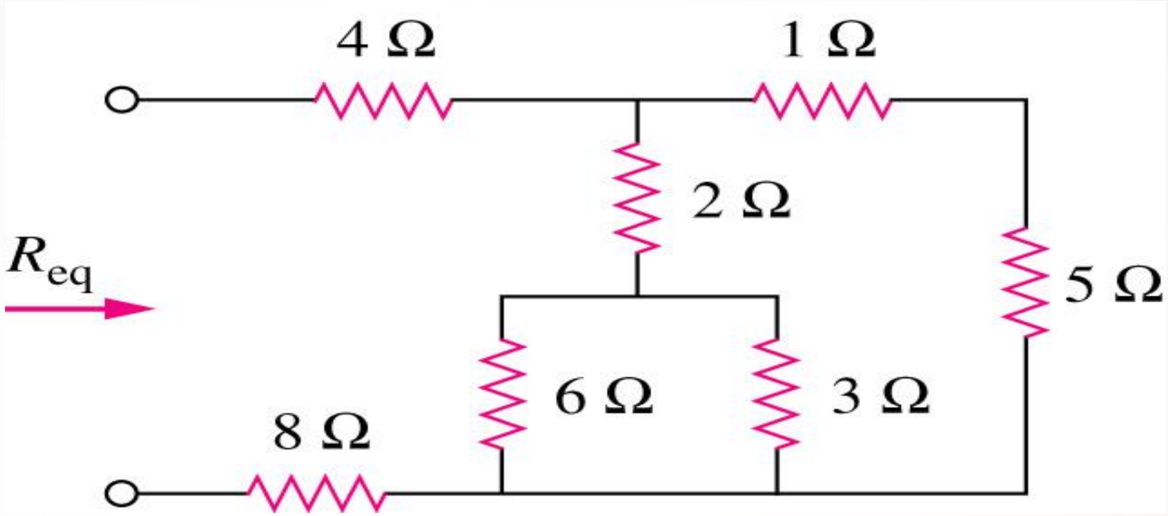
Voltage sources placed in Series



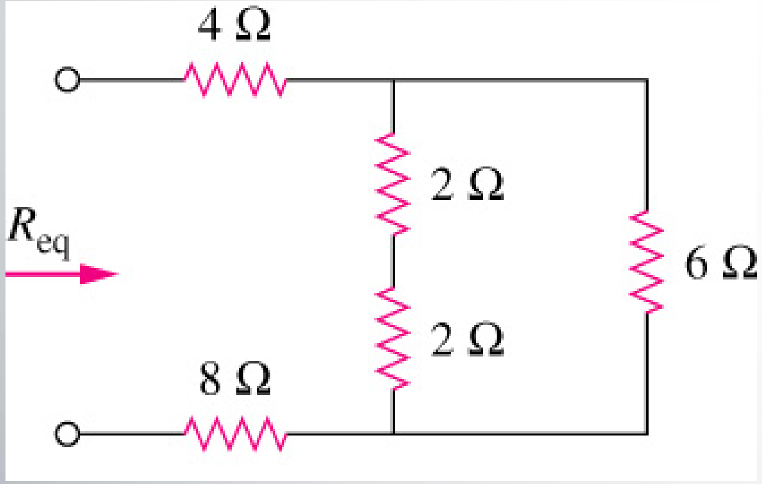
Note the voltage source sign when obtaining their equivalence

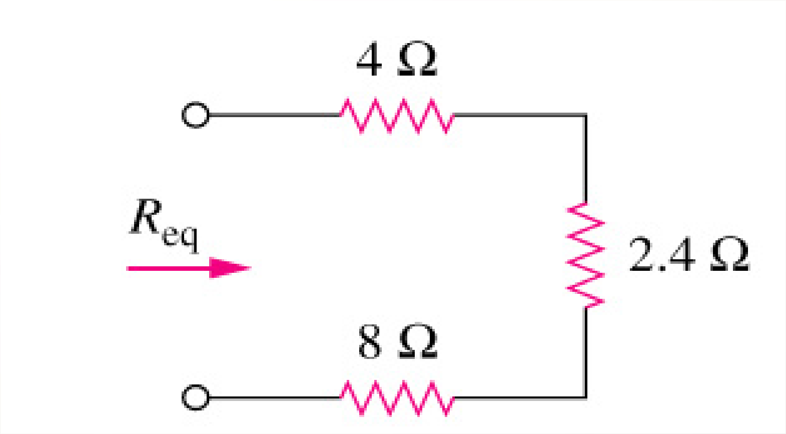
Example(10)

Find R_{eq} for the shown resistor arrangement



Solution

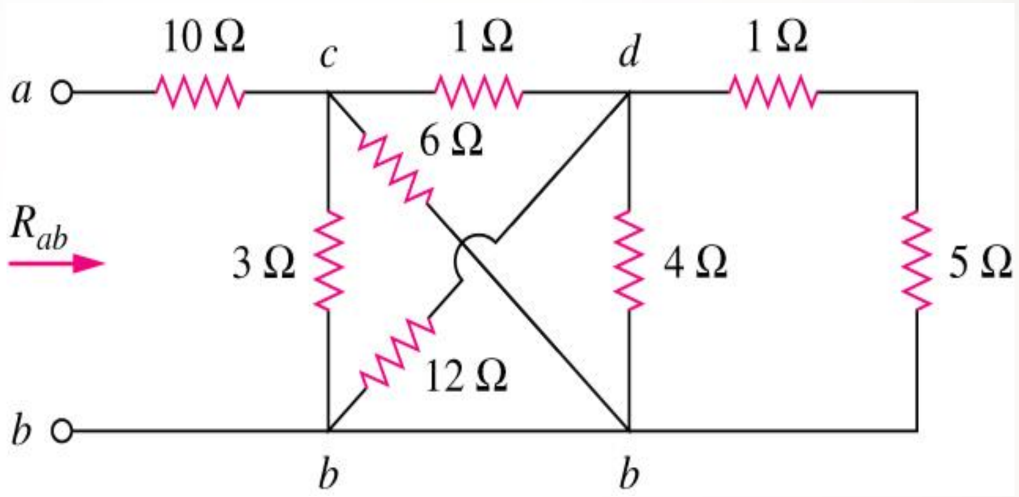




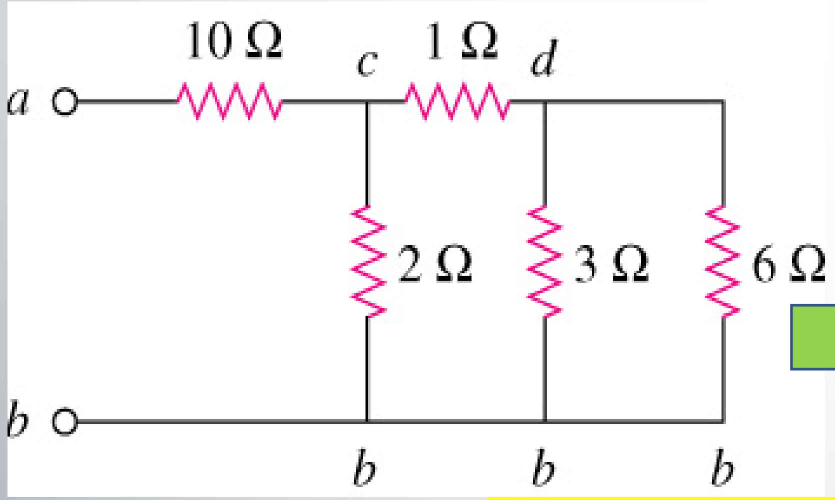
$R_{eq} = 4 + 8 + 2.4 = 14.4\ \Omega$

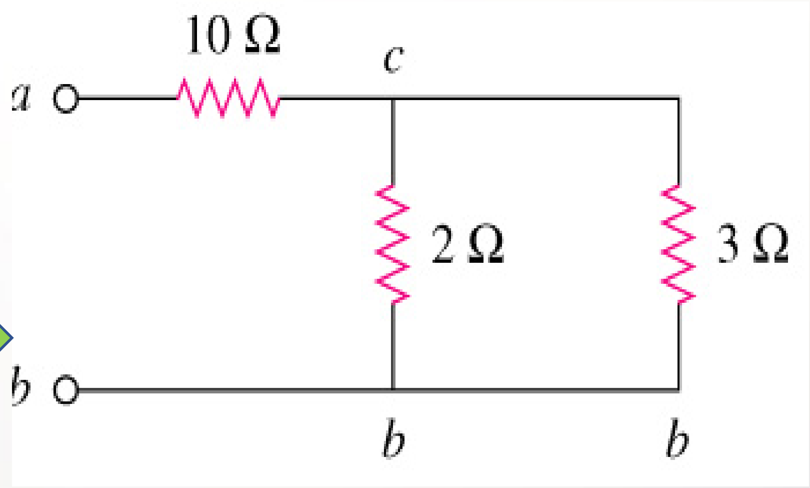
Example(11)

Find R_{eq} for the shown resistor arrangement



Solution

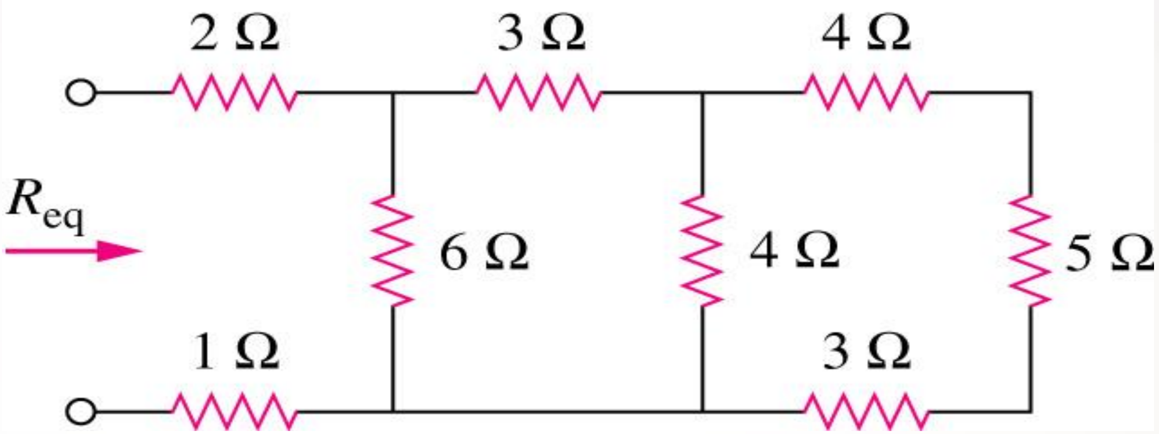




$R_{eq} = 10 + (3\times2)/(3+2) = 11.2\ \Omega$

Example(12)

Find R_{eq} for the shown resistor arrangement



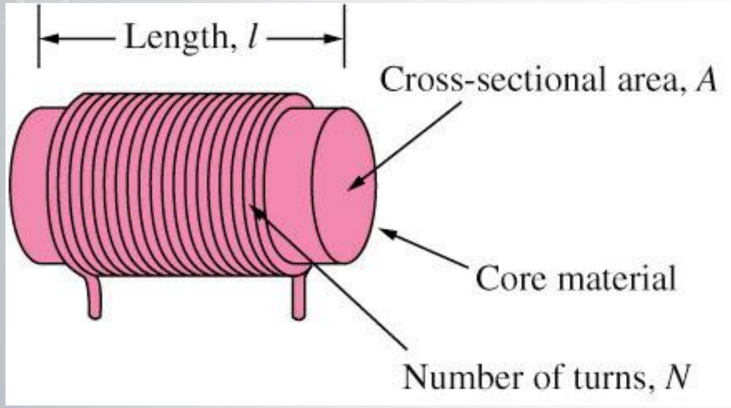
Solution

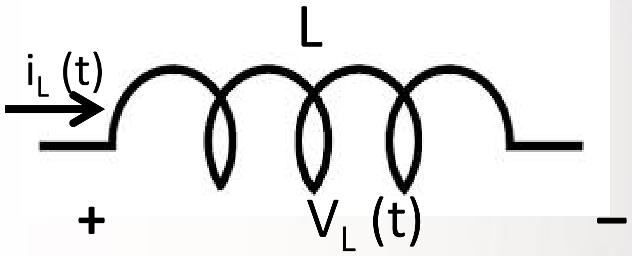
Try to solve

Final answer 6 Ω

Inductors

An inductor is a two terminal passive element designed to store energy in its magnetic field.





Follow passive sign convention

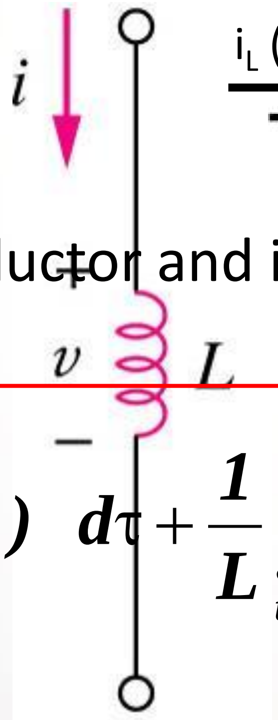
An inductor consists of a coil of conducting wire.

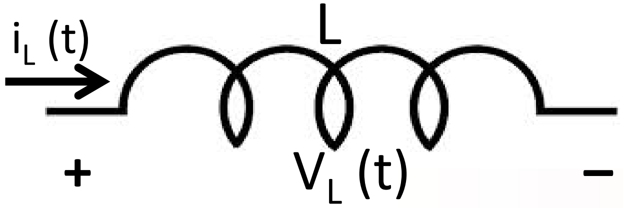
- **Inductors store electromagnetic energy.**
- **They may supply stored energy back to the circuit, but they cannot create energy.**
- **They must abide by the passive sign convention**

Current voltage relationship of an inductor

- The inductor voltage is proportional to the derivative of the current passing through it.

$$v_L(t) = L\frac{di_L(t)}{dt}$$





Where L is the inductance of the inductor and its unit is Henry (H).

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt = \frac{1}{L}\int_{-\infty}^{t_o} v_L(t)\ d\tau + \frac{1}{L}\int_{t_o}^{t} v_L(t)\ d\tau$$

- An inductor acts like a short circuit in a DC circuit since di/dt = 0.
- The inductor current cannot change abruptly.

Example 7.1 (page 274)

Find the voltage across the Inductor.

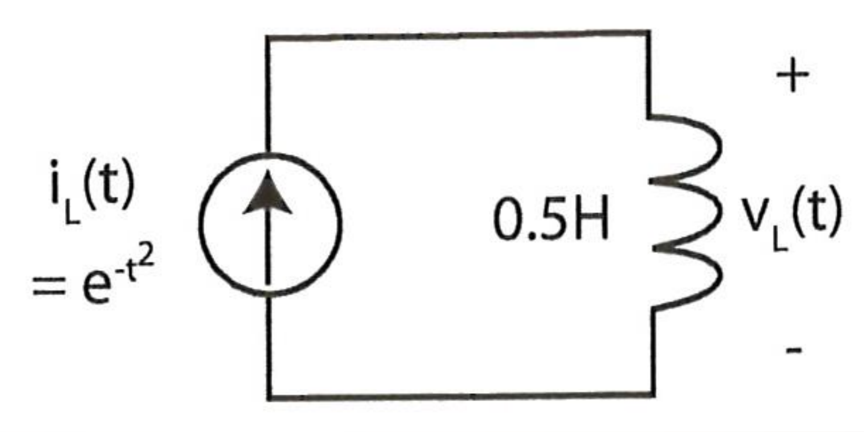
$$i_L(t) = e^{-t^2}$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$v_L(t) = (0.5)\frac{d(e^{-t^2})}{dt}$$

$$v_L(t) = (0.5)(-2t)e^{-t^2}$$

$$v_L(t) = -te^{-t^2} \quad V$$



Series and parallel inductors

The equivalent inductance of series-connected inductors is the sum of the individual inductances.

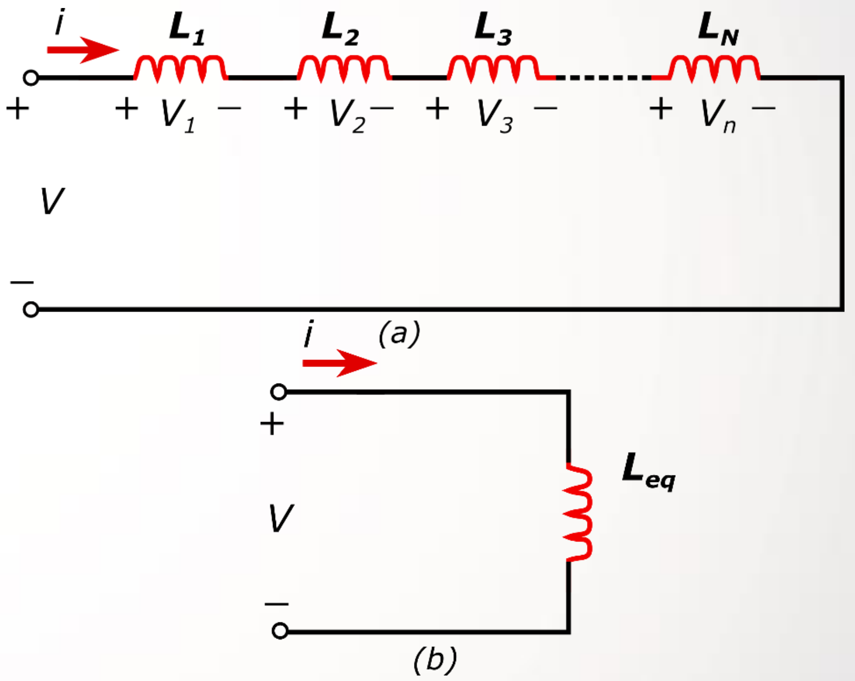
$$v = v_1 + v_2 + v_3 + \ldots + v_N$$

Substituting $v_k = L_k \frac{di}{dt}$ *results* in:

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \ldots + L_N \frac{di}{dt}$$

$$= (L_1 + L_2 + L_3 + \cdots + L_N) \frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k \right) \frac{di}{dt} = L_{eq} \frac{di}{dt} \Longrightarrow \boxed{L_{eq} = L_1 + L_2 + L_3 + \ldots.. + L_N}$$



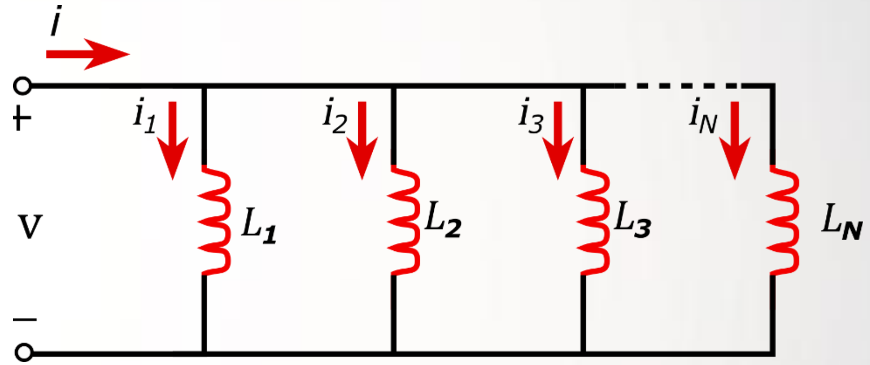
Series and parallel inductors

The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

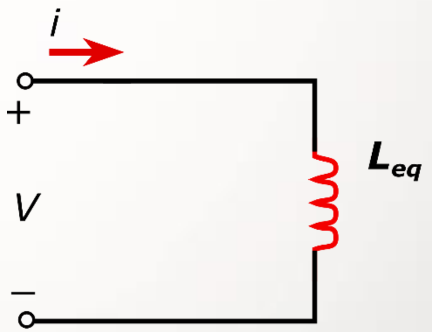
$$i = \frac{1}{L_1}\int_{t0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t0}^{t} v\,dt + i_2\,(t_0) + \cdots + \frac{1}{L_N}\int_{t0}^{t} v\,dt + i_N\,(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0) + \ldots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i\,(t_0)$$



(a)

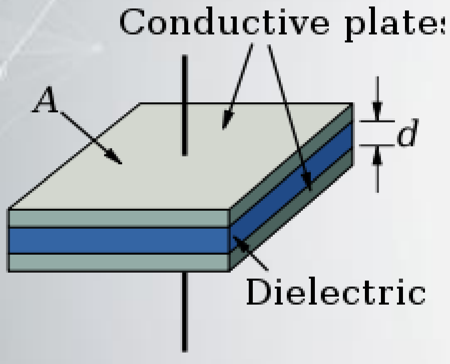


(b)

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\ldots + \frac{1}{L_N}}$$

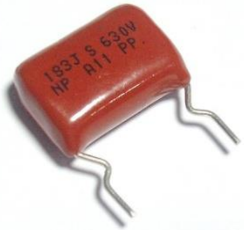
Capacitors

A capacitor is a passive element designed to store energy in its electric field. It consists of two conducting plates separated by an insulator (or dielectric).



Fixed capacitors

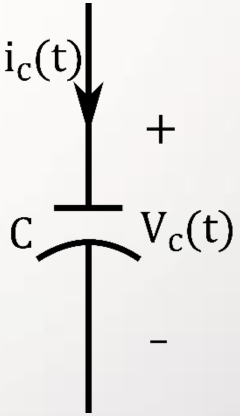
Polyester capacitor



Ceramic capacitor

Electrolytic capacitor

Capacitance of a capacitor is the ratio of the charge on one plate to the voltage difference between the two plates, measured in farads (F).

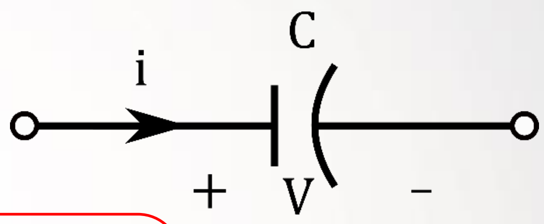


$$C = q / v$$

NOTE: 1 farad = 1 coulomb/volt.

The current-voltage relationship of a capacitor

- The charge, voltage and capacitance can be mathematically presented as: $q = C\,v$



$$\frac{dq}{dt} = C\,\frac{dv}{dt} \;\Rightarrow\;$$ The current in the capacitors is: $$\boxed{i_c = C\,\frac{dv}{dt}}$$

where C is the capacitance of the capacitor measured in farad (F)

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(\tau)\,d\tau = \frac{1}{C}\int_{-\infty}^{t_o} i_c(\tau)\,d\tau + \frac{1}{C}\int_{t_o}^{t} i_c(\tau)\,d\tau$$

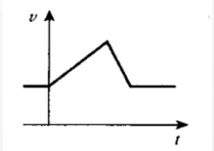
$$v_c(t) = v_c(t_o) + \frac{1}{C}\int_{t_o}^{t} i_c(\tau)\,d\tau$$

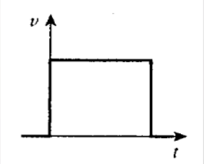
NOTE

If a DC voltage is applied, the capacitor will be an **open circuit.**

The capacitor voltage **cannot change abruptly**.

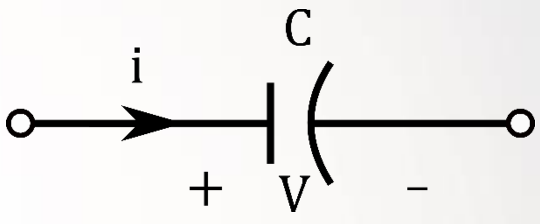
allowable voltage change





Not allowable voltage change

Example Find the voltage across a capacitor of 2 micro Farads holding 100 micro Coulomb of charge.



$$\boldsymbol{C = q / v}$$

$$v = q / C = \frac{10 \times 10^{-5}}{2 \times 10^{-6}} = 50V$$

Example Determine the voltage across a 2 μF capacitor if the current passing through it is: ***$i(t) = 6e^{-3000t}$ mA.*** Assume v(0) =0.

$$v_c(t) = v_c(t_o) + \frac{1}{C}\int_{t_o}^{t} i_c(\tau)\; d\tau$$

$$v_c(t) = 0 + \frac{1}{2\times 10^{-6}}\int_{0}^{t}(6\times 10^{-3})e^{-3000t}dt$$

$$v_c(t) = 0 + \frac{6\times 10^{-3}}{-3000\times 2\times 10^{-6}}e^{-3000t}\Big|_0^t = (1 - e^{-3000t})V$$

Series and parallel capacitors

The equivalent capacitance of *N* parallel-connected capacitors is the sum of the individual capacitances.

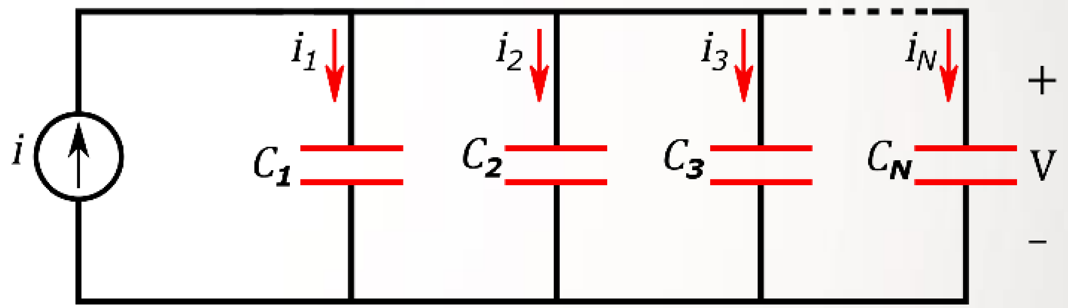
$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

Since $i_k = C_k \frac{dv}{dt}$:

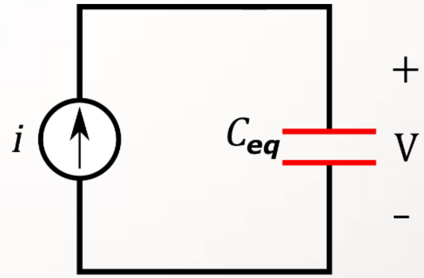
$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

$$\boxed{C_{eq} = \sum_{k=1}^{N} C_k = C_1 + C_2 + C_3 + \cdots + C_N}$$



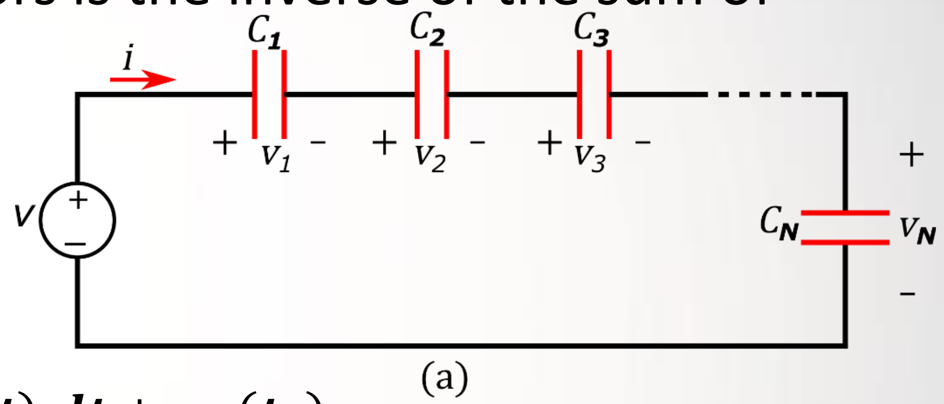
(a)



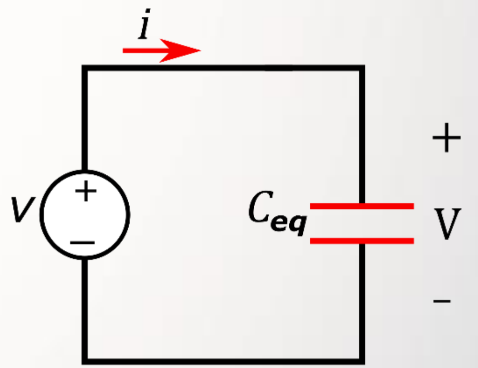
(b)

Series and parallel capacitors

The equivalent capacitance of *N* series-connected capacitors is the inverse of the sum of the inverses of the individual capacitances.



(a)



(b)

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

Since $v_k = \frac{1}{C_k}\int_{t_0}^{t} i(t)dt + v_k(t_0)$:

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(t)\,dt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(t)\,dt + v_2(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(t)\,dt + v_N(t_0)$$

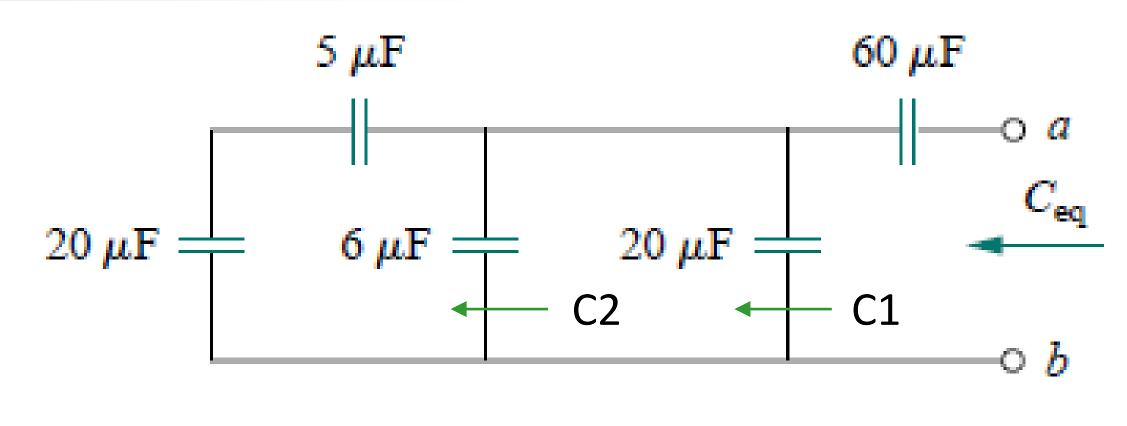
$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(t)\,dt + v_1(t_0) + v_2\,(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(t)\,dt + v(t_0)$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$

Example

Find the equivalent capacitance seen between terminals a and b



C2=(20*5/25)+6=10 μF

C1=C2+10=30 μF

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20\ \mu\text{F}$$