American University Of The Middle East

Laplace Transform Analysis

Linear Circuit Analysis II
EECE 202

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Announcement

1. Project Proposal will be released this week
2. Form project groups by the end of this week
3. Quiz during Week 4

Recap

1. Laplace Transform Analysis

New Material

1. Laplace transform analysis
2. Properties of Laplace transform
3. Laplace transform of sinusoidal signals

Learning Outcomes

2. an ability to compute responses of linear circuits with and without initial conditions via one-sided Laplace transform techniques. SO [1]

Laplace Transform Analysis

The Laplace Transform of a Signal, a Function, or an Excitation is given by:

$$\boldsymbol{F(S) = L[f(t)] = \int_{0^-}^{\infty} f(t)e^{-st}dt}$$

s =σ + jω is a complex variable

j=$\sqrt{-1}$

Time domain	Laplace domain
$u(t)$	$\dfrac{1}{s}$
$\mathrm{r}(t) = t \times u(t)$	$\dfrac{1}{s^2}$
$t^2 \times u(t)$	$\dfrac{2}{s^3}$
$t^n \times u(t)$	$\dfrac{n!}{s^{n+1}}$

Time domain		Laplace domain
$u(t-T)$	$\longrightarrow$	$\dfrac{e^{-Ts}}{s}$
$u(t-2)$	$\longrightarrow$	$\dfrac{e^{-2s}}{s}$
$e^{-at} \times u(t)$	$\longrightarrow$	$\dfrac{1}{s+a}$
$e^{-at} \times r(t)$	$\longrightarrow$	$\dfrac{1}{(s+a)^2}$

Laplace transform properties

Time shift property

$$u(t) \longrightarrow \frac{1}{s}$$

$$u(t-T) \longrightarrow \frac{e^{-Ts}}{s}$$

Frequency shift property

$$e^{-at} \times f(t) \longrightarrow F(s+a)$$

Example 12.17 (p. 576)

Find F(s) = L[f (t)] when f (t) = sin(ωt)u(t).

Apply Euler formula: for t ≥ 0

$$\mathrm{f}(t) = \sin(\omega t)u(t) = \left[\frac{e^{j\omega t}}{2j} - \frac{e^{-j\omega t}}{2j}\right]u(t)$$

Since the Laplace Transform of e^{-at} u(t) is $\frac{1}{s+a}$, *then*

$$\mathbf{F(s)} = \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right] = \frac{1}{2j}\left[\frac{s+j\omega-s+j\omega}{s^2+\omega^2}\right] = \frac{\omega}{s^2+\omega^2}$$

Exercise (p. 576)

Find F(s) = L[f (t)] when f (t) = cos(ωt)u(t).

Recall LT properties and LT pairs

Apply Euler formula: for t ≥ 0

$$\mathrm{f}(t) = \cos(\omega t)u(t) = \left[\frac{e^{j\omega t}}{2} + \frac{e^{-j\omega t}}{2}\right]u(t)$$

Since the Laplace Transform of e^{-at} u(t) is $\frac{1}{s+a}$, *then*

$$\mathbf{F(s)} = \frac{1}{2}\left[\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right] = \frac{1}{2}\left[\frac{s+j\omega+s-j\omega}{s^2+\omega^2}\right] = \frac{\boldsymbol{s}}{\boldsymbol{s}^2+\omega^2}$$

Additional example 3

Find F(s) for the following function $f(t)=10e^{-2t}\cos(4t)\,u(t)$

$$\mathbf{F(s)} = \frac{10\,(s+2)}{(s+2)^2+4^2} = \frac{10(s+2)}{s^2+4s+20}$$

Additional example 4

Find $F_6(s)$ for the following function $f_6(t)=20t\,e^{-2t}\sin(4t)\,u(t)$

$$f_6(t) = 20t\,e^{-2t}\sin(4t)\,u(t) = \mathbf{t}\,\boldsymbol{f_a(t)}$$

where $f_a(t) = 20e^{-2t}\sin(4t)\,u(t)$

$$F_a(s) = 20\frac{4}{(s+2)^2+16} = \frac{80}{(s+2)^2+16}$$

$$F_6(s) = -\frac{d}{ds}F_a(s)$$

$$F_6(s) = -\frac{d}{ds}\left(\frac{80}{(s+2)^2+16}\right) = \frac{160(s+2)}{[(s+2)^2+16]^2}$$

Example 5

Find the Laplace transforms for: g(t) = 6cos (4(t −1))u(t-1)

$$G(s) = 6\frac{s}{s^2+4^2}e^{-s} = \frac{6s}{s^2+16}e^{-s}$$

Example 6

Find the Laplace transforms for: $h(t)$ = (6sin(3t)+ 8cos(3t))$u(t)$

$$H(s) = 6\frac{3}{s^2+9} + 8\frac{s}{s^2+9} = \frac{8s+18}{s^2+9}$$

Practice

1. Explain how the Laplace Transform simplifies solving differential equations. Explain the significance of the time-shift property of the Laplace transform and how it applies to a delayed signal.

2. How does the frequency shift property affect the Laplace transform of a sinusoidal signal?

3. Discuss the importance of initial conditions when solving linear circuit problems using Laplace transforms.

4. Solve activity on Moodle

Summary

- Transforms a time-domain function into the s-domain for easier analysis of linear systems.

- Key Properties:
  - Linearity:
  - Time Shift:
  - Frequency Shift

- Applications in Circuit Analysis:
  - Helps in analyzing systems with initial conditions.
  - Used to solve differential equations by transforming them into algebraic equations.

Suggested Additional Problems for Ch. 12:

Exercises 1, 2, 4, 6 (p. 592), 16, 17 (p. 595) ?