American University Of The Middle East

Impedance and Admittance

Linear Circuit Analysis II
EECE 202

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Announcement

1. GCA1 during Week 5 Second Lecture

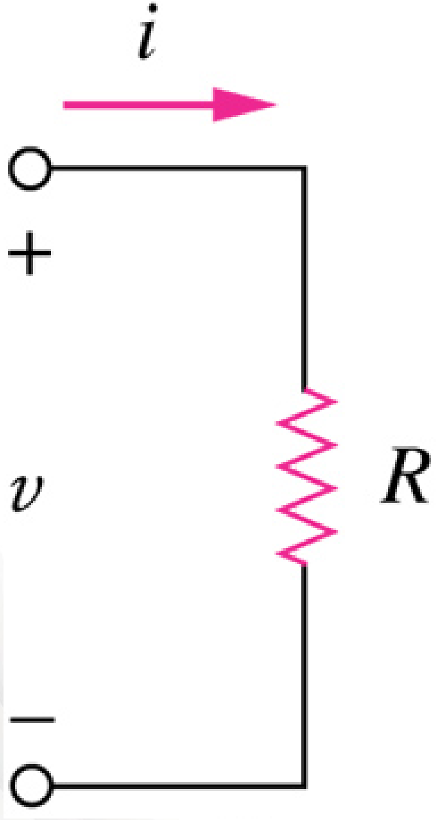
Recap

1. Solving equation differential and integrals using LT

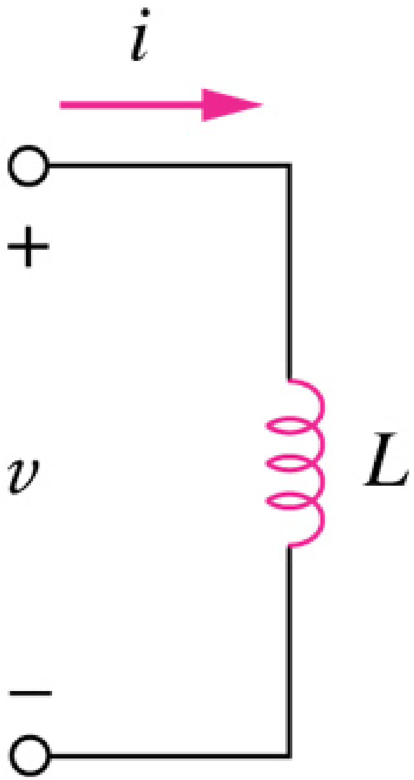
New Material

1. Impedances
2. Admittances

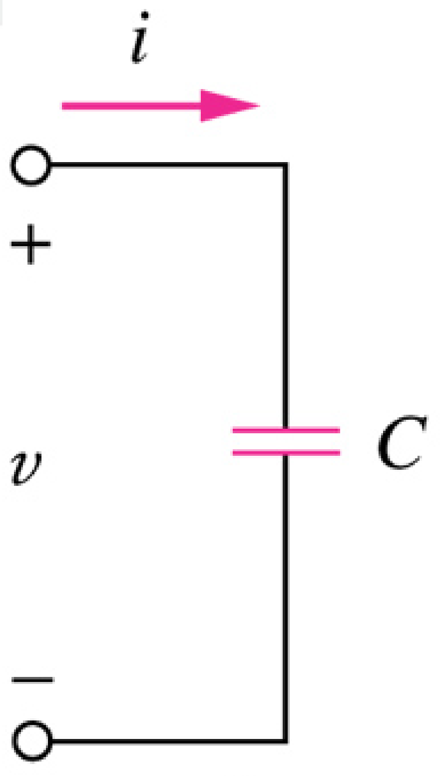
Passive Elements



Resistor



Inductor



Capacitor

What is an Impedance??

Impedance is a generalized Resistance

that is frequency dependent

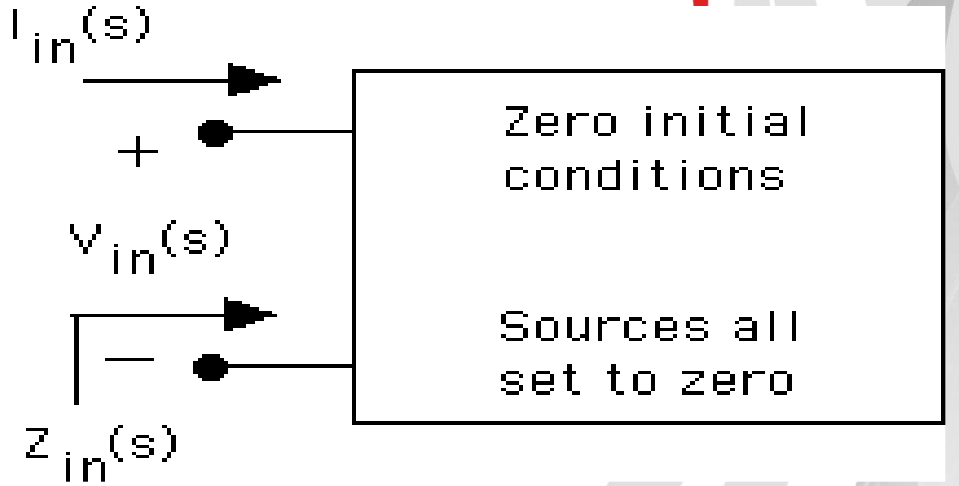
Impedance, denoted $Z_{in}(s)$, in the s---world,

in the <u>total absence of initial conditions</u>, is:

$$Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} \qquad \Omega$$

$$V_{in}(s) = Z_{in}(s) \times I_{in}(s)$$

$$Y_{in}(s) = \frac{1}{Z_{in}(s)} = \frac{I_{in}(s)}{V_{in}(s)} \qquad \Omega^{-1}$$



Resistance Impedance/Admittance

In the total absence of initial conditions

In time domain, $v_R = Ri_R$

In "s" domain, $V_R(s) = RI_R(s) \triangleq Z_R(s)I_R(s)$

$$\Longrightarrow I_R(\mathrm{s}) = \frac{1}{R} \times V_R(\mathrm{s}) \triangleq Y_R(s) \times V_R(\mathrm{s})$$

Result: $Z_R(s) = R$ and $Y_L(s) = \frac{1}{R}$

Note: At s = 0, the impedance of the inductor is zero meaning the inductor looks like a short circuit

Inductance Impedance/Admittance

In the total absence of initial conditions

In time domain, $v_L = L\dfrac{di_L}{dt}$

In "s" domain, $V_L(s) = LsI_L(s) \triangleq Z_L(s)I_L(s)$

$$\Longrightarrow I_L(\mathrm{s}) = \frac{1}{Ls} \times V_L(\mathrm{s}) \triangleq Y_L(s) \times V_L(\mathrm{s})$$

Result: $Z_L(s) = Ls$ and $Y_L(s) = \dfrac{1}{Ls}$

Note: At s = 0, the impedance of the inductor is zero meaning the inductor looks like a short circuit

Capacitance Impedance/Admittance

In the total absence of initial conditions

In time domain, $i_C = C\dfrac{dv_C}{dt}$

In "s" domain, $I_C(s) = CsV_C(s) \triangleq Y_C(s)V_C(s)$

$$\Rightarrow V_C(s) = \frac{1}{Cs} \times I_C(s) \triangleq Z_C(s) \times I_C(s)$$

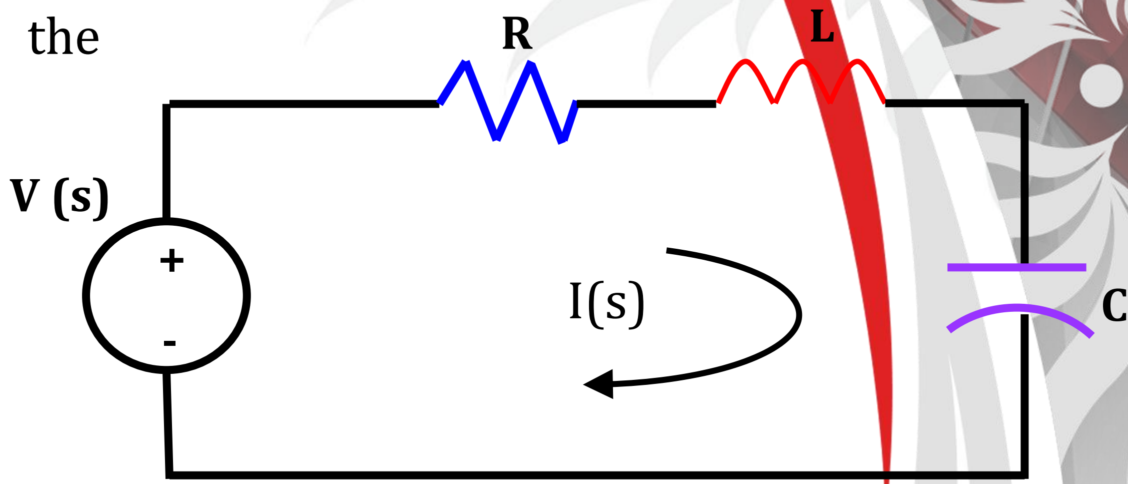
Result: $Z_C(s) = \dfrac{1}{Cs}$ and $Y_L(s) = Cs$

Note: At s = 0, the impedance of the capacitor is infinite meaning the capacitor looks like an open circuit

	Impedance	Admittance
Resistor	$Z_R = R$	$Y_R = \frac{1}{R}$
Inductor	$Z_L = Ls$	$Y_L = \frac{1}{Ls}$
Capacitor	$Z_C = \frac{1}{Cs}$	$Y_C = Cs$

Series Circuit **Manipulation RULES**

In the total absence of the initial conditions, find the equivalent impedance and admittance for the shown series connection :

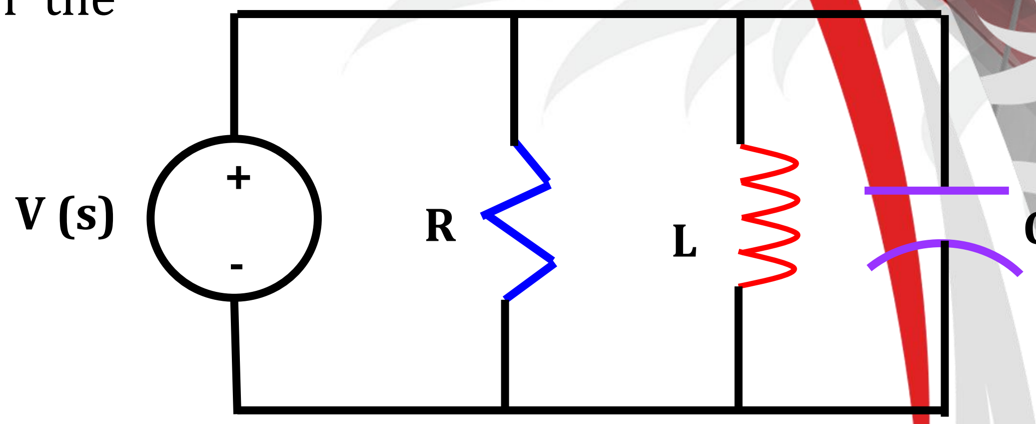


$$Z(s) = R + Ls + \frac{1}{Cs} \quad \Omega$$

$$Y(s) = \frac{1}{Z(s)} = \frac{1}{R + Ls + \frac{1}{Cs}} \quad \Omega^{-1}$$

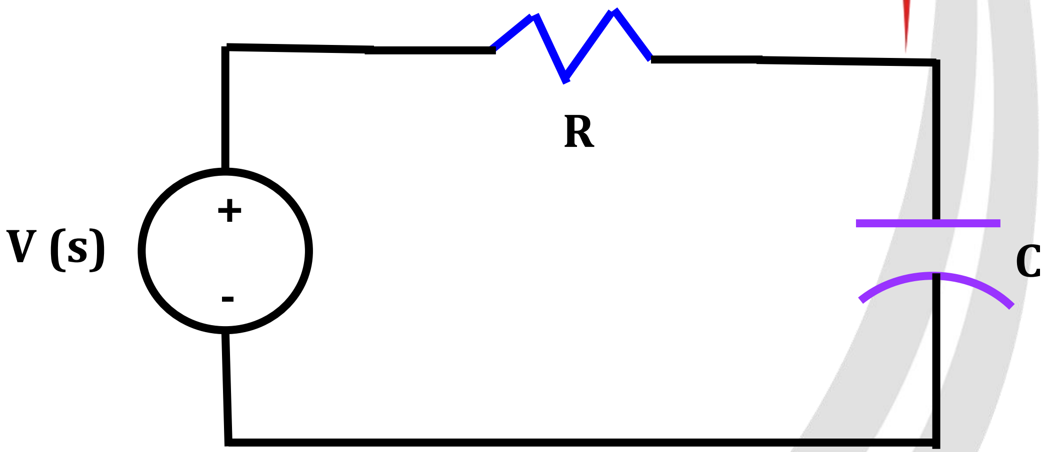
Parallel Circuit Manipulation RULES

In the total absence of the initial conditions, find the equivalent impedance and admittance for the shown parallel connection :



$$Y(s) = Y_R + Y_L + Y_C$$

$$= \frac{1}{R} + \frac{1}{Ls} + Cs \quad \Omega^{-1}$$



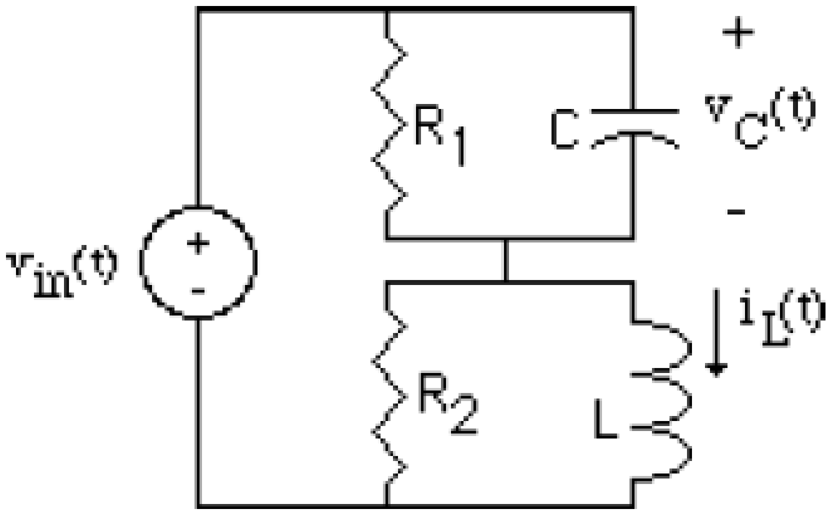
$$Z(s) = \frac{1}{Y(s)}$$

Example 1

Assume zero initial conditions, find the input impedance seen by the source. Assume all parameter values are *1.*

$$Z_{in}(s) = \frac{R_1 \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} + \frac{R_2 Ls}{R_2 + Ls}$$

$$Z_{in}(s) = \frac{1}{s+1} + \frac{s}{s+1} = 1$$



Example 13.14 (p. 613)

For the shown circuit, Compute the following:

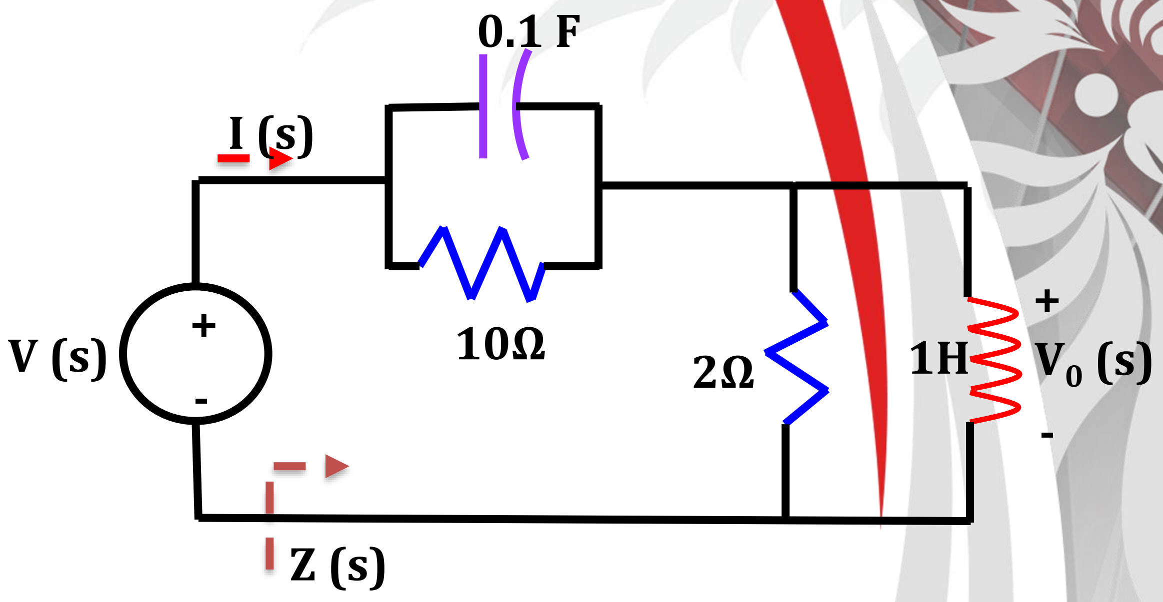
1-Draw the circuit in the S-domain.

2- Find Z(s).

3-Find $V_0(s)$ in terms of V(s).

4-Given that v(t) = u(t), find $v_0(t)$.

Assume zero initial conditions



$$Z_{R1} = R_1 = 10\ \Omega$$

$$Z_{R2} = R_2 = 2\ \Omega$$

$$Z_L = Ls = 1s\ \Omega$$

$$Z_C = 1/(sC) = 1/(0.1s) = 10/s\ \Omega$$

Example 13.14 (contin…)

$Z_1(s) = Z_C // Z_{R1}$

OR

$$Y_1(s) = Y_C + Y_{R1} = sC + \frac{1}{R_1} = 0.1s + \frac{1}{10} = \frac{s+1}{10}$$

$$\boxed{Z_1(s) = \frac{10}{s+1}}\ \Omega$$

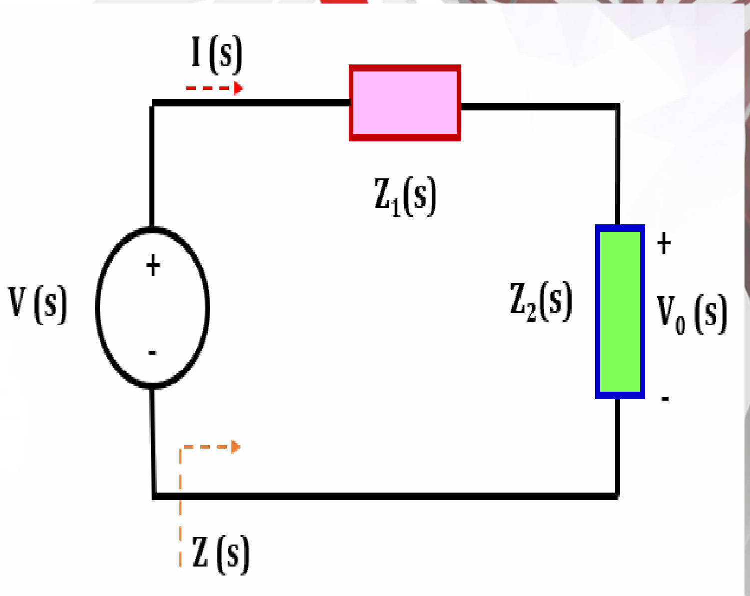
Similarly,

$Z_2(s) = Z_L // Z_{R2}$

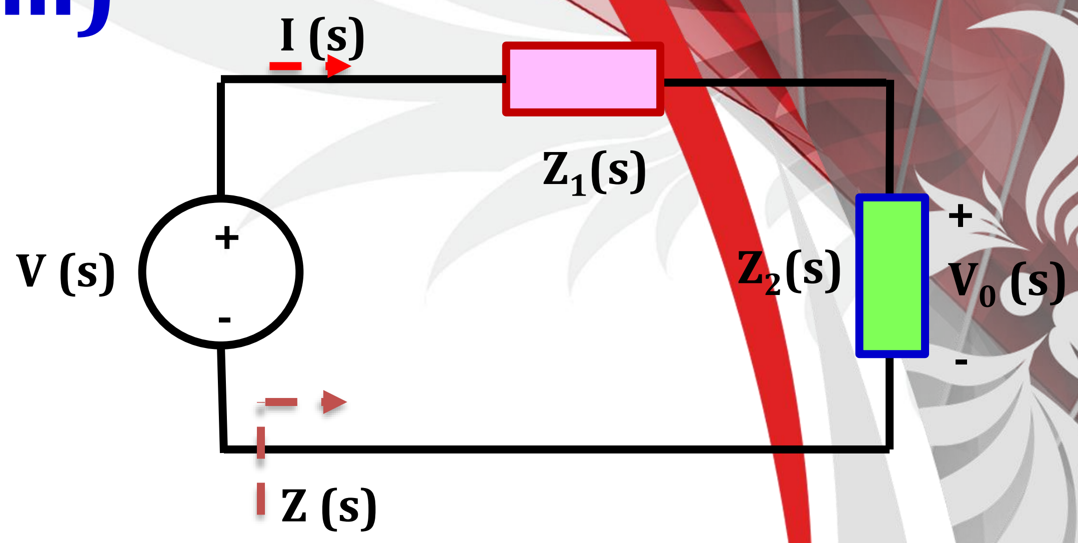
OR

$$Y_2(s) = Y_L + Y_{R2} = \frac{1}{sL} + \frac{1}{R_2} = \frac{1}{1s} + \frac{1}{2} = \frac{s+2}{2s}$$

then, $\boxed{Z_2(s) = \frac{2s}{s+2}}\ \Omega$



Example 13.14 (contin…)



$$Z_1(s) = \frac{10}{s+1} \quad \Omega$$

$$Z_2(s) = \frac{2s}{s+2} \quad \Omega$$

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z(s) = \frac{10(s+2)+2s(s+1)}{(s+1)(s+2)} = \frac{2s^2+12s+20}{(s+1)(s+2)} \quad \Omega$$

Example 13.14 (contin…)

$$V_o(s) = \frac{Z_2(s)}{Z(s)} V(s)$$

$$V_o(s) = \frac{\frac{2s}{s+2}}{\frac{(2s^2+12s+20)}{(s+1)(s+2)}} V(s)$$

$$V_o(s) = \frac{2s(s+1)(s+2)}{(s+2)(2s^2+12s+20)} V(s)$$

$$V_o(s) = \frac{2s(s+1)}{(2s^2+12s+20)} V(s)$$

$$V_o(s) = \frac{s(s+1)}{(s^2+6s+10)} V(s)$$

Given that v(t) = u(t), find $v_o(t)$

$$V_o(s) = \frac{s(s+1)}{(s^2+6s+10)} V(s)$$

And given that v(t) = u(t), then V(s) = 1/s, then

$$V_o(s) = \frac{s(s+1)}{(s^2+6s+10)} \frac{1}{s}$$

$$V_o(s) = \frac{(s+1)}{(s^2+6s+10)} = \frac{(s+1)}{((s+3)^2+1^2)}$$

Has complex roots $-3 \pm j1$

$$V_o(s) = \frac{s}{((s+3)^2+1^2)} + \frac{1}{((s+3)^2+1^2)}$$

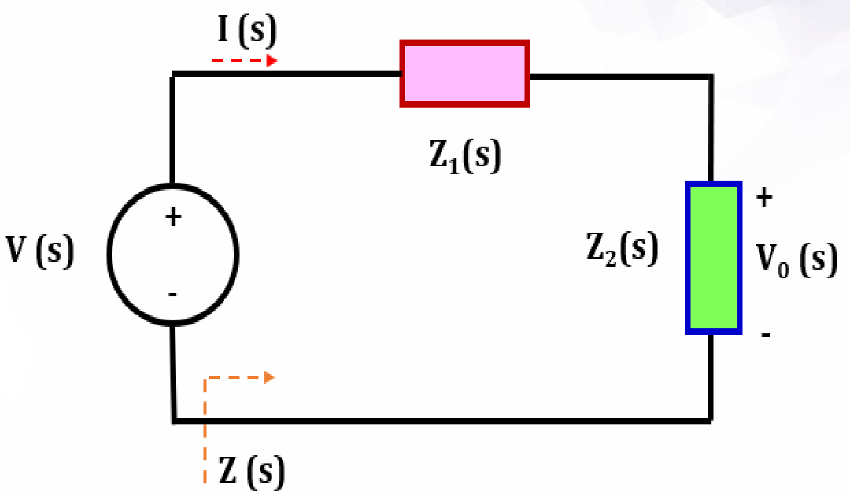
$$V_o(s) = \frac{s+3-3}{((s+3)^2+1^2)} + \frac{1}{((s+3)^2+1^2)}$$

$$V_o(s) = \frac{(s+3)}{((s+3)^2+1^2)} + \frac{1-3}{((s+3)^2+1^2)}$$

$$V_o(s) = \frac{(s+3)}{((s+3)^2+1^2)} + \frac{-2}{((s+3)^2+1^2)}$$

$$V_o(s) = \frac{(s+3)}{((s+3)^2+1^2)} - 2\frac{1}{((s+3)^2+1^2)}$$

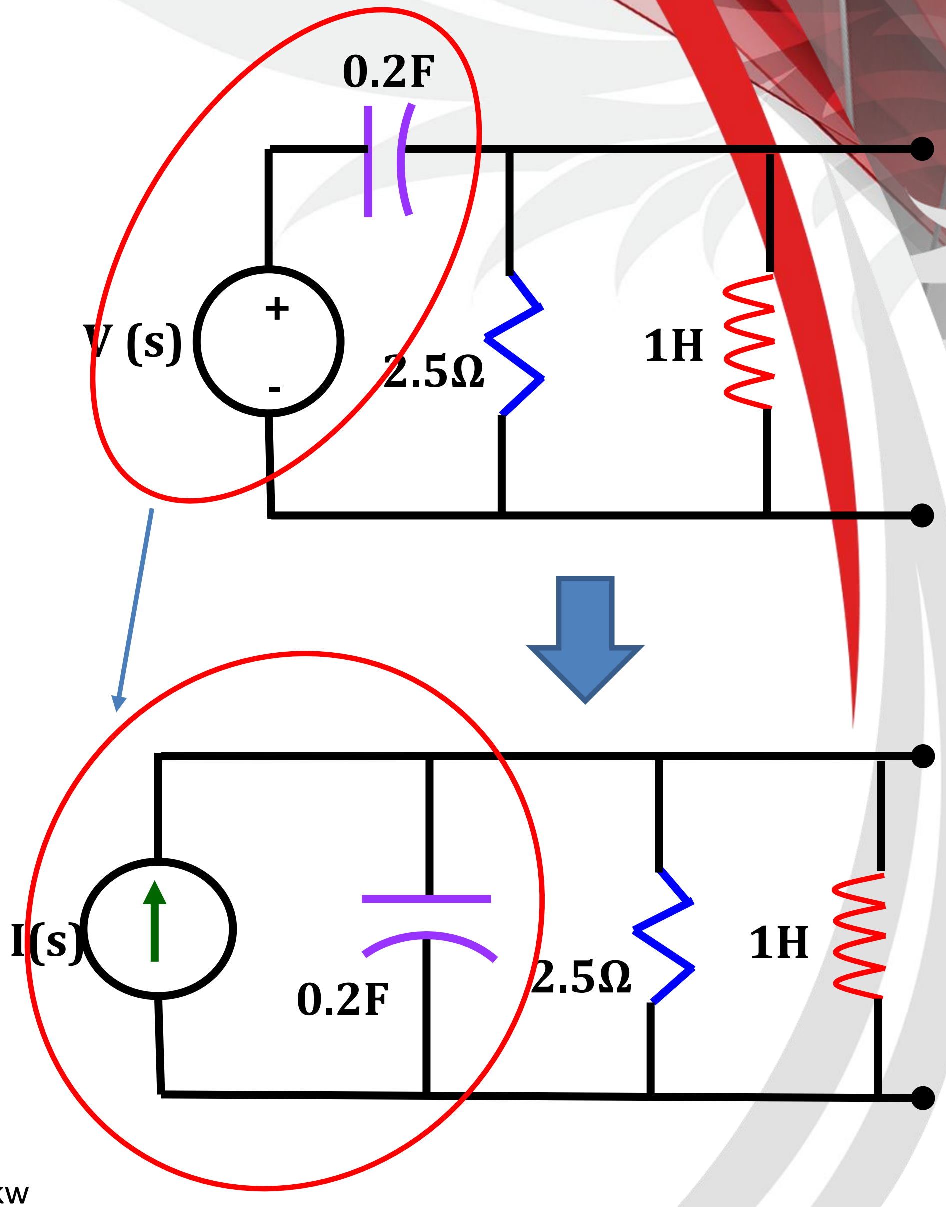
$$\boxed{\boldsymbol{v_o(t) = e^{-3t}Cos(1t)u(t) - 2e^{-3t}Sin(1t)u(t)\ \text{V}}}$$



Repeat example 13.14 for R_2=10 Ω and C= 0.01 F

Example 1

Assume zero initial conditions, Use source transformation to determine Z_{th} (s) for the indicated terminals



By source transformation :

$$I(s) = \frac{V(s)}{\frac{1}{0.2s}} = 0.2V(s)$$

Turn the current source off :

$$Y_{th}(s) = Y_R(s) + Y_C(s) + Y_L(s)$$

$$Y_{th}(s) = \frac{1}{2.5} + 0.2s + \frac{1}{s} = \frac{s^2+2s+5}{5s}$$

$$Z_{th}(s) = \frac{5s}{s^2+2s+5}$$

Use AI and answer following questions

1. What is the difference between impedance and admittance in a circuit?
2. How does frequency affect the impedance of an inductor and a capacitor?
3. Why does the impedance of an inductor behave like a short circuit at low frequencies?
4. How does admittance simplify the analysis of parallel circuits?
5. Solve Activity

Summary

- Inductance Impedance
- Capacitance Impedance
- Series circuit manipulation rules
- Series circuit manipulation rules

Suggested Additional Problems for Ch. 13:

Example 13.1 (p. 609), 13.2 (p. 611), 13.3 (p. 612), Exercise (p. 606), (p. 607)???