Sinusoidal Steady State Analysis

Linear Circuit Analysis II
EECE 202

Announcement

- PD2 (Technical Report) due Thursday Week 12

Recap

1. Sinusoidal steady state (SSS) sources
2. Laplace Transform approach to SSS

New Material

1. Resonance
2. Series RLC circuit
3. Parallel RLC circuit

Resonance

In an electrical circuit, resonance exists when the inductive reactance and the capacitive reactance are of equal magnitude thus Input and output voltages/signals are in phase.

$Y_{in}(j\omega)$ and $Z_{in}(j\omega)$ ARE **REAL** AT $\omega = \omega r$

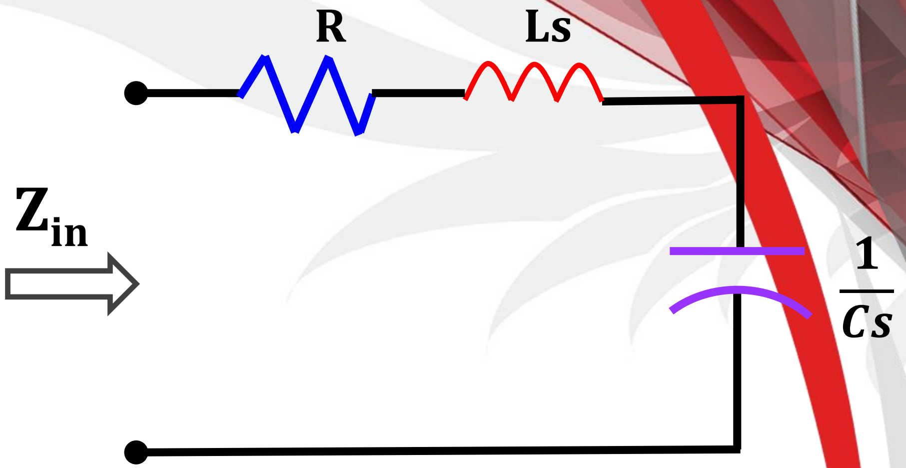
THE RESONANT FREQUENCY

Series RLC Circuit

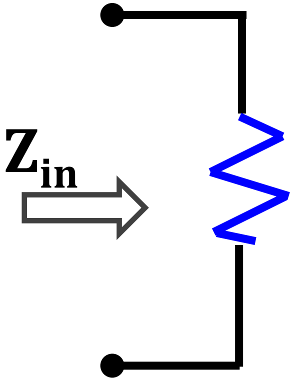
$$Z_{in}(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega_r L - \frac{1}{\omega_r C} = 0 \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$$



At resonance



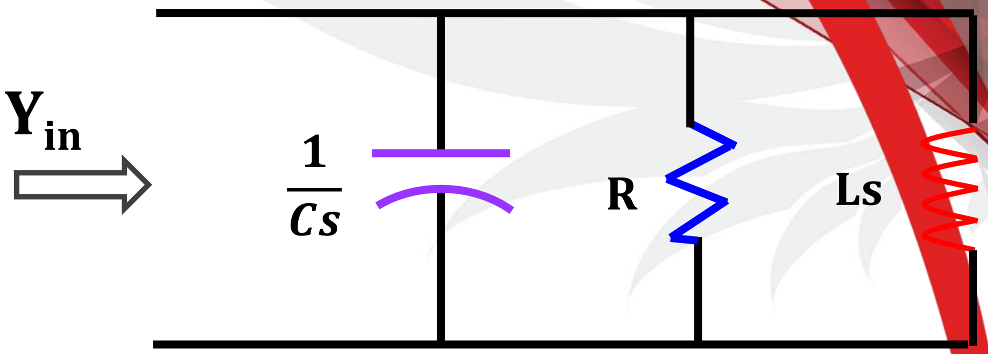
At Resonance, series LC acts as a short circuit and $Z_{in}(jw) = R$

Parallel RLC Circuit

$$Y_{in}(s) = \frac{1}{R} + Cs + \frac{1}{Ls},$$

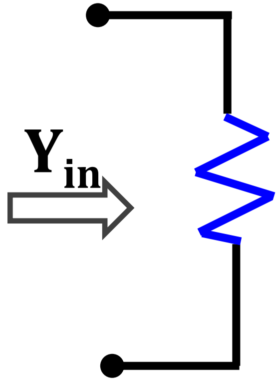
then $Y_{in}(jw) = \frac{1}{R} + jwc + \frac{1}{jwl}$

$$= \frac{1}{R} + j\left(wc - \frac{1}{wL}\right)$$



$$\omega_r C - \frac{1}{\omega_r L} = 0 \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$$

At resonance



At resonance, the parallel LC part is an open circuit and $Y_{in}(j\omega_r) = \frac{1}{R}$

Example -1

For the circuit shown, which represents the equivalent circuit of an amplifier circuit, given that RL=20KΩ, gm=2ms, the magnitude of Vin is 0.1 V with f=10MHz. calculate the following:

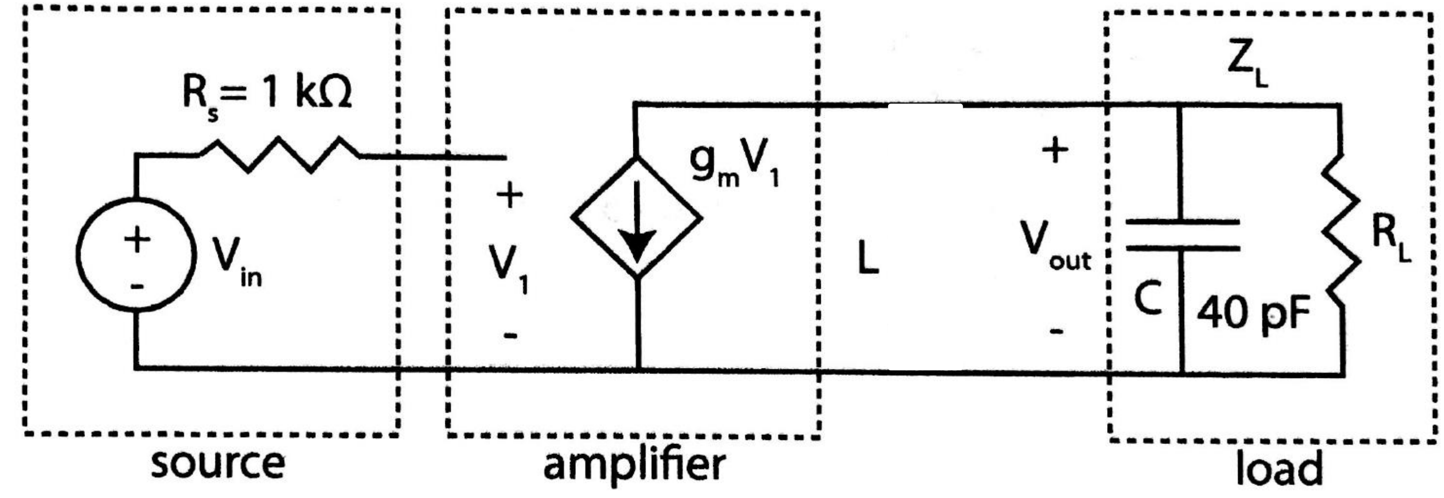
1- The magnitude of the output voltage across RL.

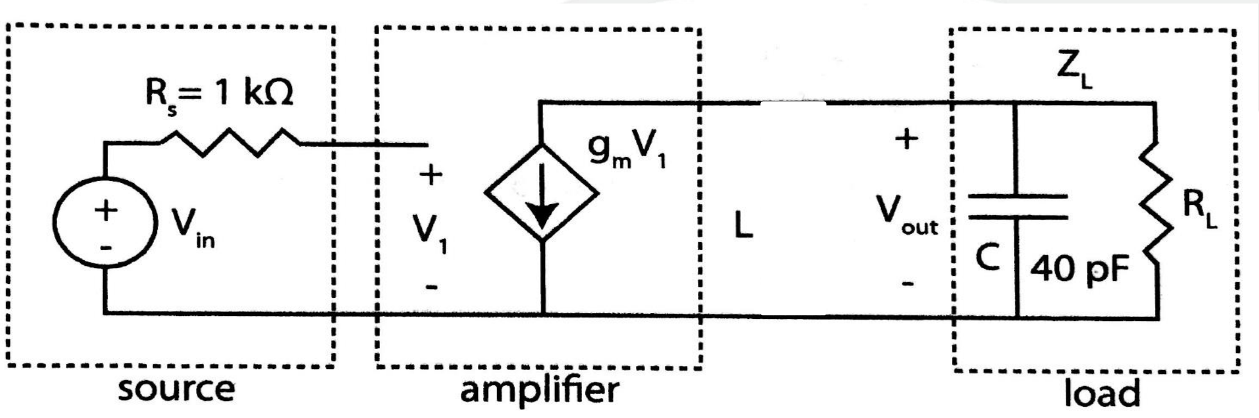
2- Calculate the gain (Vout / Vin).

3- In order to maximize the gain, an inductor will be connected to resonate with the capacitor at the load side. Calculate the value of "L", which is required to resonate with "c".

4- Calculate the output voltage after connecting the inductor "L".

5-Calculate the new gain.





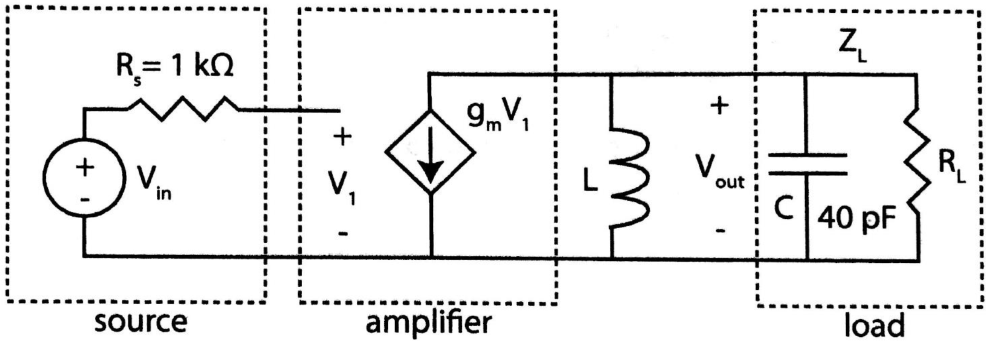
1- The magnitude of the output voltage across RL.

$$\boldsymbol{Z_L(jw) = 397.8\,\underline{\lfloor -88.9^o}\,\Omega}$$

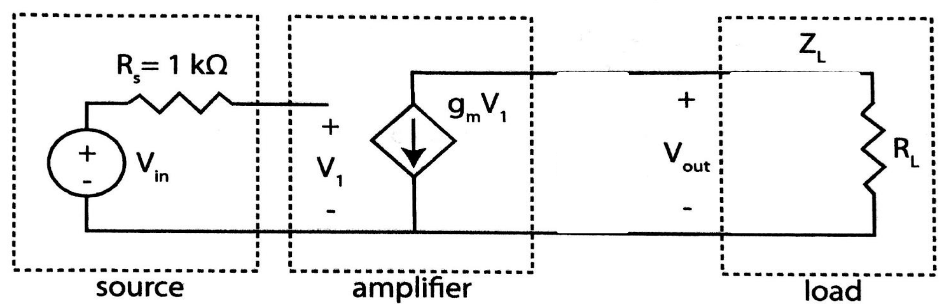
$$\boldsymbol{|V_{out}| = 0.1 * 0.002 * 397.8 = 0.0796\,V}$$

2- Calculate the gain (Vout / Vin).

$$\boldsymbol{Gain = \left(\frac{0.0796}{0.1}\right) = 0.796}$$



At resonance



3- Calculate the value of "L", which is required to resonate with "c".

$$w_r = \frac{1}{\sqrt{LC}} \qquad L = \frac{1}{w_r^2 C} = 6.33 * 10^{-6} H$$

4- Calculate the output voltage after connecting the inductor "L".

$$|V_{out}| = 0.1 * 0.002 * 20000 = 4\ V$$

5-Calculate the new gain.

$$Gain = \left(\frac{4}{0.1}\right) = 40$$

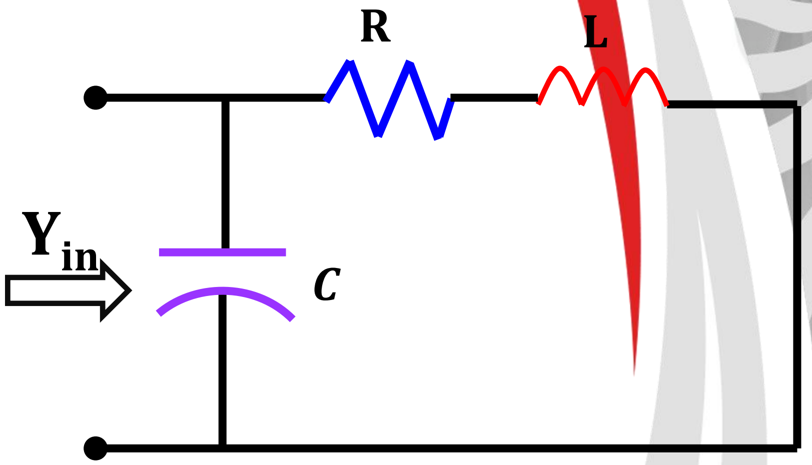
Example -2

For the coupling circuit shown in the figure, find the following:

$Y_{in}(j\omega)$, ω_r and $Y_{in}(j\omega_r)$.

$$Y_{in}(j\omega) = j\omega C + \frac{1}{R_L + j\omega L} = j\omega C + \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2}$$

$$= \frac{R_L}{R_L^2 + \omega^2 L^2} + j\omega\left[C - \frac{L}{R_L^2 + \omega^2 L^2}\right]$$



At Resonance.

$$C - \frac{L}{R_L^2 + \omega_r^2 L^2} = 0 \qquad R_L^2 + \omega_r^2 L^2 = \frac{L}{C}$$

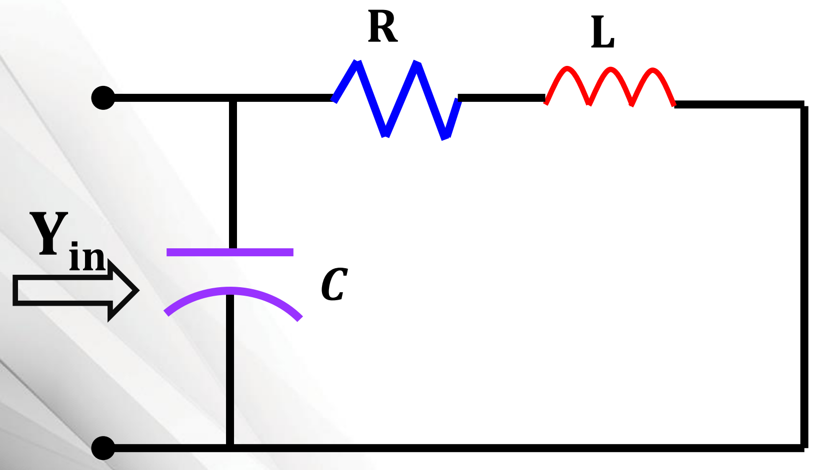
$$Y_{in}(j\omega_r) = \frac{R_L}{R_L^2 + \omega^2 L^2}$$

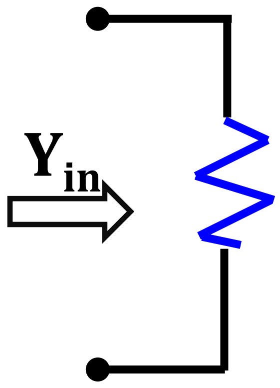
$$\boxed{R_L^2+\omega_r^2 L^2=\frac{L}{C}}$$

$$\omega_r^2=\frac{1}{CL}-\frac{R_L^2}{L^2}\Rightarrow\omega_r=\sqrt{\frac{1}{CL}-\frac{R_L^2}{L^2}}=\frac{1}{\sqrt{LC}}\sqrt{1-\frac{CR_L^2}{L}}$$

ω_r exists (is real) only when $\dfrac{CR_L^2}{L}<1$

At resonance



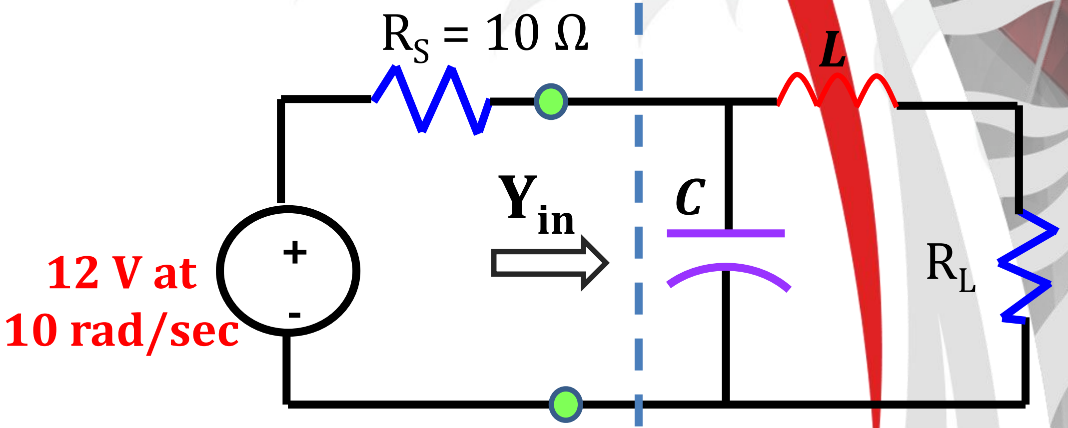


$$Y_{in}(j\omega_r)=\frac{R_L}{R_L^2+\omega^2L^2}$$

Example -3

Design a coupling (matching) network to achieve maximum power transfer from the source to the load R_L = 5 Ω. The source is v_{in} = 12 V_{rms} at 10 rad/s.

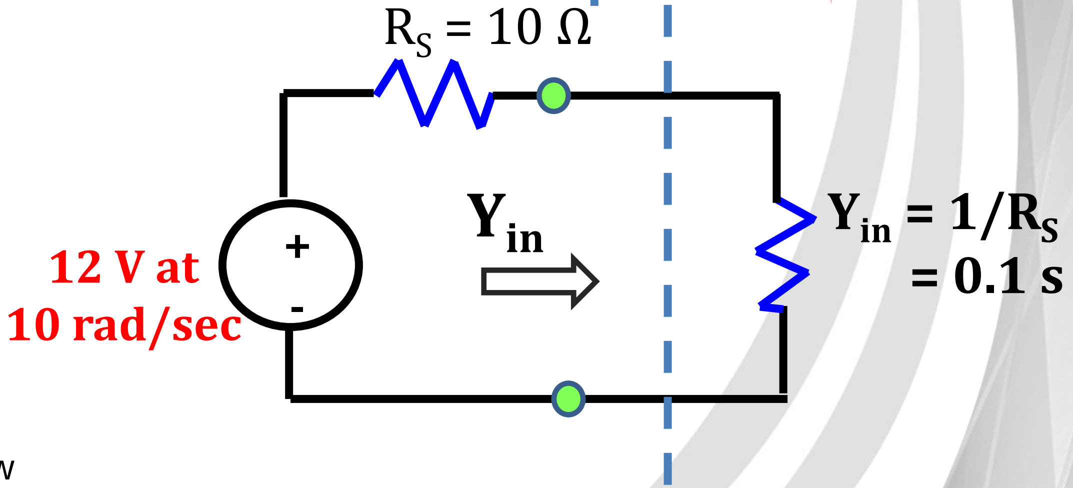
To achieve the maximum power Zin has to equal to Rs or Yin has to equal to (1/Rs) at resonance.



$$Y_{in}(jw_r) = 1/R_s$$

$$Y_{in}(jw_r) = \frac{1}{5 + j10L} + j10C$$

$$= \frac{5 - j10L}{25 + 100L^2} + j10C$$



This has to equal to $= 0.1 + j0$

$$0.1 = \frac{5}{25+100L^2} \qquad \Rightarrow L = 0.5\ \mathrm{H}$$

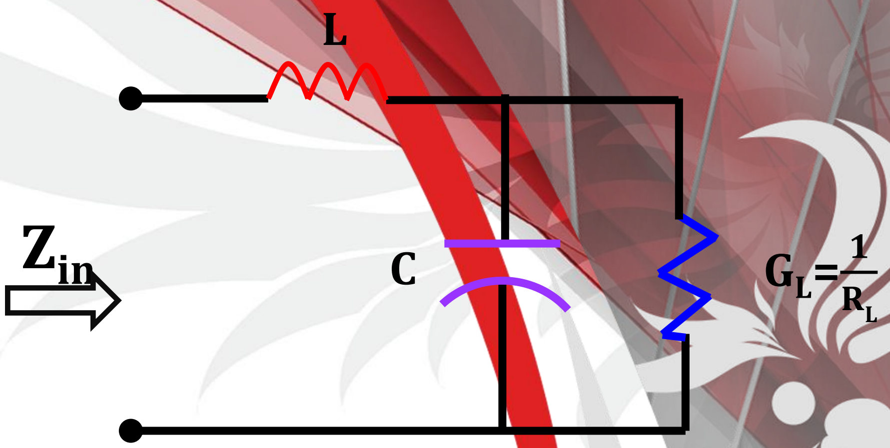
$$\frac{-j10L}{25+100L^2} = -j10C$$

$$C = \frac{0.5}{25+(100\times 0.5^2)} = 0.01\ F$$

Example -4

For the coupling circuit shown in the figure, find the following:

$Z_{in}(j\omega)$, ω_r and $Z_{in}(j\omega_r)$.



$$Z_{in}(j\omega) = j\omega L + \frac{1}{G_L + j\omega C}$$

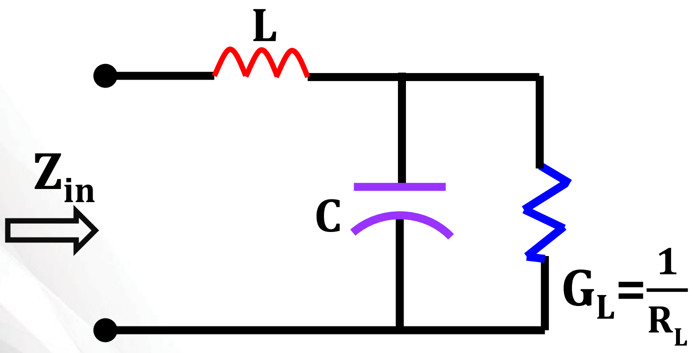
$$= j\omega L + \frac{G_L - j\omega C}{G_L^2 + \omega^2 C^2}$$

$$= \frac{G_L}{G_L^2 + \omega^2 C^2} + j\omega \left(L - \frac{C}{G_L^2 + \omega^2 C^2} \right)$$

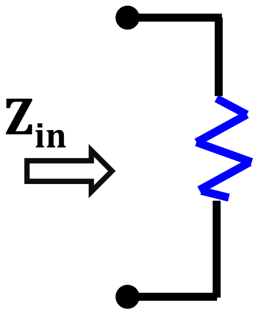
$$Z_{in}(j\omega) = \frac{G_L}{G_L^2 + \omega^2 C^2} + j\omega\left(L - \frac{C}{G_L^2 + \omega^2 C^2}\right)$$

$$L - \frac{C}{G_L^2 + \omega_r^2 C^2} = 0$$ **then** $$\omega_r = \sqrt{\frac{1}{LC} - \frac{G_L^2}{C^2}}$$

$$Z_{in}(j\omega_r) = \frac{G_L}{G_L^2 + \omega^2 C^2}.$$



At resonance

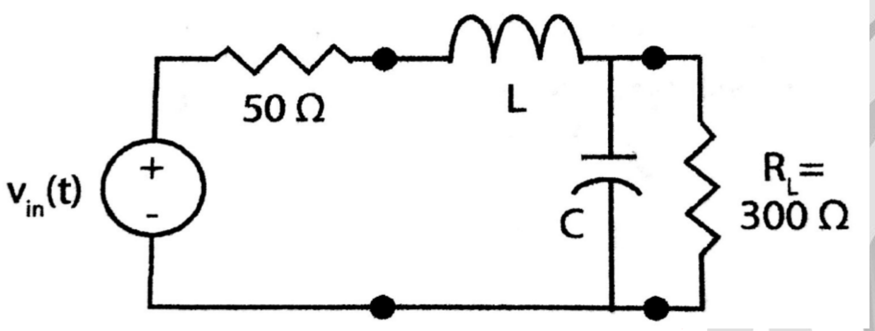


$$Z_{in}(j\omega_r) = \frac{G_L}{G_L^2 + \omega^2 C^2}$$

Example -5

For the circuit shown, design a coupling (matching) network to achieve maximum power transfer from the source to the load R_L. G iven that $v_{in}(t) = 100\sqrt{2}\,cos(2\pi * 10^6 t)$ V. Also, calculate the power across RL.

To achieve the maximum power Zin has to equal to Rs or Yin has to equal to (1/Rs) at resonance.

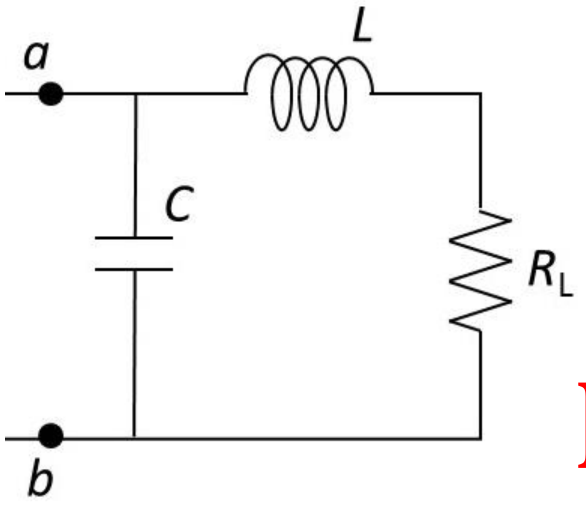
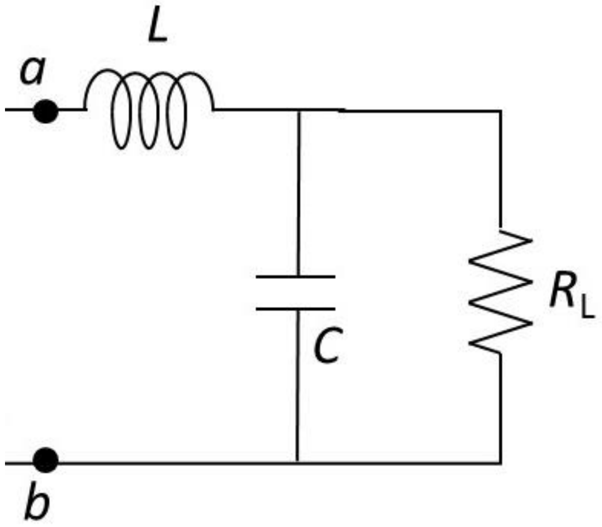


$$Z_{in}(j\omega) = \frac{G_L}{G_L^2 + \omega^2 C^2} + j\omega\left(L - \frac{C}{G_L^2 + \omega^2 C^2}\right)$$

$$L - \frac{C}{G_L^2 + \omega_r^2 C^2} = 0 \iff \boxed{L = 17.9\ \mu H}$$

$$Z_{in}(j\omega_r) = \frac{G_L}{G_L^2 + \omega^2 C^2}. \iff \boxed{C = 1186pF}$$

Coupling Networks

 $R_L < R_s$	 $R_L > R_s$
$Y_{in}(j\omega) = \frac{R_L}{R_L^2+\omega^2L^2} + j\omega\left[C - \frac{L}{R_L^2+\omega^2L^2}\right]$	$Z_{in}(j\omega) = \frac{G_L}{G_L^2+\omega^2C^2} + j\omega\left(L - \frac{C}{G_L^2+\omega^2C^2}\right)$

Answer the following questions using ChatGPT

1. Explain the condition for resonance in a series RLC circuit and a parallel RLC circuit. Why is the input impedance purely real at the resonant frequency?
2. In a matching network, explain the significance of matching the input impedance Zin to the source impedance Rs at the resonant frequency.
3. How does this impact power transfer efficiency?
4. Provide two practical scenarios where resonance is used to enhance circuit performance, and explain the role of resonance in each case.

Summary

- Resonance occurs when the inductive reactance (XLX_LXL) and capacitive reactance (XCX_CXC) in a circuit are equal in magnitude but opposite in phase.
- Series RLC Circuit - Minimum impedance and maximum current.
- Parallel RLC Circuit - Maximum impedance and minimum current.
- Energy oscillates between the inductor's magnetic field and the capacitor's electric field at the resonant frequency.
- Resonance is a vital principle in circuit design, enabling precise control over frequency, energy transfer, and amplification.