American University Of The Middle East

Filters – Part A

Linear Circuit Analysis II
EECE 202

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Announcement

1. Midterm during week 8
2. PD 1 Voice Over PPT due in Week 10

Recap

1. Transfer functions
2. Initial and final value theorem

New Material

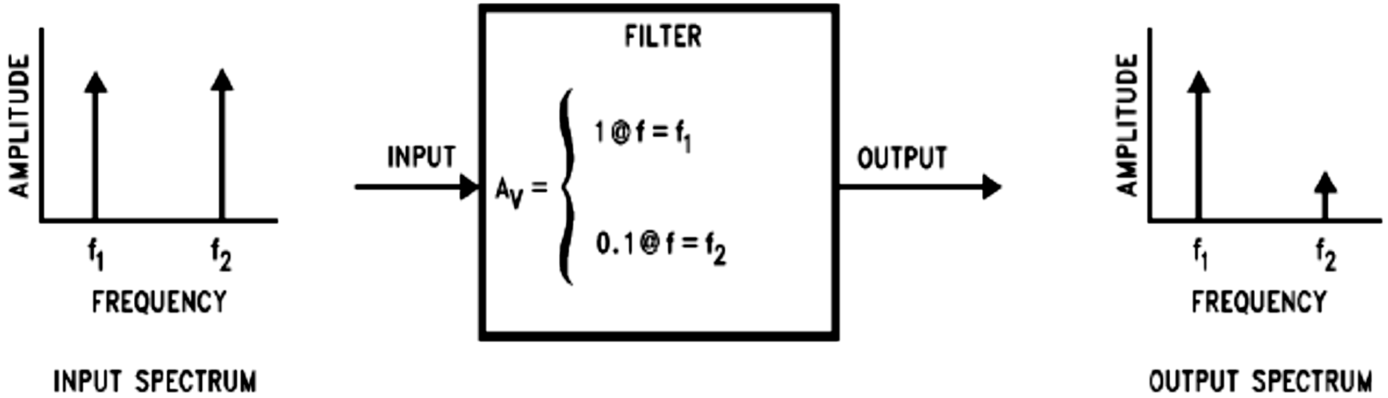
1. Filters
2. Filter Type
3. Transfer function of a filter
4. Cutoff frequency

Filters

- The water filter separates water from other impurities like sand, salts, solid particles, etc.
- Likewise, **Electric/Electronic Filter**s separate the “required signal” from other “unwanted signals”.
- In **Electric/Electronic Filter**s, we separate signals on the basis of their frequencies.
- In other words, The **Electric/Electronic Filter** is a circuit or device that passes a signal with a certain frequency and blocks signals with all other frequencies.
- Also, The **Electric/Electronic Filter** can pass signals with a certain range of frequencies and blocks what is outside this range.

Filters and Signals: What Does a Filter Do?

Ideally, a filter will not add new frequencies to the input signal, nor will it change the component frequencies of that signal, but it will change the relative amplitudes of the various frequency components and/or their phase relationships



Consider a situation where a useful signal at frequency f_1 has been contaminated with an unwanted signal at f_2. If the contaminated signal is passed through a circuit that has very low gain at f_2 compared to f_1, the undesired signal can be removed, and the useful signal will remain.

Filters Types

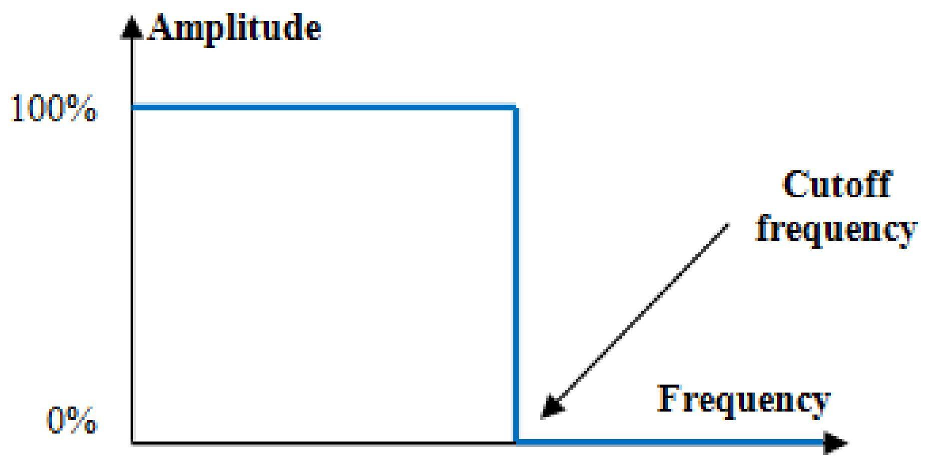
- Low Pass Filter (LPF)
- High Pass Filter (HPF)
- Band Pass Filter (BPF)

Filters Description

- A filter can be described through one of the following :
  - Transfer function $H(s)$
  - Frequency Response $H(j\omega)$
  - Gain Magnitude $|H(j\omega)|$
  - Gain in dB $G_{dB}(\omega)$

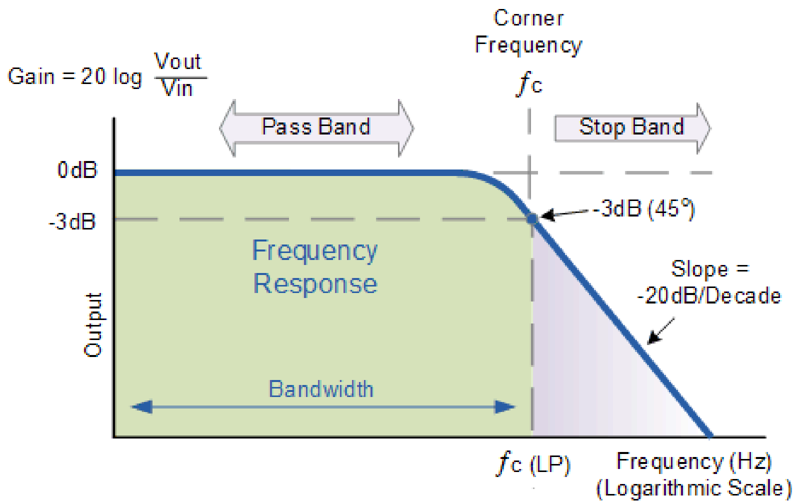
Low Pass Filter (LPF)

- The ideal Low pass filter passes signals with frequencies below a certain value (f_c), and blocks frequencies above this value. This value (f_c) as shown in the figure (shows ideal filter) is called the cutoff frequency.



- In the real Low pass filter the signal at the cutoff frequency (f_c) is attenuated to 70.7% (-3 dB) of its original amplitude.

- The frequencies below this f_c is called "Pass Band", and the frequencies above the f_c is called the "Stop Band".

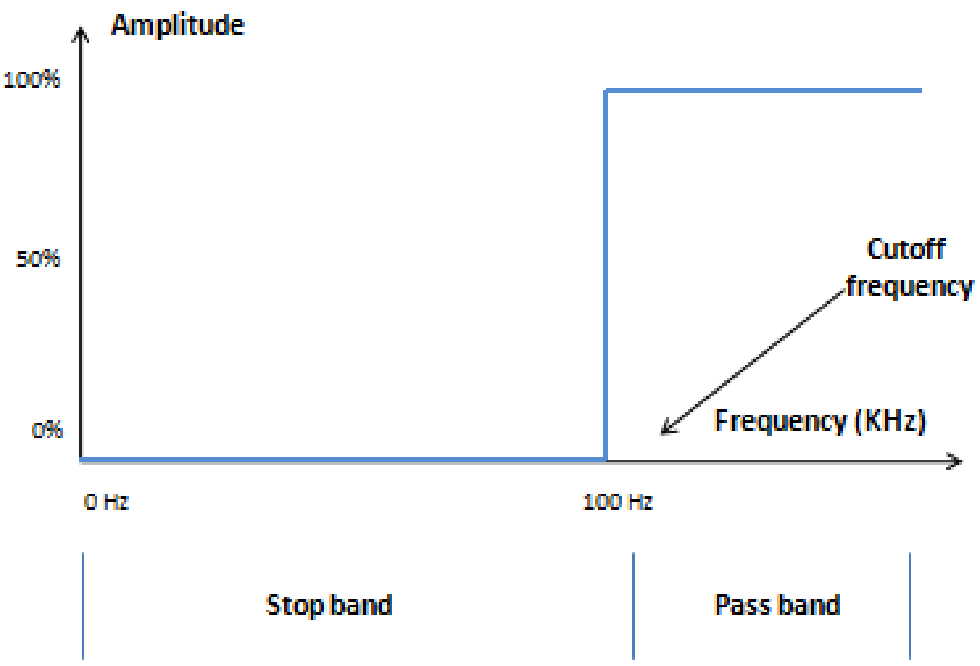


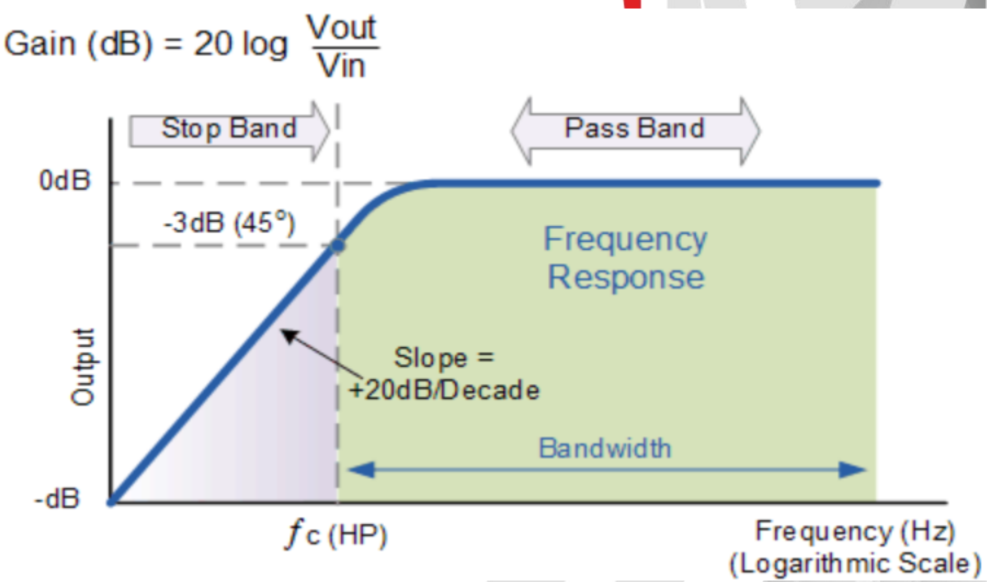
High Pass Fiter (HPF)

- The ideal High pass filter passes signals with frequencies above a certain value (f_c), and blocks frequencies below this value. This value (f_c) as shown in the figure (shows ideal filter) is called the cutoff frequency.

- In the real High pass filter the signal at the cutoff frequency (f_c) is attenuated to 70.7% (-3 dB) of its original amplitude.

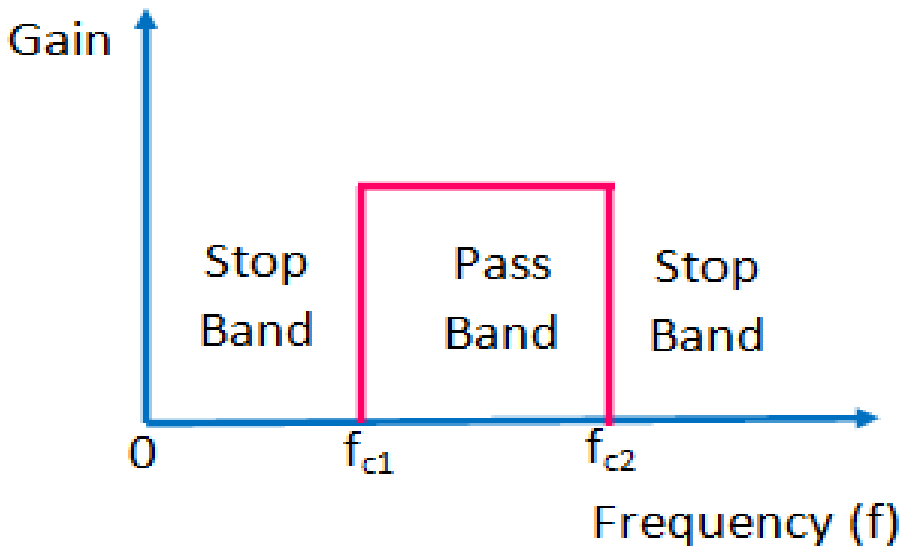
- The frequencies above this f_c is called "Pass Band", and the frequencies below the f_c is called the "Stop Band".





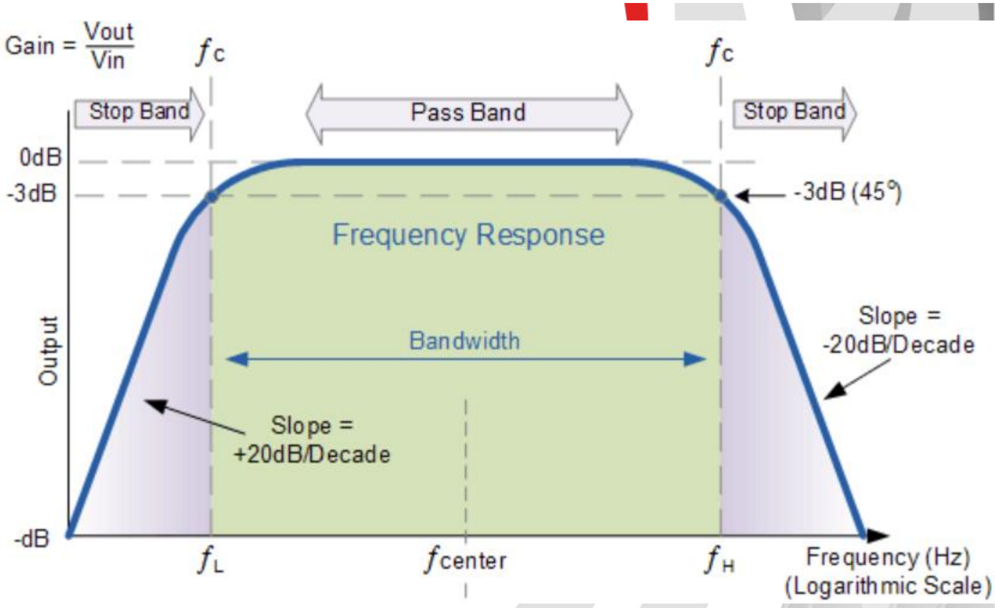
Band Pass Filter (BPF)

- The ideal Band pass filter passes signals with frequencies between certain values (f_{c1}, f_{c2}), and blocks frequencies outside these values. These values (f_{c1}, f_{c2}) as shown in the figure (shows ideal filter) is called the cutoff frequencies.



- In the real band pass filter the signal at the cutoff frequency (f_{c1} and f_{c2}) are attenuated to 70.7% (-3 dB) of its original amplitude.

- The frequencies in between these f_{c1} and f_{c2} are called "Pass Band", and the frequencies outside the f_{c1} and f_{c2} are called the "Stop Band".



Low Pass Filter (LPF)

A low-pass filter passes low frequency signals, and rejects signals at frequencies above the filter's cutoff frequency,

$$H(S) = \frac{K\omega_o^2}{S^2 + 2\sigma S + \omega_o^2}$$

The poles of the TF are : $P_{1,2} = -\sigma \pm j\omega_d$

ω_0: is the magnitude of pole frequency like in BP filter, and **σ** is the real part of the pole frequency

The peak frequency is : $\boldsymbol{\omega_0 = \sqrt{\sigma^2 + \omega_d^2}}$

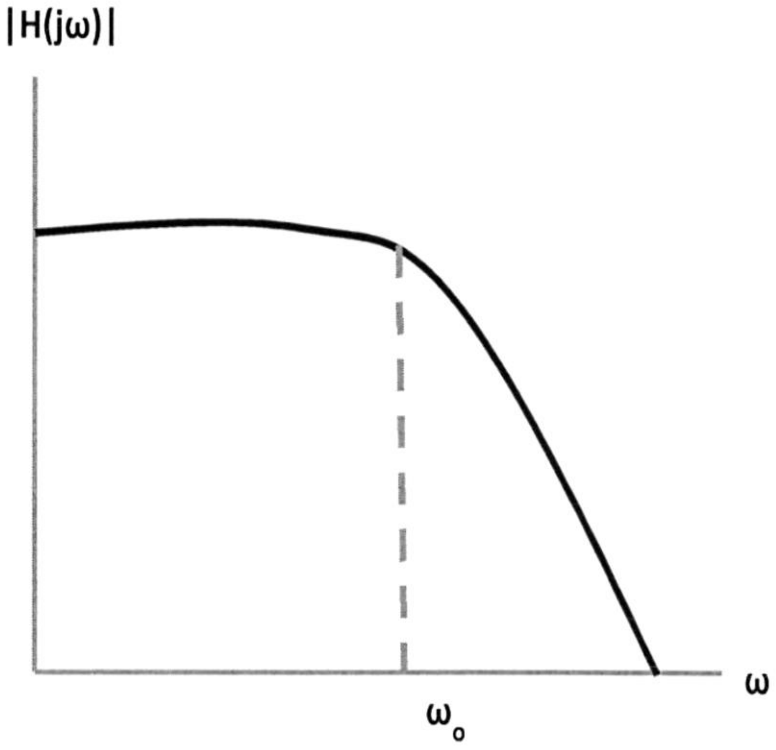
Low Pass Filter (LPF)

The frequency response is given by: $H(j\omega) = \frac{K\omega_0^2}{-\omega^2+2\sigma jw+\omega_0^2}$

The gain magnitude (amplitude) is given by:

$$|H(j\omega)| = \left|\frac{K\omega_0^2}{-\omega^2+2\sigma jw+\omega_0^2}\right| = \frac{K\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+4\sigma^2\omega^2}}$$

$$= \frac{K}{\sqrt{(1-\frac{\omega^2}{\omega_0^2})^2+4(\frac{\sigma\omega}{\omega_0^2})^2}}$$

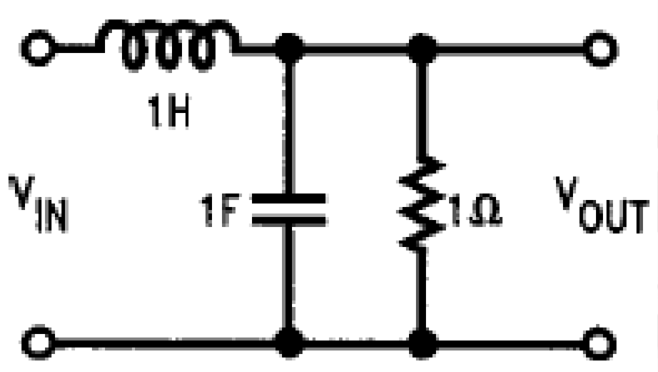


The phase response is given by:

$$\theta(\omega) = \arg H(s) = -\tan^{-1}\left[\frac{2\sigma\omega}{(\omega_o^2-\omega^2)}\right]$$

Example - Low Pass Filter (LPF)

1-Find the transfer function of the shown circuit.
2-What kind of filter does it represent ?
3-Find the peak frequency (ω_o) and the gain factor "K".



Consider $Z_{in} = sL = S$, and $Z_{out} = \frac{1}{s} || 1 = \frac{1}{S+1}$

Then the TF will be: $\frac{V_{out}}{V_{in}} = \frac{Z_{out}}{Z_{in}+Z_{out}} = \frac{\frac{1}{S+1}}{s+\frac{1}{S+1}} = \frac{1}{s^2+s+1}$

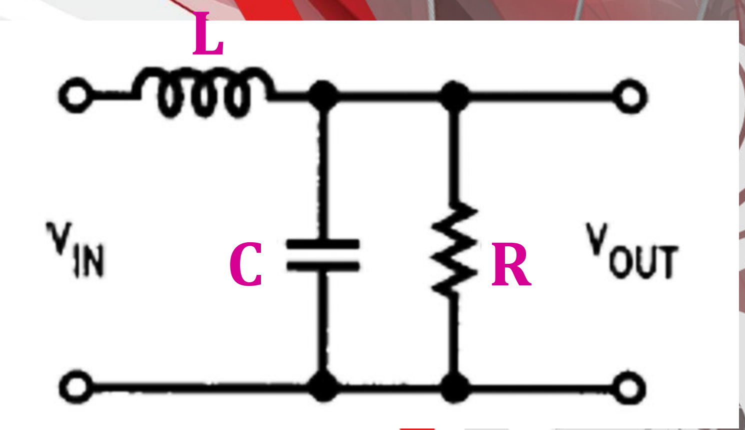
There is no "s" on the numerator, then this is a **Low Pass Filter**.

By matching the below 2 equations, we can easily find ω_0=1, and K =1

$$H(S) = \frac{K\omega_o^2}{S^2 + 2\sigma S + \omega_o^2} \quad \& \quad \frac{V_{out}}{V_{in}} = \frac{1}{s^2+s+1}$$

Example - Low Pass Filter (LPF)

Design the filter shown in the figure so that the peak frequency "f_o" is 1000 Hz.



This is a LPF with the following generic TF:

$$H(S) = \frac{K\omega_o^2}{S^2 + 2\sigma S + \omega_o^2}$$

Using circuit to calculate $\frac{V_{out}}{V_{in}}$ we get:

$$H(s) = \frac{\frac{R}{RCs+1}}{Ls + \frac{R}{RCs+1}} = \frac{R}{LRCs^2 + Ls + R}$$

$$= \frac{\frac{R}{LRC}}{s^2 + \frac{Ls}{LRC} + \frac{R}{LRC}} = \frac{\frac{1}{LC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

Then we find the following TF based on circuit analysis:

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

By comparing the above TF with the general TF of that of the LPF.

$$H(S) = \frac{K\omega_o^2}{S^2 + 2\sigma S + \omega_o^2}$$

We find that: $$\omega_0^2 = \frac{1}{LC} = (2\pi 1000)^2$$

Since we have 3 unknowns and one equation. We can assume the values of 2 unknowns and compute the 3rd.

Assume R=1Ω and L=1H.

$$\therefore \ \frac{1}{LC} = 3.94 \times 10^7, substitute\ for\ L = 1\ \therefore C \cong 25\, nF$$

Answer the following questions using ChatGPT

- What is the primary function of an electronic filter in a circuit
- How does a Low Pass Filter (LPF) affect signals above its cutoff frequency?
- What is the significance of the -3 dB point in a filter's frequency response?
- How does the step response of a system differ from its impulse response, and what does it indicate about the system's stability?
- How can you find out the type of filter?

Summary

- Filters in electronics separate required signals from unwanted ones based on frequency.
- They selectively pass certain frequencies (pass band) and block others (stop band)
- Low Pass Filter (LPF): Passes signals below a cutoff frequency, blocking higher frequencies.
- High Pass Filter (HPF): Passes signals above a cutoff frequency, blocking lower frequencies.
- Band Pass Filter (BPF): Passes signals between two cutoff frequencies, blocking signals outside this range
- Transfer Function $H(s)$ represents the mathematical relationship between input and output.
- Frequency Response $H(j\omega)$ describes how the filter responds to different frequencies.