American University Of The Middle East

Filters – Part B

Linear Circuit Analysis II
EECE 202

www.aum.edu.kw

Announcement

1. Midterm during week 8
2. PD 1 Voice Over PPT due in Week 10

Recap

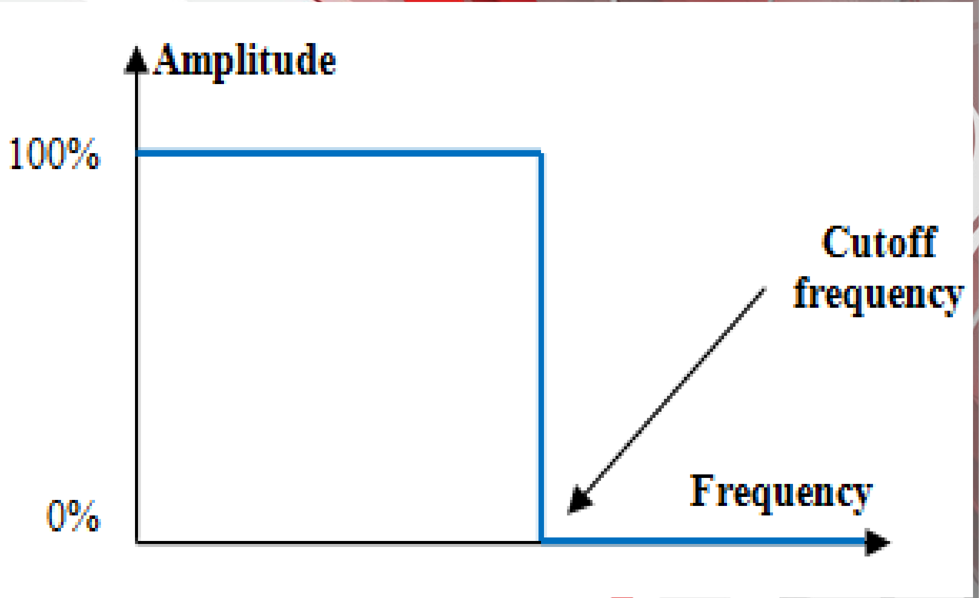
1. Filters
2. Filter Type
3. Transfer function of a filter
4. Cutoff frequency

New Material

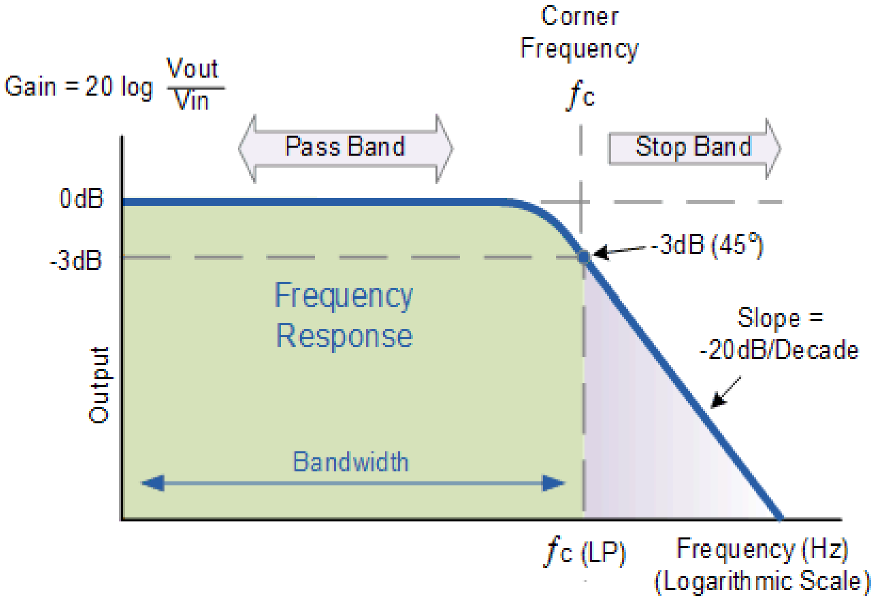
1. Transfer function of LP, BP, and HP filter

Low Pass Filter

- The Low pass filter passes signals with frequencies below a certain value (f_c), and blocks frequencies above this value. This value (f_c) as shown in the figure (shows ideal filter) is called the cutoff frequency.



- The characteristic shown in the figure represents the real response of the low pass filter, where the signal at the cutoff frequency (f_c) is attenuated to 70.7% (-3 dB) of its original amplitude.



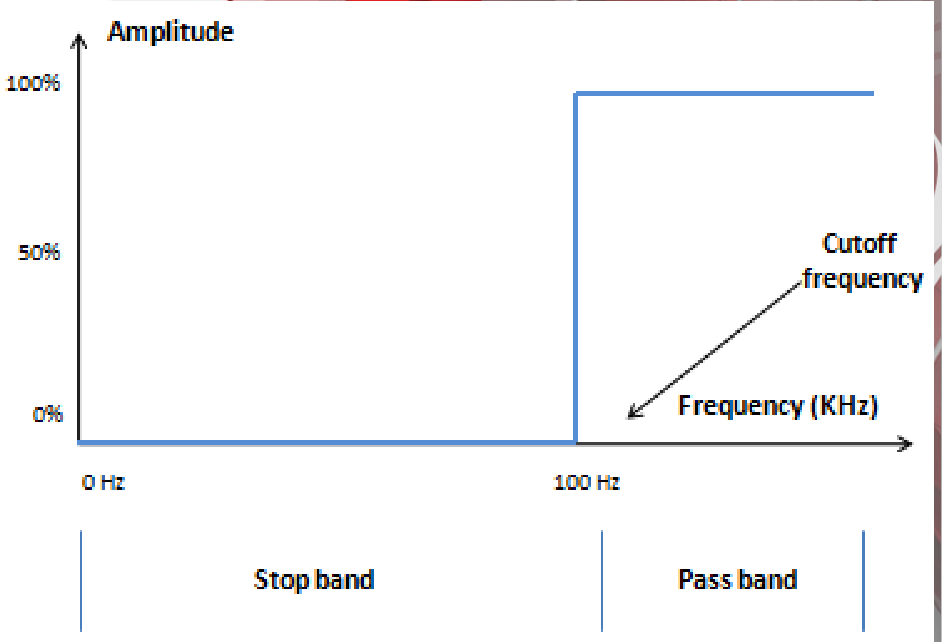
- The frequencies below this f_c is called "Pass Band", and the frequencies above the f_c is called the "Stop Band".

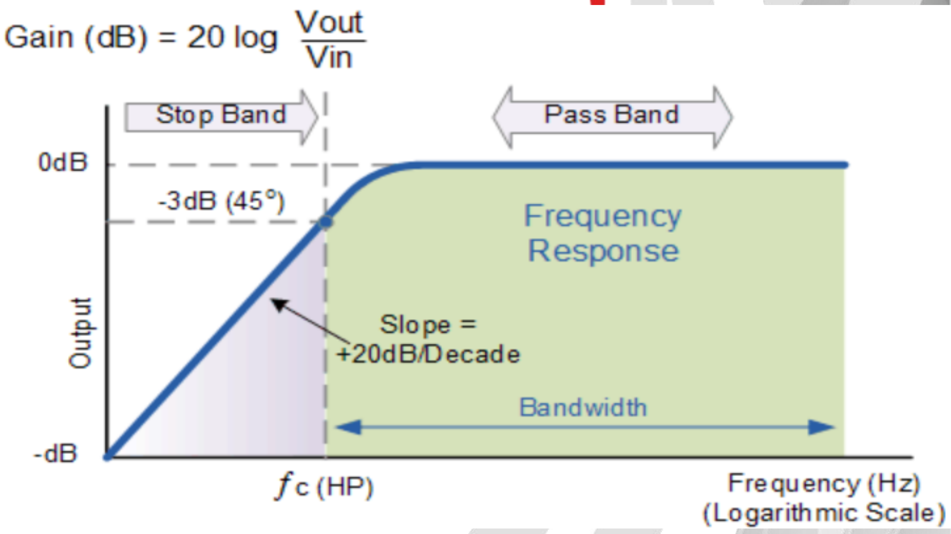
High pass filter

- The High pass filter passes signals with frequencies above a certain value (f_c), and blocks frequencies below this value. This value (f_c) as shown in the figure (shows ideal filter) is called the cutoff frequency.

- The characteristic shown in the figure represents the real response of the high pass filter, where the signal at the cutoff frequency (f_c) is attenuated to 70.7% (-3 dB) of its original amplitude.

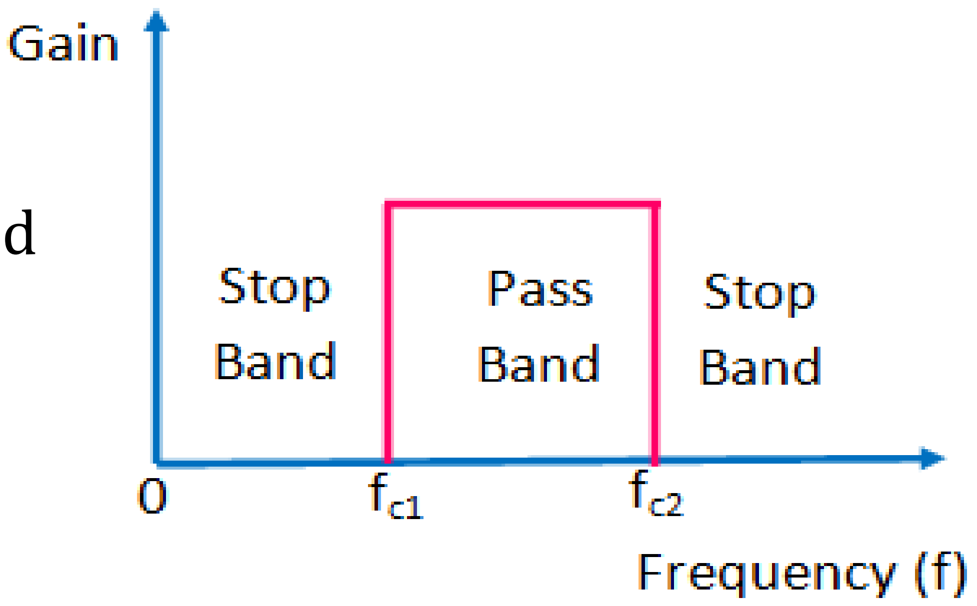
- The frequencies above this f_c is called "Pass Band", and the frequencies below the f_c is called the "Stop Band".



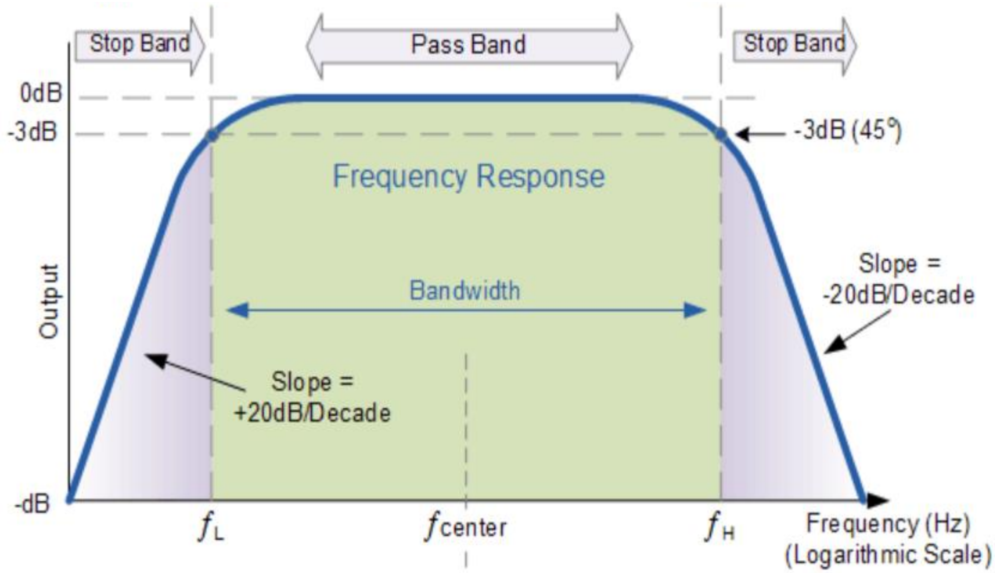


Bandpass Filter

- The Band pass filter passes signals with frequencies between certain values (f_{c1}, f_{c2}), and blocks frequencies outside these values. These values (f_{c1}, f_{c2}) as shown in the figure (shows ideal filter) is called the cutoff frequencies.



- The characteristic shown in the figure represents the real response of the band pass filter, where the signal at the cutoff frequency (f_{c1} and f_{c2}) are attenuated to 70.7% (-3 dB) of its original amplitude.



- The frequencies in between these f_{c1} and f_{c2} are called "Pass Band", and the frequencies outside the f_{c1} and f_{c2} are called the "Stop Band".

High Pass Filter (HPF)

The opposite of the low-pass is the high-pass filter, which rejects signals below its cutoff frequency.

$$H(S) = \frac{KS^2}{S^2 + 2\sigma S + \omega_o^2}$$

The poles of the TF are : $P_{1,2} = -\sigma \pm j\omega_d$

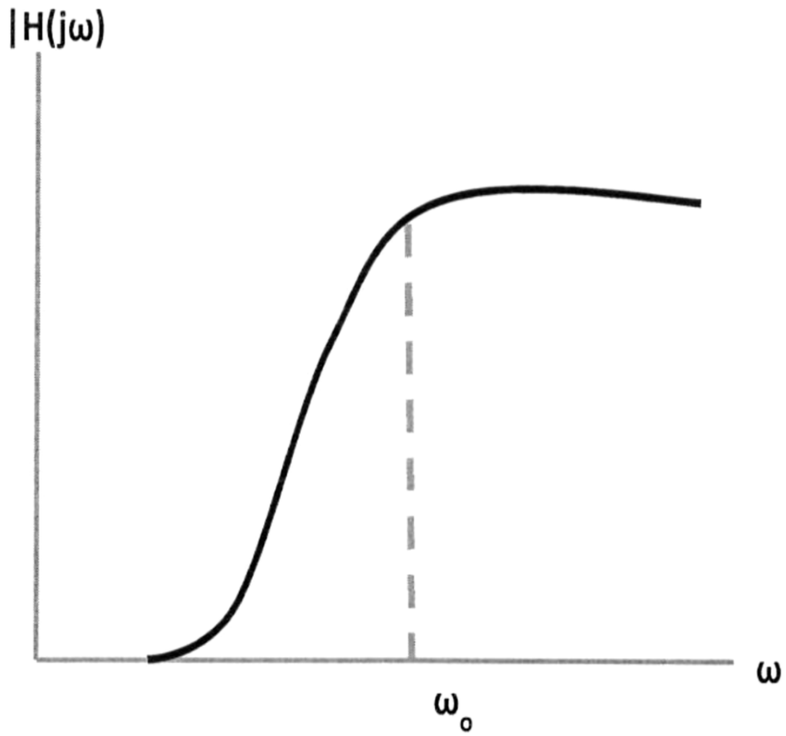
ω_0: is the magnitude of pole frequency like in BP filter, and σ is the real part of the pole fre

The peak frequency is : $\omega_o = \sqrt{\sigma^2 + \omega_d^2}$

The gain (amplitude) of the frequency response is given by:

$$|H(j\omega)| = \left|\frac{-K\omega^2}{-\omega^2 + 2\sigma j\omega + \omega_0^2}\right| = \frac{K\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\sigma^2\omega^2}}$$

$$= \frac{K(\frac{\omega}{\omega_0})^2}{\sqrt{(1 - \frac{\omega^2}{\omega_0^2})^2 + 4(\frac{\sigma\omega}{\omega_0^2})^2}}$$

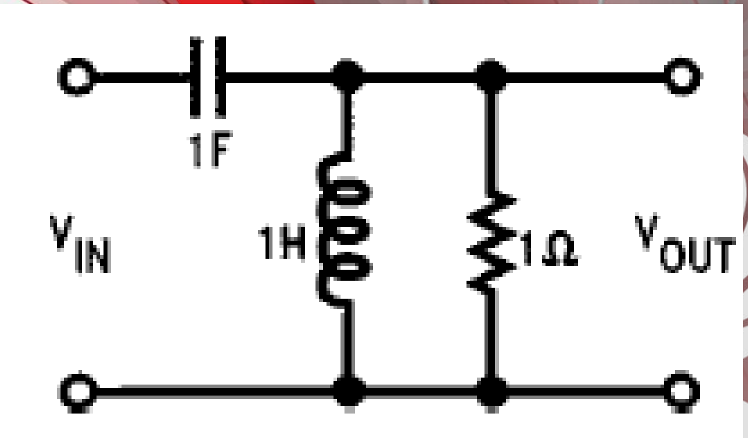


The phase response is given by:

$$\theta(\omega) = \arg H(s) = 180 - \tan^{-1}\left[\frac{2\sigma\omega}{(\omega_o^2 - \omega^2)}\right]$$

Example

1- Find the transfer function of the shown circuit.
2- What kind of filter does this circuit represent.
3-Find the peak frequency (w_o) and the gain factor “K”.



Consider $Z_{in} = \frac{1}{sc} = \frac{1}{s},$ and $Z_{out} = \text{sL}||1 = \text{s}||1 = \frac{S}{S+1}$

Then the TF will be: $\frac{V_{out}}{V_{in}} = \frac{Z_{out}}{Z_{in}+Z_{out}} = \frac{\frac{s}{s+1}}{\frac{1}{s}+\frac{s}{s+1}} = \frac{\frac{s}{s+1}}{\frac{s+1}{s(s+1)}+\frac{s\times s}{s(s+1)}} = \frac{\frac{s}{s+1}}{\frac{s^2+s+1}{s(s+1)}} = \frac{s^2}{s^2+s+1}$

There is an “s^2” on the numerator, then this is a **High Pass Filter**.

By matching the below 2 equations, we can easily find ω_0=1, and K =1

$$H(S) = \frac{KS^2}{S^2 + 2\sigma S + \omega_o^2} \qquad \& \qquad \frac{V_{out}}{V_{in}} = \frac{s^2}{s^2+s+1}$$

Band Pass (BP) Filter

The general TF of a 2nd order BPF

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{K*S}{S^2 + (\frac{\omega_p}{Q_P})S + \omega_P^2} = \frac{K*S}{S^2 + 2\sigma S + \omega_P^2}$$

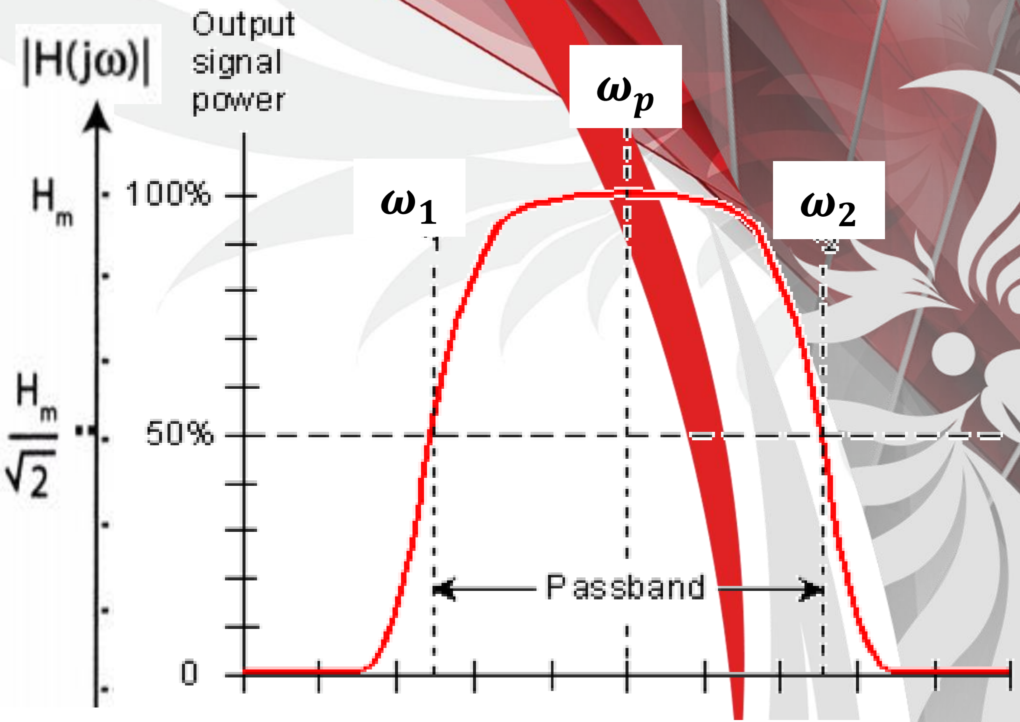
The poles of the TF are : $P_{1,2} = -\sigma \pm j\omega_d$

The peak frequency is : $\omega_p = \sqrt{\sigma^2 + \omega_d^2}$

The Bandwidth is : $B_\omega = \frac{\omega_P}{Q_P} = 2\sigma = \omega_2 - \omega_1$

The maximum value: $H_m = \frac{K}{2\sigma} = \frac{K}{B_\omega} = \frac{K*Q_P}{\omega_p}$

The Quality factor: $Q_P = \frac{\omega_P}{B_\omega}$



$\omega\ (rad/sec)$

$$\omega_{1,2} = \omega_P\left(\sqrt{1 + \frac{1}{4Q_P^2}} \pm \frac{1}{2Q_P}\right)$$

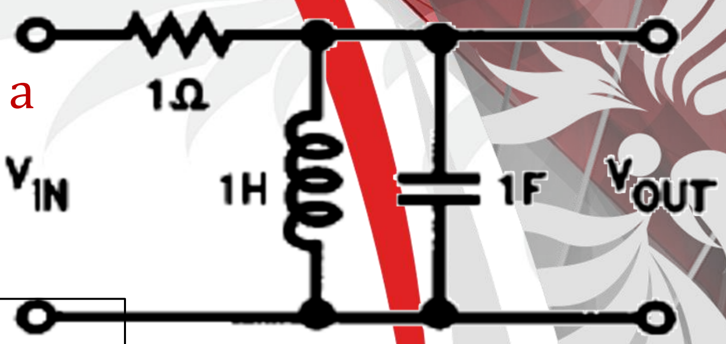
Example -1

For the circuit shown, answer the following:

1-Find the transfer function H(s).

2-What kind of filter does this circuit represent.

3-Find the magnitude and the phase of the transfer function as a function of "ω".



Solution

$$H(s) = \frac{\dfrac{s \times \frac{1}{s}}{s + \frac{1}{s}}}{1 + \dfrac{s \times \frac{1}{s}}{s + \frac{1}{s}}} = \frac{\dfrac{1}{\frac{s^2+1}{s}}}{1 + \dfrac{1}{\frac{s^2+1}{s}}} = \frac{\frac{s}{s^2+1}}{1 + \frac{s}{s^2+1}} = \frac{\frac{s}{s^2+1}}{\frac{s^2+1+s}{s^2+1}} = \frac{s}{s^2+s+1}$$

H(s) is like $\frac{K*S}{S^2+2\sigma S+\omega_P^2}$

so the circuit represents a band pass filter

$$A(\omega) = |H(s)| = \left|\frac{j\omega}{-\omega^2 + j\omega + 1}\right| = \frac{\omega}{\sqrt{\omega^2 + (1-\omega^2)^2}}$$

$$\theta(\omega) = \arg H(s) = 90^\circ - \tan^{-1}\frac{\omega^2}{(1-\omega^2)}$$

Example -2

Suppose a second-order BP-filter circuit having the transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{2S}{S^2 + 2S + 256}$$

Find H_m, ω_p, B_w and $\omega_{1,2}$. Also, draw the response and indicate the obtained values.

$$H(s) = \frac{K * S}{S^2 + 2\sigma S + \omega_P^2} = \frac{K * S}{S^2 + \frac{\omega_p}{Q_p} S + \omega_P^2}$$

$K=2,$

$$\boldsymbol{\omega_p} = \sqrt{256} = 16 \frac{rad}{s},$$

$$B_{\boldsymbol{\omega}} = 2\boldsymbol{\sigma} = 2 rad/s,$$

$$Q_p = \frac{\boldsymbol{\omega_p}}{2\boldsymbol{\sigma}} = \frac{16}{2} = 8,$$

$$\boldsymbol{\omega_1} = 16\sqrt{1 + \frac{1}{4 \times 64}} - \frac{1}{2 \times 8} = 15\, rad/s$$

$$\boldsymbol{\omega_2} = 16\sqrt{1 + \frac{1}{4 \times 64}} + \frac{1}{2 \times 8} = 17\, rad/s$$

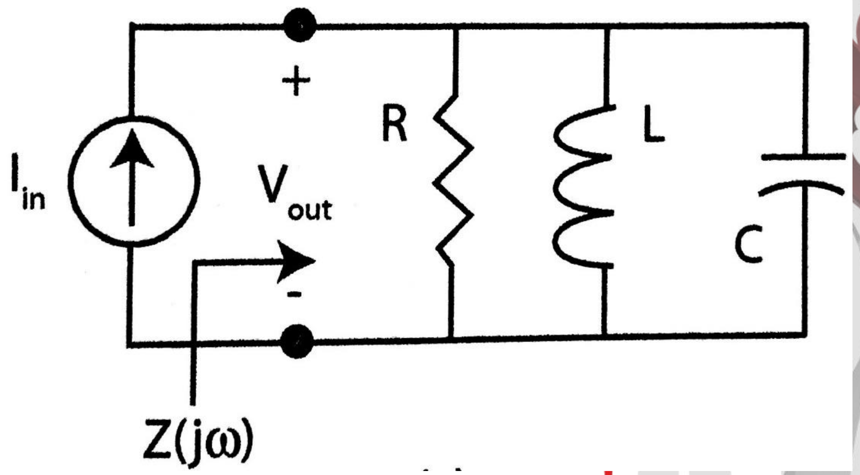
ω_p=16 rad/sec, B_W=2 rad/sec, H_m=1, ω_1=15 rad/sec and ω_2=17 rad/sec.

Example -3

For the shown circuit, find the transfer function

$$\frac{V_{out}(s)}{I_{in}(s)} = Zin(\text{s}) .$$

Use R=2.5K Ω, L=0.1H, C=0.1uF



Also, find H_m, ω_p, B_w and $\omega_{1,2}$.

Draw the response and indicate the obtained values.

Example -3

$$\frac{V_{out}(s)}{I_{in}(s)} = Z(s) = \frac{1}{Y(s)} = \frac{1}{Cs + \frac{1}{R} + \frac{1}{Ls}} = \frac{1}{\frac{RLCs^2 + Ls + R}{RLs}} = \frac{RLs}{RLCs^2 + Ls + R}$$

$$\rightarrow Z(s) = \frac{RLs}{RLCs^2 + Ls + R} = \frac{\frac{RLs}{RLC}}{\frac{RLCs^2}{RLC} + \frac{Ls}{RLC} + \frac{R}{RLC}} = \frac{\frac{1}{C}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Compare this equation: $\dfrac{\frac{1}{\mathrm{C}}\mathrm{s}}{\mathrm{s}^2 + \frac{1}{\mathrm{RC}}\mathrm{s} + \frac{1}{\mathrm{LC}}}$

With the BPF equation: $\dfrac{\boldsymbol{K\,s}}{\boldsymbol{s^2 + 2\sigma s + \omega_p^2}}$

Example 3

$$\sigma = \frac{1}{2RC}$$

$$\omega_p = \frac{1}{\sqrt{LC}}$$

$$Q_p = \frac{\omega_p}{2\sigma} = \frac{RC}{\sqrt{LC}} = R\sqrt{\frac{L}{C}}$$

$$\omega_p = \frac{1}{\sqrt{LC}}$$

$$\frac{\frac{1}{C}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$K = \frac{1}{C}$$

Now, replace each parameter with its value in the example.

$$\frac{K\,s}{s^2 + 2\sigma s + \omega_p^2}$$

$$H_m = \frac{K}{2\sigma} = \frac{1/C}{1/RC} = R$$

$$\omega_{1,2} = \frac{1}{\sqrt{LC}}\left(\sqrt{1 + \frac{1}{4R^2\frac{C}{L}}} \mp \frac{1}{2R\sqrt{\frac{C}{L}}}\right) = \frac{1}{\sqrt{LC}}\left(\sqrt{\frac{4R^2C + L}{4R^2C}} \mp \frac{1}{2R\sqrt{\frac{C}{L}}}\right)$$

$$= \left(\sqrt{\frac{4R^2C + L}{4R^2LC^2}} \mp \frac{1}{2R\sqrt{LC\frac{C}{L}}}\right) = \boxed{\sqrt{\frac{1}{LC} + (\frac{1}{2RC})^2} \mp \frac{1}{2RC}}$$

Answer the following questions using ChatGPT

- How do filters affect the amplitude of signals within their pass band and stop band.

- Give an example where each filter type might be practically applied in electronic or communication systems

- Which property of a filter describes its ability to distinguish between closely spaced frequencies.

- Which filter blocks only a specific narrow band of frequencies while passing all others.

Summary

- Low-Pass Filter (LPF) allows frequencies below the cutoff frequency (f_c) to pass and attenuates frequencies above fc. Used to remove high-frequency noise from signals.
- High-Pass Filter (HPF) allows frequencies above the cutoff frequency (f_c) to pass, attenuates frequencies below fc, useful in applications that require eliminating low-frequency noise or interference.
- Band-Pass Filter (BPF) passes frequencies within a specific range (f_{c1} to f_{c2}), attenuates frequencies outside this range, and ideal for isolating specific frequency bands.
- Notch (Band-Stop) Filter attenuates a narrow band of frequencies, passes frequencies outside this band, used to eliminate unwanted frequencies.
- Key Concepts
- At cutoff frequency signal amplitude drops to 70.7% (-3 dB).
- Pass Band & Stop Band: Frequency ranges that are passed or attenuated by the filter.
- Quality Factor (Q) indicates the filter's selectivity and ability to distinguish between frequencies.