Laplace Transform Analysis

Linear Circuit Analysis II
EECE 202

American University Of The Middle East

Announcement

1. GCA 1 (individual) - Week 5 (Second Lecture)

Recap

1. Types of Partial Fraction Expansions
2. Repeated Poles
3. Applying the Inverse Laplace Transform to the Decomposed Terms

New Material

1. Solution of Integral-Differential Equations
2. Time Differentiation
3. Integration Property
4. Apply Laplace Transform to given differential or integral equation.

Learning Outcomes

2. an ability to compute responses of linear circuits with and without initial conditions via one-sided Laplace transform techniques. SO [1]

Time Differentiation Property

$$L[\frac{df}{dt}]=sF(s)-f(0^-)$$

$$L[\frac{d^2f}{dt^2}]=s^2F(s)-sf(0^-)-\dot{f}(0^-)$$

$$L[\frac{d^nf}{dt^n}]=s^n\times F(s)-s^{n-1}\times f(0^-)-s^{n-2}\times \dot{f}(0^-)-\cdots f^{(n-1)}(0^-)$$

Differentiation in the time domain is equivalent to multiplication by s in the s-domain.

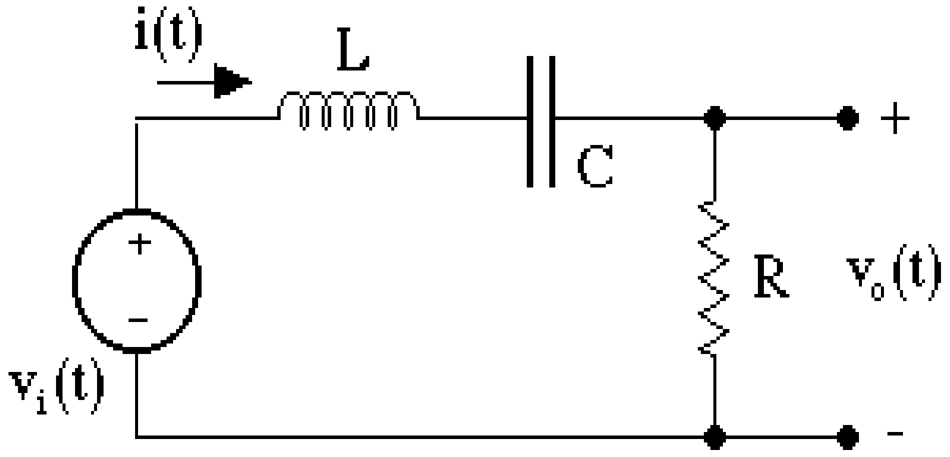
Example 1

For the circuit shown:

Given that $i(t) = 4e^{-2t}\sin(6t)u(t)$A, and L=0.5H, Calculate the voltage across the inductor L, v_L(t). (*assume zero initial conditions*)

1-Use the time domain.

2-Use the S domain.



$$1)\mathrm{v_L}(\mathrm{t}) = L\frac{di(t)}{dt}$$

$$2)\mathrm{V_L}(s) = L\left(sI_L(s) - I_L(0^-)\right) = LsI_L(s)$$

$$= 0.5s\frac{4\times6}{(s+2)2+6^2} = \frac{12s}{(s+2)2+6^2} = \frac{12(s+2-2)}{(s+2)2+6^2} = \frac{12(s+2)}{(s+2)2+6^2} - \frac{24}{(s+2)2+6^2}$$

$$v_L(t) = 12e^{-2t}\cos(6t)u(t) - 4e^{-2t}\sin(6t)u(t)\ V$$

Integration Property

Integration in the time domain is equivalent to division by s in the S-domain

$$L\left[\int_{0^-}^{t} f(t)dt\right] = \frac{F(s)}{s}$$

$$L\left[\int_{-\infty}^{t} f(t)dt\right] = \frac{F(s)}{s} + \frac{\int_{-\infty}^{0^-} f(t)dt}{s}$$

Example 2

Find the solution to the following differential equation

$$16\int_{0^-}^{t} f(q)dq + \frac{d}{dt}f(t) = 2u(t)$$

Apply Laplace transform to the given formula

$$16\frac{F(s)}{s} + sF(s) - f(0^-) = \frac{2}{s}$$

$$F(s)\left(\frac{16}{s} + s\right) - f(0^-) = \frac{2}{s}$$

$$F(s)\left(\frac{16+s^2}{s}\right)=\frac{2}{s}+f(0^-)$$

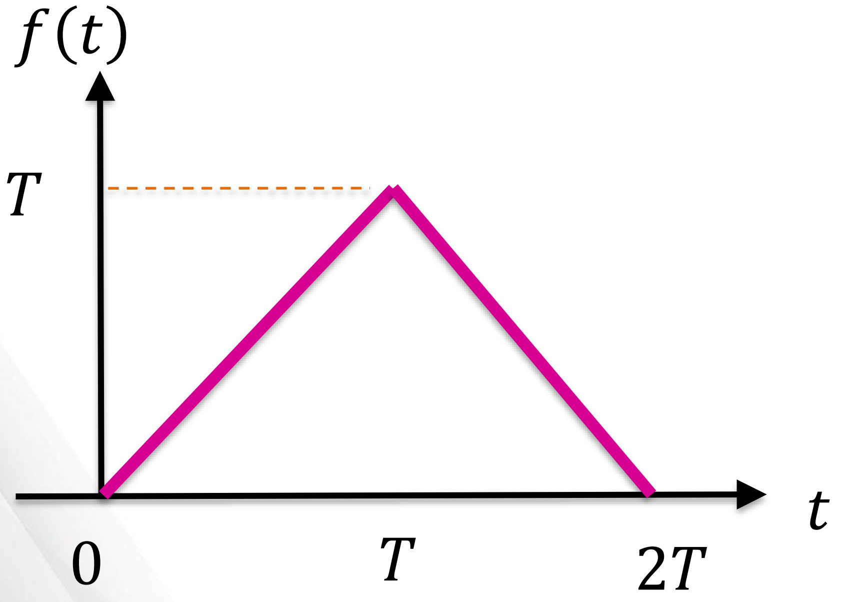
$$F(s)=\frac{2}{(16+s^2)}+\frac{sf(0^-)}{(16+s^2)}$$

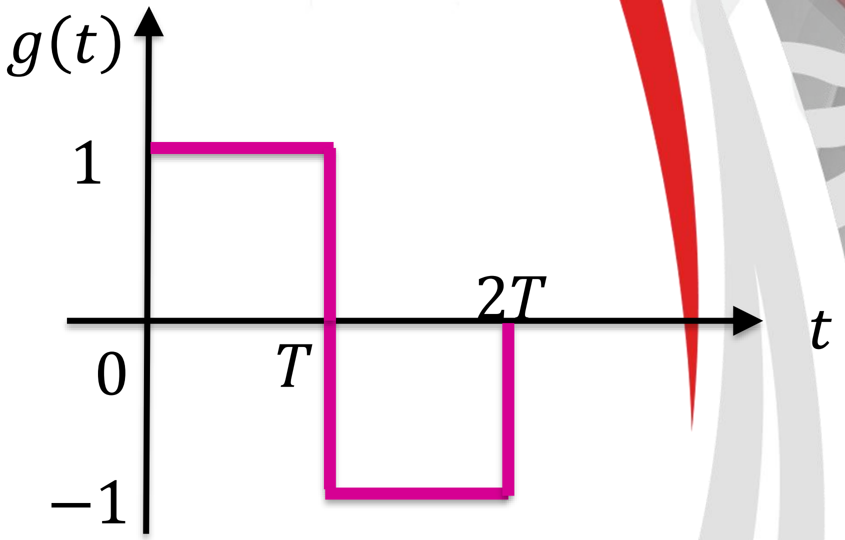
$$F(s)=0.5\,\frac{4}{(s^2+4^2)}+\frac{sf(0^-)}{(s^2+4^2)}$$

$$\mathrm{f}(t)=0.5\sin(4t)\,u(t)+f(0^-)\cos(4t)\,u(t)$$

Example 12.21 (p. 580)

Find the Laplace transform of the signal *f(t)* sketched in Figure below using the integration property.





Note that the triangular waveform *f(t)* is the integral of the square wave *g(t)*

Step 1. Represent the square wave *g(t)* in terms of steps and shifted steps:

$$g(t) = (u(t) - u(t-T)) - (u(t-T) - u(t-2T))$$
$$= u(t) - 2u(t-T) + u(t-2T)$$

Step 2. Obtain the Laplace transform of *g(t)*

$$L[g(t)] = \frac{1}{s}[1 - 2e^{-sT} + e^{-2sT}]$$

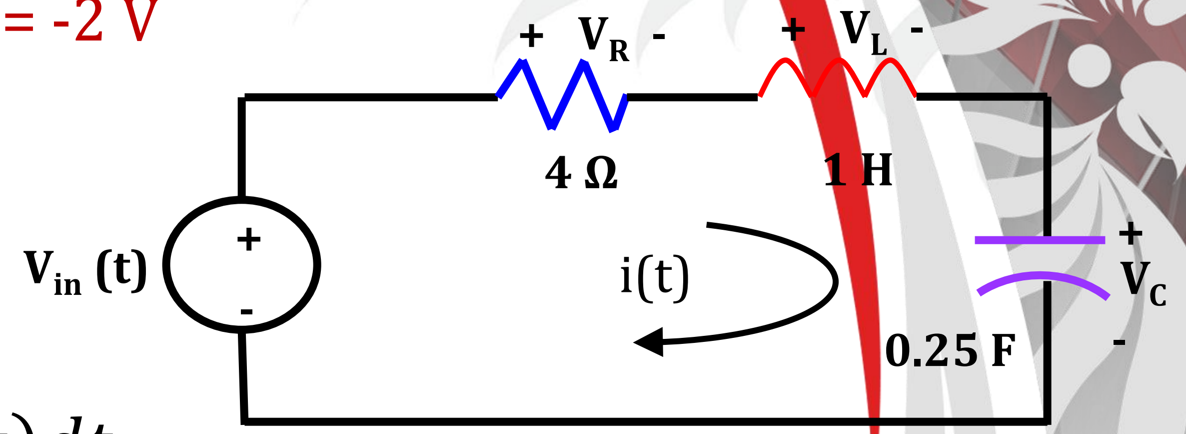
Step 3. Apply the integration property using $f(t) = \int_{-0}^{t} g(q)dq$

$$\boldsymbol{L[f(t)] = L\left[\int_{-0}^{t} g(q)dq\right] = \frac{1}{s}L[g(t)] = \frac{1}{s^2}[1 - 2e^{-sT} + e^{-2sT}]}$$

Example 12.26 (p. 587)

Find the current (i(t)) flowing in the circuit below,

given that: V_{in} (t)= δ(t), $i_L(0^-)$ = 1A, $V_C(0^-)$= -2 V



$$V_{in} = V_R + V_L + V_C$$

$$V_{in}(t) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

$$\delta(t) = i(t)4 + 1\frac{di(t)}{dt} + \frac{1}{0.25}\int_{-\infty}^{t} i(t)dt$$

Since V_{in} (t)= δ(t)

By applying LT to the obtained equation

$$1 = 4I(s) + 1 \times [sI(s) - i(0^-)] + 4\frac{1}{s}I(s) + \frac{v_c(0^-)}{s}$$

$$1 + 1 + \frac{2}{s} = I(s)[4 + s + 4\frac{1}{s}]$$

$$\frac{2S + 2}{s} = I(s)[\frac{4s + s^2 + 4}{s}]$$

Solve for I(s)

$$I(s) = \frac{2s+2}{4s+s^2+4} = \frac{2s+2}{(s+2)^2} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2}$$

B=-2, A= 2

Apply inverse LT to get i(t)

$$i(t) = 2e^{-2t}u(t) - 2e^{-2t}r(t) \qquad A$$

Practice – Use AI and answer

1. How does differentiation in the time domain translate to the s-domain using the Laplace transform?

2. Explain how the integration property of Laplace transforms is used to simplify solving circuits with capacitors and inductors.

3. After solving an equation in the s-domain, what is the process of using the inverse Laplace transform to find the time-domain solution?

4. Why is the Laplace transform particularly useful for handling initial conditions in circuits? Give an example of how it simplifies solving a differential equation.

5. Solve activity on Moodle

Summary

- **Time Differentiation Property**
  - Differentiation in time domain = Multiplication by s in the s-domain.
- **Integration Property**:
  - Integration in time domain = Division by s in the s-domain.
- **Steps to Solve Differential Equations**:
  - Apply the Laplace transform to both sides of the equation.
  - Solve for the unknown function in the s-domain (e.g., F(s)F(s)F(s)).
  - Apply the inverse Laplace transform to obtain the solution in the time domain.

Suggested Additional Problems for Ch. 12:

Example 12.27 (p. 588), Exercise (p. 588) ???