Mutual Inductance, Dot Convention, Differential Equations, and S Domain Model

Linear Circuit Analysis II
EECE 202

Announcement

- PD2 (Technical Report) due Thursday Week 12

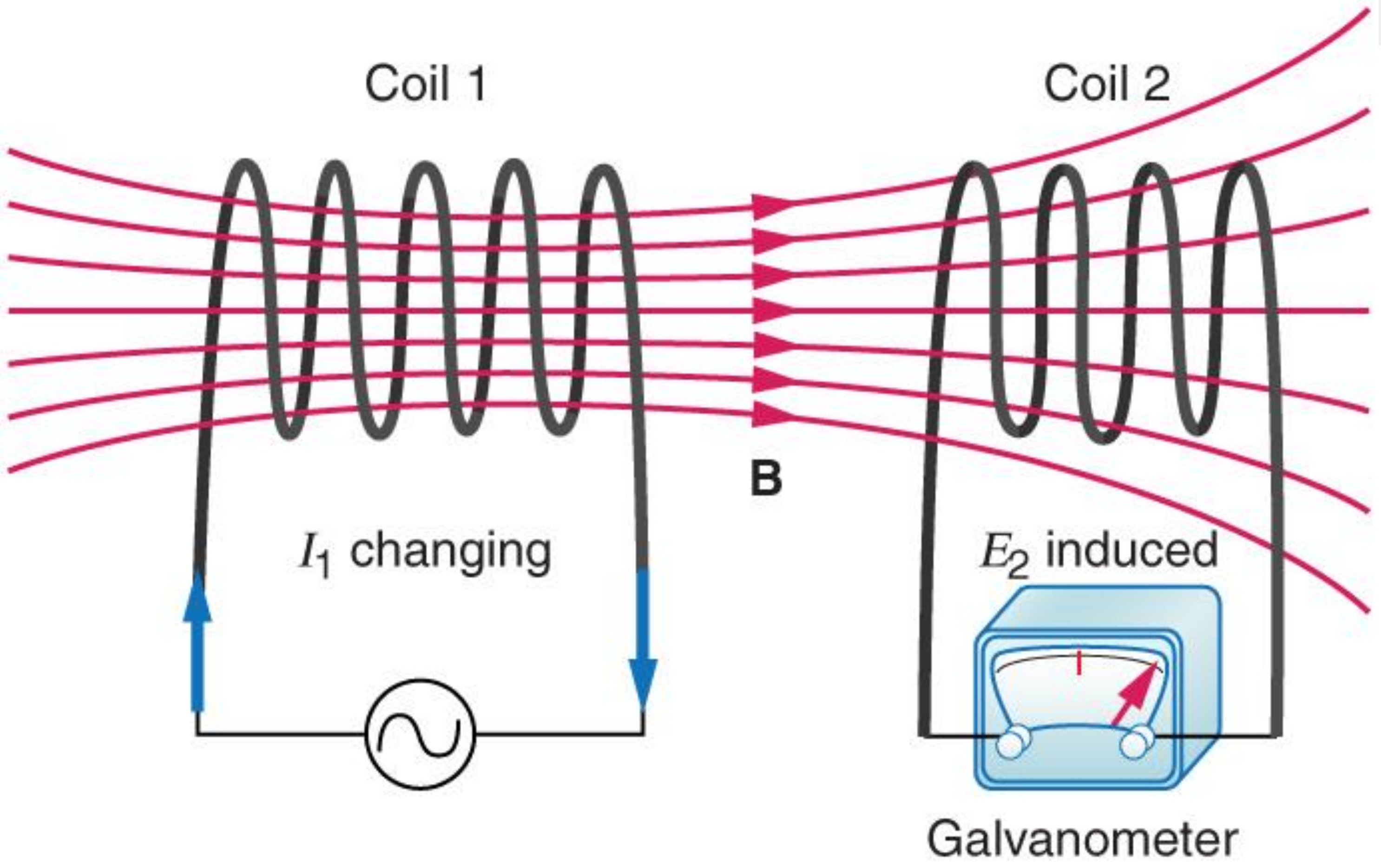
Recap

1. Resonance
2. Series RLC circuit
3. Parallel RLC circuit

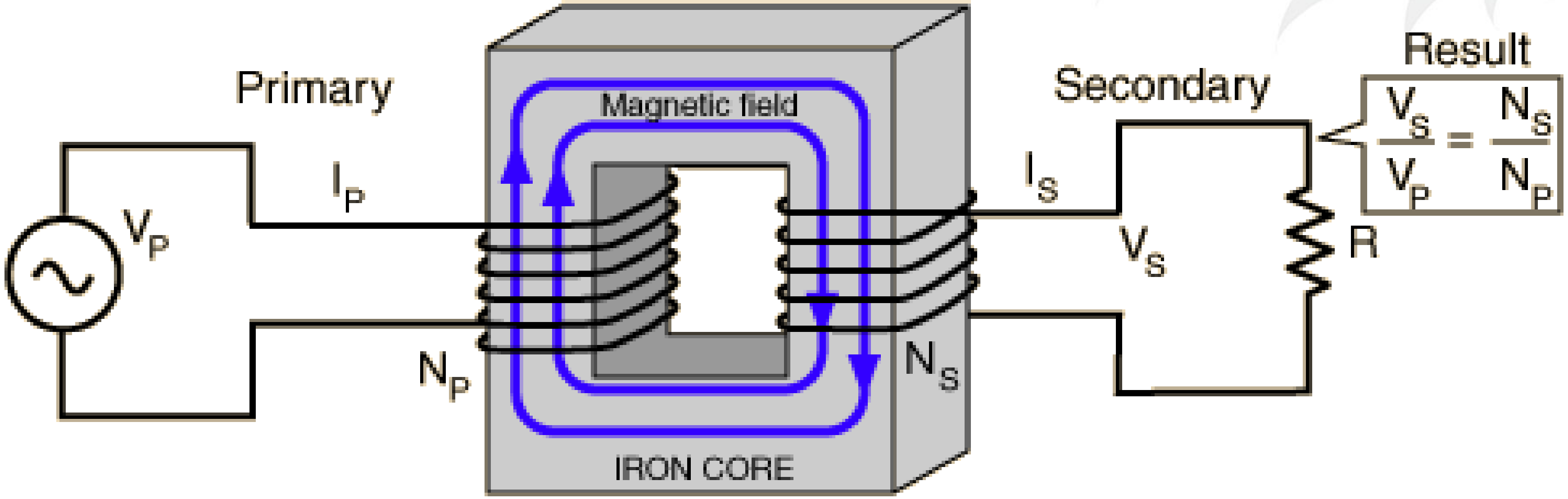
New Material

1. Mutual inductance and its equations in time and s domain
2. Transformers
3. Dot conventions

Mutual inductance

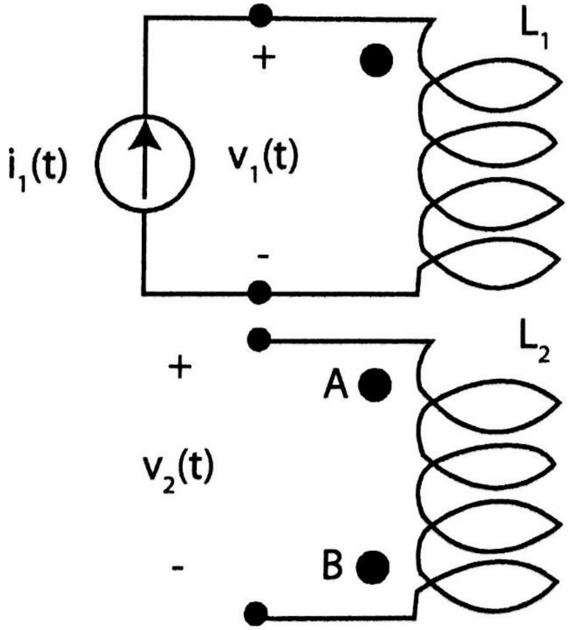


Transformers



https://www.youtube.com/watch?v=d7InRZokfzY&t=11s

Mutual inductance



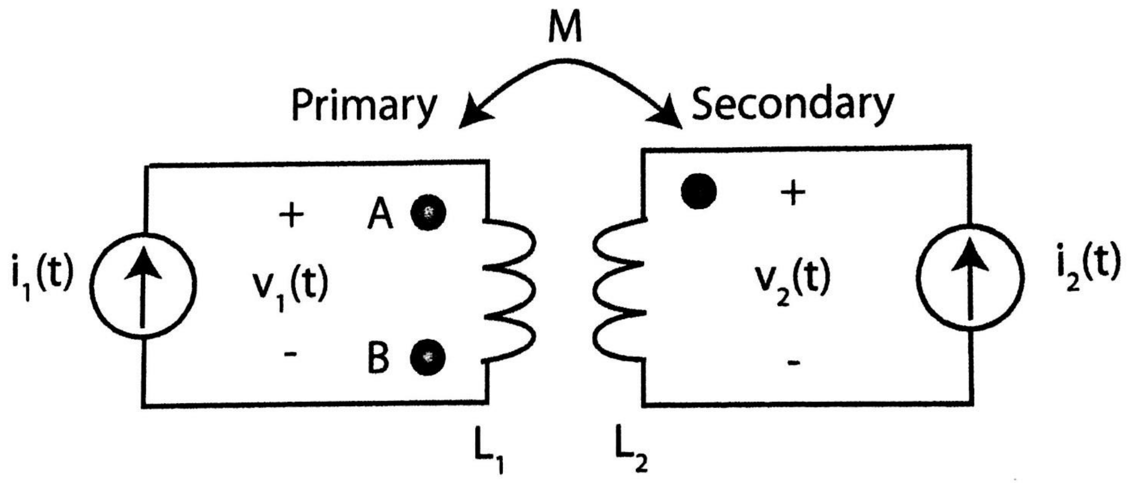
$$V_2(t) = \pm M_{12}\frac{di_1(t)}{dt}$$

L - is the coil inductance
M - is the mutual inductance

Mutual inductance equations in the time domain

$$v_1(t) = L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}$$

$$v_2(t) = \pm M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$



The coupling coefficient

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

The energy stored at t=T

$$W(T) = 0.5L_1I_1^2 + 0.5L_2I_2^2 \pm MI_1I_2$$

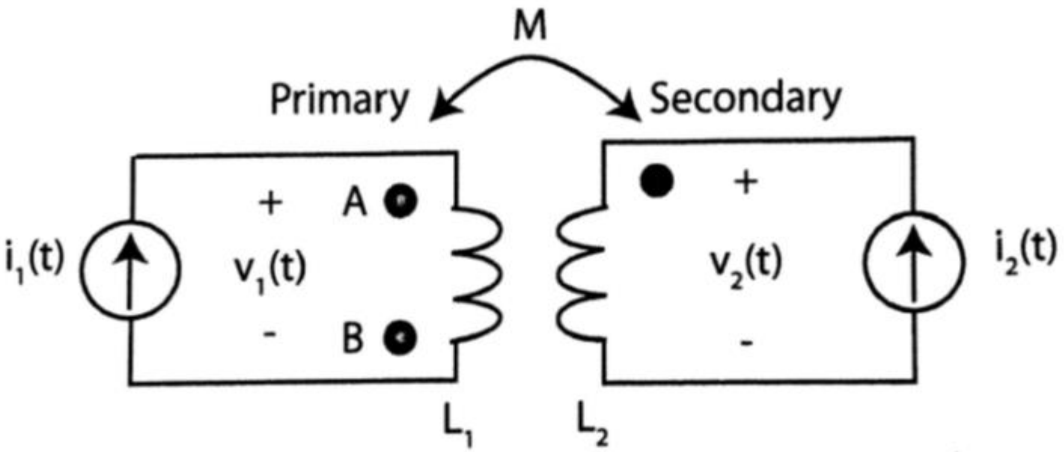
Mutual inductance equations in the Laplace domain

$$v_1(t) = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$V_1(s) = L_1 s I_1 \pm M s I_2\,(s)$$

$$v_2(t) = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$V_2(s) = \pm M s I_1\,(s) + L_2 s I_2(s)$$



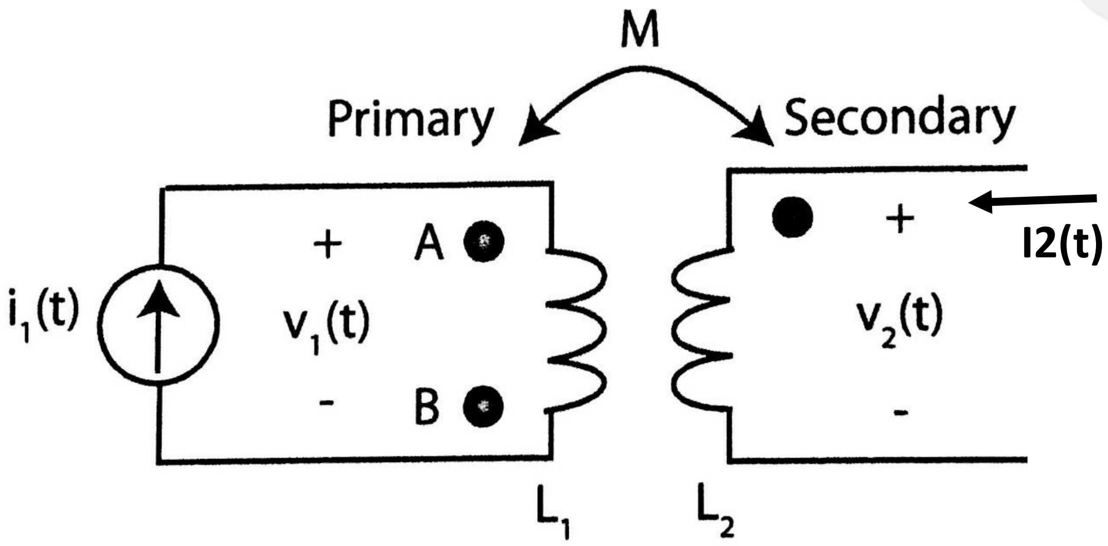
Example

In the shown figure, if i1(t)=2tu(t) A and M=0.005H. Determine the value of V2(t).

(The dot is at position A)

$$V_2(t) = M\frac{di_1(t)}{dt}$$

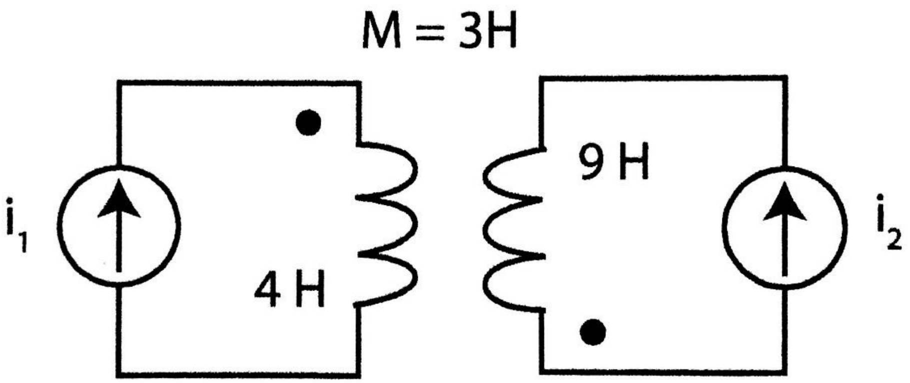
$$V_2(t) = 0.005 * 2u(t)$$
$$= 0.01u(t)$$



Example

For the shown figure, i1(T)=6A and i2(T)=2A . Find the coupling coefficient (k) and the energy stored by the inductors at t=T.

The coupling coefficient



$$k = \frac{M}{\sqrt{L_1 L_2}} = 0.5$$

The energy stored at t=T

$$W(T) = 0.5L_1 I_1^2 + 0.5L_2 I_2^2 \pm M I_1 I_2$$

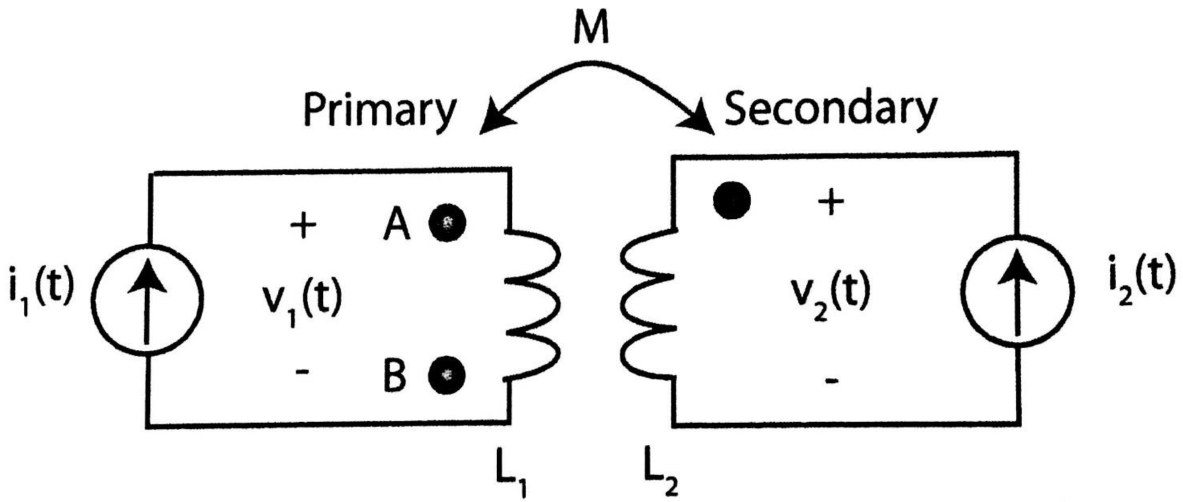
Mutual inductance equations in S-domain

$$v_1(t) = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$V_1(s) = L_1 s I_1 \pm M s I_2 (s)$$

$$v_2(t) = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$V_2(s) = \pm M s I_1 (s) + L_2 s I_2(s)$$



Example

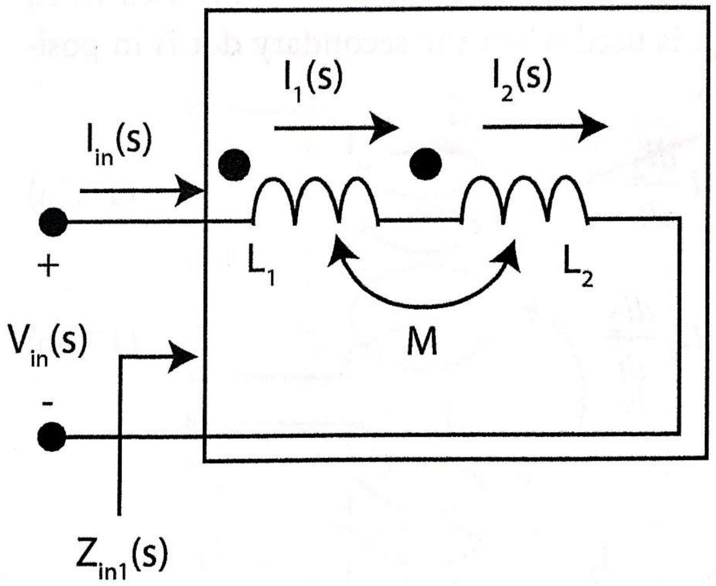
For the circuit shown, Find the input impedance Zin.

$$V_{in}(s) = V_{L1}(s) + V_{L2}(s)$$

$$= (L_1 s + Ms) I_{in}(s) + (Ms + L_2 s) I_{in}(s)$$

$$= (L_1 + L_2 + 2M) s I_{in}(s)$$

$$\boxed{Z_{in1}(s) = \frac{V_{in}(s)}{I_{in}(s)} = (L_1 + L_2 + 2M) s}$$



Example

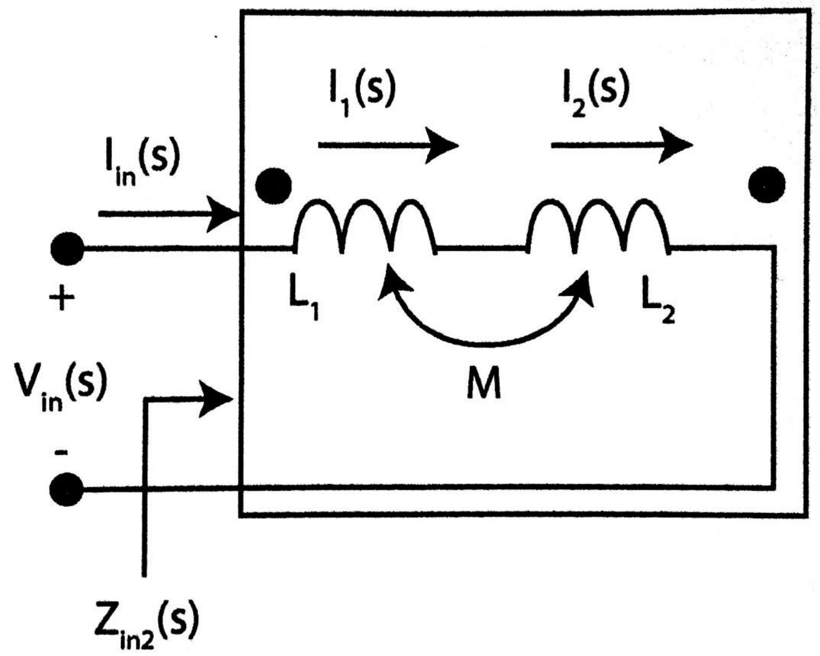
For the circuit shown, Find the input impedance Zin.

$$V_{in}(s) = V_{L1}(s) + V_{L2}(s)$$

$$= (L_1 s - Ms) I_{in}(s) + (-Ms + L_2 s) I_{in}(s)$$

$$= (L_1 + L_2 - 2M) s I_{in}(s)$$

$$Z_{in2}(s) = \frac{V_{in}(s)}{I_{in}(s)} = (L_1 + L_2 - 2M) s$$



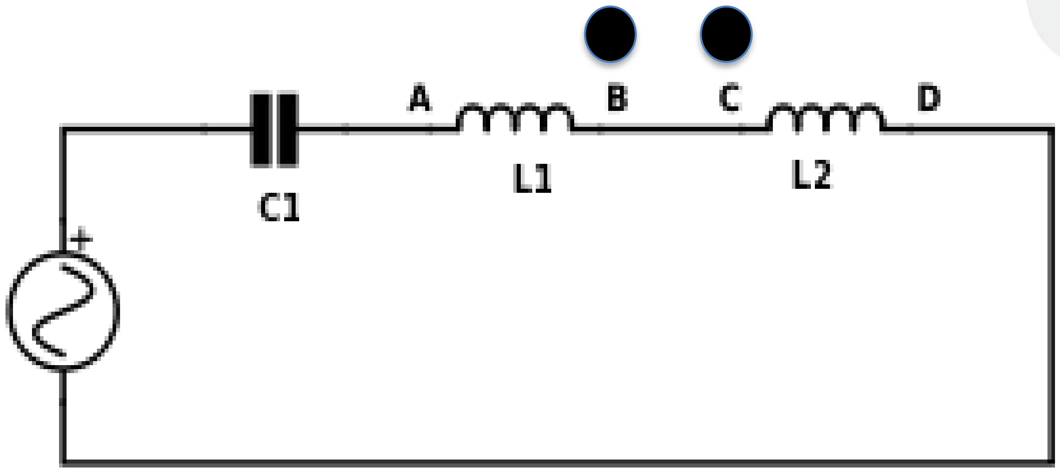
Example

For the circuit shown, Find the input impedance Zin(s). Consider the mutual inductance effect between L1 and L2 where the dots are at B and C.

$$-V + \left(L_1 sI \pm MsI + L_2 sI - MsI + \frac{1}{C_1 s} I \right)$$

$$V = \left(L_1 sI \pm MsI + L_2 sI - MsI + \frac{1}{C_1 s} I \right)$$

$$Z_{in}(s) = \left((L_1 + L_2 - 2M)s + \frac{1}{C_1 s} \right)$$

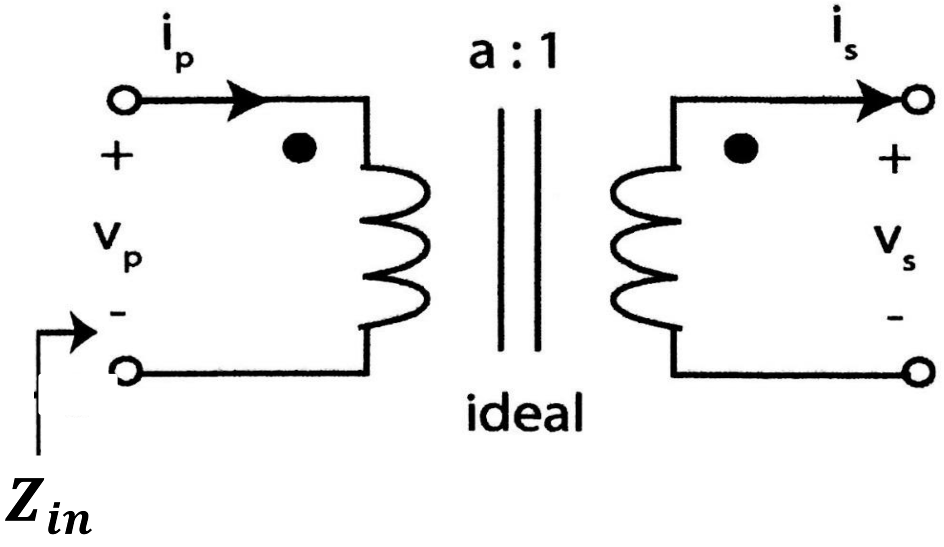


Ideal Transformers

$$\frac{v_p(t)}{v_s(t)} = a, \quad \frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

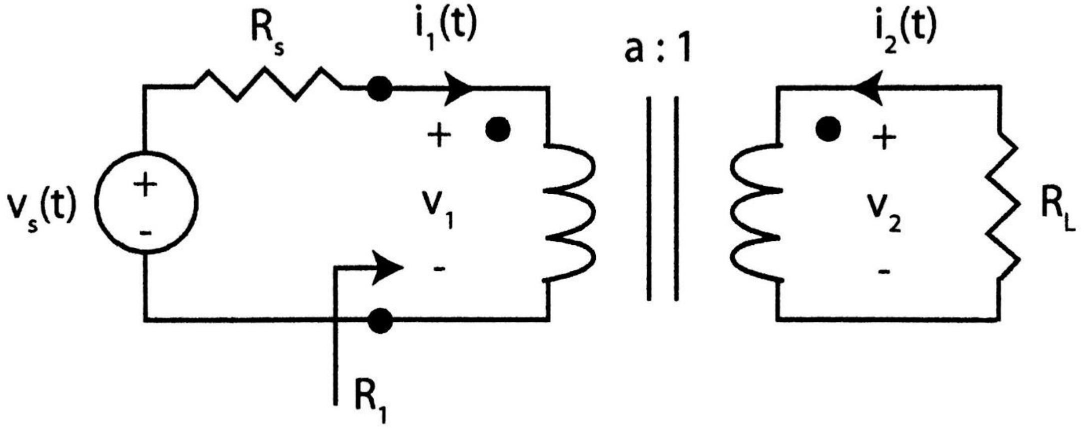
$$a = \frac{N1}{N2}$$

$$Z_{in} = \left(\frac{N1}{N2}\right)^2 * Z_L = a^2 * Z_L$$

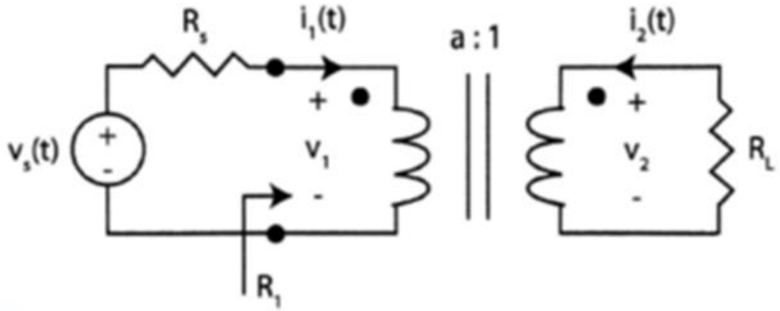


Example

For the circuit shown, Rs=10Ω, a=0.1, RL=1kΩ and Vs=20cos(1000t). Find R_1, V_1, V_2, I_1 and I_2.



$$Z_{in} = R_1 = a^2 * Z_L = 10\ \Omega$$



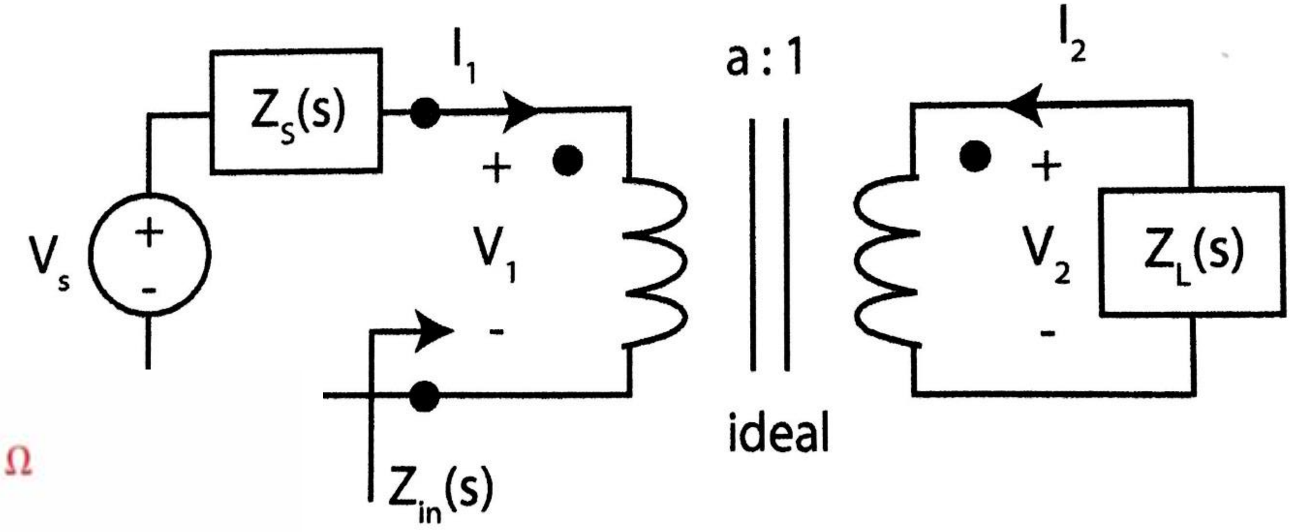
$$V_1(t) = V_s(t)\left(\frac{R_1}{R_s + R_1}\right) = 10\cos(1000t)\ V$$

$$\frac{V_1(t)}{V_2(t)} = a \qquad V_2(t) = \frac{V_1(t)}{a} = 100\cos(1000t)\ V$$

$$I_1(t) = \frac{V_1(t)}{R_1} = \cos(1000t)\ A \qquad I_2(t) = aI_1(t) = 0.1\cos(1000t)\ A$$

Example

For the circuit shown, if Zs=10Ω, Z_L=(1/S), Vs(t)=10u(t) and a=2. find I_1(s) and I_1(t).



$$Z_{in} = a^2 * Z_L = \left(\frac{4}{s}\right)\Omega$$

$$I_1(s) = \left(\frac{V_s(s)}{Z_s + Z_{in}}\right) = \frac{\left(\frac{10}{s}\right)}{10 + \frac{4}{s}} = \frac{10}{10s + 4} = \frac{1}{s + 0.4}V$$

$$I_1(t) = e^{-0.4t}u(t)A$$

Answer the following questions using ChatGPT

1. What is mutual inductance, and how does it relate to the inductance of individual coils?
2. Explain the significance of the dot convention in mutual inductance.
3. How does the position of dots affect the induced voltage polarity?
4. Describe the conditions under which mutual inductance can occur between two coils.
5. What happens to mutual inductance if the coils are moved farther apart?
6. How does the mutual inductance term affect the input impedance in the S-domain for a circuit?
7. Explain how the turns ratio *a* in a transformer affects the primary and secondary voltages and currents.

Answer the following questions using ChatGPT

1. Mutual Inductance is the phenomenon where a changing current in one coil induces a voltage in a nearby coil
2. This depends on the geometry, distance, and magnetic coupling of the coils.
3. The dot convention indicates the polarity of induced voltages in mutually coupled coils and determines
4. The coupling coefficient k, ranging from 0 to 1, represents the efficiency of magnetic coupling between two inductors
5. Coupled inductors store energy in their magnetic fields, and mutual inductance facilitates energy transfer between the coils
6. Transformers use mutual inductance to transfer power between circuits, with ideal transformers simplifying analysis through turns ratio �a and input-output impedance relationships.