Signals

Linear Circuit Analysis II
EECE 202

Announcement

1. Project Proposal will be released next week
2. Form project groups.

Recap

1. Syllabus introduction
2. Linear Circuit Analysis I revision of important concepts

New Material

1. Signals

Learning Outcomes

2. an ability to compute responses of linear circuits with and without initial conditions via one-sided Laplace transform techniques. SO [1]

Basic Signals

1- Unit step function.

2- Impulse function.

3- Ramp function.

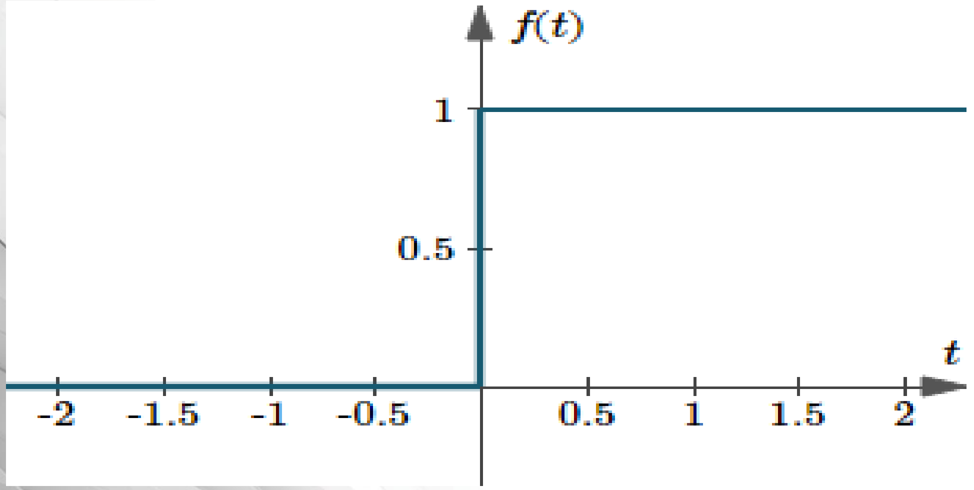
Unit Step Function

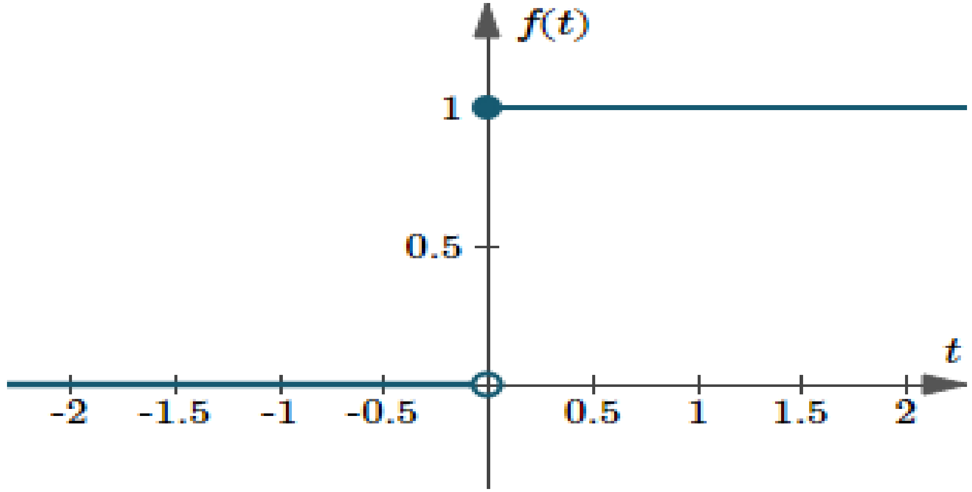
When a voltage is switched on or off in an electrical circuit at a specified value of time t.

The switching process can be described mathematically by the function called the Unit Step Function.

Unit step function: $u(t)=\begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

Unit step function: $u(t)=\begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

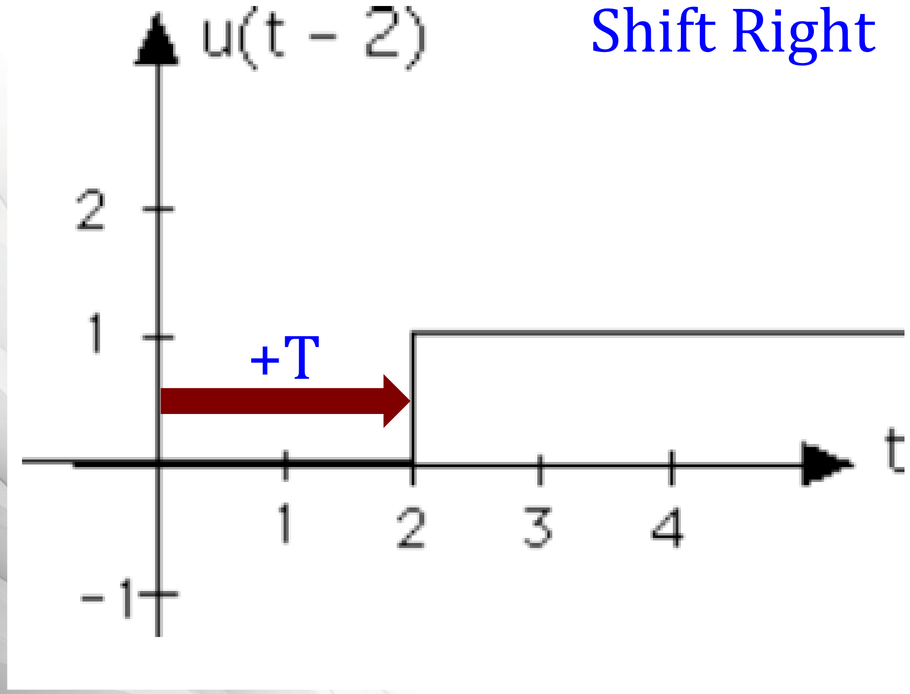


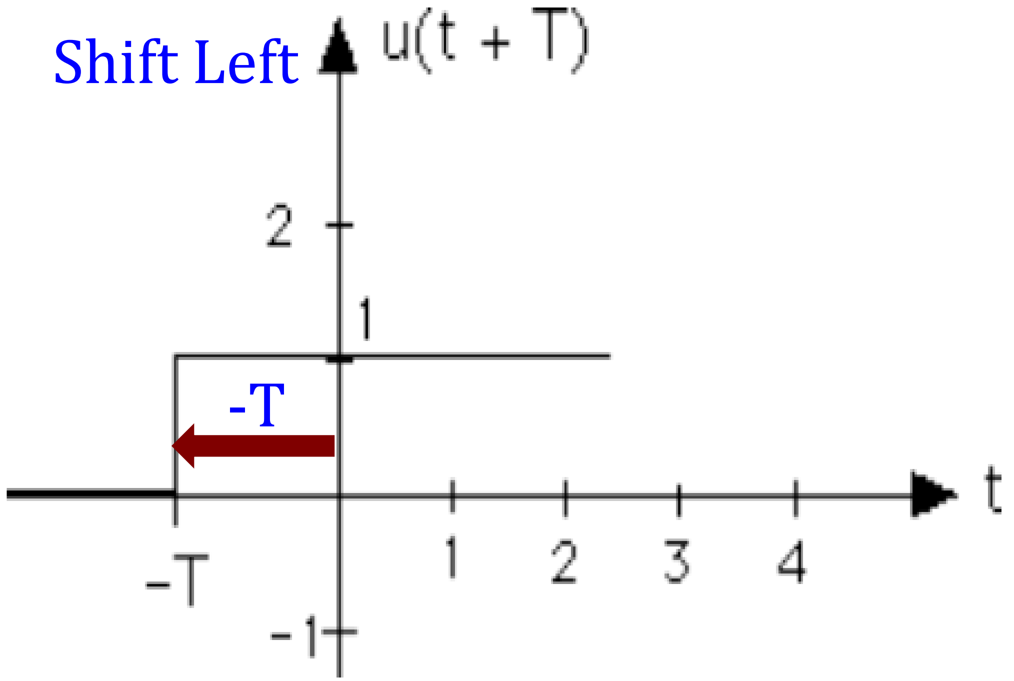


Shifted Step Function

$f(t)=u(t-T)$, time-delayed unit step signal

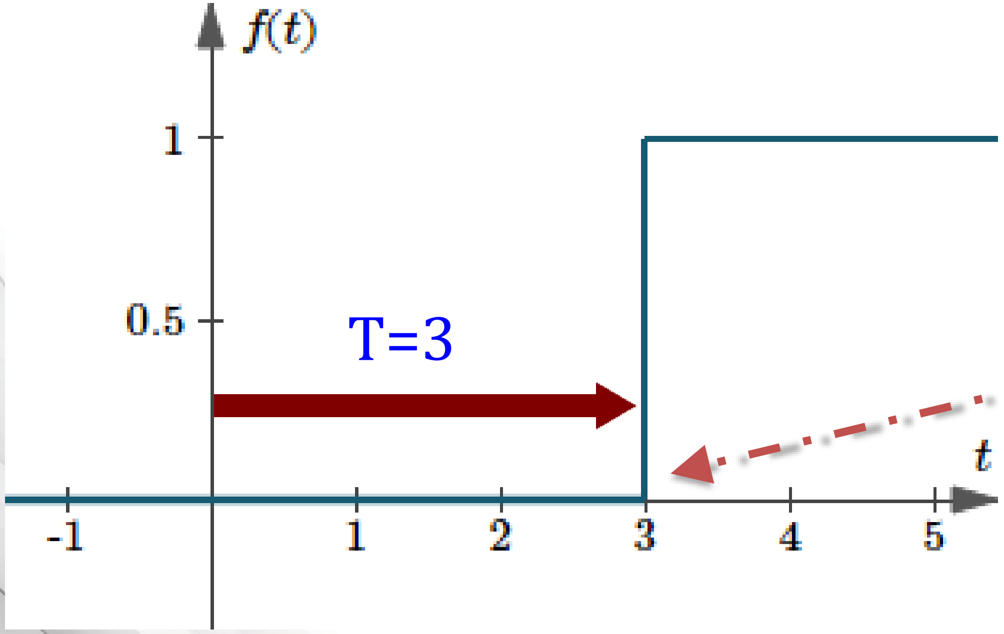
Often represent voltages that turn on after a prescribed time period *T*.





Example

Obtain the suitable function for the following graph



Shifted step:

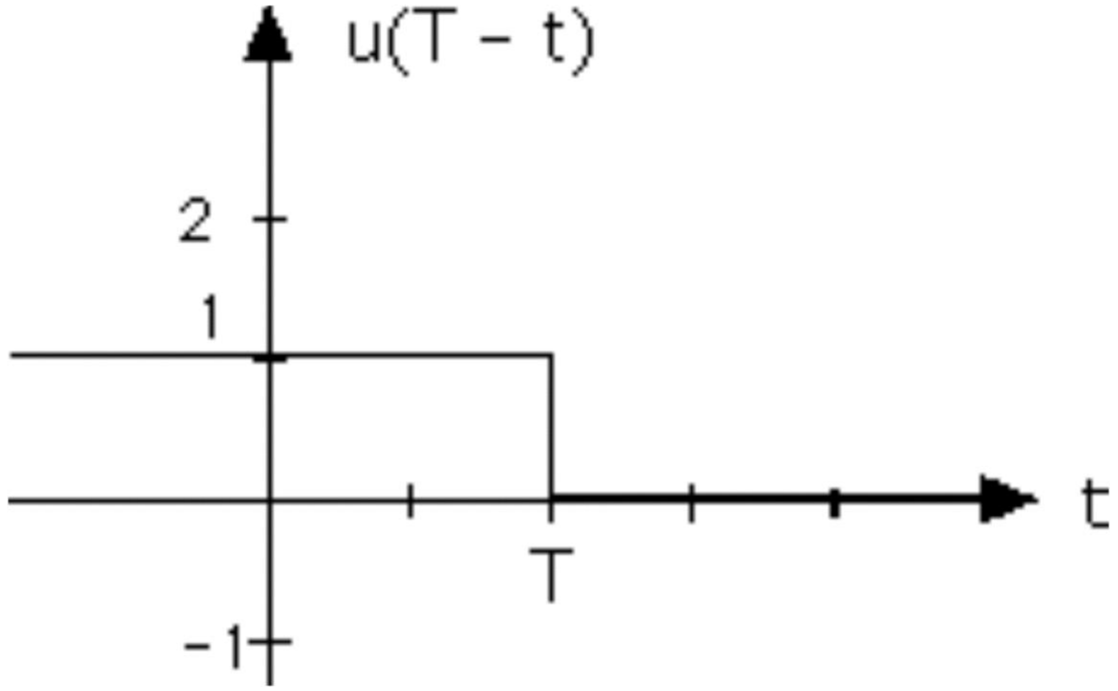
$f(t)=u(t-T)$

$$\boxed{f(t) = u(t-3)}$$

Flipped Step Function

In flipped step function, the step takes on the value of unity for time $t \leq T$.

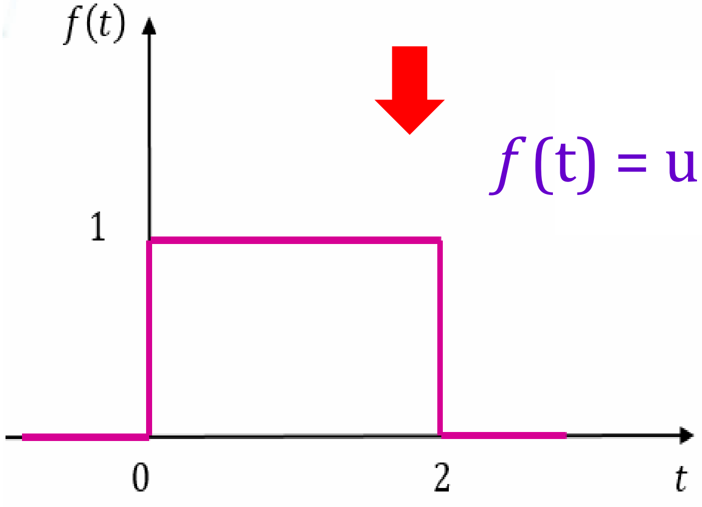
$$f(t)=u(T-t)$$



Exercise (p. 549)

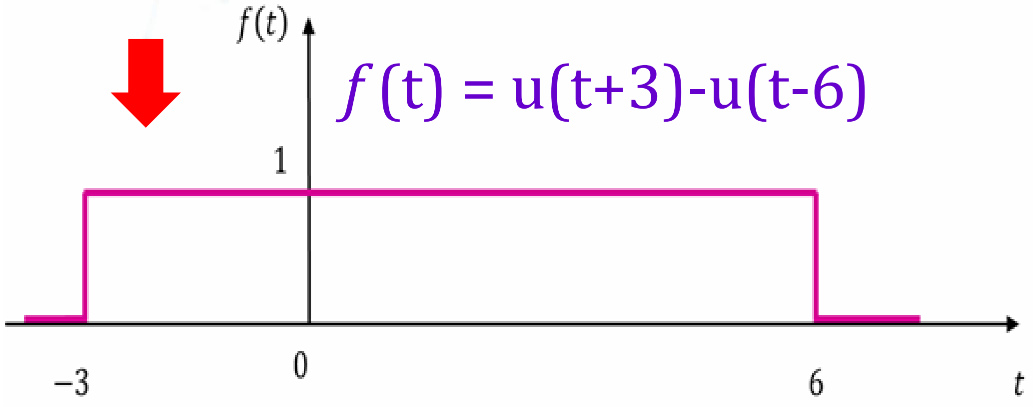
Represent each of the following functions as sums of step functions:

$$(i)\ f(t) = \begin{cases} 1 & 0 \le t \le 2 \\ 0 & otherwise \end{cases}$$



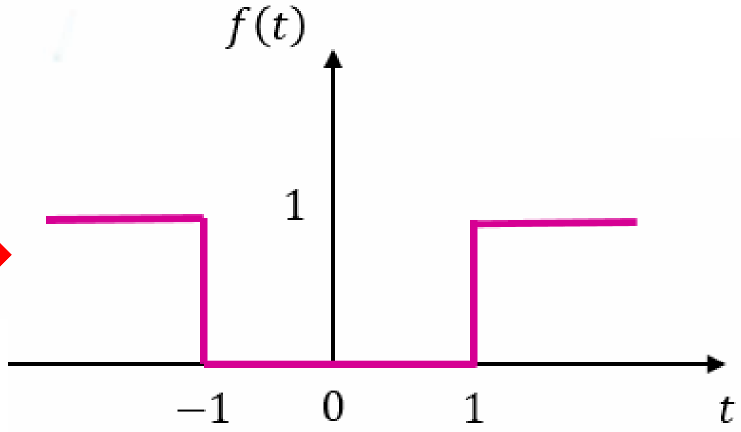
f(t) = u(t)-u(t-2)

$$(ii)\ f(t) = \begin{cases} 1 & -3 \le t \le 6 \\ 0 & otherwise \end{cases}$$



f(t) = u(t+3)-u(t-6)

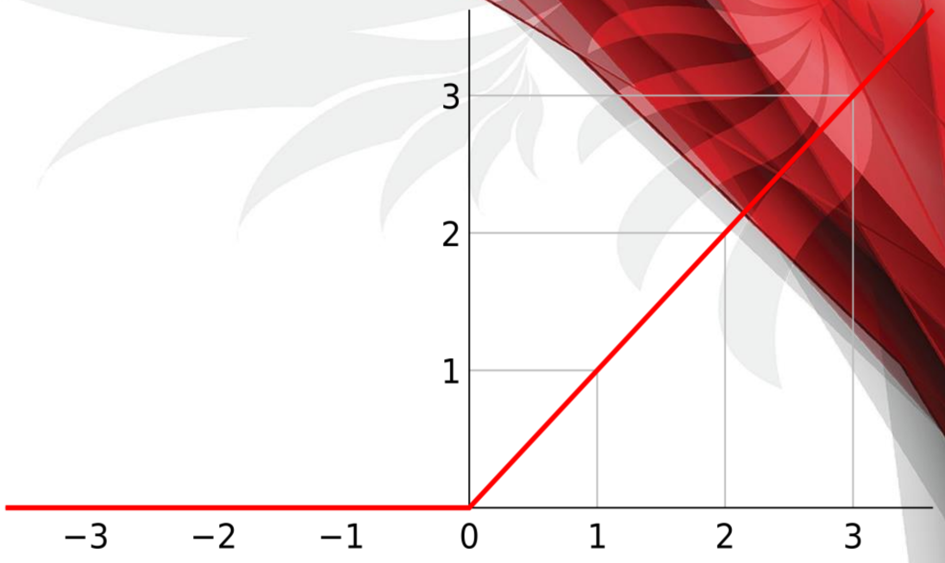
$$(iii)\ f(t) = \begin{cases} 1 & t \le -1 \\ 0 & -1 < t < 1 \\ 1 & t \ge 1 \end{cases}$$



f(t) = u(-1-t)+u(t-1)

Ramp Function

A Ramp function models signals having a constant rate of increase



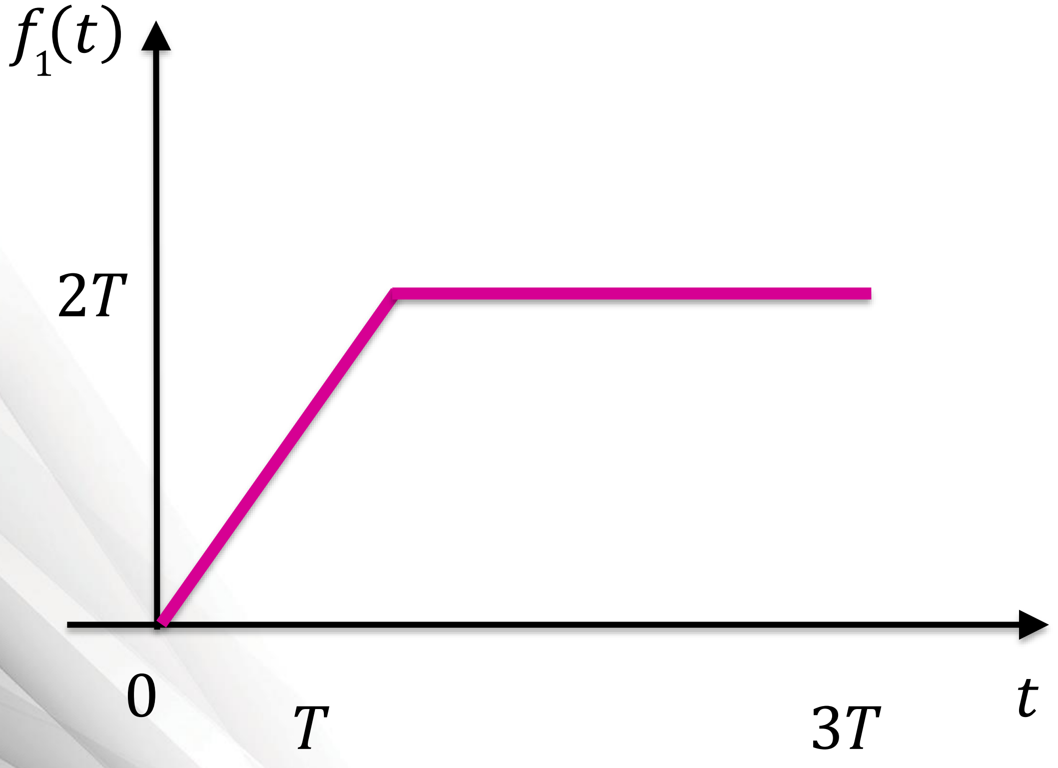
A Ramp function is the integral of the unit step function

$$r(t) = tu(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

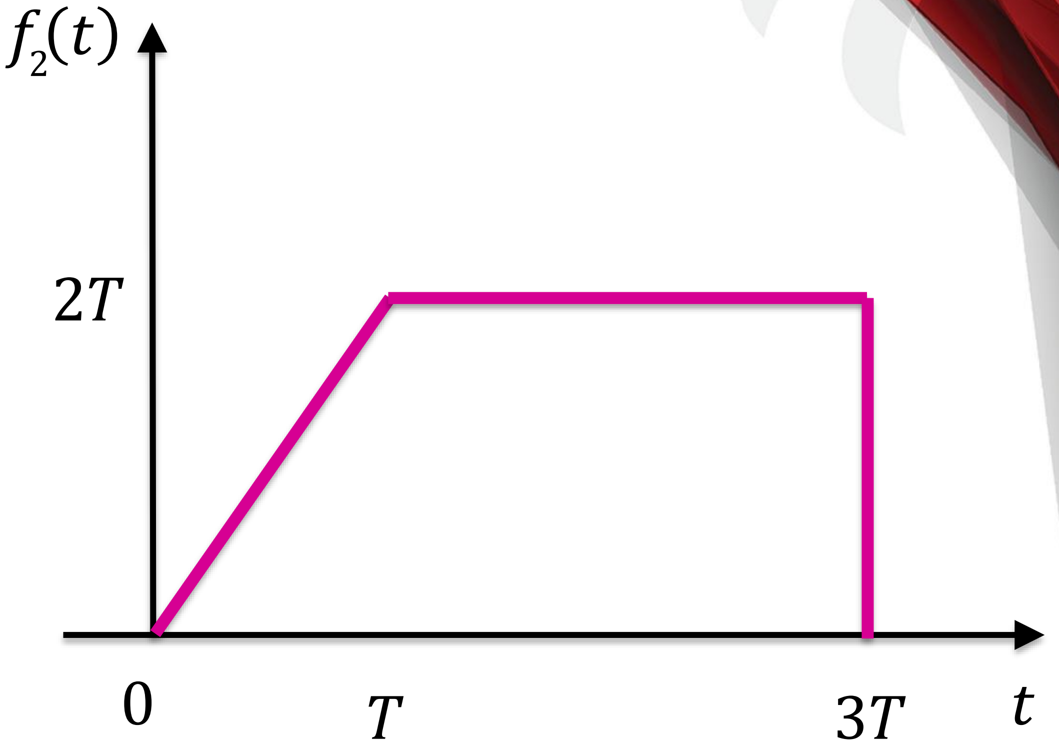
$\boldsymbol{\tau}$ *is a dummy variable of integration*

Example 12.2 (p. 550)

Express the following figures in terms of steps and ramps



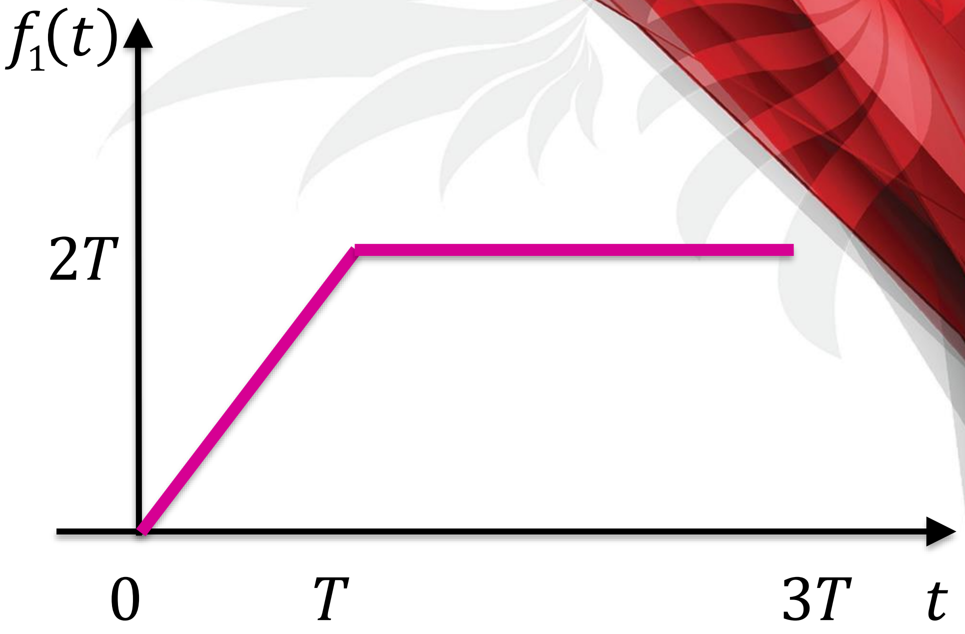
(a)

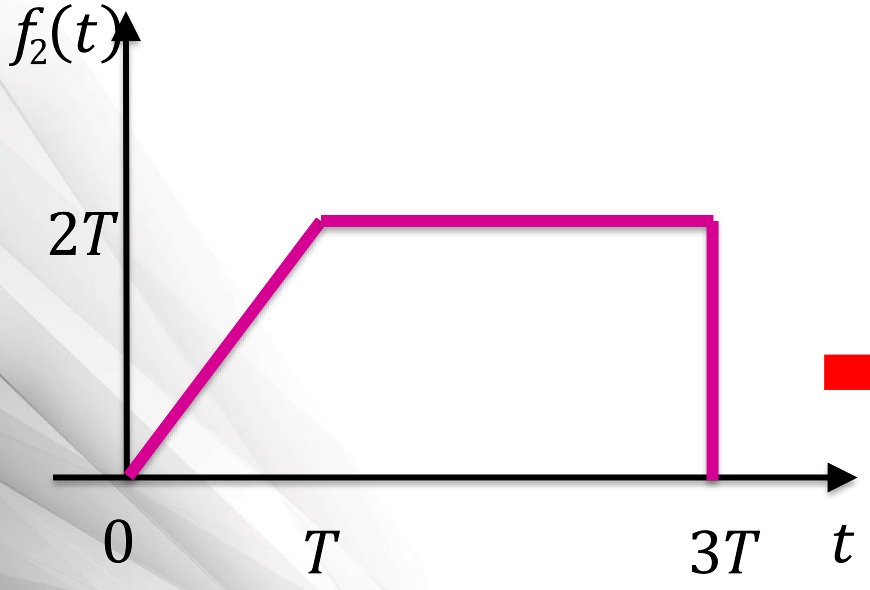


(b)

Example 12.2 (p. 550) cont.

$$f_1(t) = \frac{2T}{T}r(t) - \frac{2T}{T}r(t-T)$$



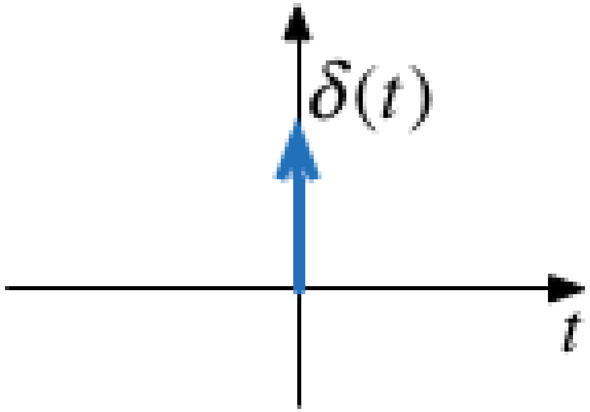


$$\boxed{f_2(t) = \frac{2T}{T}r(t) - \frac{2T}{T}r(t-T) - 2Tu(t-3T)}$$

Impulse Function

The Impulse function is also named **Delta Function (δ(t))**

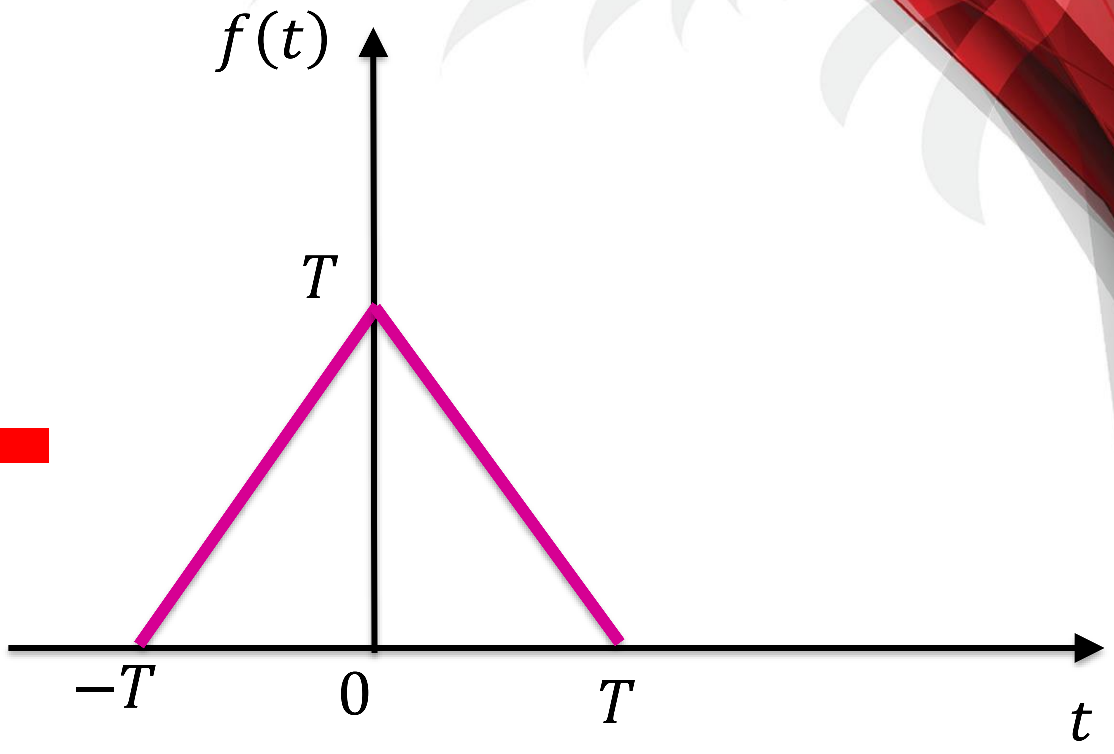
$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$



The delta function has infinite height, zero width, and a well defined area of 1 unit

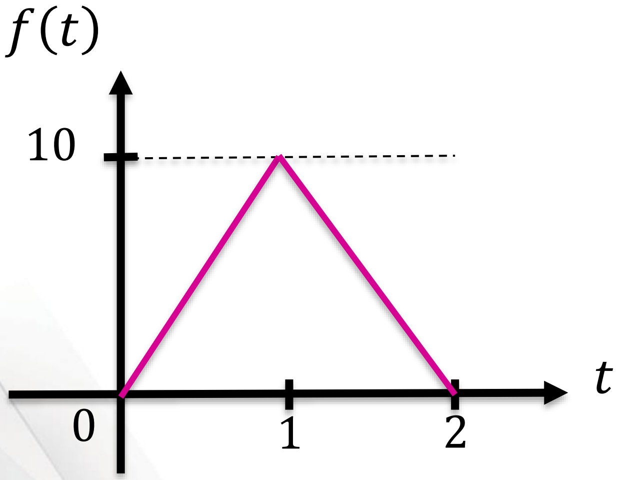
Example 12.3 (p. 551)

$f(t) = r(t+T) - 2r(t) + r(t-T)$



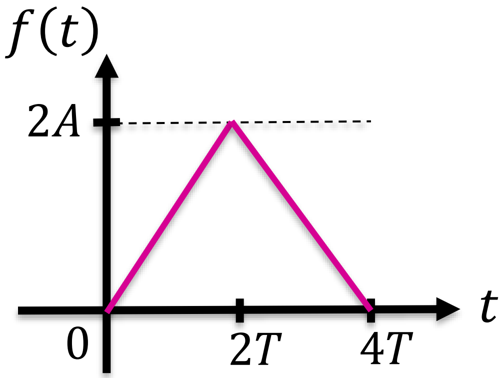
Exercise 8 (p. 593 (a))

Represent *f* (*t*) as a sum as steps, ramps, shifts of basic signals



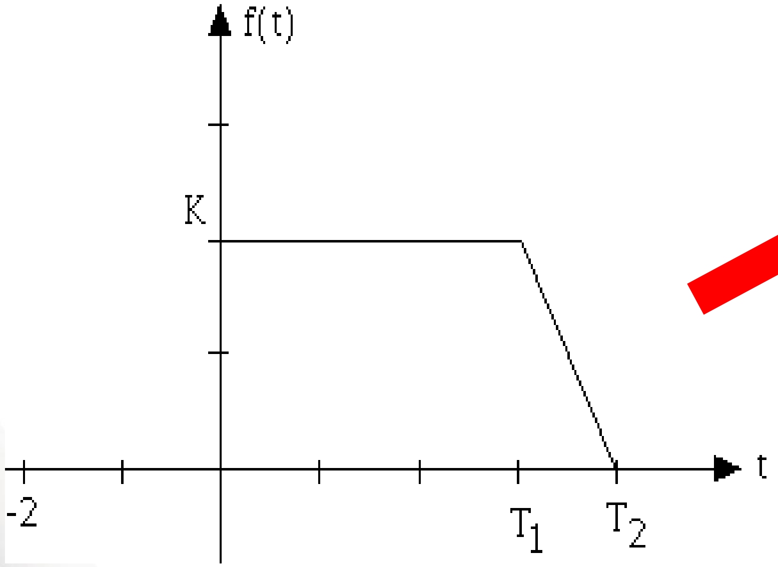
$$f(t) = 10r(t) - 20r(t-1) + 10r(t-2)$$

$$f(t) = \frac{2A}{2T} r(t) - 2\frac{2A}{2T} r(t-2T) + \frac{2A}{2T} r(t-4T)$$

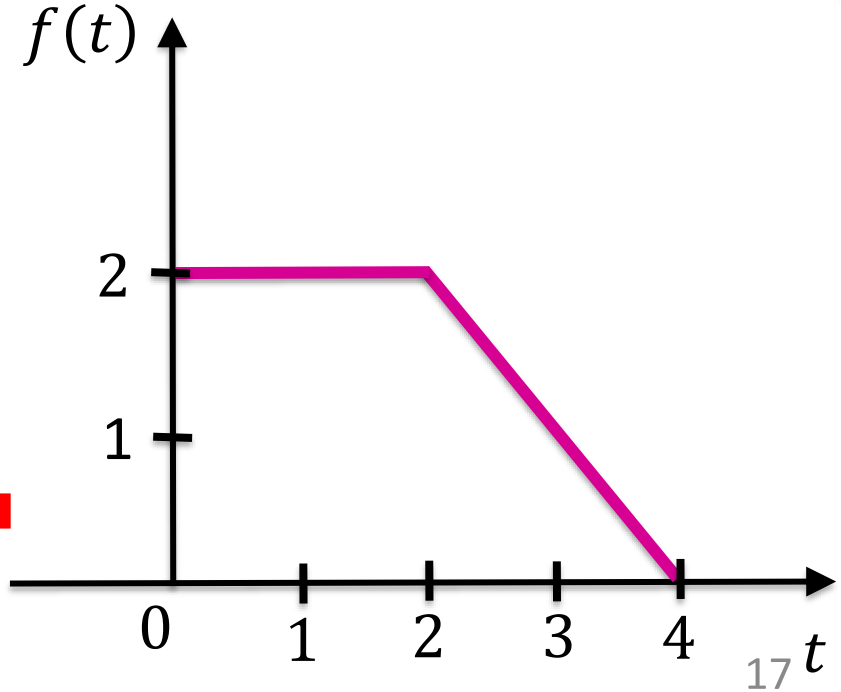


Exercise 10 (p. 594 (b))

Represent $f(t)$ as a sum as steps, ramps, shifts of basic signals



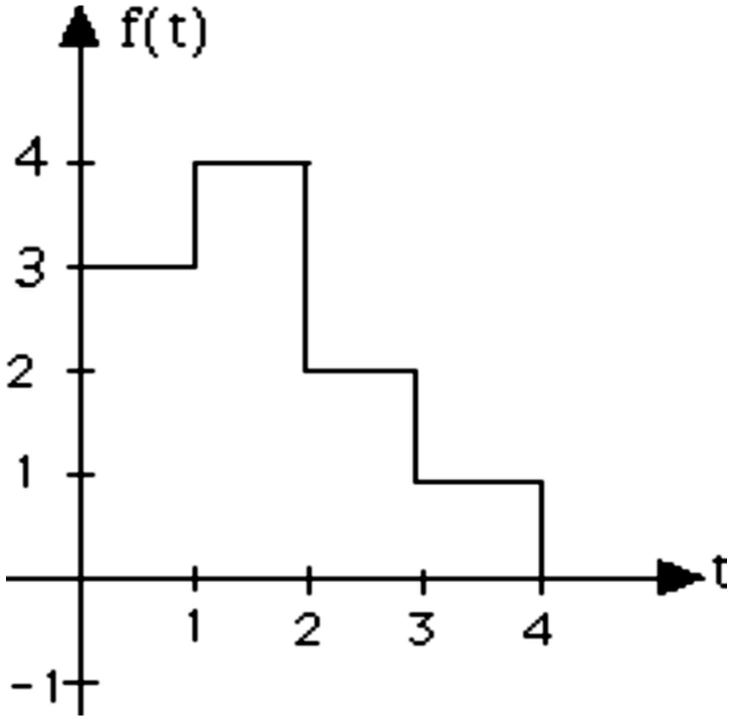
$$f(t) = Ku(t) - \frac{K}{T2-T1} r(t-T_1) + \frac{K}{T2-T1} r(t-T_2)$$



$$f(t) = 2u(t) - \mathbf{1}\, r(t-T_1) + \mathbf{1} r(t-T_2)$$

Additional example 1

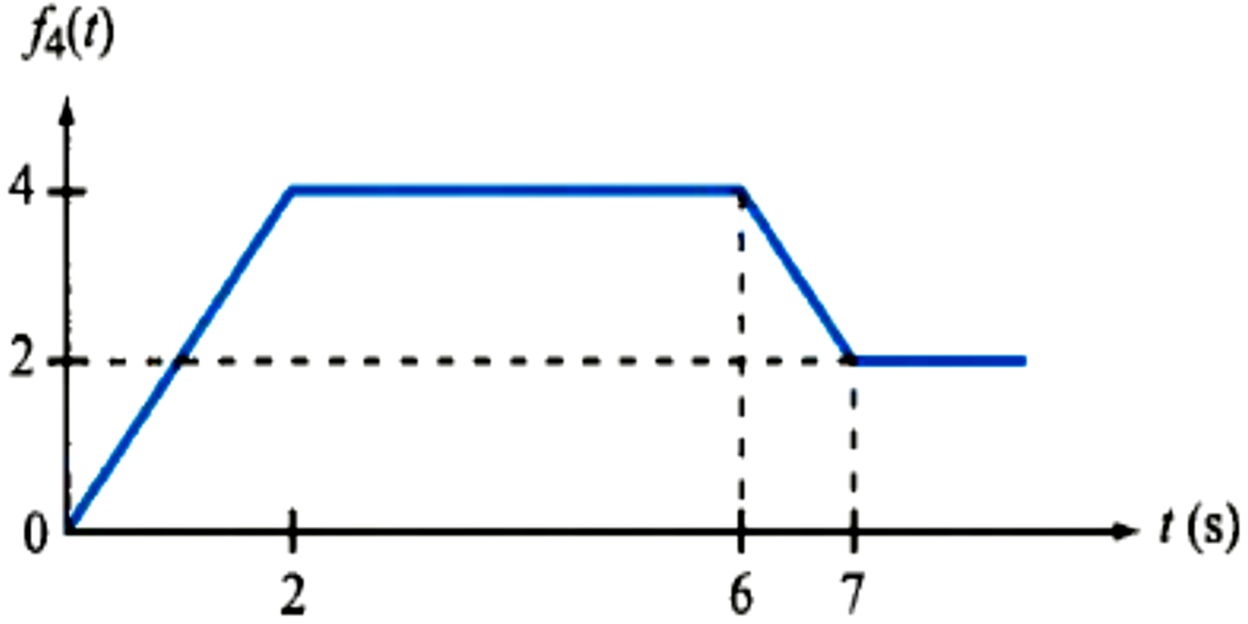
Represent $f(t)$ as a sum as steps, ramps, shifts of basic signals



$$\mathbf{f(t)} = \mathbf{3}u(t) + u(t-1) - \mathbf{2}u(t-2) - u(t-3) - u(t-4)$$

Example 2

Represent $f_4\ (t\ )$ as a sum as steps, ramps, shifts of basic signals



$$f_4(t) = 2t\,u(t) - 2(t-2)\,u(t-2) - 2(t-6)\,u(t-6) + 2(t-7)u(t-7)$$

Practice

Use ChatGPT to answer following:

1. Explain the Unit Step Function and its significance in circuit analysis.

2. What is a flipped step function, and how is it different from the standard unit step function?

3. Define the ramp function and explain its relationship with the unit step function.

4. What is the impulse function, and how does it differ from the unit step function?

Summary

- **Unit Step Function**
  - Describes voltage switching on/off in a circuit
  - Example: $f(t)=u(t-T)$
- **Ramp Function**
  - Models signals with constant increase rate
  - Integral of the Unit Step Function
  - Impulse (Delta) Function
  - Represents an instantaneous change in signal with zero width and infinite height
- **Shifted and Flipped Step Functions**
  - Shifted Step: $f(t)=u(t-T)$
  - Flipped Step: $f(t)=u(T-t)$
- **Signal Composition**
  - Representing complex functions using sums of steps, ramps, and shifts
  - Example: $f(t)=r(t) - 2r(t-2T) + r(t-4T)f(t)=r(t)-2r(t-2T)+r(t-4T)$

Suggested Additional Problems for Ch. 12:

Exercise 8 (p. 593 (b)), 10 (p. 594 (a)), 7