# Sinusoidal Steady State Analysis

## Linear Circuit Analysis II
## EECE 202

# Announcement

1. GCA2 – Group Week 11 Lecture 2
2. PD2 (Technical Report) due Thursday Week 12

# Recap

1. Switching
2. Series combination
3. Parallel combination

# New Material

1. Sinusoidal steady state (SSS) sources
2. Laplace Transform approach to SSS
3. Laplace Transform approach to SSS

# Sinusoidal Steady-State Sources

- Sources in which the value of the voltage and/or current varies with time

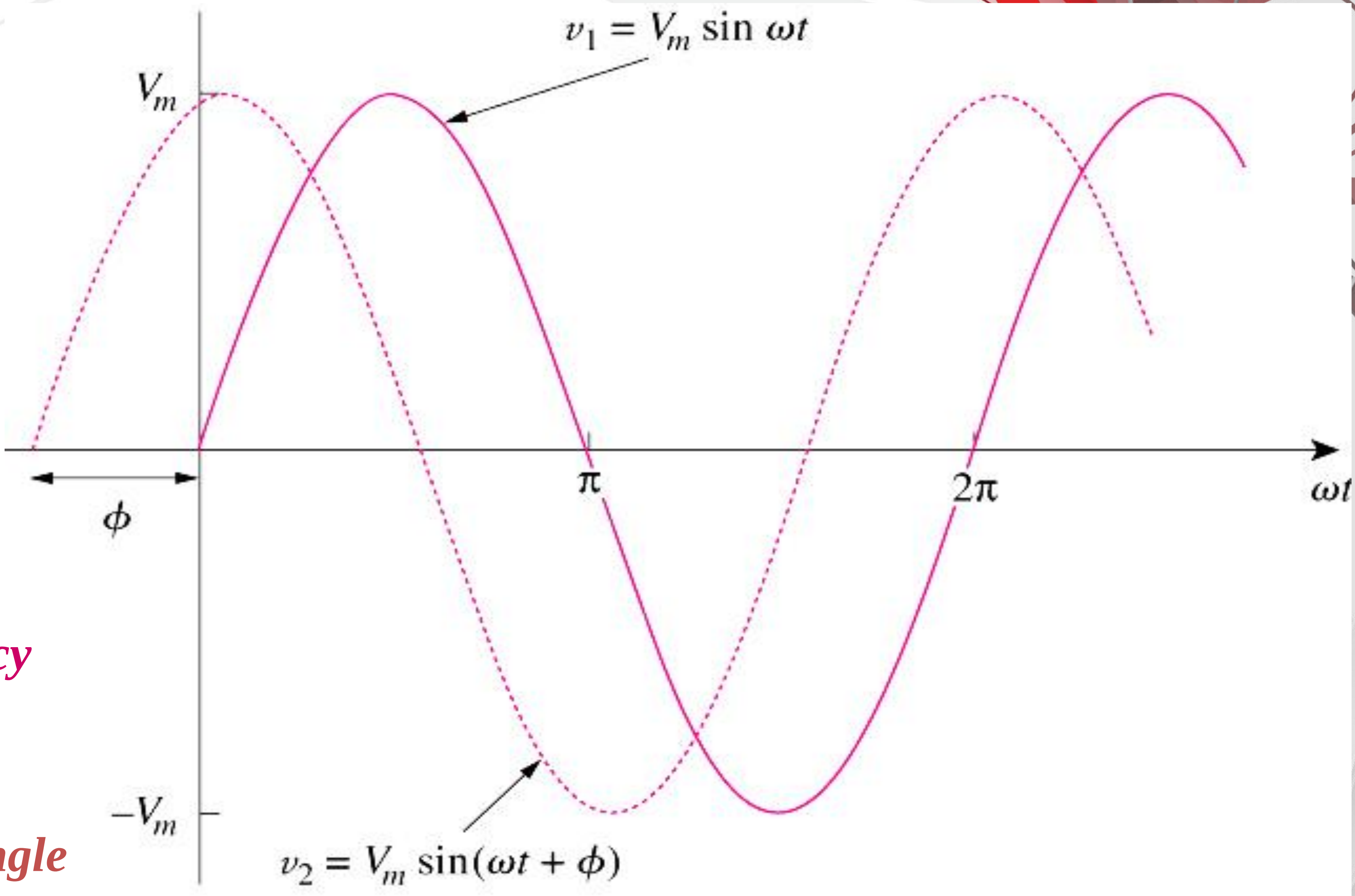
## Why Sinusoidal Sources???

- Generation, Transmission, Distribution and Consumption occur under sinusoidal Steady-state behavior

- Understanding the behavior of Sinusoidal circuits makes it easy to predict The behavior of circuits with non-sinusoidal ones

- Steady state sinusoidal behavior often simplifies the design of electrical systems

# Sinusoidal waveform




*Maximum Value*

*Time Period*

*Source Frequency*

*Angular Frequency*

*Phase Angle*

# How to express a sinusoidal varying function ?

$V = V_m \cos(wt + \varphi)$

**Cosine Function**

$V = V_m \sin(wt + \varphi)$

**Sine Function**

$\sin(wt + \theta) = \cos(wt + \theta - 90^0)$

$V = V_m \cos(wt + \varphi)$

**Maximum Value**

**Angular frequency**

$W = 2\pi F$ (rad/sec)

**Phase angle**

**Source Frequency: F (Hz)**

**Time Period: T (sec)**

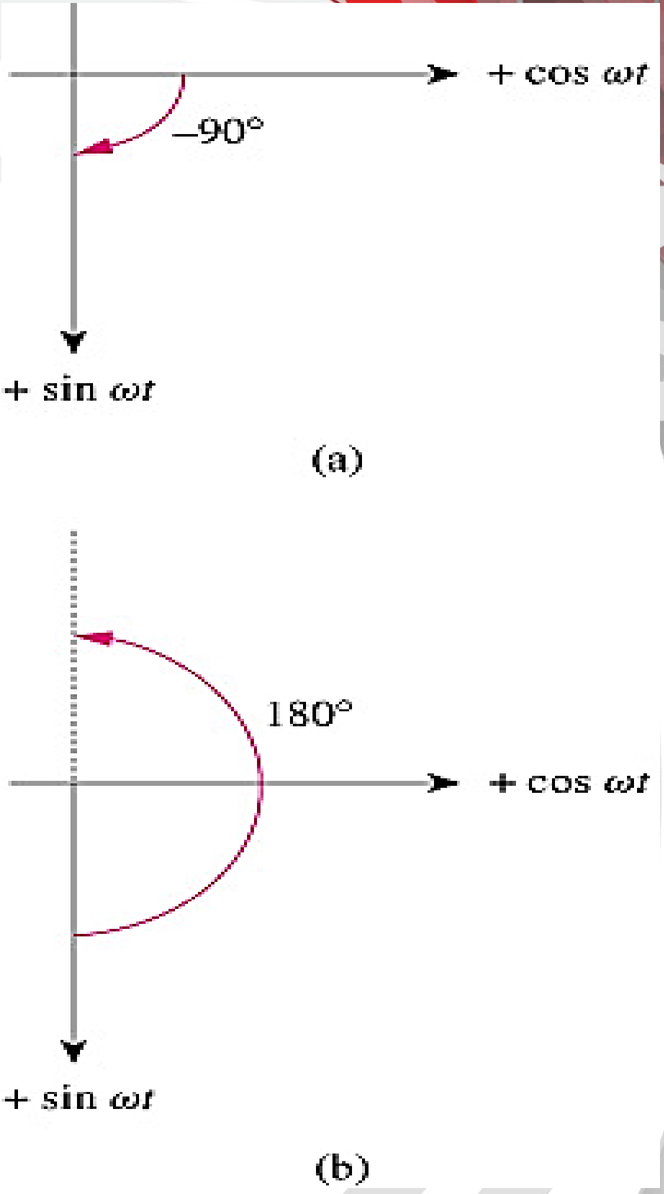
**T= 1/F**

# Sine – Cosine Conversion

$\text{Sin}(wt+\theta) = \text{Cos}(wt+\theta-90^{0})$

$\text{Sin}(wt \pm 180^{0}) = -\text{Sin}\, wt$

$\text{Cos}(wt \pm 180^{0}) = -\text{Cos}\, wt$



# Example-1

**A Sinusoidal voltage is given by the expression $V = 300 \cos(120\pi t + 30^0)$**

What is the period of the voltage in milliseconds?

What is the frequency in Hertz?

What is the magnitude of V at t =2.778 ms?

As $w = 2\pi F = 120\pi = 2\pi/T$, then $T = 1/60 = 16.667$ ms

As $F = 1/T$, then $F = 1/16.667 = 60$ Hz

As $w = 2\pi/T$, then $W = 2\times3.14/16.667$ rad/sec, then at $t = 2.778$ ms

$wt = (2\times3.14/16.667)\times2.778 = 1.047$ rad; or $wt = 1.047\times180^0/3.14 = 60^0$ degrees

Then $V = 300 \cos(60+30) = 300 \cos 90^0 =$ Zero V

## Example-2

**A Sinusoidal current has a maximum amplitude of 20 A, the current passes through one complete cycle in 1 ms, the magnitude of the current at zero time is 10A.**

- What is the frequency of current in Hz?
- What is the value of angular frequency?
- Write an expression for i(t) using the cosine function, express φin degrees

- $F=1/T=1/1\times10^{-3}=1000$ Hz
- $w=2\pi F=2000\,\pi$ rad/sec
- $i(t)=I_m \cos(wt+\varphi)=20\cos(2000\pi t+\varphi)$, but as at t= 0, I=10A, then

$10=20\cos(0+\varphi)$……. $\varphi=\cos^{-1}0.5=60^0$, then $i(t)=20\cos(2000\,\pi t+60^0)$

# Phasor domain representation

So: $V(t) = V_m \cos(wt+\varphi)$ ⟺ $V = V_m \angle \varphi$

| *Time domain* | *Phasor domain* |
|---|---|
| $V_m \cos(wt+\varphi)$ | $V = V_m \angle \varphi$ |
| $V_m \sin(wt+\varphi)$ | $V = V_m \angle \varphi - 90^0$ |
| $I_m \cos(wt+\varphi)$ | $I = I_m \angle \varphi$ |
| $I_m \sin(wt+\varphi)$ | $I = I_m \angle \varphi - 90^0$ |

# Laplace Transform approach to SSS

$K\cos(\omega t+\theta)$

Stable Circuit with TF H(s)
s= jω

$M(\omega)\cos(\omega t+\phi(\omega))$

$M(\omega) = K\,|H(j\omega)|$
$\phi(\omega) = \angle H(j\omega) + \theta$

***You just need to replace a stable H(s) by H(jw) to compute the SSS.***

# Example -3

Suppose a second order linear circuit having the transfer function:

$$\boldsymbol{H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{s^2 - 0.5s + 5}{s^2 + 0.5s + 5.7321}}$$

The transfer function is driven by a sinusoidal input $\boldsymbol{V_{in}(t) = \sqrt{2}\,Cos(2t + 45^o)}$ . Find the steady-state response.

$$\boldsymbol{H(j2) = \frac{(j2)^2 - 0.5(j2) + 5}{(j2)^2 + 0.5(j2) + 5.7321} = \frac{\sqrt{8}}{4} \angle -75^{\circ}}$$

$$\boldsymbol{Ans. \quad V_{ss}(t) = Cos(2t - 30^o)}$$

# Example -4

Suppose a second order linear circuit having the transfer function:

$$\boldsymbol{H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{2s+6}{s(s^2+4s+16)}}$$

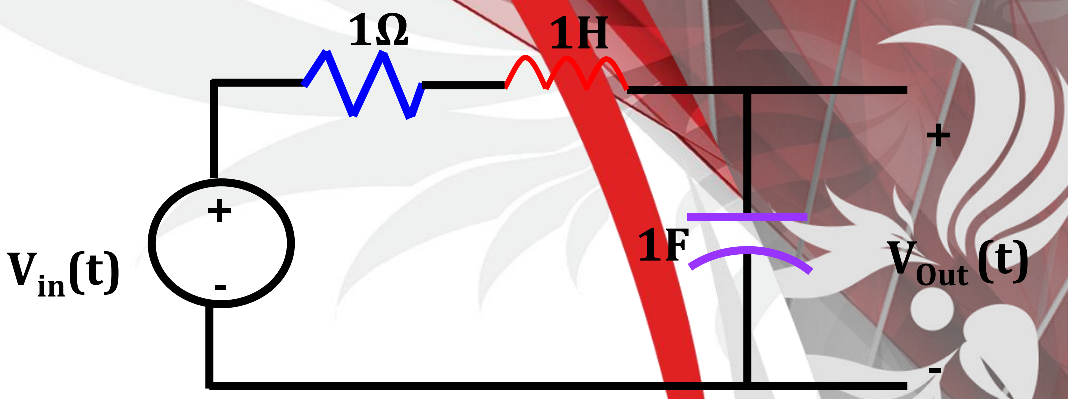
The transfer function is driven by a sinusoidal input $\boldsymbol{V_{in}(t) = 2\ Cos(2t + 45^o)}$ . Find the steady-state response.

$$\boldsymbol{H(j2) = \frac{2(j2)+6}{s((j2)^2+4(j2)+16)} = \quad 0.25\angle -90^\circ}$$

$$\boldsymbol{Ans. \quad V_{ss}(t) = 0.5\ Cos(2t - 45^o)}$$

# Example -5

Find $V_{out,ss}$ (t) for the circuit below when $v_{in}(t) = K\cos(\omega t)$ V, K > 0. All passive RLC circuits are stable since there are no active/dependent sources.




***Step 1. Find H(s)***

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{s}}{1+s+\frac{1}{s}} = \frac{1}{s^2+s+1}$$

***Step 2. Find $V_{out,ss}(s)$***

$$\boldsymbol{V_{out,ss}(jw) = H(jw) * V_{in}(jw)}$$

$$\boldsymbol{V_{out,ss}(jw) = \frac{1}{(jw)^2 + (jw) + 1} * K\angle 0^o}$$

$$\boldsymbol{V_{out,ss}(jw)}=\left(\frac{K}{1-w^2+jw}\right) = M(\omega)\angle\varphi(\omega)$$

$$M(\omega)=\frac{K}{\sqrt{\left(1-\omega^2\right)^2+\omega^2}}$$

$$\varphi(\omega)=-\tan^{-1}\left(\frac{\omega}{1-\omega^2}\right)$$

$$\boxed{\boldsymbol{V_{out,ss}(t) = M(w)cos(wt + \omega(w))}}$$

# Answer the following questions using ChatGPT

1. What are sinusoidal sources, and why are they commonly used in electrical systems?
2. How does the time period (T) relate to the frequency (f) of a sinusoidal waveform?
3. Why is understanding sinusoidal steady-state behavior important for analyzing circuits with non-sinusoidal inputs?
4. Explain the relationship between sine and cosine functions in terms of phase shifts.
5. Why is phasor representation particularly useful in AC circuit analysis?
6. How does the Laplace transform facilitate sinusoidal steady-state (SSS) analysis?

# Summary

- Voltage or current varies with time following a sinusoidal function.
- Sinusoidal Waveform Parameters (amplitude, frequency, angular frequency, phase)
- Phasors represent sinusoidal signals as complex numbers for simplified circuit analysis.
- Replacing stable H(s) with H(jω) for steady-state analysis, simplifies computation of outputs for sinusoidal inputs
- Facilitates understanding of resonance and impedance in RLC circuits.

# Suggested examples

- Page 704, example 14.8
- Page 749, example 31
- Page 750, example 32
- Page 751, examples 36 and 37