Poles, Zeroes, S-Plane Plot, and Stability.

Linear Circuit Analysis II
EECE 202

American University Of The Middle East

Announcement

1. PD 1 Voice Over PPT due in Week 10
2. GCA2 – Group Week 11 Lecture 2

Recap

1. Active vs passive filters
2. Types of active filters

- Butterworth filter

New Material

1. Transfer function revision
2. Stability in s-plane

- Effect of pole location

Revision (Transfer functions)

$$\boldsymbol{H(s) = \frac{Output(s)}{Input(s)}}$$

$$\boxed{\boldsymbol{Output(s) = H(s) \times Input(s)}}$$

Impulse response

$$Input(s) = 1, \qquad Output(s) = H(s)$$

Unit step response or the step response

$$Input(s) = \frac{1}{s}, \qquad Output(s) = H(s) \times \frac{1}{s}$$

Transfer functions

As defined, the transfer function is a rational function in the complex variable $s = \sigma + j\omega$, that is

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0} \qquad \text{....(1)}$$

It is often convenient to factor the polynomials in the numerator and denominator, and to write the transfer function in terms of those factors:

$$H(s) = \frac{n(s)}{d(s)} = K \frac{(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)} \qquad \text{....(2)}$$

Transfer functions cont.

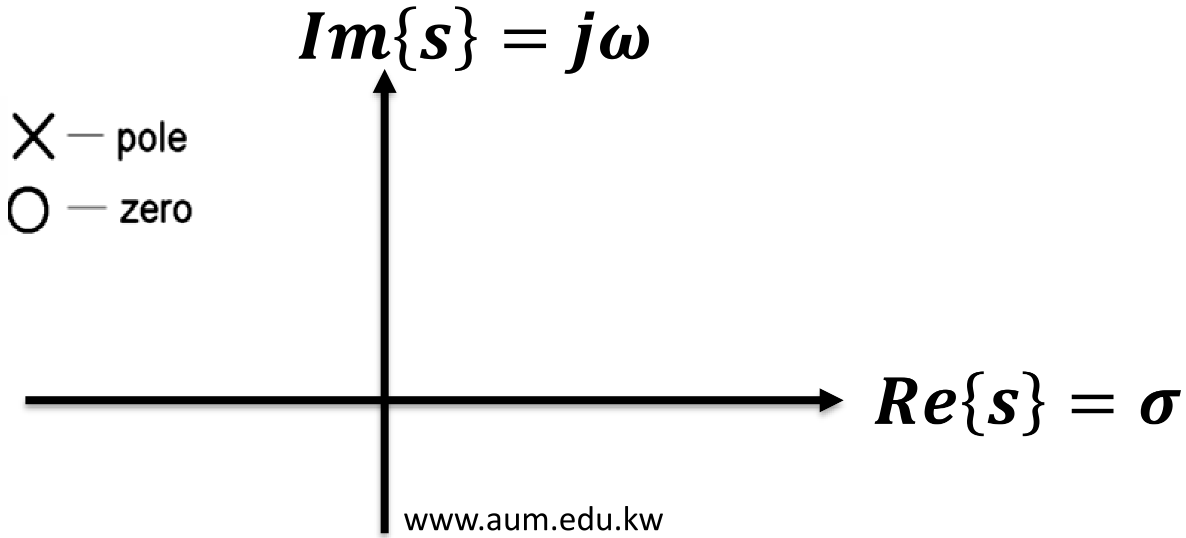
$$H(s) = \frac{n(s)}{d(s)} = K\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

Zeros

Poles

- $z_1, z_2, \cdots z_m$ are zeros of *H(s)*
- $p_1, p_2, \cdots p_n$ are poles of *H(s)*
- *K* is the gain

s-plane.



Transfer functions cont.

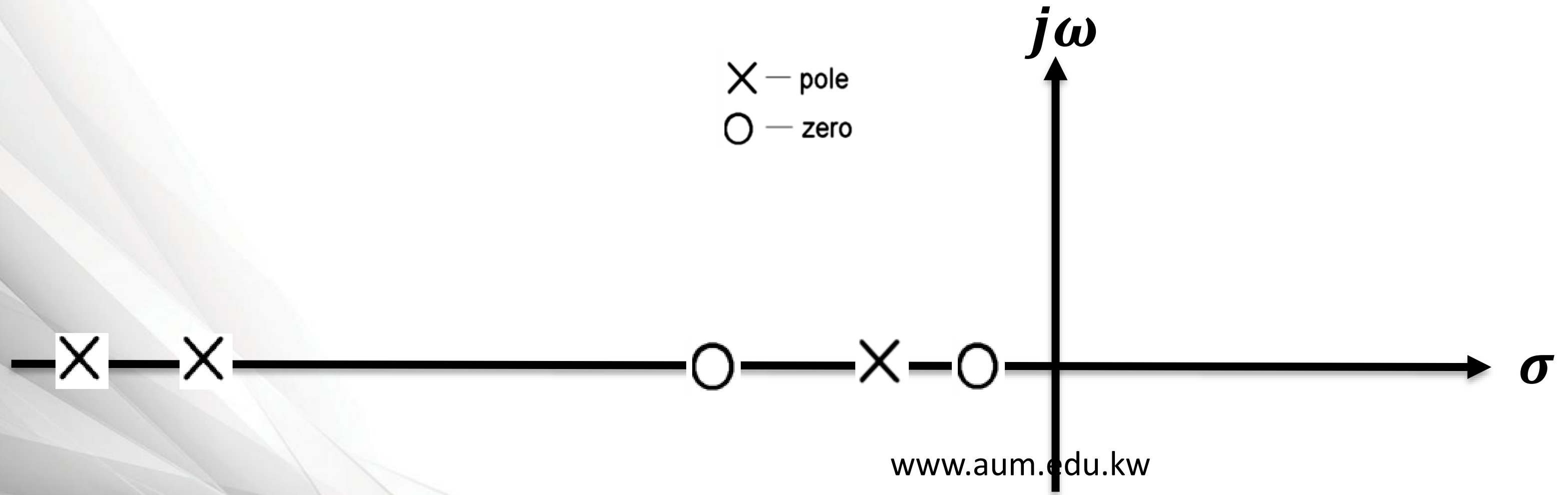
Example

Find zeros and poles for the following transfer function, map poles and zeros on s-plane.

$$H(s) = \frac{5(s+1)(s+4)}{(s+2)(s+9)(s+11)}$$

Real zeros at : $s = -1, s = -4$

Real poles: $s = -2, s = -9, s = -11$



Example -1

Determine the zeros and poles for the following transfer function:

$$H(s) = \frac{2s+1}{s^2+5s+6}$$

The given transfer function can be rewritten as

$$H(s) = \frac{2(s+\frac{1}{2})}{(s+3)(s+2)} = \frac{2\left(s-\left(-\frac{1}{2}\right)\right)}{(s-(-3))(s-(-2))}$$

Real zero at : $s = -1/2$

Real poles: $s = -3, s = -2$

Example -2

Find K, zeros and poles for the following transfer function, map the poles and zeros on s-plane.

$$H(s) = \frac{6(s + 1.4)(s + 2.6)}{s(2s + 1)(3s - 3.3)(s^2 + 9)}$$

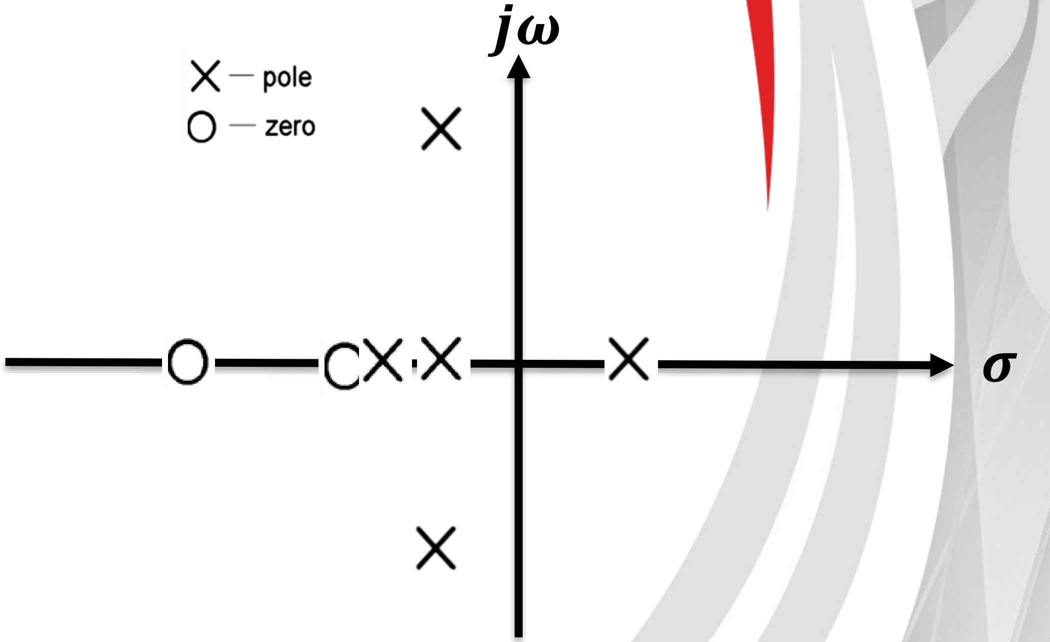
Solution:

Two real zeros: $s = -1.4, s = -2.6$

Two real poles: $\boldsymbol{s = 0, s = -1/2, s = 1.1}$

Two complex poles: $\boldsymbol{s = -j3, s = +j3}$

$\mathbf{K} = 1$

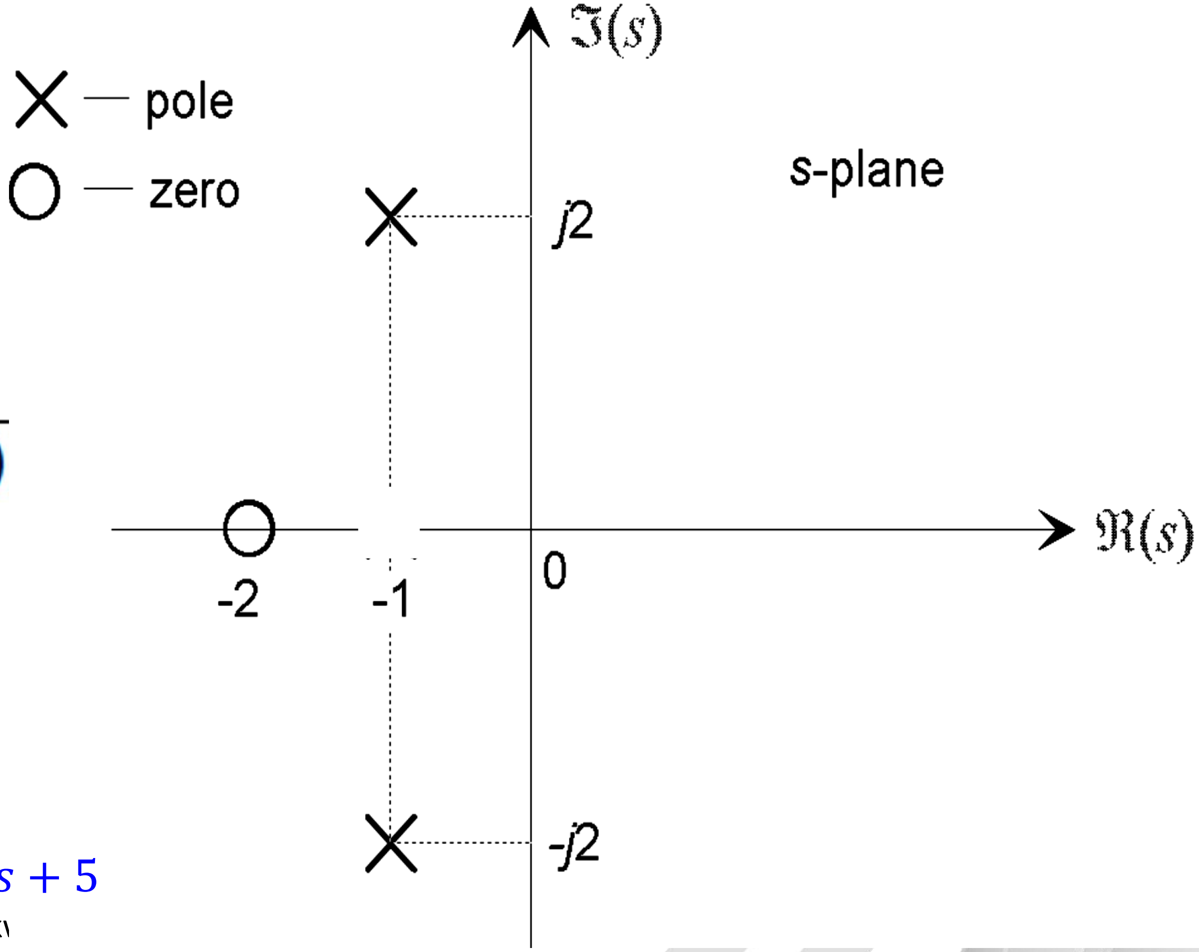


Example -3

A system has a pair of complex conjugate poles $p_1, p_2 = -1 \pm j2$, a single real zero $z_1 = -2$, and a gain factor $K = 3$. Find the transfer function representing the system.

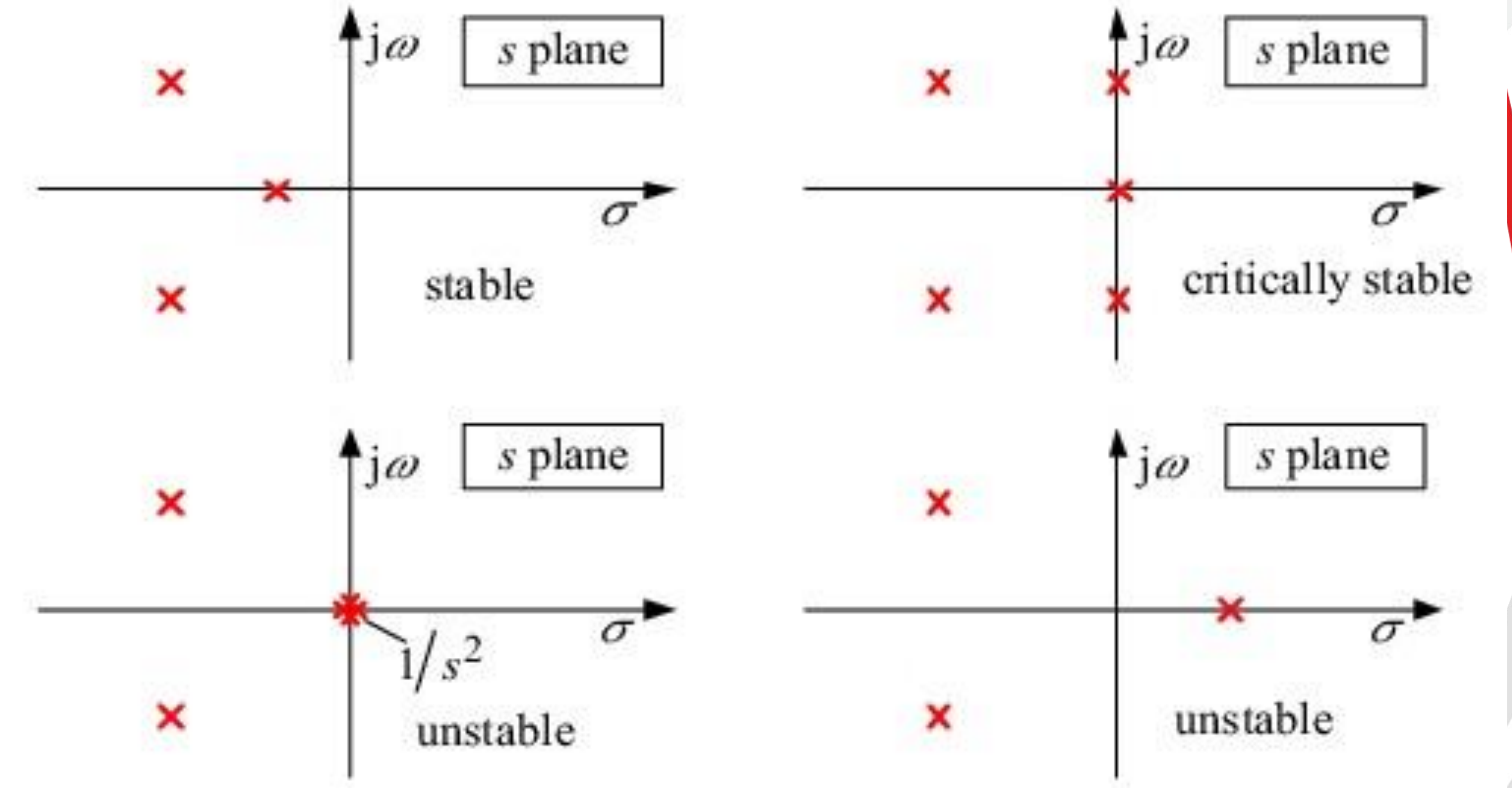
The transfer function is

$$
\begin{aligned}
H(s) &= K\frac{s-z}{(s-p_1)(s-p_2)} \\
&= 3\frac{s-(-2)}{(s-(-1+j2))(s-(-1-j2))} \\
&= 3\frac{(s+2)}{s^2+2s+5}
\end{aligned}
$$



Note: $(s-(-1+j2))(s-(-1-j2)) = (s+1)^2+2^2 = s^2+2s+5$

Stability in the S-plane (characterized by pole locations)

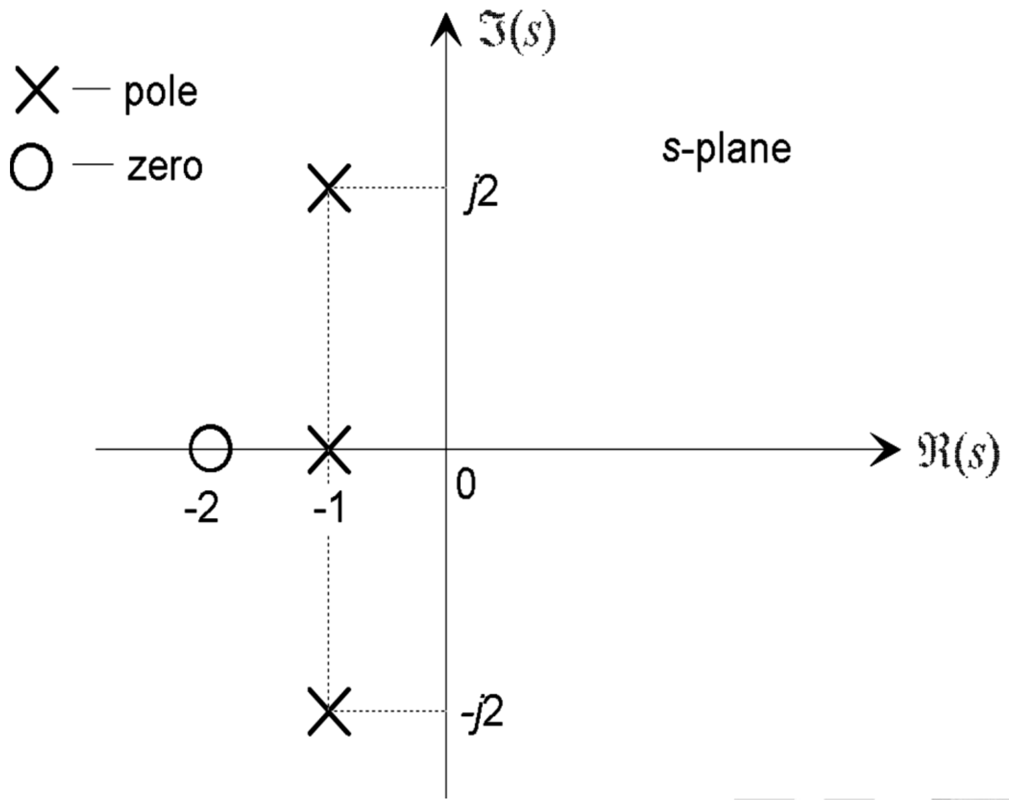


Example -4

Check the stability of the following systems.

$$H(s) = \frac{6(s+1.4)(s+2.6)}{s(2s+1)(3s-3.3)(s^2+9)}$$

System is unstable because of the pole at s=1.1



System is stable because all poles are at the LHP

Example -5

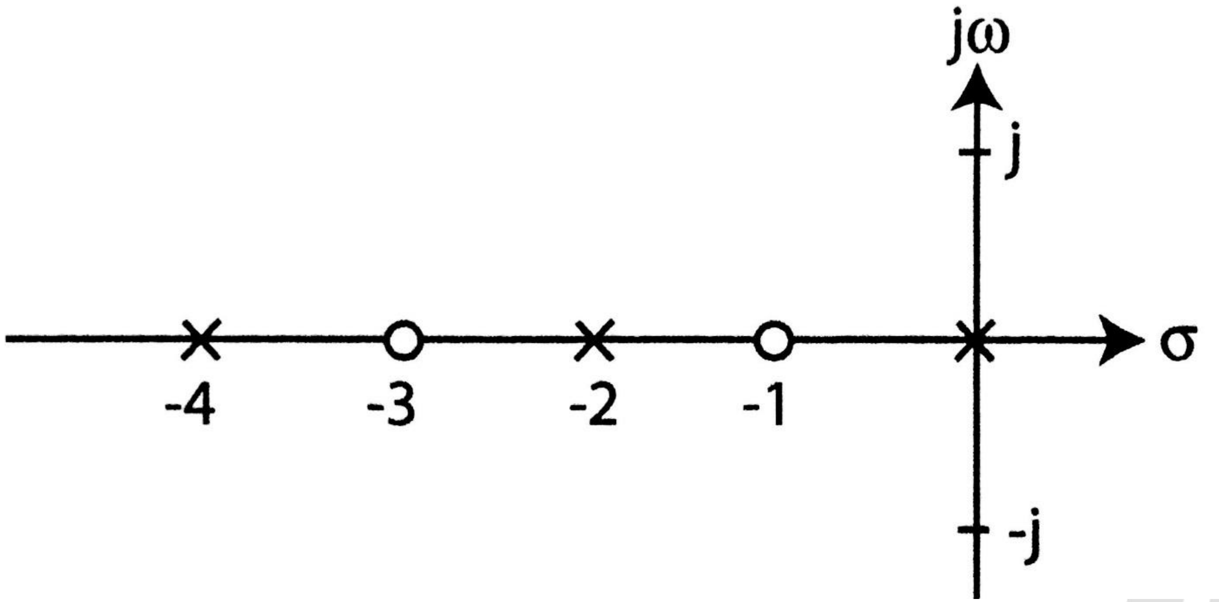
Find the transfer function for the system that has poles at 0, -2 and -3 and zeros at -1, -4 and -6, note that H(1)=21. Check the stability of the system.

$$H(s) = \frac{3.6(s+1)(s+4)(s+6)}{s(s+2)(s+3)}$$

Critically Stable because two poles are negative and one at 0

Example -6

Find the transfer function for the system that has poles and zeros shown in the figure, note that H(1)=8. Check the stability of the system.



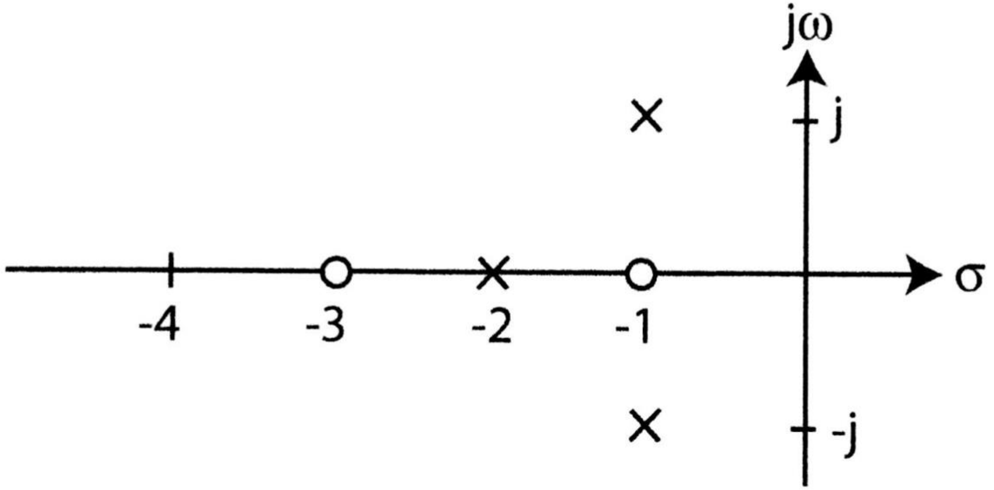
$$H(s) = \frac{15(s+1)(s+3)}{s(s+2)(s+4)}$$

Critically Stable

Example -7

Find the transfer function for the system that has poles and zeros shown in the figure, note that H(0)=3. Check the stability of the system.

$$H(s) = \frac{4(s+1)(s+3)}{(s+2)(s^2+2s+2)}$$

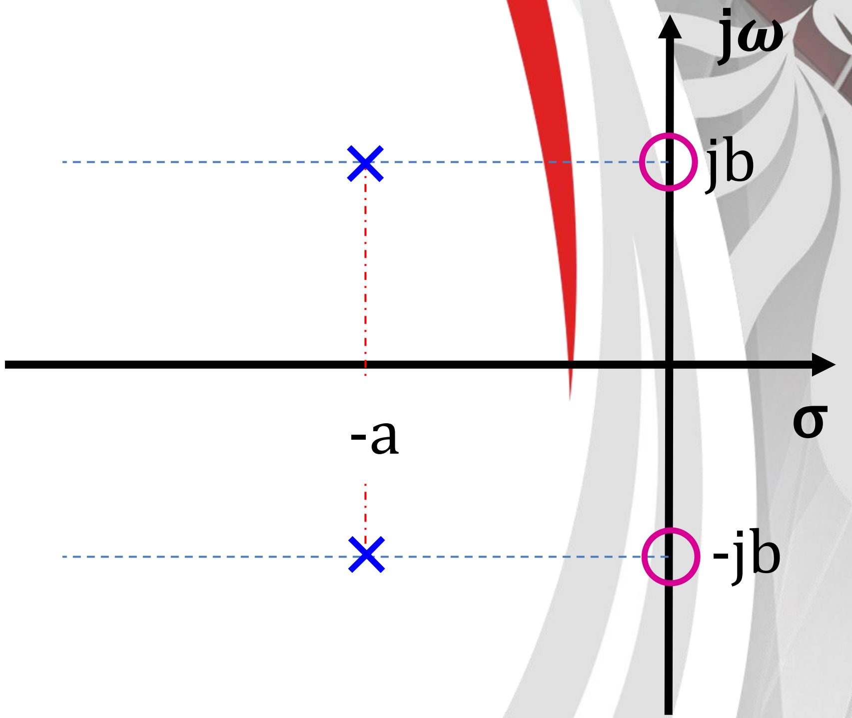


Stable

Example -8

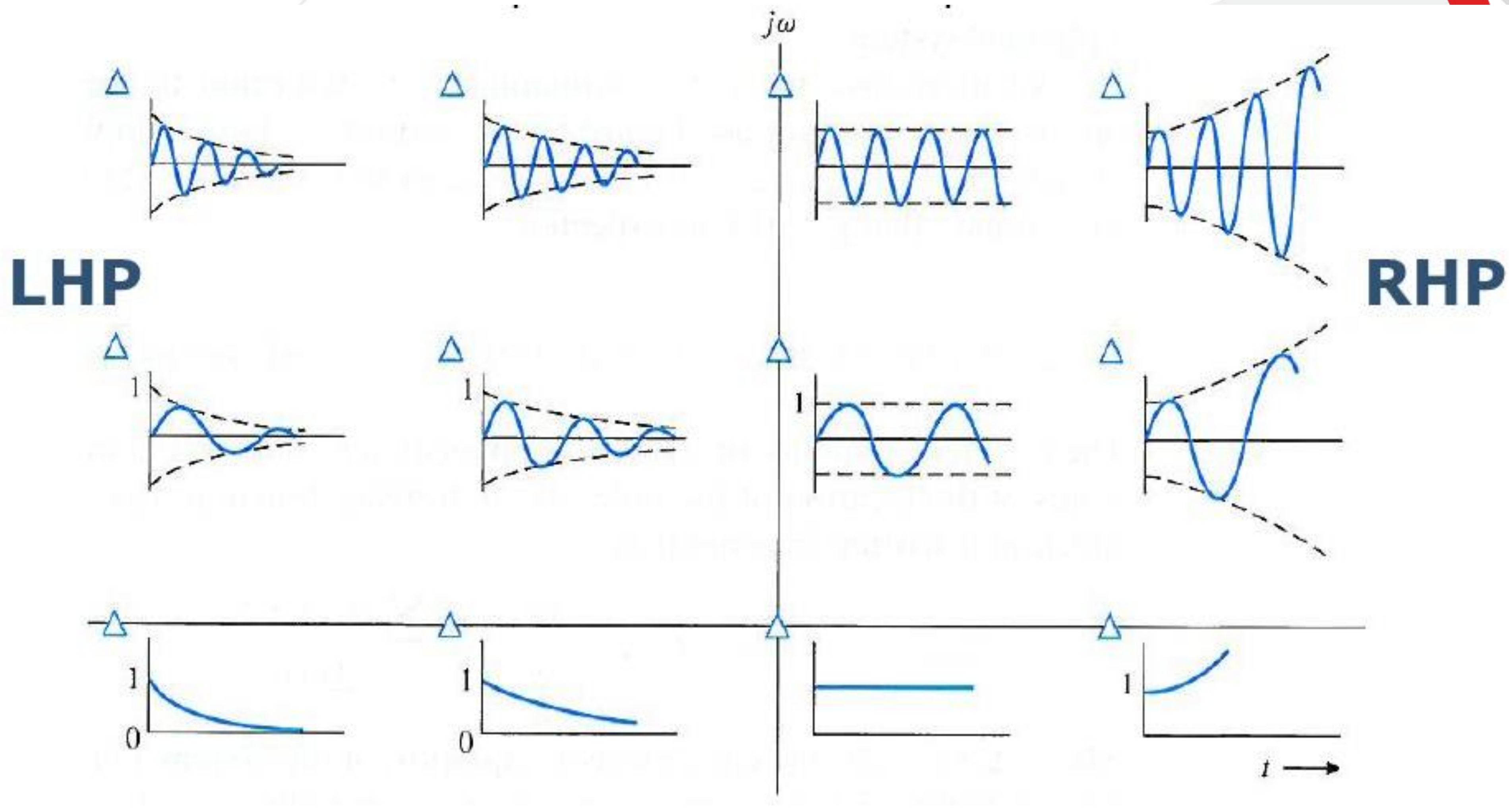
Find $H(s)$ in terms of a and b from the pole-zero plot below assuming $H(0) = 5$.

$$H(s) = K\frac{(s+jb)(s-jb)}{(s+a+jb)(s+a-jb)} = K\frac{s^2+b^2}{(s+a)^2+b^2}$$



$$H(0) = 5 = K\frac{b^2}{a^2+b^2} \Rightarrow K = 5\,\frac{a^2+b^2}{b^2}$$

Effect of Pole Location on the Impulse Response



Example -9

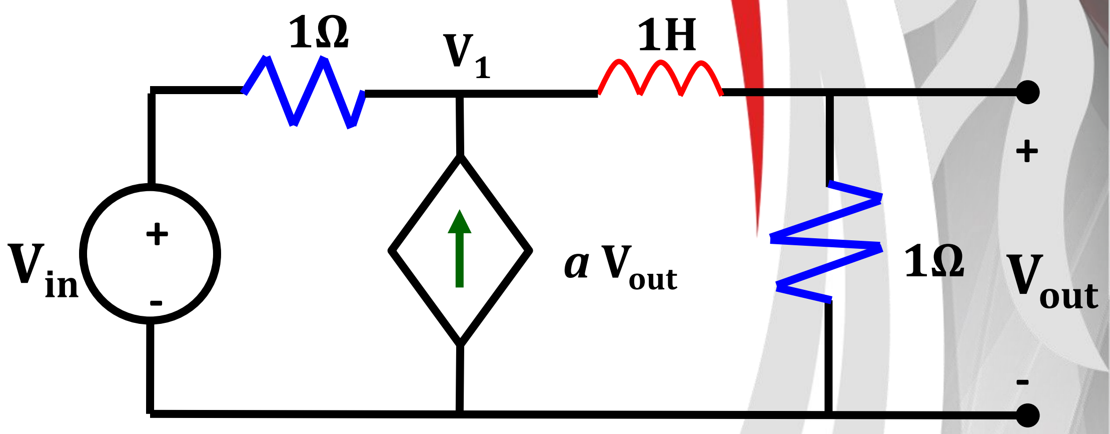
Find the Range of "a" that achieves stability for the transfer function of the circuit shown.

By applying Nodal analysis

Node V_{out} $$1 \times V_{out} + \frac{1}{s}\left(V_{out} - V_1\right) = 0$$

Multiply by s $$sV_{out} + V_{out} - V_1 = 0$$

$$(s+1)V_{out} = V_1 \quad \textbf{Eq. 1}$$



Node V_1

$$\left(V_1 - V_{in}\right) - aV_{out} + \frac{1}{s}\left(V_1 - V_{out}\right) = 0$$

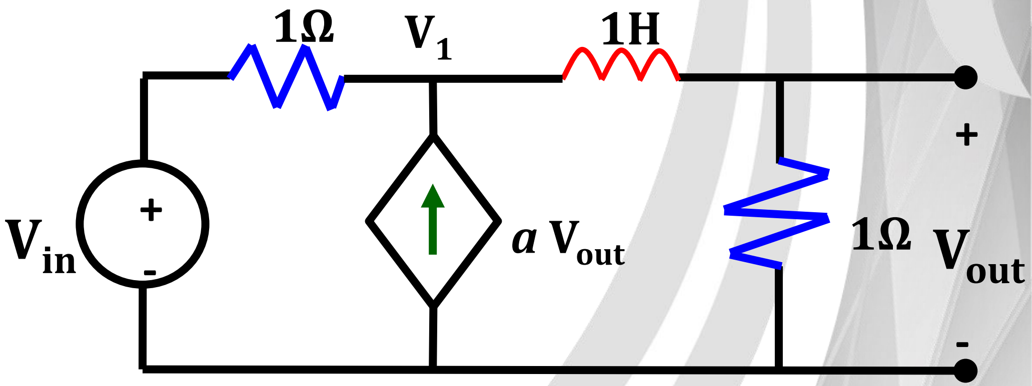
$$s\,(V_1 - V_{in}) - asV_{out} + V_1 - V_{out} = 0$$

$$V_{out}\,(as+1) - (s+1)V_1 = -sV_{in}$$ Eq. 2

By substituting eq. 1 into eq. 2

$$V_{out}\,(as+1) - (s+1)^2 V_{out} = -sV_{in}$$

$$V_{out}(as + 1 - s^2 - 2s - 1) = -sV_{in}$$



$$V_{out}(as - s^2 - 2s) = -sV_{in}$$

$$V_{out}(a - s - 2) = -V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{s + (2 - a)}$$

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{s + 2 - a}$$

For the system to be stable, (a-2)< 0, then a<2

Answer the following questions using ChatGPT

1. Explain the physical significance of poles and zeros in a system.

2. How does the location of poles on the s-plane affect the system’s stability and response?

3. What does the s-plane represent in control system analysis?

4. Why is mapping poles and zeros onto the s-plane useful?

5. How does moving poles closer to the imaginary axis affect the system's response?

Summary

- Zeros: Values of s that make $H(s)=0$.
- Poles: Values of s that make $H(s)\rightarrow\infty$.
- Mapped on the s-plane to analyze system behavior.
- Stability depends on pole locations:
  - Poles in the Left Half Plane (LHP): Stable.
  - Poles in the Right Half Plane (RHP): Unstable.
  - Poles on the imaginary axis: Critically stable.
- Impulse response shape is influenced by pole location:
  - Closer to the origin: Faster response.
  - Farther from the origin: Slower response or oscillatory.

Suggested examples

- Page 685, example 14.1
- Page 687, exercise
- Page 745, examples: 16, 17, 18

Page 686, example 14.2
Page 742, examples: 3, 5