American University Of The Middle East

Transfer Function

Linear Circuit Analysis II
EECE 202

Announcement

1. Midterm during week 8
2. PD 1 Voice Over PPT due in Week 10

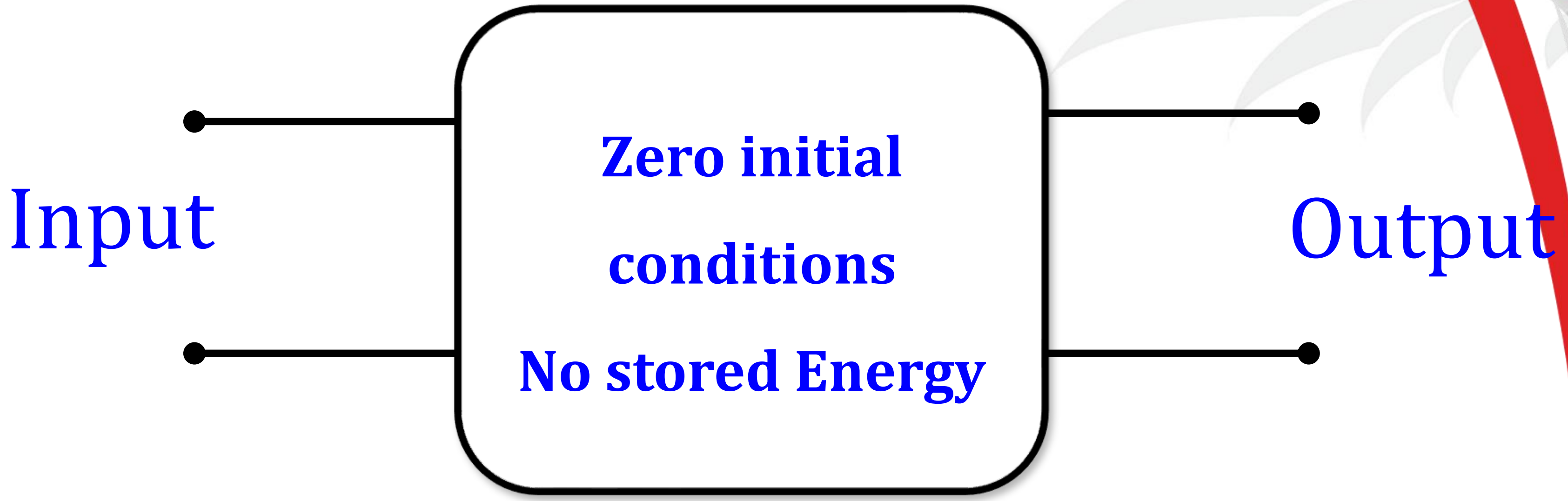
Recap

1. Impedances
2. Admittances

New Material

1. Transfer functions
2. Initial and final value theorem

Transfer Function

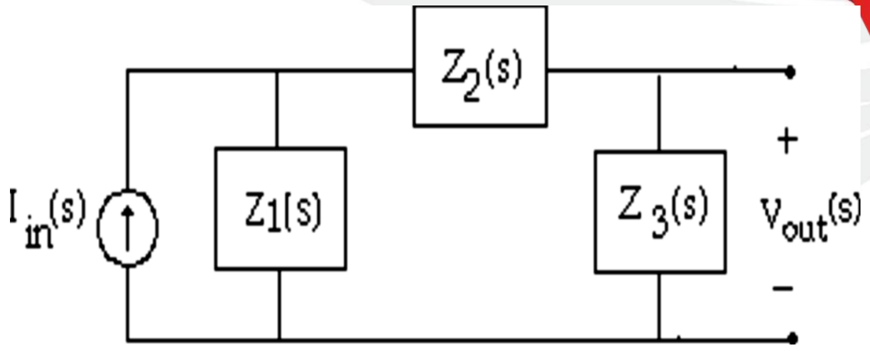


$$H(s) = \frac{L[output\ signal]}{L[input\ signal]}$$

Example -1

Calculate H(s) for the shown circuit

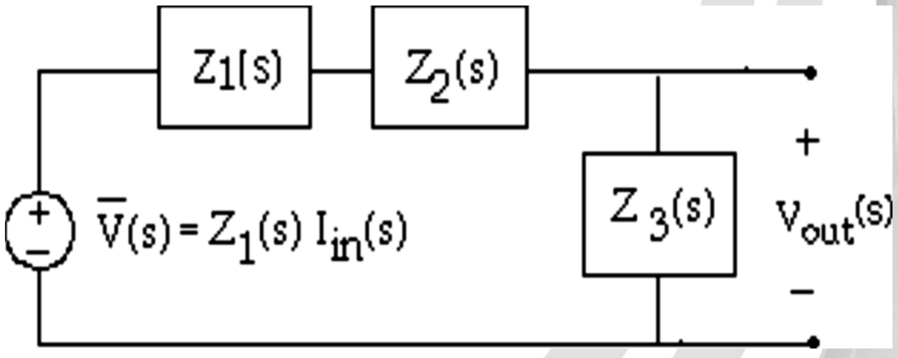
$$H(s) = \frac{V_{out}(s)}{I_{in}(s)}$$



Using source transformation and voltage division

$$V_{out}(s) = \frac{Z_3(s)}{Z_1(s)+Z_2(s)+Z_3(s)}\bar{V}(s) = \frac{Z_3(s)Z_1(s)I_{in}(s)}{Z_1(s)+Z_2(s)+Z_3(s)}$$

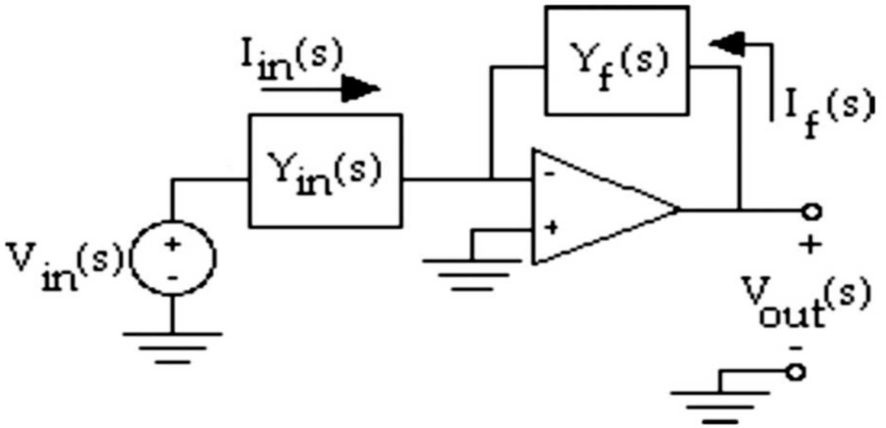
$$\boxed{H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{Z_3(s)Z_1(s)}{Z_1(s)+Z_2(s)+Z_3(s)}}$$



Example -2 Transfer function for Op Amp Circuits

For the circuit shown, show that

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Y_{in}(s)}{Y_f(s)}$$



By applying KCL at the inverting input node noting that this is the virtual ground node

$$I_{in}(s) = -I_f(s)$$

$$Y_{in}(s)V_{in}(s) = -Y_f(s)V_{out}(s)$$

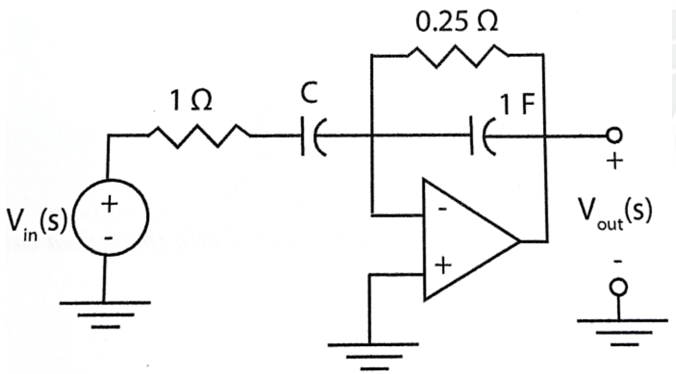
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Y_{in}(s)}{Y_f(s)}$$

Note: You can use nodal analysis at the virtual ground node. You must get the same result.

Example -2

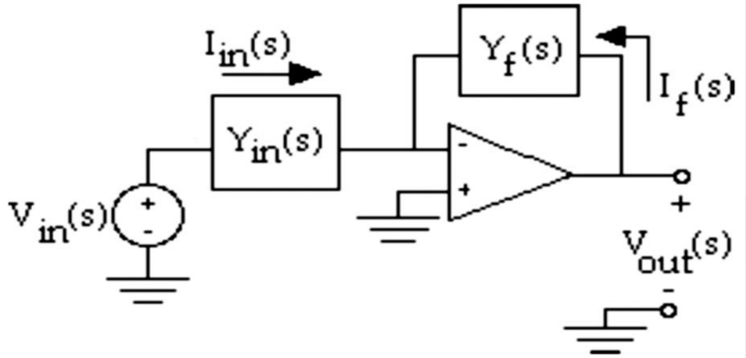
In the Op Amp Circuit shown in figure, find the value of C for which the transfer function should equal to:

$$\boldsymbol{H(s) = \frac{-s}{(s+2)(s+4)}}$$



By comparing the circuit of example 2 with the general inverting amplifier circuit shown below

$$Z_{in}(s) = 1 + \frac{1}{Cs} = \frac{Cs+1}{Cs}\text{ , }then\ Y_{in}(s) = \frac{Cs}{Cs+1}$$



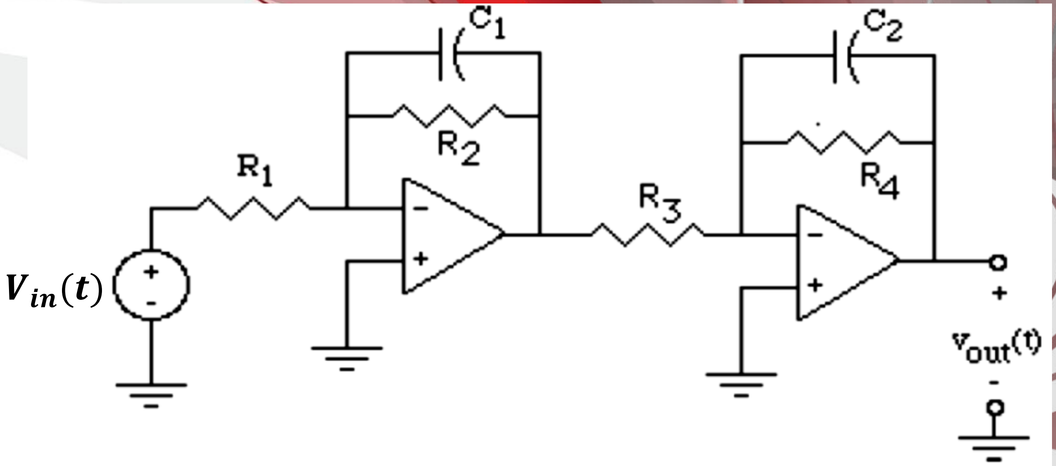
$$Y_f(s) = 4 + s$$

$$H(s) = -\frac{Y_{in}(s)}{Y_f(s)} = \frac{\frac{-Cs}{Cs+1}}{(4+s)} = \frac{-Cs}{(Cs+1)(s+4)} = \frac{-s}{(s+\frac{1}{C})(s+4)} = \boldsymbol{\frac{-s}{(s+2)(s+4)}}$$

Then $\boldsymbol{C = 0.5\,F}$

Example -3

In the Op Amp Circuit shown in figure, find the values of R1, R2, R3, R4, C1 and C2 so that the transfer function should equal to:



$$H(s) = \frac{V_{out}}{V_{in}} = \frac{30}{(s+5)(s+6)}$$

Also, find the impulse and step response.

Solution

$$H(s) \triangleq H_1(s)H_2(s)$$

As what have been done in the previous example, we can find H1(s) and H2(s)

$$Z_{in1}(s) = R_1, \qquad Y_{in}(s) = \frac{1}{R_1} = G_1 \qquad Y_{f1}(s) = \frac{1}{R_2} + C_1 s = (G_2 + C_1 s)$$

$$Z_{in2}(s) = R_3, \qquad Y_{in}(s) = \frac{1}{R_3} = G_3 \qquad Y_{f2}(s) = \frac{1}{R_4} + C_2 s = (G_4 + C_2 s)$$

Example -3 (Contin…)

$$H_1(s) = \frac{-Y_{in1}(s)}{Y_{f1}(s)} = \frac{G_1}{(C_1 s + G_2)}$$

$$H_2(s) = \frac{-Y_{in2}(s)}{Y_{f2}(s)} = \frac{G_3}{(C_2 s + G_4)}$$

$$H(s) = H_1(s) * H_2(s) = \frac{G_1 G_3}{(C_1 s + G_2)(C_2 s + G_4)}$$

$$\boxed{= \frac{30}{(s+5)(s+6)}}$$

Since we have 6 unknowns and 3 equations, we can assume the values of 3 variables

$$\boldsymbol{Assume\ C1 = C2 = 1F\ and\ G1 = 6s}$$

$$\text{Then, G2} = 5\text{s}, \text{G4} = 6\text{s and G3} = 5\text{s}.$$

$$R1 = \frac{1}{6}\Omega,\ R2 = 0.2\ \Omega,\ R3 = 0.2\ \Omega,\ R4 = \frac{1}{6}\Omega,\ C1 = C2 = 1F.$$

Impulse Response

$$h(t) \triangleq L^{-1}\left[H(s)L\left[\delta(t)\right]\right] = L^{-1}\left[H(s)\right]$$

$$h(t) = L^{-1}\left[H(s)\right] = L^{-1}\left[\frac{30}{(s+5)(s+6)}\right]$$

$$= L^{-1}\left[\frac{30}{s+5}\right] - L^{-1}\left[\frac{30}{s+6}\right] = 30e^{-5t}u(t) - 30e^{-6t}u(t)$$

Step Response

$$\text{Step Response} \triangleq L^{-1}\left[H(s)L\left[u(t)\right]\right] = L^{-1}\left[\frac{H(s)}{s}\right]$$

$$L^{-1}\left[\frac{H(s)}{s}\right] = L^{-1}\left[\frac{30}{s(s+5)(s+6)}\right]$$

$$= L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{6}{s+5}\right] + L^{-1}\left[\frac{5}{s+6}\right]$$

$$\textbf{Step Response} = \left[1 - 6e^{-5t} + 5e^{-6t}\right]u(t)$$

Initial and Final Values

Initial Value Theorem.

$$\lim_{s\to\infty} sF(s) = \lim_{t\to 0^+} f(t) = f(0^+)$$

Final Value Theorem:

$$\lim_{s\to 0} sF(s) = \lim_{t\to\infty} f(t) = f(\infty)$$

Example -4

Find f(0+) and f(∞) for the following function

$$F(s) = \frac{(2s+7)(3s+14)}{s(s+7)(s+14)}.$$

Solution

$$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{(2s+7)(3s+14)}{(s+7)(s+14)} = 6$$

$$f(\infty) = \frac{(2s+7)(3s+14)}{(s+7)(s+14)}\Big]_{s=0} = \frac{98}{98} = 1$$

Example -5

The Laplace transform of a capacitor voltage is given by:

$$V_C(s) = \frac{2}{s} - \frac{1}{5s+2}$$

Find the initial capacitor voltage $v_c(0^-)$

Solution

By applying the initial value theorem

$$\boldsymbol{V_C(0^+) = \lim_{s \ldots \infty} sV_C(s) = \lim_{s \ldots \infty} \left[\frac{2s}{s} - \frac{s}{5s+2}\right] = 2 - 0.2 = 1.8\ V}$$

Answer the following questions using ChatGPT

- What is the primary purpose of a transfer function in analyzing linear circuits?
- Why are initial conditions often assumed to be zero when calculating a transfer function?
- What is the significance of finding the impulse response of a circuit from its transfer function?
- How does the step response of a system differ from its impulse response, and what does it indicate about the system's stability?

Summary

- Transfer function
- Impulse response
- Step response
- Initial and final value

- Page 626, example 13.9
- Page 629, example 13.11
- Page 636, example 13.16
- Page 640, exercise
- Page 662, example 21
- Page 663, example 26
- Page 664, example 29
- Page 665, example 31