
Final project - Matlab

Numerical analysis

- December 30, 2020

By:

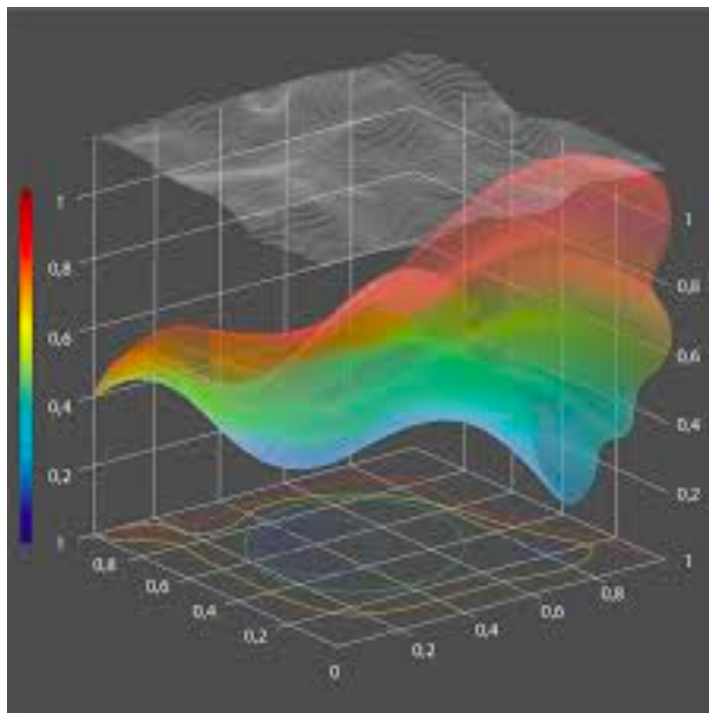
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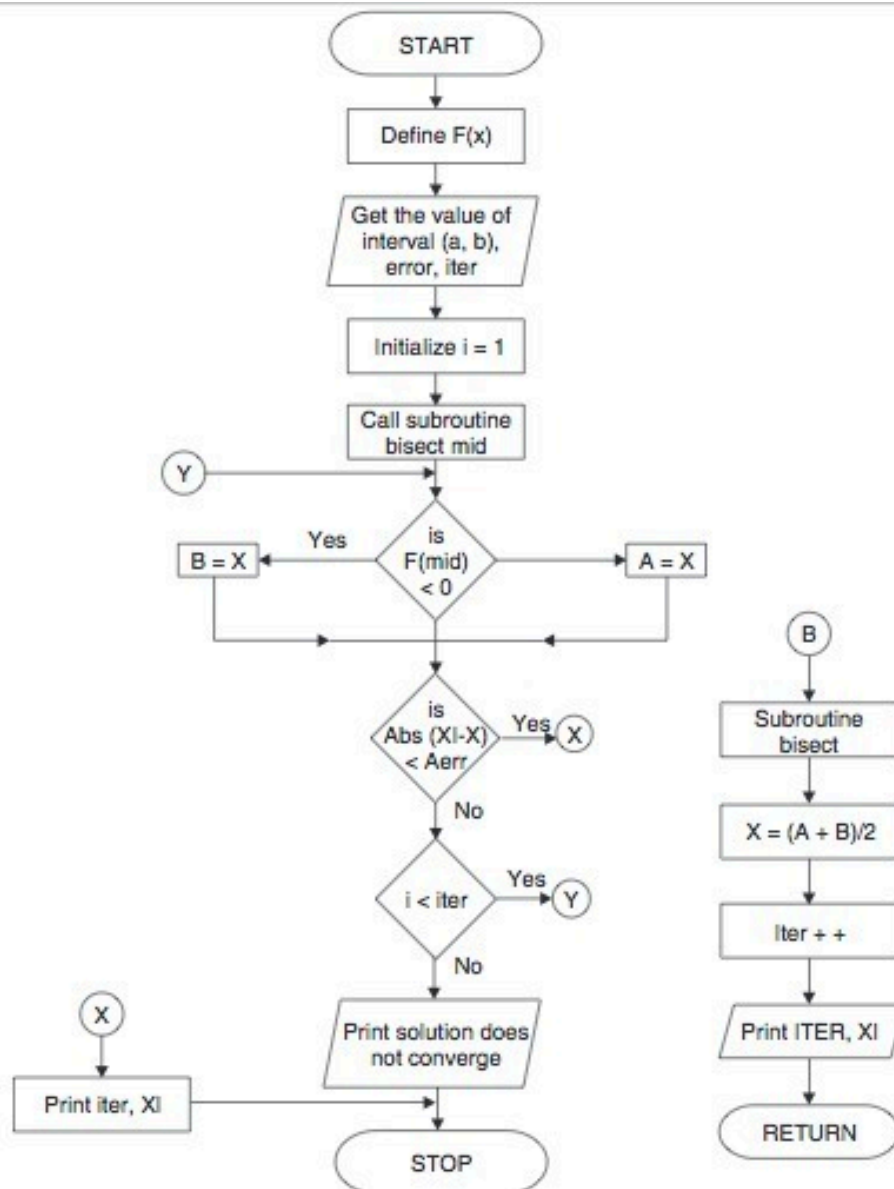
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Bisection:

The algorithm we used:

1. Start
2. Define function $f(x)$
3. Input
 - a. Lower and Upper guesses x_0 and x_1
 - b. tolerable error e
4. If $f(x_0)*f(x_1) > 0$
 print "Incorrect initial guesses"
 goto 3
End If
5. Do
 $x_2 = (x_0+x_1)/2$
 If $f(x_0)*f(x_2) < 0$
 $x_1 = x_2$
 Else
 $x_0 = x_2$
 End If
 while $\text{abs}(f(x_2)) > e$
6. Print root as x_2
7. Stop

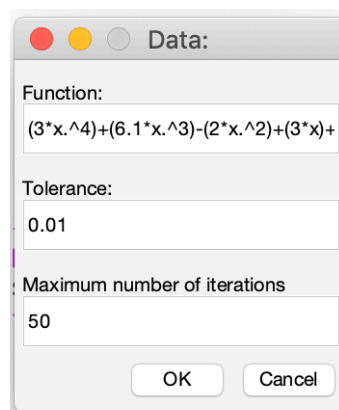


Examples:

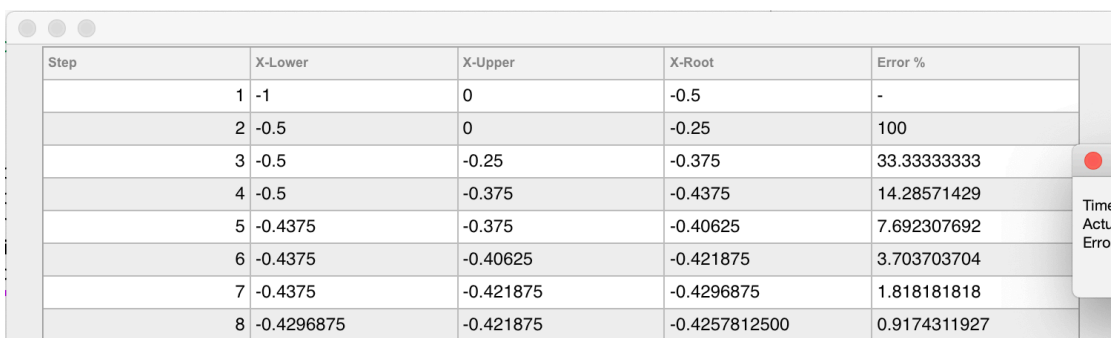
$$1) f(x) = 3x^4 + 6.1x^3 - 2x^2 + 3x + 2$$

Tolerance : 10^{-2}

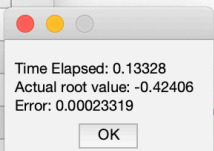
Interval $[-1,0]$



A dialog box titled "Data:" with three input fields and two buttons. The first field is labeled "Function:" and contains the expression $(3*x.^4)+(6.1*x.^3)-(2*x.^2)+(3*x)+$. The second field is labeled "Tolerance:" and contains the value "0.01". The third field is labeled "Maximum number of iterations" and contains the value "50". At the bottom are "OK" and "Cancel" buttons.



Step	X-Lower	X-Upper	X-Root	Error %
1	-1	0	-0.5	-
2	-0.5	0	-0.25	100
3	-0.5	-0.25	-0.375	33.33333333
4	-0.5	-0.375	-0.4375	14.28571429
5	-0.4375	-0.375	-0.40625	7.692307692
6	-0.4375	-0.40625	-0.421875	3.703703704
7	-0.4375	-0.421875	-0.4296875	1.818181818
8	-0.4296875	-0.421875	-0.4257812500	0.9174311927



A small dialog box with the following text: "Time Elapsed: 0.13328", "Actual root value: -0.42406", "Error: 0.00023319", and an "OK" button.

2) $f(x) = x^3 - x - 1$
Tolerance: 10^{-5}
Interval [1,2]

Data:

Tolerance: 0.00001

Maximum number of iterations: 50

OK Cancel

MENU

Input function as text

Read from file

input.txt

1 $(x.^3) - x - 1$

Step	X-Lower	X-Upper	X-Root	Error %
1	1	2	1.5	-
2	1	1.5	1.25	20
3	1.25	1.5	1.375	9.090909091
4	1.25	1.375	1.3125	4.761904762
5	1.3125	1.375	1.34375	2.325581395
6	1.3125	1.34375	1.328125000	1.176470588
7	1.3125	1.328125000	1.320312500	0.5917159763
8	1.320312500	1.328125000	1.324218750	0.2949852507

Time Elapsed: 0.16562
Actual root value: 1.3247
Error: 5.181e-07

OK

3) $f(x) = x^2 - \sin(x) - 0.5$
Tolerance : 10^{-3}
Interval [0, 2]

Step	X-Lower	X-Upper	X-Root	Error %
1	0	2	1	-
2	1	2	1.5	33.33333333
3	1	1.5	1.25	20
4	1	1.25	1.125	11.11111111
5	1.125	1.25	1.1875	5.263157895
6	1.1875	1.25	1.21875	2.564102564
7	1.1875	1.21875	1.203125000	1.298701299
8	1.1875	1.203125000	1.195312500	0.6535947712

Time Elapsed: 0.62675
Actual root value: 1.1961
Error: -2.7791e-06

OK

Regula Falsi:

The algorithm we used:

1. Start

2. Define function $f(x)$

3. Input

- a. Lower and Upper guesses x_0 and x_1
- b. tolerable error e

4. If $f(x_0)*f(x_1) > 0$

 print "Incorrect initial guesses"

 goto 3

End If

5. Do

$$x_2 = x_0 - ((x_0 - x_1) * f(x_0)) / (f(x_0) - f(x_1))$$

 If $f(x_0)*f(x_2) < 0$

$x_1 = x_2$

 Else

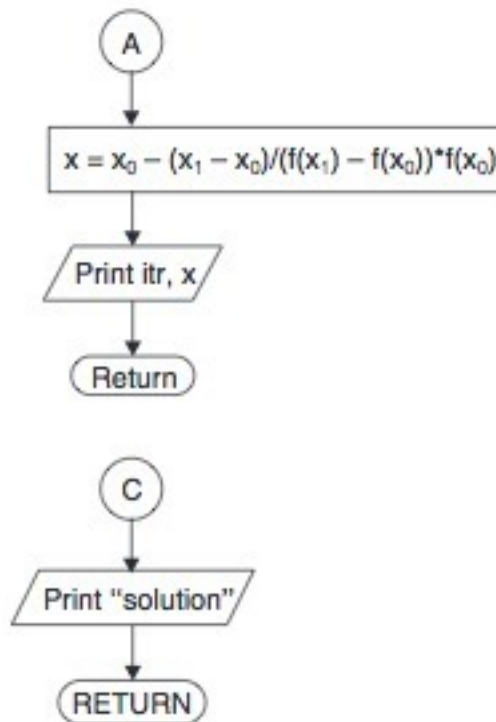
$x_0 = x_2$

 End If

While $\text{abs}(f(x_2)) > e$

6. Print root as x_2

7. Stop



Examples:

1) $f(x) = 2x^3 - 2x - 5$

Tolerance : 10^{-3}

Interval [1,2]

Step	X-Lower	X-Upper	X-Root	F(Xr)	Error %
1	1.8	1.9	1.827676752	-0.006990635541	-
2	1.827676752	1.9	1.829285619	-4.018435564e-4	0.08795058484
3	1.829285619	1.9	1.829377981	-2.305403140e-5	0.005048815939

Time Elapsed: 0.17077
Actual root value: 1.8294
Error: -5.6209e-06
OK

2) $f(x) = e^x + 2^{-x} + 2 \cos(x) - 6$
Tolerance : 10^{-3}
two initial guesses [1.8,1.9]

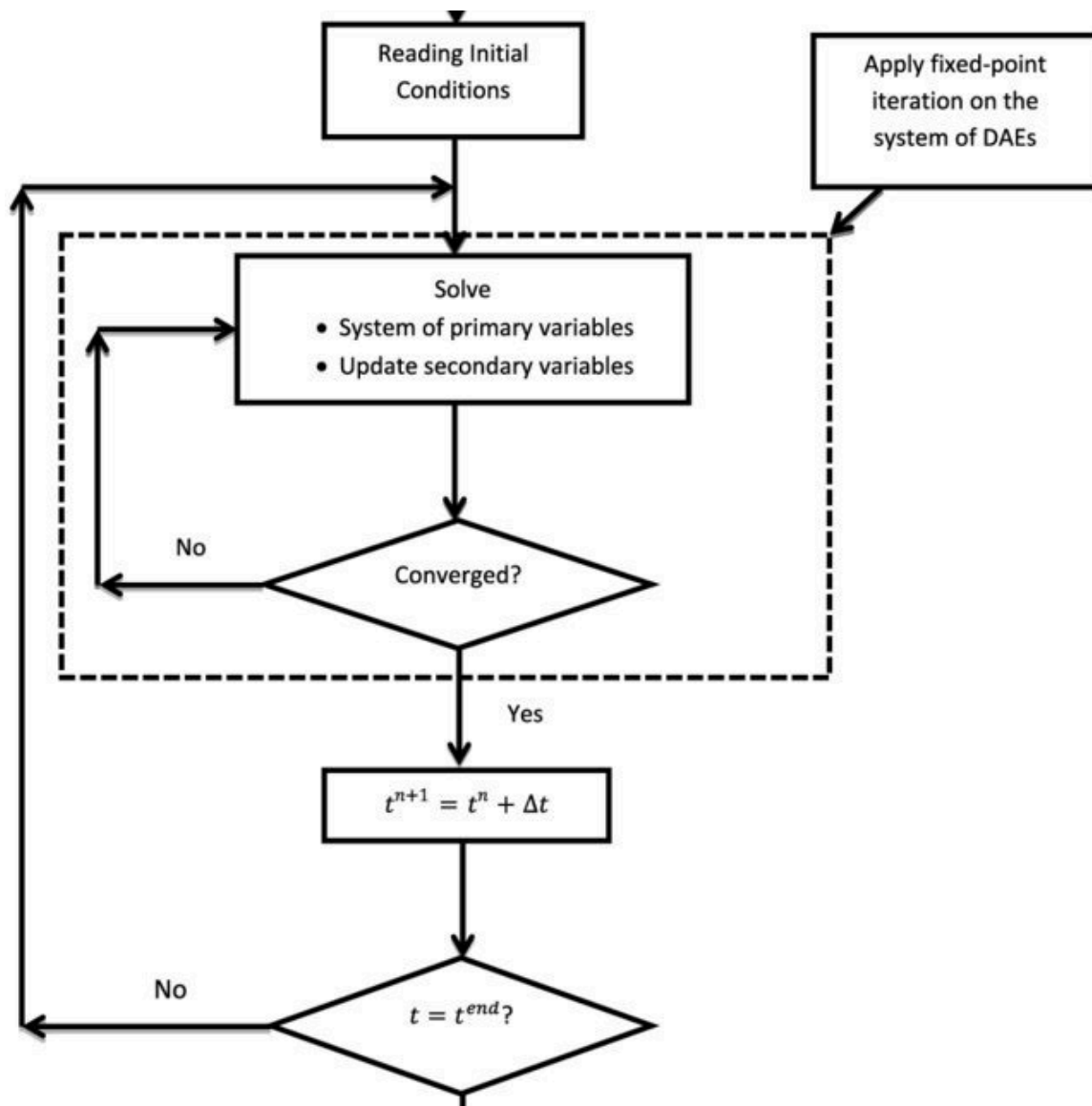
Step	X-Lower	X-Upper	X-Root	F(Xr)	Error %
1	1.8	1.9	1.827676752	-0.006990635541	-
2	1.827676752	1.9	1.829285619	-4.018435564e-4	0.08795058484
3	1.829285619	1.9	1.829377981	-2.305403140e-5	0.005048815939
4	1.829377981	1.9	1.829383279	-1.322476398e-6	2.896313492e-4

Fixed point Approximation

The algorithm we used:

1. Start
2. Define function as $f(x)$
3. Define convergent form $g(x)$
4. Input:
 - a. Initial guess x_0
 - b. Tolerable Error e
 - c. Maximum Iteration N
5. Initialize iteration counter: $\text{step} = 1$
6. Do
 - $x_1 = g(x_0)$
 - $\text{step} = \text{step} + 1$
 - If $\text{step} > N$
 - Print "Not Convergent"
 - Stop
 - End If
 - $x_0 = x_1$
 - While $\text{abs } f(x_1) > e$
7. Print root as x_1
8. Stop

Note: $g(x)$ is obtained by rewriting $f(x)$ in the form of $x = g(x)$

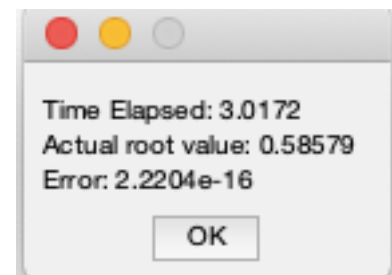


Examples:

$$1) f(x) = x^3 - 7x^2 + 14x - 6$$

$$X_0 = 0.5$$

$$\text{Interval} = [0, 1]$$

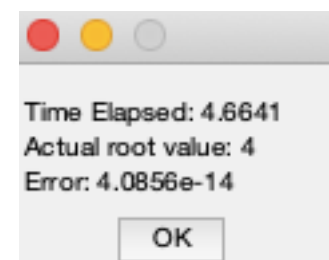


Step	X(i)	F(Xi)	F'(Xi)	X(i+1)	Error%
1	0.5	-0.625	7.75	0.5806451613	13.88888889
2	0.5806451613	-0.03524554396	6.882414152	0.5857662632	0.8742568817
3	0.5857662632	-1.377620357e-4	6.828638661	0.5857864373	0.003443944206
4	0.5857864373	-2.133754506e-9	6.828427128	0.5857864376	5.334386937e-8

$$2) f(x) = x^2 - 2x - 8$$

$$\text{Interval} = [-2, 4]$$

$$X_0 = 3.9$$



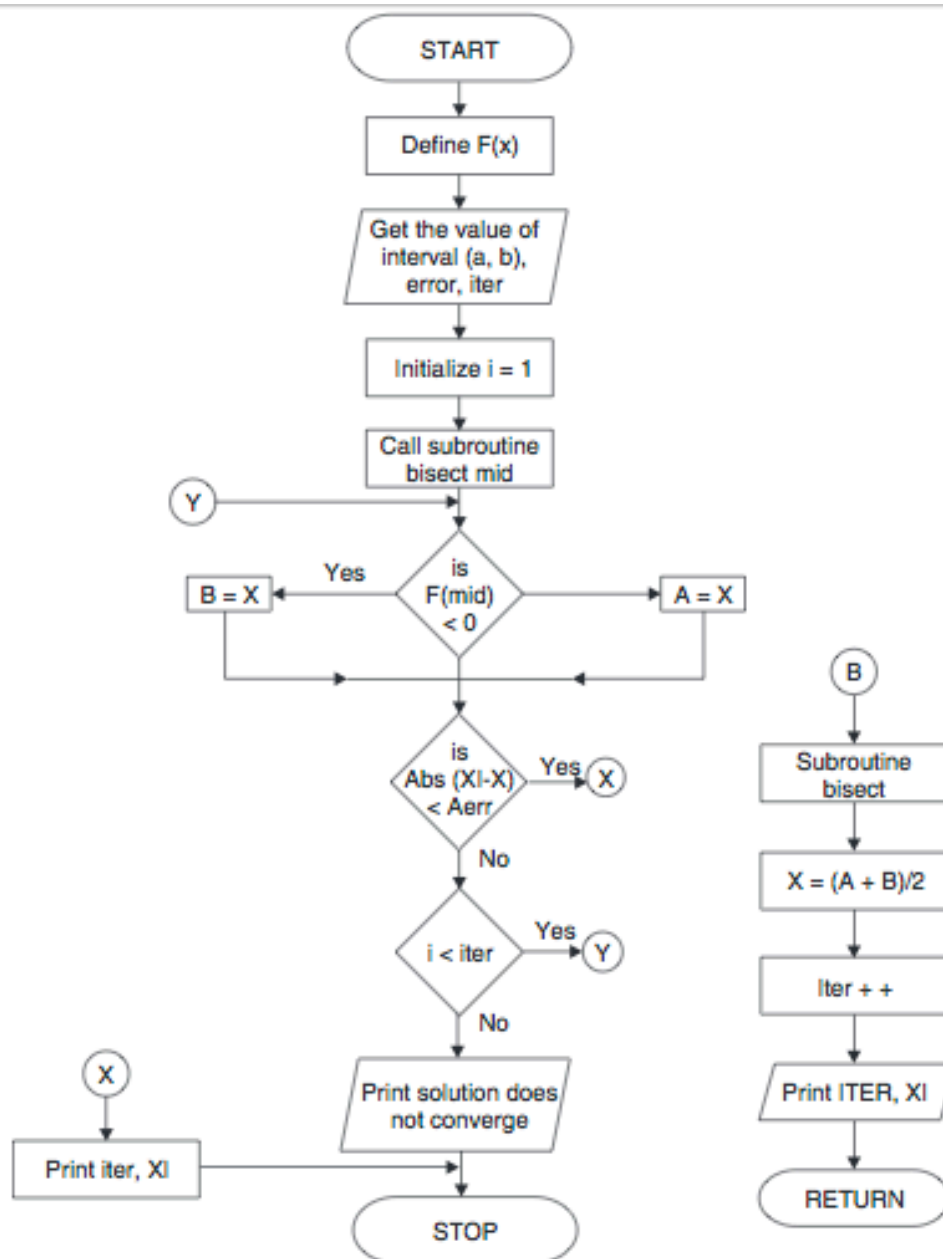
Step	X(i)	F(Xi)	F'(Xi)	X(i+1)	Error%
1	3.9	-0.59	5.8	4.001724138	2.542007755
2	4.001724138	0.01034780024	6.003448276	4.000000495	0.04309106401
3	4.000000495	2.970944413e-6	6.000000990	4	1.237893301e-5

Newton Rasphon:

The algorithm we used:

1. Start
2. Define function as $f(x)$
3. Define derivative of function as $g(x)$
4. Input:
 - a. Initial guess x_0
 - b. Tolerable Error e
 - c. Maximum Iteration N
5. Initialize iteration counter $step = 1$
6. Do
 - If $g(x_0) = 0$
 - Print "Mathematical Error"
 - Stop
 - End If
 - $x_1 = x_0 - f(x_0) / g(x_0)$
 - $x_0 = x_1$
 - $step = step + 1$
 - If $step > N$
 - Print "Not Convergent"
 - Stop
 - End If
 - While $abs\ f(x_1) > e$
7. Print root as x_1

8. Stop



Examples:

$$1) f(x) = x - 0.75 - 0.2 \sin(x)$$

Tolerance : 10^{-1}

Interval $[0, \pi/2]$

$$X_0 = \pi/4$$

Step	X(i)	F(Xi)	F'(Xi)	X(i+1)	Error%
1	0.7853981630	-0.1060231932	0.8585786437	0.9088850375	13.58663300
2	0.9088850375	0.001121248336	0.8770748730	0.9076066422	0.1408534460
3	0.9076066422	1.288734144e-7	0.8768732890	0.9076064952	1.619305902e-5

$$2) f(x) = x^3 - 2x^2 - 4x + 8$$

Tolerance: 10^{-1}

$$X_0 = 1.5$$

Time Elapsed: 66.8979
Actual root value: 0.90761

Step	X(i)	F(Xi)	F'(Xi)	X(i+1)	Error%
1	1.5	0.875	-3.25	1.769230769	15.21739130
2	1.769230769	0.2007282658	-1.686390533	1.888259109	6.303602058
3	1.888259109	0.04854890687	-0.8564690456	1.944944061	2.914477248
4	1.944944061	0.01195774231	-0.4313540392	1.972665472	1.405276805
5	1.972665472	0.002968282016	-0.2164346964	1.986379918	0.6904241303
6	1.986379918	7.394999129e-4	-0.1084041356	1.993201613	0.3422481004
7	1.993201613	1.845580704e-4	-0.05424844407	1.996603702	0.1703938368
8	1.996603702	4.610017463e-5	-0.02713577653	1.998302573	0.08501568705

Secant:

The algorithm we used:

1. Start
2. Define function as $f(x)$
3. Input:
 - a. Initial guess x_0, x_1
 - b. Tolerable Error e
 - c. Maximum Iteration N
4. Initialize iteration counter $step = 1$
5. Do
 - If $f(x_0) = f(x_1)$
 - Print "Mathematical Error"
 - Stop
 - End If

```
x2 = x1 - (x1 - x0) * f(x1) / ( f(x1) -  
f(x0) )  
x0 = x1  
x1 = x2  
step = step + 1  
If step > N  
    Print "Not Convergent"  
    Stop  
End If
```

```
While abs f(x2) > e
```
6. Print root as x_2

7. Stop

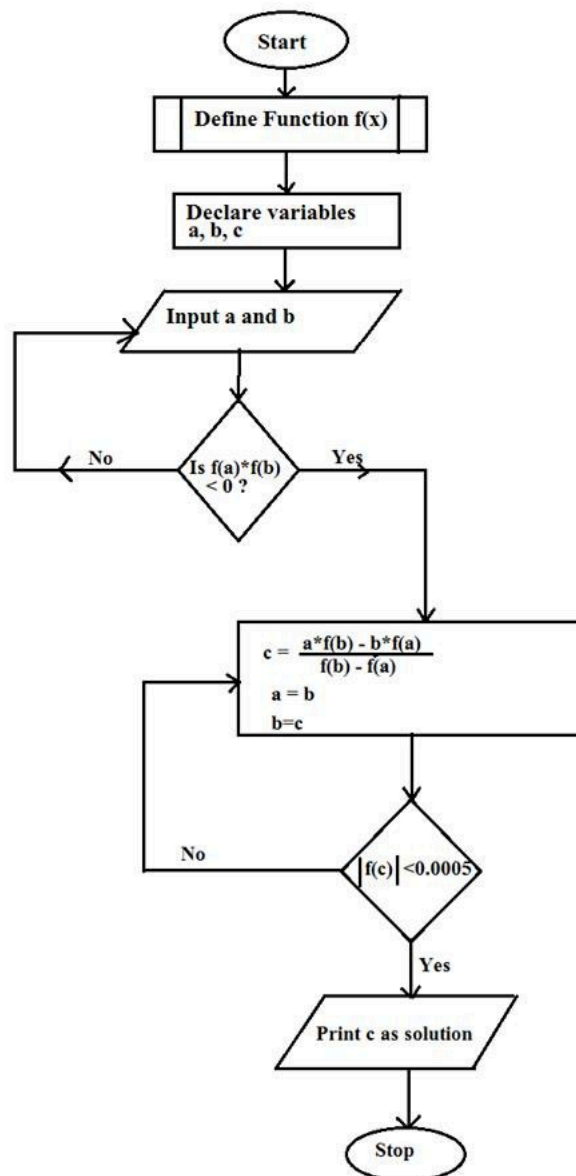


Fig. Flow Chart for Secant Method

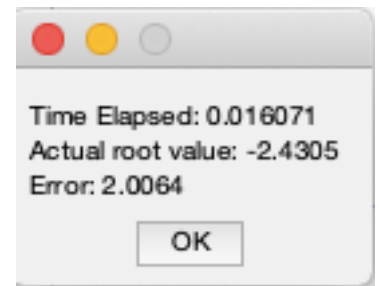
codewithc.com

Examples:

$$2) f(x) = 3x^4 + 6.1x^3 - 2x^2 + 3x + 2$$

Interval $[-1, -3]$

Tolerance : 10^{-2}

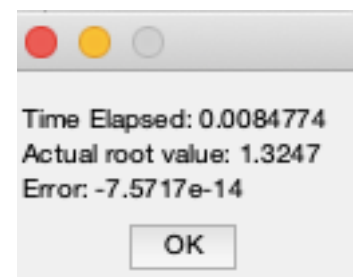


Step	X(i-1)	X(i)	F(X(i-1))	F(X(i))	X(i+1)	Error%
1	-1	-3	-6.1	53.3	-1.205387205	-148.8826816
2	-3	-1.205387205	53.3	-8.872216564	-1.461485415	-17.52314509
3	-1.205387205	-1.461485415	-8.872216564	-12.01165338	-0.4816399503	-203.4394083
4	-1.461485415	-0.4816399503	-12.01165338	-0.4289832965	-0.4453497568	-8.148695009
5	-0.4816399503	-0.4453497568	-0.4289832965	-0.1535174205	-0.4251251941	-4.757319253
6	-0.4453497568	-0.4251251941	-0.1535174205	-0.007531149433	-0.4240818480	-0.2460246971
7	-0.4251251941	-0.4240818480	-0.007531149433	-1.452235312e-4	-0.4240613336	-0.004837620113
8	-0.4240818480	-0.4240613336	-1.452235312e-4	-1.423944722e-7	-0.4240613134	-4.748035557e-6

$$2) f(x) = x^3 - x - 1$$

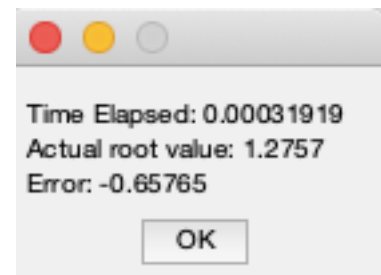
Interval $[1, 2]$

Tolerance: 10^{-4}



Step	X(i-1)	X(i)	F(X(i-1))	F(X(i))	X(i+1)	Error%
1	1	2	-1	5	1.166666667	71.42857143
2	2	1.166666667	5	-0.5787037037	1.253112033	6.898454746
3	1.166666667	1.253112033	-0.5787037037	-0.2853630296	1.337206446	6.288812988
4	1.253112033	1.337206446	-0.2853630296	0.05388058668	1.323850096	1.008901951
5	1.337206446	1.323850096	0.05388058668	-0.003698115440	1.324707937	0.06475692647
6	1.323850096	1.324707937	-0.003698115440	-4.273426281e-5	1.324717965	7.570533496e-4
7	1.324707937	1.324717965	-4.273426281e-5	3.458221465e-8	1.324717957	6.121414230e-7

3) $f(x) = x^5 - 5x + 3$
Interval [0.5,1.5]
Tolerance : 10^{-3}



Step	X(i-1)	X(i)	F(X(i-1))	F(X(i))	X(i+1)	Error%
1	0.5	1.5	0.53125	3.09375	0.2926829268	412.5
2	1.5	0.2926829268	3.09375	1.538733132	-0.9019914327	-132.4485263
3	0.2926829268	-0.9019914327	1.538733132	6.912905294	0.6347421246	242.1036036
4	-0.9019914327	0.6347421246	6.912905294	-0.07067518890	0.6191900830	2.511674858
5	0.6347421246	0.6191900830	-0.07067518890	-0.004933950793	0.6180228855	0.1888599141
6	0.6191900830	0.6180228855	-0.004933950793	4.741677498e-5	0.6180339959	0.001797692490
7	0.6180228855	0.6180339959	4.741677498e-5	-3.037805918e-8	0.6180339887	1.150973372e-6

Data Structures:

1- B=ArrayFun(func,A)

applies the function `func` to the elements of `A`, one element at a time. `arrayfun` then concatenates the outputs from `func` into the output array `B`, so that for the i th element of `A`, $B(i) = func(A(i))$. The input argument `func` is a function handle to a function that takes one input argument and returns a scalar. The output from `func` can have any data type, so long as objects of that type can be concatenated. The arrays `A` and `B` have the same size.

2-HPF decimal class

Very often I see people asking for a tool that offers more than 16 digits or so of accuracy

Good practices of numerical analysis are worth far more than any high precision tool. Even so, there are times when you will have a use for a bit of extra precision. And some of you will just want to play in the huge number sandbox. While some of you may use tools like that written by Ben Barrowes, HPF is written purely in MATLAB, so no compiles are needed. For all of you, whatever your reasons, I offer HPF, a High Precision Floating point tool.

3-Tic

`tic` works with the `toc` function to measure elapsed time. The `tic` function records the current time, and the `toc` function uses the recorded value to calculate the elapsed time.

`timerVal = tic` stores the current time in `timerVal` so that you can pass it explicitly to the `toc` function. Passing this value is useful when there are multiple calls to `tic` to time different parts of the same code. `timerVal` is an integer that has meaning only for the `toc` function.

4-Toc

`toc` reads the elapsed time since the stopwatch timer started by the call to the `tic` function. MATLAB® reads the internal time at the execution of the `toc` function and displays the elapsed time since the most recent call to the `tic` function without an output. The elapsed time is expressed in seconds.

`toc(timerVal)` displays the elapsed time since the call to the `tic` function corresponding to `timerVal`.

Functions used:

`Feval` : evaluates a function using its name or its handle, and using the input arguments x_1, \dots, x_M .

`fzero`: Root of nonlinear function

Gui functions:

`uifigure`: Create figure for designing apps

`uitable`: Create table user interface component

`msgbox`: Create message dialog box

`inputdlg`: Create dialog box to gather user input

`uiwait`: Block program execution and wait to resume

`num2str`: Convert numbers to character array

`str2double`: Convert strings to double precision values