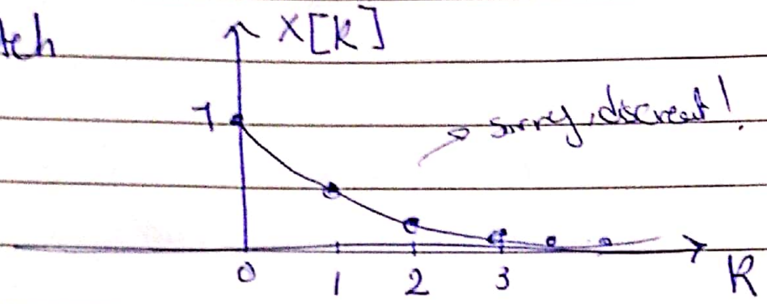
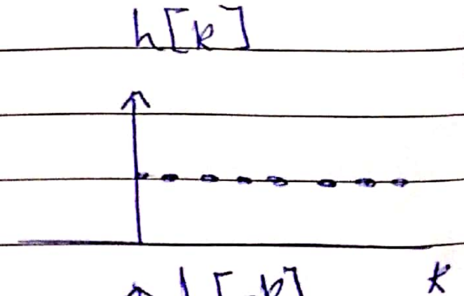
$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

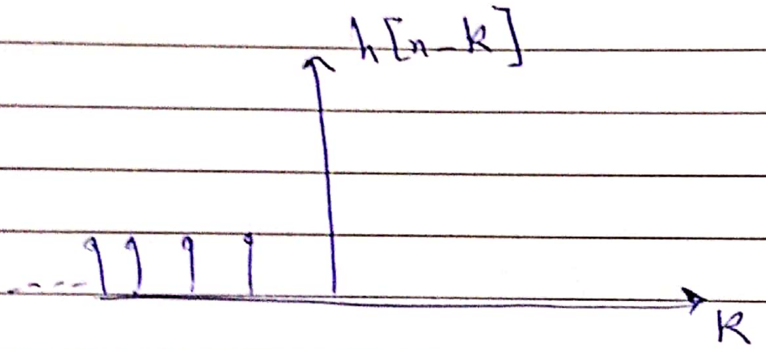
$x[n] = \alpha^n u[n]$ $\quad 0 < \alpha < 1$

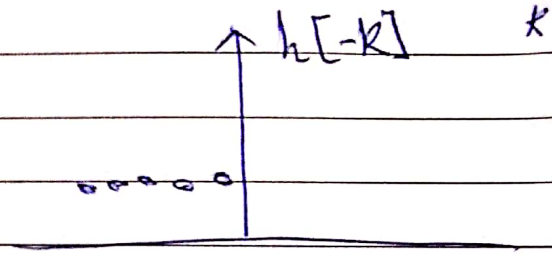
$h[n] = u[n]$ $\quad$ Find $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

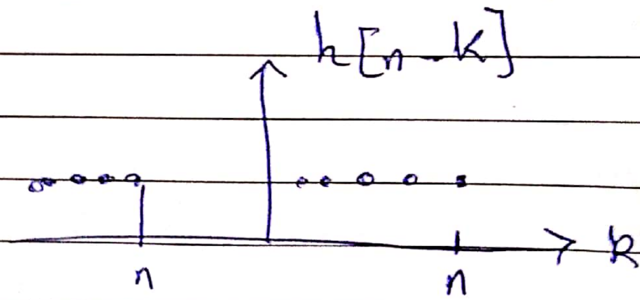
1) sketch



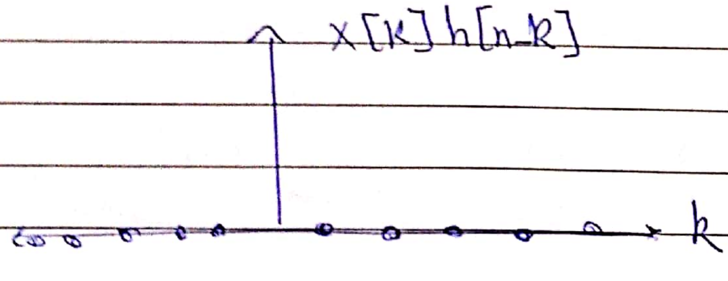




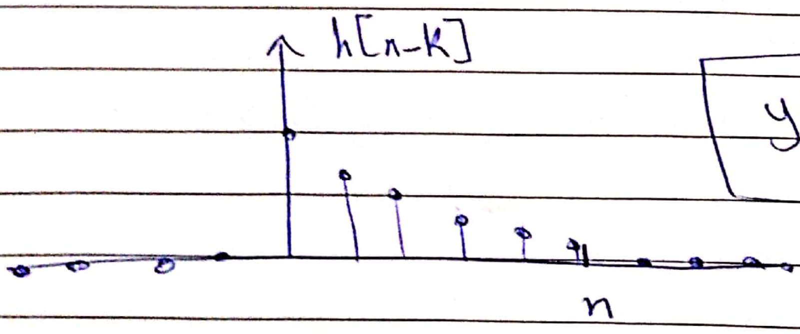




$n<0$



$y[n]$ = zero + zero + zero + ...etc = zero



$$\boxed{y[n] = \sum_{k=0}^{n} \alpha^k \qquad n \geq 0}$$

Properties of LTI systems

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) \circledast h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Conuatative

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

order doesn't matter..

$$y(t) = x(t) \circledast h(t) = h(t) * x(t)$$

$$h(t) \longrightarrow \boxed{x(t)} \longrightarrow y(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

Distribtive

$$x(t) \longrightarrow \begin{array}{c} \boxed{h_1(t)} \\ \\ \boxed{h_2(t)} \end{array} \longrightarrow y(t)$$

$$=$$

$$x(t) \longrightarrow \boxed{h_1(t) + h_2(t)} \longrightarrow y(t)$$

$$y(t) = x(t) \circledast h_1(t) + x(t)\, h_2(t) = x(t) * [h_1(t) + h_2(t)]$$

Associativity

$$x(t) \rightarrow \boxed{h_1(t)} \rightarrow \boxed{h_2(t)} \rightarrow y(t)$$

$$x(t) \rightarrow \boxed{h_1(t) \circledast h_2(t)} \rightarrow y(t)$$

$$y(t) = \big(x(t) \circledast h_1(t)\big) \circledast h_2(t)$$

$$y(t) = x(t) \circledast [h_1(t) \circledast h_2(t)]$$

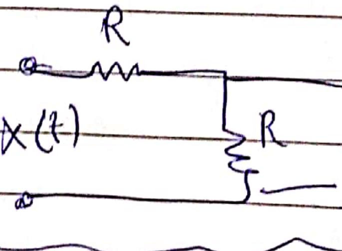
Memory

You need memory, you need all past & future values of the input $\int_{-\infty}^{\infty}$

- there is one exception ~~when~~ without memory

$$h(t) = k\,\delta(t) \Rightarrow y(t) = \int_{-\infty}^{\infty} k(\tau)\,\delta(t-\tau)\,d\tau$$

$$= k\,x(t) \quad \text{"amplifier"}$$



$y(t) = 0.5\,x(t)$

Causality

LTI systems are generally non causal as we depend on future values.

$$y(t) = \int_{-\infty}^{t} x(\tau)\,h(t-\tau)\,d\tau \quad \text{(causal)}$$

exceptions: $h(t-\tau) = 0$ when $\boxed{\tau > t}$

$h(-ve) = 0 \rightarrow u(t)$

if $h(t) = \square\, u(t)$

$u(t-\tau) = 0$

$\tau > t$

Invertibility

$x(t) \rightarrow \boxed{h(t)} \xrightarrow{y(t)} \boxed{h_2(t)} \rightarrow x(t)$

$y(t) = x(t) \circledast h(t)$ $\qquad$ $x(t) = y(t) * h_2(t)$

$$\boxed{x(t) = x(t) \circledast (h(t) \circledast h_2(t))} \rightarrow = \delta(t)$$

$$x(t) = x(t) \circledast \delta(t) \rightarrow x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

The Condition is $h_1(t) \circledast h_2(t) = \delta(t)$
(The only way to become invertable)

Stability

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

$y(t) = x(t) \circledast h(t)$

$x(t) \leq B$
when $x(t)$ is Bounded
when
$|y(t)| < \infty$

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(t-\tau)\,h(\tau)\,d\tau \right|$$

$$|y(t)| \leq \int_{-\infty}^{\infty} |x(t-\tau)\,h(\tau)|\,d\tau$$

$$|y(t)| \leq \int_{-\infty}^{\infty} B\,|h(\tau)|\,d\tau$$

maximum value that
X can take.

$$\boxed{\text{if } \int_{-\infty}^{\infty} |h(\tau)|\,d\tau = \infty}$$

then the System will be unstable
even if the input is finite

$$\boxed{\text{stable } \int_{-\infty}^{\infty} |h(\tau)|\,d\tau < \infty}$$