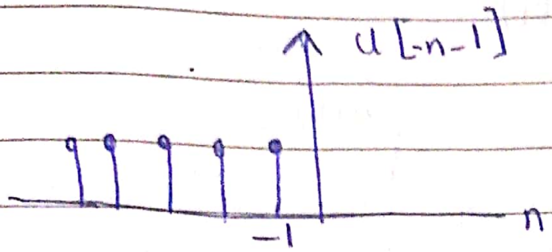
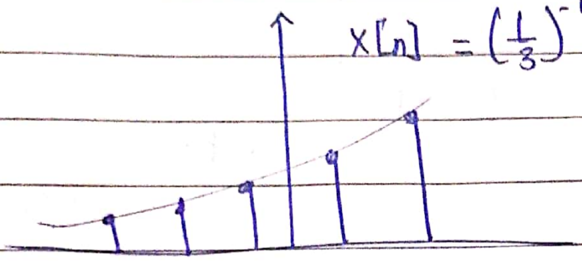
Question (1)

$X[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$





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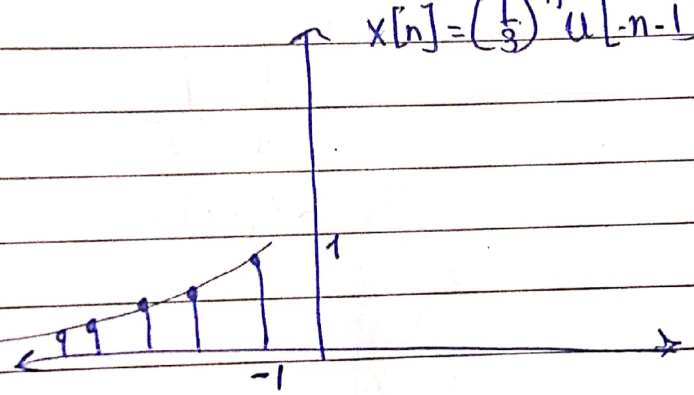
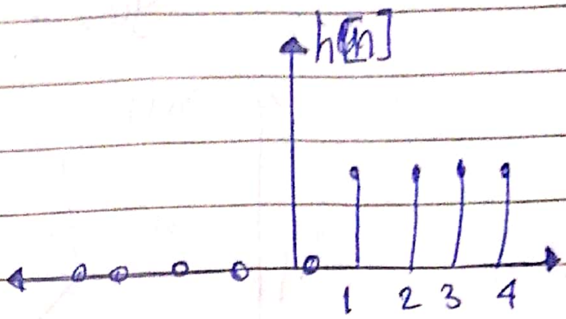


$h[n] = u[n-1]$



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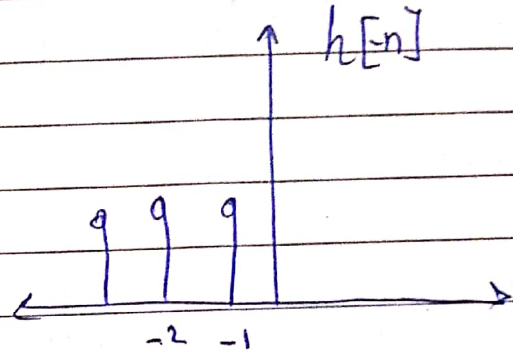


$$y[n] = \sum_{k=-\infty}^{\infty} x[n]\, h[n-k]$$

if $n-1 \leq -1 \longrightarrow n \leq 0 \longrightarrow y[n] = 0$

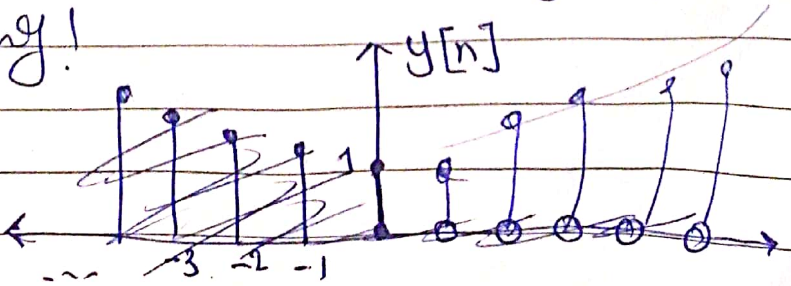
$n-1 = +1 \longrightarrow n = 2 \longrightarrow y[2] = \left(\frac{1}{3}\right)^0 = 1$

$n-1 = +2 \longrightarrow n = 3 \longrightarrow y[3] = \left(\frac{1}{3}\right)^0 + \left(\frac{1}{3}\right)^1 = 1 + \frac{1}{3} = 4/3$

$n-1 = +3 \longrightarrow n = 4 \longrightarrow y[4] = 4/3 + \left(\frac{1}{3}\right)^2 = 13/9$

$n-1 = +4 \longrightarrow n = 5 \longrightarrow y[5] = 13/9 + \left(\frac{1}{3}\right)^3 = 40/27$

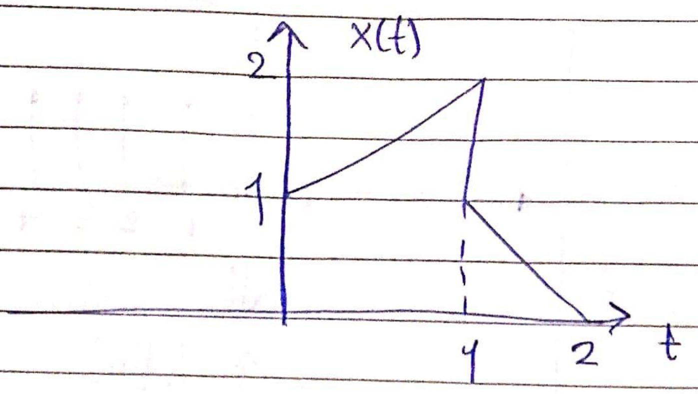
increasing!

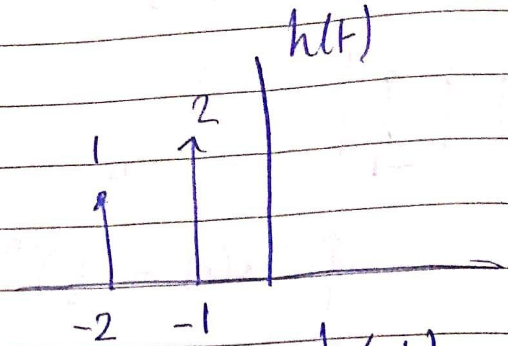


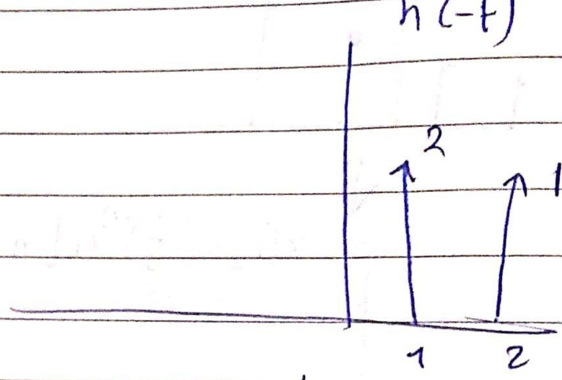
$\boxed{\text{Question (2)}}$

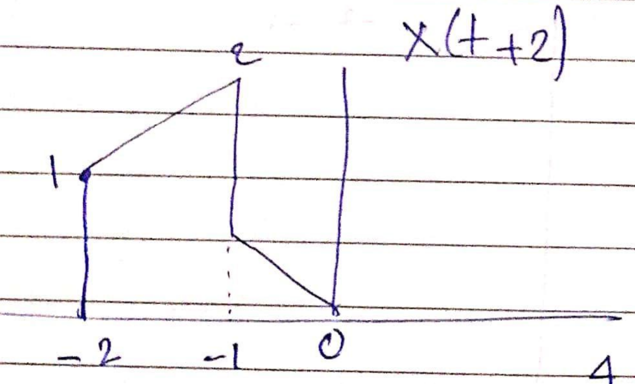
$$h(t) = \delta(t+2) + 2\delta(t+1)$$

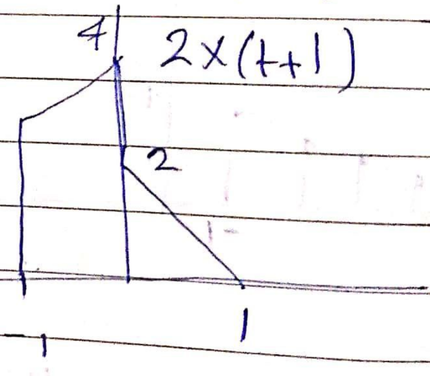
$$y(t) = x(t+2) + 2x(t+1)$$

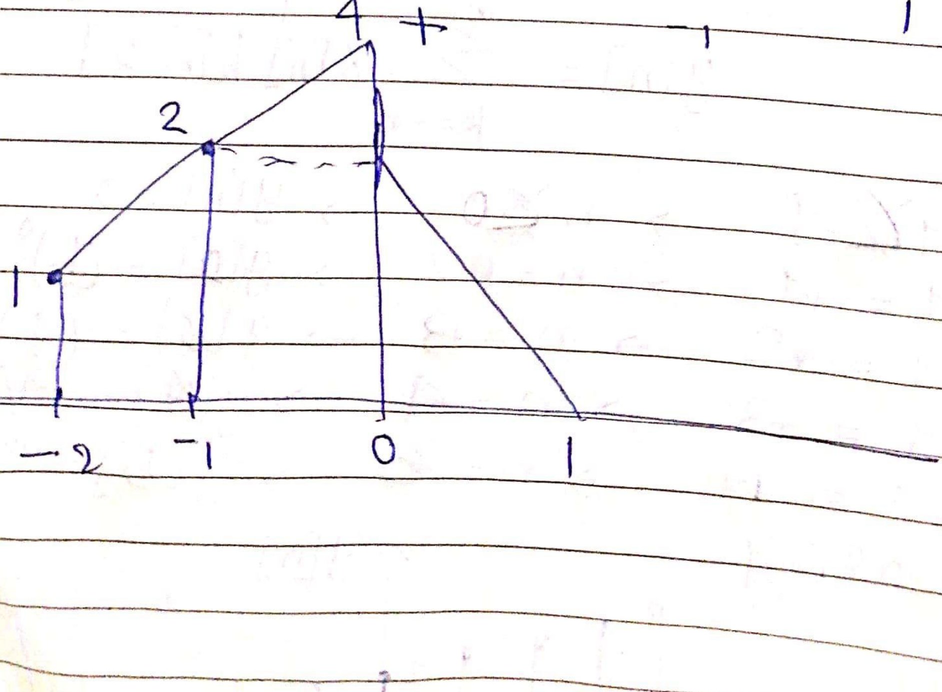












Question (3)

Stable if $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$, Causal $h(t) = 0 \quad t < 0$

[1] $h(t) = e^{-(1-2j)t}\, u(t) = \begin{cases} e^{-(1-2j)t} & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$\int_{0}^{\infty} e^{-(1-2j)t}\, dt = \frac{1}{-(1-2j)}\, e^{-(1-2j)t} \Big]_{0}^{\infty} = \left[0 + \frac{1}{1-2j}\right] < \infty$$

$\boxed{\text{Stable}}$ & $\boxed{\text{Causal}}$

[2] $h(t) = e^{-t} \cos(2t)\, u(t) = \begin{cases} e^{-t}\cos(2t) & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$\int_{0}^{\infty} e^{-t} \cos(2t)\, dt = \infty$$

$\boxed{\text{not stable}}$ & $\boxed{\text{Causal}}$

I	D
e^{t}	$\cos(2t)$
e^{t}	$-2\sin(2t)$

[3] $h[n] = n \cos\left(\frac{\pi}{4} n\right) u[n] = \begin{cases} n\cos\left(\frac{\pi}{4}n\right) & n \geq 0 \\ 0 & n < 0 \end{cases}$

$\boxed{\text{Causal}}$

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty = \sum_{-\infty}^{\infty} n \cos\left(\frac{\pi}{4} n\right) = \infty$$

$\boxed{\text{not Stable}}$

[4] $h[n] = 3^{n}\, u[-n+10] = \begin{cases} 3^{n} & n \leq 10 \\ 0 & n > 10 \end{cases}$

$\boxed{\text{non Causal}}$

$$\sum_{n=-\infty}^{\infty} 3^{n} = \infty$$ $\boxed{\text{non Stable}}$

Question (4)

① $X[n] = \alpha^n u[n] \qquad h[n] = B^n u[n]$

$$y[n] = \sum_{K=-\infty}^{\infty} \alpha^K u[K] * B^{n-K} u[n-K]$$

$$= \sum_{K=0}^{n} \alpha^K * B^n * B^{-K}$$

$$= B^n \sum_{K=0}^{n} \left(\frac{\alpha}{B}\right)^K \qquad n \geq 0$$

$$= B^n \left(\frac{1 - \left(\frac{\alpha}{B}\right)^{n+1}}{1 - \left(\frac{\alpha}{B}\right)} \right) u[n]$$

② $X[n] = \alpha^n u[n] \qquad h[n] = \alpha^n u[n]$

$$y[n] = \sum_{K=0}^{n} \alpha^n * \alpha^{n-K}$$

$$= \alpha^{2n} \sum_{K=0}^{n} \alpha^{-K} = \alpha^{2n} \sum_{K=0}^{n} \left(\frac{1}{\alpha}\right)^K$$

$$= \alpha^{2n} * \frac{1}{1 - \frac{1}{\alpha}}$$

$$\frac{\alpha^{2n} * \alpha}{\alpha - 1} = \frac{\alpha^{2n+1}}{\alpha - 1}$$

$$y[n] = \alpha^n \left[\sum_{K=0}^{n} (1)^K \right] u[n] = (n+1)\alpha^n u[n]$$

Question(5)

$\boxed{2}$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{0}^{2} h(t-\tau)\, d\tau - \int_{2}^{5} h(t-\tau)\, d\tau$$

$$y(t) = \begin{cases} \int_{0}^{2} e^{2(t-\tau)} d\tau - \int_{2}^{5} e^{2(t-\tau)} d\tau & t \leq 1 \\ \int_{t-1}^{2} e^{2(t-\tau)} d\tau - \int_{2}^{5} e^{2(t-\tau)} d\tau & 1 \leq t \leq 3 \\ \int_{t-1}^{5} e^{2(t-\tau)} d\tau & 3 \leq t \leq 6 \\ 0 & 6 \leq t \end{cases}$$

$$\Downarrow$$

$$y(t) = \begin{cases} \frac{1}{2}\left[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}\right] & t \leq 1 \\ \frac{1}{2}\left[e^{2} + e^{2(t-5)} - 2e^{2(t-2)}\right] & 1 \leq t \leq 3 \\ \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right] & 3 \leq t \leq 6 \\ 0 & 6 < t \end{cases}$$

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

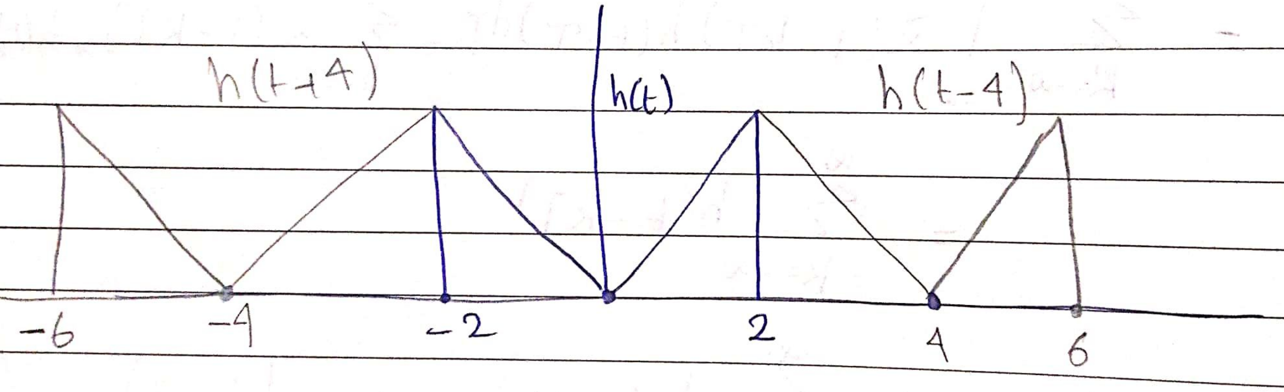
$$y(t) = \left(\sum_{k=-\infty}^{\infty} \delta(t-kT)\right) * h(t) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(\tau-kT)\, h(t-\tau)\, d\tau$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\tau-kT)\, h(t-\tau)\, d\tau = \sum_{k=-\infty}^{\infty} \delta(t-kT) * h(t)$$

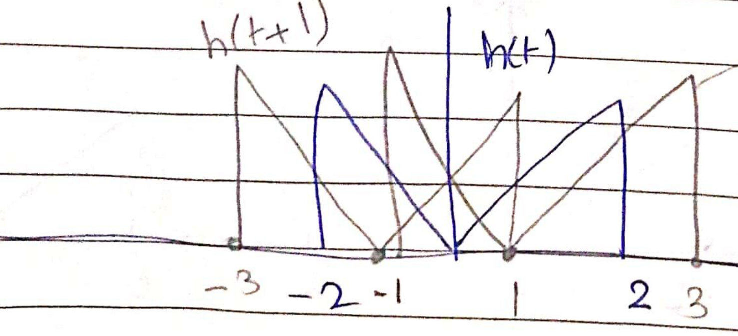
$$= \sum_{k=-\infty}^{\infty} h(t-kT)$$

[a] when $\boxed{T=4}$

$$y(t) = \sum_{k=-\infty}^{\infty} h(t-4k) = \dots + h(t+4) + h(t) + h(t-4)$$

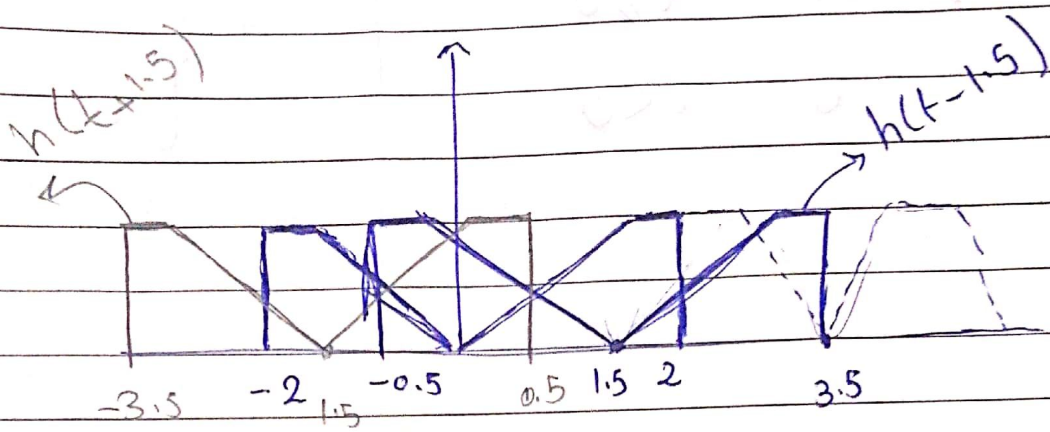


[b] when $\boxed{T=1}$ $\quad y(t) = \sum_{k=-\infty}^{\infty} h(t-k) = \dots h(t+1) + h(t) + h(t-1)$



$\boxed{c}$ $T = 3/2$ $\quad y(t) = \sum_{k=-\infty}^{\infty} h(t - 1.5k)$

$= \dots + h(t+1.5) + h(t)$
$+ h(t-1.5)$



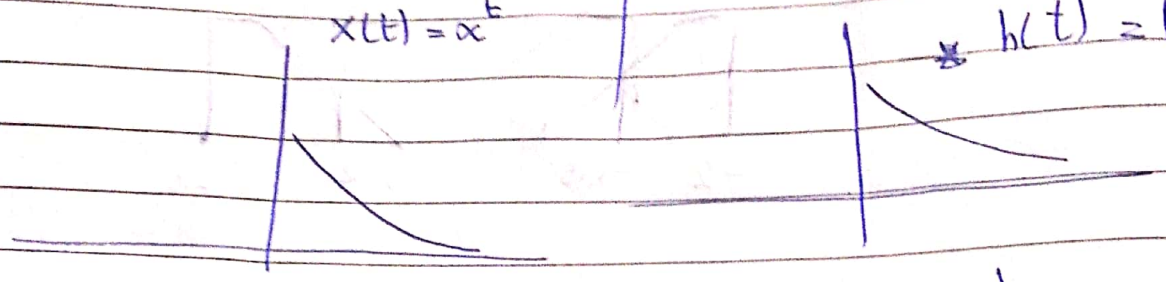
Q5

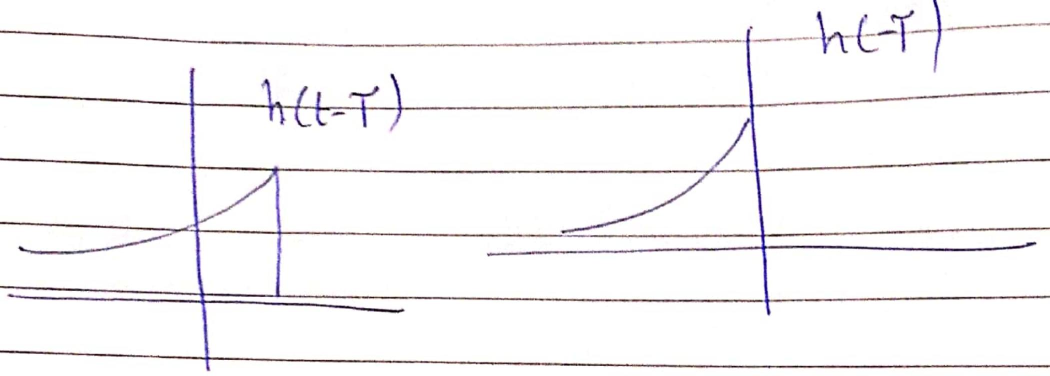
(a) $x(t) = \alpha^t u(t)$

$$= \begin{cases} \alpha^t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$h(t) = \beta^t u(t)$

$$= \begin{cases} \beta^t & t \geq 0 \\ 0 & t < 0 \end{cases}$$





$$y(t) = \int_0^t \beta^{t-\tau} \alpha^\tau \, d\tau = \beta^t \int_0^t \left(\frac{\alpha}{\beta}\right)^\tau d\tau = \cancel{\beta^t}$$

$$= \beta^t \left[\frac{\left(\frac{\alpha}{\beta}\right)^\tau}{\ln\left(\frac{\alpha}{\beta}\right)}\right]_0^t = \beta^t \left(\frac{\left(\frac{\alpha}{\beta}\right)^t}{\ln\left(\frac{\alpha}{\beta}\right)} - \frac{1}{\ln\left(\frac{\alpha}{\beta}\right)}\right)$$

if $\alpha = \beta$

$$y(t) = \int_0^t \alpha^\tau \alpha^{t-\tau} \, d\tau$$

$$= a^t \int_0^t d\tau = t a^t$$