DATE: SUBJECT:

- $x(t) \longrightarrow [\,TI\,] \longrightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$ "The Performance doesn't Change with time"

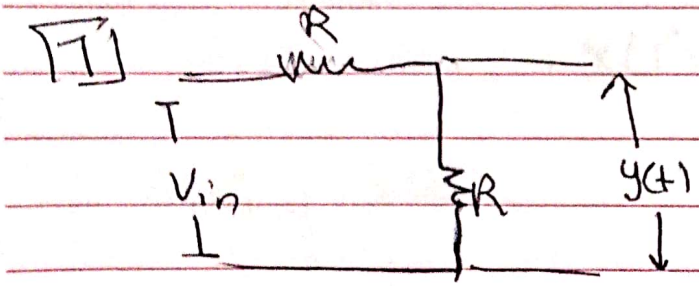
- $y(t) = \frac{dx(t)}{dt} \Rightarrow \lim_{\Delta \to 0} \frac{x(t) - x(t+\Delta)}{\Delta}$

needs memory

Linear time invariant system (LTI)

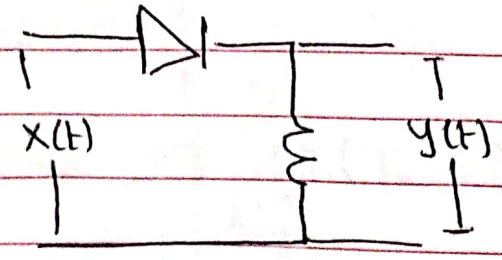
you can analyse the system without knowing the inside

[TI]



Knowing the Circuit!

$$y(t) = \frac{R}{R+R} x(t) = \frac{1}{2} x(t)$$



$$y(t) = \begin{cases} x(t) & x(t) \geq 0 \\ 0 & x(t) < 0 \end{cases}$$

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* How can you tell the relation without knowing the inside?

1) Apply input ($\delta(t)$), $y(t)$ = impulse response

$\delta(t) \longrightarrow h(t)$

$A\delta(t-\Delta) \longrightarrow Ah(t-\Delta)$

$\delta(t) \longrightarrow$ [LTI] $\longrightarrow h(t)$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = \lim_{\Delta T \to 0} \sum_{n=-\infty}^{\infty} x(\tau_n)\,\delta(t-\tau_n)\,\Delta\tau$$

$$= x(\tau)$$

$\delta(t)$	$h(t)$
$\delta(t-\tau_n)$	$h(t-\tau_n)$
$x(\tau_n) * \delta(t-\tau_n)$	$x(\tau_n) \cdot h(t-\tau_n)$
$\lim \sum_{-\infty}^{\infty} x(\tau_n)\delta(t-\tau_n)\Delta\tau$	$\lim \sum_{-\infty}^{\infty} x(\tau_n)\delta(t-\tau_n)\,\Delta\tau$
$x(t)$	$y(t)$

LTI system equation $y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$

"Convolution Integral"

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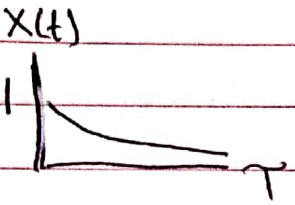
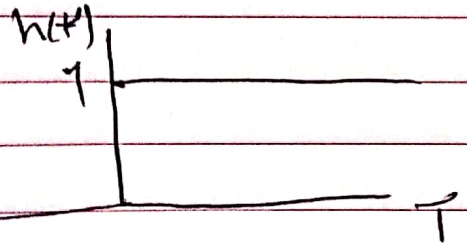
ex: $x(t) = e^{-\alpha t} u(t)$, $h(t) = u(t)$

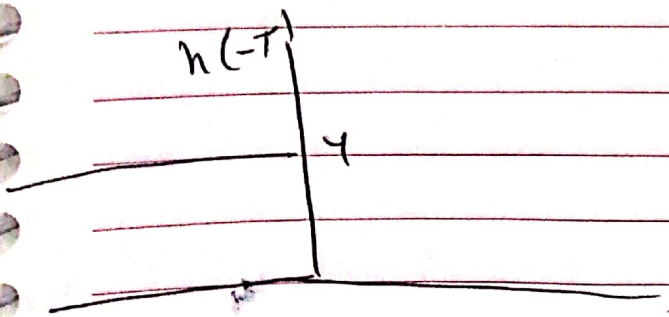
$$y(t) = \int_{-\infty}^{\infty} e^{-\alpha\tau} u(\tau)\, u(t-\tau)\, d\tau$$

How to do Convolution integral?

1) Sketch 2) Multiply 3) Do integration

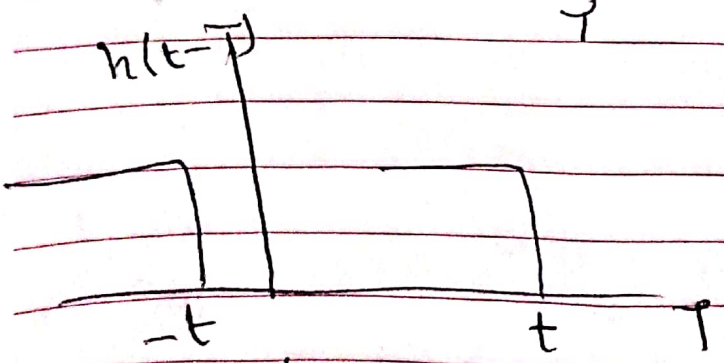






$u(t+\tau)$

t	$u(t)$
≥ 0	1
< 0	0



τ	$t-\tau$	$u(t-\tau)$
$\tau \leq t$	≥ 0	1
$\tau > t$	< 0	0

depend on t

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2)

$$y \begin{cases} X(\tau)h(t-\tau) = \text{zero} & \text{when } t<0 \\ \int_0^t e^{-\alpha\tau}\,d\tau = \frac{1}{\alpha}e^{-\alpha\tau}\Big]_0^t = \frac{1}{\alpha}[1-e^{-\alpha t}] & t \geq 0 \end{cases}$$