

Question (1)

$$[1] \quad X(t) = e^{-2t} u(t)$$

$$X(t) = \begin{cases} e^{-2t} & \text{when } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$E = \lim_{T \rightarrow \infty} \int_0^T e^{-4t} dt = \left[\frac{1}{-4} e^{-4t} \right]_0^T$$

$$= \lim_{T \rightarrow \infty} \left[-\frac{1}{4} e^{-4T} + \frac{1}{4} \right]$$

$$= 0 + \frac{1}{4} = \left[\frac{1}{4} \right]$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[-\frac{1}{4} e^{-4T} + \frac{1}{4} \right]$$

$$= \lim_{T \rightarrow \infty} \frac{-\frac{1}{4} e^{-4T}}{2T} + \frac{1}{8T} = \frac{0}{\infty} + \frac{1}{\infty} =$$

Zero

$$X(t) = \sin(t)$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T \sin^2(t) dt$$

$$= \lim_{T \rightarrow \infty} \int_{-T}^T \left[\frac{1}{2} - \frac{1}{2} \cos(2t) \right] dt$$

$$= \lim_{T \rightarrow \infty} \left[\frac{1}{2}t - \frac{1}{4} \sin(2t) \right]_{-T}^T$$

$$= \lim_{T \rightarrow \infty} \left[\frac{1}{2}T - \frac{1}{4} \sin(2T) - \left(-\frac{1}{2}T + \frac{1}{4} \sin(2T) \right) \right]$$

$$= \lim_{T \rightarrow \infty} \left[T - \frac{1}{2} \sin(2T) \right]$$

$$= \boxed{\infty}$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T X^2(t) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \times \left[T - \frac{1}{2} \sin(2T) \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2} - \frac{1}{2} \frac{\sin(2T)}{T}$$

$$= \frac{1}{2} - \frac{1}{2} \times \frac{2}{1} = \boxed{-\frac{1}{2}}$$

Question (2)

$$X[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$X[n] = \begin{cases} \left(\frac{1}{2}\right)^n & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$E = \lim_{N \rightarrow \infty} \sum_{n=-N}^N X^2[n] = \lim_{N \rightarrow \infty} \sum_{n=0}^N \left(\frac{1}{2}\right)^{2N} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

Geometric series $= \frac{1}{1-r} = \frac{1}{1-\frac{1}{4}}$

$$= \left[\frac{4}{3} = 1.33 \right]$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N X^2[n] = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N \left(\frac{1}{4}\right)^n$$

$$= \frac{1}{\infty} * \frac{4}{3} = \boxed{\text{Zero}}$$

Question (2)

$$x[n] = \cos\left[\frac{\pi}{4}n\right]$$

$$E = \lim_{N \rightarrow \infty} \sum_{n=-N}^N x^2[n] = \lim_{N \rightarrow \infty} \sum_{n=-N}^N \cos^2\left[\frac{\pi}{4}n\right]$$

$$= \infty$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N \frac{1}{2} + \frac{1}{2} \cos\left(\frac{\pi}{2}n\right) = \boxed{\frac{1}{2}}$$

$$\therefore -\frac{1}{2} \leq \frac{\cos\left(\frac{\pi}{2}n\right)}{2} \leq \frac{1}{2}$$

$$+\frac{1}{2} - \frac{1}{2} \leq \frac{1}{2} + \frac{1}{2} \cos\left(\frac{\pi}{2}n\right) \leq \frac{1}{2} + \frac{1}{2}$$

$$0 \leq \frac{1}{2} + \frac{1}{2} \cos\left(\frac{\pi}{2}n\right) \leq 1$$

Question (3)

$$[4] \quad y(t) = x(t-2) + x(2-t)$$

- needs memory because of $x(t-2)$

- invertible as I can get the ~~out~~ input from the output again by doing some operation

$$y_1 = x(t-2) \quad \text{invertible}$$

$$y_2 = x(2-t) \quad \text{invertible}$$

Non

- Causal \rightarrow depends on past signal as well as future signals ^{ex:} when $y(0) = x(-2) + x(2) \rightarrow$ future

- Stable \rightarrow output finite when input is finite.

$$- \quad y(t) \xrightarrow{t_0} y(t-t_0) \rightarrow \underline{x(t-t_0-2) + x(2-t+t_0)}$$

$$- \quad x(t) \xrightarrow{t_0} x(t-t_0) \xrightarrow{\text{eye}} \underline{x(t-t_0-2) + x(2-t-t_0)}$$

time variant system

- it's a linear system

$$y_1(t) + y_2(t) = \underline{x_1(t-2) + x_1(2-t) + x_2(t-2) + x_2(2-t)}$$

$$x_1(t) + x_2(t) = \underline{x_1(t-2) + x_1(2-t) + x_2(t-2) + x_2(2-t)}$$

DATE: _____ SUBJECT: _____

$$x(t) \xrightarrow{\text{eye}} y(t) \xrightarrow{"K"} Ky(t) = K[() + ()]$$

$$x(t) \xrightarrow{"K"} Kx(t) \rightarrow K() + K() = K[() + ()]$$

The system follows the law of homogeneity.

it's a linear system as it follows the law of superposition.

$$y(t) = X\left(\frac{t}{3}\right)$$

- it needs memory as ex. $y(9) = X(\underline{3})$
Past

$$\begin{aligned} y(t) &\longrightarrow y(t - t_0) = X\left(\frac{t - t_0}{3}\right) \\ X(t) &\longrightarrow X\left(\frac{t}{3} - t_0\right) = X\left(\frac{t - 3t_0}{3}\right) \end{aligned}$$

The system is time variant

$$\begin{aligned} y(t) &\xrightarrow{"K"} Ky(t) = KX\left(\frac{t}{3}\right) \\ X(t) &\xrightarrow{"K"} KX\left(\frac{t}{3}\right) \end{aligned}$$

This system is linear.

non

- Causal as it depends on the past input and the future as well $y(-3) = X(-1) \rightarrow$ future

- stable \rightarrow Finite input \rightarrow Finite output

DATE: _____

SUBJECT: _____

$$\boxed{3} \quad y(t) = \frac{d}{dt} x(t)$$

- Memoryless

$$y(t-t_0) \rightarrow \frac{d}{dt} x(t-t_0)$$

$$x(t-t_0) \rightarrow \frac{d}{dt} x(t-t_0)$$

This is time variant

$$\begin{array}{lcl} y(t) & \xrightarrow{"K"} & Ky(t) = K \cdot \frac{d}{dt} x(t) \\ x(t) & \xrightarrow{"K"} & K \cdot \frac{d}{dt} x(t) \end{array}$$

This is a linear system

- Causal systems, the output depends on the current input

non stable \rightarrow ~~finite input~~ \rightarrow ~~finite output~~
it may be undefined for some functions at finite input

Question (4)

$$y[n] = x[n-2] - 2x[n-8]$$

- needs memory

$$\begin{aligned} y[n-n_0] &\rightarrow x[n-n_0-2] - 2x[n-n_0-8] \\ x[n] &\rightarrow x[n-n_0] \rightarrow x[n-n_0-2] - 2x[n-n_0-8] \end{aligned}$$

This system is Time invariant

$$\begin{aligned} y[n] &\rightarrow Ky[n] \rightarrow K[x[n-2] - 2x[n-8]] \\ x[n] &\rightarrow Kx[n] \rightarrow Kx[n-2] - 2Kx[n-8] \end{aligned}$$

Follows superposition \rightarrow linear!

- Causal, depends on previous "Past" input

- stable \rightarrow finite input \rightarrow finite output

DATE: _____

$$[2] \quad y[n] = n^2 x[n]$$

- Memoryless

$$\begin{array}{lcl} y[n] & \rightarrow & y[n-n_0] \rightarrow (n-n_0)^2 x[n-n_0] \\ x[n] & \rightarrow & x[n-n_0] \rightarrow n^2 x[n-n_0] \end{array}$$

Time Variant

$$\begin{array}{lcl} y[n] & \rightarrow & k y[n] \rightarrow k n^2 x[n] \\ x[n] & \rightarrow & k x[n] \rightarrow n^2 k x[n] \end{array}$$

linear

- Causal, depends on the current signal

- Stable \rightarrow finite input \rightarrow finite output

DATE: _____

SUBJECT: _____

$$y[n] = x[4n+3]$$

- needs memory

$$\begin{aligned} y[n-n_0] &\longrightarrow x[4n-4n_0+3] \\ x[n-n_0] &\longrightarrow x[4n-n_0+3] \end{aligned}$$

Time Variant

$$\begin{aligned} - K y[n] &= K x[4n+3] \\ K x[n] &\longrightarrow K x[4n+3] \end{aligned}$$

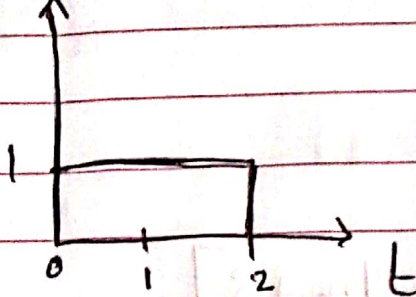
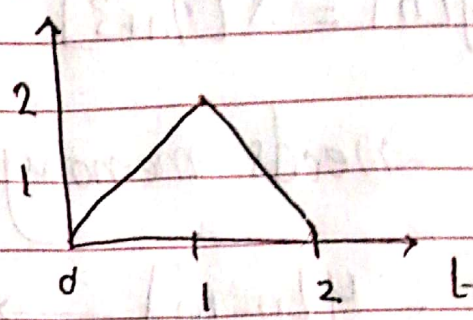
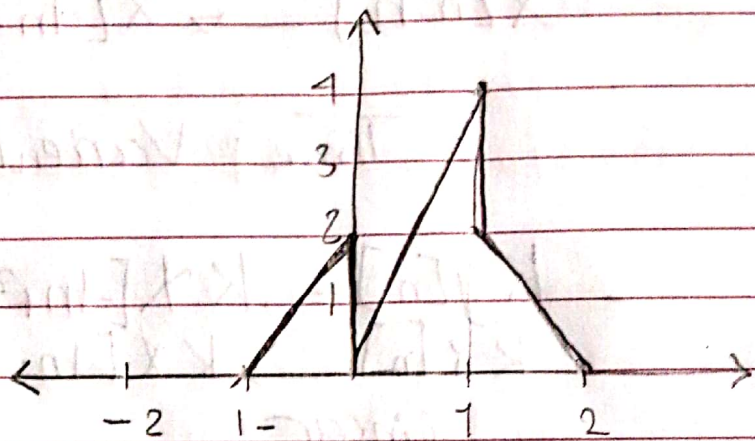
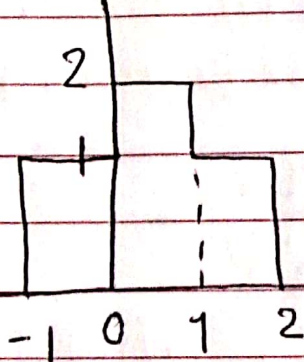
linear

- non Causal \longrightarrow depends on the future

- stable finite input \longrightarrow finite output.

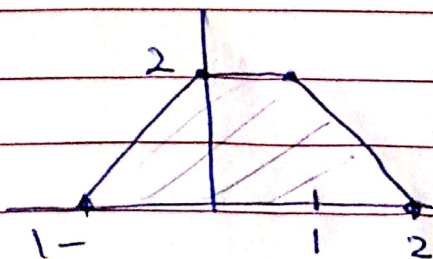
DATE: _____

SUBJECT: _____

 $x_1(t)$  $y_1(t)$  $x_2(t)$ 

another answer: if I miss understood the concept of linearity above (dealing with each part of the function as linear, not the whole function)

$$x_2(t) = x_1(t) + x_1(t+1)$$



$$y_2(t) = y_1(t) + y_1(t+1)$$

$$y_2(-1) = y_1(0) = 0$$

$$y_2(0) = y_1(0) + y_1(1) = 2$$

$$y_2(1) = y_1(1) + y_1(2) = 2$$

$$y_2(2) = y_1(2) + y_1(3) = 0$$