1 Nested Quantifiers

Let the domain be all real numbers:

$\forall x \exists y (x + y) = 0$
This is the same as:
$\forall x q(x)$ where q(x) is $\exists y p(x, y)$ and p(x,y) is x+y=0

$\forall x \forall y (x + y = y + x)$
$\forall x \forall y \forall z (x + (y + z) = (x + y) + z)$
$\forall x \forall y ((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$

2 Quantification as Loop

1. $\forall x \forall y p(x, y) \rightarrow$ For every x, for every y

   While looping through x, each x loops through y
   If p(x,y) is true for all x and y, then the statement is true.
   However, if x for p(x, y) is false, then the statement is false.

2. $\forall x \exists y p(x, y) \rightarrow$ For every x, y exists

   Loop through x until a y makes p(x,y) true.
   If for every x there is a y that makes p(x,y) true, then the statement is true.
   However, if some x's doesn't have a y that makes p(x,y) true, then the statement is false.

3. $\exists x \forall y p(x, y) \rightarrow$ There exists an x, for every y

   Loop through x until there is an x for which p(x,y) is always true when we loop through all values for y.
   It is true if there is an x that meets this criteria. False if there is no x.

4. $\exists x \exists y p(x, y) \rightarrow$ For an x that exists, y exists

   Loop through x for where each x, loop through y until there is an x where y makes p(x,y) is true.
   This is false if there is no x with a y that makes p(x,y) true.

3 Order of Quantification

Let p(x,y) be "x + y = y + x" and the domain is all real numbers.

Is the statement "$\forall x \forall y p(x, y) \equiv \forall y \forall x p(x, y)$" true?

$\forall x \forall y p(x, y) \rightarrow$ For all real numbers of x, for all real numbers of y, x + y = y + x.
Since p(x,y) is true for all numbers x and y, $\forall x \forall y p(x, y)$ is true.

$\forall y \forall x p(x, y) \rightarrow$ For all real numbers of y, for all real numbers of x, x + y = y + x.
This has the same meaning as the statement above, therefore the statements are equivalent.

4 Translating Mathematical Statements

The sum of two positive integers is always positive.

Translating this would be:

$\forall x \forall y((x > 0 \land y > 0) \rightarrow (x + y > 0))$ and the domain is all integers for both variables.
$\forall x \forall y(x + y > 0) \rightarrow$ For every x that is a positive integer, adding x to every y that is a positive integer will always be greater than 0.

5 Translating Statements into English

$$\forall x(c(x) \lor \exists y(c(y) \land f(x, y)))$$

1. c(x) = "x has a computer", f(x,y) = "x and y are friends", domain = all students at the school

2. For every x student in the school, x student has a computer or there exists y student with a computer and is friends with x student.

6 Negating Nested Quantifiers

$$
\begin{aligned}
&\neg\forall x\exists y(xy = 1)\\
&\equiv \exists x\neg\exists y(xy = 1)\\
&\equiv \exists x\forall y\neg(xy = 1)\\
&\equiv \exists x\forall y(xy \neq 1)
\end{aligned}
$$