



UNIVERSIDAD NACIONAL
DE SAN MARTÍN

Machine Learning - Week 3

Maestría en Ciencia con mención de Tecnología de la información

**Yonatan Carlos CARRANZA ALARCÓN,
Ph. D. in Machine Learning,
ycarranza.alarcon@gmail.com**

<https://salmuz.github.io/>

October 16, 2021

Overview

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

Overview

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

Supervised vs. Unsupervised learning [Patil, 2018].

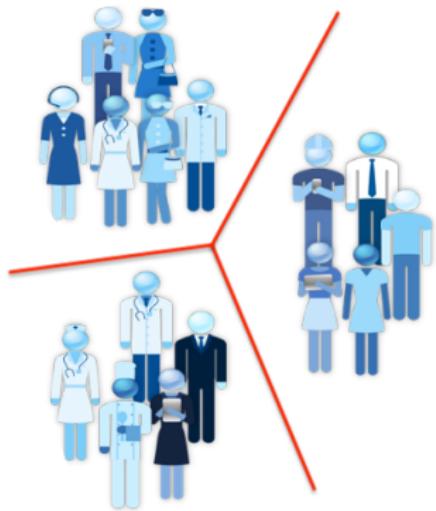


Figure: Supervised learning

- Labeled dataset $(\mathbf{x}_i, y_i)_{i=1}^N$.
 - Learn a decision boundary model.

Supervised vs. Unsupervised learning [Patil, 2018].

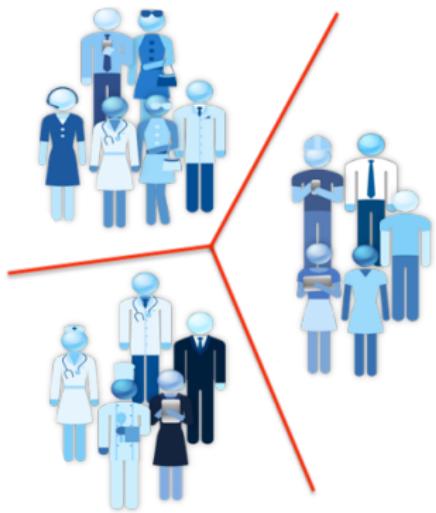


Figure: Supervised learning

- Labeled dataset $(\mathbf{x}_i, y_i)_{i=1}^N$
 - Learn a decision boundary model.



Figure: Unsupervised learning

- Unlabeled dataset $(\mathbf{x}_i)_{i=1}^N$.
 - Discover hidden patterns in dataset without human intervention.

Supervised vs. Unsupervised learning [Patil, 2018].

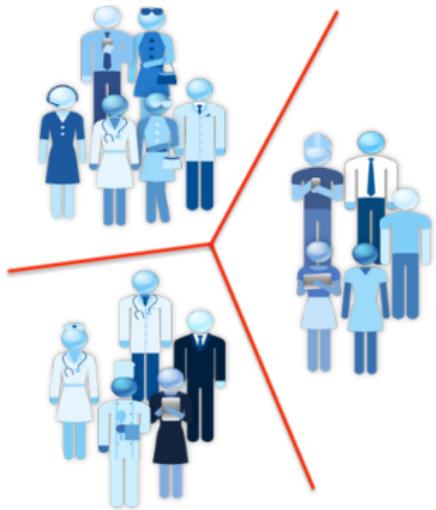


Figure: Supervised learning

- Labeled dataset $(\mathbf{x}_i, y_i)_{i=1}^N$
 - Learn a decision boundary model.

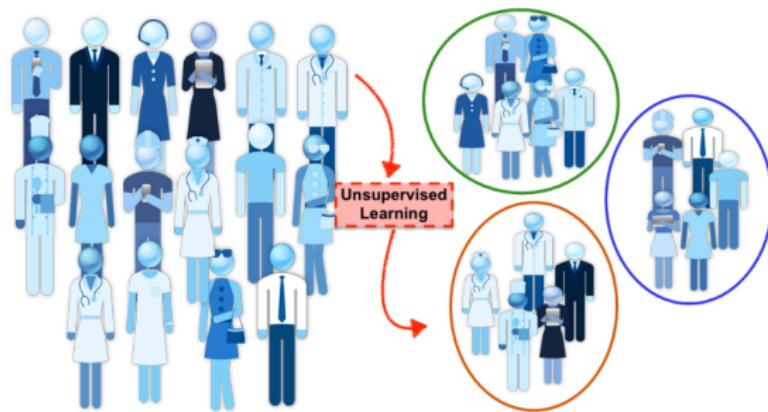


Figure: Unsupervised learning

- Unlabeled dataset $(\mathbf{x}_i)_{i=1}^N$.
 - Discover hidden patterns in dataset without human intervention.

Unsupervised learning - Motivation.

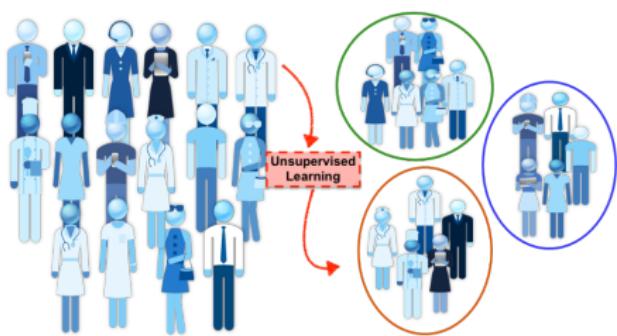


Figure: Unsupervised learning

- Unlabeled dataset $(x_i)_{i=1}^N$.
 - Discover hidden patterns in dataset without human intervention.

Why would you use unlabeled data?

- To label data is expensive in time and money (e.g. Biology).
 - Often labeled data is not available.
 - In BigData, it is difficult properly to label all data (e.g. Crowdsourcing).

Unsupervised learning.

Unsupervised learning is a type of machine learning in which the algorithm is **not provided** with any **pre-assigned labels** or **scores** for the training data [Hinton et al., 1999].

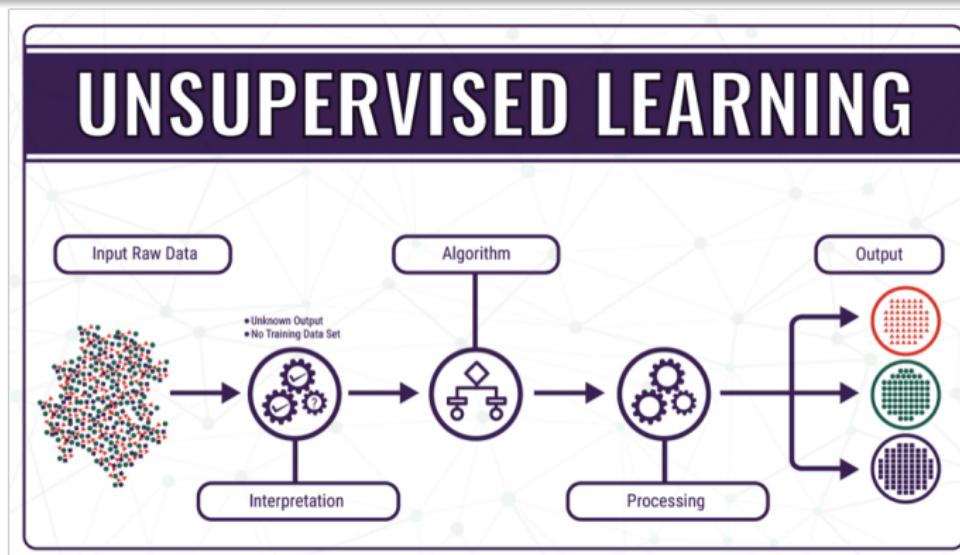


Figure: Outline of Unsupervised learning scheme

Unsupervised learning.

Objective

It may be to discover groups of similar examples within the data (i.e. clustering),

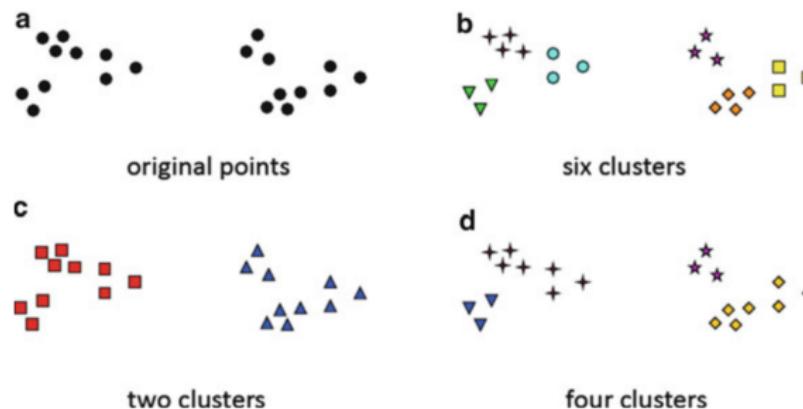


Figure: Different ways of clustering the same set of points [Dougherty, 2012].

Unsupervised learning.

Objective

It may be to **discover groups of similar examples** within the data (i.e. clustering),

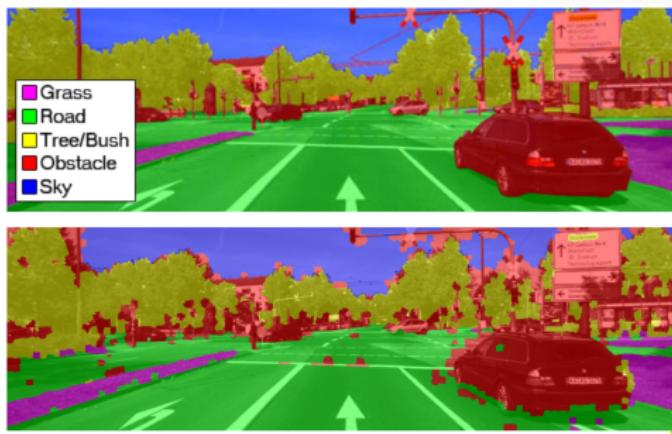


Figure: U-Net architecture for image segmentation Ronneberger et al. [2015].

Unsupervised learning.

Objective

It may be to **discover groups of similar examples** within the data (i.e. **clustering**), or to **determine the distribution of data** within the input space (i.e. **density estimation**),

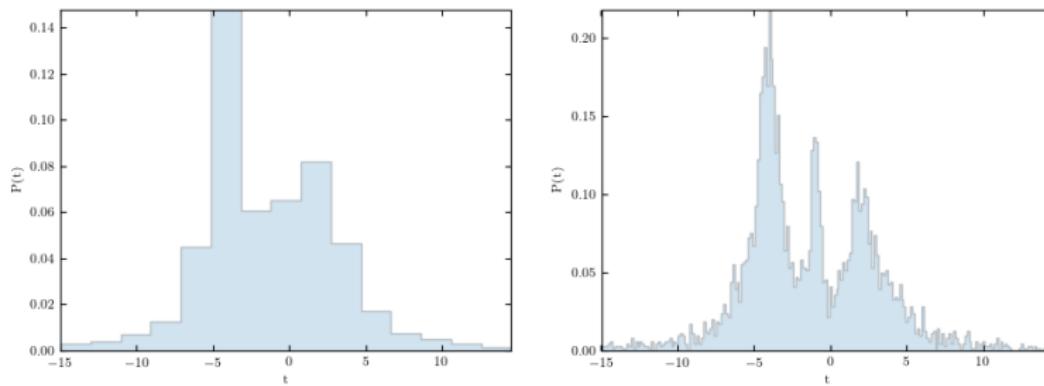


Figure: Density estimation from a raw data set.

Unsupervised learning.

Objective

It may be to **discover groups of similar examples** within the data (i.e. **clustering**), or to **determine the distribution of data** within the input space (i.e. **density estimation**), or to **project the data from a high-dimensional space** down to two or three dimensions for the purpose of **visualization**.

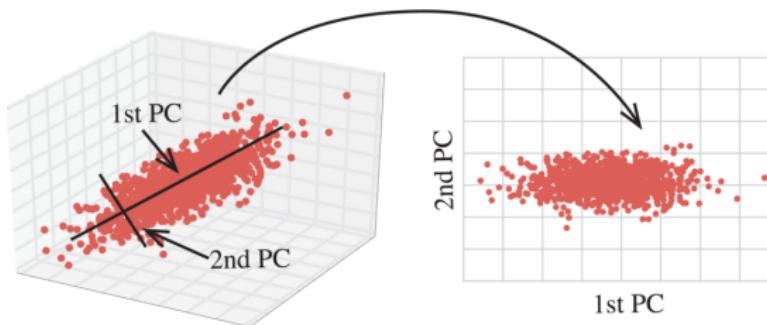


Figure: Principal Component Analysis for dimensionality reduction

Mathematical formulation

Let $\mathcal{D} = \{\mathbf{x}_i | i = 1, \dots, N\} \subseteq \mathcal{X}^p$ be a data set generated from an unknown joint probability distribution $\mathbb{P}_{\mathcal{X}}$

- A vector of p predictors \mathbf{x} (also called inputs, features, attributes, explanatory variables)

Objective

The goal is to build a function $\varphi : \mathcal{X} \rightarrow \mathcal{O}$ that finds or discovers hidden and interesting patterns (in an \mathcal{O} output space) in unlabeled data by minimizing or maximizing a specific criterion $\mathcal{C} : \mathcal{X}^{\otimes N} \rightarrow \mathbb{R}$.

For instances: Learning labeled from raw data.

There are several algorithmic, **statistical**, and **mathematical** methods!

Overview

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

Clustering

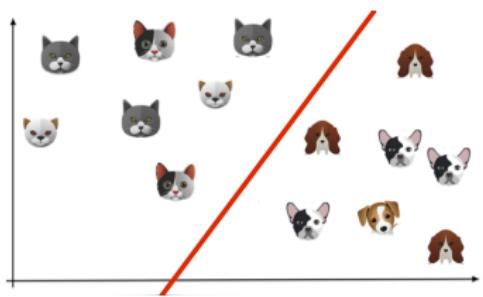


Figure: Labeled samples

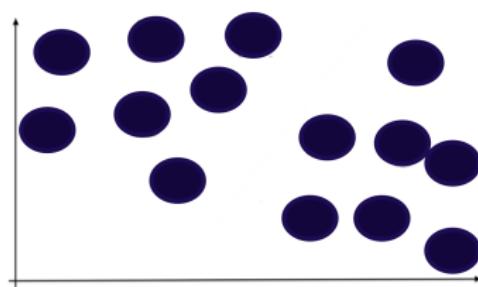


Figure: Unlabeled samples

- How can we discover hidden patterns ?
 - How can we measure the dissimilarity between two samples?

Clustering

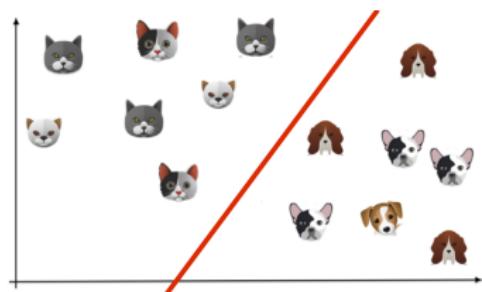


Figure: Labeled samples

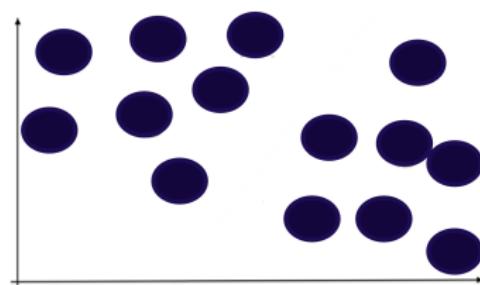


Figure: Unlabeled samples

- How can we discover hidden patterns ?
- How can we measure the dissimilarity between two samples?

Dissimilarity measure

It corresponds to the intuitive idea of a distance between two objects: the larger it is, the farther the objects are.

$$D(x_i, x_j) = \text{dist}(x_i, x_j), \quad i, j \in \{1, \dots, N\} \quad (1)$$

Clustering

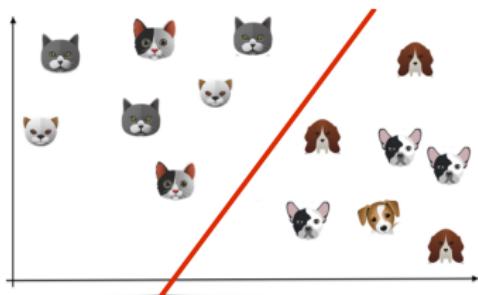


Figure: Labeled samples

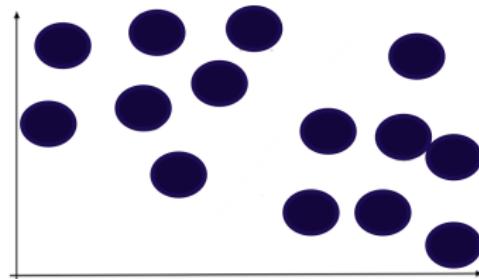


Figure: Unlabeled samples

$$D(x_i, x_j) = \text{dist}(x_i, x_j), \quad i, j \in \{1, \dots, N\}$$

- Which dissimilarity measure or topological space?
→ For practical purposes, we use an euclidian topological space!

Clustering

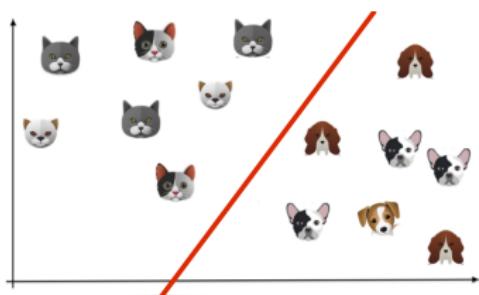


Figure: Labeled samples

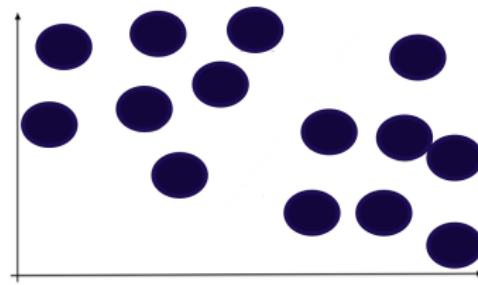


Figure: Unlabeled samples

- Which dissimilarity measure or topological space?
→ For practical purposes, we use an euclidian topological space!

Euclidean distance

It is the distance between the two points in n -dimensional Euclidean space:

$$D(\mathbf{x}_i, \mathbf{x}_j) = \|\mathbf{x}_i - \mathbf{x}_j\|^2, \quad i, j \in \{1, \dots, N\} \quad (1)$$

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Example of clustering in python

Dimensionality reduction

Other Unsupervised Learning methods

Bibliography

K-means clustering

The K-means algorithm is one of the most popular iterative descent clustering methods used often for variables of quantitative type [Friedman et al. \[2001\]](#).

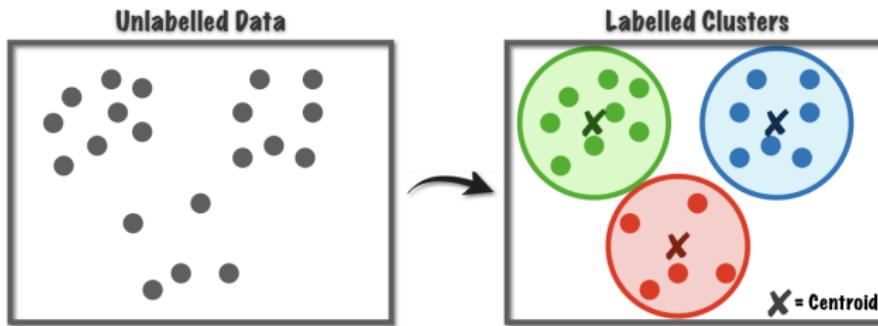
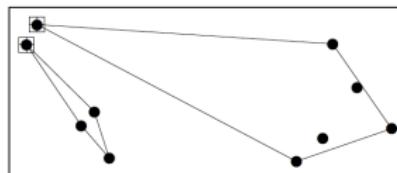


Figure: K-means clustering example

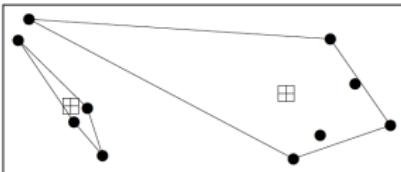
K-means clustering (Algorithm)



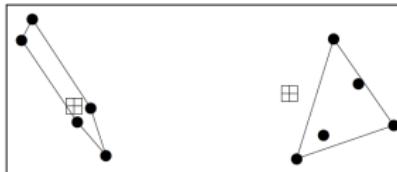
(a) Step 1: Choice 2 random points



(b) Step 2: Assignment of each point to the nearest center



(c) Step 3: Calculate the new centers of gravity.



(d) Step 4: Assignment of each point to the nearest center



(e) Step 5: Calculate the new centers of gravity.

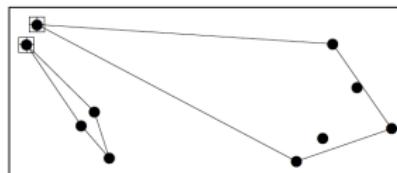


(f) Step 6: Assignment of each point to the nearest center

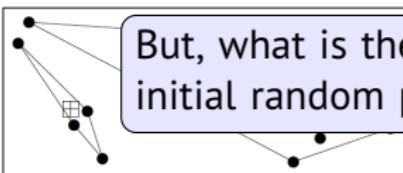
K-means clustering (Algorithm)



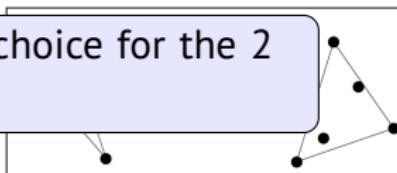
(a) Step 1: Choice 2 random points



(b) Step 2: Assignment of each point to the nearest center



(c) Step 3: Calculate the new centers of gravity.



(d) Step 4: Assignment of each point to the nearest center



(e) Step 5: Calculate the new centers of gravity.



(f) Step 6: Assignment of each point to the nearest center

K-means clustering (Quality of a partition)

How can we obtain good quality of a clustering partition?

Inertia and variance

Inertia measure the dispersion of observations relative to a reference point u in a metric space.

$$I_u(\{\xi_i, w_i\}_{i=1}^n) = \sum_{i=1}^n w_i \|\xi_i - u\|_M = \sum_{i=1}^n w_i \|x_i\|_M = I_0(\{x_i, w_i\}_{i=1}^n)$$

where $w_i = 1/n$ and M is often choice as the matrix identity.

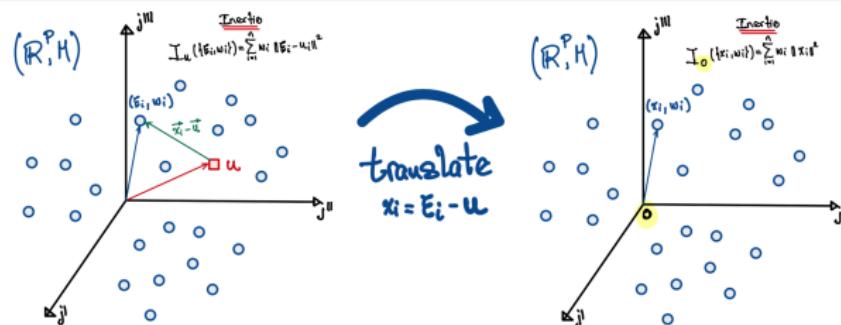


Figure: Translation of observations into u as origine.

K-means clustering (Quality of a partition)

How can we obtain good quality of a clustering partition?

Huygens' theorem

Total inertia $I_O(\{x_i, w_i\}_{i=1}^n)$ of points on a partition $P = (P_1, \dots, P_K)$ can be written as follows

Total inertia of the points = Inertia Intra-cluster + Inertia Inter-cluster.

$$I_O(\{x_i, w_i\}_{i=1}^n) = \sum_{k=1}^K w^k \|\bar{x}_k\|_M^2 + \sum_{k=1}^K \sum_{i \in P_k} w_i \|x_i - \bar{x}_k\|_M^2, \quad (\text{Inertie totale})$$

$$\sum_{k=1}^K w^k \|\bar{x}_k\|_M^2, \quad \text{where: } w^k = \sum_{i \in P_k} w_i \quad (\text{Inertia Intra-cluster})$$

$$\sum_{k=1}^K \sum_{i \in P_k} w_i \|x_i - \bar{x}_k\|_M^2 = \sum_{k=1}^K I_{\bar{x}_k}(P_k), \quad (\text{Inertia Inter-cluster})$$

K-means clustering (Quality of a partition)

Geometrical Insights of Huygens' theorem

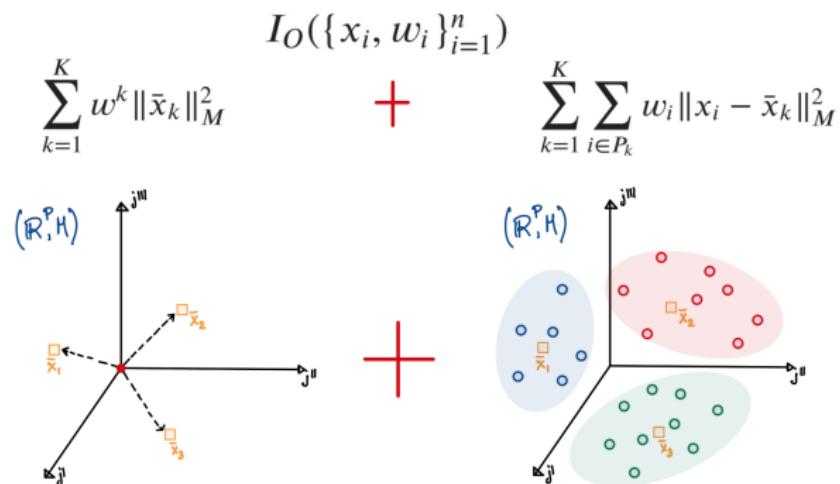


Figure: Huygens' theorem

K-means clustering (Quality of a partition)

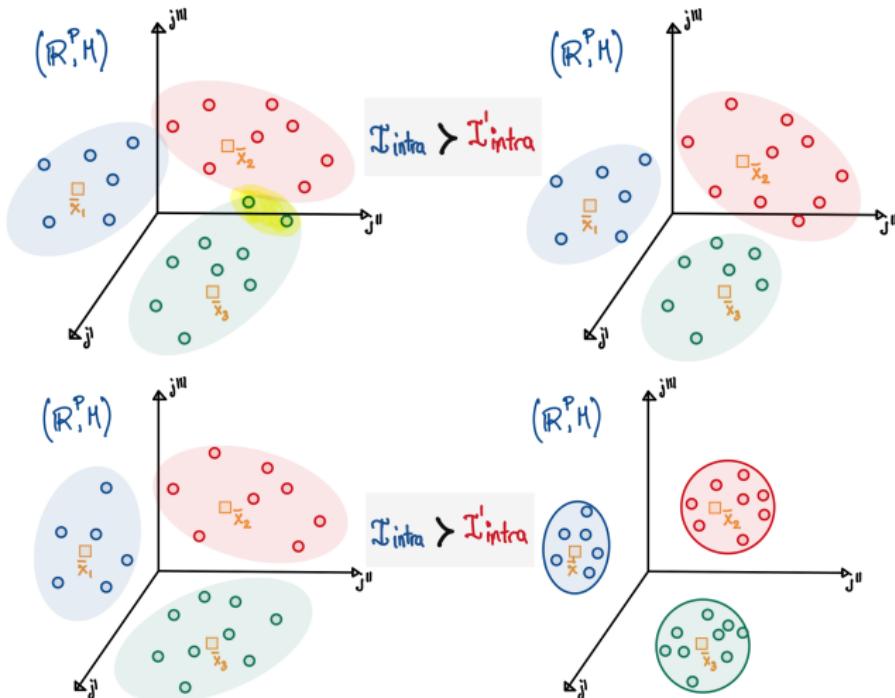


Figure: What is the better partition?

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Other Unsupervised Learning methods

Bibliography

Hierarchical clustering

Let us consider N observations (or objects) $\mathcal{O} = \{o_1, \dots, o_N\}$ and a dissimilarity measure between pairs of observations $d(\cdot, \cdot)$ on \mathcal{O} .

In Hierarchical clustering, we also define, based on $d(\cdot, \cdot)$, a [linkage criterion](#) $\mathcal{C}(\cdot, \cdot)$ which specifies the dissimilarity of sets as a function of the pairwise distances of observations in the sets.

Let us consider in this course tree linkage criterions:

- Minimum or single-linkage clustering

$$\mathcal{C}(A, B) = \min \{d(\mathbf{x}, \mathbf{x}'), \mathbf{x} \in A \text{ and } \mathbf{x}' \in B, A, B \subseteq \mathcal{O}\}$$

- Maximum or complete-linkage clustering

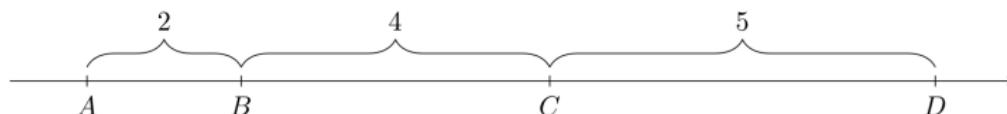
$$\mathcal{C}(A, B) = \max \{d(\mathbf{x}, \mathbf{x}'), \mathbf{x} \in A \text{ and } \mathbf{x}' \in B, A, B \subseteq \mathcal{O}\}$$

- Unweighted average linkage clustering

$$\mathcal{C}(A, B) = \frac{1}{n_A * n_B} \sum_{\mathbf{x} \in A, \mathbf{x}' \in B} d(\mathbf{x}, \mathbf{x}')$$

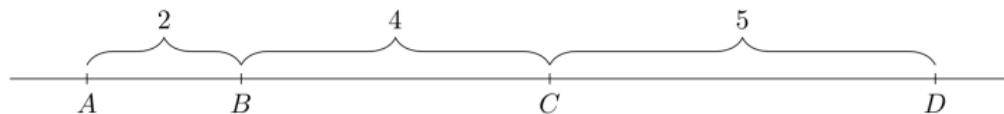
Hierarchical clustering

Let us consider 4 points (or objects) separated by some distance



Hierarchical clustering

Let us consider 4 points (or objects) separated by some distance



	A	B	C	D
A	0			
B	2	0		
C	6	4	0	
D	11	9	5	0

	{A, B}	C	D
{A, B}	0		
C	4	0	
D	9	5	0

	{A, B, C}	D
{A, B, C}	0	
D	5	0

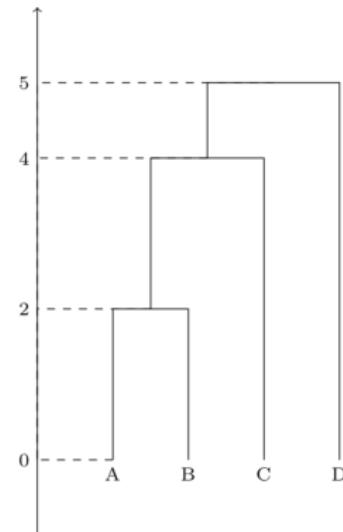
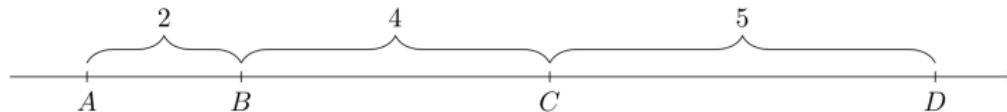


Figure: Minimum or single-linkage clustering

Hierarchical clustering

Let us consider 4 points (or objects) separated by some distance



	A	B	C	D
A	0			
B	2	0		
C	6	4	0	
D	11	9	5	0

	{A, B}	C	D
{A, B}	0		
C	6	0	
D	11	5	0

	{A, B}	{C, D}
{A, B}	0	
{C, D}	11	0

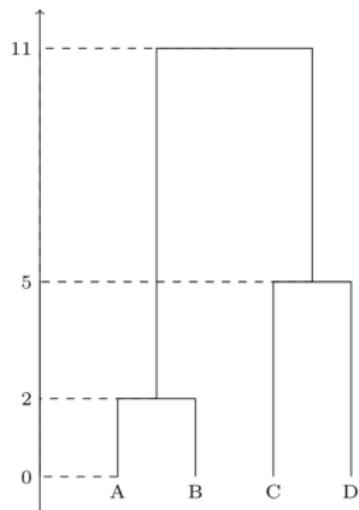
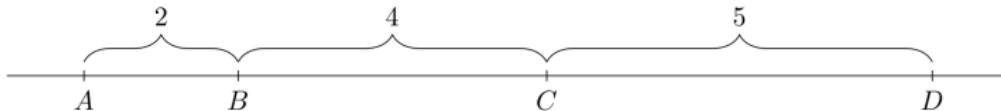


Figure: Maximum or complete-linkage clustering

Hierarchical clustering

Let us consider 4 points (or objects) separated by some distance



	A	B	C	D
A	0			
B	2	0		
C	6	4	0	
D	11	9	5	0

	{A, B}	C	D
{A, B}	0		
C	5	0	
D	10	5	0

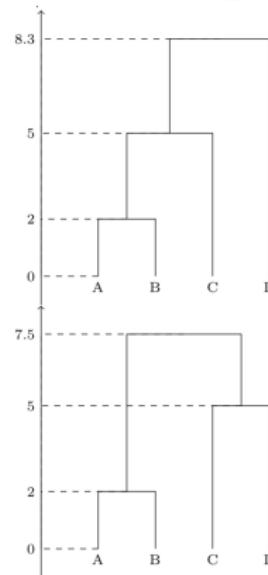
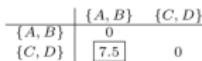
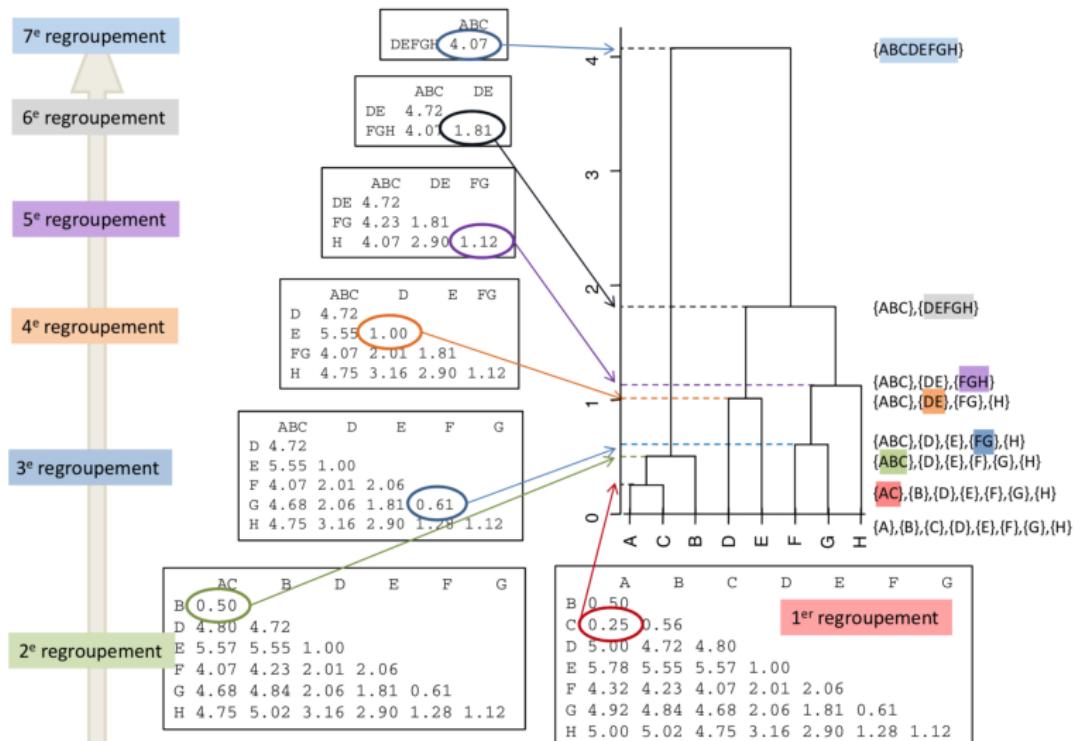


Figure: Unweighted average linkage clustering

Hierarchical clustering (Single-linkage Algorithm)

Algorithme



Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Other Unsupervised Learning methods

Bibliography

EM for the Gaussian Mixture Model (GMM)

GMM is a probabilistic clustering assuming that unlabelled observations $\{\mathbf{x}_i\}_{i=1}^N$ are generated by a mixture of K Gaussian distributions, whose unknown parameters are estimated by an iterative method known as Expectation-Maximization (the EM algorithm).

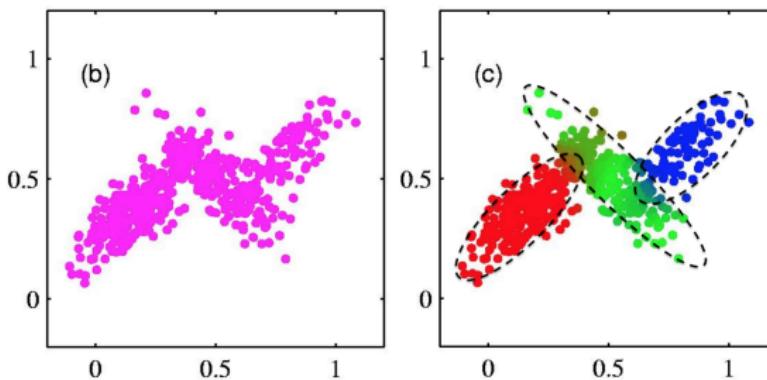


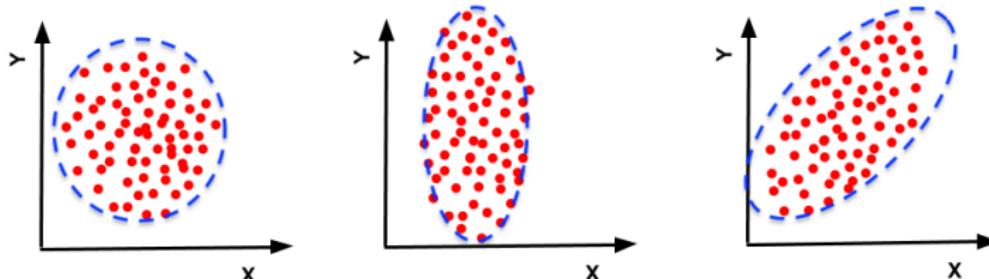
Figure: Examples of Gaussian mixture models

EM for the Gaussian Mixture Model (GMM)

GMM is a probabilistic clustering assuming that unlabelled observations $\{\mathbf{x}_i\}_{i=1}^N$ are generated by a mixture of K Gaussian distributions, whose unknown parameters are estimated by an iterative method known as Expectation-Maximization (the EM algorithm).

Remark

K-means algorithm is a particular case where all Gaussian distributions are assumed to have **the same diagonal covariance matrix**, with infinitely small variance

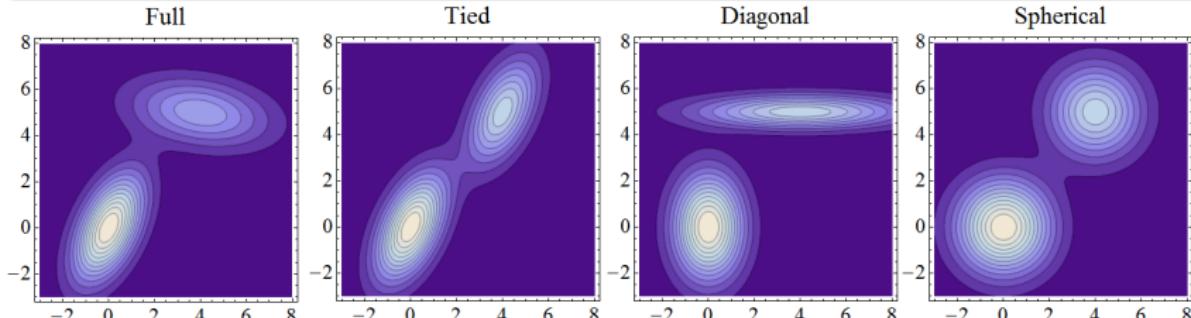


EM for the Gaussian Mixture Model (GMM)

GMM is a probabilistic clustering assuming that unlabelled observations $\{\mathbf{x}_i\}_{i=1}^N$ are generated by a mixture of K Gaussian distributions, whose unknown parameters are estimated by an iterative method known as Expectation-Maximization (the EM algorithm).

Remark

K-means algorithm is a particular case where all Gaussian distributions are assumed to have **the same diagonal covariance matrix**, with infinitely small variance



EM for the Gaussian Mixture Model (GMM)

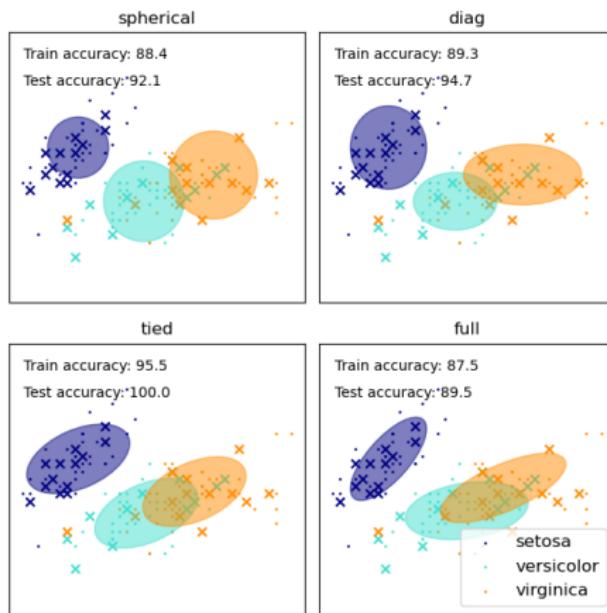


Figure: Gaussian mixture models on Iris Dataset

The GaussianMixture comes with different options to constrain the covariance of the different classes estimated: **spherical, diagonal, tied or full covariance**.

EM Algorithm - Mathematical formulation

- Let $\mathcal{D} = \{x_i\}_{i=1}$ be a unlabelled data set generated i.i.d. from X random variable with probability distribution

$$X \sim \sum_K^{k=1} \pi_k \mathcal{N}(\mu_k, \Sigma_k)$$

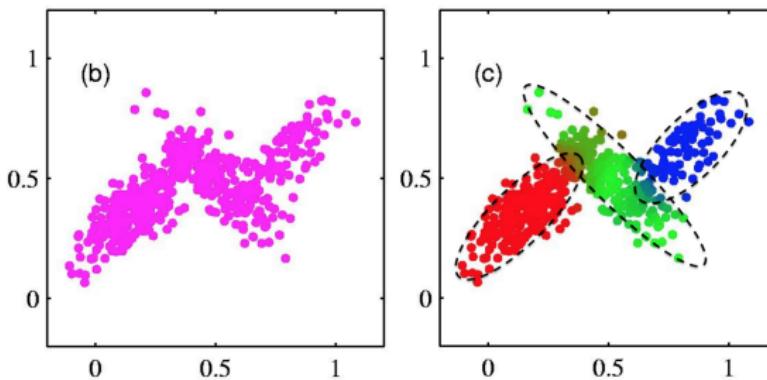


Figure: X follows a Gaussian mixture models

EM Algorithm - Mathematical formulation

- Let $\mathcal{D} = \{\mathbf{x}_i\}_{i=1}$ be a unlabelled data set generated i.i.d. from X random variable with probability distribution

$$X \sim \sum_K^{k=1} \pi_k \mathcal{N}(\mu_k, \Sigma_k)$$

- Let's assume that there is a latent random variable $Y \in \{1, \dots, K\}$ of K classes with π_1, \dots, π_K probabilities, such that the condition distribution

$$X|Y = k \sim \mathcal{N}(\mu_k, \Sigma_k)$$

EM Algorithm - Mathematical formulation

- Let $\mathcal{D} = \{\mathbf{x}_i\}_{i=1}$ be a unlabelled data set generated i.i.d. from X random variable with probability distribution

$$X \sim \sum_K^{k=1} \pi_k \mathcal{N}(\mu_k, \Sigma_k)$$

- Let's assume that there is a latent random variable $Y \in \{1, \dots, K\}$ of K classes with π_1, \dots, π_K probabilities, such that the condition distribution

$$X|Y = k \sim \mathcal{N}(\mu_k, \Sigma_k)$$

Objective

Estimate the unknown parameters of GMM $\sum_K^{k=1} \pi_k \mathcal{N}(\mu_k, \Sigma_k)$

$$\theta = (\pi_1, \dots, \pi_K, \mu_1, \dots, \mu_K, \Sigma_1, \dots, \Sigma_K)$$

How to estimate θ ? \implies Maximum likelihood estimate with EM.

EM Algorithm - Mathematical formulation

Objective

Estimate the unknown parameters of GMM $\sum_K^{k=1} \pi_k \mathcal{N}(\mu_k, \Sigma_k)$

$$\theta = (\pi_1, \dots, \pi_K, \mu_1, \dots, \mu_K, \Sigma_1, \dots, \Sigma_K)$$

How to estimate θ ? \Rightarrow Maximum likelihood estimate with EM.

Expectation-Maximization Algorithm

E-Step:

$$\begin{aligned} Q(\theta, \theta^{(t)}) &= \mathbb{E}_{Y|X, \theta^{(t)}} [\log L(\theta; X, Y)] \\ &= \sum_Y P(Y|X, \theta^{(t)}) \log(L(\theta; X, Y)) \end{aligned}$$

M-Step:

$$\theta^{(t+1)} = \arg \max_{\theta} Q(\theta, \theta^{(t)})$$

Loop: $|\ell(\theta^{(t)}; X) - \ell(\theta^{(t+1)}; X)| < \epsilon$

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Other Unsupervised Learning methods

Bibliography

Examples of Clustering

Let us do Machine Learning
Code source - [Link]

Overview

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

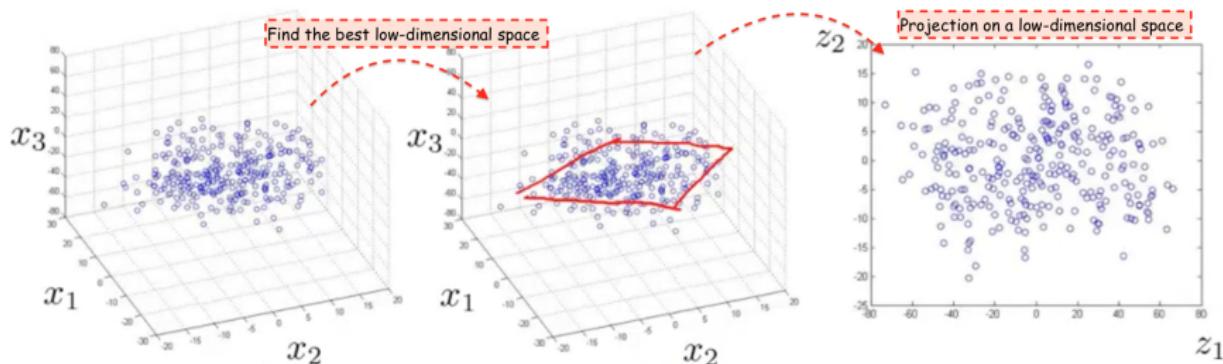
Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

Dimensionality reduction

Dimensionality reduction is the transformation of data from a **high-dimensional space** into a **low-dimensional space** so that the low-dimensional representation retains some meaningful properties of the original data, ideally close to its intrinsic dimension.



Dimensionality reduction

Dimensionality reduction is the transformation of data from a **high-dimensional space** into a **low-dimensional space** so that the low-dimensional representation **retains some meaningful properties of the original data**, ideally close to its intrinsic dimension.

Dimensionality reduction - Mathematical formulation

Let \mathcal{X}^p be the original input space of p -dimensional space. Dimensionality reduction aims to find a map function φ^k such that the target output space is of k -dimensional space, with $p > k$, i.e.:

$$\varphi : \mathcal{X}^p \rightarrow \mathcal{Y}^k$$

Note that the topological space can be a **euclidian** space or a **manifold** space, or whatever other space.

Unsupervised learning

Clustering

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

Principal component analysis (PCA)

PCA is a technique for reducing the dimensionality, increasing interpretability but at the same time minimizing information loss.

- Retains geometric properties.
 - No losing too much of information or variability.

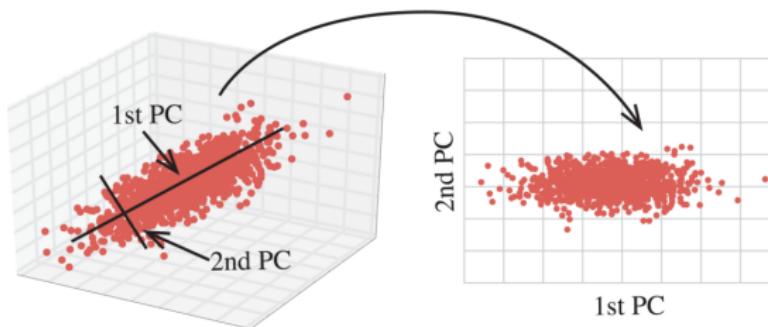


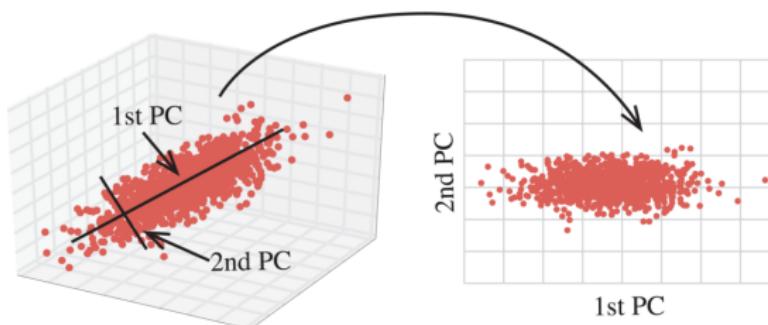
Figure: Example PCA 3D to 2D

Principal component analysis (PCA)

Objective of PCA

PCA aims to:

1. identify hidden pattern in a data set,
 2. reduce the dimensionality of the data by removing the noise and redundancy in the data,
 3. identify correlated variables

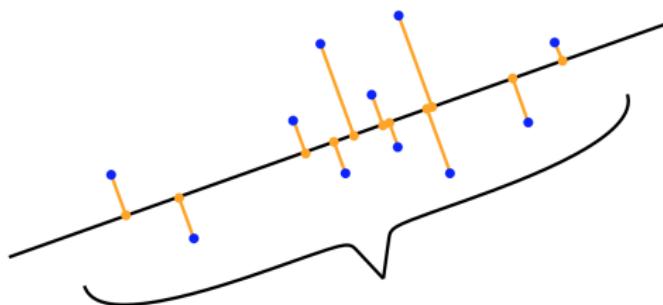


YC Carranza-Alarcón, Ph.D. • Machine Learning • October 16, 2023

Principal component analysis (PCA)

How to minimize the information loss and to maximize the conservation of variability (geometric properties)?

In a Euclidian space, we use the **inertia measure** to calculate the conservation of variability (variance), i.e. the information loss or not.



Direction with Maximal Variance

Figure: Maximal Inertia or Variance

Principal component analysis (Inertia measure)

Let be a data set of 20-dimensional space.

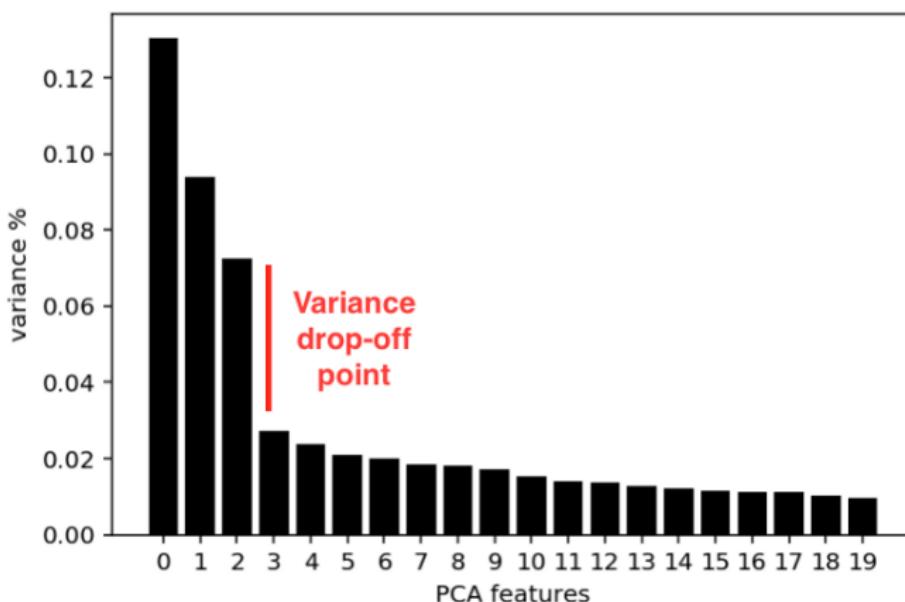


Figure: Variability retains par component

PCA as an identifier of hidden patterns

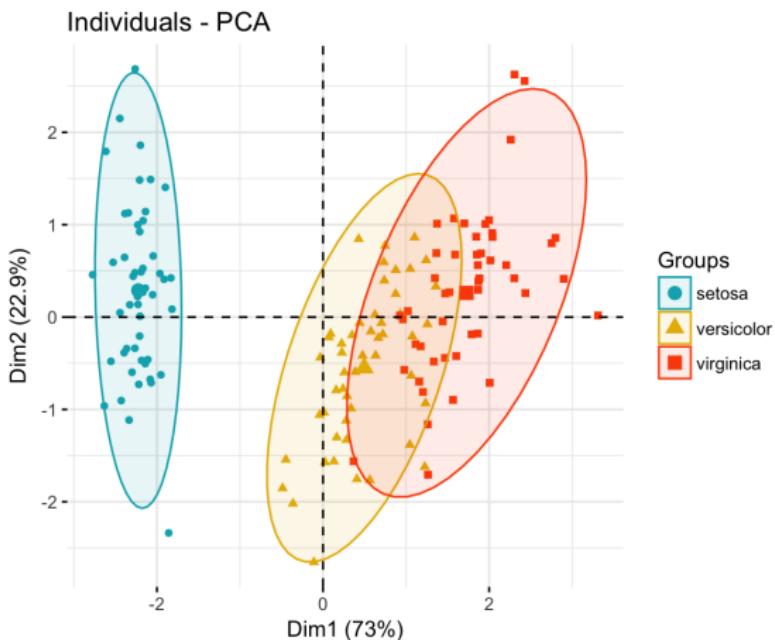


Figure: PCA applied to Iris data set [Kassambara et al. \[2017\]](#).

PCA as an identifier of hidden patterns

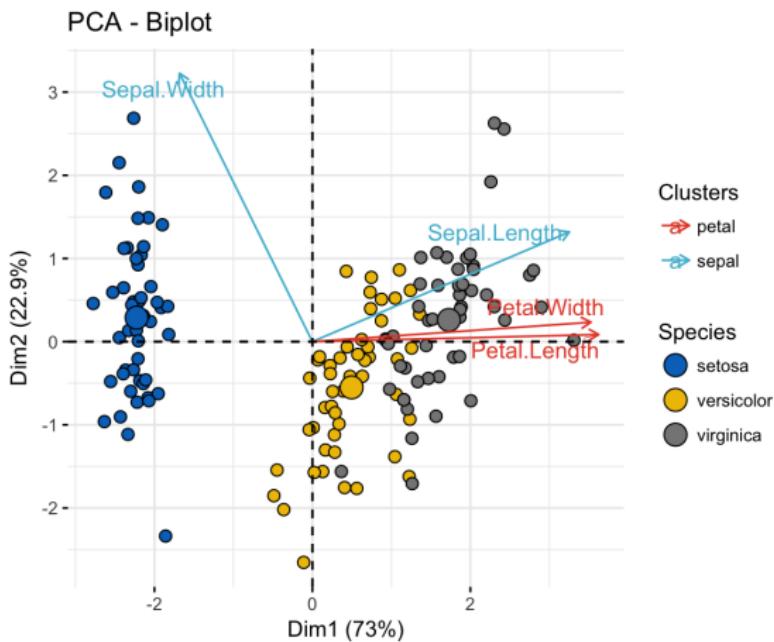


Figure: PCA applied to Iris data set [Kassambara et al. \[2017\]](#).

PCA as an identifier of hidden patterns

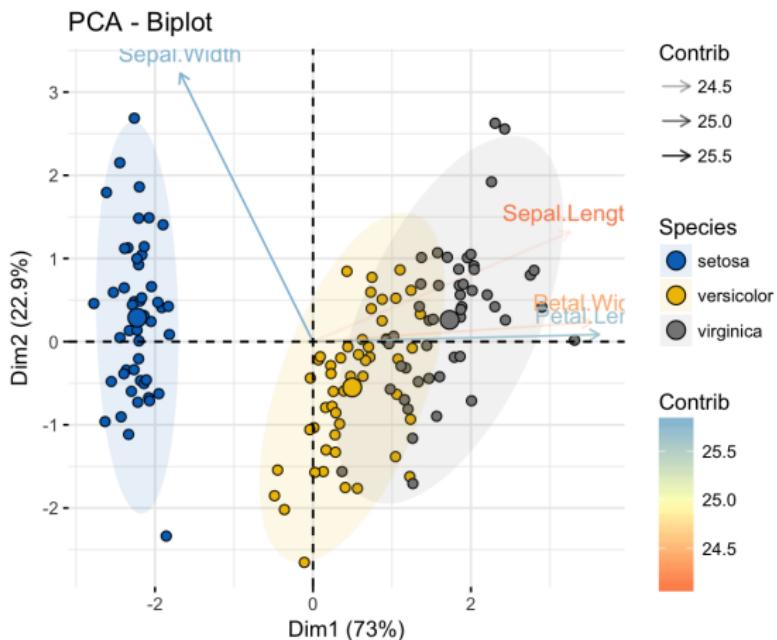


Figure: PCA applied to Iris data set Kassambara et al. [2017].

Principal component analysis (Drawbacks)

PCA sometime does not reduce very well the origin space.



Figure: Which animal is?

Principal component analysis (Drawbacks)

PCA sometime does not reduce very well the origin space.

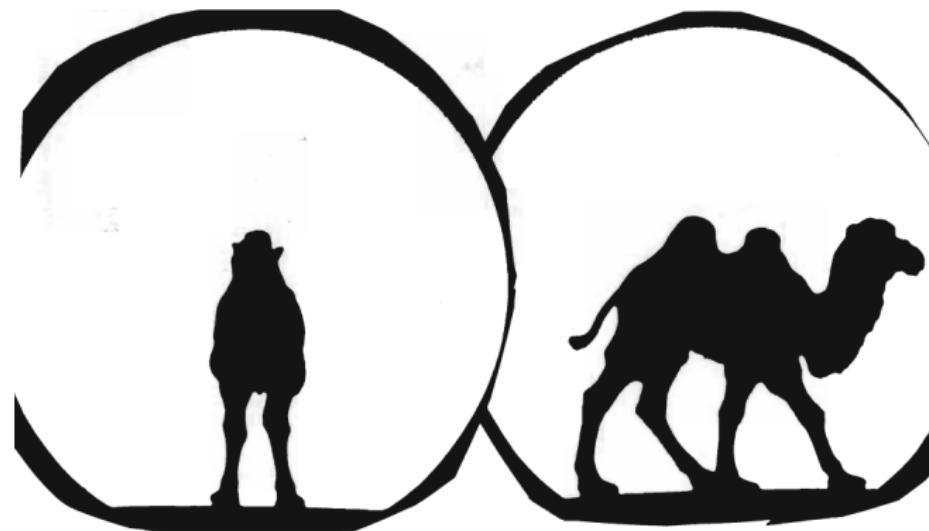


Figure: Which animal is?

Principal component analysis (Drawbacks)

PCA sometime does not reduce very well the origin space.

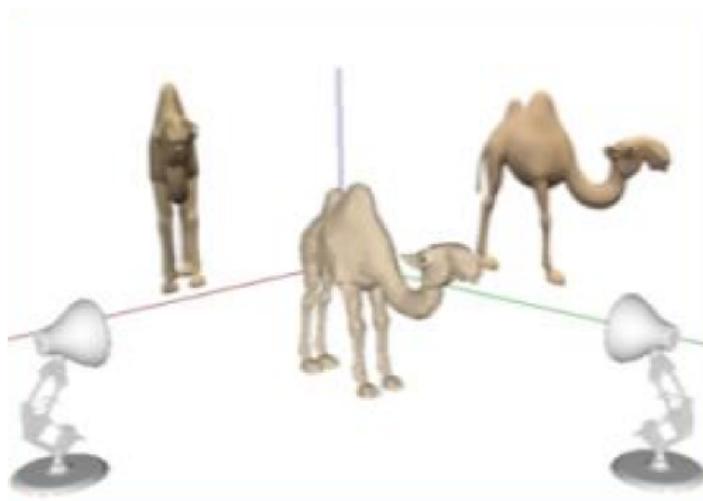


Figure: Which animal is?

Unsupervised learning

Clustering

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

t-SNE

PCA is not suitable for non linear data, but T-SNE can work with that.

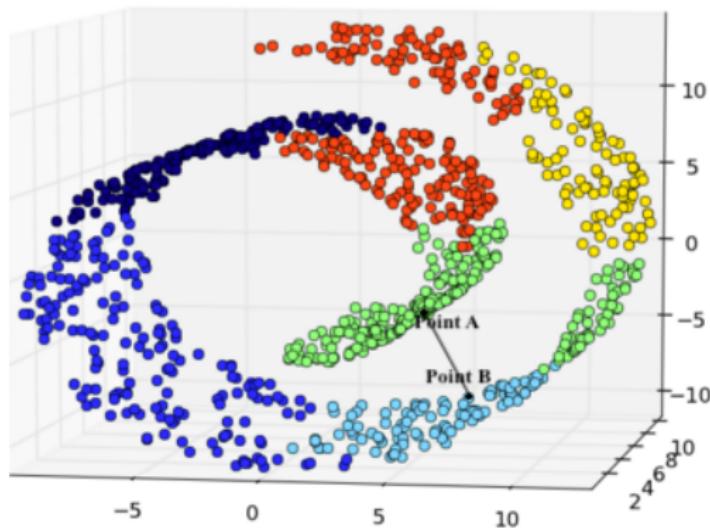


Figure: Non-linear data set

t-SNE

Dimensionality reduction

It is a statistical method for visualizing high-dimensional data by giving each datapoint a location in a two or three-dimensional map.

Non-linear data

It is a nonlinear dimensionality reduction technique well-suited for embedding high-dimensional data for visualization in a low-dimensional space of two or three dimensions

t-SNE

t-SNE algorithm (two stages) [Wikipedia \[2021b\]](#)

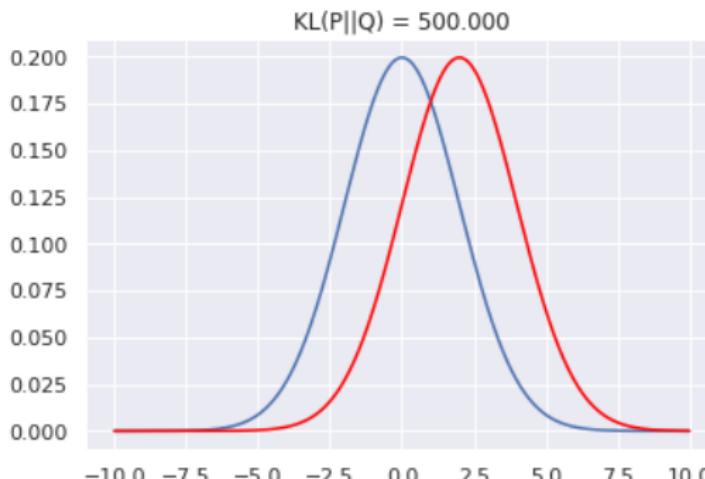
1. to construct a probability distribution over pairs of high-dimensional objects in such a way that similar objects are assigned a higher probability while dissimilar points are assigned a lower probability.



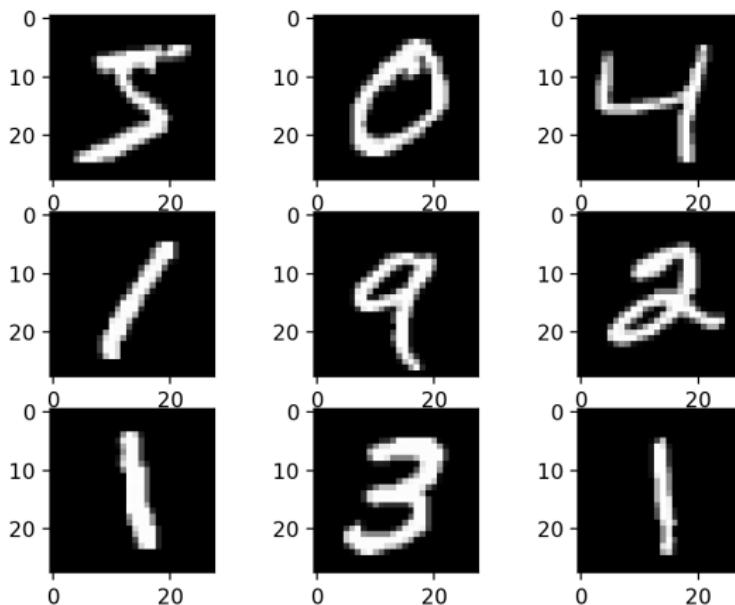
t-SNE

t-SNE algorithm (two stages) [Wikipedia \[2021b\]](#)

1. to define a similar probability distribution over the points in the low-dimensional map, and it minimizes the **Kullback–Leibler divergence** between the two distributions with respect to the locations of the points in the map.



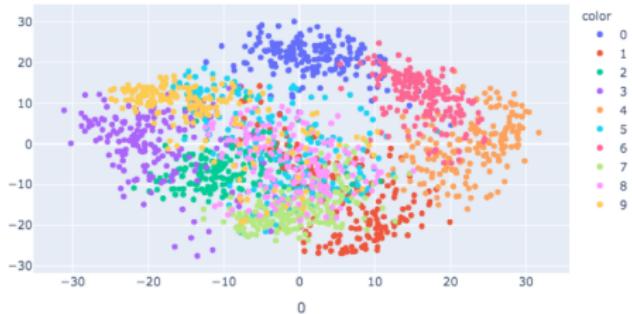
t-SNE - MNIST Digits Dataset¹



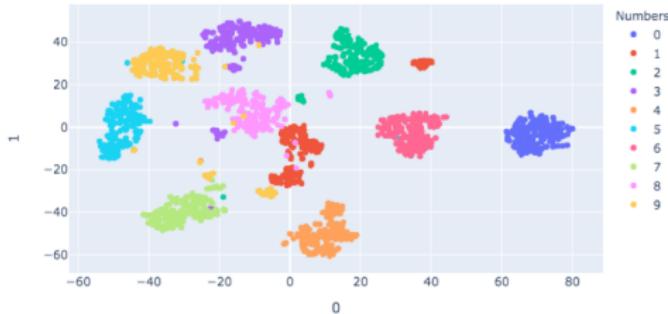
¹<https://observablehq.com/@robert-browning/t-sne-t-distributed-stochastic-neighbor-embedding>

t-SNE - MNIST Digits Dataset¹

PCA 2D Reduction of Digits Dataset

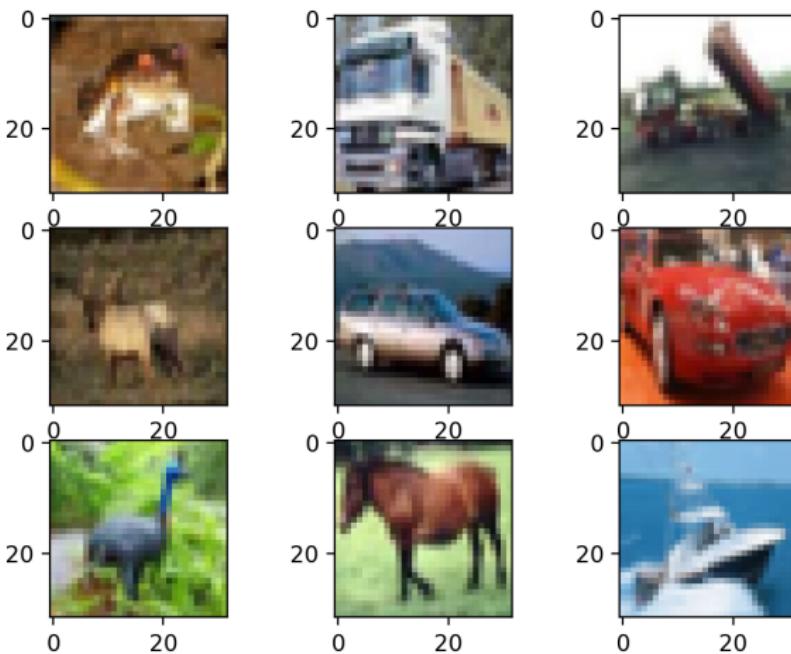


t-SNE 2D Reduction of Digits Dataset



¹<https://observablehq.com/@robert-browning/t-sne-t-distributed-stochastic-neighbor-embedding>

t-SNE - CIFAR10 Dataset

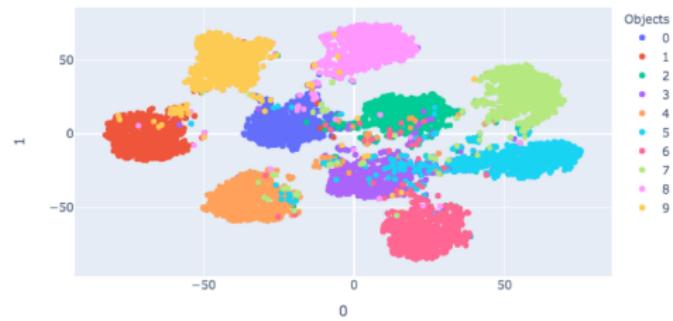


t-SNE - CIFAR10 Dataset

PCA 2D Reduction of CIFAR10 Dataset Feature Vectors [512x1]



t-SNE 2D Reduction of CIFAR10 Dataset Feature Vectors [512x1]



Unsupervised learning

Clustering

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

Examples of dimensionality reduction

Let us do Machine Learning

Code source - [Link]

Overview

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

Anomaly detection

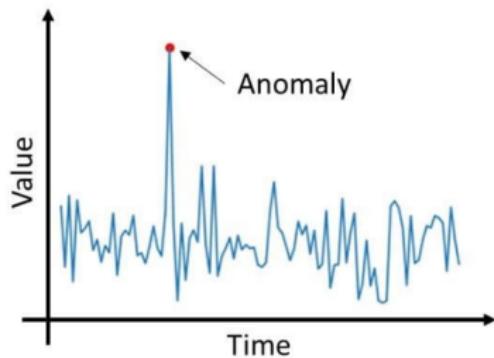
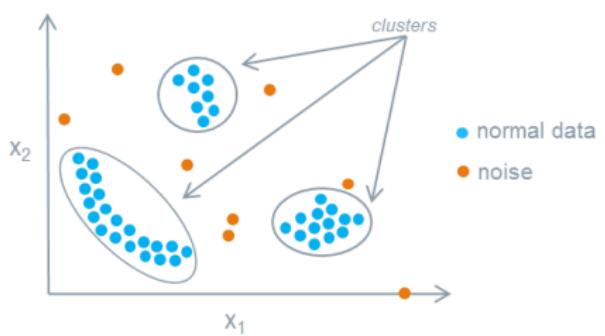


Figure: Examples of Anomaly detection

Neural network - Generative adversarial network



Figure: Examples of GAN

Neural network - Autoencoders

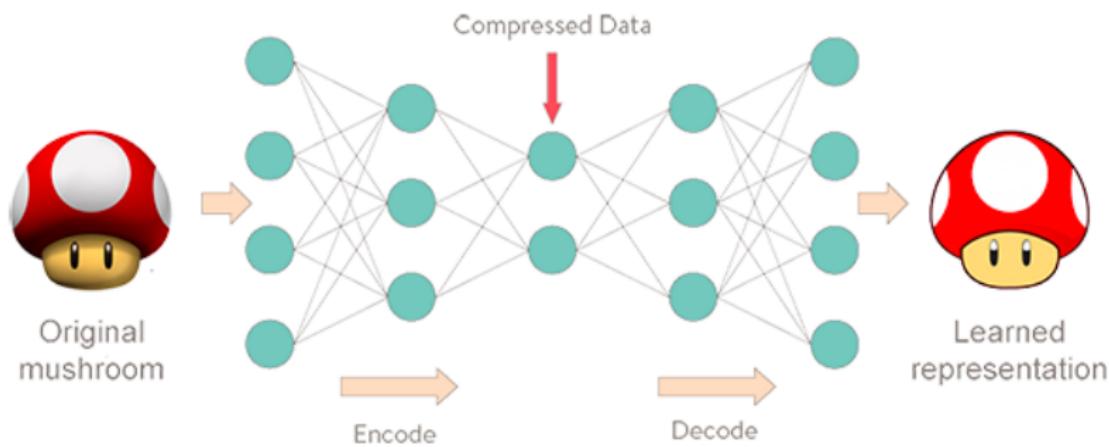
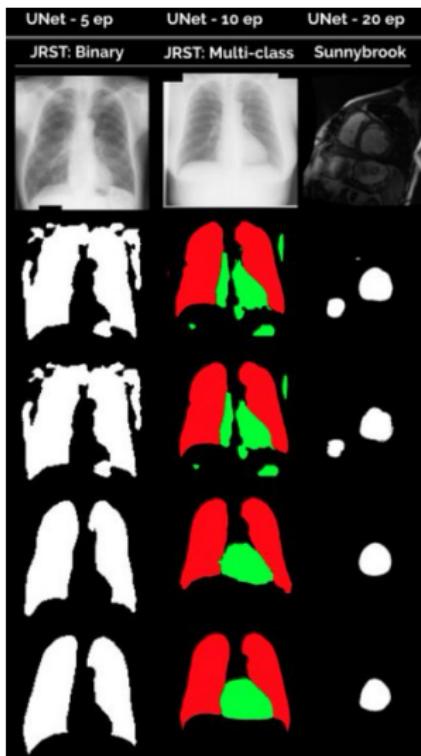


Figure: Examples of autoencoders

Neural network - Semantic Segmentation



Non-negative matrix factorization

Everything is personalized



Over 75% of what people watch comes from a recommendation

Figure: Examples of recommendation system

Unsupervised learning in Action

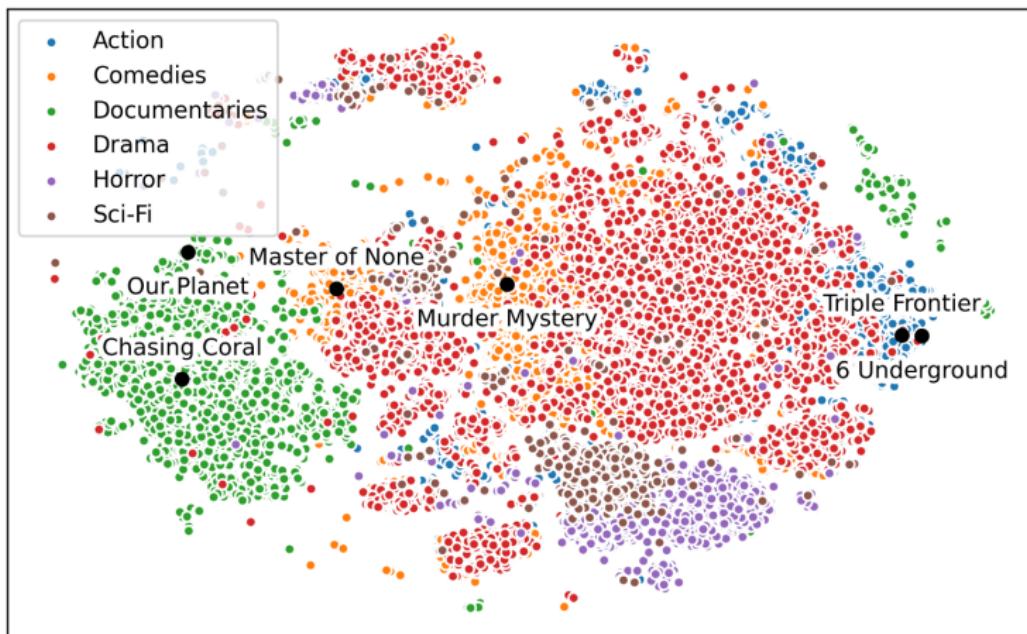


Figure: Examples of real data set

Overview

Unsupervised learning

Clustering

K-means clustering

Hierarchical clustering

Expectation-Maximization for the Gaussian Mixture Model

Example of clustering in python

Dimensionality reduction

Principal component analysis

T-distributed stochastic neighbor embedding (T-SNE)

Examples of dimensionality reduction in python

Other Unsupervised Learning methods

Bibliography

Bibliography I

Dougherty, G. (2012). *Pattern recognition and classification: an introduction*. Springer Science & Business Media.

Friedman, J., Hastie, T., and Tibshirani, R. (2001). *The elements of statistical learning*. Springer New York Inc.

Hinton, G. E., Sejnowski, T. J., et al. (1999). *Unsupervised learning: foundations of neural computation*. MIT press.

Kassambara, Visitor, T. t., Visitor, Agathe, Enrique, and Julie (2017).
Pca - principal component analysis essentials.

Loon, A. R. v., Raut, A. S., Matthews, A. K., Sajjad, A. F., NT, A. B., Jeevan, A. M., and Guest, A. (2019). Machine learning explained: Understanding supervised, unsupervised, and reinforcement learning.

Bibliography II

Patil, P. (2018). K means clustering : Identifying f.r.i.e.n.d.s in the world of strangers.

Ronneberger, O., Fischer, P., and Brox, T. (2015). U-net: Convolutional networks for biomedical image segmentation. In *International Conference on Medical image computing and computer-assisted intervention*, pages 234–241. Springer.

whuber (<https://stats.stackexchange.com/users/919/whuber>).
Different covariance types for gaussian mixture models. Cross
Validated. URL:<https://stats.stackexchange.com/q/326678> (version:
2018-02-03).

Wikipedia (2021a). Dimensionality reduction.

Wikipedia (2021b). T-distributed stochastic neighbor embedding.

Thank You for Your Attention!