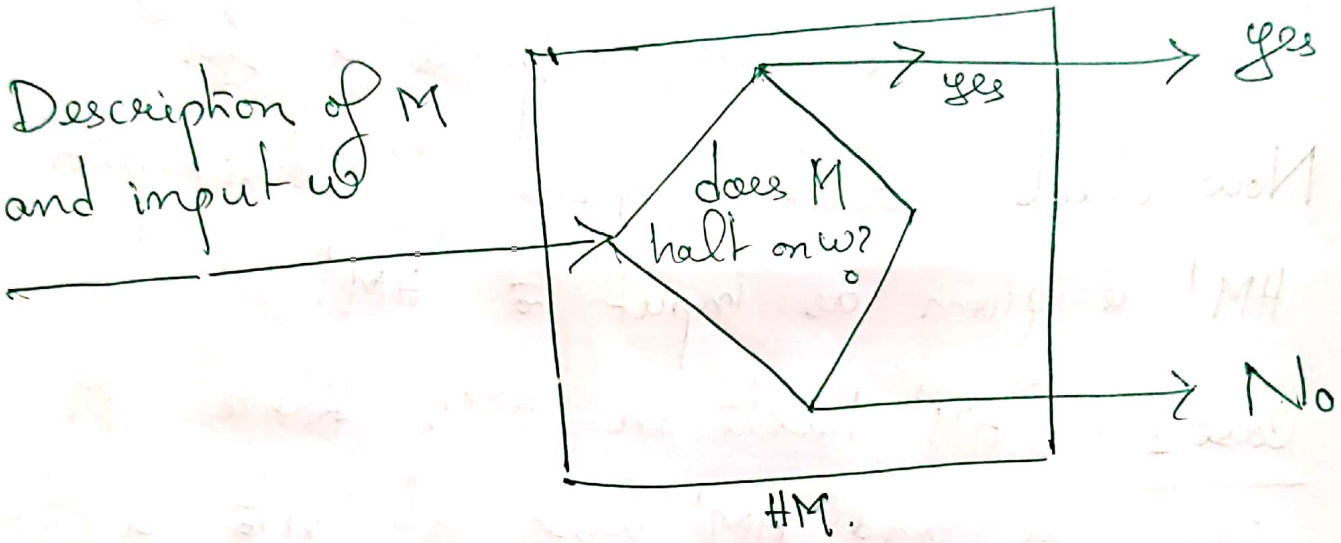
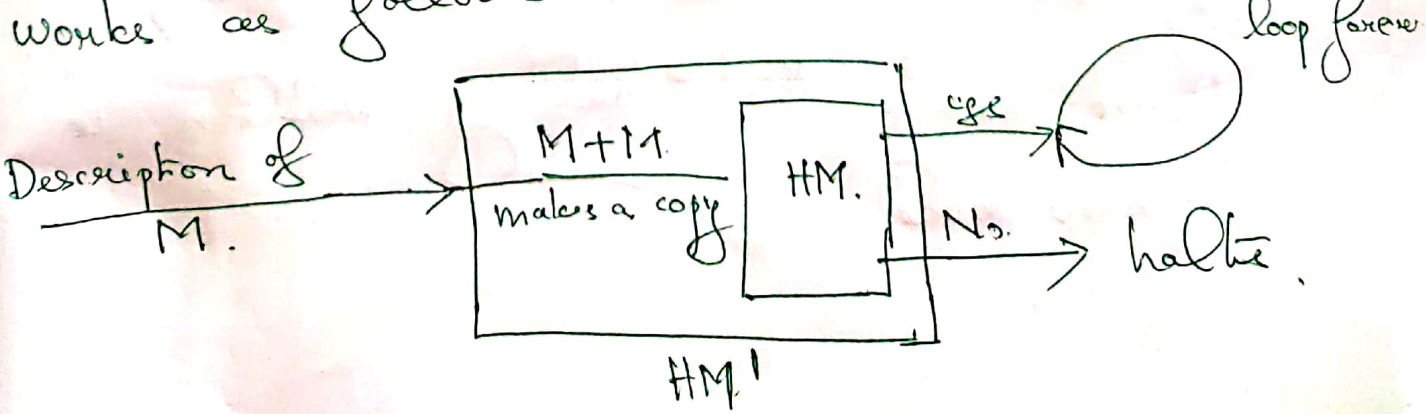
Halting problem Theorem :

Let us assume that halting problem is decidable. Then there exist a TM HM, that solves the halting problem. The halting machine HM works as follows.



1) Input to HM : Description of a TM M and input string w.
2) Output of HM : HM says yes, if M halts on w. and HM says No, if M does not halt on w.

Now we can construct a TM HM' which works as follows.



1) Input to HM' : Description of a TM, M.

2) processing :- HM' makes a copy of its input & gives both copies to HM as input.

3) output of HM' : HM' halts when HM outputs a 'No' and HM' loops forever when HM outputs a 'Yes'.

Now check what happens if description of HM' is given as input to HM'.

Case 1 : HM' halts on HM'. Then HM will say yes and HM' will get into a loop forever.

case 2 : HM' does not halt on HM'. Then HM will say no & HM' will halt.

In both cases, we arrive at a contradiction & hence our assumptions was incorrect, i.e, HM never exists.

Hence proved that halting problem is undecidable.