Regulation and Trade in the Secondary Mortgage Market*

Preliminary and Incomplete

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Abstract

Mortgage originators sell, or securitize, in the secondary market close to 70 percent of all mortgages they originate. Because of this, fluctuations, and sometimes collapses, in the volume of issuances of mortgage-backed securities in the secondary market are associated to large fluctuations in the volume of new issuance of mortgage loans to households in the primary market. These dynamics are a concern for policymakers because of the government mandate of providing affordable and stable mortgage credit to households. I develop a quantitative model of financial intermediation and securitization that accounts for these large fluctuations based on a theory of adverse selection in the secondary market. I extend a standard credit model, in which borrowers and lenders face exogenous income and default risk, by adding a secondary market where lenders can trade loans. The environment features heterogeneity in lenders' loan origination technology, and private information about the quality of loans traded, which generates an adverse selection problem. Large shocks on the default rate can lead to a collapse of trade, which in turn generates a contraction of credit in the primary market. I calibrate the model to match key moments of the cross-section of mortgage lenders, and time series of aggregates for households and lenders in the U.S. mortgage market. The model accounts for two-thirds of the collapse of the mortgage market during the great recession. Then, I study policy interventions in the secondary market, and find that policies that compensate investors against losses from default and adverse selection can generate welfare gains by improving liquidity and stabilizing trade. Borrower households benefit from lower interest rates but face a higher tax burden.

Keywords: Mortgage market; Securitization; Private information; Adverse selection; Credit cycles

JEL: D82, G11, G21

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1 Introduction

In the United States, mortgage originators sell, or securitize, in the secondary market close 70% of all mortgages they originate in a given year. Secondary markets allow the reallocating of these illiquid resources -long term debt- from mortgage originators to mortgage investors. By doing this, mortgage originators free up capital and can extend new mortgages to households. The volume of new issuance in the secondary market is volatile and sometimes collapses abruptly, and when this happens the volume of lending in the primary market follows in tandem. From 2006 to 2010 the aggregate volume of new issuance of residential mortgage-backed securities, and the volume of new issuance of mortgage loans to households fell by 40%. These large fluctuations are a concern for policymakers due to the government mandate of providing affordable and stable mortgage credit to households.

I develop a quantitative general equilibrium model of financial intermediation and securitization that is able to account for large fluctuations in the volume of sales in the secondary market due to fluctuations in the default rate on long-term debt and household's income, and transmit those fluctuations into the supply of mortgage lending to households. I add a secondary market for loans to a standard credit model in which lenders and borrowers exchange resources through long-term debt contracts. Households borrow to smooth consumption of non-durables goods and housing services in the presence of aggregate income and default risk on their debt. A continuum of lenders own households' debt and are able to trade it in a secondary market among themselves. Lenders have two main characteristics: first, they have heterogeneous technology to issue new loans, some lenders are better than others at issuing loans. This heterogeneity gives a reallocation role to the secondary market, lenders with the most efficient loan origination technology sell their portfolio of illiquid loans, free up resources and issue new loans to households. Whereas lenders with the least efficient technology purchase loans in the secondary market instead of issuing new ones. This capture the shift of the mortgage industry to fund mortgages through capital markets instead of deposits. Second, lenders have private information about the probability of default of the loans held in their portfolio[1]. I model the secondary market as an anonymous and centralized market in which all loans trade at a pooling price as in [22] Kurlat (2013). In this environment, the presence of private information gives rise to an adverse selection problem that hinders trade by reducing the return on the assets purchased in the secondary market, which discourages some lenders to participate in the market[2]. Fluctuations in default rate shock generate fluctuations in the volume of trade in secondary markets due to an endogenous response of the adverse selection problem. I show that given the cross-sectional characteristics of the U.S. mortgage market such fluctuations induce large fluctuations in mortgage credit supply

[1] This motivated by a large body of literature that documents that mortgage originators are better informed than investors, about the probability of default of the loans they originate. Furthermore, mortgage originators take advantage of such information by retaining the loans with lower probability of default, and selling first those with higher probability of default which adversely affect investors. See [1]Adelino, Gerardi, and Hartman-Glaser (2017), [21]Keys, Mukherjee, Seru, and Vig (2010) find evidence of presence of the adverse selection problem in different segments of the secondary market of mortgages.

[2] Theoretical work starting with [4]Akerlof (1970), Mayers and Majluf (1984), Bernanke and Gertler (1989) have studied how asymmetries of information about asset qualities can generate large fluctuations and market breakdowns at the firm and at the aggregate level. Recent theoretical works include: [17]Guerrieri and Shimer (2012), [22]Kurlat (2013), and [10]Chari, Shourideh and Zetlin-Jones (2014).

to households.

The model is calibrated to match key moments of the U.S. mortgage market, both in the cross-section and the time series for the period 1990 to 2006. Then, I evaluate how the model performs during the great recession by simulating the model for the sequence of default rate and income shocks as observed in the data. The model is able to account for two-thirds of the collapse of the primary and secondary mortgage market during the great recession. I use the model to evaluate policies in which the government provide a subsidy to loan buyers in the secondary market in order to reduce the losses from adverse selection and default risk and stabilize the volume of trade. The government finances such policy with a charge on mortgage originators, and non-distorting taxes on households. I find that the subsidy policy can generate welfare gains to both borrower households and lenders by stabilizing trade and reducing the probability of collapses in the secondary market. Better functioning secondary markets allow for more efficient reallocation of illiquid resources among lenders and reduce the interest rate that borrowers face. Borrowers benefit from lower mortgage interest rates and lower volatility of consumption but face higher taxes.

The paper is structured as follows: Section 2 presents main features of the mortgage market at the institutional level, cyclical patterns, and cross-sectional characteristics of mortgage originators. Section 3 presents the model. Sections 4 and 5 presents the model's characterization and main properties, an important part of the results, and the mechanism of the model are explain there. Section 6 presents the calibration and a simulation of the model under different policy scenarios, and Section 8 concludes.

2 Trends in the Mortgage Market

This section briefly reviews on the main trends in the mortgage market, the business cycle properties of aggregates, and the cross-sectional characteristics of the mortgage market during the period 1990-2018. I calibrate the model to match several of these cross-sectional and time series moments during the period of analysis. Then, I use the calibrated model to perform policy analysis.

2.1 Mortgage Market Structure

The mortgage market in the United States is comprised by two markets; a primary mortgage market, where mortgages are originated, and a secondary market, where mortgages are sold, bundled and transformed into mortgage backed securities, process known as securitization. The origination market links homebuyers and mortgage originators, while the secondary market brings together mortgage originators and investors. Most of these investors are financial institutions that manage large pools of savings, such as pension funds, mutual funds, and insurance companies.

The rise of securitization started in the mid 1980s due changes in regulation, and financial innovation that led to rapid growth of the secondary mortgage market[3]. Mortgage originators,

[3] The main regulatory changes: i) bank deregulation acts, ii) regulation changes to Fannie Mae and Fredie Mac that allowed them to purchase non-FHA mortgages. See Campbell and Hercowitz(2005) and Landovigt (2016)

traditionally banks and savings & loans institutions, started to sell their mortgages to large securitizers, who bundled and transformed them into mortgage backed securities, purchased by investors in the secondary market. . During the last three decades, as documented below, the origination market is dominated by large originators, banks and non-banks, who specialize in originating mortgages but do not hold them in their balance sheet. This shift from deposit funding to capital market funding through the secondary market is known as the *originate-and-distribute* model of mortgage credit intermediation. [the mid-80s x% of all loans were securitized, while currently y% of all loans are securitized.]

The secondary market of mortgages in the United States can be broadly divided in two main segments: the agency segment and the private-label segment. The agency segment correspond to segment dominated by the Government Sponsored Enterprises (GSEs), which securitize pools of residential mortgage loans into Agency MBSs, and guarantee the coupon and interest payments to investors against borrowers' default. Private-label segment of the market, correspond to private financial institutions that issue MBSs with no guarantee of principal and interest payments, thus investors do face default risk. The other main characteristics of these markets is the type of mortgages that are pooled, the agency segment mostly purchases loans that conform to specific guidelines set up by the GSEs[4]: loan amount, credit score and loan-to-value (LTV) ratios, these loans are known as conforming loans. Those that do not conform with GSEs guidelines are usually called jumbo loans, typically although not exclusively private label MBSs are backed by non-conforming loans.

2.2 Mortgage Market characteristics

2.2.1 Data

The analysis is based in three large loan-level data sets that cover virtually the entire U.S. residential mortgage market.

Home Mortgage Disclosure Act Data (HMDA), which requires covered depository and non-depository institutions to collect and publicly disclose information about their mortgage lending activity. The information includes characteristics of the mortgages the originate or purchase during a calendar year. The loans reported are estimated to represent the majority of home lending in the US[5]. I merge the Loan/Application Registries (LARs) with the Panel of Reporters, both HMDA databases, to form a panel with aggregated variables at the level of mortgage originators from 1990 to 2017. I restrict the sample of loans to conventional, 1-4 family, owner occupied, and include both home purchases and refinance. The limitations of this dataset is that there is no information about the type of contract, the contracted interest rate, and the type of securitization of the loans. I use HMDA to look at aggregate volumes of i) originations, and ii) sales of loans in the secondary market.

CoreLogic Loan Performance (LP), also known as Private Label Securities (PLS), which covers nearly all privately securitized mortgage loans, i.e the non-agency segment of the secondary

for a categorization of the stages of mortgage finance in the U.S. according to financial regulation changes.

[4]GSEs guidelines specify limits to the amount of the loan, to the ratio of the loan to the value of the collateral, among other characteristics.

[5]Residential Mortgage Lending in 2016: Evidence from the HMDA data, Federal Reserve Bulletin 2017.

Table 1: Volume of originations and trade, 1990-2017

Average 1990-2017	sales as % of originated volume	correlation
All purchases	0.58	0.64
GSEs	58.6	-0.19
Priv Institutions	41.4	0.78

Source: HMDA LARs and Reporter Panel 1990-2017

market; loans have flags indicating its classification as prime conforming, prime jumbo, subprime, and alternative-A. It includes two datasets, a static version that collects characteristics of the loans and borrowers at the moment of origination, and a dynamic version that follows the life of each loan recording changes in interest rates, and performance status in a monthly basis.

Residential Mortgage Servicing Database (RMSD), maintained by McDash Analytics. This dataset provides information about mortgages sold to GSES and loans that are retained in originator's portfolio. It covers the universe of mortgage issued by the 12 largest mortgage originators in the U.S., which represents about two-thirds of the residential mortgage market, including both agency and non-agency securitized loans.

2.2.2 Time Series characteristics

I use the HMDA dataset to document: i) time series characteristics of aggregates in the entire mortgage market, and ii) cross sectional characteristics of the distribution of mortgage originators. Then, I use LP and RMSD to measure volatility of aggregate quantities, and average prices in the agency and non-agency segments of the market.

Using HMDA I document a positive time series correlation between the volume of loan sales in secondary markets and the volume of mortgage originations in the primary market, see Figure **??**. Table 1 report that on average 58% of all newly originated loans are sold in the secondary market within the first 9 months of origination[6]. Two measures of volume of originations are constructed: i) the total number of new loans originated by a lender in given year; ii) the total dollar amount of origination of a lender in given year[7]. For the period of analysis HMDA covered on average 8,127 lenders that reported mortgage loans. The volume of trade in the secondary market can be measured by: the number of loans sold, the fraction of dollar amount of sales, and also by looking at the dollar amount of issuance of MBS.

I restrict the sample of loans in the LP and RMSD databases, to mortgages with following characteristics: first, 15-year and 30-year fixed rate loans. Both the 15-year and 30-year mortgages constitute more than 90% of the total mortgages in these databases. Second, I focus on home purchases, and refinance, since these are the most common type of loans contracted at the terms above. Third, I compare only loans categorized as prime in both LP and RMSD datasets. The idea is to study whether there are differences in prices and quantities for the same type of product across the agency, and non-agency segment of the market.

[6] This is consistent with patterns documented for similar markets, Ivashina and Sharfstein (2010)[19] look at the secondary market for syndicated loans in the U.S. and find a strong time series correlation between volume of originations and the fraction retained by originators.

[7] All reported dollar amounts in tables are transformed to constant prices of 2015.

2.2.3 Cross-sectional characteristics

The group of large originators in the mortgage industry is comprised by: Large Banks, Affilitiated Mortgage Companies, and Independent Mortgage Companies. This composition have been a consistent pattern for the entire sample 1990-2017. The number of lenders has declined over time, going from more than 9,000 in the early 90s to less than 6,000 by 2017. Most of the reduction in the number of originators is due to a reduction in the number of small banks and credit unions reporting home mortgage lending activity[8].

Big players dominate the mortgage market. The market share of lenders reporting more than one thousands loans is above 75% for the entire sample 1990-2017. This is both the case for volume of lending in dollar amount and loan count. Table 2 shows the averages for the sample period, Large Banks, Affiliated Mortgage Companies and Individual mortgage companies dominate the market.

Table 2: Composition of Lenders by type of institution, averages 1990-2017

Description	**Small Bank**	**Large Bank**	**Credit Union**	**Aff. Mtg Co.**	**Ind. Mtg Co.**	**All**
All institutions (number)	4,384	641	1,878	308	916	8,127
(%) Inst reporting <= 1 thousand loans	0.97	0.46	0.96	0.60	0.60	0.86
(%) Inst. reporting <= avg vol loans*	0.98	0.56	0.98	0.65	0.68	0.90

* Refers to the average number of loans across all reporters in a given year. Source: HMDA LARs and Reporter Panel 1990-2017

The cross sectional distribution of the volume of lending measured as the log of USD originations by lender is remarkably stable in the period of analysis 1990-2014. Due to the high concentration in the lending market the distribution displays positive skewness[9]. Table 3shows the average moments for the period of analysis. These results are very similar if one restricts the set of loans to those that are Home Purchase, conventional, 1-4family property, owner occupied, i.e the most common home mortgage loan.

Table 3: Moments of the distribution of Mortgage Lending, HMDA 1990-2017

Moments	All loans	Home Purchase
Mean/median	1.007	1.007
skewness	0.33	0.31
Fraction of Inst. reporting > avg vol loans (USD)	0.10	0.10
Mkt shr of Inst. reporting > avg vol loans (USD)	0.89	0.91

Source: HMDA LARs and Reporter Panel 1990-2017

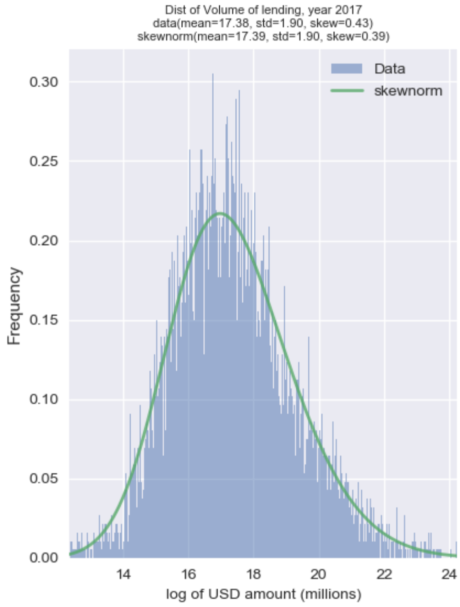
Figure 1 shows the fitted cross sectional distribution for year 2017, See 10 for the cross-sectional distribution for years 1990, 2000, and 2010.

[8]

1. This pattern is consistent with changes in banking regulation that relaxed branching restrictions in the mid 90s. Corbae and D'Erasmo (2013), McCord and Prescott(2014), Janicki and Prescott (2006) document a decline in the number of commercial banks as a trend that has marked the commercial banking industry during the last three decades.

[9]USD amount of lending follows an exponential distribution in the cross section, taking logarithm converts the distribution to normal. Alternatively, the USD amount of lending follows a log-normal distribution.

Figure 1: Cross Section Distributions of Mortgage Lending, 2017



Source: HMDA LARs and Reporter Panel 1990-2017

2.3 Private Information in the mortgage market

Ample information about the underlying mortgages backing the MBS is usually available to the MBS prospective buyers, this includes characteristics of the borrowers and the loans, distribution of interest rates, maturities, loan-to-value ratios, credit scores, among others. However, there are several reasons why a mortgage originator may have superior information to the mortgage buyer, or to the ultimate MBS buyer, often investors like mutual funds, pension funds, or insurance companies. Shimer (2014)[28] analyzes three main sources of superior information on the originator side: i) low documentation loans, ii) borrower's misrepresentation/ misreporting, iii) differences in technologies at valuing mortgages: mortgage originators may have a superior technology at valuating mortgages due to gains from specialization.

This type of private information is relevant to the extent that it can help to predict more accurately the performance of mortgages, which determines the ultimate quality of the loans, and the derivatives traded in the secondary market. The fact that a mortgage originator is better informed about the quality of mortgages gives him an advantage over potential buyers. In particular, an originator might exploit this informational advantage by retaining the higher quality assets, and trading the lowest quality assets, which adversely affect a buyer, and trade in the secondary market. This is the adverse selection problem, also known as the problem of the lemons.

There are several empirical papers documenting significant presence of adverse selection in secondary market of mortgages. Keys, Mukerjee, Seru, Vig (2010)[21] find evidence that MBS originators not only have superior information that its buyer counterpart but also exploit such information for their own advantage. Using a sample of more than one million home purchase loans during the period 2001–2006, the authors empirically confirm that when an originator expects to retain rather than sell a loan, they screening it more carefully. This issue is particularly relevant for low documentation mortgage loans[10], in which mortgage originators rely on information about the borrower's prospects that cannot be easily quantified, and hence is not reported to mortgage buyer[11]. Adelino, Gerardi, and Hartman-Glaser (2017)[1] find evidence of severe presence of adverse selection in the private segment of the residential mortgage market. They report a strong relation between ex-post mortgage performance and time-to-sale, mortgages loans sold 5 months or more after origination are approximately 5 percentage points less likely to default relative to loans sold immediately after origination.

2.4 Government interventions: institutional background

Starting in the mid fifties the U.S. public policy made the provision of affordable housing for citizens one of its main goals[12]. The creation of the government-sponsored entities in the 70's was a fundamental cornerstone in accomplishing such goal. Since then, these institutions have intervened in the mortgage market with the mandate to: i) provide stability and liquidity to the mortgage market by buying loans; ii) insure or guaranteed mortgage loans against default, iii) develop mortgage-backed securities such as pass-through and collateralized obligations, and guarantee these products against default risk; iv) standardize mortgage loan terms and documentation[13].

Significant changes since the development of the secondary market of mortgages in the 70's with the creation of the Government Sponsored Enterprises (GSEs) going from what is known as the *originate and hold* to the *originate and distribute* model of mortgage origination. The GSEs are comprised by three hybrid, public-private institutions, the Federal Home Loan Mortgage Corporation (FHLMC) known as Freddie Mac, the Federal National Mortgage Association (FNMA), known as Fannie Mae, and the Federal Home Loan Bank System, the bulk of the activities is carried out by Freddie and Fannie, so I will refer only to these two entities. Fannie Mae was formally created in 1968, when FNMA was partitioned into a private corporation, Fannie Mae, and a publicly owned institution, Ginnie Mae[14]. Two years later in 1970, the U.S. congress created Freddie Mac to provide competition to Fannie Mae. In the following decades the GSEs were successful to modify the model of mortgage origination from what is known as the *originate and hold* to the *originate and distribute* model. This changed funding of mortgages to a combination of deposits and capital markets instead of deposits only as was the case in the early 50s.

[10] Low documentation loans refer to loans that required little or no documentation about the borrower's income and financial position. Usually this type of loans rely on a collateral that suppose to protect the mortgage originator against borrower's default.

[11] This information is known as "soft" information, as opposed to "hard" information that can be easily verified and quantified. Shimer (2014) argues that to the extend that this information is relevant to predict borrowers repayment capacity this could potentially be a quantitatively significant source of private information.

[12] See Fabozzi and Modigliani (1992) for a historical review of government interventions in the housing market.

[13] The Federal National Mortgage Association Charter Act of 1954 describes the purposes assigned to the GSEs.

[14] This marks the start of the pass-through securitization era (1970-1984) as defined by Campbell and Hercowitz (2005), and Landovigt (2016) who do a brief review of financial regulation changes.

3 Model

This section presents the quantitative model. It is a standard model of credit provision between borrowers and lenders expanded to include a market for debt. There are two main markets of interest: primary market, where lenders extend credit to borrowers, and the secondary market for debt claims, where lenders exchange debt claims among themselves. For the characterization of trading decisions under asymmetric information I follow the approach presented in Kurlat2013. In this environment, trade in secondary market is anonymous, and all debt claims are traded at a pooling price.

3.1 Environment

3.1.1 Preferences and Endowments

Time is discrete and infinite. There are three type of agents: a government, and two type of households with different discount rates as is standard in exchange economies[15]. The impatient household is modeled as a representative consumer[16], with discount rate $\beta^B > \beta^L$, where β^L is the discount factor of patient households, which are modeled as a continuum of mass one. In equilibrium patient households save and lend resources to impatient households, who become borrowers. Hereinafter, we will refer to them simply as borrower and lenders.

Preferences. The representative borrower has preferences over a final numeraire consumption good, C_t, and over the housing services provided from owning a housing stock, K_t.

$$u(C_t, K_t) = (1 - \theta) \log C_t + \theta \log K_t$$

where θ represents the valuation of housing services relative to other non-housing consumption goods.

Lenders are atomistic and denoted by lower case letters and superscript j, each lender j has preferences only over the final consumption good[17]:

$$u(c_t^j) = \log c_t^j$$

Endowments. The borrower households receives an exogenous *income endowment* every period in units of the numeraire good, Y_t. Each period an income realization is drawn from a finite-state Markov process with strictly positive support, and a Markov transition matrix Π_y. Borrowers can transform units of this endowment into final consumption good C_t, or store it in the form of a housing stock K_t that provides housing services proportional to the stock. Lenders

[15] This a standard assumption to ensure exchange of resources among agents in equilibrium. See Kiyotaki and Moorse (1997), Campbell and Hercowitz (2009), Justianiano, Primiceri, and Tambalotti (2014, 2015), among others.

[16] See Werning (2015): "Incomplete Markets and Aggregate Demand", under log preferences (as used here), and certain assumptions regarding the structure of idiosyncratic shocks, an incomplete markets economy with idiosyncratic risk can yield an aggregation consistent with a representative agent's Euler equation, with the effects of market incompleteness inducing a change from the individual to the aggregate discount factor.

[17] As documented by Landvoigt et al, (2013) housing markets are highly segmented, which implies that in practice there is little trading of housing goods between borrowers and savers households. This formulation is equivalent to assuming a rigid housing demand by lender that derive services from a constant housing stock.

do not have an income endowment, they receive income in the form of cash flows from their debt claims extended to borrowers.

3.1.2 Assets Technology

Markets are incomplete, there is only one financial asset available to borrowers and lenders in this economy, which is the mortgage loan. A distinction is made between the *existing* mortgage balances denoted by B_t (aggregate), and *newly issued* mortgages denoted by N_t (aggregate). Hence, B_t represents the aggregate stock of debt for the borrower. Lender $j's$ holdings of debt claims are denoted b_t^j so the aggregate stock of debt claims is $B_t = \int b_t^j d\Gamma$

Debt Structure. The mortgage is modeled as long-term debt contract with a fixed-rate and perpetual geometrically declining payments as in Chatterjee and Eyigungor (2015). This motivated by the fact that the most prominent mortgage contract in the US is the fixed-rate 30 years mortgage. Under this type of contract, a fraction ϕ of the remaining principal balance becomes due each period, so that next period's principal balance and payment decay by factor $(1-\phi)$. Additionally, the borrower pays a coupon $\gamma > 0$ proportional to the outstanding value, borrower's recurring payment is $\hat{\phi} = \phi + (1-\phi)\gamma$ fraction of B. Thus, parameters $\{\phi, \gamma\}$ characterize the mortgage bond. New mortgage loans are priced competitively at discounted price q_t, so that at origination a lender gives the borrower q units of the numeraire good, and in exchange the lender receives a stream of payments $\phi(1-\phi)^{s-1}$ units of consumption at time $t+s$[18].

Aggregate default. Default is modeled as given exogenously to the economy by a rate $\lambda \in (0,1)$[19]. At the beginning of each period t, an aggregate exogenous default rate λ_t realizes, it is observed by all agents in the economy, it is assumed to apply at the level of the individual lender, and to the aggregate debt liabilities of borrowers. The interpretation is that a fraction λ of all debt claims in the economy will default with probability one at the beginning of next period, additionally I assume that upon default lenders recover nothing from the defaulting debt claims. A lender j looses fraction λ of its mortgage cash flow in the current period, receiving $(1-\lambda)\hat{\phi}b^j$, and internalizes that the stock of outstanding debt claims rolling onto next period will be $(1-\lambda)(1-\phi)b^j$, see problem of the lender in the next subsection. For the borrower, the default rate reduces its aggregate liabilities but it also reduces its aggregate holdings of the stock of housing good K, so that the default shock does not represent a windfall. This aims to capture in a simplified way the loss of housing equity a borrower experience upon default by entering into foreclosure. The debt structure and the aggregate default rate imply that the the following law of motion for the aggregate amount of debt in the economy:

[18]This representation has the advantage that face value of the all coupon payments is: $F_t = \sum_{t=0}^{\infty} \phi(1-\phi)^t = 1$. Furthermore, after making the first coupon payment ϕ the amount of outstanding debt remaining next period is $F_{t+1} = \sum_{t=1}^{\infty} \phi(1-\phi)^t = 1-\phi$.

[19]Although this a strong assumption, it allows for a clean and stark characterization of the dynamics that connect the primary, and secondary market. Making default endogenous is an exciting task left for future research agenda.

$$B_{t+1} = (1-\phi)(1-\lambda_t)B_t + \int n_t d\Gamma \tag{1}$$
$$= (1-\phi)(1-\lambda_t)B_t + N_t$$

Notice that going forward, a unit bond issued $t \geq 1$ periods in the past has exactly the same payoff structure as another unit bond issued $t' > t$ periods in the past. Thus we only need to keep track of the total number of bonds B, which reduces the number of state variables.[20]Assume λ is drawn from a finite -state Markov process with strictly positive support, and Markov transition Π_λ.

Origination Technology. At the beginning of each period t, a lender draws a lending productivity z_t, which distributes $z \sim i.i.d.$ across time and agents with continuous cdf $F(z)$ in a bounded support $[z_a, z_b]$. The lending technology is linear, each lender j originates new loans size n_t^j at a gross cost $n_t^j z_t^j$. This is a source of idiosyncratic risk for each lender.

3.1.3 Secondary Market for debt claims

There is a secondary market to trade *existing* (outstanding) debt claims $(1-\phi)b_t$ in which only lenders participate, since they are the only holders of debt claims. The market is assumed to be anonymous, and all debt claims trade at a competitive pooling price p. [21]

Private Information. Each lender j has the capacity to identify which of his own debt claims b_t^j are affected by the aggregate default rate λ_t in the current period, but the rest of agents in the economy cannot. This is motivated by a large body of literature that documents that mortgage originators are better informed about the probability of default of the loans they hold than any potential investor. Shimer (2014) summarizes the evidence on sources of private information in the mortgage market in the US. Additionally, it is also assumed that lender $j's$ productivity z_t^jat each period t is private information.

3.1.4 Aggregate Resource constraint:

The aggregate

$$C_t + \int c_t^j dF + \zeta(N_t) + K_{t+1} - (1-\delta)(1-\lambda)K_t \leq Y_t \tag{2}$$

where $\zeta(N_t) = \int (z_t^j - 1)n_t^j dF$ represents aggregate cost of lending in the economy.

3.1.5 State Variables

The exogenous aggregate states of the economy are: $\{\lambda_t, Y_t\}$. Additionally, the joint distribution of debt holdings and lenders' productivity $\Gamma_t(b_t, z_t)$ is also an aggregate state variable[22]. Let X

[20]See Hatchondo and Martinez (2009), Chaterjee and Eyigungor (2012), Bianchi and Mondragon (2018) use this representation of long-term debt contract structure to model sovereign foreign debt.

[21]Why anonymity is fundamental?, what is the interpretation of the anonymity assumption. Elaborate more on the structure of markets.

[22]In the presence of aggregate shocks market-clearing prices change every period, thus households need to forecast prices. The distribution becomes a state variable because prices are a function of aggregates, which are computed integrating the joint distribution.

denote the set of aggregate states in the economy, then $X = \{\lambda, Y, \Gamma\}$.

3.2 Decision Problems

3.2.1 Borrower Household

$\{B_t, K_t\}$ are the endogenous states that characterize the problem of the representative borrower, the recursive formulation of the borrowers household problem is:

$$
\begin{aligned}
V(B,K;X) &= \max_{\{C,N,B'\}} u(C,K) + \beta^B \mathbb{E}_{X'} V(B',X')|X && (3)\\
s.t. & \\
C + K' - (1-\delta)(1-\lambda)K &= Y - \hat{\phi}(1-\lambda)B + q(X)N && (4)\\
B' &= (1-\phi)(1-\lambda)B + N && (5)\\
B' &\leq \pi K' && (6)\\
& \quad N \geq 0,\ K \geq 0 \quad \text{given } \{B_0, K_0\}
\end{aligned}
$$

The borrower problem is to choose sequence of consumption C_t, housing K_{t+1}, and new mortgage debt N_t to maximize life-time utility subject to its budget constraint, eq(4). Notice that the exogenous default rate affects the borrower in three ways: first, it reduces his current payment of debt $(1-\lambda_t)\hat{\phi}B_t$ which translate in a loss to lenders who holds the corresponding debt claims; second it reduces the remaining stock of debt in the law of motion; and third it also reduces the current stock of un-depreciated housing good $(1-\lambda_t)(1-\delta)K_t$ so that the default rate does not represent a windfall for borrowers[23]. Additionally, the stock of housing good depreciates at rate δ. Borrower sources of income are his endowment, Y_t, and the amount of new debt N_t discounted at price q_t.

Equation (5) represents the evolution of the outstanding mortgage debt, at the end of the period the outstanding mortgage debt is the sum of the remaining mortgage debt after default and new borrowing N_t. The borrowing constraint in equation (6) restricts the total amount of debt B_{t+1}at the end of the period to a fraction π of new level of housing stock.

3.2.2 Problem of the lender

Given that lenders are atomistic, the set of individual endogenous states that characterize a lender's problem is $\{b^j, z^j\}$, then the recursive representative the problem of a lender is:

[23] This is simple way of modeling exogenous default. Given that mortgage loans are collateralized debt contracts, when a borrower defaults on its debt obligations the collateral gets foreclosed by the lender. Thus, effectively in the aggregate borrowers lose an equivalent fraction of its housing equity.

$$
\begin{aligned}
V(b,z;X) &= \max_{\{c,n,b',d,s_G,s_B\}} u(c)+\beta^L \mathbb{E}_{X'}V(b',z',X')|X && (7)\\
s.t. & \\
c+znq+pd &\leq (1-\lambda)\hat{\phi}b+p(s_G+s_B) && (8)\\
b' &= (1-\phi)(1-\lambda)b+n+(1-\mu)d-s_G && (9)\\
& n\geq 0 \quad d\geq 0 \\
& s_G\in[0,(1-\lambda)(1-\phi)b] && (10)\\
& s_B\in[0,\lambda(1-\phi)b] && (11)\\
& \text{given } b_0>0
\end{aligned}
$$

I suppress the dependence of each variable on history X^t for ease of notation[24]. Equation 8 describes lender's flow of funds constraint, a lender j's income are the mortgage payments due this period after taking into account losses incurred by the default shock, and the cash a lender obtains for selling good and bad loans in the secondary market at price p[25]. Lender allocates resources to consumption c, origination of new loans n at discounted price q using his linear origination technology z, and to purchases d of loans in the secondary market .

Equation 9 represents lender's law of motion of debt claims, tomorrow's portfolio will be given by: the outstanding non-defaulting $(1-\phi)(1-\lambda)b^j$ debt claims, plus new originated loans n, plus new purchases of loans in the secondary market d taking into account that only a fraction $(1-\mu)$ will payoff due to the adverse selection problem present in the secondary market. Notice that s_B does not show up on the law of motion of debt claims, this is because of the assumption that the defaulting fraction λ has a recovery value of zero, i.e a lender effectively writes it off the book next period, so it will not show up tomorrow in the portfolio. However, s_B shows up today in the budget constraint because a lender can sell the debt claim in the secondary market before it is taken off the books. The assumption of private information about which debt claims are affected by λ is key, $\{s_B, s_G\}$ is a distinction that each individual lender makes on his own books, while an outsider will only observes s_B+s_G.

Lastly, the last restrictions correspond to the non-negativity over origination, purchases, and restrictions over the amount of debt claims a lender can sell.

$\{p,q,\mu\}$ are equilibrium objects, μ is known in the literature as the *adverse selection discount*, it correspond to the aggregate fraction of bad loans supplied in the secondary market.

[24] Notice that due to asymmetric information lenders do not observe the full state of the economy, meaning they do not observe other lenders' productivity, they only observe a subset of the state of the economy that includes aggregate shocks and their realized productivity $\{\lambda_t, Y_t, z_t^j\}$. Thanks to the assumption that lenders' idiosyncratic shocks distribute *i.i.d* in time the endogenous aggregate variables $\{q_t, p_t, \mu_t\}$ that are relevant for a lender's decision depend only on aggregate states in X.

[25] Remember that for outstanding amount of debt b_t^j, borrower's coupon obligations are $\gamma(1-\phi)b^j$ and payment-of-principal obligations will be ϕb^j, let $\hat{\phi}b^j = [\phi+(1-\phi)\gamma]b^j$ represent total income from outstanding debt claims received by a lender.

3.3 Equilibrium Definition

An equilibrium in this economy consist of sequences of: prices $\{q(X), p(X)\}$; adverse selection discount $\{\mu(X)\}$; allocations for borrowers $\{C, N, B', K'\}$; allocations for lenders $\{c^j, n^j, d^j, s^j_G, s^j_B\}_{j\in J}$ such that given initial endowments $\{B^B_0, \{b_0\}\}$; a law of motion $\Gamma'(X)$, its transition density $\Pi(X'|X)$; and value functions $\{V(B, K; X), V(b, z; X)\}$:

1. Borrowers' allocations solve the program in (3) taking as given $\{q(X), p(X)\}$.

2. Lenders' allocations solve the programs in (7) taking as given $\{q(X), p(X), \mu(X)\}$.

3. the price of loans $q(X) > 0$ clears the *Credit Market*:

$$N(q) = \int n(q, p)dF(z) \tag{12}$$

4. whenever $p(X) > 0$ is the *secondary market for debt claims* clears:

$$\int d(b, z, X)dF(z) = \int s(b, z, X)dF(z) \tag{13}$$

   and the *adverse selection discount* $\mu(X)$ is determined in equilibrium by:

$$\mu(X) = \frac{S_B(p)}{S(p)} \tag{14}$$

5. the law of motion of Γ is consistent with individual decisions: $\Gamma'(b, z) = \int_{b'(\hat{b},\hat{z},X)\leq b} d\Gamma(\hat{b}, \hat{z})F(z)$.

The notation for aggregates in the *Secondary Market of debt claims* is as follow: S_G denotes the total supply of good loans, S_B the total supply of bad loans, and S be the total supply of all loans traded in the secondary market, each of these object is defined by:

$$S_G(X) = \int s_G(b, z, X)d\Gamma(b, z)$$
$$S_B(X) = \int s_B(b, z, X)d\Gamma(b, z)$$
$$S(X) = S_G + S_B$$

and let D denote the demand of loans

$$D(X) = \int d(b, z, X)d\Gamma(b, z)$$

4 Model Characterization

This section develops the characterization of the model. Given the environment it is possible to characterize the individual lender's decisions in the secondary market.

4.1 Characterization of lender's decisions

The dynamic problem in 7 shares similar characteristics to the problem exposed in [22] Kurlat (2013), the main difference is that in this environment lenders must also take into account the price of debt q into their policy functions. However, it is still possible to characterize lender's decisions in closed form. Here, I follow the strategy developed by [22] Kurlat (2013), namely: first, I show that all lender's policy functions are linear in debt holdings b. Second, I show that given choices of c and b', decisions $\{n, d, s_{G,}, s_B\}$ are obtained by solving a linear program which leads to corner solutions. Third, I transform the problem of the lenders into a relaxed problem that allows a simple characterization of the consumption-savings decisions, and how to derive demand, and supply of debt claims in the secondary market. Fourth, I show how the dynamics in the secondary market are connected to the primary market and borrower's decisions.

4.1.1 Aggregate states and linearity of policy functions

Aggregate states X follow a joint distribution $\Theta(X) \equiv \Theta(\lambda, Y, \Gamma)$ with law of motion $\Theta'(X') = \int \Pi(X'|X)d\Theta(X)$, where $\Pi(X'|X)$ is the transition density associated to the law of motion. Additionally, the law of motion of Γ to need be consistent with individual decisions: $\Gamma'(z, b)(X) = \int_{b'(\tilde{b},\tilde{z},s)} d\Gamma(\tilde{b}, \tilde{z})F(z)$.

Under the assumption that $z \sim i.i.d$ across agents, the joint distribution of debt holdings and idiosyncratic shocks $\Gamma(b, z)$ at time t can be written as the product of the distribution of idiosyncratic shocks F and the distribution of asset holdings G across lenders: $\Gamma_t(b_t, z_t) = G(b_t) \cdot F(z_t)$ this means that the stock of a lender's debt holdings does not affect its probability of obtaining a particular realization of z.

Furthermore, assuming that $z \sim i.i.d$ across time implies that z does not correlate with $\{\lambda, Y\}$ then:

$$\begin{aligned}
\Theta(\lambda', Y', \Gamma'|\lambda, Y, \Gamma) &= \Theta(\lambda', Y', \Gamma'|\lambda, Y) \\
&= \Pi(\lambda', Y'|\lambda, Y)\Gamma(b', z') \\
&= \Pi(\lambda', Y'|\lambda, Y)G(b')F(z')
\end{aligned}$$

Thus, the law of motion of the aggregate states will be determined by the law of motion of the exogenous states $\Pi(\lambda', Y'|\lambda, Y)$.

Linearity of policy functions. The recursive problem in 7 has two main properties: i) the constraint set is linear in debt claims holdings b, and ii) preferences are homothetic, given the assumption $u(c) = \log(c)$. Hence, lender's consolidated wealth is proportional to assets holdings, and preferences imply that consumption, and saving decisions will be a constant fraction of wealth. This implies that policy functions for all lender's decisions $\{c, b', s_G, s_B, d\}$ are linear in holdings of debt claims b. This is summarized in Lemma 1.

Lemma 1. Aggregate debt B is a sufficient statistic to predict prices and aggregate quantities, in particular these do not depend on the distribution of debt claim holdings only on aggregate debt B.

Furthermore, Lemma 1 implies that the minimum relevant set of states needed to predict aggregate debt holdings next period is:

$$X = \{\lambda, Y, B, K\} \tag{15}$$

4.1.2 Origination and trading policy functions $\{n, d, s_G, s_B\}$

It is useful to characterize trading decisions in the secondary market separately from consumption-savings decisions $\{c, b'\}$. Taking savings $\{b'\}$ as given the problem of a lender j, eq 7, consist in maximizing consumption c by choosing $\{d, n, s_G, s_B\}$ which implies solving a linear program. This can be seen by combining the budget constraint eq 8, and the law of motion of debt holdings eq9 together, which yields

$$c^j = (1-\lambda)b^j\left[\hat{\phi} + (1-\phi)z^j q\right] + s_B^j p + s_G^j\left[p - z^j q\right] - d^j\left[p - z^j q(1-\mu)\right] - z^j q b'^j$$

Since, each lender j takes as given prices $\{p, q\}$ and secondary market conditions μ, trading decision are easily derive by comparing static payoffs: sales of bad loans s_B will be positive as long as $p > 0$, since a lender have not incentive to keep a non-performing loan with zero recovery value, hence it will choose to sell all the bad loans in his portfolio hitting the corner in eq11: $s_B = (1-\phi)\lambda b$. The decision to sell good loans s_G will be based on how lender's origination productivity qz^j compares to the price of selling loans p, respecting budget set constraint eq 10:

$$s_G = \begin{cases} (1-\lambda)(1-\phi)b & \text{if} \quad z < \frac{p}{q} \\ 0 & \text{if} \quad z \geq \frac{p}{q} \end{cases}$$

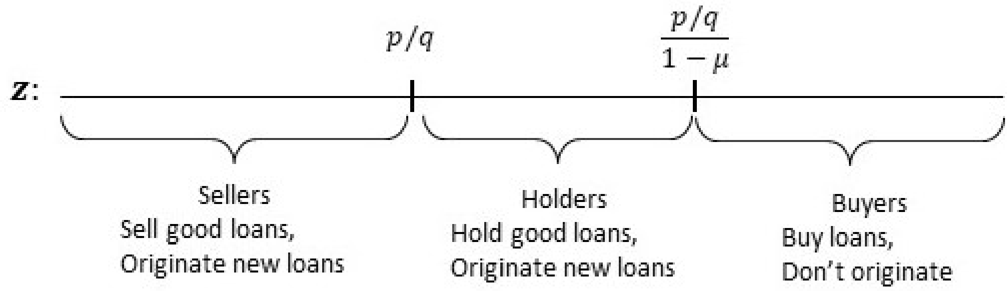
The decision to purchase loans in the secondary market d also depends on how lender's origination productivity qz^j compares to the cost of obtain a unit of an existing loan in the secondary market $\frac{p}{1-\mu}$. A lender understands that he is buying a composite of all the loans supplied in the secondary market, and because all participants have incentives to sell all their bad loans a fraction μ of them will not perform[26]. Consequently, the effective cost of obtaining loans in the market $\frac{p}{1-\mu}$ is higher than the market price p:

$$d = \begin{cases} > 0 & \text{if} \quad z > \frac{p}{q}\frac{1}{(1-\mu)} \\ 0 & \textit{otherwise} \end{cases}$$

Figure 2 summarizes lenders' trading decisions in the secondary market according to cut-offs: $\{\frac{p}{q}, \frac{p}{q}\frac{1}{(1-\mu)}\}$ which split the support of z in three intervals.

[26]Recall that μ is defined as the ratio of total supply of bad loans to the total supply of loans in the market. Thus, effectively it is the rate of non-performing loans obtained in the market.

Figure 2: Lenders' trading decisions in the Secondary Market



Lenders' trading decisions lead them to self-classify in three groups: sellers, holders, and buyers. Sellers are lenders with low-z, $z \in [z_a, p/q]$, they can originate new loans at a low cost, because of this, they have incentives to sell their entire outstanding portfolio in the secondary market and use the proceeds to originate new ones. Buyers are lenders with a high-z, $z \in (\frac{p/q}{1-\mu}, z_b]$, for them originating new loans is very costly, the market gives them opportunity to buy loans from other lenders at a lower price -after considering the adverse selection discount- relative to their origination cost. Thus, they choose to buy instead of originating new loans. Holder, are lenders that fall between the cut-offs, $z \in [p/q, \frac{p/q}{1-\mu}]$, given their z-productivity the market price is not high enough to induce them to sell good loans, and due to the adverse selection, the discount the price they must pay for buying is too high, so they choose to hold onto their good loans and do not buy loans in the secondary market. In consequence, if they decide to originate new loans the must do so using their technology.

Lemma 2. Given a lender's savings b', if there exists a positive market price for loans $p > 0$, optimal trading decisions $\{n, d, s_G, s_B\}$ are given by:

Table 4: Trading and lending decisions

	$z < p/q$	$z \in [p/q, \frac{p/q}{1-\mu}]$	$z > \frac{p/q}{1-\mu}$
d	0	0	$\frac{b'-(1-\lambda)(1-\phi)b}{1-\mu}$
s_G	$(1-\lambda)(1-\phi)b$	0	0
s_B	$\lambda(1-\phi)b$	$\lambda(1-\phi)b$	$\lambda(1-\phi)b$
n	b'	$b' - (1-\lambda)(1-\phi)b$	0

if there is no positive price that clears the secondary market, trading decisions are $d = s_G = s_B = 0$, and a lender origination decision is $n = b' - (1-\lambda)\phi b$. Respecting the non-negativity constraints $n \geq 0, \quad d \geq 0$.

Lemma 2 summarizes trading and lending decisions for lenders. Trade in secondary markets is essentially an alternative saving technology to origination. When the secondary market is active, some lenders can specialize in originating loans, and others simply buy existing loans, meaning they find profitable to save through the market instead of originating new loans. If the secondary market is not active, this alternative technology is not available to any lender. Consequently, all lenders save using their own origination technology.

4.1.3 Consumption savings policy functions $\{c, b'\}$

As shown before differences in the origination technology across lenders induce different trading decisions, in turn different trading decisions imply different budget sets for a lender. Figure XXX in Appendix A.2 shows that the budget set for lenders that turn out to be buyers or holders is not convex. This is because the marginal rates of substitution is not only different across agents but also different between possible equilibrium outcomes in the secondary market. [22]Kurlat (2013) shows that in this environment it is possible to characterize consumption-savings policy functions by defining an extended budget set that is convex. For this Kurlat introduces the concept of an agent's *virtual wealth*. This approach allows to: i) set up a relaxed problem for any lender type j, and derive consumption-savings policy rules as functions of the an agent's virtual wealth before the realization of their idiosyncratic productivity z; and ii) analytical characterization of aggregate supply and demand of loans in the secondary market. Furthermore, the solution to the relaxed problem will coincide with the solution of the original problem whenever the secondary market for loans is active, i.e there is a positive price p that clears the market. If there is no such positive price, the relaxed problem can also be used to obtain consumption-savings policy functions without trade in the secondary market.

Define a lender's virtual wealth as:

$$W(b, z, X) = b\left[(1-\lambda)\hat{\phi} + \lambda(1-\phi)p(X) + (1-\lambda)(1-\phi)\max\{p, \min\{zq, p/(1-\mu)\}\}\right] \quad (16)$$

The virtual wealth represents lender's consolidated wealth as a generic function of its own productivity z, prices $\{q, p\}$, and originating and trading decisions $\{n, d, s_G, s_B\}$. It consolidates lender's sources income, income from its maturing debt claims, from selling bad loans in the secondary market, and the virtual valuation of its portfolio loans, either at the market price, or at the lender's internal rate which depends on its own origination productivity. Using eq 16 we can define a convex budget set that is weakly larger that the original budget set in program 7, this relaxed program is given by:

$$V(b, z; X) = \max_{\{c, b'\}} \left[\log(c) + \beta^L \mathbb{E}_{X'|X} V(b', z'; X')\right] \quad (17)$$

$$s.t.$$

$$c + b' \min\{zq, p/(1-\mu)\} \leq W(b, z, X)$$

Given lender's logarithm utility function, the optimal consumption-saving decision rule will be to save a constant fraction β^L of its wealth, and consume the rest. Lemma 3 presents the policy functions for a lender j as function of their virtual wealth.

Lemma 3. The optimal consumption and savings policy functions that solve program 17 are given by:

$$c = (1-\beta^L)W(b, z, X) \quad (18)$$

$$b' = \beta^L \max\{zq, p/(1-\mu)\}W(b, z, X) \quad (19)$$

4.1.4 Equilibrium in Secondary Loan Market

Supply of loans in the secondary market is given by integrating the policy function of sales of good loans, and bad loans from all lenders. Eq 20 is obtained by using sales decisions, as described in Lemma 2, across the distribution of lenders[27].

$$\begin{aligned}S(p,q;X) &= S_B(X) + S_G(X) \\ &= \int s_B^j(b,z,X)\ d\Gamma(b,z) + \int s_G^j(b,z,X)\ d\Gamma(b,z) \\ &= B(1-\phi)\left[\lambda + (1-\lambda)F(p/q)\right]\end{aligned} \tag{20}$$

The adverse selection discount μ is defined by eq (14):

$$\mu(p,q;X) = \frac{\lambda}{\lambda + (1-\lambda)F(p/q)} \tag{21}$$

Demand of loans comes from integrating purchases from buyers, for this we use lender's savings policy function, eq 19, and purchasing decisions from Lemma 2[28].

$$\begin{aligned}D(p,q;X) &= \int d^j(b,z,X)\ d\Gamma(b,z) \\ &= \int_{\frac{p/q}{1-\mu}}^{z_b} \int_b \frac{b' - (1-\lambda)(1-\phi)b}{1-\mu}\ dG(b)dF(z) \\ &= B\left(1 - F\left(\frac{p/q}{1-\mu}\right)\right)\left[\beta[(1-\lambda)(\frac{\hat{\phi}}{p} + \frac{1-\phi}{1-\mu}) + \lambda(1-\phi)] - \frac{(1-\lambda)(1-\phi)}{1-\mu}\right]\end{aligned} \tag{22}$$

Notice that demand is only well defined for $\mu < 1$, when $\mu = 1$, demand will be zero.

Market clearing requires:

$$S(p,q;X) \geq D(p,q;X) \text{ holding strict whenever} p > 0 \tag{23}$$

Lemma 4. $D > 0$ only if the solutions to programs eq(7), and eq(17) coincide for all lenders.

The solutions to programs eq(7), and eq(17) will differ whenever a lender chooses an allocation outside its budget set in eq(7). If this is the case, then demand for loans in the secondary market will be zero, and the price must also be zero.

Volume of trade in Secondary Markets

The volume of trade is computed as the fraction of assets belonging to sellers and holders that is sold in the secondary market. I restrict to sellers and holders because those are the mass of agents that originate loans, in the data I measure volume of trade as the volume of loan sales from originators into the secondary market. Two statistics are computed, the fraction of all loans traded x_T -which includes good and bad loans-, and the fraction of good loans traded x_G.

[27] See Appendix A.1 for derivation.

[28] See Appendix A.1 for derivation.

$$x_T = \frac{S(X)}{(1-\phi)\int_b \int_{z_a}^{\frac{p/q}{1-\mu}} b^j \; d\Gamma} \tag{24}$$

$$x_G = \frac{S_G(X)}{(1-\lambda)(1-\phi)\int_b \int_{z_a}^{\frac{p/q}{1-\mu}} b^j \; d\Gamma} \tag{25}$$

The denominator of x_T, and x_G, represent integral of holdings of outstanding loans across sellers and lenders.

An alternative measure correspond to the ratio of all assets traded in the secondary market to all assets oustanding in the economy:

$$x = \frac{S(X)}{(1-\phi)B}$$

4.1.5 Primary Credit Market

The equilibrium in the primary market is determined by the market clearing condition, eq (12), that equates borrowers demand of credit to lenders supply of credit. Borrower's demand of credit is derived by obtaining borrower's policy function of aggregate debt $B'$from solving program eq(3). Using the law of motion of borrower's aggregate debt yields its demand of credit:

$$N^D(q;X) = B'(B,K;X) - (1-\lambda)(1-\phi)B \tag{26}$$

The supply of credit is derived by aggregating lender's lending decisions as given by Lemma 2, where lending policy functions are defined for two scenarios. One in which the secondary market for loans is active -loans trade at a positive price-, and another when it is not active and the price of loans is zero. In the first case, lenders that become sellers and holders originate new loans, the total mass of originators is given by the mass over the interval $[z_a, \frac{p/q}{1-\mu(p/q)}]$, see Figure 2. Hence, the total supply of credit when the market is active corresponds to eq(27). For the case in which the secondary market is not active, aggregate supply will be given by the integral of lending decisions over all lenders in the interval $[z_a, \bar{z}(q)]$, where $\bar{z}(q) = \min\{z_b, \frac{\beta^L\hat{\phi}}{(1-\beta^L)(1-\phi)}\frac{1}{q(X)}\}$. This is summarized in Lemma 5.

Lemma 5. Credit supply in the primary credit market is contingent on the equilibrium outcome achieved in the secondary market.

1. *If there is a price $p > 0$ that clears the secondary market, eq(23), credit supply in the primary market is given by*

$$N^S(p,q;X) = \int_{z_a}^{\frac{p/q}{1-\mu}} n^j d\Gamma(b,z) \tag{27}$$

2. *If instead, the only price that satisfy eq(23) is $p = 0$, credit supply in the primary market*

is given by

$$N^S(p,q;X) = \int_{z_a}^{\bar{z}} n^j d\Gamma(b,z) \tag{28}$$

3. *Credit supply is strictly increasing in p, whenever* $p > 0$.

4. *Credit supply is strictly decreasing in* q.

Market clearing in the primary market requires:

$$N^D(q;X) = N^S(p,q;X) \tag{29}$$

Proposition 1.

1. *In any recursive equilibrium, functions* $\{p(X), q(x)\}$ *satisfy market clearing conditions: eq(23)-(29) for all* X.

2. *There exists at least one pair of functions* $\{p(X), q(X)\}$ *that satisfies eq(23)-(29), with* $p(X) \geq 0$, *and* $q(X) > 0$.

Proposition 1 establishes that a recursive equilibrium exists, and that there is always a pair of prices $\{p, q\}$ that satisfies the market clearing conditions in both the primary credit market, and the secondary loan market. The weak inequality for p was established in Lemma 4, the price of loans in the secondary market can be positive, or it can be zero if the solutions to programs eq(7), and eq(17) do not coincide. The price of lending in the primary credit market q is always strictly positive.

The model is fully characterized by the solution to the representative borrower's problem, eq(3), the policy functions to each individual lender problem eq(18)-(19), the market clearing conditions for each market eq(12)-(13), and the aggregate resource constraint of the economy, eq(2). I jointly solved for equilibrium prices $\{p(X), q(X)\}$, and the problem of the borrower. See section B for the computational algorithm to solve the model.

5 Model Properties

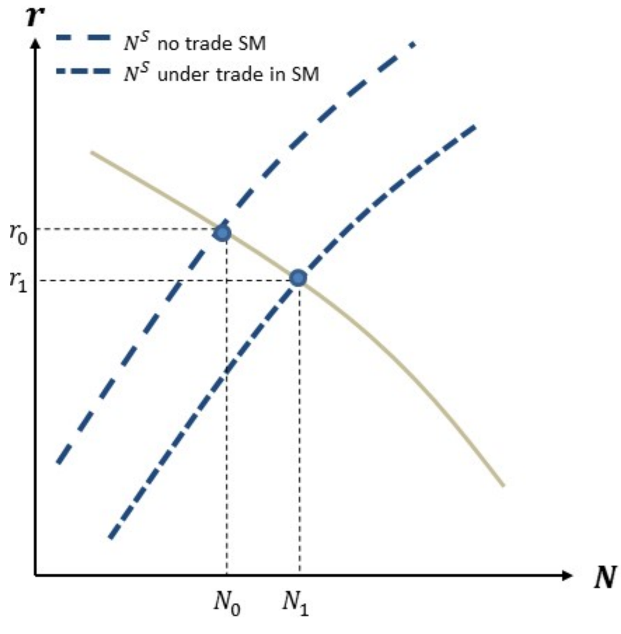
5.1 The role of the Secondary Market

The secondary market serves two main purposes in the economy: first, it performs liquidity transformation of a lender's iliquid loan portfolio, and second, it allows for an efficient reallocation of resources among lenders. These two intertwined roles determine the balance sheet position of lenders, and the technology available to them for saving. Which in turns, affect the aggregate supply of credit in the primary market, and in equilibrium the price of borrowing and lending in the economy.

Liquidity transformation occurs by allowing lenders with low valuation of their outstanding portfolio to sell it in the market, and obtain cash that they can use in the current period to originate new loans and consume. The consolidated wealth of a lender at any point in time is given by the cash payments from maturing loans, plus the private value of their holdings of outstanding loans, which are iliquid due to the nature of long-term debt. Without a secondary market to trade these iliquid assets, the resources available to a lender to consume and lend are limited to the cash payments from their maturing portfolio. These feature of the model captures the maturity transformation that occurs to loans in the process of securitization.

The reallocation of iliquid resources among lenders occurs due to lender's heterogeneous valuation of their portfolio. The assumption of idiosyncratic shocks that determine individual portfolio valuations introduces motives for trade among lenders. The most productive lenders -those hit by low z - have low valuation of their current portfolio because they have access to a cheap saving technology. For the least productive lenders -those hit by high z - holding iliquid assets is a better saving technology than originating new loans, because their loan origination technology features high cost. In this sense, the secondary market allows a reallocation of iliquid assets from low-z-lenders to high-z-lenders, so that only the low cost -most efficient- lenders originate new loans, and the high cost -least efficient- lenders save by buying existing loans. The reallocation feature is connected to the idea of specialization in financial markets, some financial institutions specialize in originating loans but not on holding them in their portfolios, whether others specialize in holding and investing in long-term assets.

Figure 3: The role of the Secondary Market in Credit Supply



To the extent that this reallocation is carried over, the aggregate costs of lending in the economy will be lower than in the absence of secondary markets. The economy as a whole would be lending and borrowing at a lower aggregate cost. To see this recall that, as characterized in section 4.1.2, high-z-lenders called *buyers* do not originate loans, for these lenders lending directly to a borrower by originating new loans is very costly. In the aggregate, new lending is carried out by the lenders with lowest cost, i.e z productivity to the left of the cut -off $\frac{p/q}{1-\mu(p/q)}$, eq(27). This saves resources burned out in the lending process compared to the scenario in which all lender were to use their origination technology absent the secondary loan market. Now, since lenders consume and save in fixed proportions a fraction of those extra resources will increase savings. In the aggregate, this will be reflected by an expansion of credit supply to borrowers in the primary market as shown in Figure 3.

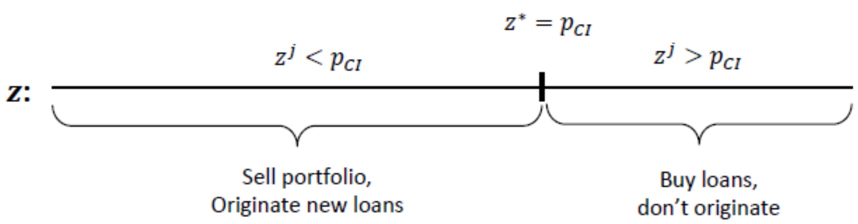
In general equilibrium, changes in the aggregate cost of lending have implications over the price of borrowing and lending, q. From the perspective of lenders, the higher is the cost of lending the higher will be the interest rate charge to borrowers (lower q), and vice-versa. Thus, by facilitating liquidity transformation and reallocation of resources the secondary market also reduces the aggregate cost of lending, which translates into lower interest rates to borrowers.

5.2 The effect of private information

The information friction in this environment reduces the extent to which lenders can trade in the secondary market by introducing a wedge between the price that a seller receives, and the cost that a buyer faces when purchasing a unit of the composite of all loans traded. While a buyer pays p for a unit purchased, she only obtains $1-\mu$ units, because fraction $\mu > 0$ of all traded loans in the market will payoff zero next period. The impossibility of publicly identifying bad loans in the secondary market creates an adverse selection problem, sellers are better informed about the quality of the loans they sell, and they use this information advantage on their benefit by always selling the bad loans, and retaining the good loans if the market price is below their valuation.

Consider the case in which there is complete information, i.e let us assume that all loans affected by λ can be identified by all lenders in the secondary market. Given that bad loans pay zero upon default with certainty their market value will be zero. In this case, there is no adverse selection in the secondary market, since only good loans will be traded at a price different than zero if there is an positive equilibrium price. Under complete information the market discount due to adverse selection is $\mu_{CI} = 0$, so there is no wedge between the price a lender receive and the cost to a buyer from purchasing composite claims in the secondary market. Figure 4 show lenders decisions under complete information. In this case, there is only one cut-off, trading and origination decisions are as before: low-z-lenders sell their entire portfolio in the secondary market to obtain cash and originate new loans; high-z-lenders retain their portfolio, buy debt claims in the secondary market and do not originate new loans.

Figure 4: Lenders trading decisions under complete information



Notice, when the default rate is low, the effects of private information in disrupting trade can be small. However, as default rate shocks increase the adverse selection problem becomes very acute leading to a complete disruption of trade in secondary markets. The following section presents comparative statics of how fluctuations in the aggregate default rate induces fluctuations in the volume of credit, and in the price of credit faced by borrowers.

5.3 Cyclical fluctuations and comparative statics

We want to establish how aggregate shocks to the default rate μ affect aggregate outcomes in the secondary market, and in the credit market. This is the main focus of this paper. First, let $\hat{z}$ be a market cut-off, in equilibrium $\hat{z} \equiv \frac{p}{q}$.

***Lemma** 6. the share of bad loans in the market $\mu(p/q)$ is a decreasing function of λ, and is decreasing in $\hat{z}$.*

See Proof. Notice that Lemma 6 is a result of features of the environment, it indicates that when the default rate λ is high, if there is an equilibrium price, which defines the cut-off $\hat{z}$ the proportion of bad loans in the market will be high. Also, when the equilibrium price in the market is high, i.e loans are highly valued, then proportion of bad loans in the market is low. This is the case because if loans are traded at higher prices in the secondary market, more lenders will sell their entire portfolio, which increases the fraction of good loans in the market and turn reduces the fraction of bad loans.

Assumption A1: $\forall \hat{z} \in [z_a, z_b]$*:*

$$m(\hat{z}) > \frac{1}{\hat{z}}\left[1 + \frac{1-\lambda}{\lambda}F(\hat{z})\right]$$

where $m(\hat{z}) = \frac{F(\hat{z})}{f(\hat{z})}$ *is the mills ratio or hazard rate of* $\hat{z}$*.*

***Lemma** 7. Under assumption A1, the second equilibrium cut-off $\frac{\hat{z}}{1-\mu(\hat{z})}$ is decreasing in $\hat{z}$.*

This Lemma states that as secondary market conditions improve, loans are traded at a higher price (higher $\hat{z}$), the cut-off that indicates the real price paid by buyers gets closer to the price seller receives (first cut-off), in other words the private information wedge is lower. As explained before, this wedge represent the extent to which information frictions in the market impede trade. Thus, shocks that increase the value of loans in the secondary market attenuate the frictions imposed by private information.

Proposition 2. A shock that increases the default rate λ leads to:
Secondary Market

1. *higher proportion of bad loans.*

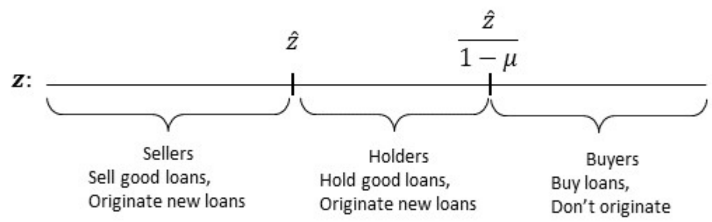
2. *fall in the volume of trade: sales of good loans and purchases.*

Credit Market

1. *fall in aggregate lending.*
2. *increase in the interest rate (fall in the price of lending).*
3. *fall in borrowers consumption.*

Shocks that increase the default rate disrupt the functioning of the secondary market by increasing the proportion of bad loans, which follows from Lemma 1. A higher μ will in turn increase the real price paid by buyers for loans in the secondary market which contracts demand, i.e second cut-off $\frac{p/q}{1-\mu}$ moves to the right in Figure 5. Due to lower demand of loans price must fall in order for the secondary market to clear[29], which implies moving to new equilibrium in which both demand and supply are lower. Consequently, the volume of trade is lower, are a lower price more lenders retain loans, i.e more lenders become holders.

Figure 5: Effects of λ shock



Furthermore, as explained in 5.1, the aggregate cost of lending increases when the default rate λ is high, due to a larger mass of lenders holding on their loans and using their own z-technology to originate new loans, less trade in secondary market implies less reallocation of iliquid resources among lenders. In the aggregate, the cost of lending has increased, which induces lenders to reduce lending in the aggregate, i.e lenders would like to save less and consume more today. In the primary market, borrowers needs for credit have not change, and if something have increased due to the λ shock. Therefore, in order to induce lenders to originate the same amount of lending as before the λ shock the price of credit in the primary market must adjust. Lenders will demand a higher interest rate for every unit of consumption lend. Graphically, after both cut-offs in Figure 5 have move outwards, a fall in the price of lending q can move cut-offs in the opposite direction increasing trade and volume of lending.

Secondary market collapse. A corollary of Proposition 1, is that there exists a threshold $\bar{\lambda}(X)$ such that for $\lambda_t > \bar{\lambda}(X)$:

[29]Notice, that Lemma 2 ensures that upon a drop in the price of loans the second cut-off $\frac{p/q}{1-\lambda^m}$ will move to the right.

1. *there is no trade in secondary markets, each lender uses its own technology to originate new loans.*

2. *aggregate cost of lending, and interest rate are higher than when the secondary market operates.*

3. *welfare of borrowers and lenders is lower.*

In the presence of private information, the equilibrium in the secondary market is achieved for shocks below a $\bar{\lambda}$ threshold, this is verified in the numerical simulations, Figure XX in appendix XX shows the distribution of volume of lending across originators for the case in which the secondary market is active and the secondary is not active. Bullet point 2 follows from the analysis of the role of secondary market in the economy, sec 5.1, to allocate iliquid resources efficiently in the presence of heterogeneous productivity shocks. Bullet point 3, is the outcome from numerical simulation after calibrating the model to match key features of the mortgage market in the U.S.

5.4 Policy interventions

This section analyzes government interventions that aim to capture current government policies in the secondary market. I model a government that have access to the two policy instruments: i) an insurance fee on a originated loan paid by lenders, and ii) the degree of market coverage of the insurance policy. I assume government deficit is financed by lump sum transfer to borrowers each period, so that government always balances its budget.

The insurance fee. Government entities insure loans by charging an insurance fee to the originator, the fee is expressed as a surge charge -in basis points- added to the loan interest rate contracted with the borrower. Thus, a lender obtains $r^g = r^* - g_{\text{fee}}$for every insured loan, where r^* is the interest rate contracted with the borrower. In the model, the discounted price of the long-term bond is given by q, let $\gamma(g_{\text{fee}})$ represent the insurance fee in units of the discounted price, then a lender must give up $\tilde{q} = q + \gamma$ units in order to lend a unit of resources.

Insurance coverage in the secondary market. I model the government as insuring investors in the secondary market against losses induced by the default rate due to the adverse selection discount, μ. The way government undoes this losses is by providing a subsidy $\tau > 0$ to buyers of loans in the secondary market. This will reduce the average cost of investors when purchasing loans by reducing the discount from the adverse selection problem, notice that μ and τ are opposing forces. Since $\mu(X)$ is an equilibrium object that depends on the realization of aggregate states X, the insurance policy implies that $\tau(\mu(X))$ must be a function of degree of adverse selection in the secondary market. I model $\tau(X) = \omega\mu(X)$, where $\omega \in [0, 1]$ corresponds to the degree of insurance provided by government policy. When $\omega = 1$, government policy completely offset the adverse selection discount, and all lenders trade. When $\omega = 0$, there is no provision of insurance at all.

Government flow of funds is given by:

$$\gamma N + T^B = \tau p \int_{\frac{p/\tilde{q}}{1-\mu}(1-\tau)}^{z_b} d(b,z)dF(z), \qquad \text{where } \tau = \omega\mu \tag{30}$$

The term γN represents aggregate government revenue from collecting the insurance fee γN. The right hand side represents government expenditures from providing subsidy τ to buyers of loans in the secondary market with market coverage ω. I assume that the government closes its fiscal position by levying lump sum transfers T^B to borrowers, so that every period this lump sum transfers adjust to satisfy government's budget constraint. Thus, government's vector of policy instruments is given by $\{\gamma, \omega\}$.

The budget constraint of a lender j, in program eq 7, modified to include policy interventions changes to:

$$c + zn\tilde{q} + pd(1-\tau) \leq (1-\lambda)\hat{\phi}b + p(s_G + s_B), \qquad \text{where } \tilde{q} = q + \gamma$$

notice that $\tilde{q}$ is the relevant price for lender's decisions, while q is the relevant price for borrower's decisions. The budget constraint of borrower's program, eq 4 , is updated to:

$$C + K' - (1-\delta)(1-\lambda)K = Y - \hat{\phi}(1-\lambda)B + q(X)N + T^B$$

6 Calibration and Numerical Simulations

6.1 Calibration

The model is calibrated in annual frequency for the period 1990-2006. The calibrated parameters are described in Table **??**.

Preferences parameters

For borrowers, the discount rate β^B is set to 0.86 to match the ratio of consumption of non-durables -and services- to disposable personal income from the Flow of Funds, which equals 0.79. The housing preference parameter θ is set to 0.20, so that the ratio of borrower's housing good to non-durable consumption good C/K matches the ratio of consumption of non-durables and services to the ratio of residential real estate 0.4 from NIPA. Annual housing depreciation rate is set to $\delta = 0.03$ as is standard in the literature[30]. The parameter governing the borrowing constraint π is set to 0.45 in order to match the ratio of household's mortgage debt to the stock of residential real estate in the Flow of Funds. Table **??**reports time series averages for all ratios of aggregates.

For lenders, the discount rate $\beta^L = 0.985$ to match the average real risk-free rate[31] obtained from a 1 year T-Bill, for 1990-2006 the average is 1.57%. Table reports average risk free and mortgage rates for the period of analysis.

[30] Campbell and Hercowitz (2005)

[31] In the model lenders do not have access to a risk-free bond, however it is possible to compute the risk-free rate corresponding to a 1 period risk-free bond by computing their stochastic discount factor, based on the aggregate consumption that the family of lenders obtains: $\frac{1}{1+r^f} = \beta^L \mathbb{E}_{X'|X}[U_{c'}/U_c]$, where $U_c = \frac{1}{\int c^j d\Gamma(b,z)}$.

Technology parameters

The mortgage bond is characterized by two parameters $\{\phi, \gamma\}$, ϕ governs the duration of the bond, and γ represents the mortgage coupon payment. Here, I borrow from [14] Elenev, Landovigt and Van Niewerburgh (2016) who estimate these parameters from an mortgage index in the market in order to match duration and the coupon payments structure of a representative mortgage bond given by the Barclay MBS index.

The distribution of productivity shocks across lenders, $F(z)$, is calibrated using microdata from the Home Mortgage Disclosure Act (HMDA) database[32]. I aggregate the volume of mortgage originations in dollar amounts for every lender, and for every year in the database from 1990 to 2017. Tables 10-11 report the average moments of the cross-sectional distribution. In order to match key moments of the distribution, $F(z)$ is modeled as a Beta distribution $F(z; \alpha, \beta)$ characterized by shape parameters (α, β) in a bounded support $[z_a, z_b]$.

- The shape parameters (α, β) are chosen by methods of moments to match the market share of large originators, which is 0.90, and the ratio of mean to median volume of origination 1.01. The choice of these moments is motivated by the analysis of section 2.2.3 where I argue that the main features of the cross sectional distribution of volume of originations are: i) positive skewness driven by the presence of large originators that dominate the market, ii) low mas of large originators: 10% of originators originate volumes above the mean, iii) high concentration in the volume of lending, 90% of the dollar amount of lending is originated by large originators in the market.

- The bounds in the support of the distribution $[z_a, z_b]$ are chosen by calibrating the scale: $sc = z_b - z_a$, and location $lc = z_a$parameters. I normalize the scale to $sc = 1$, and set the location parameter to match the average real mortgage rate of 4.5% for the period 1990-2006. See table 15

Government policy parameters

Government's vector of policy instruments is given by $\{\gamma, \omega\}$. The discounted price fee γ is a time varying function that satisfies:

$$r^g(\tilde{q}_t) = r^*(q_t) - g_{\text{fee}}$$

where $r^*(q_t)$ is the interest rate implied in the discounted price q_t faced by borrowers, and $r^g(\tilde{q}_t)$ is the net interest rate obtained by the lender after subtracting the guarantee fee. Using the definition of $\tilde{q} = q + \gamma$, a relation between γ and the g_{fee}can be derived:

$$\gamma = \tilde{q} - \left(\frac{g_{\text{fee}}}{\hat{\phi}} + \frac{1}{\tilde{q}} \right)^{-1}$$

I calibrate the guarantee fee, g_{fee}, to 20 basis points, since this was the average for the period 1990 to 2006, as reported by Fannie Mae. The parameter governing the degree of insurance in

[32] HMDA requires covered depository and non-depository institutions to collect and publicly disclose information about applications for, originations of, and purchases of home purchase loans, home improvement loans, and refinancing.

the secondary market, ω, is set to the average market share of GSEs of all sales of mortgages in the secondary market which is about 70% for the period 1990 to 2006.

Aggregate Exogenous Processes

The exogenous processes in the model are given by borrower's income process Y, and the default rate λ. For income I use the cyclical component of the Disposable Personal Income from the Flow of Funds account. The default rate corresponds to the national delinquency rate for mortgage loans that are 90 or more days delinquent, or went into foreclosure. I estimate a Vector Autoregression model of first order, VAR(1), for the period of analysis 1990-2006.

Then, I approximate a joint Markov process for the VAR using the estimated matrix of autocorrelations, and the covariance matrix associated to the residuals[33]. See section A.0.3 in the appendix for the estimation results, and the state space and transition matrix that characterize the Markov process.

Table **??** summarizes the calibration of the model.

6.2 Model performance

The model fits the data in three dimensions: i) in the primary market the model does a good job fitting the skewness and the market share of large originators -mass of lenders above the cross-sectional mean- of the cross-sectional distribution; ii) the fit for aggregates in the secondary market is remarkably close given that none of those moments are targeted, and iii) the model falls off from fitting the volatility of the mortgage rate, but it fits well the level.

[33] The discretization procedure is similar to the usual discretization process presented in Tauchen (1986), but extended for the general-multivariate-normal case, see [29] Terry and Knoteck II (2011).

Table 5: Model vs Data, moments

Description	var	value	Target	Var	Data	Model
Preferences						
lenders discount factor	β^L	0.985	risk free rate, 90-06.	r^f	1.57%	1.52%
borrowers discount factor	β^B	0.92	FoF, cons. ndur + serv to DPI, 90-06.	C/Y	0.79	0.84
housing expenditure share	θ^B	0.20	FoF, cons. ndur + serv to RE, 90-06.	C/K	0.40	0.41
housing good depreciation rate	δ	0.03	FoF, RE invest to RE stock, 90-06	I/K	0.03	0.035
Loan to value ratios	π	0.43	FoF, mortgage debt to RE ratio, 90-06.	B/K	0.43	0.43
Mortgages						
life of mortgage pool	ϕ	0.30	Average maturity of mortg bond index.	b_{dur}	3.7	3.7
mortg coupon payment	γ	0.12				
Lenders technology						
$F(z)$ shape	α	4.2	mkt shr (large inst)		0.90	0.84
	β	2.25	mean/med volume of loan issuance		1.01	0.95
$F(z)$ location	lc	0.65	Freddie Mac, 30Y FRM real, 90-06. Scaled parameter normalized, $sc = 1$.	r^m	4.50%	4.61%

Table 6: Model vs Data, moments

Description	var	Target	Variable	Data	Model
Exogenous processes, $s = (\lambda, Y)$					
State Space $(n_\lambda, n_y) = (3, 3)$	$\mathcal{S}_{n_Y \times n_\lambda}$	λ, default rate, 90-06	mean	1.33%	1.33%
		""	std	0.19%	0.20%
Transition Matrix	Π	Y, cyclical component DPI, 90-06	mean	1.0	1.0
		""	std	0.88%	0.88%
		correlation	corr	-0.21	-0.18
Gov policy					
Policy coverage	$\omega = 0.6$	Market share of GSEs of mortgage sales in the SM, 90-03 & 90-16			-
Insurance fee	$g_{\text{fee}} =$ 20 bps	Average insurance fee, Fredie Mac and Fannie Mae, 90-06			-

NOntargeted moments

Table 7: Non-targeted moments

Description	Model	Data	Description
fraction of sales in SM	0.90	0.62	% home mortg sold in SM, HMDA 90-06
corr (sales, lending)	0.81	0.98	Time series, HMDA, 90-06
New lending, N/K	0.13	0.05	FoF, new lending to GDP, 90-06
Value added of lenders	1.78%	3.07%	BEA, VA credit intermd to GDP, 90-06

6.3 The great recession

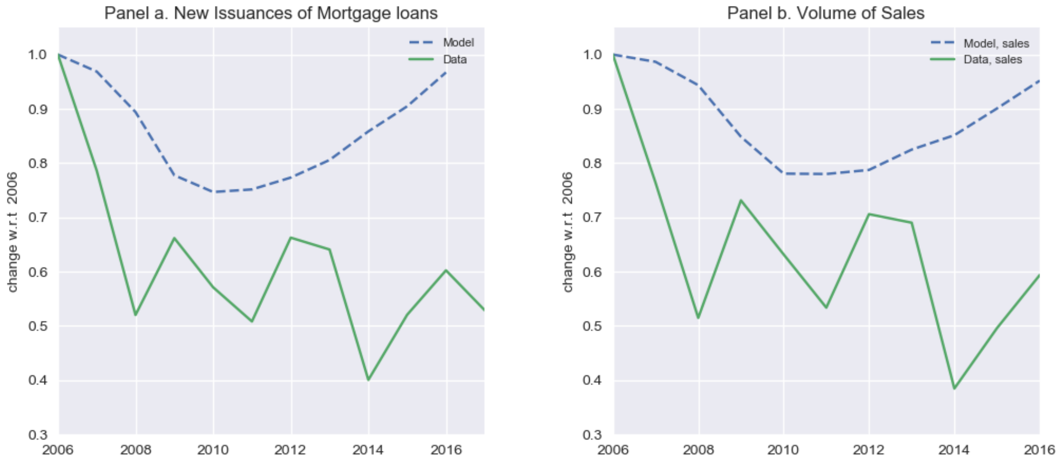
The goal of this exercise is to study the out of sample predictions of the model. The baseline calibration corresponds to the period 1990-2006 as described in the previous section. In order to evaluate the performance of the model during the great recession I feed into the model the sequence of realized shocks for aggregate households' income and default rate from 2001 to 2016 as shown in Figure 6.

Figure 6: Income and default processes

Panel a. Household Income corresponds to the cyclical component of Disposable Personal Income from NIPA.
Panel b. Default rate corresponds to the percentage of delinquent mortgage loans 90 days or more, or in foreclousure. Source: National Mortgage Database, FHFA.

The model is consistent with the magnitude of the decline of macroeconomic aggregates during the great recession. Figure 7 shows percentage changes with respect to 2006, the volume of sales in the secondary market fell by 40% on average between 2006 and 2013, the model predicts a decline of 20% during the same period. In the primary market, the volume of new issuance of mortgage loans also fell by 40% on average from 2006 to 2013, the model predicts a contraction of 25% during the same period.

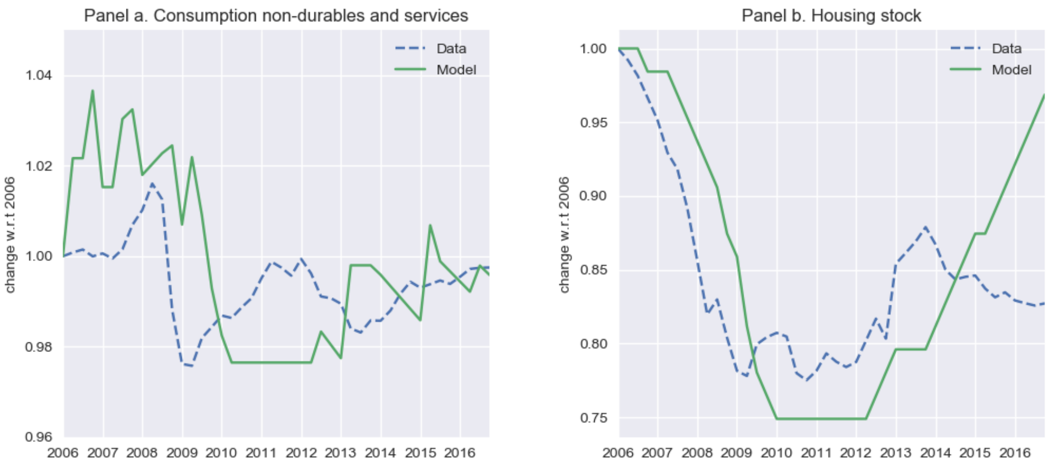
Figure 7: The Mortgage market during the great recession



Panel a: Data, N Agg is the aggregate volume of new mortgage issuance in a given year in dollar amount. Data N x-section is the average volume of new mortgage issuance per mortgage reporter in a given year in dollar amount. Source: HMDA database.
Panel b. Data, Sales corresponds to the aggregate volume of sales of mortgage loans in the secondary market in a given year in dollar amount. Source: HMDA Database. Data, RMBS issuance is the aggregate volume of new issuance of residential mortgage backed securities in dollar amount. Source: Security Industry and Financial Association (SIFMA). All data series have been deflated to prices of 2015. Figure shows percentage change with respect to 2006.

Figure 8 shows percentage changes with respect to 2006 for households' aggregates. The consumption of non-durables falls by 2% in 2009 then it dwindles until 2016 when it reaches the 2006 levels. The model generates fall of the same magnitude with a one year lag, consumption falls by 2% starting in 2010, it stays down until 2013 and then slowly recovers to 2006 levels by the end of 2016. For the ratio of residential real estate to income the patterns of data and model are remarkably similar. The data shows a fall of 23% starting in 2009 until 2013, the model is able to generate a fall of 25% for the same period. Afterwards the model predicts a recovery, which in the data is interrupted from 2014 onward.

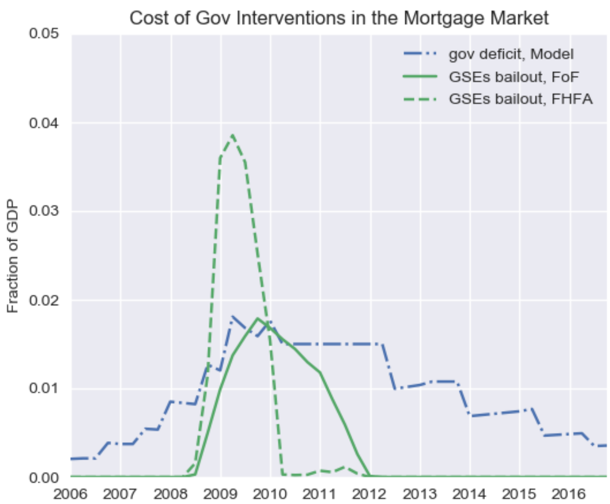
Figure 8: Households aggregates during the great recession



Panel a: Data is the ratio of consumption of non-durable plus services to Disposable Personal Income from NIPA.
Panel b: Data is the stock of residential real estate in prices of 2015 from the Flow of Funds. All series are shown in percentage change with respect to 2006.

Figure 9 shows an estimate of the cost of the intervention predicted by the model versus the aggregate cost of government interventions related to the GSEs during the great recession. I present two estimates; the first is *GSEs bailout, FHFA* which corresponds to the all Federal government resources committed to support mortgage markets through the GSEs as reported by the Department of Treasury and the Federal Reserve System. It includes: i) Senior Preferred Stock Purchase Agreements, ii) purchases of MBS guaranteed by GSEs, and iii) purchases of debt securities issued by GSEs. The second, GSEs bailout, FoF corresponds to an aggregate of Federal government holdings of Agency and GSEs-backed securities as reported in the Flow of Funds account, table L.106. In the model, *government deficit* corresponds to the transfers levied on borrowers in order to provide insurance in the secondary market net of the revenues collected from the insurance fee.

Figure 9: Cost of interventions in the Secondary Market



The model predicts the cost of the intervention is around 2% of aggregate output in the economy from 2009 until 2012, which is in the ballpark of estimate outlays made by the the Federal government during the great recession episode.

7 Policy experiments

7.1 Increasing Insurance fee

Many academics and policy makers, have argued that the levels of the guarantee fee did not reflect the risk-adjusted price of providing full insurance in the secondary market, which implied a net transfer of resources from borrower's to lenders. The first experiment consist in evaluating the effect of increasing the guarantee fee so that it reflects the cost of the policy to lenders.

Table 8: Policy experiment: increasing guarantee fee

	Benchmark $g_{\text{fee}} =$ 20 *bps*	$g_{\text{fee}} = 50$ *bps*	$g_{\text{fee}} = 85$ *bps*
Mortgage spread %, $r^m - r^f$	3.07	3.36	3.71
Fraction of loans traded	0.90	0.90	0.90
Gov. policy: subsidy to purch, τ^s	3.2%	3.2%	3.2%
Borrower's share of Tax bill	0.76	0.41	0.00
Δ% Borrower welfare	-	-0.34	-0.75
Δ% Non-durable consumption	-	0.01%	0.04%
Δ% Housing good consumption	-	-1.27%	-2.87%
Δ% Debt levels	-	-1.26%	-2.86%

*Moments obtained from simulating the model for long time series (10 thousands periods).

Changes in welfare are computed in consumption equivalent units. Results from the simulations indicate that: first, the increase on the insurance fee is passed to borrowers in the form of higher interest rates. Second, borrower's face lower tax burden, since now the government collects a higher guarantee fee. Third, borrower's consumption of non-durable good increases, but its consumption of housing good falls. The dominant on welfare effect comes from lower consumption of housing good, which leads to negative changes in welfare.

7.2 Market coverage

The second policy experiment consist on studying the effects of market coverage of the insurance policy. This resembles a policy in which the only secondary market for mortgages is the agency segment. For this, I simulate the model for different levels of market coverage, ω. The benchmark calibration corresponds to the case in which $\omega = 0.7$.

Table 9: Policy experiment: increasing government coverage

Description	**Benchmark** $\omega = 70\%$	$\omega = 100\%$	$\omega = 0\%$
Mortgage spread %, $r^m - r^f$	3.07	2.71	3.74
Mortgage spread, std %	0.12	0.06	0.27
Fraction of loans traded	0.90	1.00	0.70
Gov. policy: $\tau^s = \omega\mu$	3.2%	4.4%	0.0
Borrower's share of Tax bill	0.76	0.83	0.0
Δ% Borrower welfare	-	+0.15	-0.21
Δ% Non-durable consumption	-	-0.35	+1.16
Δ% Housing good consumption	-	+2.04	-5.90
Δ% Debt levels	-	+2.04	-5.91
Δ% Lenders' welfare	-	+2.34	-6.65

*Moments obtained from simulating the model for long time series (10 thousands periods). Insurance fee is kept fixed at $g_{\text{fee}} = 20bps$.

Changes in welfare are computed in consumption equivalent units. Results from the simulations indicate that: first, borrower's mildly benefit from a complete coverage of the market. The main gains are in reduction of interest rates, and lower volatilty. Second, borrower's face a higher tax burden in order to finance the policy. Third, lender's significantly benefit from complete coverage policy. Their welfare increases by 2.5%, this due to an increase in liquidity and volume of trade in the secondary market.

8 Conclusion

This paper develops a framework that connects dynamics in the lending and secondary mortgage markets in a dynamic general equilibrium model. I calibrate the model to match key moments of the cross section of mortgage lenders, and time series of aggregates for households and lenders in the U.S. mortgage market. The model can replicate the collapse of the secondary mortgage market during the great recession episode. Then, I study policy interventions in the secondary market, and find that policies that provide insurance against default risk to investors improve liquidity and stabilize the market. Borrower's benefit from lower mortgage interest rates but face a higher tax burden.

This paper contributes to the literature that studies the role of government interventions in the mortgage market by evaluating the potential benefit of government policies to stabilize mortgage credit and by quantifying the welfare benefits and costs.

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A Calibration Appendix

A.0.1 Cross Sectional distribution of Mortgage Lending activity

- Average cross sectional moments

Table 10: Market share of lenders by volume of origination, average 1990-2017

	Small Bnk	**Large Bnk**	**Credit U.**	**Aff. Mtg Co.**	**Ind. Mtg Co**	**All**
Volume: loan count in thousands						
All institutions	1,163	4,686	454	2,984	3,463	12,751
Share of Inst reporting >= 1K loans	0.37	0.98	0.45	0.98	0.95	0.90
Volume: USD amount in millions						
All institutions	168,066	1,007,588	59,581	599,329	712,250	2,546,875
Share of Inst reporting >= 1K loans	0.41	0.97	0.50	0.98	0.93	0.91

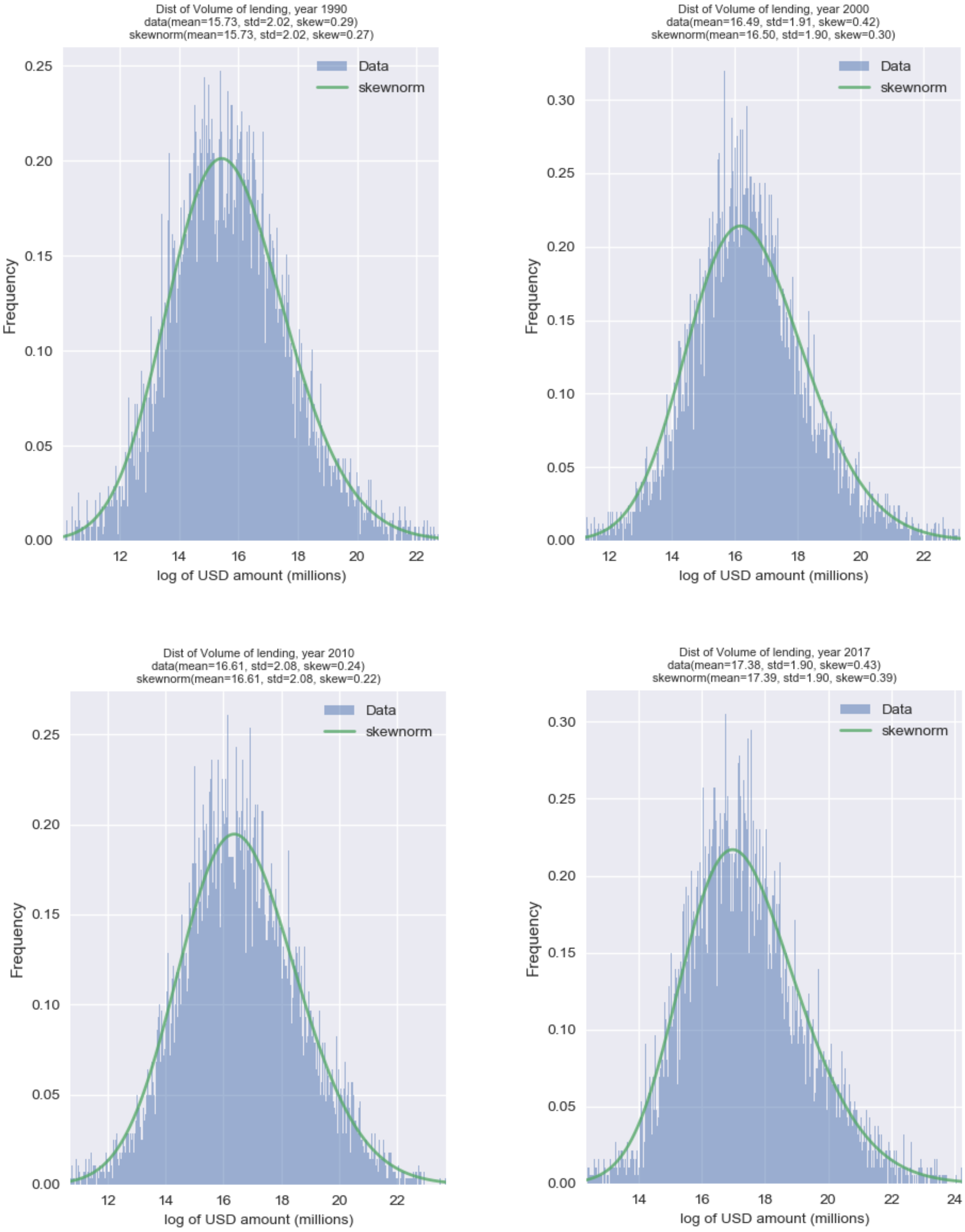
Table 11: Moments of the distribution of Mortgage Lending, HMDA 1990-2017

Moments	All loans	Home Purchase
Mean/median	1.007	1.007
	(0.41)	(0.32)
std	2.00	2.10
	(0.08)	(0.08)
skewness	0.33	0.31
	(0.08)	(0.07)
Mkt shr of Inst. > avg vol	0.89	0.91
Fraction of Inst. reporting > avg vol loans (USD)	0.10	0.10

Source: HMDA LARs and Reporter Panel 1990-2017

- Kernel density for selected years

Figure 10: Cross Section Distributions of Mortgage Lending, selected years



Source: HMDA LARs and Reporter Panel 1990-2017

A.0.2 Value added of the Financial sector

A.0.3 Estimation of exogenous processes

The exogenous processes in the model are given by borrower's income process Y, and the default rate λ. For income I use the cyclical component of the Disposable Personal Income from the Flow of Funds account. The default rate corresponds to the national delinquency rate for mortgage loans that are 90 or more days delinquent, or went into foreclosure. I estimate the following Vector Auto-regression model of first order, VAR(1), for the period of analysis 1990-2006.

$$S_t = A_0 + A_1 S_{t-1} + \epsilon_t \qquad \epsilon \sim N(0, Q)$$

Table 12: Average Value Added Financial Sector (%GDP)

Variable	**90-06**	**60-18**	**90-18**
Financial Sector	6.85	5.77	6.92
Credit Intermediation	3.07	2.70	3.05
Securities & investment	1.19	0.83	1.23
Insurance	2.44	2.15	2.53
Funds,trusts & others	0.14	0.09	0.13

Source: Burea of Economic Analysis (BEA), Value Added by Industry as a Percentage of Gross Domestic Product.

Table 13: Joint Markov Process for Income and default rates

State	1	2	3	4	5	6	7	8	9
Y	0.986	0.986	0.986	1.000	1.000	1.000	1.014	1.014	1.014
λ	0.008	0.012	0.016	0.008	0.012	0.016	0.008	0.012	0.016
Stationary Prob									
Prob	0.020	0.109	0.054	0.119	0.395	0.119	0.054	0.109	0.020

where $S_t = [Y_t, \ \lambda_t]^T$, and Q is the covariance matrix of the disturbances. The above estimation yields

$$A_0 = \begin{bmatrix} 0.031 \\ 0.010 \end{bmatrix} \qquad A_1 = \begin{bmatrix} 0.695^{***} & -0.003 \\ 2.052 & 0.921^{***} \end{bmatrix} \qquad Q = \begin{bmatrix} 0.0026e^{-3} & -0.032e^{-3} \\ -0.032e^{-3} & 6.31e^{-3} \end{bmatrix}$$

*** indicates parameters significant at 1%.

The discretization of the VAR into a Markov chain of first order yields:

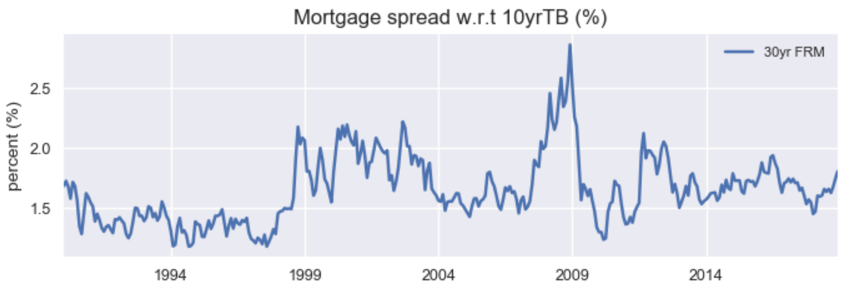
The Markov process fits well the unconditional means and standard deviations for income, and default rate, and the negative correlation between income and delinquency rates. Table 14 shows the moments obtained from a simulated time series of 100,00 periods versus the data moments

Table 14: Fitted moments for time series

	mc simulation	data, 90-06
Y mean	0.0	0.0
Y std	0.0086	0.0075
λ mean	1.22%	1.22%
λ std	0.20%	0.17%
corr(Y,λ)	-0.18	-0.26

A.0.4 Mortgage Interest rates

Figure 11: Historic Mortgage Interest Rates



Source: Freddie Mac Primary Mortgage Market Survey 2018.

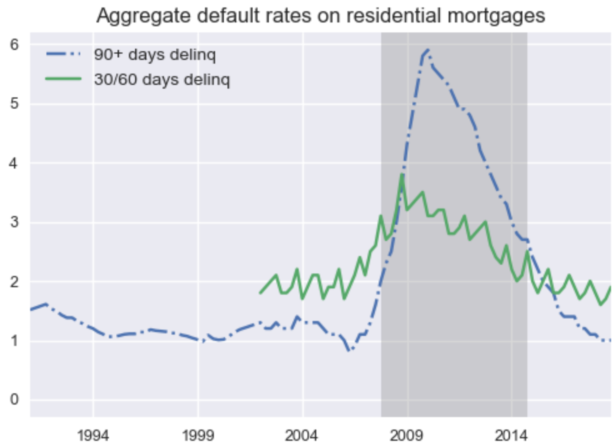
Table 15: Historic Mortgage Interest rates, averages

	90-06	**90-18**	**00-18**
1 year T-bill real			
mean	1.57	0.09	-1.10
std	1.57	2.30	1.86
30 year FRM real			
mean	4.50	3.26	2.30
std	1.10	1.78	1.36
30 year FRM nominal			
mean	7.45	6.24	5.26
std	1.22	1.81	1.28

*Source: Freddie Mac Primary Mortgage Market Survey 2018. FRM is Fixed Rate Mortgage.
**Real rates corresponds to the nominal rate minus expected inflation from the Survey of Consumers, University of Michigan.

A.0.5 Mortgage Default rates

Figure 12: National Delinquency rates



Source: National Mortgage Database (NMDB), Federal Housing Finance Agency

Table 16: National Delinquency rates, 2002-2018

Period	30/60 days delinq	90+ days delinq
mean	2.35%	2.50%
std	0.56	1.61

Source: National Mortgage Database (NMDB), Federal Housing Finance Agency

A Proofs Appendix

A.1 Proof of Lemma 1.

1. Given that we assume $z \sim i.i.d.$, and the linearity of policy functions on b, the aggregate supply and demand of debt claims in the secondary market $\{S, D\}$ do not depend on the distribution of b, this can be shown by working out the expressions for supply and demand in the secondary market from the definitions.

   (a) Supply

$$\begin{aligned}
S(X) &= S_B(X) + S_G(X) \\
&= \int s_B^j(b,z,X)\ d\Gamma(b,z) + \int s_G^j(b,z,X)\ d\Gamma(b,z) \\
&= \int_z \int_b s_B(b,z,X)\ dG(b)dF(z) + \int_z \int_b s_B(b,z,X)\ dG(b)dF(z) \\
&= \int_z \int_b \lambda(1-\phi)b\ dG(b)dF(z) + \int_z \int_b (1-\lambda)(1-\phi)b\ dG(b)dF(z) \\
&= \lambda(1-\phi)\int_{z_a}^{z_b}\left[\int_b bdG(b)\right] dF(z) + (1-\lambda)(1-\phi)\int_{z_a}^{p/q}\left[\int_b bdG(b)\right] dF(z) \\
&= \lambda(1-\phi)\int_{z_a}^{z_b} BdF(z) + (1-\lambda)(1-\phi)\int_{z_a}^{p/q} BdF(z) \\
&= B(1-\phi)\left[\lambda\int_{z_a}^{z_b} dF(z) + (1-\lambda)\int_{z_a}^{p/q} dF(z)\right] \\
&= B(1-\phi)\left[\lambda + (1-\lambda)F(p/q)\right]
\end{aligned}$$

   (b) Demand

$$\begin{aligned}
D(X) &= \int d^j(b,z,X)\ d\Gamma(b,z) \\
&= \int_z \int_b d(b,z,X)\ dG(b)dF(z) \\
&= \int_{z^m/q}^{z_b} \int_b \frac{b' - (1-\lambda)(1-\phi)b}{1-\mu}\ dG(b)dF(z) \\
&= \frac{1}{1-\mu(X)}\left[\int_{z^m/q}^{z_b}\int_b b'\ dG(b)dF(z) - (1-\lambda)(1-\phi)\int_{z^m/q}^{z_b}\int_b b\ dG(b)dF(z)\right] \\
&= \frac{1}{1-\mu(X)}\left[\int_{z^m/q}^{z_b}\int_b b'\ dG(b)dF(z) - (1-\lambda)(1-\phi)B\int_{z^m/q}^{z_b} dF(z)\right] \\
&= \frac{1}{1-\mu(X)}\left[\int_{z^m/q}^{z_b}\frac{\beta}{z^m(X)}\Big((1-\lambda)\hat{\phi} + \lambda(1-\phi)p(X) + (1-\lambda)(1-\phi)z^m(X)\Big)\left[\int_b b\ dG(b)\right] dF(z)\right] \\
&\quad -\frac{1}{1-\mu(X)}(1-\lambda)(1-\phi)B\int_{z^m/q}^{z_b} dF(z) \\
&= \frac{1}{1-\mu(X)}\left[\int_{z^m/q}^{z_b}\frac{\beta}{z^m(X)}\Big((1-\lambda)\hat{\phi} + \lambda(1-\phi)p(X) + (1-\lambda)(1-\phi)z^m(X)\Big)BdF(z)\right] \\
&\quad -\frac{1}{1-\mu(X)}(1-\lambda)(1-\phi)B\int_{z^m/q}^{z_b} dF(z) \\
&= \frac{1-F(z^m/q)}{1-\mu(X)}B\left[\frac{\beta}{z^m(X)}[(1-\lambda)(\hat{\phi} + (1-\phi)z^m(X)) + \lambda(1-\phi)p(X)] - (1-\lambda)(1-\phi)\right]
\end{aligned}$$

It follows that the market clearing values of $\{p, \mu\}$ do not depend on the distribution of b either. Definition $z^m = \frac{p}{(1-\mu)}$.

2. The price of debt q does not depend on the distribution of debt holdings across lenders because the market clearing condition in the Credit Market is a function only of the aggregate level of debt B.

   (a) Demand of credit from borrowers depends only on aggregates states $\{B, K, \lambda, Y\}$ through the policy function of $B'(B, K, X)$. Hence, the distribution of debt claims is irrelevant from the stand of the borrower.

   $$N^B = B'^B - (1-\lambda)(1-\phi)B^B$$

   (b) Supply of credit from lenders correspond to the integral across the individual originations n^j. Given that lending policy functions are linear in b, the aggregate supply of lending is linear in the aggregate amount of debt claims in the economy B. This can be seen from the aggregation of the origination decisions.

   $$N^L = \int n^j(b, z, X) d\Gamma(b, z)$$

   There are two possible expressions for the aggregate supply of credit. The first case when the secondary market is active meaning $p > 0$,

$$
\begin{aligned}
N^{\text{seller}} &= \int n^j(b,z,X)d\Gamma(b,z) \\
&= \int_{z_a}^{p/q}\int_b b'(b,z,X)\ dG(b)dF(z) \\
&= \int_{z_a}^{p/q}\frac{\beta}{zq}\left[(1-\lambda)\hat{\phi}+(1-\phi)p(X)\right]\left[\int_b b\ dG(b)\right]dFz \\
&= B\frac{\beta}{q}\left[(1-\lambda)\hat{\phi}+(1-\phi)p(X)\right]\int_{z_a}^{p/q}\frac{1}{z}dFz \\
N^{\text{holder}} &= \int n^j(b,z,X)d\Gamma(b,z) \\
&= \int_p^{z^m/q}\int_b\left[b'(b,z,X)-(1-\lambda)(1-\phi)b\right]\ dG(b)dF(z) \\
&= \int_p^{z^m/q}\frac{\beta}{zq}\left[(1-\lambda)\hat{\phi}+\lambda(1-\phi)p(X)+(1-\lambda)(1-\phi)zq(X)\right]\left[\int_b b\ dG(b)\right]dFz \\
&\quad -\int_p^{z^m/q}(1-\lambda)(1-\phi)\left[\int_b b\ dG(b)\right]dFz \\
&= B\frac{\beta}{q}\left[(1-\lambda)\hat{\phi}+\lambda(1-\phi)p(X)\right]\int_p^{z^m/q}\frac{1}{z}dFz+\beta(1-\lambda)(1-\phi)B\int_p^{z^m/q}dFz \\
&\quad -(1-\lambda)(1-\phi)B\int_p^{z^m/q}dFz \\
&= B\frac{\beta}{q}\left[(1-\lambda)\hat{\phi}+\lambda(1-\phi)p(X)\right]\log(z)f(z)|_p^{z^m/q} \\
&\quad -B(1-\beta)(1-\lambda)(1-\phi)\left(F(z^m/q)-F(p/q)\right) \\
N^L &= N^{\text{seller}}+N^{\text{holder}}
\end{aligned}
$$

the second case when the secondary market is inactive and each lender originates using its own technology

$$
\begin{aligned}
N^L &= \int n^j(b,z,X)d\Gamma(b,z) \\
&= \int_{z_a}^{z_b}\int_b\left[b'(b,z,X)-(1-\lambda)(1-\phi)b\right]\ dG(b)dF(z) \\
&= \int_{z_a}^{z_b}\frac{\beta}{zq}\left[(1-\lambda)\hat{\phi}+(1-\lambda)(1-\phi)zq(X)\right]\left[\int_b b\ dG(b)\right]dF(z) \\
&\quad -\int_{z_a}^{z_b}(1-\lambda)(1-\phi)\left[\int_b b\ dG(b)\right]dFz \\
&= B(1-\lambda)\left[\frac{\beta}{q}\hat{\phi}\int_{z_a}^{z_b}\frac{1}{z}dF(z)-(1-\beta^L)(1-\phi)\right]
\end{aligned}
$$

A.2 Proof of Lemma 2

The analysis in section 4.1.2 it follows that if there is a price $p > 0$ in the secondary market, then:

- Seller. For a lender j that such that $z \in [z_{a,}p/q)$, trading decisions are: $\{d^j = 0,\ s_G^j = (1-\lambda)(1-\phi)b^j,\ s_B^j = \lambda(1-\phi)b\}$. By replacing these policy functions in the law of motion of debt holdings eq9 it follows that the origination decision for a seller is $n^j = b'^j$.

- Buyer. For a lender j such that $z^j \in (\frac{p}{q}\frac{‘1}{1-\mu}, z_b]$, trading decisions are $\{d^j > 0,\ s_G^j = 0,\ s_B^j = \lambda(1-\phi)b^j\}$. Notice that n^j and d^j are alternative ways of saving resources. Originating one loan today costs $z^j q$ and pays off one unit tomorrow, while purchasing one loan in the secondary market today costs p and pays off $(1-\mu)$ units tomorrow. Hence, the return on saving by originating loans is $\frac{1}{z^j q}$, while the return for purchasing a loan is $\frac{1-\mu}{p}$. Given that $z^j > \frac{p}{q}\frac{1}{1-\mu}$, the optimal decision is to set $n^j = 0$, and accumulate loans by purchasing existing loans in the secondary market. By replacing these decisions in the law of motion of debt holdings eq9 yields the policy function for purchases, $d^j = \frac{b'^j-(1-\phi)(1-\lambda)b^j}{1-\mu}$.

- Holder. For a lender j such that $z^j \in [\frac{p}{q},\ \frac{p}{q}\frac{‘1}{1-\mu}]$, trading decisions are $\{d^j = 0,\ s_G^j = 0,\ s_B^j = \lambda(1-\phi)b^j\}$. By replacing these decisions in the law of motion of debt holdings eq9 obtains $n^j = b'^j - (1-\lambda)(1-\phi)b^j$.

In the case in which there is no positive price that clears the secondary market, the secondary market will not be active. Trading decisions for all lenders are trivial: $\{d^j = 0,\ s_G^j = 0,\ s_B^j = 0\}$. By replacing these decisions in the law of motion of debt holdings eq9 obtains the origination decision: $n^j = b'^j - (1-\lambda)(1-\phi)b^j$.

Budget sets by type of lender

By replacing optimal origination and trading decisions of Lemma 2 in the budget constraint and law of motion of the lenders problem, eq 7, obtains:

- Buyers:

$$\begin{aligned} c + p(X)\left[\frac{b' - (1-\lambda)(1-\phi)b}{1-\mu(X)}\right] &= (1-\lambda)\hat{\phi}b + \lambda(1-\phi)p(X)b \\ c + z^m(X)b' &= \Big[(1-\lambda)\hat{\phi} + \lambda(1-\phi)p(X) + (1-\lambda)(1-\phi)z^m(X)\Big]\, b \end{aligned}$$

  where $z^m(X) = p(X)/(1-\mu(X))$.

- Sellers:

$$\begin{aligned} c + zq(X)\left[b'\right] &= (1-\lambda)\hat{\phi}b + \lambda(1-\phi)p(X)b + (1-\lambda)(1-\phi)p(X)b \\ c + zq(X)b' &= \left[(1-\lambda)\phi + (1-\phi)p(X)\right] b \end{aligned}$$

- Holder:

$$\begin{aligned} c + zq(X)[b' - (1-\lambda)(1-\phi)b] &= (1-\lambda)\hat{\phi}b + \lambda(1-\phi)p(X)b \\ c + zq(X)b' &= \left[(1-\lambda)\phi + \lambda(1-\phi)p(X) + (1-\lambda)(1-\phi)zq(X)\right] b \end{aligned}$$

A.3 Proof of Lemma 3

We will derive $\{c, b'\}$ policy functions by guess and verify. Taking F.O.C w.r.t to b' to program eq 17 obtains:

$$
\begin{aligned}
u_c \min\{zq, p/(1-\mu)\} &= \beta^L \mathbb{E}_{X'|X}\left[V_{b'}(b', z', X')\right] \\
&= \beta^L \mathbb{E}_{X'|X}\left[u_{c'} W_{b'}(b', z', X')\right]
\end{aligned}
$$

where the second equation holds because of the Envelope theorem, and $W_b = \frac{\partial W(b,z,X)}{\partial b}$ is the marginal change in a lender's virtual wealth, eq 16, of increasing debt claims in one unit. Given that preferences are assumed to be logarithm:

$$
\frac{1}{c}\min\{zq, p/(1-\mu)\} = \beta^L \mathbb{E}_{X'|X}\left[\frac{1}{c'} W_{b'}(b', z', X')\right]
$$

Guess that the policy function for consumption has the form: $c = \alpha W(b, z, X)$, where $\alpha \in \mathbb{R}$. Then, from budget constraint in 17 it implies:

$$
b' = \max\{\left\{\frac{1}{zq}, \frac{1-\mu}{p}\right\}\}(1-\alpha)W(b, z, X)
$$

and,

$$
\begin{aligned}
c' &= \alpha W(b', z', X') \\
&= \alpha W_{b'}(b', z', X')b' \\
&= \alpha W_{b'}(b', z', X')\left[\max\{\left\{\frac{1}{zq}, \frac{1-\mu}{p}\right\}\}(1-\alpha)W(b, z, X)\right]
\end{aligned}
$$

Replacing expression for c' in the Euler equation obtains:

$$
\begin{aligned}
\frac{1}{c} &= \beta^L \max\{\left\{\frac{1}{zq}, \frac{1-\mu}{p}\right\}\mathbb{E}_{X'|X}\left[\frac{W_{b'}(b', z', X')}{\alpha W_{b'}(b', z', X')\left[\max\{\left\{\frac{1}{zq}, \frac{1-\mu}{p}\right\}\}(1-\alpha)W(b, z, X)\right]}\right] \\
\frac{1}{\alpha W(b, z, X)} &= \beta^L \mathbb{E}_{X'|X}\left[\frac{1}{\alpha(1-\alpha)W(b, z, X)}\right] \\
\alpha &= 1 - \beta^L
\end{aligned}
$$

which yields:

$$
\begin{aligned}
c &= (1-\beta^L)W(b, z, X) \\
b' &= \beta^L \max\{\left\{\frac{1}{zq}, \frac{1-\mu}{p}\right\} W(b, z, X)
\end{aligned}
$$

B Computational algorithm to solve the model

B.1 Algorithm A. To solve the General Equilibrium model

I solve the model in a discrete state space for endogenous and exogenous state variables. Exogenous states are characterize by a joint state space $(\lambda, Y) \in \{* \times \mathcal{Y}\}$, and an associated transition Π_s matrix. The aggregate endogenous states for debt and housing holdings are given by the space $\mathcal{B} \times \mathcal{K}$. The space of all aggregate state is given by $\mathcal{X} \equiv * \times \mathcal{Y} \times \mathcal{B} \times \mathcal{K}$. Solving the model consists on finding: i) policy, and value functions for borrower's problem, eq(3), ii) schedule of prices $\{q(X), p(X)\}$ for all realizations of the aggregate state vector $X \in \mathcal{X}$ as defined in eq(15).

Notice that given the closed form characterization of lenders decision rules developed in section 4 we can obtained analytical expressions for all objects in the system of equations(23)-(29) except for borrower's demand of credit. Hence, the only unknown function is borrower's borrowing policy function which defines credit demand. Notice that given the assumption of exogenous default rate borrower's debt hast not an effect on default probability, the borrower observes a price schedule $q(B', B; X)$ that is a function of current realized aggregate states, and tomorrow's debt level. Hence, it is possible to solve the model in two steps: first we obtain the set of price schedules $\{q, p\}$ for all states X and transitions to (B', X'), and then we use them to solve borrower's problem. The algorithm is as follow:

1. Jointly compute price schedules $\{q(X), p(X)\}$, for every $X \in \mathcal{X}$.
   - (a) Notice that for all realizations $X \in \mathcal{X}$, we can compute borrower's credit demand, $N^D = B'(B, K, X) - (1-\lambda)(1-\phi)B$, from transitioning from every combination of current states $\{B, X\} \to \{B'; X\}$.
   - (b) For every combination in a) solve the system of equations from the market clearing conditions eq(23)-(29).
2. Given the set of price schedules $q(B', X)$, solve for borrower's policy functions $\{C(X), K'(X), B'(X)\}$.
   - (a) I solve borrower's problem by grid search global solution methods iterating over the value function .