

Numerical study of the effect of secondary electron emission on the dynamics of electron clouds in gyrotron guns

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In this document, behavior of ions inducing electrons in presence of magnetic and strong electric fields is reviewed. Ion-induced electron-emissions (IIEE) are implemented in the FENNECS code [LB22]. Results are planned to be compared with the Trapped Electrons Experiment (TREX).

Keywords: Gyrotron, electron cloud, trapped electron cloud, ionisation, IIEE, FENNECS

I. INTRODUCTION

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II. THEORY

A. Gyrotron guns and trapping phenomena

Gyrotrons are microwaves generator devices used in plasma heating and current drive applications for fusion purposes [PPZ⁺16]. The operating mode of gyrotron guns is based on the electron cyclotron maser instability (CMI). This instability occurs when a beam of relativistic electrons, gyrating around magnetic field lines, interact with the transverse component of the electric field. The transverse electrons' energy is then converted in electromagnetic waves in an open, resonant cavity. More precisely, the interaction takes place between the electron beam and a transverse electric mode $\text{TE}_{m,p,q}$ supported by the cavity, see Fig.(1) - Left. The subscripts m, p, q denote respectively the azimuthal, radial and longitudinal mode numbers. When the pulsation of the EM wave inside of the cavity equals the relativistic electron cyclotron (EC) frequency, that is $\omega = \omega_e/\gamma$, a group of electrons with initially no phase relation, will, under the action of the field, regroup and gyrate in phase, as a whole. This is called 'phase bunching', see Fig.(1) - Right. Note that we define ω , ω_e as the EM wave and the EC frequencies respectively, and γ the Lorenz factor. Then, increasing the frequency ω , after an integer number of periods, the electrons will be in a region such that their transverse velocity is on average positive, $\langle v_x \rangle > 0$ [Alb91]. Using that the electrons energy W varies as $dW/dt = -|e|v_x E_x$, the energy variation is negative, and by conservation, the latter is converted in EM energy.

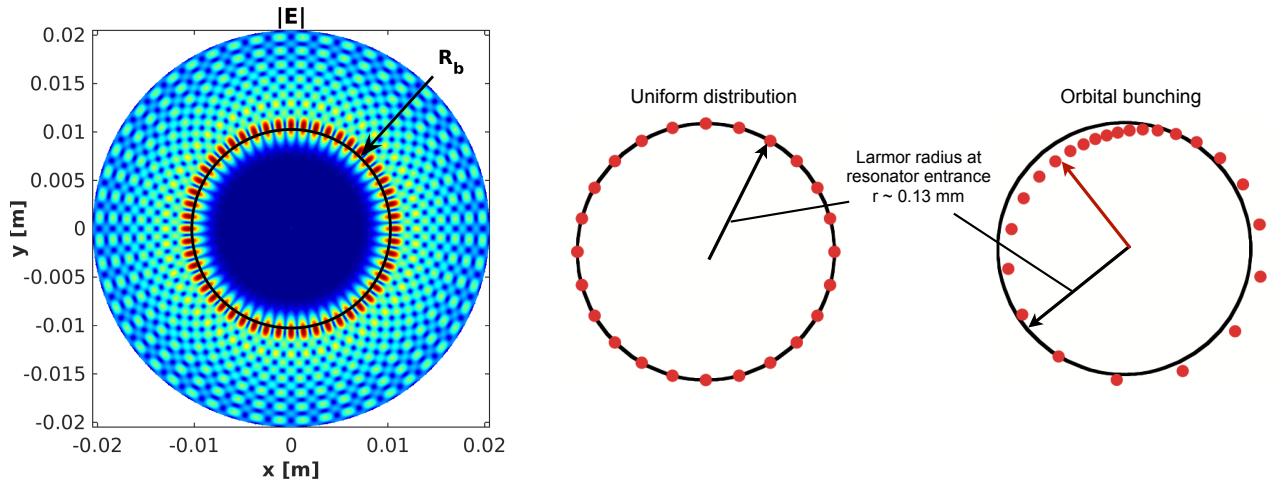


FIG. 1. Left: Amplitude of the electric field profile in the transverse cross-section for the mode $\text{TE}_{26,7}$ of Tokamak à Configuration Variable (TCV) dual-frequency gyrotron. [Gen19] The black circle represents the annular electron beam. - Right: Orbital bunching mechanism of electrons in the cavity (resonator). Electrons are shown by red dots, and the black circle corresponds to a Larmor orbit. Source: courtesy of S. Alberti.

On top of the cavity lies a mode converter so that the frequency of the emitted microwaves matches one of the plasma resonance frequencies. The radiation is then reflected by several mirrors and directed through a diamond window so it can penetrate the plasma, see Fig.(2). The most common heating frequency corresponds to the electrons' gyrofrequency ω_e , or some of its harmonics. In that case, we speak of electron cyclotron resonance heating (ECRH). It is when the electromagnetic wave has a component along the toroidal magnetic field that it can induce a current. The latter phenomenon is called electron cyclotron current drive (ECCD).

All the previously described events occur in the gyrotron cavity, or after the latter. However, the beam quality as it enters the cavity is very important, since it will, among others, participate to the interaction efficiency [PPZ⁺16]. The beam is generated by an electron gun, also called magnetron injection gun (MIG). An axial cross section of the MIG is depicted in the left part of Fig.(2). It consists in a cylindrical cathode around a coaxial insert, and a cylindrical anode of a larger radius around it. The beam electrons are produced by thermo-emission. The electrons are accelerated by the electric field and guided through the exit of the MIG by mean of the magnetic field present as shown in Fig.(2), so they can reach the cavity where they will achieve the collective gyro-motion from the CMI. Nevertheless, multiple phenomena can disrupt the functioning of the MIG, among which the formation of trapped

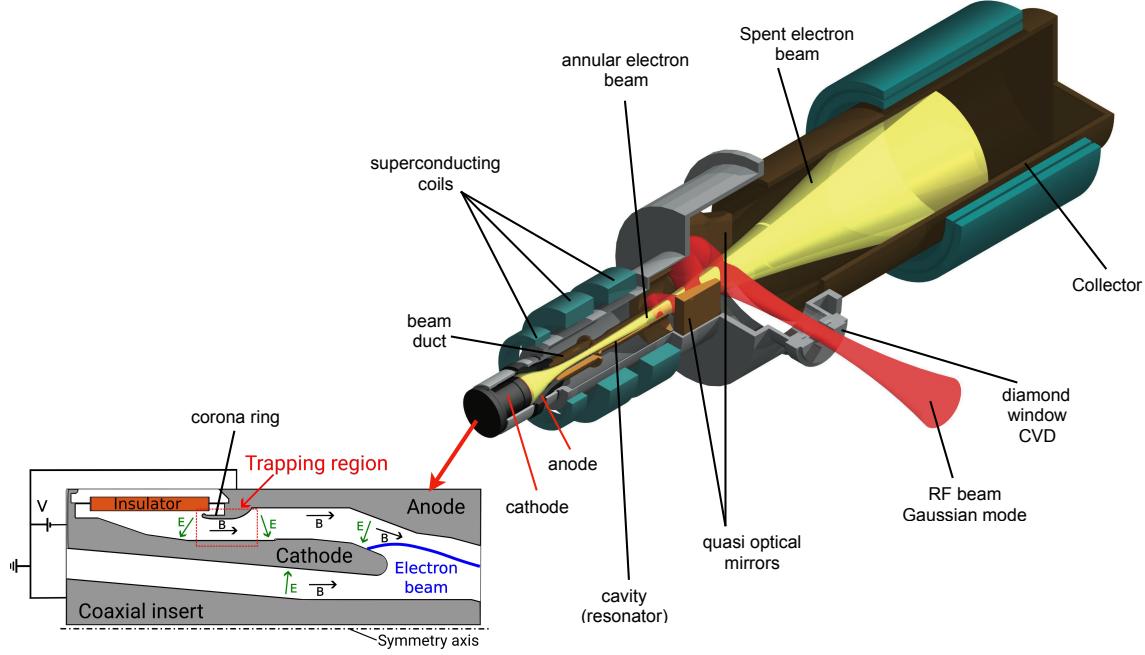


FIG. 2. Right: Schematic of a gyrotron gun and its different constituents. Source: courtesy of S. Alberti - Left: Zoomed view of the MIG from the gyrotron [RAH⁺22].

electron clouds in the rear part of it [PPZ¹⁶, LBHL²²]. Indeed, as the local population of electrons increase inside the MIG cavity, discharges phenomena can occur when a certain critical value is reached. Thus, voltage stand-off problems can follow from discharges [HGA⁰⁹], that were later attributed to the presence of these trapped electrons. Regarding the trapping in itself, electrons can accumulate in a cloud for several reasons, as among others, the so-called adiabatic trap, or the potential well trap. In this report, we will focus on the second. Let us first briefly summarise this type of trapping mechanism.

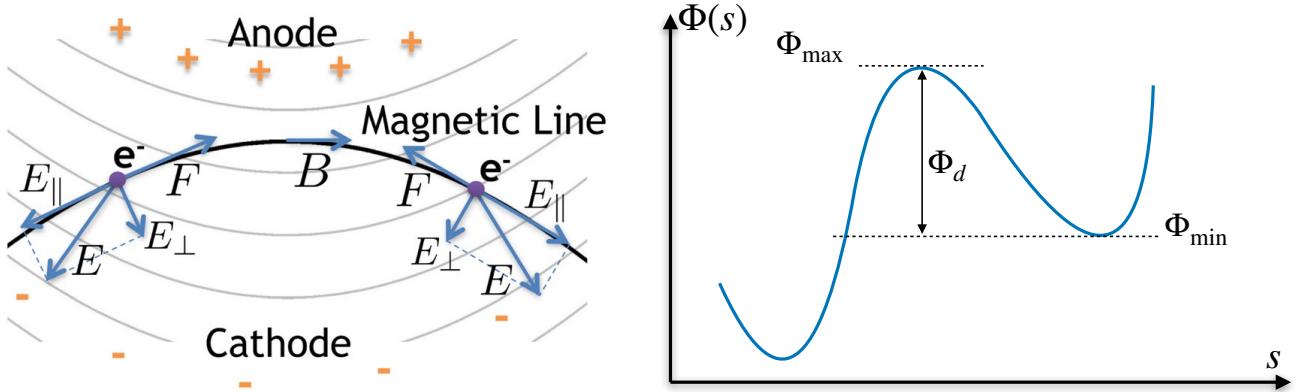


FIG. 3. Left: configuration favorable to the formation of a potential well. The equipotentials are depicted by plain, grey lines, while the magnetic field line is represented by the black one [PPZ¹⁶] - Right: Potential-well definition. The curvilinear abscissa along the field line is denoted by s .

A magnetic potential well can form every time a magnetic line intersects twice an equipotential as depicted in Fig.(3). Indeed, in such a configuration, the electrons are accelerated on one side of the well and decelerated on the other side by the Coulomb force projected along the field line $F_{||} = -eE_{||}$. The potential well is characterised by its location as well as its depth, which is defined locally as the difference between the local maximum Φ_{max} and the highest of the local minima on both side of Φ_{max} . Hence, the depth reads $\Phi_d = \Phi_{max} - \max \{\Phi_{min}\}$. Thus, the electrons

oscillate inside that well around its center, while the component E_{\perp} of the electric field contributes to an azimuthal drift $\mathbf{v}_d \propto \mathbf{E}_{\perp} \times \mathbf{B}$, causing the electrons to gyrate around the MIG symmetry axis, giving the cloud an annular shape.

B. TRapped Electrons eXperiment TREX

In order to emphasize experimentally the mechanism of electron trapping by magnetic potential well as described in II A, an experiment is, at the time of writing, being built at the Swiss Plasma Center (SPC) [RAH⁺22]. The TREX experiment aims at studying the formation of electron clouds under multiple gyrotron's magnetic, electric and neutral pressure configurations. One sees indeed that the central electrode designed as shown in Fig.(4) is an approximate replica of the corona-ring from MIG.

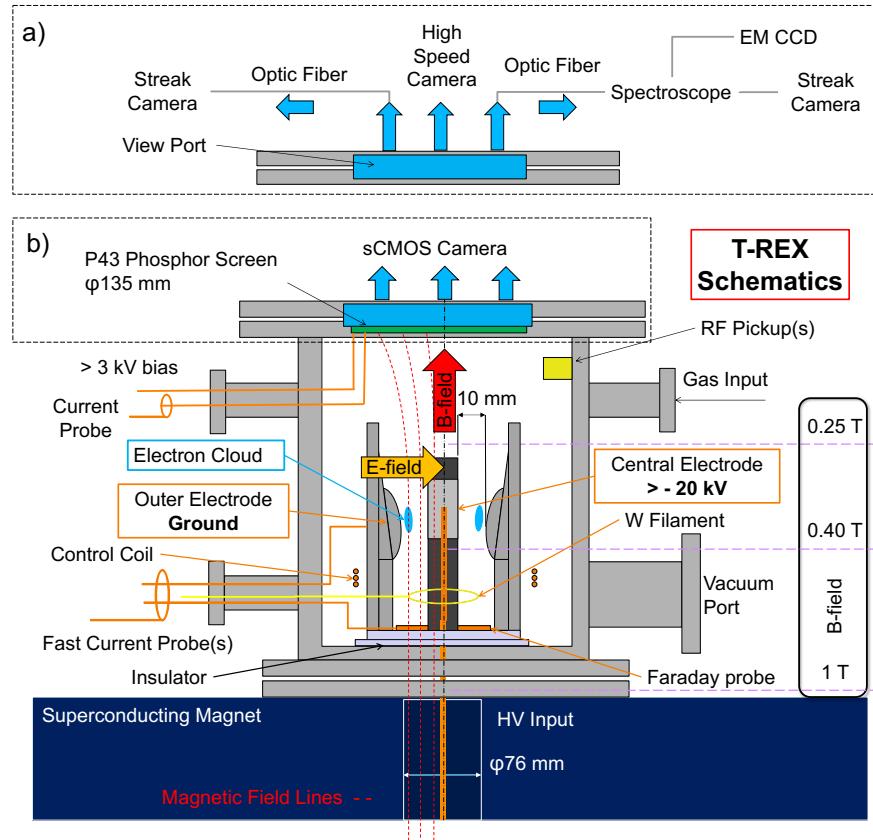


FIG. 4. Schematic of the TREX experiment [RAH⁺22].

Regarding the experiment design in itself, the chamber is set on top of a superconducting magnet, such as the magnetic field lines run through the chamber axially. The potential bias between the two electrodes is planned to be of about $\Delta\Phi = 20$ kV and the electric field topology is changed locally by mean of amovible and interchangeable electrodes. The magnetic field intensity is kept between 0.25 – 0.4 T, and the neutral pressure should range as $p = 10^{-7}$ to 10^{-5} mbar. A tungsten filament can act as a source of electrons, although it has been shown by [LBHL⁺22] that background radiation could be sufficient in certain geometries, to provoke the formation of a cloud. The optical diagnostic methods consist in either a sCMOS camera on top of a biased phosphor screen, or streak camera working together with a high speed camera as well as a spectroscope. When neutralising the cloud by switching off the HV, the cloud would follow the field line and be directed towards the phosphor screen, enabling to measure the current through it, and the image would be taken by the sCMOS camera. The experiment being built currently, the design is guided by some FENNECS code results, and some numerical characteristics as the expected cloud density were deduced from simulations too. The experimental results are to be confronted to the code ones.

C. The FENNECS code

In order to model the electron trapping in various geometrical situations, the FENNECS code [LB22] was used. It is a 2D3V particle-in-cell code, that solves the Poisson-Vlasov system Eq.(1) in cylindrical geometry, for the distribution function f_e and the electrostatic potential ϕ . It has been developed in order to show the self-consistent formation of electron-clouds in gyrotron guns, due to the ionisation of the residual neutral gas (RNG) present in the gyrotron cavity [LBHL⁺22].

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{x}} + \frac{e}{m_e} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial}{\partial \mathbf{v}} \right) f_e(\mathbf{x}, \mathbf{v}, t) &= 0, \\ \Delta\Phi(\mathbf{x}, t) &= -\frac{e}{\epsilon_0} \int f_e(\mathbf{x}, \mathbf{v}, t) d^3\mathbf{v}. \end{aligned} \quad (1)$$

FENNECS is a highly parallelised code in the sense that it uses both multiple nodes (MPI) and multiple core per node (openMP). Regarding numerical methods, the code uses cylindrical coordinates, and axisymmetry is assumed, so that $\partial\theta = 0$, θ being the poloidal coordinate. The poisson equation is solved using finite elements, based on weighted extended b-splines, whereas the collisional Vlasov equation is solved by sampling the electron distribution f_e with macro-particles, then calculating the trajectory of each macro-particle by mean of the Boris algorithm [LBHL⁺22, BLVV85]. So far, the code only takes account for primary ionisations coming from electron-neutral collisions, where electrons collide with the RNG. In fact, the ions having a Larmor radius of the order of the meter, they are collected at the cathode, during times much shorter than the average ionisation time. Indeed, being generated with approximately zero velocity (the RNG atoms have zero velocity in the code), the ions characteristic 'living' time is roughly $\tau_l \sim \Delta r / v_{\mathbf{E} \times \mathbf{B}}$, Δr being of the order of the distance between the electrodes (\sim cm), which is much shorter than the ionisation time. Taking $B \sim 0.2$ T, $\Delta\Phi \sim 2 \cdot 10^4$ V and $\Delta r \sim 10^{-2}$ m, with $\Delta\Phi$ the electrostatic potential bias between the electrodes, one gets that $\tau_i \sim 10^{-9}$ s. Although this estimate is very rough, for a pressure of 10^{-6} mbar, characteristic of gyrotron guns, the ratio of the ions living time to the characteristic ionisation time is of about $\tau_l/\tau_{io} \sim 10^{-6}$ [LBHL⁺22]. Thus, the ions-neutrals collisions are neglected, and the electrons-neutrals collisions are implemented using a Monte-Carlo method.

However, although ions-neutral collisions are neglected in the ionisation process, their contribution in the formation of electron-clouds can be non-negligible, since when they impinge on the cathode, some electrons can be generated at the metallic surface, by various emission phenomena [Bar93]. The ion-induced electron-emissions will be the object of the following study, having first to be implemented in the code, before to be compared with situations where electron clouds form self-consistently, without taking account for IIEE.

D. Ion-Induced Electron-Emissions

In order to take into account IIEE in the FENNECS code, the question of choosing the appropriate physical model arose. Indeed, expected energy distribution of the ions in the TREX experiment was ranging between 0 and 20-30 keV. These values correspond to roughly the minimal and maximal values that could be reached by ions accelerated in an electric field perpendicular to the magnetic field lines, in vacuum, and over a distance of about 1cm. Fig.(5) shows the initial distribution of protons in the coaxial geometry with the (\mathbf{E}, \mathbf{B}) configuration described previously, where the electric equipotentials are plotted in blue, and the magnetic field lines in black. In this particular configuration, in vacuum, the acquired energy of the protons when they reach the cathode follows

$$E(r_0) = \Delta\Phi \frac{\log\left(\frac{r_0}{r_a}\right)}{\log\left(\frac{r_b}{r_a}\right)}, \quad (2)$$

where r_0 is the initial proton radial position, $\Delta\Phi$ the potential bias between the cathode located at r_a and the anode located at r_b . Thus, taking numerical values such as $\Delta\Phi = 20$ kV and evaluating $E(r_a)$ and $E(r_b)$, one get that $E \in [0, 20]$ keV. This way, a model describing IIEE in this energy range had to be found.

Regarding the model in itself, one had to consider that IIEE would be dependent on the material constituting the electrodes. Indeed, protons impinging on stainless steel would not have the same effect as if they were striking a copper electrode. The electron yield would also depend on the energy of the incident particle. It was expected that the number of released electrons would be different if the ion hit the cathode at high or low energy. At this point,

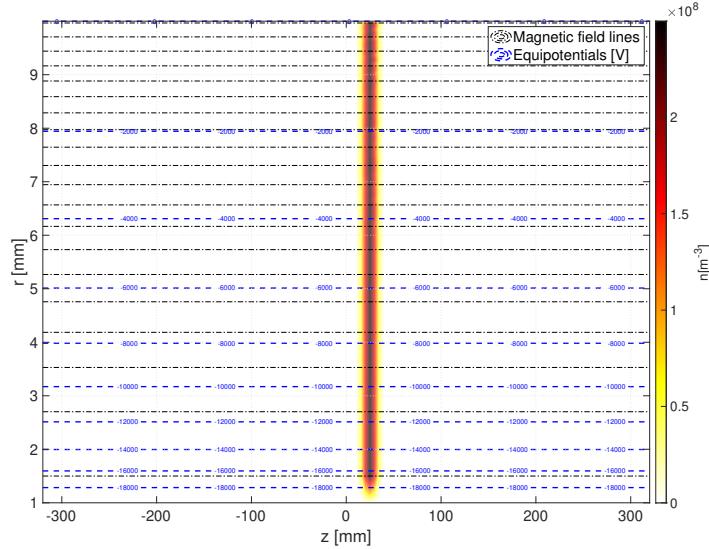


FIG. 5. Initial ion configuration in (R, Z) plane. The geometry is coaxial. Azimuthal symmetry assumed.

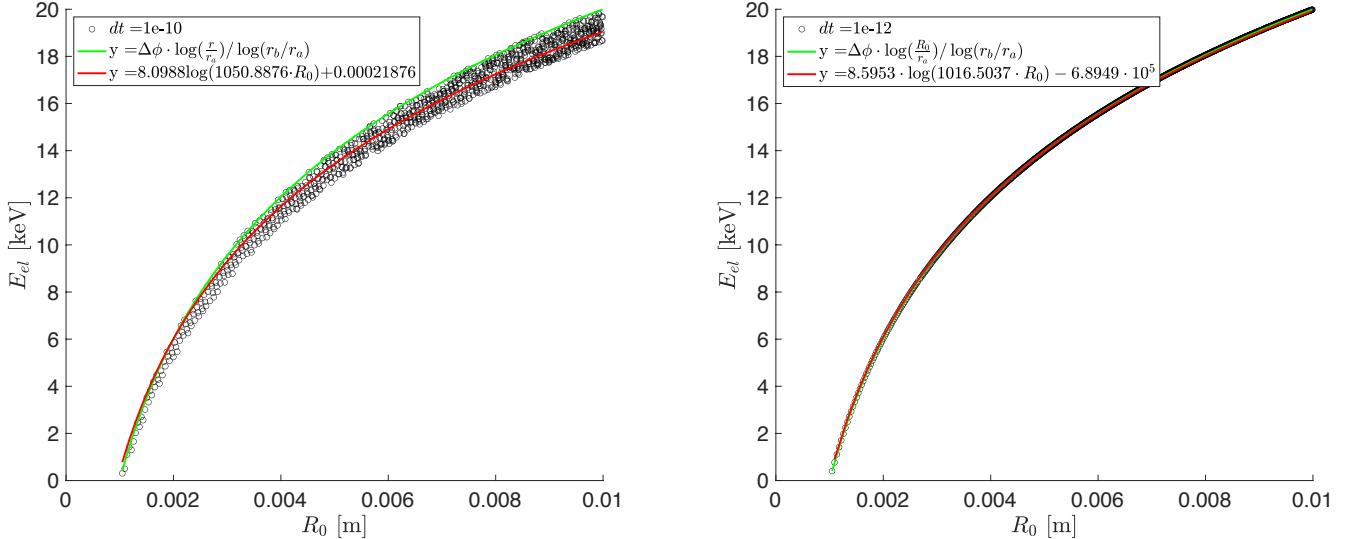


FIG. 6. Left: energy of the collected ions as they hit the cathode, $dt = 10^{-10}$ s. The red curve is the analytical expression for the energy, corresponding to the vacuum electric potential. The red curve is a fit of the data. - Right: Same with $dt = 10^{-12}$.

nothing ensured that the type of interaction between ion and electrode would be the same on the whole energy range. It was also expected that the electron yield would depend on ions parameters: mass m , charge q or charge to mass ratio q/m for example.

With these considerations, one model drew our attention: Schou's model, derived by Schou in 1980. Our motivation for a qualitative (at first) survey of these ion-induced electrons drove us towards this model in particular, since it distinguishes itself by its remarkable simplicity. Following the description of Schou's model from [DH], this approach of IIEE is based on the ionisation cascade theory, and a system of Boltzmann transport equations. Neglecting recoil ionisation in the material, the electron yield γ takes the following form:

$$\gamma = \Lambda \cdot D_e, \quad (3)$$

where D_e is the amount of energy deposited by inelastic collision **at the surface**, and Λ contains cross sections

dependent parameters for the interaction at a given energy and has the following form

$$\Lambda = \int_0^\infty \frac{\Gamma_m E}{4|dE_i/dx|(E + W)^2} dE. \quad (4)$$

In the above expression for Λ , $E = E_i - W$, Γ_m is a function that depends on the exponents of used power cross-sections. The term dE_i/dx corresponds to the energy loss of low-energy electrons in the material. The interesting point of this model is that it is made of two independent terms, one containing target material parameters (Λ) and the other containing the impacting particle characteristics (D_e). Another advantage of this separated description is that it can be reformulated such that it is expressed in terms of the energy loss of the incident particles inside the material, which is a quantity that can be measured easily, and for which tabulated numerical values exist. Thus, denoting by $dE/dx|_i$ the energy loss of ions in the electrode material, one can write the electron yield as follows:

$$\gamma = \Lambda \cdot \beta \cdot \left. \frac{dE}{dx} \right|_i. \quad (5)$$

In Eq.(5), β accounts for energy transport by recoiling electrons and backscattered ions. It has been shown that the product $\Lambda \cdot \beta$ is for most metals, independent of the material, and has been measured to be approximately 0.1m/MeV [DH]. Thus, the model used in our module reads:

$$\gamma = \Lambda_{exp} \cdot \left. \frac{dE}{dx} \right|_i := 0.1 \cdot \left. \frac{dE}{dx} \right|_i. \quad (6)$$

One point that it is important to be emphasized, is that Schou's model is a kinetic model that holds for substantially high energies, that is above 1keV. For energies below, Schou's theory has to be replaced by another model, in which kinetic emissions are replaced by the so-called potential emissions.

The potential emissions model that has been chosen to treat ion-induced electron emissions at low energies ($E < 1\text{keV}$) is due to Kishinevsky [Kis73]. The result is very elegant in the sense that it depends only on the Fermi energy of the material, the ionisation energy required to produce the incident ions, and the work function of the metal. Since no energy dependence of the electronic yield arises in this model, it should be **constant** in the range $[0, 1]$ keV. Comments on the validity of this approximation will be made below. Let us now briefly summarise the derivation of the result for γ . First, let us state the expression for the electronic yield as derived in [Kis73]:

$$\gamma \sim \frac{0.2}{\epsilon_F} (0.8 \cdot E_i - 2\phi), \quad (7)$$

where ϵ_F denotes the Fermi energy of the metal constituting the target material, ϕ its work function, and E_i the ionisation energy required initially to produce the incident ions. For hydrogen, this ionisation energy is $E_H \simeq 13.6$ eV. To justify this last expression, let us briefly resume the steps followed by Kishinevsky. Hagstrum had identified in [Hag54] that the electron yield induced by potential emissions followed:

$$\gamma := \int_{\epsilon_0}^\infty N_i(\epsilon_K) P_e(\epsilon_K) d\epsilon_K, \quad (8)$$

with $N_i(\epsilon_K)$ the energy distribution of Auger electrons inside the metal. The energy of the electrons is denoted by ϵ_K , and ϵ_0 is defined as $\epsilon_0 := \epsilon_F - \phi$. Note that $P_e(\epsilon_K)$ is the probability that the electron with energy ϵ_K overcomes the potential barrier at the surface of the metal. Studying experimentally the dependence of the yield on several parameters, it has been shown that the yield is a function of only three parameters $\gamma \equiv \gamma(\epsilon_F, \phi, E_i)$.

Kishinevksy then reviewed yield values at fixed parameters. Fixing (ϕ, E_i) , he managed to show that $\gamma_{(\phi, E_i)} \simeq \frac{cst.}{\epsilon_F}$. Applying the same procedure for ϵ_F and E_i , he constrained γ as follow:

$$\begin{aligned} \gamma &\simeq \frac{0.23}{\epsilon_F} (0.75 \cdot E_i - 2\phi), \\ \gamma &\simeq \frac{0.18}{\epsilon_F} (0.83 \cdot E_i - 2\phi). \end{aligned} \quad (9)$$

These expressions are very close to each other and for $E_i > 3\phi$, they differ by about $\pm 10\%$ [Kis73]. Hence, still with an accuracy of about 10%, he deduced Eq.(7).

E. Numerical implementation

In order to implement numerically Schou's model from Eq.(6), it was necessary to obtain reference values for the electronic yield, as a function of incident ions' energies. Tabulated values for the energy loss dE/dx of protons in various materials were extracted from [Jan82a, Jan82b]. To be consistent with the TREX experiment plans (See II B), attention was drawn on 304 stainless steel (^{304}SS), copper (Cu) and aluminum (Al).

From the tabulated values for dE/dx obtained, energy loss curves for protons in various materials were derived, and a primary tendency for the electronic yield appeared. Indeed, for all three materials, $d^2E/dx^2 < 0$, that is the energy loss is a concave function of the incident energy, in the range of interest. Since the electronic yield is directly proportional to the energy loss, one deduces that it is increasing yet concave on our restricted ions energy range. Fig.(7) shows the electronic yield from the tabulated values, based on a proportionality constant $\Lambda_{exp} = 0.1 \text{ cm}/\text{MeV}$. One then remarks that the yield is the highest for ^{304}SS at higher energies, and lowest for Al.

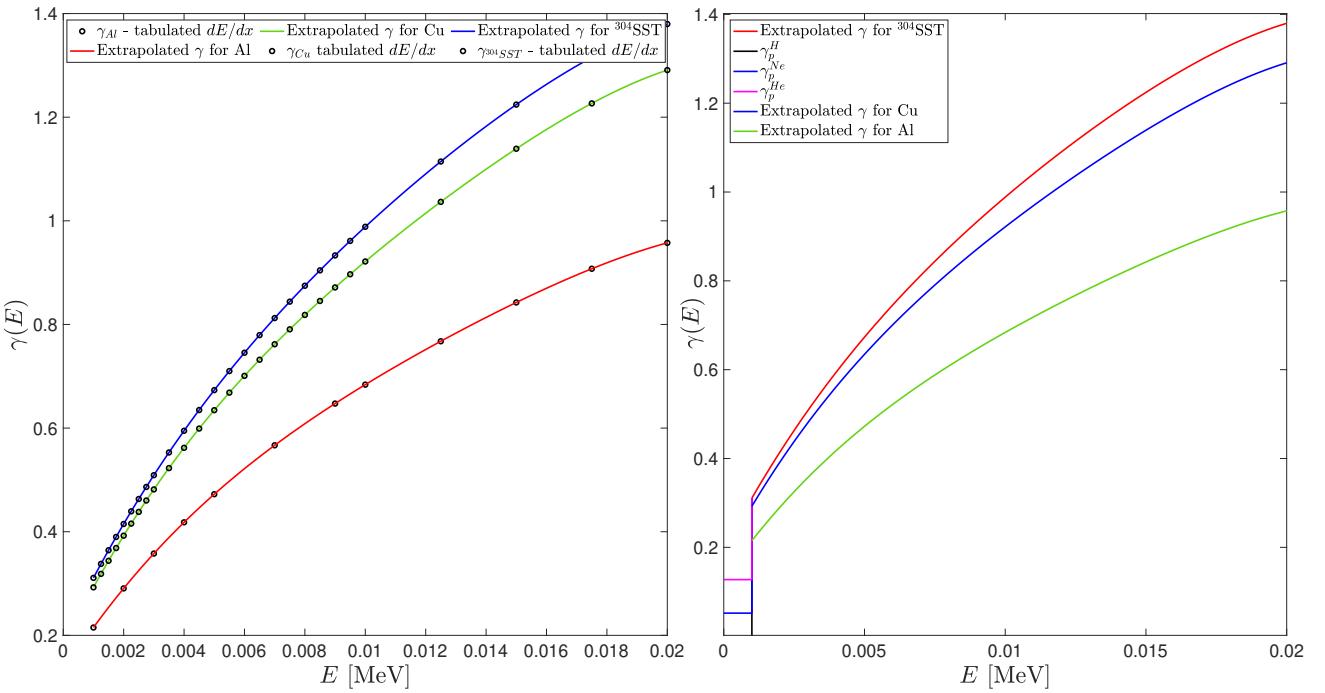


FIG. 7. Electronic yield γ obtained from tabulated values of dE/dx from [Jan82a, Jan82b] and extrapolated between the points, for the 3 possible electrode materials. Left: Let appear the tabulated values - Right: Extrapolated $\gamma(E)$ over the full energy range taking account for the potential emissions model (constant yield) from Eq.(7).

To numerically implement the previous results, the yield curve $\gamma(E)$ had to be polynomially interpolated between the points. Since the required degree for a polynomial fit over the full energy range would have been somewhat too high to be implemented in the code, for numerical complexity reasons, the energy range has been split so that the curve could be fitted by several degree-3 polynomials in the kinetic emissions region, as shown in Fig.(8). Note that quadratic polynomials would have underfitted the curve, while degree 4 polynomials would overfit for the desired precision, and weigh down the numerical treatment. Regarding the potential emissions region, the yield curve was linearly interpolated between the bottom of the kinetic emissions region to the constant value from Kishinevksy's model. Thus the yield curve simply reduced to piecewise polynomials, whose coefficients were implemented in the FENNECS code.

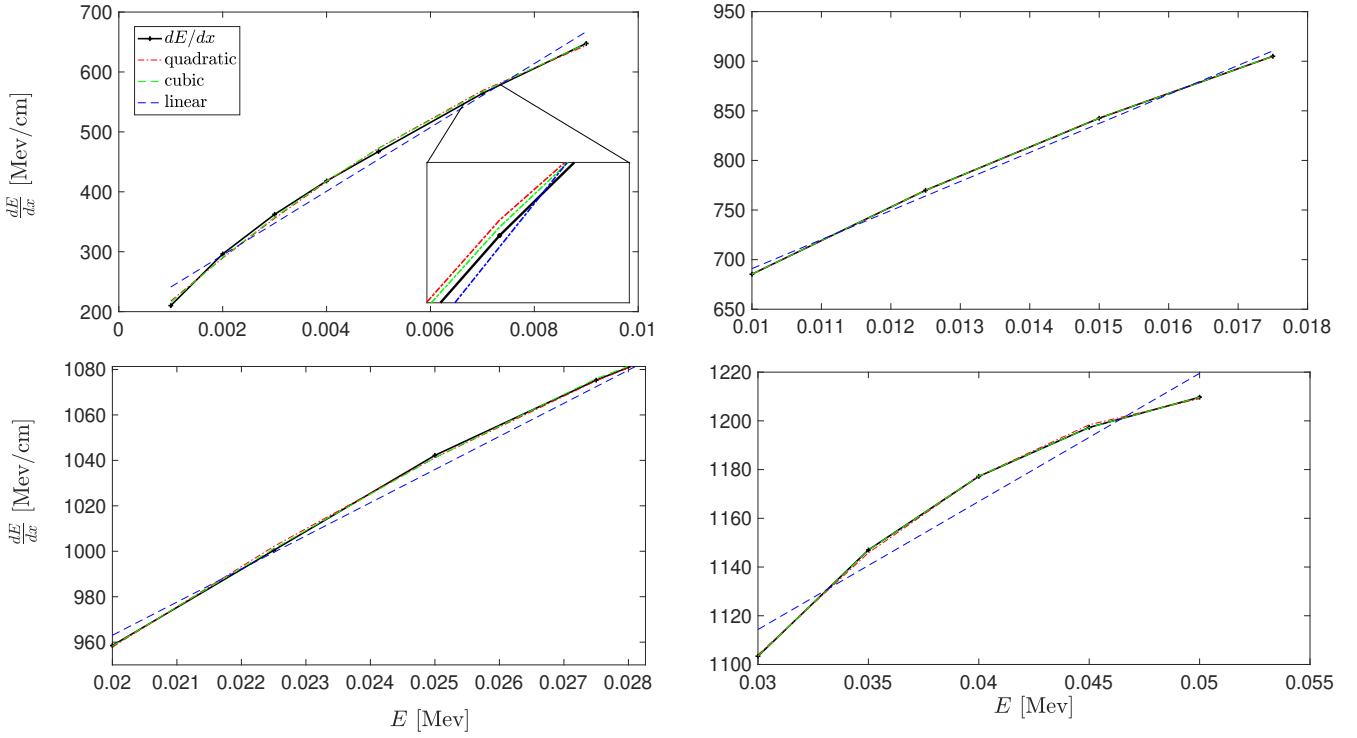


FIG. 8. Several fitting polynomials for the energy loss curve dE/dx , over the kinetic energy range. For all four intervals, the best fitting was, as shown in the zoom part of the first plot, cubic.

Regarding the emission of electrons in itself, since electron-emissions are discrete and qualified as rare, such events ought to follow a Poisson law. Considering that the average number of electrons emitted per ion is fully determined by the yield curve $\gamma(E)$, the parameter of the chosen Poisson law must be $\lambda(E) = \gamma(E)$. To sum things up, each ion impinges on the cathode surface with an energy E . Depending on its value, the emission phenomenon will be either potential or kinetic. The average number of emitted electrons, for a large number of collisions with the electrode, at this fixed value E , will be $\gamma(E)$. Then, since the number of emitted electrons has to be discrete, the probability to release k electrons, with $k \in \mathbb{N}$ must be following a Poisson law such that its average λ equals $\gamma(E)$. Hence, the probability for an ion of energy E to release k electrons is

$$P(k) = \frac{e^{-\gamma(E)}}{k!}. \quad (10)$$

The cumulative distribution function corresponding to the PDF in Eq.(10) is then given by the following expression

$$C(k) = \sum_{j=0}^{\lfloor k \rfloor} \frac{\gamma(E)^j}{j!}. \quad (11)$$

Now that the probability for each event to occur is known, remains to implement the electron generator in the FENNECS code. To do so, a random number generator has been programmed such that the integers produced follow a Poisson law specified by $\lambda := \gamma(E)$. In order to achieve this random generation, random numbers are generated uniformly in $[0, 1]$. Since the cumulative distribution function (CDF) of the probability law ranges in $[0, 1[$, the distance from these numbers to the pre-images of the CDF is evaluated. The integer k taken as number of emitted electrons is then given by the min value of the previous distance. Mathematically, this reads as follows:

- i) Generate a random number r uniformly in $[0, 1[$.
- ii) Evaluate the CDF of the law with $\lambda = \gamma(E)$ as in Eq.(11).

iii) Take k as $k := \min_{\tilde{k}} |r - C(\tilde{k})|$.

In order to make sure that this way of generating random number according to the CDF in Eq.(10) is correct, a statistical survey over a large number of tests is ran, with several values for $\lambda = \gamma(E)$, and the obtained number of counts for each value of k is represented in Fig.(9). As expected for $\lambda = 1$, the number of counts for $k = 0$ and $k = 1$ are equal. Results show good agreements with theory for other values of λ , ensuring then that our Poisson random generator works fine.

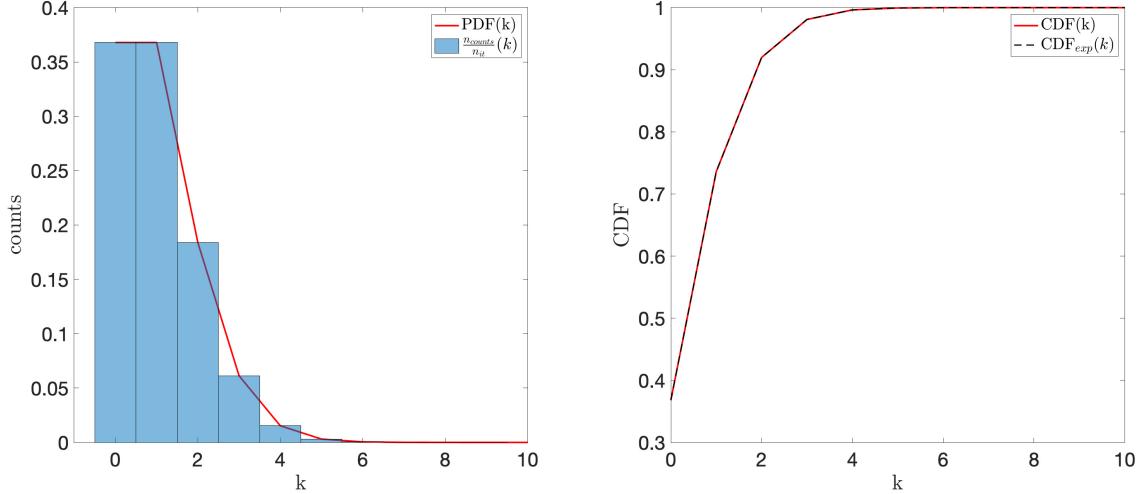


FIG. 9. Left: Normalised histogram for the number of counts obtained for each k following the Poisson law of parameter $\lambda = 1$ with the expected $P(k)$ (red) - Right: $C_{exp}(k)$ obtained by summing the contributions from each bin of the histogram and $C(k)$ the theoretical cumulative distribution function for $\lambda = 1$.

One more thing that had to be treated is the angular distribution of the emitted electrons, and their energy. Indeed, all the electrons are not generated with zero velocity, and the emission direction can also vary. Regarding the energy distribution, according to [DH, PPZ⁺16], it seems that the emitted electrons' energy follows a gamma law, such that it averages at 2 eV. Recall that the gamma distribution varies in shape and in peak localisation according two parameters κ and θ , with κ the shape parameter and θ the scale parameter. Recall too that the mean is given by $m = \kappa\theta$.

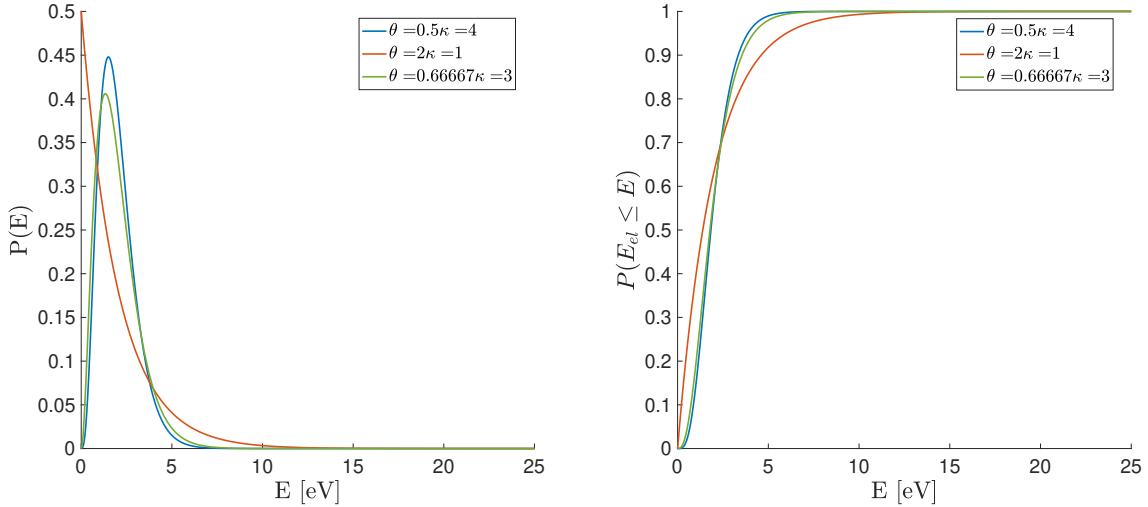


FIG. 10. Left: Gamma PDF for several (κ, θ) couples - Right: Corresponding CDF.

Fig.(10) shows several gamma distributions with different couples (κ, θ) such that $m = 2$ eV. The couple that has been kept is $(\kappa, \theta) = (0.5, 4)$, so the peak probability would be closer to 2 eV. Recall that the probability density function and the cumulative distribution for a gamma distribution have the following forms:

$$P(E) = \frac{1}{\Gamma(\kappa)\theta^\kappa} E^{\kappa-1} e^{-\frac{E}{\theta}}, \quad (12)$$

$$C(E) = \frac{1}{\Gamma(\kappa)} \gamma\left(\kappa, \frac{E}{\theta}\right), \quad (13)$$

where $\gamma\left(\kappa, \frac{E}{\theta}\right)$ is the lower incomplete gamma function.

Regarding the numerical implementation of the electron energy distribution in the FENNECS code, the same procedure as to produce a random number of electrons was applied, but in that case, the random numbers generated would not be integer anymore, and would follow a gamma distribution instead of Poisson. The difference in the number generation lies mostly in the discretisation of the range of values that could be taken. For Poisson generated numbers, only integer values can be taken, and hence, the construction conducted previously leads to exactly Poisson distributed values. In the case of the electrons' energy, the latter can physically vary on a continuum. Our energy-values generator is hence limited by the degree of precision in the discretisation of the energy range. However, for our purpose, that is of a qualitative survey of the behavior of ion-induced electrons, such precision is not required, since according to the large electric field values, no significant effect of the initial electrons' energy is to be expected.

- i) Generate a random number r uniformly in $[0, 1]$.
- ii) Evaluate the CDF of the law with $(\kappa, \theta) = (0.5, 4)$ in the range $[0, 15]$ eV with $N = 500$ points.
- iii) Take E as $E := \min_{\tilde{E}} |r - C_\Gamma(\tilde{E})|$.

Fig.(11) shows that our gamma random-number generator functions correctly. Indeed, following the same procedure as to statistically test the Poisson generator, one sees that over a large number of counts, the obtained distribution matches the gamma PDF.

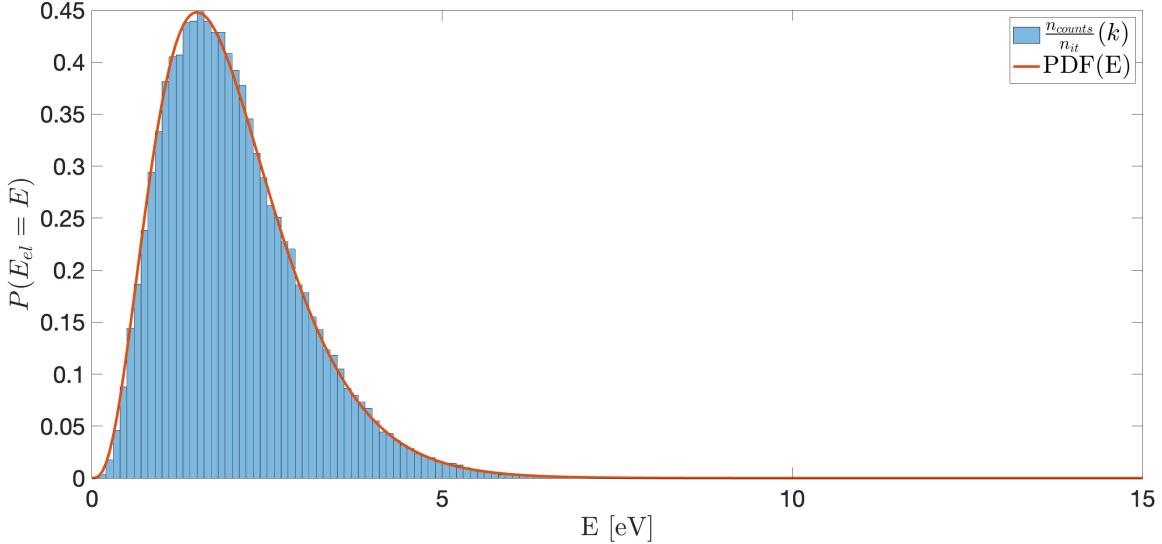


FIG. 11. Statistical results obtained from our gamma random-number generator. The red curve represents the analytical probability density function for this particular gamma distribution.

To put the module functioning in a nutshell, the process of generating electrons out of the ions loss consisted simply in combining the previous steps each time an ion was to hit the cathode. The full numerical treatment consisted in identifying the ions that would hit the cathode and apply the external module taking account for electron emissions.

Summing up the steps, each ion that disappeared would be identified evaluating the geometric weight, which would state that the particle is no longer in the simulation domain. Then, its energy would be evaluated, so that the average yield $\gamma(E)$ would serve in generating a random number of electrons, for which some additional memory would have been allocated. These electrons would then be placed initially at the last position *inside* the domain, and be given an energy randomly generated according to the gamma law from above. Then ion could then safely be removed from the particles, and the electrons would contribute to the newly updated population of particles. In light of that, it is important that the time step is short enough so the last recorded ion positions before they get removed, are close enough to the electrode, to avoid any spread effect as in Fig.(6). However, since the electrons are to be tracked to evaluate their potential contribution to any electron cloud, the time step should be already little enough. Fig.(12) illustrates the numerical treatment of cathode colliding ions.

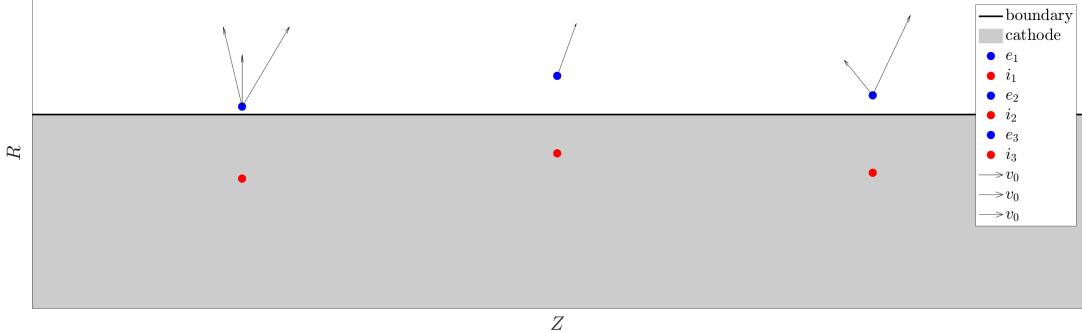


FIG. 12. Schematics of the situation where several ions are collected at the cathode, and a group of electrons is produced each time. Note that the number of electrons per incident ion is different, and the initial velocity vectors have different lengths due to the initial energy distribution

III. RESULTS

A. Code testing

The IIEE module being implemented, some tests were required in order to check that the code behaves accordingly. To do so, several tests were realised. The first one consisted of statistical results for the yield. To derive a statistics, the simple geometry from ?? was implemented, as well as several horizontal slices of ions. These ions were initialised at different radial distances from the cathode. Making use of Eq.(2), their energy as they hit the electrode is known, and so is the average yield per incident ion. Recall that the yield is a function of the incident ion energy only, through the energy loss in the material,

$$\gamma(E) = \Lambda_{exp} \cdot \frac{dE}{dx} \Big|_i . \quad (14)$$

It is also important to note that the energy loss tables used to implement the yield curves as in Fig.(7) contained values for protons impinging on different materials. In order to obtain results exploitable for the TREX experiment, the yield for incident protons had to be replaced by the yield for other ions, as H_2^+ for example. Approximatively, since H_2 is a diatomic molecule formed of hydrogen atoms, which are protons when ionised, one can deduce the yield for H_2^+ cations through $dE(\tilde{E})/dx|_{H_2^+} \sim 2dE(\tilde{E}/2)/dx|_{H^+}$ [?]. Hence, Eq.(14) reads

$$\gamma_{H_2^+}(\tilde{E}) = \Lambda_{exp} \cdot \frac{dE(\tilde{E})}{dx} \Big|_{H_2^+} = 2\Lambda_{exp} \cdot \frac{dE(\tilde{E}/2)}{dx} \Big|_{H^+} , \quad (15)$$

and then

$$\gamma_{H_2^+}(E) \simeq 2 \cdot \gamma_{H^+}(E/2). \quad (16)$$

For all three materials, the slices of ions were initialised along the axial direction, thus enabling to show that the electron generation from the code, over the loss of ions, is independent of the axial position, as expected. Fig.(13) shows the results for H_2^+ cations impinging on the cathode made either of ^{304}SS , Cu or Al. The potential bias applied between the two electrodes is $\Delta\phi = 20$ kV. The magnetic field is uniform, with field lines parallel to the electrodes, and $B = 0.21$ T, as it is a value of the order of magnitude used in gyrotrons. The radial positions of the electrode were $r_a = 0.001$ m for the cathode and $r_b = 0.01$ m for the anode, and the ions were generated at $r_1 = 0.003$ m, $r_2 = 0.005$ m and $r_3 = 0.008$ m. Using the very simple equation Eq.(16), the expected yield could be estimated and the comparison of the latter with the one obtained from the IIEE module is exposed in Table.(I).

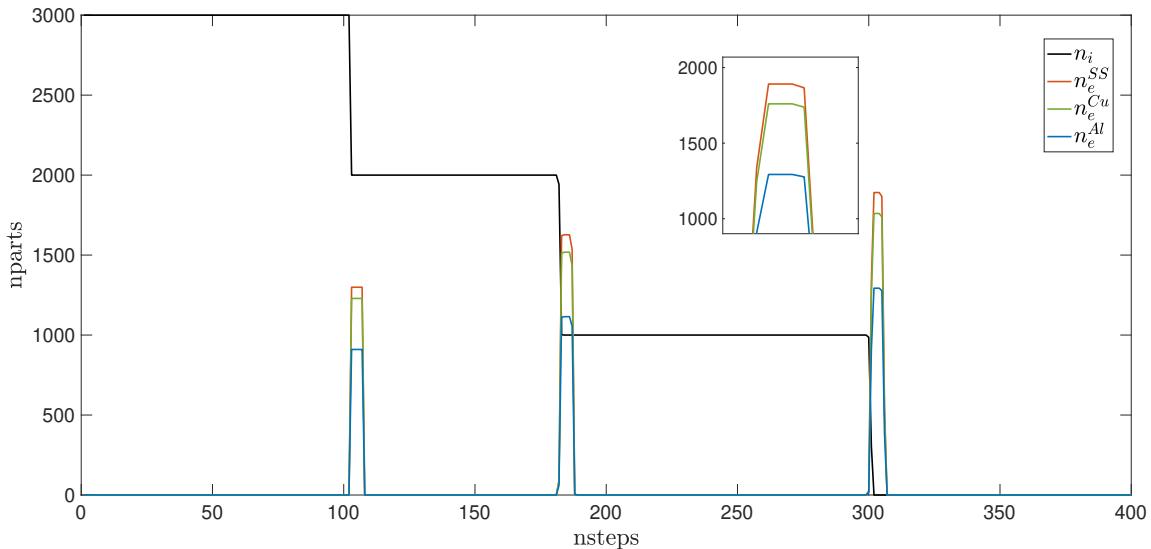


FIG. 13. Plot of the number of particles over the number of time-steps. The black curve represents the number of ions, while the colored curves show the number of electrons produced in the impingement of H_2^+ ions over ^{304}SS , Cu and Al.

One notes on Fig.(13) that the ions population curve is discontinuous. It is due to the fact that the ions are initially configured in three slices at distinct radii, as shown in Fig.(14), and together with the symmetry of the system, they reach the cathode and disappear at distinct discrete times, hence the steps. Regarding the peaked shape of the electronic population curves, they disappear few time after being generated because in this geometry, the magnetic field lines are parallel to the electrodes. Thus, of a Larmor gyration, the electrons are not moving away from the cathode, and hence, they are recaptured few after being emitted.

The statistical results presented in Table.(I) are interesting in the sense that the theoretical model seems to have been implemented correctly. Regarding the relative errors, they are always lesser or equal to 1.6%, which is an acceptable value considering the fact that the model is approximative in several ways. Indeed, the yield conversion from H^+ to H_2^+ exposed in Eq.(16) is an approximation. The fact that the yield in the potential emission region was constant (see Eq.(7)), and that we linearly interpolated this constant value to the bottom of the kinetic emission model, constitutes one more source of approximation. However, this description is sufficient for a simple estimate of the influence of IIEE on electron clouds formation and behavior. Indeed, the yield being of the order of $\sim 1 - 2$, the approximations made should not influence or disturb greatly the expected results, in terms of electronic densities for example.

The last phase of the module testing was to determine whether or not some of the electrons generated by ions colliding with the electrodes would be kept in the simulation domain, or if no matter the geometry, they would be lost, as on Fig.(13). To do so, an initial non-zero ion density was implemented in a region that was known to trap electrons, in a geometry where the cathode has half an ellipse dug inside it, as shown in Fig.(15) - Left. This is one of the geometry planned to be mounted in the TREX experiment. No electron cloud was present at the time, so the potential well naturally present due to the topology of the \mathbf{E} and \mathbf{B} fields was not affected by the presence of an

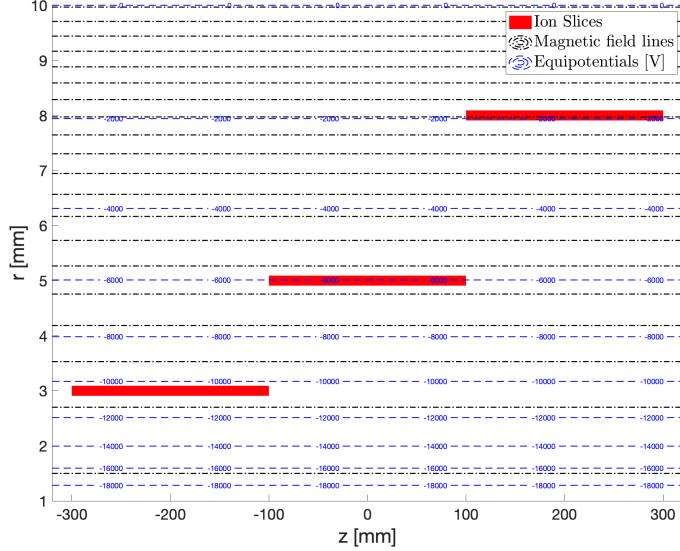


FIG. 14. Initial ionic configuration used to produce the statistics from Table.(I). The red stripes denote the initial ions positions. Note that by azimuthal symmetry, the ions are disposed on annuli.

	E_1	E_2	E_3
^{304}SS			
γ_{th}	1.311	1.623	1.870
γ_{iiee}	1.299	1.627	1.891
ϵ_{rel}	0.9%	0.2%	1.1%
Cu			
γ_{th}	1.237	1.522	1.746
γ_{iiee}	1.229	1.518	1.760
ϵ_{rel}	0.6%	0.3%	0.8%
A1			
γ_{th}	0.920	1.133	1.297
γ_{iiee}	0.910	1.115	1.293
ϵ_{rel}	1.0%	1.6%	0.3%

TABLE I. Yield statistics for H_2^+ ions impinging on the three materials

electron cloud. All the electrons produced at the electrode by IIEE were tracked down in a specific species in the code, so they would not be mixed up with the other electrons, coming from ionisations of the RNG. The right plot of Fig.(15) shows the orbits of two among all the electrons that have not left the simulation domain. One notes that they were trapped in the potential well, bouncing at its axial limits, while they were drifting azimuthally over several full poloidal periods. The radial width of the orbit is defined by the Larmor radius. With this information in hand, it appears that ion induced electron emissions could produce interesting results regarding the formation of clouds. For example, the formation time could be influenced, the maximum density too. This will be treated in the next subsection.

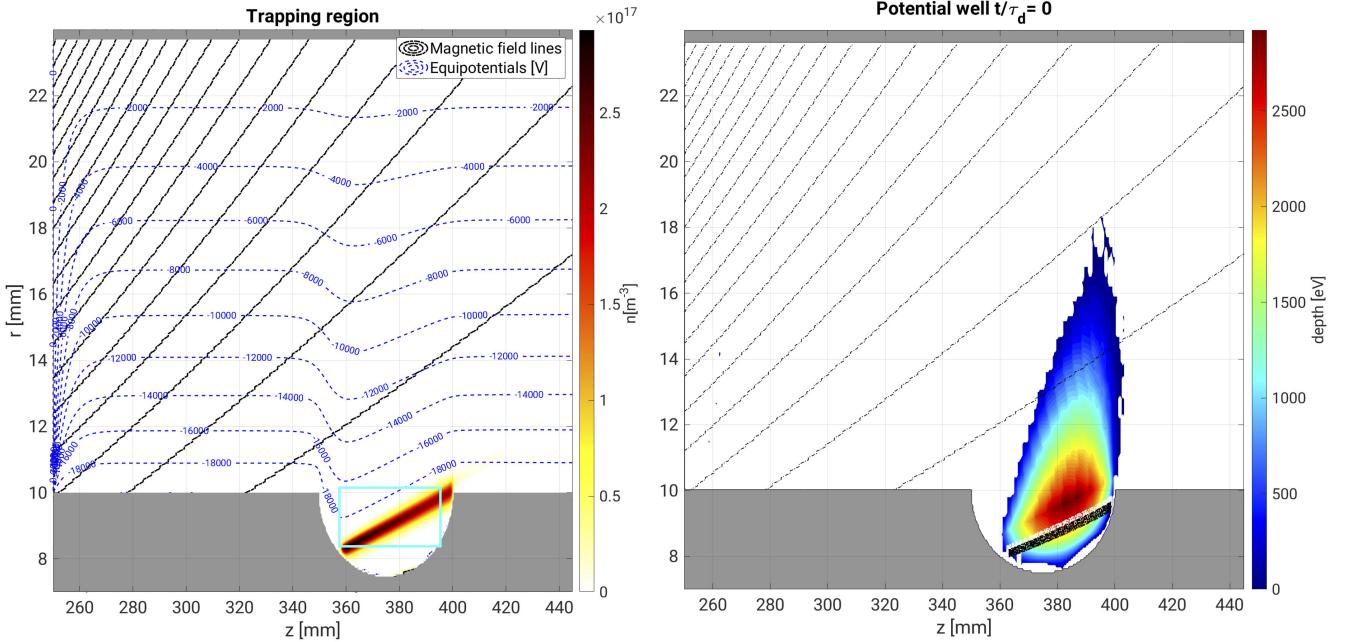


FIG. 15. Left: trapping region from the TREX extrude geometry. A cloud is present and the initial ion distribution is emphasized by the blue rectangle. - Right: vacuum potential well and two trapped electrons orbits.

B. Cloud formation

Now, let us deal with results concerning the clouds formation and dynamic, with and without taking account for IIEE. In first considerations, two particular TREX geometries have been implemented. Let us start with results in the *slanted* geometry, that is with a prominent half-ellipse coming out of the anode, as in Fig.(4).

TREX *slanted* geometry

Several characteristics are of interest in this case. As mentioned before, the characteristic time of cloud formation might be affected by the IIEE. The peak cloud density could also be increased. It can be interesting to look up the currents present at the boundaries too. Indeed, a strong electric current is as much as the source will not have to supply. To obtain these informations, two situations are considered. The geometry is the same for both, that is with the ellipse protruding from the anode. The anode is grounded, and the cathode is at $\Phi = -20$ kV. The magnetic field intensity is of about $B = 1.15$ T on the high field side (low z), and decreases down to $B = 0.15$ T on the low field side (high z). The magnetic field topology and the initial electronic configuration are shown together in Fig.(16). Since the bottom of the ellipse is known to be a trapping region, a rectangle distribution of electrons is initialised below it, in order to act as a source of ionisation, to produce the ions by collisions with the RNG. Now two cases are to be distinguished: the case with and the case without IIEE.

Let us first compare how fast the steady state is reached in both situations. Recall that the steady-state is characterized by the ions loss equal to the electrons loss. Hence, this is supposed to imply that the ionic current and the electronic current are constant over time, and of the same order of magnitude. Regarding the number of particles, it is supposed to reach a maximum and stagnate. The time required to reach this maximum corresponds to the cloud formation time. Plotting the total electronic charge over time can be a great indicator of the state of formation of the cloud. It can also enable to compare the formation in the two situations, where IIEE are considered and not.

On the left part of Fig.(17), the absolute value of the total electronic charge in the domain is represented over time, the time being normalised by the collision time, to remove the pressure dependency. The blue curve represents the charge when the ion induced emissions are taken into account. One notes that the formation times are similar, since the inflexion point of each curve is located at about the same time, and so is the peak value. As expected the charge is higher, since more electrons have been produced. Note that this information alone is not sufficient

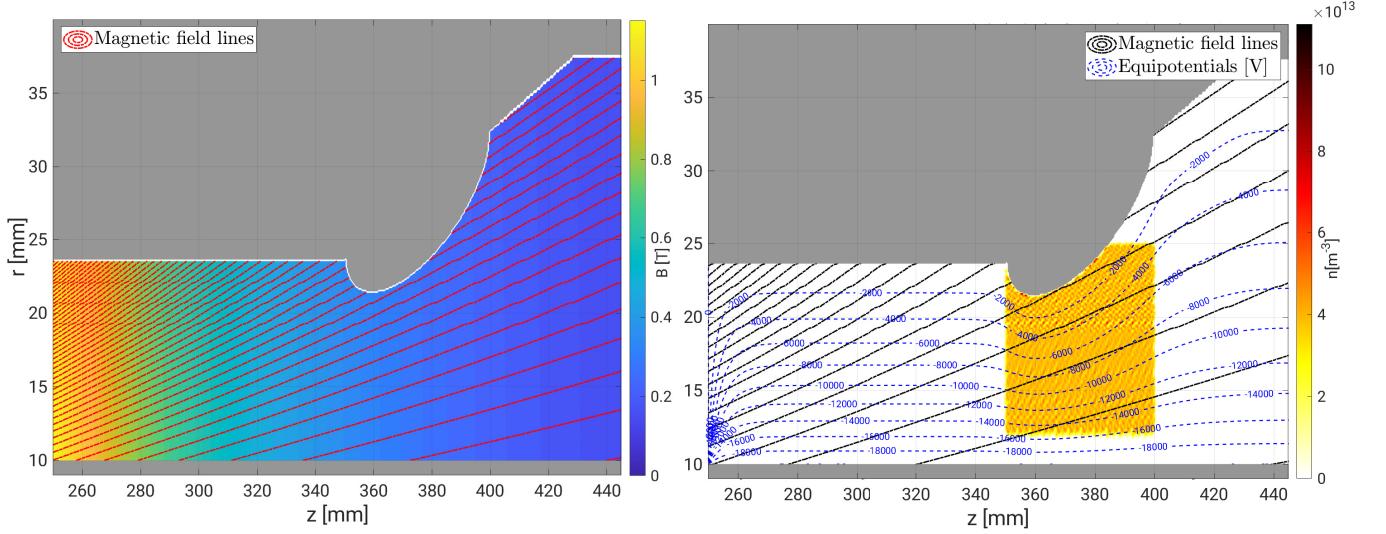


FIG. 16. Left: magnetic configuration from the TREX slanted geometry. - Right: initial configuration of electrons, leading to the formation of an electron cloud.

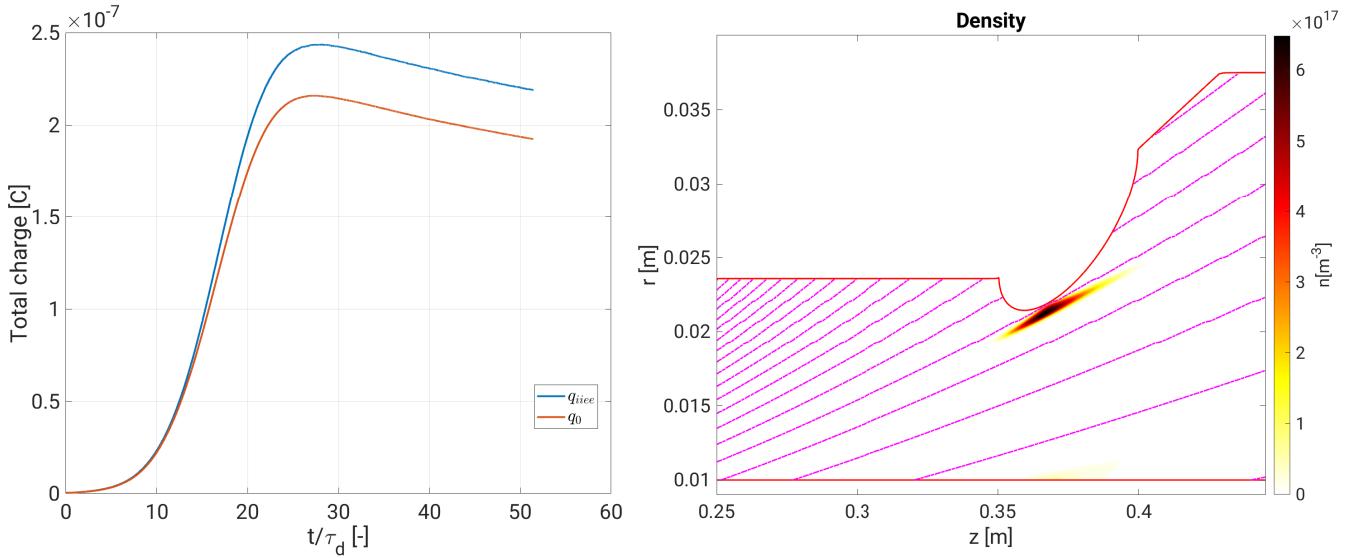


FIG. 17. Left: evolution of the total electronic charge inside the domain, over time normalised by the collision time. - Right: electron density in steady state.

to determine whether the total charge is higher because more electrons have accumulated in the cloud, or if they are present elsewhere in the domain. This is why the electron density is plotted in steady state, on the right plot of Fig.(17). Although it is not represented, both clouds, with and without IIEE had the same density and shape (hence, for the sake of saving space, only the situation with IIEE has been shown). This is consistent with our expectation, since the ion-induced electrons are formed at the cathode, and the potential well is located right by the anode. It is clear that these electrons, due to their Larmor motion along field lines, cannot reach the cloud and be trapped too. They will simply leave the domain axially, and contribute to a source of axial current. Now, one question arises: what is the order of magnitude of this ion-induced electronic current ?

To answer this question, the current collected at all the boundaries of the domain has been plotted over time. The boundaries are the axial limits, $z = 250 \text{ mm}$ and $z = 445 \text{ mm}$, and the metallic electrodes. The curves shown in Fig.(18) corresponds to the currents collected in the case where the IIEE were not considered. The purple curve

corresponds to the electronic current collected in the region of the ellipse, as the cloud is progressively compressing against the electrode and leaking electrons towards the metal. The plain green line corresponds to the electronic current at the cathode, which is consistently approximately zero. The green dashed line corresponds to the ionic current at the anode, while the yellow plain line shows the axial current at the low field side, that is electrons leaking from the potential well as it is progressively squeezed against the anode (see Fig.(23) for the evolution of the potential well). The black plain curve represents the total current, that is the sum of all these contributions. The dashed black curve corresponds to the sum of all currents from Fig.(19), for the sake of comparison. The blue (plain) line shows the evolution of the electron density n_e over time, in order to visualise when the steady state is reached, while the dashed one corresponds to the electrons density in the lower radial half of the domain. Note that the time has been normalised here by the collision time τ_d . As predicted, in steady state, the ionic (dashed green) and electronic (purple + yellow) are constant and of the same order. Of course, no current is collected on the high field side.

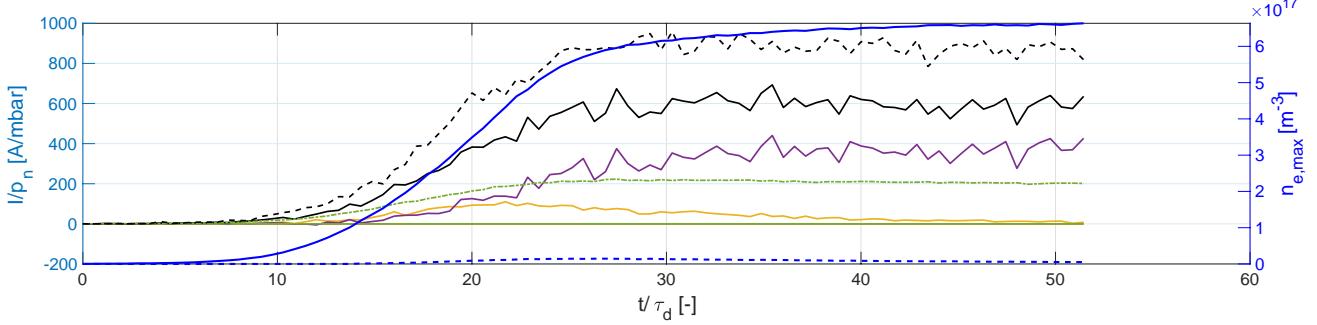


FIG. 18. Currents collected at all the boundaries of the domain, in absence of ion induced electron emissions. The colors correspond to the domains highlighted at the bottom part of Fig.(19)

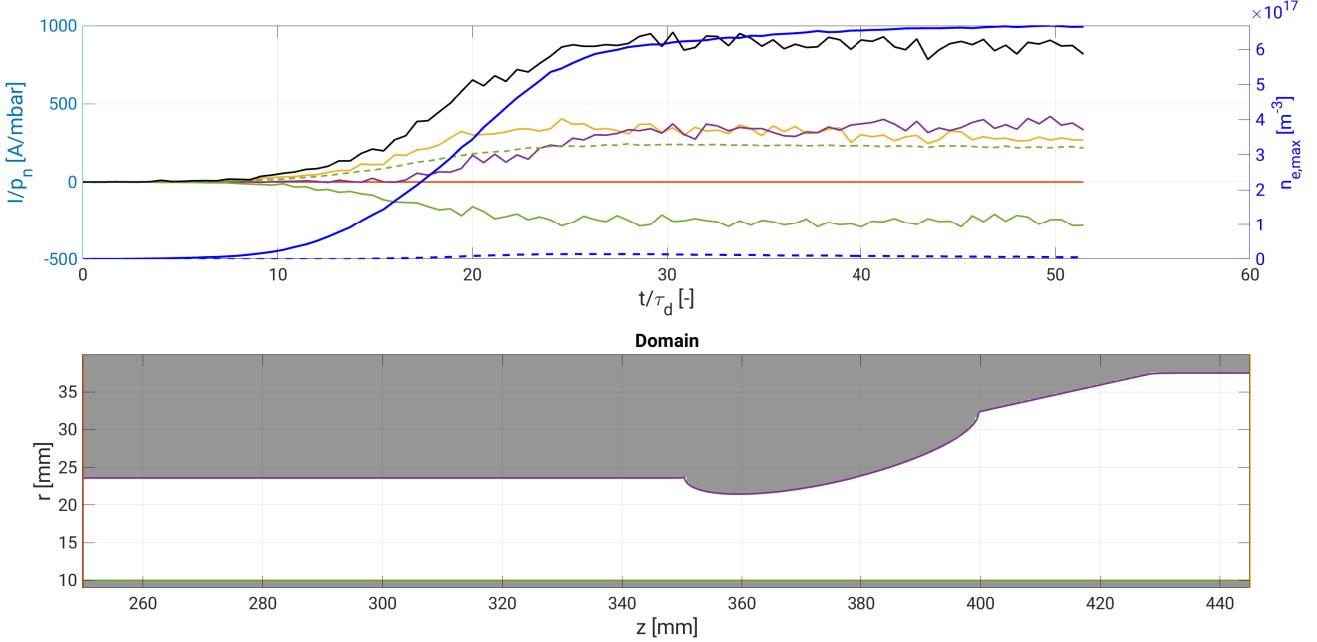


FIG. 19. Top: Currents collected at all boundaries of the domain, taking account for IIEE. - Bottom: Domain and boundaries definitions. The high field side is in red on the left, the low field side in yellow, the cathode in green and the anode in purple.

The high plot from Fig.(19) represents the same information as in Fig.(18), with the exact same color code. However, the contributions to the different currents might change a little bit. Still, no current is collected on the high field

side. The electronic current collected at the anode is approximately the same in both situations, emphasising that the cloud behavior does not depend on IIEE in this geometry. However, now a contribution (negative) of the total current at the cathode is present (green, solid line). This contribution is due to the electrons leaving the cathode as they are produced by the ions. Note that since the yield is on average, of the order of 1-1.5, the ionic current (green dashed) is approximately equal and of opposite sign as the electronic current leaving the cathode. The generation of electrons at the electrode being discrete events, the currents fluctuates a bit more around its average value.

Since the ions and the electrons have opposite charges, and they flow in opposite directions, the negative current at the cathode, coming from the electrons, will contribute to a new net current towards the grounded region. Indeed, all electrons being generated at the cathode are flowing towards the low field size to leave the domain. Thus, it is expected that the axial current is at least as important as the electronic current induced by electrons leaving the cathode. It turns out that it is even greater, since it contains the contribution from the electrons that are leaking from the cloud, see yellow line on Fig.(18). The total current flowing from the cathode (at $\Phi = -20$ kV) to the grounded regions (low field side and anode), plotted in black, is now much greater than the total current observed without ion induced emissions.

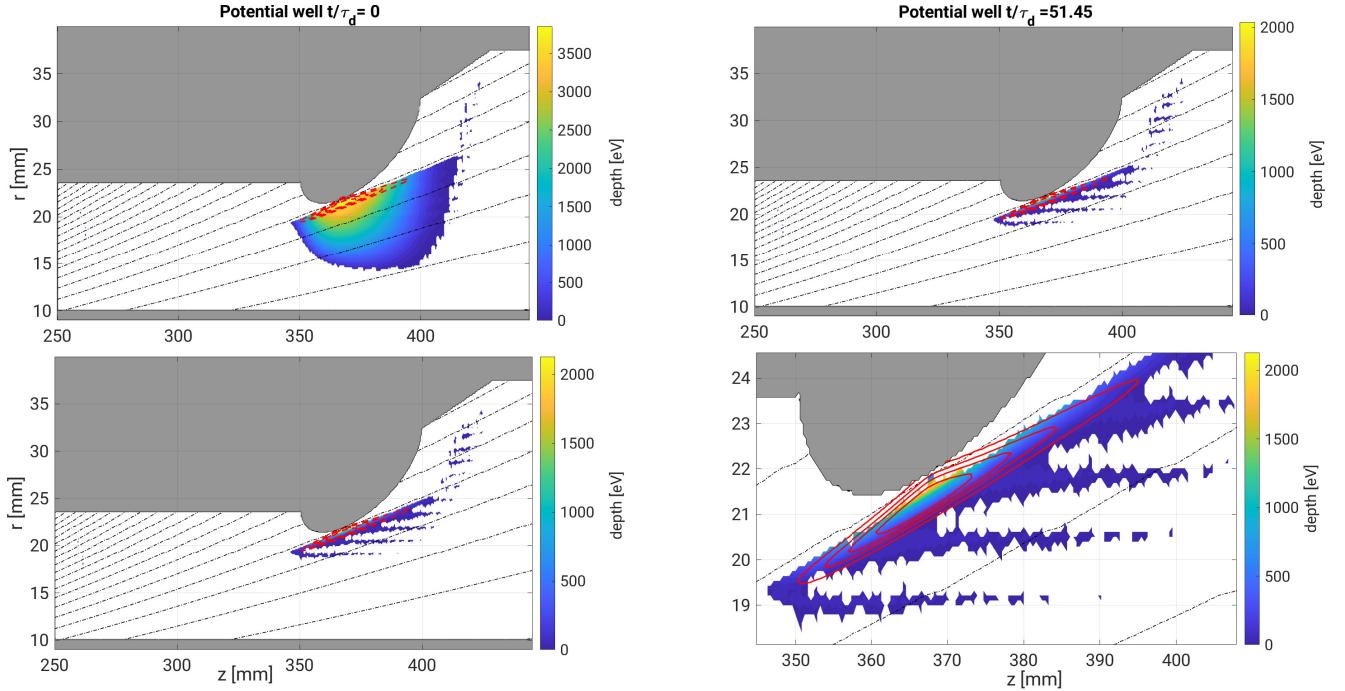


FIG. 20. Potential well dynamics as the electron cloud forms. Up Left: Vacuum potential well - Up Right: Potential well in presence of the cloud (IIEE) - Down Left: Potential well in presence of the cloud (without IIEE) - Down Right: Zoomed view

The dynamic of the potential well as the cloud is forming is shown in Fig.(23). The upper left corner plot shows the vacuum potential well, that is the well formed naturally by the magnetic field line crossing the equipotentials, in absence of the cloud. For the sake of readability, the equipotentials have not been shown. The dashed red contours show where the cloud forms inside the potential well. The upper right and lower left corners plots show the potential well in presence of the cloud, with IIEE and without them respectively. Note that the wells are identical, as expected from the previous considerations. The lower right corner plots shows a zoomed view of the potential well, with the contours ouf the cloud. Note that the outer contour intersects the anode, explaining the purple curve in Fig.(18-19), since some electrons from the cloud are collected at the anode. The potential well depth is remarkably affected by the cloud (see colorbar).

So to sum things up, the IIEE in this particular geometry seem not to influence neither the cloud formation time, nor the density. The clouds behave the same both with and without the ion induced electrons. As said previously, this can be explained by the magnetic topology, preventing the electrons leaving the cathode to reach the cloud's potential well. However, it is important to note that the electrons produced by the ions contribute to a non-negligible net current component, that adds up on top of the other contributions, coming from the cloud leaking electrons, and

the ions. Hence, this new contribution has to be taken into account in the dimensioning and the design of the power supply for the MIG.

TREX *extrude* geometry

Now, let us focus on one of the other designed geometries for TREX, the so called *extrude* geometry, that is with half an ellipse dug inside the cathode. The electrode geometry has been shown previously in Fig.(15). Here again, the anode is grounded, and the cathode is set at the potential $\Phi = -20$ kV.

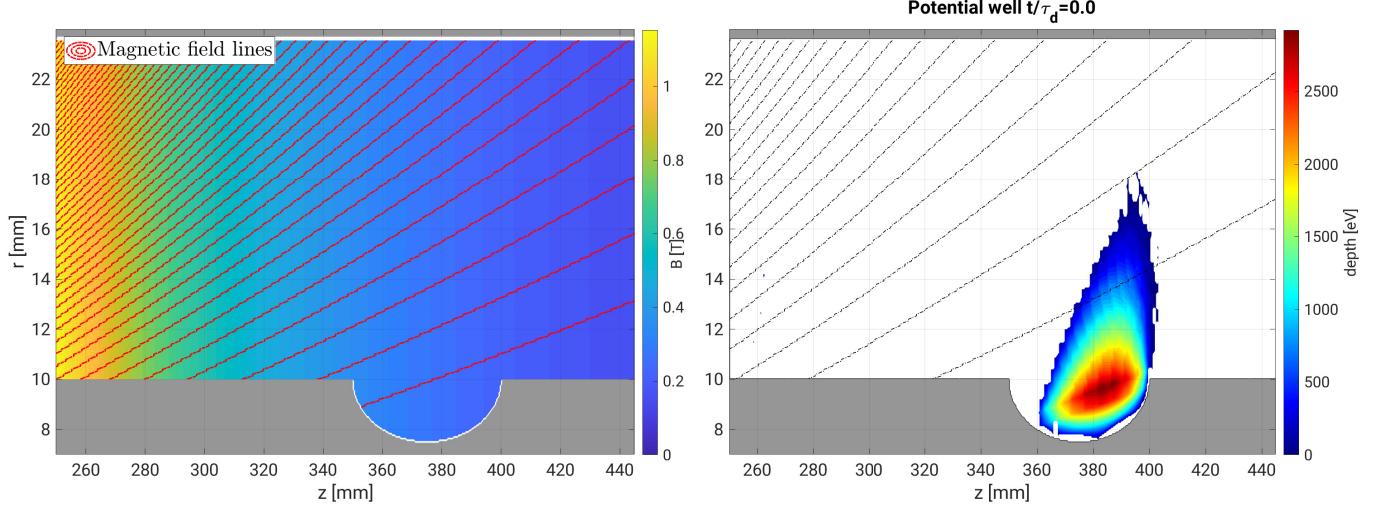


FIG. 21. Potential well dynamics as the electron cloud forms. Up Left: Vacuum potential well - Up Right: Potential well in presence of the cloud (IIEE) - Down Left: Potential well in presence of the cloud (without IIEE) - Down Right: Zoomed view

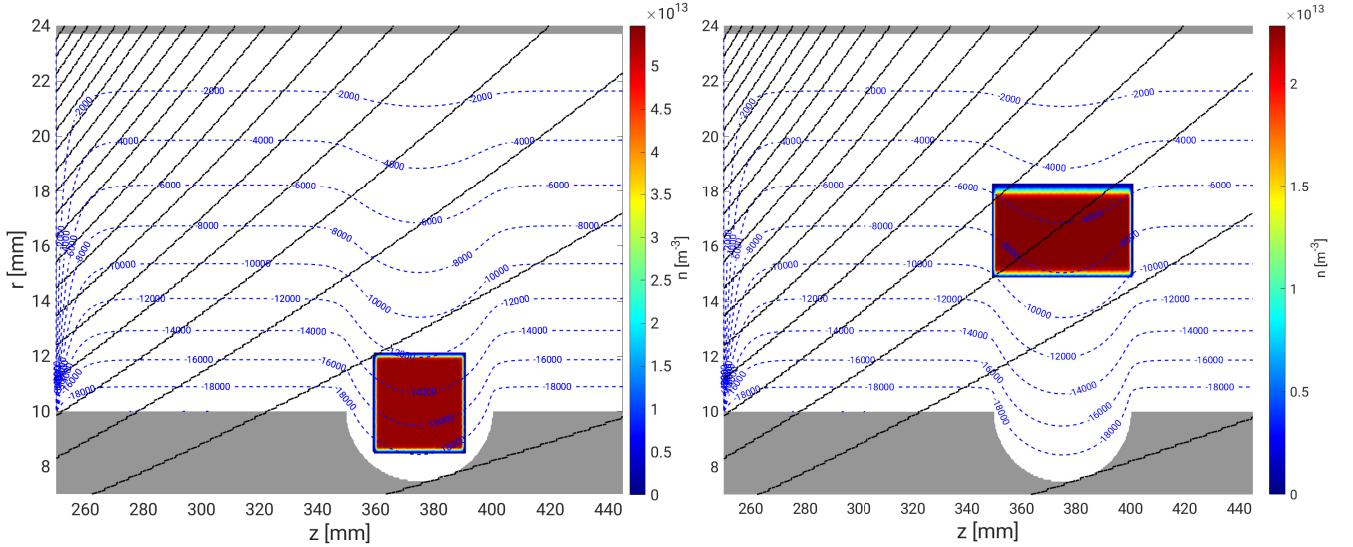


FIG. 22. Potential well dynamics as the electron cloud forms. Up Left: Vacuum potential well - Up Right: Potential well in presence of the cloud (IIEE) - Down Left: Potential well in presence of the cloud (without IIEE) - Down Right: Zoomed view

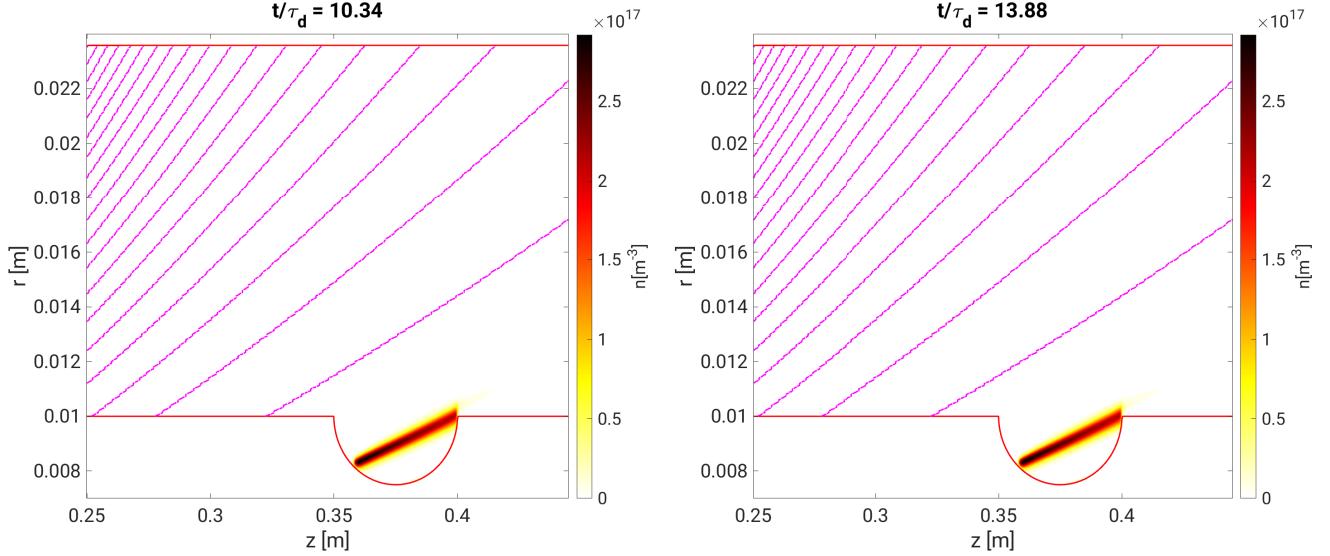


FIG. 23. Potential well dynamics as the electron cloud forms. Up Left: Vacuum potential well - Up Right: Potential well in presence of the cloud (IIEE) - Down Left: Potential well in presence of the cloud (without IIEE) - Down Right: Zoomed view

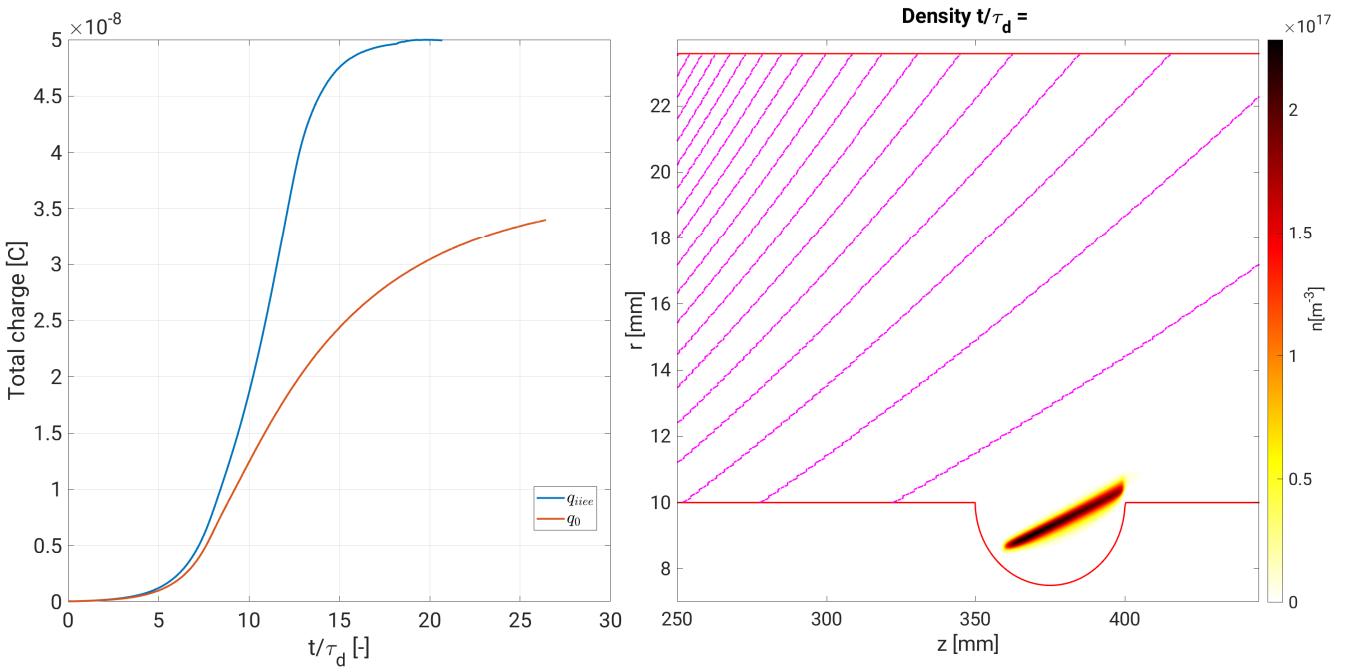


FIG. 24. Potential well dynamics as the electron cloud forms. Up Left: Vacuum potential well - Up Right: Potential well in presence of the cloud (IIEE) - Down Left: Potential well in presence of the cloud (without IIEE) - Down Right: Zoomed view

C. Further implementation

IV. CONCLUSION

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