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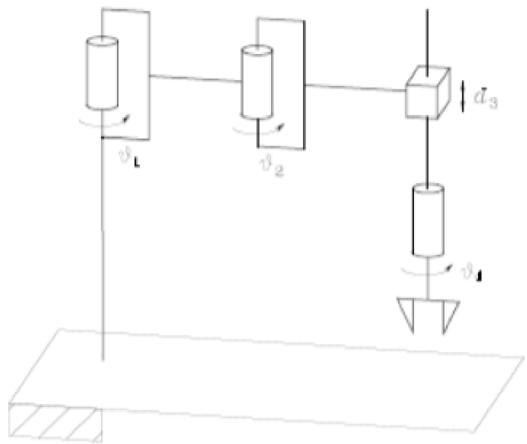
PROJECT 1 REPORT

The project given was to solve for the joint values of the SCARA Manipulator given to using the inverse differential kinematics.



The manipulator parameters are

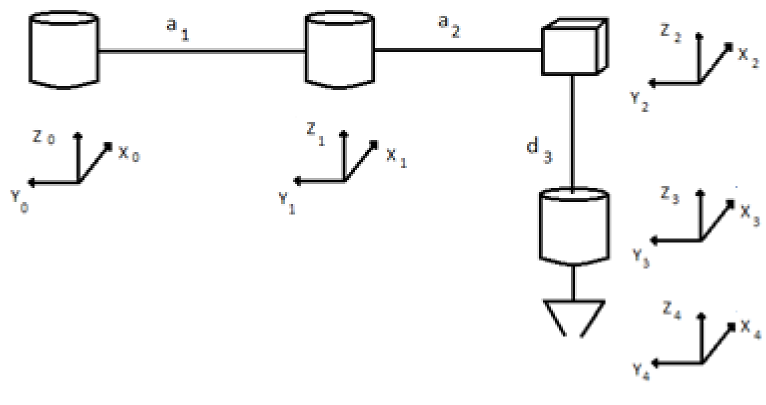
$$d_0 = 1\ m,\ a_1 = a_2 = 0.5\ m,\ l_1 = l_2 = 0.25\ m$$

$$\theta_{1_{min}} = -\pi/2\ rad,\ \theta_{1_{max}} = \pi/2\ rad,\ \theta_{2_{min}} = -\pi/2\ rad,\ \theta_{2_{max}} = \pi/4\ rad$$

$$d_{3_{min}} = 0.25\ m,\ d_{3_{max}} = 1\ m,\ \theta_{4_{min}} = -2\pi\ rad,\ \theta_{4_{max}} = 2\pi\ rad$$

Question 1.1:

The frames assigned are shown below:



The D-H Parameter Table is as follows:

Θ	α	a	d
Θ_1	0	a_1	d_0
Θ_2	0	a_2	0
0	0	0	$-d_3$
Θ_4	0	0	0

The generalised direct kinematics Equation is:

$$\boldsymbol{x} = \boldsymbol{k}(\boldsymbol{q})$$

The below equation is basically the direct kinematics equation as the final transformation matrix will be a function of the q i.e. the joint variables.

$$^{0}H_4 = {}^{0}H_1 * {}^{1}H_2 * {}^{2}H_3 * {}^{3}H_4$$

$^{0}H_4=$

$$\begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}H_4 = \begin{bmatrix} c_{124} & -s_{124} & 0 & a_1c_1 + a_2c_{12} \\ s_{124} & c_{124} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & d_0 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 1.2:

The Differential Kinematics equation represents the linear and angular velocity of the end effector as a function of joint variables.

$$v_e = \begin{bmatrix} p_e \\ \omega_e \end{bmatrix} = J(q)(\dot{q})$$

Here is the representation of the Differential Kinematics Equation.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ -\dot{d}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

Let $a_1 = a_2 = 0.5$

$$\frac{-s_{12}\dot{\theta}_2}{2} - \frac{(s_1 + s_{12})\dot{\theta}_1}{2} = \dot{\boldsymbol{x}}$$

$$\frac{C_{12}\dot{\theta}_2}{2} + \frac{(C_1 + C_{12})\dot{\theta}_1}{2} = \dot{\boldsymbol{y}}$$

$$-\dot{d}_3 = \dot{\boldsymbol{z}}$$

$$\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_4 = \dot{\boldsymbol{\omega}}_{\boldsymbol{z}}$$

Question 1.3:

In the geometric Jacobian there is no possibility of rotation around the x and y axes hence we can conclude that the 6x4 Matrix can be reduced to a 4x4 Matrix. This is the Analytical Jacobian. So, for this case the Geometrical and Analytical Jacobian is the same. Advantage of using this Analytical Jacobian is that we avoid redundancy. The Geometric Jacobian is represented as follows:

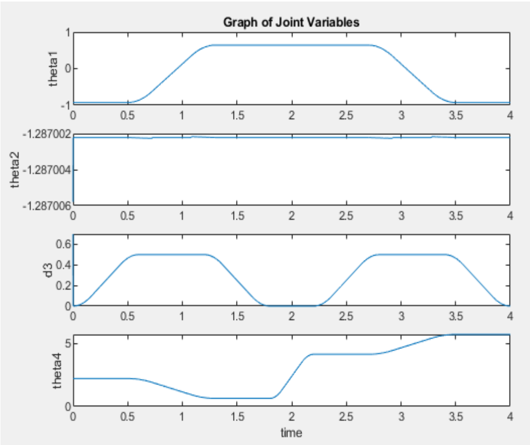
$$\boldsymbol{J} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

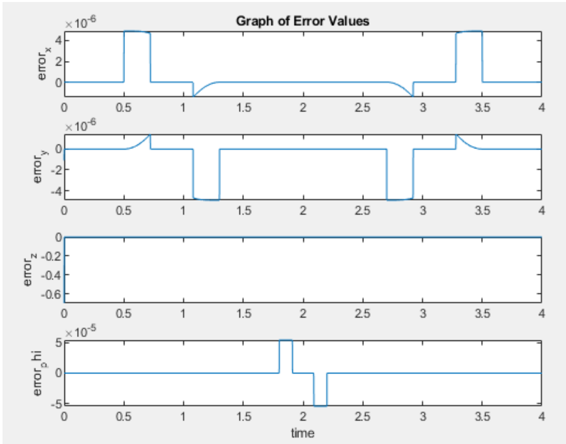
Here the Jacobian matrix (*J*) shows that the 4th and 5th rows are zero by magnitude hence we can eliminate them and obtain the new Jacobian matrix (J_A) as given below:

$$J_A = \begin{bmatrix} -a_1s_1 - a_2s_{12} & -a_2s_{12} & 0 & 0 \\ a_1c_1 + a_2c_{12} & a_2c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

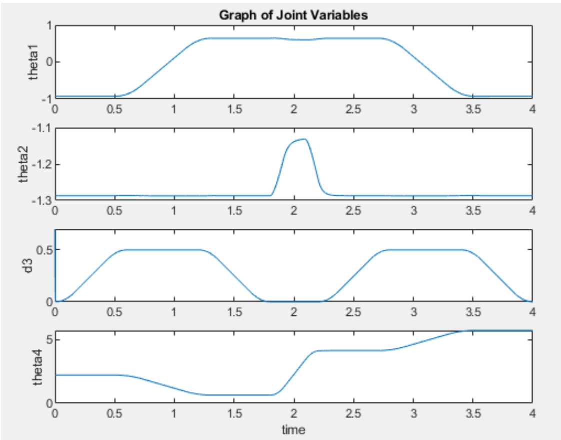
Question 1.4

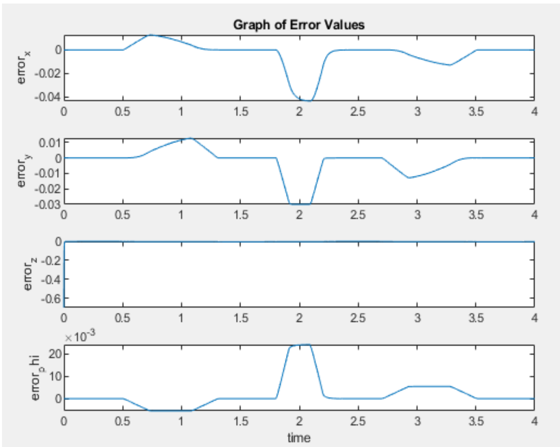
Inverse





Transpose





Question 2.1

The phi is the sum of all the rotations namely $\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_4$. When we try to relax the phi component then essentially, we realize that there is no rotation about the Z axis since the sum of $\dot{\theta}_1, \dot{\theta}_2$ *and* $\dot{\theta}_4$ is the angular velocity of the Z axis. Hence, we eliminate the 4th row from the Analytical Jacobian and this gives us a 3x4 matrix. Since the new Analytical matrix is not a square matrix anymore then direct inverse is not possible for a non-square matrix, we calculate the pseudo-inverse for the matrix.

$$\boldsymbol{J}^{\dagger} = \boldsymbol{J}^{\boldsymbol{T}}(\boldsymbol{J}\boldsymbol{J}^{\boldsymbol{T}})^{-\mathbf{1}}$$

Question 2.2

When we relax the phi component, the fourth row of the analytical jacobian is eliminated. The new J_A is reduced to a 3x4 Matrix

$$J_{A1} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

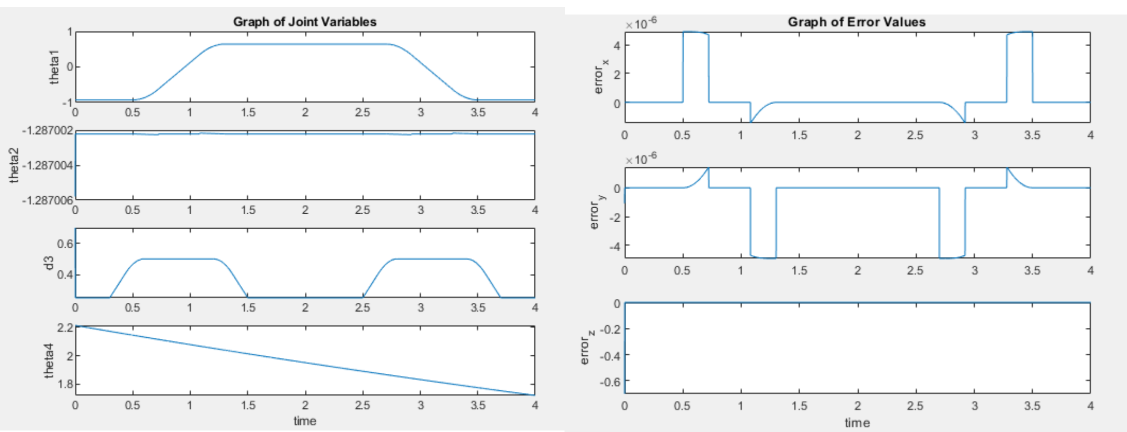
The maximum distance once we have relaxed phi is given as:

$$w(q) = \frac{-1}{2n} \sum_{i=1}^{n} \left(\frac{q_1 - \bar{q}_\iota}{q_1 M - q_i m} \right)^2$$

$$w(q) = \frac{-1}{8} \left[\frac{q_1^2}{\Pi^2} + \frac{q_2^2 + (\pi|8)^2 - q_2 \Pi \,/\, 8}{\frac{9\pi^2}{16}} + \frac{q_4^2}{16\Pi^2} + \frac{q_3^2}{2 \cdot 25} \right]$$

Question 2.3

Graph when the phi component is relaxed:



Question 2.4

Once we assume to relax the z component of the 3rd row of the analytical Jacobian, then we get the Jacobian pseudo inverse.

$$J_{A2} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Question 2.5

The distance from the obstacle is defined as:

$$w(q) = \min_{p,o} \|p(q) - 0\|$$

$$w(q) = \begin{bmatrix} c_{124} & -s_{124} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{124} & c_{124} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \cdot 4 \\ -0.7 \\ 0 \cdot 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 1 \end{bmatrix}$$

$$w(q) = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ d_3 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \cdot 2 \\ -0.9 \\ 0 \cdot 3 \\ 1 \end{bmatrix}$$

Question 2.6

Graph when the z component is relaxed: