Homework 5

1] (a) $f(n) = n^2 \log n$

$a = 4$ & $b = 2$

$\therefore n^{\log_b a} = n^2$

So by applying ~~T(n) = a(T(n/~~

$T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$

$\Rightarrow \Theta(n^2 \log^2 n)$.

(b) $f(n) = n \log n$

$a = 8$ & $b = 6$

$\therefore n^{\log_b a} = n^{\log_6 8}$

$f(n) = n \log n = O(n^{\log_6 8 - \epsilon})$ - (applying Case 1)

$T(n) = \Theta(n^{\log_6 8})$

(c) $a = \sqrt{6000}$ $f(n) = n^{\sqrt{6000}}$

$b = 2$

$n^{\log_b a} = n^{\log_2 \sqrt{6000}}$

$\because af(n/b) = \sqrt{6000} \left(\frac{n}{2}\right)^{\sqrt{6000}} \leq c n^{\sqrt{6000}}$

$T(n) = \Theta(n^{\sqrt{6000}})$

(d) $a = 10$ $b = 2$ & $f(n) = 2^n$

$n^{\log_b a} \Rightarrow n^5$

$af(n/b) \Rightarrow 10 \cdot 2^{(n/2)} \leq c \cdot 2^n$

$T(n) = \Theta(2^n)$

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(c) ~~n=2~~ we change $n = 2^d$ & so

$T(2^d) = 2T(2^{d/2}) + \log_2 2^d$

$= 2T(2^{d/2}) + d$

~~$f(d) = d$~~

we change $T(2^d)$ to $S(d)$

$S(d) = 2S(d/2) + d$

$a = 2$ & $f(d) = d$

$b = 2$

$\therefore d^{\log_2 2} = d$ & $f(n) = d$

$\therefore S(m) = \Theta(d \log d)$

$\therefore T(2^d) = \Theta(d \log d)$

$T(n) = \Theta(\log_2 n \log_2 \log_2 n)$

4] If a is odd

$x^a = x^{a/2} * x^{a/2} * x$

& if a is even

$x^a = x^{a/2} * x^{a/2}$

It takes at most 3 calls to compute x^a.

$\therefore T(n) \leq T(n-1) + 3 \rightarrow T(n) = O(n)$

5] If a is of length n & b is J similar to a then it means that the length of b is equal to the length of a.

We use induction to prove ~~the~~ it.

When n=1 then this is trivial base case.

We now assume that we have proven this for all $n<k$ & so we now prove it for $n=K$.
There are 2 cases where b can be g-similar to a. First if they are equal then they have same length. Secondly, if $len(a_1) + len(a_2) = len(b_1) + len(b_2)$ or $len(a_1) = len(b_1)$ and $len(a_2) = len(b_2)$
thus in both cases it would mean that b has also length of n.

We will rearrange the two strings & then will prove that if they ~~project~~ are g-similar only then will they project the same string.
We split a if it is of even length into 2 halves a_1 & a_2. If it is not of even length we won't cut it. We then rearrange them into lexicographically minimum equivalents & the concatenate then.
Since we are dividing the problem into 2 halves $a=2$ & $b=2$

so $n^{\log_b a} \Rightarrow n$

& $f(n) = O(n)$

$\therefore T(n) = (n \log n)$

6) (a) We will first be checking the middle element. If the middle is the fix point we will return it o/w we will check whether the index of the middle element is greater than the value at the index. If it is then we check the lower half or else the upper half. If the size of the array = 1 then no fixed point is found so we return -1.

(b) $a = 1$ $b = 2$ $f(n) = O(1)$

$\therefore n^{\log_2 1} = n^0 = 1$

$\therefore T(n) = \Theta(\log n)$

(c) Let's say that the fixed pt is found at index i. We will then be checking i+1 & i-1 indices. If we don't find fixed pts at these indices then there can't be any other fixed points since the array is sorted & has distinct elements.

2) If more than half of the cards belong to a user we call that user a majority user.
We divide the set of cards into

two halves & then solve each half recursively to decide whether or not ~~the~~ a majority user exists. Once we solved 2 halves we then combine them to find ~~we~~ if there is a global majority user. A global majority user is someone who is the majority user in both halves. If there are different majority users then there is no global majority user present.

$\therefore a = 2$

$b = 2 \quad f(n) = \Theta(n)$

$\therefore n^{\log_a b} = n$

$\therefore T(n) = \Theta(n \log n)$

3] ~~b~~ Let $L = \{L_1, L_2 \ldots L_n\}$ be the sequence of lines sorted in increasing order of slope. We divide them into two halves & solve them recursively. When we are down with only one line, we call it visible. ~~We~~ Along with computing the set of visible lines we also compute the intersections.

We then merge the 2 recursively computed sorted lists. We then locate the intersection pt. this step can be done in $O(n)$. We then compute the set of visible lines also in $O(n)$ time.

$a = 2 \quad b = 2 \quad f(n) = O(n)$

$\therefore n^{\log_b a} = n$

$\therefore T(n) = O(n \log n)$