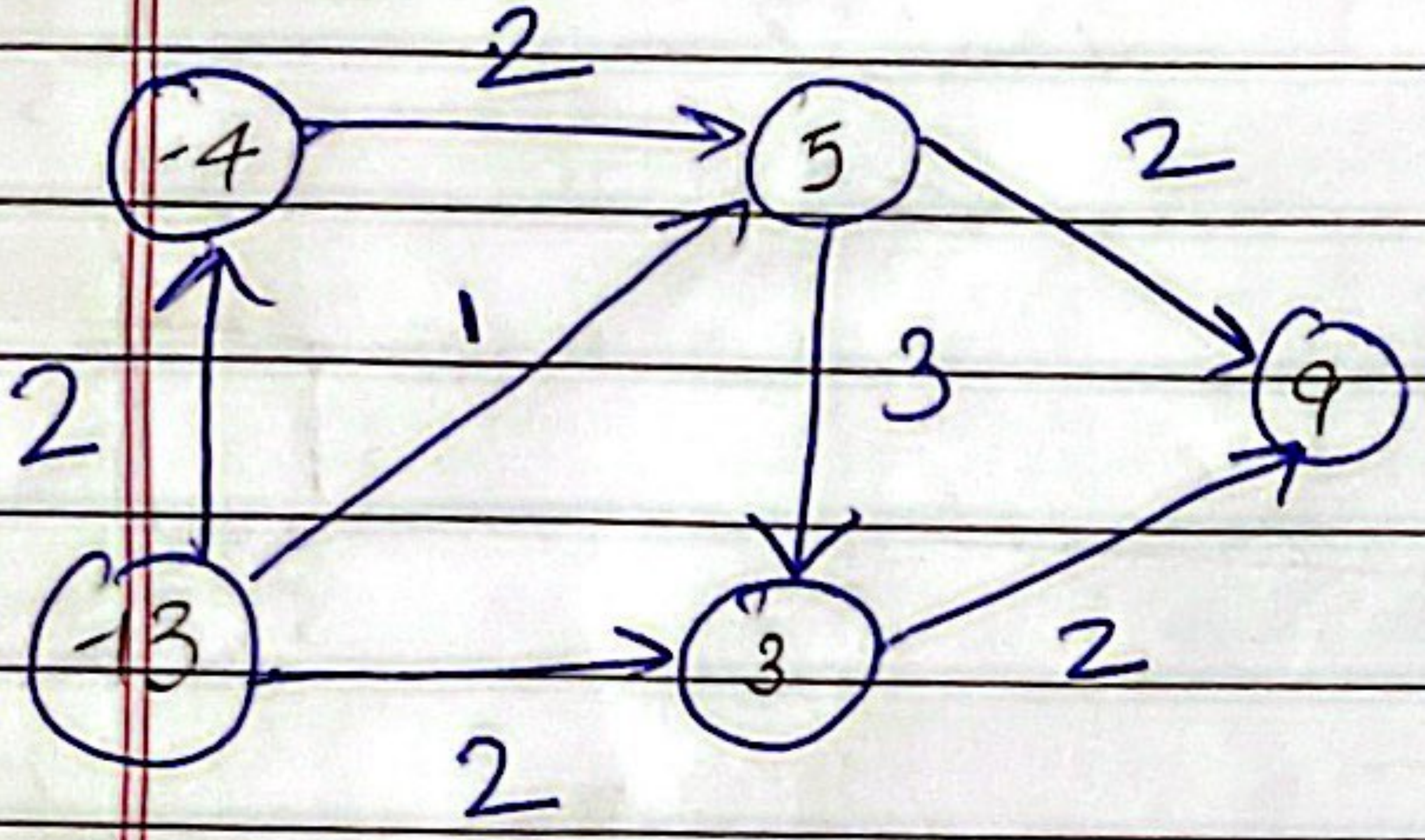
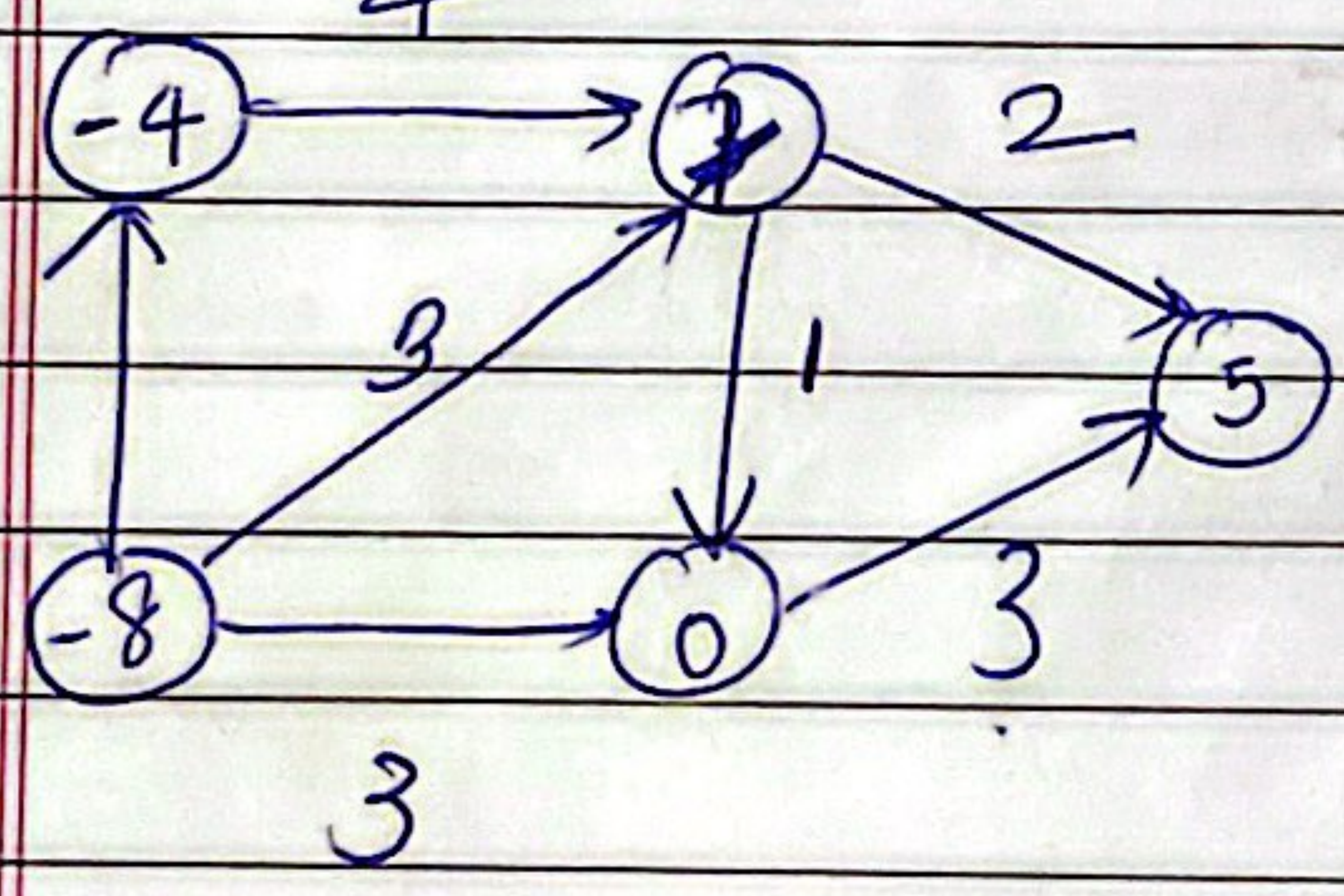
Homework 9

1] (a) We will first assigns flows equal to the lower bounds to all the edges.
After that we will calculate the Lvs and d'vs $= d_v - L_v$
Lastly the capacities change to
$C_e = C_e - l_e$.

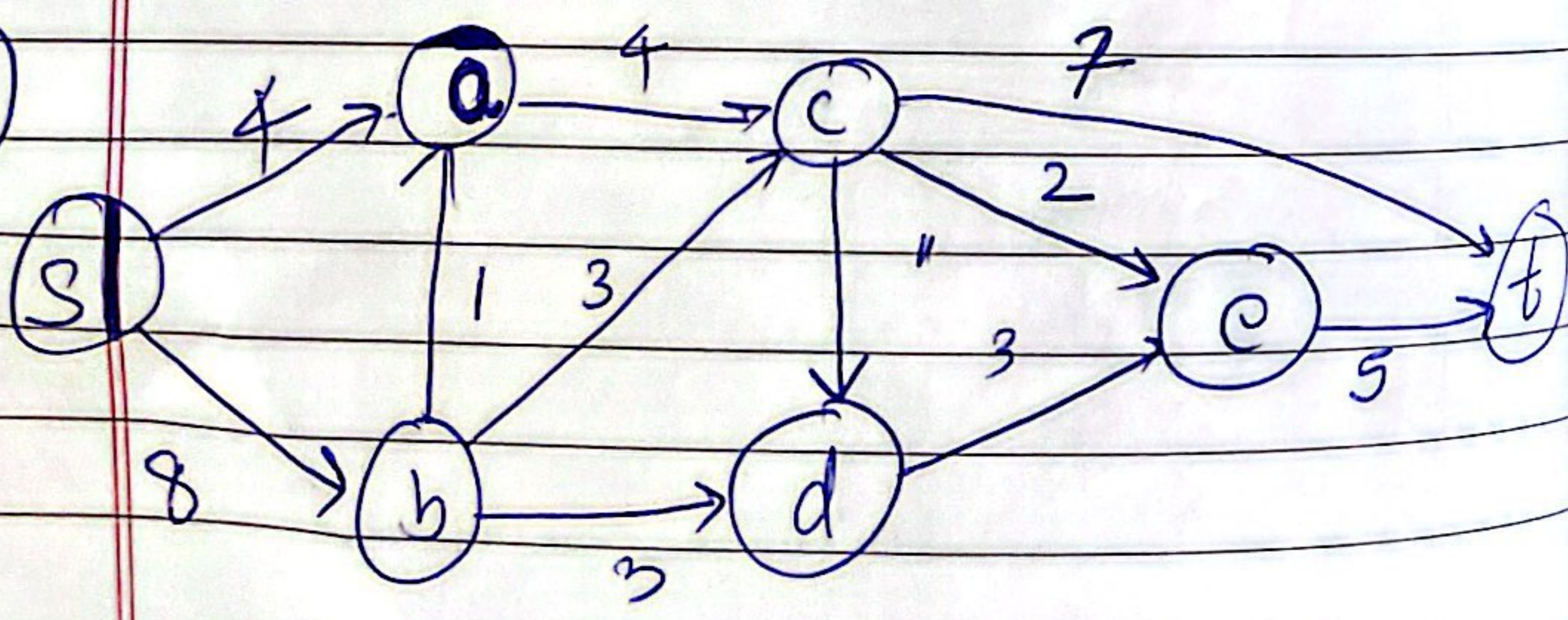
S_1



S_2



(b)



(c)

flow 1 $(f_1) = 3$

~~a~~

$s \xrightarrow{3} b \xrightarrow{3} d \xrightarrow{3} e \xrightarrow{3} t$

flow 2 $(f_2) = 4$

$s \xrightarrow{4} a \xrightarrow{4} c \xrightarrow{4} t$

flow 3 $(f_3) = 3$

$s \xrightarrow{3} b \xrightarrow{3} c \xrightarrow{3} t$

Max flow = $\underline{\underline{10}}$

$\therefore$ Max flow < Demand value

there is no feasible circulation.

3] Construct a ~~fo~~ network $G=(V,E)$. Let the two sets of vertices be that of families & tables. We join the families to the source & the tables to the sink. We add an edge from a_i family to b_j table with capacity equal to 1. We add the edges from source to families with capacities equal to g_i & tables to sink with capacities equal to h_j.

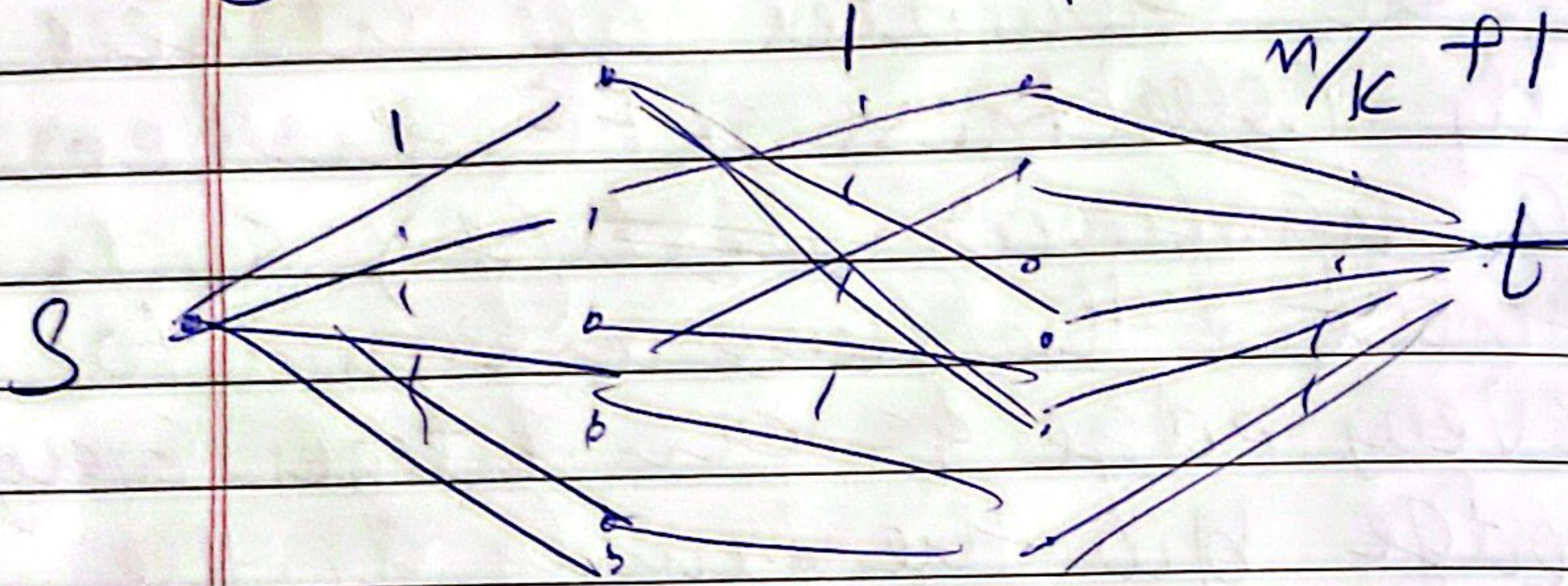
There exists a valid seating iff the value of max flow from s to t equals the $g_1+g_2+\cdots g_n$.
In order to prove this we assume that there exists a valid setting & so we construct the flow f as follows. If a member of the i^{th} family is seated at the j^{th} table then we assign the flow of 1 from (a_i, b_j) else 0.
The edge (s, a_i) is assigned a flow equaling the no. of members in the i^{th} family that are seated.

In similar way, the edge (b_j, t) is assigned ~~valid~~ a flow which equals to the no. of seats taken at b_j. The value of the flow thus equals to ~~the~~ $g_1 + g_2 + \cdots + g_n$.

Converse:- Now we assume that there is a max flow that equals to $g_1 + g_2 + \cdots + g_n$.
$\because$ the capacities are integers by correctness of FF there exists a maxflow ~~with~~ (f) such that the flow assigned to every edge is an integer. Every edge b/w the family & the table vertices has either flow of 1 or 0. We construct the seating arrangement as follows, we seat a member of the i^{th} family on the j^{th} table iff the flow going from (a_i, b_j) is 1. By construction at most one member of a family is seated at a table. $\because v(f)$ equals the capacity of the cut $(\{s\}, \{V - \{s\}\})$, every edge out of s

is Saturated. Thus by conservation at a_i, for every a_i, no. of edges out of a_i with a flow of 1 is equal to g_i. ∴ $f(b_j, t)$ out of b_j is atmost h_j, at most h_j persons are seated on b_j table. We thus have a valid seating.

4) (a) Patients hospitals



If the hospitals are within 30 min drive we connect them.

(b) Each unit flow from S to T is equal to assigning a patient to a hospital with the following restrictions:
- Each hospital gets less than $n/k + 1$ patients. Each patient will be assigned to only 1 hospital which

is located in the 30 min drive.
We use FF algo to find max flow.
The max flow is the max of this assignment & if we can assign all the patients then we can do the balance allocation.

(c) FF runs for $O(Cm)$.
$C = n$ & $m = n + nK + K$
so $O(n(n+nK+K)) = O(n^2K)$

2] We can reduce this to a circulation flow with lower bounds. We construct G as:
- For each box i, G has to 2 nodes u_i & v_i & an edge b/w them that corresponds to this box. This edge has a lower bound & capacity of 1.
- For each pair i & j, if box j can nest inside box i, there is an edge (v_i, u_j) with lower bound 0 & capacity 1.
- G also has a source node S, with demand $-K$ & a sink with demand K.

- For each box i, G has an edge (s, v_i) with lower bound 0 & capacity 1.
- For each box j, G has an edge (v_j, t) with a lower bound 0 & a capacity of 1.

We claim the following:
There is a nesting arrangement with k visible boxes iff there is a feasible circulation in G with demand -k in the source node & k in the sink.

Proof:

Suppose there is a nesting arrangement with k visible boxes. Each sequence of nested boxes inside one visible box $i_1, i_2 \ldots i_n$ defines a path from s to t as $(s, v_{i_1}, v_{i_1}, v_{i_2}, v_{i_2} \ldots t)$. Therefore we have k paths from s to t. The circulation corresponding to all these paths satisfy all demands, capacity & lower bound.

Conversely, consider a feasible circulation ntwrk. We assume that it has integer flow values. There are exactly K edges to going to t that carry 1 unit of flow. Consider one such edge (v_j, t). We know that (v_i, v_j) has 1 unit of flow. ∴ there is a unique edge into v_j that carries one unit of flow. If this edge is of kind (v_j, u_i) then put box i inside v_j & continue with box j. If this edge is of kind (s, u_i) then put box i in the back room. This box became visible. Continuing this way we pack all boxes into K visible ones. The maximum no. of iterations to find min possible no. of ~~possibles~~ boxes is n ∴ algorithm runs in polynomial time.

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