Homework 10

1] Efficient Certificate :- Given a collectio of k clauses there is a 3-Sat ~~(15/16)~~ with atleast αk clauses which are true.

Efficient certifier :- Check if every clau has 3 literals
- Check if there are atleast αk clauses which are true.

We chose 3-SAT $\leq_p$ 3-SAT (15/16)

We know that in 3SAT all the clauses are true. So in order to ~~convert~~ solve 3-SAT ~~to~~ using 3-SAT (15/16 we will add the 8 possible clauses given in the hint, which we ensure that the total clauses are now 16. (as we already know that 3-SAT has 8 clauses). Out of the 8 clauses that we add one of the clause will be false. So given a 3-SAT problem if there are atleast (15/16) clauses which are true

we can solve
then ~~all~~ 3-SAT using 3-SAT(15/16).

2] Efficient Certificate - Given a graph $G = (V, E)$ there exists a dense subgraph with atleast m edges & size atmost k.

Efficient Certifier - Check if the graph is connected
- Check if $V' \subseteq V$
- Check if the size is atmost k & atleast m edges.

We chose ~~set cover~~ independent set $\leq_p$ Dense Subgraph

We will solve ~~set cover~~ independent set using dense subgraph. If we i/p ~~a graph~~ have a subgraph of size k then we will get an independent set of size k.

3] Efficient Certificate :- Given a SAT' problem we can find whether or not there is an assignment such that m-2 clauses ~~of p~~ are satisfied.
Efficient Certifier :- Check for

m-2 clauses.

We Choose ~~3~~ SAT problem $\leq_p$ SAT'
We will add 4 more clauses to the SAT problem. $(x_1, x_2, \bar{x}_1, \bar{x}_2)$. Originally SAT problem had all true instance but now by adding these 4 new clauses if we solve SAT' and get exactly m-2 clauses to be true then ~~SAT~~ there exists an assignment which satisfies SAT. If there exists an assignment which satisfies SAT then there also exists an assignment that satisfies SAT'. Here we will get m+2 clauses to be true and we know that we are getting those because of the extra clauses we added so. SAT $\leq_p$ SAT'.

4] Efficient Certificate :- there exists a vertex cover when all the vertices in the graph are restricted to have even degree
Efficient Certifier :- Check if the vertices in the vertex cover have even degree.

We choose Vertex Cover $\leq_p$ Vertex cover even degree

Given a graph with a mixture of even & odd degree. An edge in the graph contributes to 2 degree. So there will be even no. of vertices with odd degree. We will insert a node & ~~at~~ connect it to the odd degree vertices. We will then add 2 more vertices & connect them to the additional node. This will ensure that the first new node that we added won't affect the actual vertex cover answer.
Now we run vertex cover on this & if we get a vertex cover of size k+2 then we say that vertex cover is NP complete even when all vertices in the graph have even degree.