HomeWork 1

1] Chapter 1, Exercise 1 [Tardos textbook]

- The following statement is false.

Let's consider the example:-

m prefers w to w'
m' prefers w' to w
w prefers m' to m
w' prefers m to m'

-[this is one of the example given in the book]

In the above example, we see that m prefers w but w prefers m'. So even if we had a pair like (m,w) or (m',w') or (m,w'), the ranking on the preference list of either m's or w's was not first.

Thus we prove that the ~~foll~~ statement is false and there won't be a stable matching likewise in every instance.

2] Chapter 1, Exercise 2 [Tardos Textbook]

→ The following Statement is True.
Let's consider the following Preferences:

m Prefers w to w'
m' Prefers w to w'
w Prefers m to m'
w' Prefers m' to m.

If (m', w) are a pair in S that means m is yet to propose w, because if m were to propose w then w would have rejected m' over m. Thus making (m, w) a stable pair along with (m', w'). This states that since m ranked w first and vice versa, there exist a stable pair (m, w) in S. (m', w) pair won't be stable.

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3] Let consider $n=3$, in that case we have 3 men (m, m', m'') and 3 women (w, w', w''). Let the preferences be as follows.

m prefers w to w' to w''
m' prefers w to w' to w''
m'' prefers w' to w'' to w''
w prefers m to m'' to m''
w' prefers m' to m to m''
w'' prefers m'' to m to m'

Now ~~when~~ lets say (m, w) ~~exists~~ is a pair and when m' proposes w, w rejects m' over m because m is the preferred one. Then m' proposes w', (m', w') become a pair since w' prefers m' as he is her first choice. m'' proposes to w' and w, both of them reject him over their current partners as they are ranked higher than m''. Thus (m'', w'') become a pair. Concluding that even though m, m', and m'' ~~diff~~ didn't prefer their current partners the most they are still a stable pair because they are the most preferred men by their current partners.

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7] This statement is true as far as ~~ct~~ all men have ~~different~~ their most preferred women different.

For instance, lets consider the following, m prefers w over w' and m' prefers w' over w. w prefers m over m' and w' prefers m' over m. Thus (m, w) and (m', w') would be the stable matching pair returned by GS algorithm.

4] The preferences be:

A : B > C > D
B : A > C > D
C : B > A > D
D : A > B > C

Now let's say there is a pair (A, D) leaving (B, C) as the other. Now given a chance 'A' would prefer either B or C over D because they are higher ranked in A's list of preference. Similarly, if we have (A,B) and (C, D) again C would prefer either A or B and lastly (A, C) and (B, D), over here also B would prefer anyone of A or C ~~we~~ over D. The

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above Cases Shows that there is no stable roommate matching that exists.

8] Let us prove that S is a set where $M = m_1, m_2 \dots m_N$ and $W = w_1, w_2 \dots w_N$ is a unique stable matching. If m_K prefers w_j over w_K such that $K < j$ then a highly preferred w_j won't leave her a current partner m_j over m_K. Thus there is no instability. ~~and~~
Inorder to prove that it is a unique stable matching, lets say there is another set S'. Now let's say S' is not the same as S and there exist a value for some i such that ~~m~~ w_i prefers m_K and is matched with m_K ~~w~~, here $K \neq i$. Let keep this minimum value of i as 'a'. On similar grounds, m_j is matched to w_l and let keep the minimum value of this j as 'b'. Now, we know that $w_i = m_i$ for all $i < a$ and $m_j = w_j$ for all $j < b$. We can thus say that $b = a$.

w_a matching with m_k implies $a<k$ and ~~m~~ m_b with w_l implies $b<l$ meaing a and $b < l$.
Now we know that m_a/m_b prefers ~~w~~ w_a over w_l and w_a prefers m_a over m_k, this gave rise to instability and thus we can say that S' never existed.

(5) Lets assume the following preferences of men and women.

m	m'	m''	w	w'	w''	
w''	w	w''	m	m	m'	m'
w	w''	w	m'	m'	m	m''
w'	w'	w'	m''	m''	m''	m

Let's first find out the stable matching from the true preference list of w''.
m goes to w'', w'' and m get engaged.
m' goes to w, they get engaged.
m'' goes to w'', w'' rejects m''.
m'' then goes to w, even she rejects him, m'' finally gets engaged to w'.
(m, w''), (m', w) and (m'', w') are

the stable pairs through GS algorithm. Now, lets carry out the same process but using the false preference list of w''.

m goes to w'', they get engaged. m' goes to w, they get engaged. Now, m'' goes to w'', they also get engaged and before that w'' rejects m.
m goes to w, w prefers m over m'. m' goes to w'', w'' prefers m' over m''. m'' then goes to w, w rejects him. m'' then gets engaged with w'.
Here, the pairs are (m'', w'), (w'', m') and (m, w).
As you can clearly compare, w'' got her most preferred man by using a false preference list.
So, there exist a set of preference lists for which there is a switch that would improve the partner of the woman who switched preferences.

5] Chapter 1, Exercise 4.

→ The algorithm is in lines with the Gale Shapley algorithm.
Here we consider, the hospitals to either have "positions" or "full" and the students to either "assigned" or "free".

While some hospitals h_j have available positions,
h_j prefers offering the position to S_K as he is the next on h_j's preference list.
If S_K is free then
S_K accepts it.
Else (S_K is already committed to h_i)
If S_K prefers h_i to h_j then
~~If then~~ S_K is committed to h_i
Else S_K becomes committed to h_j
and h_i position becomes available
End If
End If
End While

This algorithm would terminate in $O(mn)$ steps. Also every

~~Student~~ hospital will offers position to at most one student. If there are more than one positions available at a hospital then this algorithm would terminate in which all positions are ~~oc~~ occupied. There aren't ~~be~~ any hospital who would be having positions after the algorithm terminated because if that ~~there~~ were to happen then that would mean available positions are greater than the students which is not the case.

Now, we look into the first instability.

~~If~~ The students are s & s' & a hospital h.

s is assigned to h

s' is assigned no hospital

h prefers s' to s

If h prefers s' to s then h would have gone to s' first but at that time ~~h~~ ~~to~~ s' was already committed to h_k so ~~he~~ rejected h & thus now s is assigned to h. Thus, this

is a contradiction.
Second type of instability, there are student s and s', and hospitals h and h'.
- s is assigned to h
- s' is assigned to h'
- h prefers s' to s and
- s' prefers h to h'

If h prefers s' to s then h would have gone to s' first but at that time s' would have been committed to someone higher than h. Thus h went to s next and s accepted the offer, leaving s' with h'. Thus the contradiction.

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