Homework 11

1] Efficient Certificate: ~~T~~ graph G' is a spanning tree

Certifier: - Check if G' is connected
- all its nodes have degree $\leq K$.
- $|E'| = |V| - 1$.

We will use Hamiltonian Path to prove that it's NPC.
We find a Hamiltonian path in Graph $G(V,E)$. ~~If the path is~~ We then root it at any vertex & then check for the degrees of the vertices. If the path is of size K then the vertices in the corresponding spanning tree will have degrees $\leq K$ as the leaf nodes will have degree 1 and so on. Similarly if ~~the~~ we have a spanning tree who has nodes with degree $\leq K$ then we can use the same tree and unroot it and we will then have a Hamiltonian path. Thus it is NPC.

2] Certificate: Graph G has a zero weight cycle.
Certifier:- Check if it is a simple cycle

:- $\sum_{\text{in cycle}} \text{edges} = 0$

We will use Subset Sum to prove that this prob is NPC. We write the edges in the graph as $e_{12}: 3$ meaning from Node 1 to node 2 the weight is 3. In this manner we fill in the sets. We then select that subset which equavates to 0. While selecting so we also see if the starting element in the subset is the ending element. Once we find such a subset we then use those edges to find the cycle. If we find a subset sum whose target value is 0 we are then able to find the simple cycle in G. If we find a simple cycle in G we ~~can~~ then use it to ~~a~~ Subset sum. Thus zero weight cycle is NPC.

3] Certificate: No. of clubs
Certifier :- Check if total - k no. of clubs ~~makes~~ ~~sure that each~~ have one person in atleast 1 club.
We will use set cover to ~~solve~~ prove that this prob is NPC.
We will use the set cover's elements as list of people & make a list of clubs one of every member.
If we find a set cover of size k_s then $k = \underset{\text{clubs}}{\text{Total}} - k_s$.

If we have a yes instence of Set Cover then the k other subsets form ~~a~~ the solution for Redundant Clubs problem. And if we have k redundant clubs then the other k_s clubs would form a set cover. So set cover $\leq_p$ Redundant Clubs problem

4] Certificate: Find independent set in G.
Certifier:- Check if the size of the independent set $= |V|/2$
We will use independent set to ~~s~~ show that Half- is is NPC.
We find an independent set by

selecting alternate nodes and not violating the independent set constraints, we will end up getting $|V|/2$ size independent set which is the solution for Half-is. Similarly if we are given an independent set of size $|V|/2$ we know that it is a maximum size independent set as the no. of vertices is even. Thus I.S $\leq_p$ Half-is

5] Certificate: no. of courses taken
Certifier :- Check if the no. of courses $\geq k$

- there are no two overlapping courses

We use independent set to prove that this problem is NPC. We create a graph such that for each vertex $v_i \in V$ we create a course job i & we create an edge going from v_i to v_j & assign the interval t_{ij} to both the courses. We assume that Graph G has independent set G, thus we can ~~schedule~~ take k courses $t_{i_1} \dots t_{i_k}$ as none of them will have overlapping intervals. On the other hand if we assume that no intervals are overlapping then we can find an independent set of size k. and if there is an edge $v_{ih} v_{il} \in E$ which ~~b~~ t_{ih} and t_{il} both require the same slot & this would contradict our assumption of non overlapping intervals. Thus $I.S \leq$ this problem.