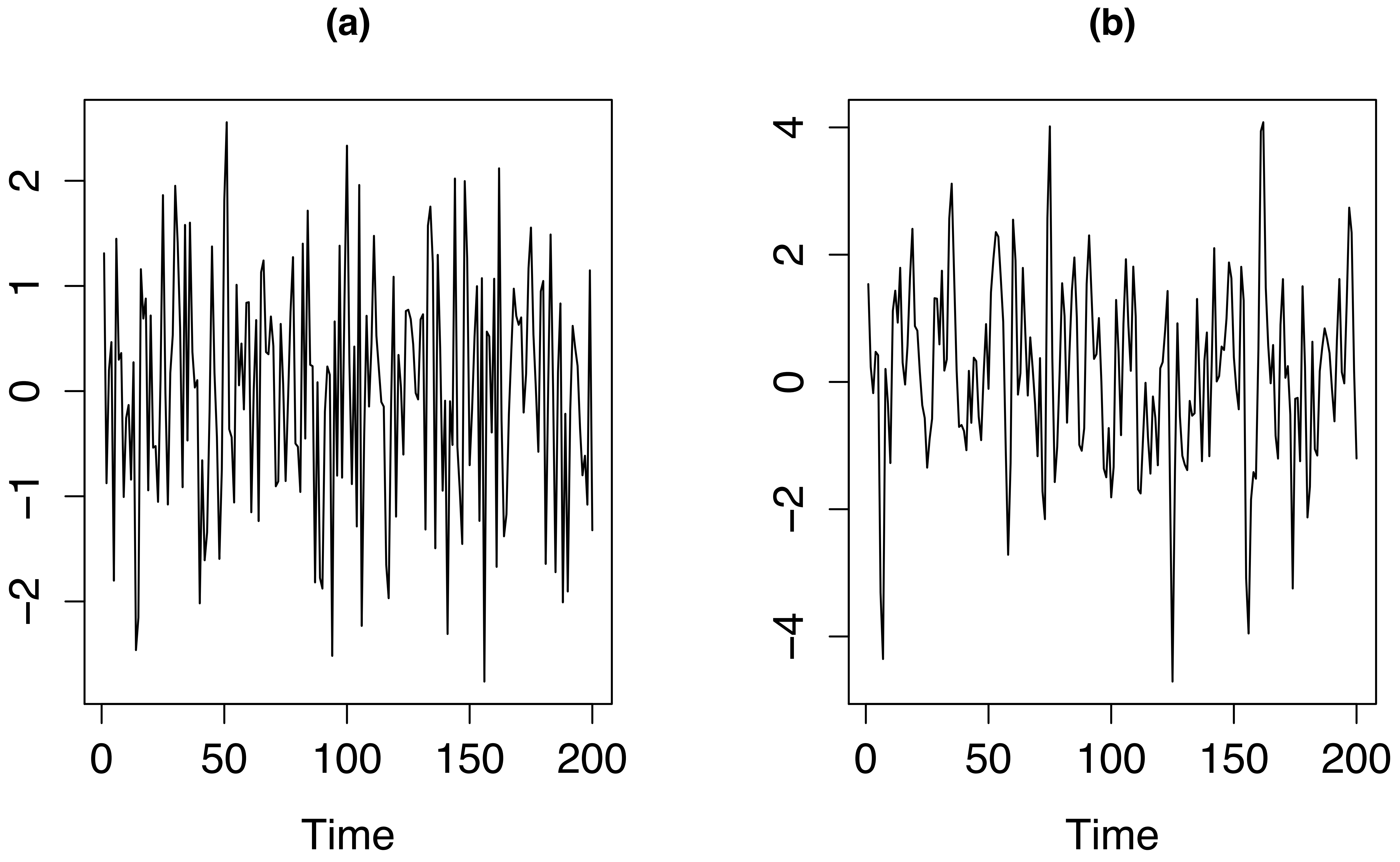
Trend Spotting!

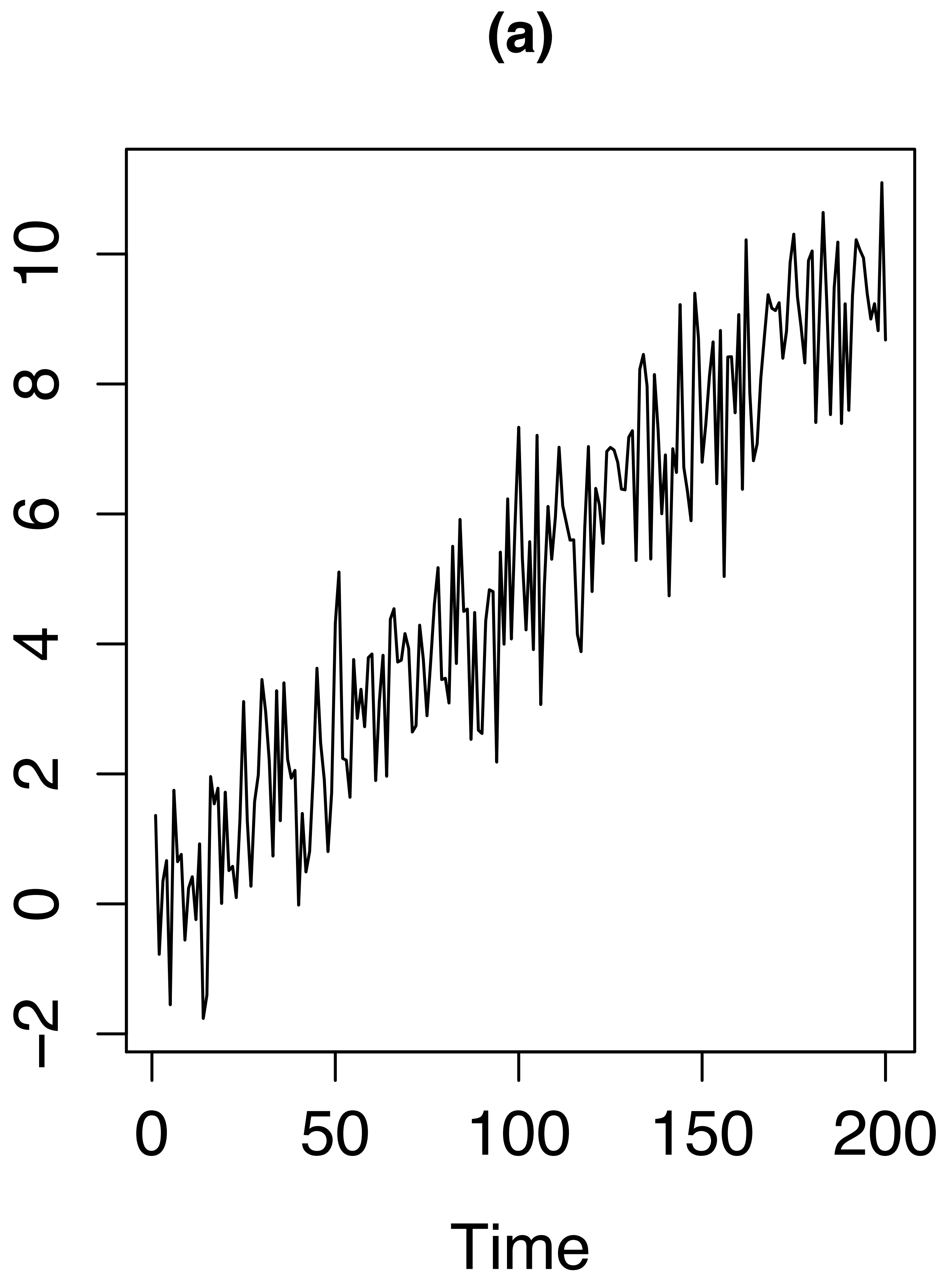
Trends

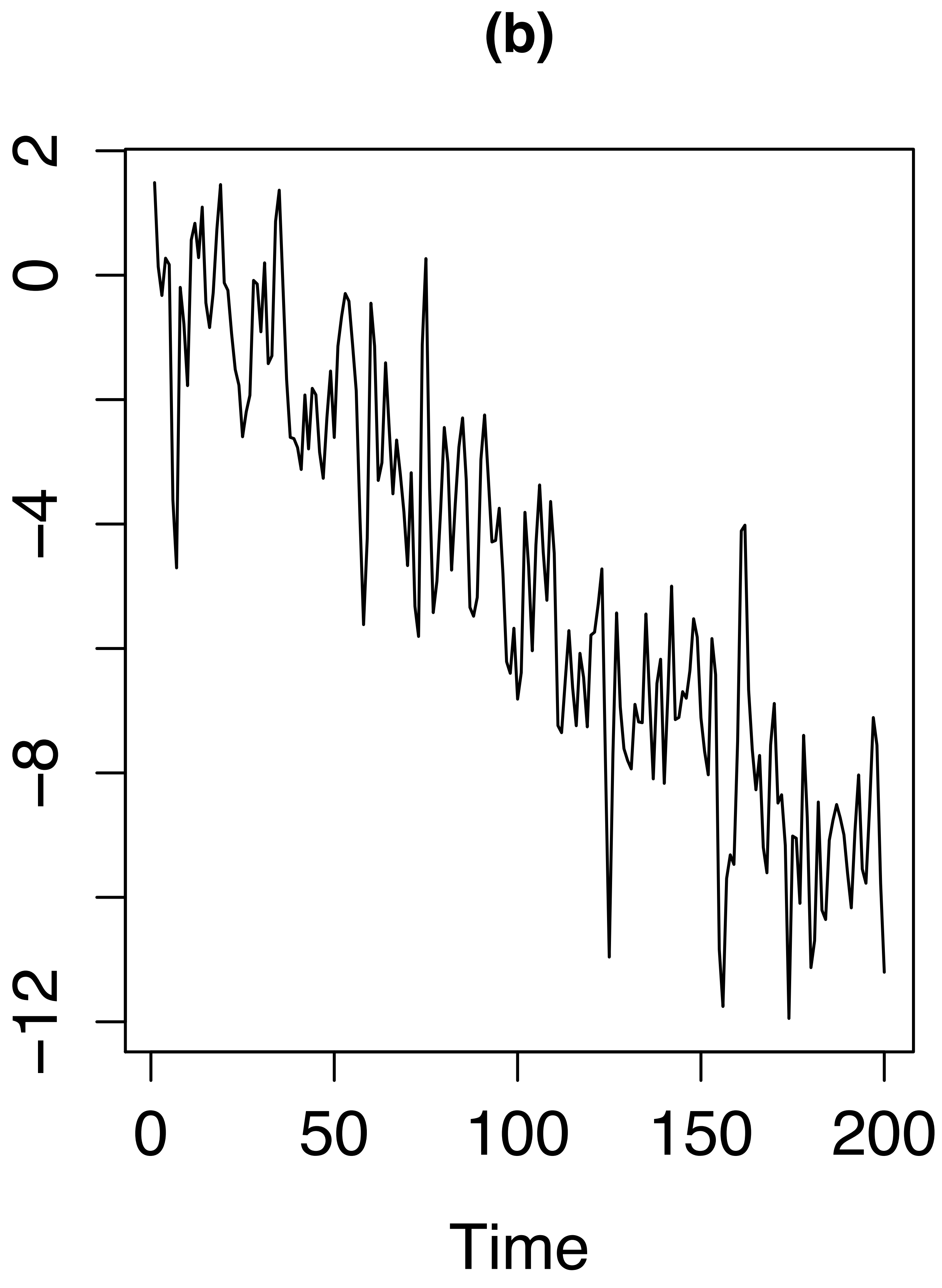
Some time series do not exhibit any clear trends over time:



Trends: Linear

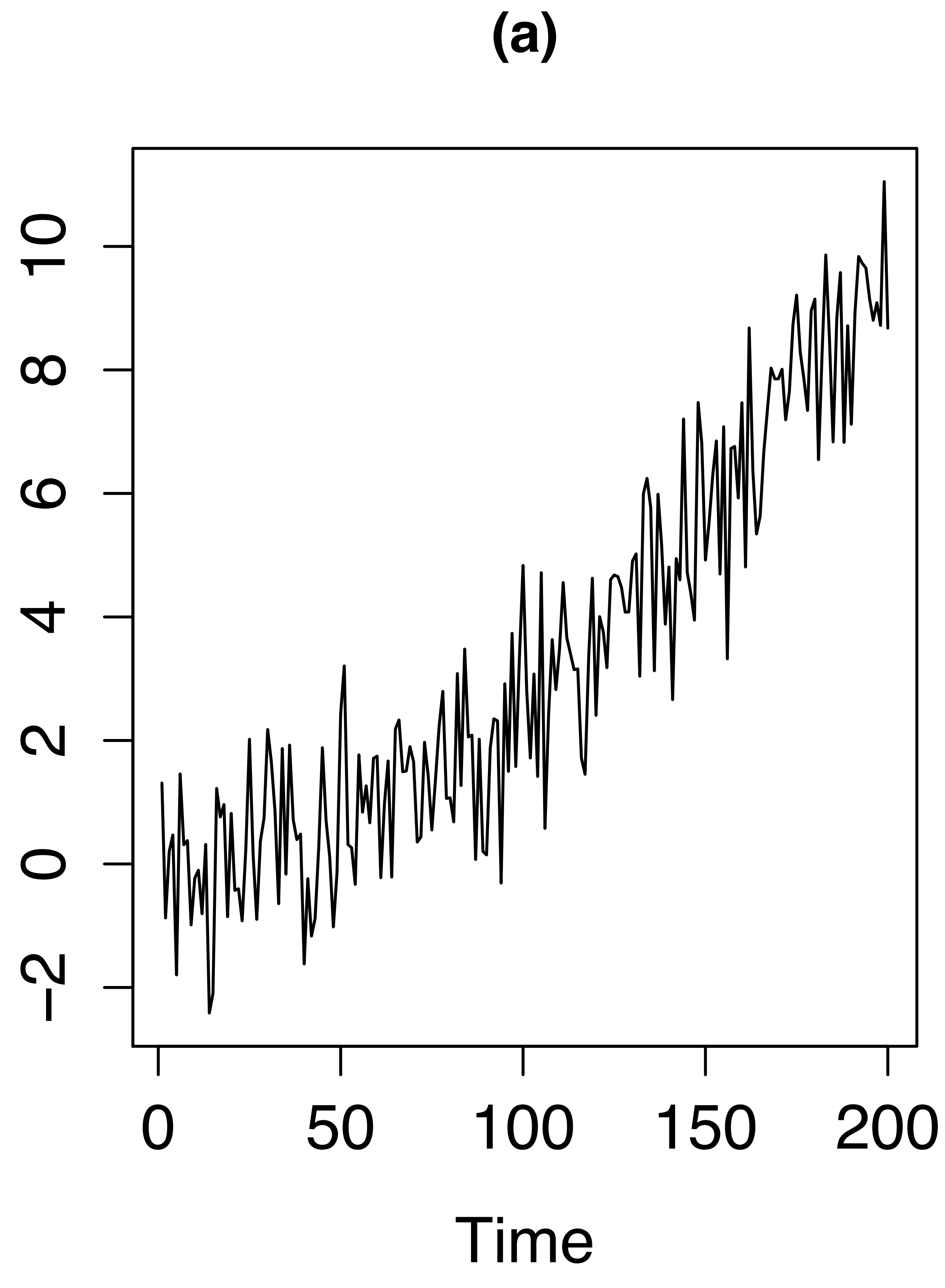
Examples of linear trends over time:

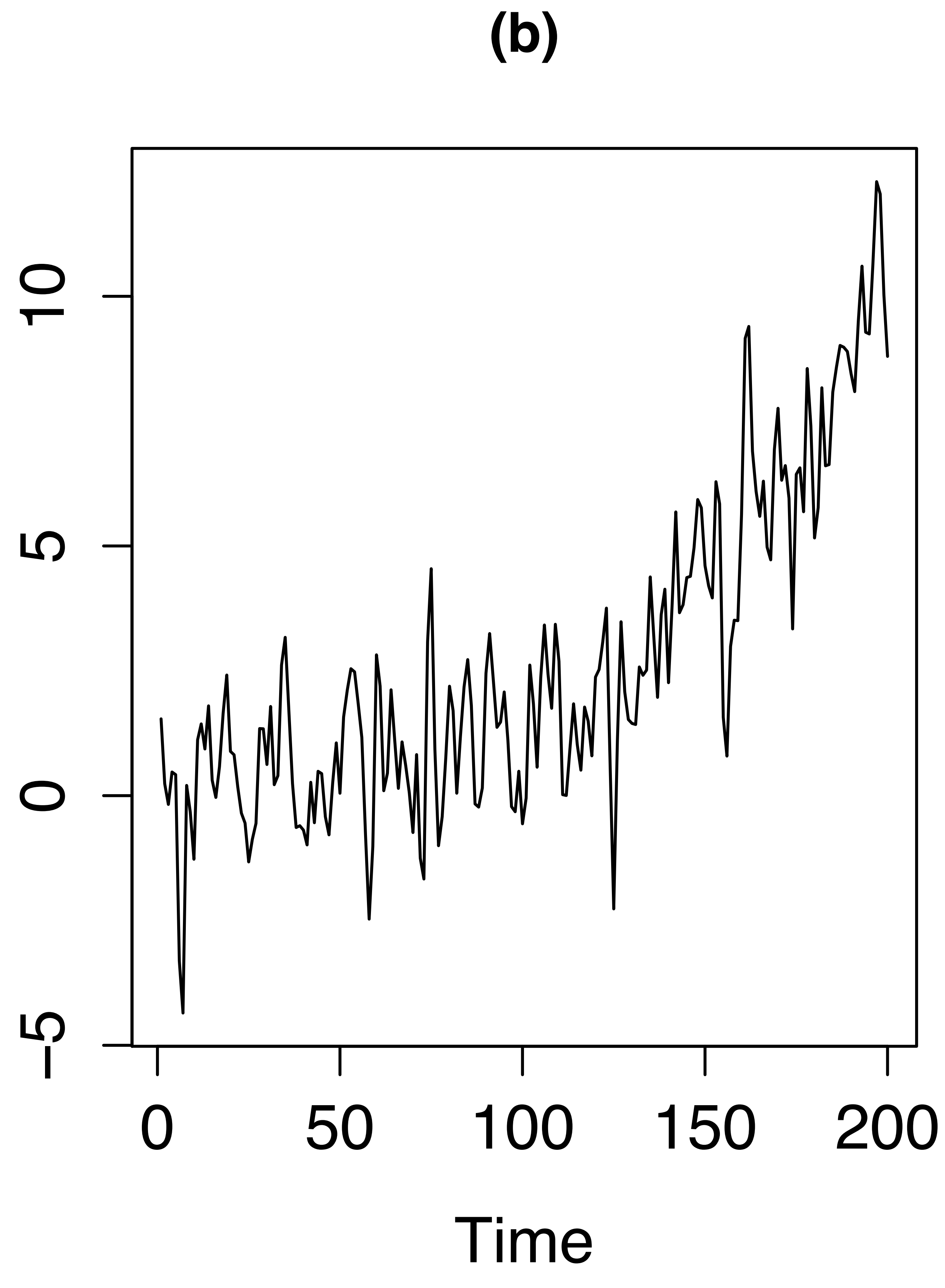




Trends: Rapid Growth

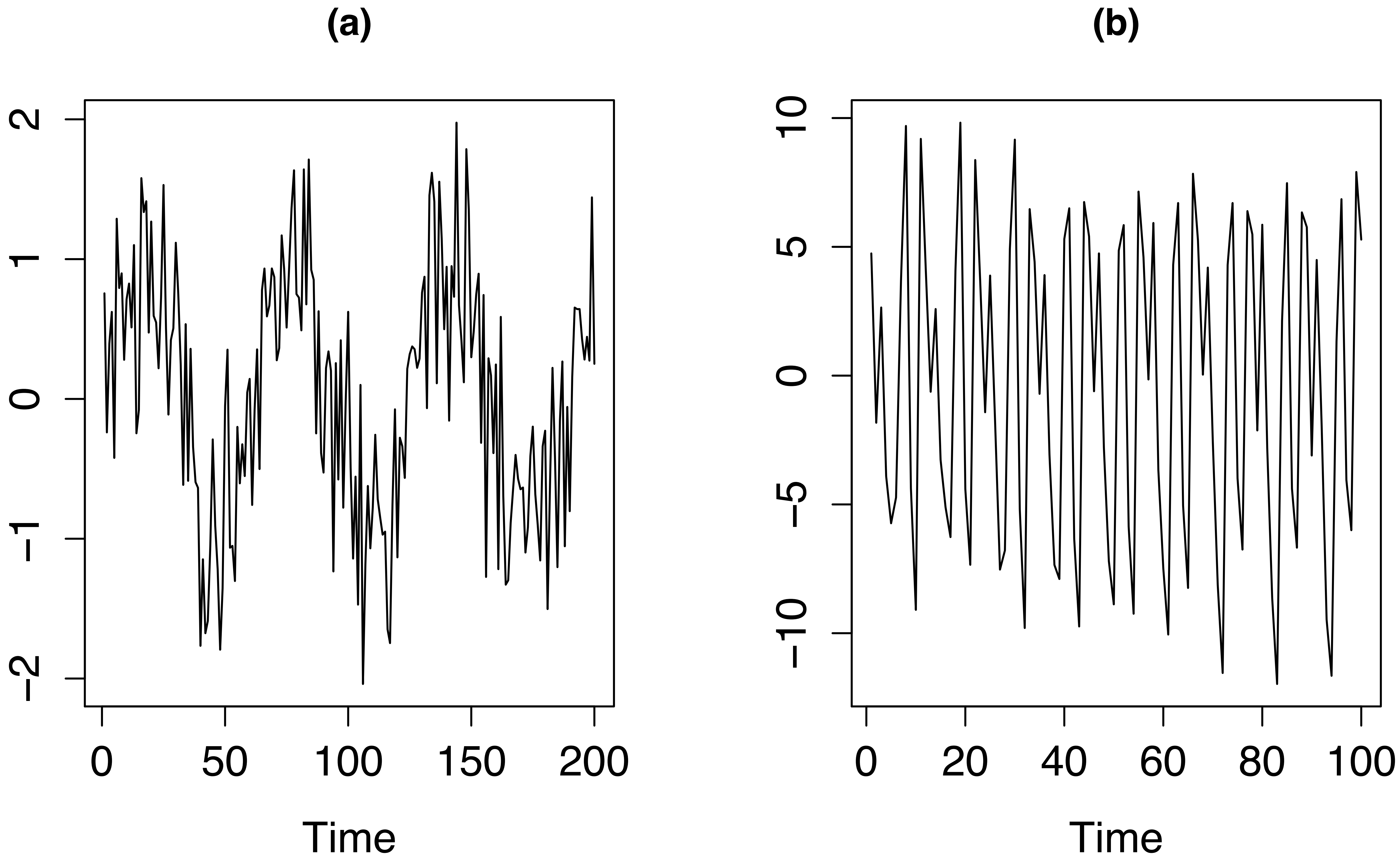
Examples of rapid growth trends over time:





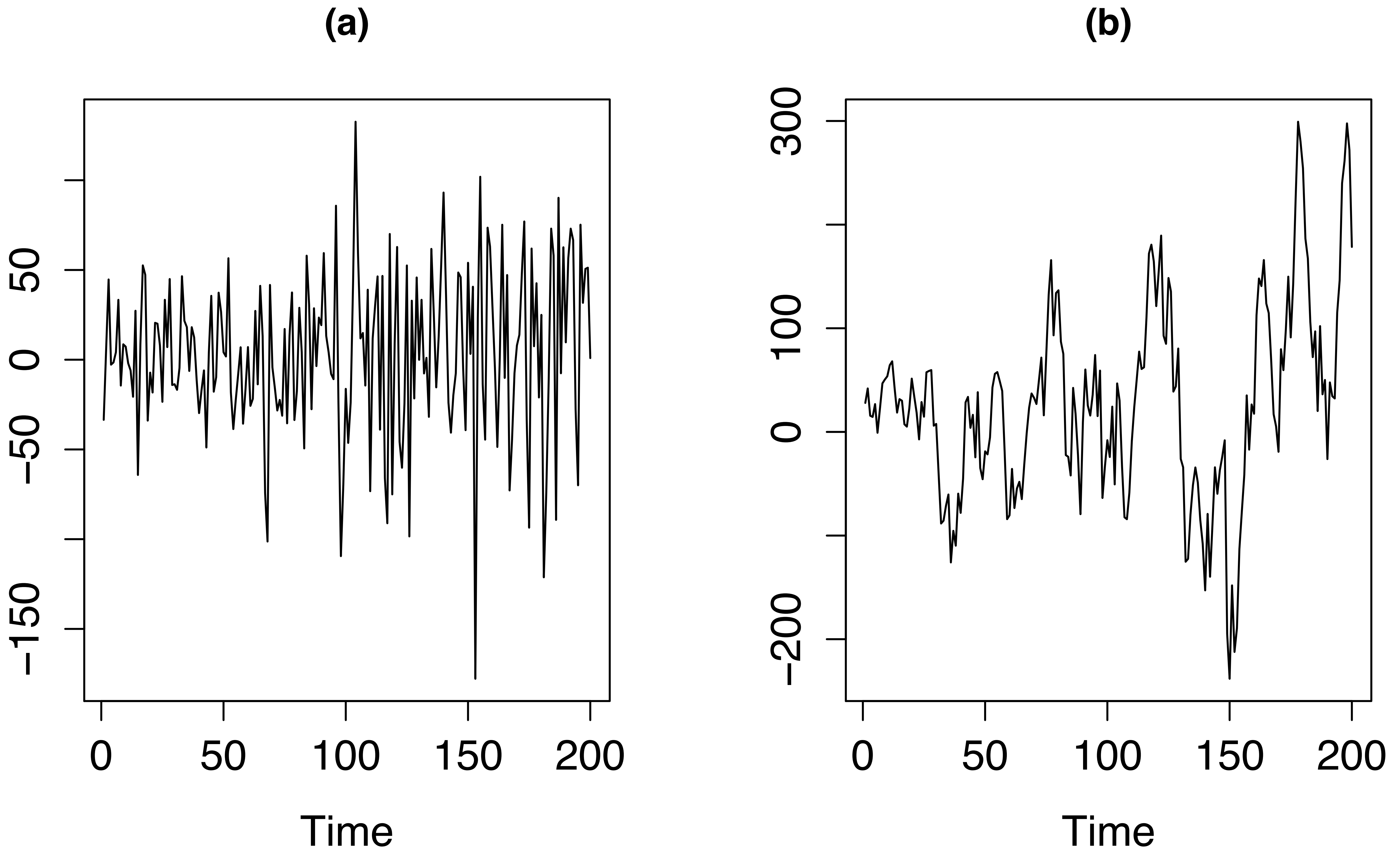
Trends: Periodic

Examples of periodic or sinusoidal trends over time:



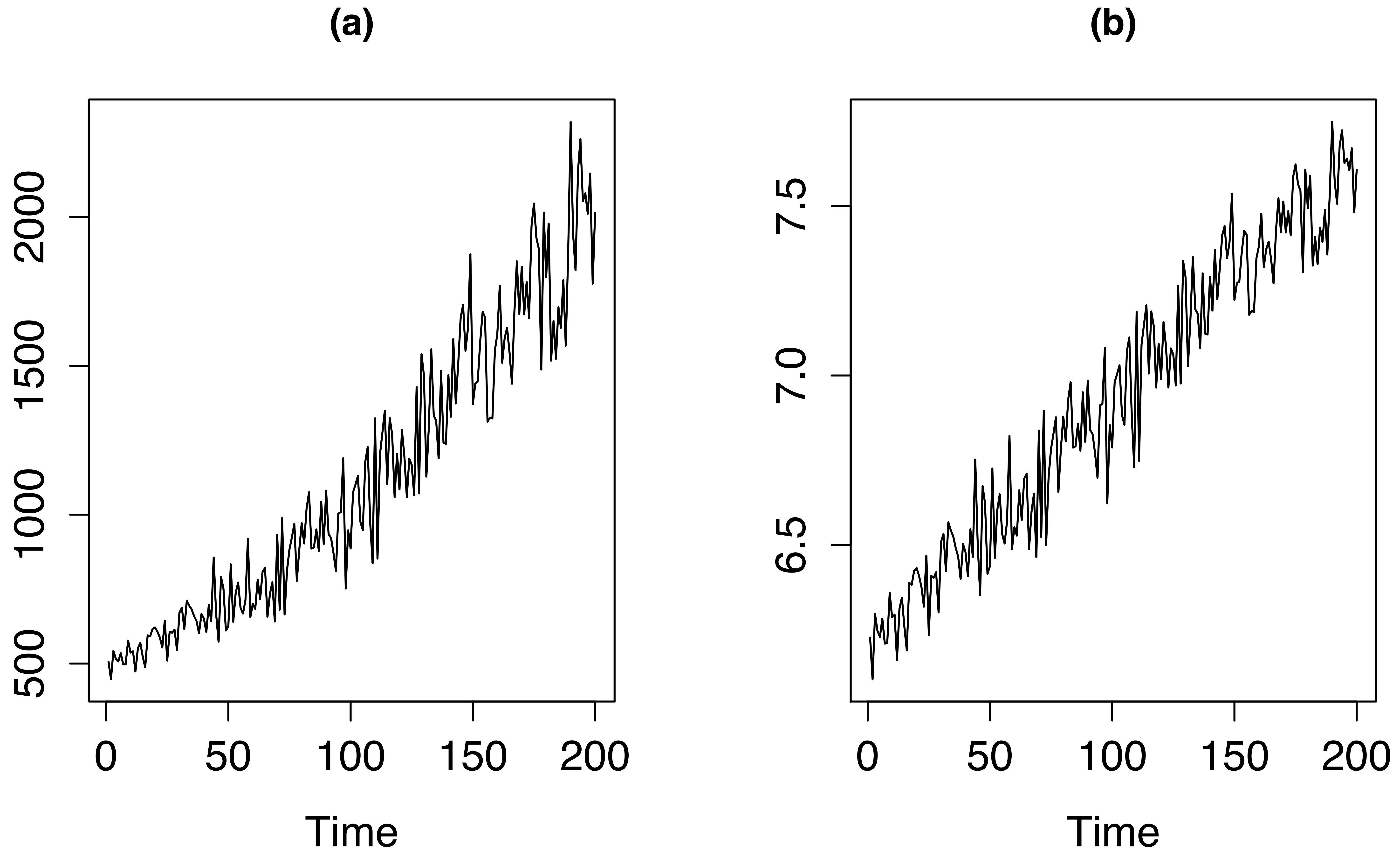
Trends: Variance

Examples of increasing variance trends over time:



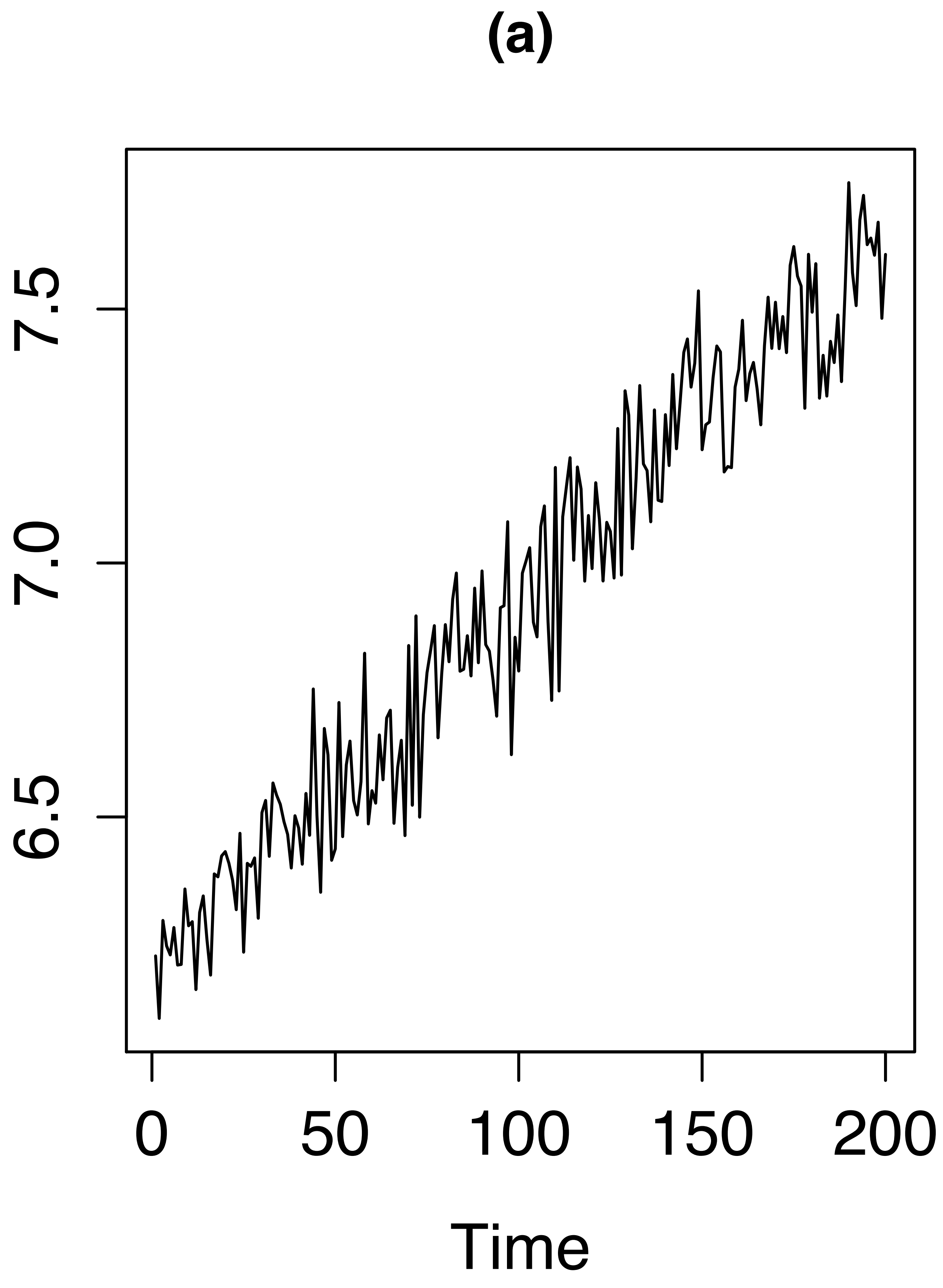
Sample Transformations: `log()`

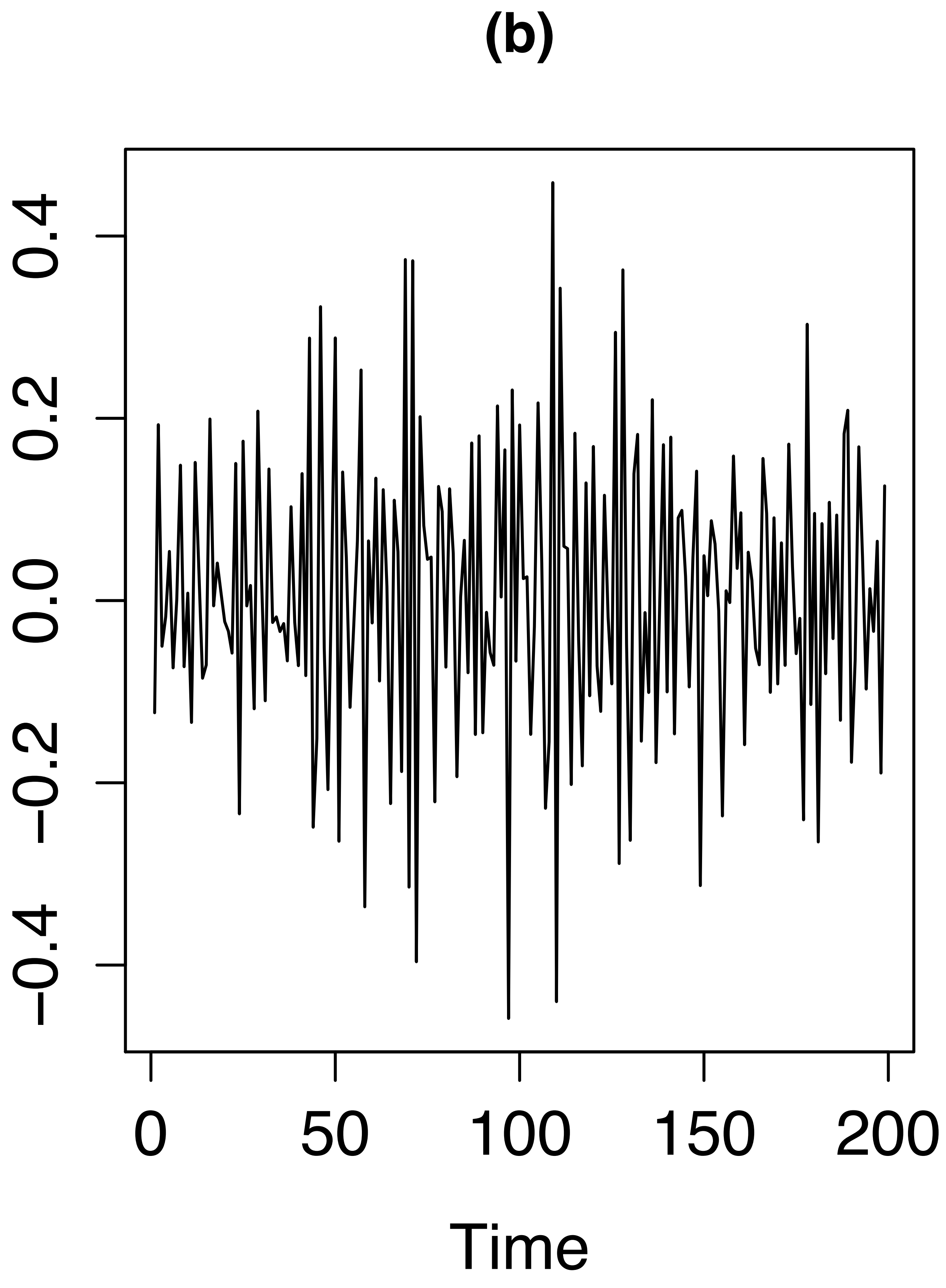
The `log()` function can *linearize* a rapid growth trend:



Sample Transformations: `diff()`

The `diff()` function can remove a linear trend:

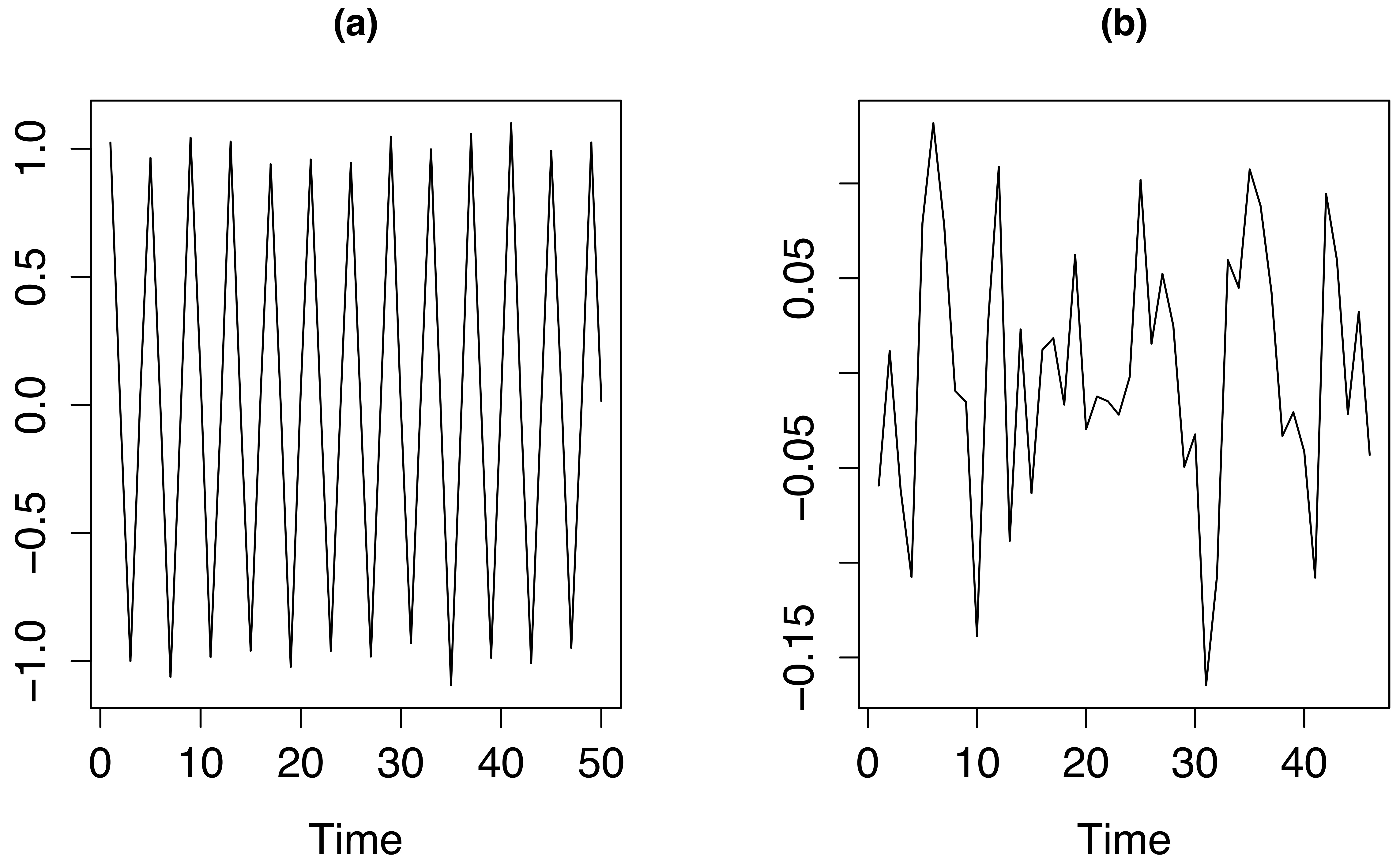




Sample Transformations: `diff(..., s)`

The `diff(..., s)` function, or *seasonal difference transformation*, can remove periodic trends.

```
> diff(x, s = 4)
```



Let's practice!

The White Noise (WN) Model

White Noise

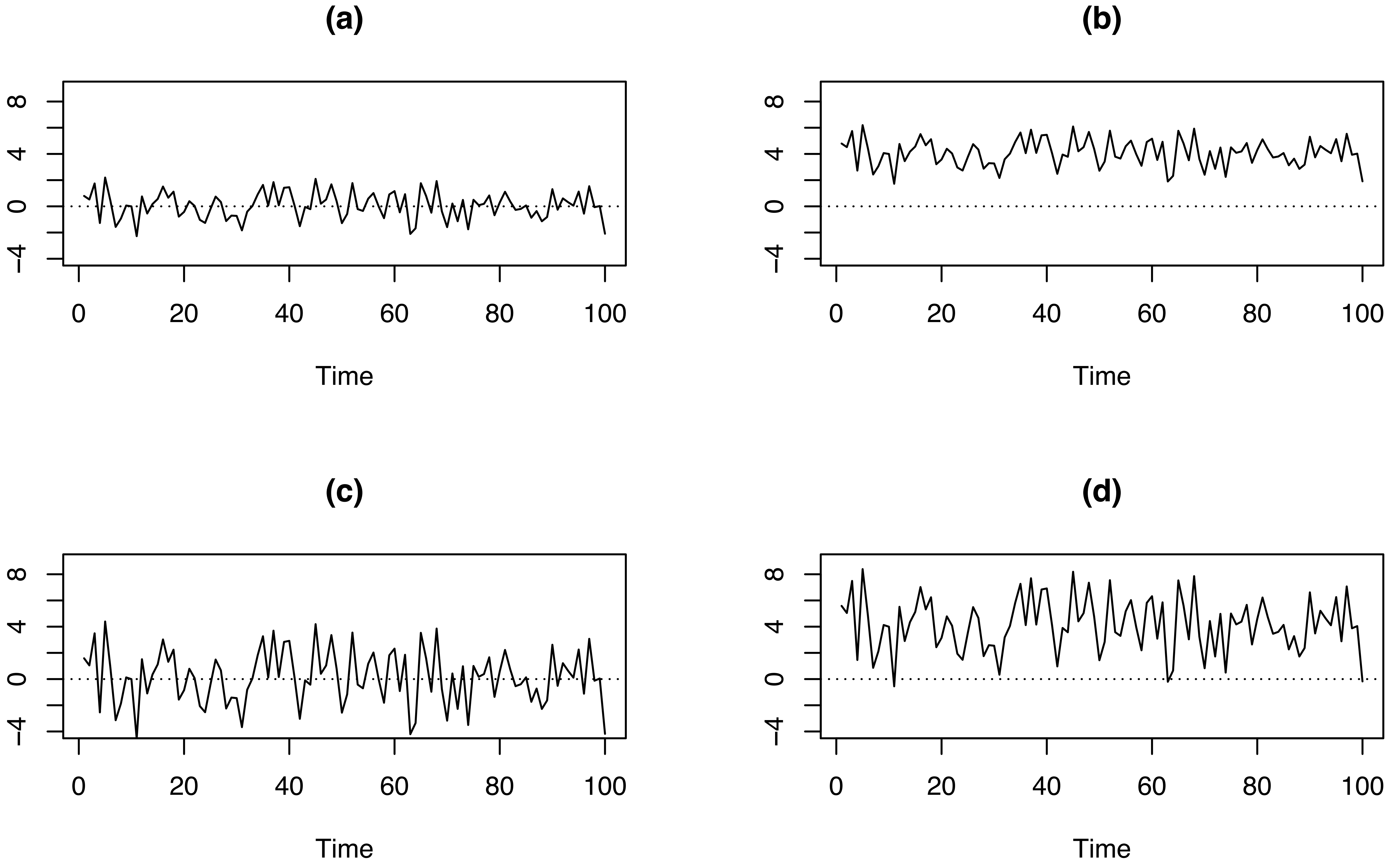
White Noise (WN) is the simplest example of a stationary process.

A *weak white noise process* has:

- A fixed, constant mean
- A fixed, constant variance
- No correlation over time

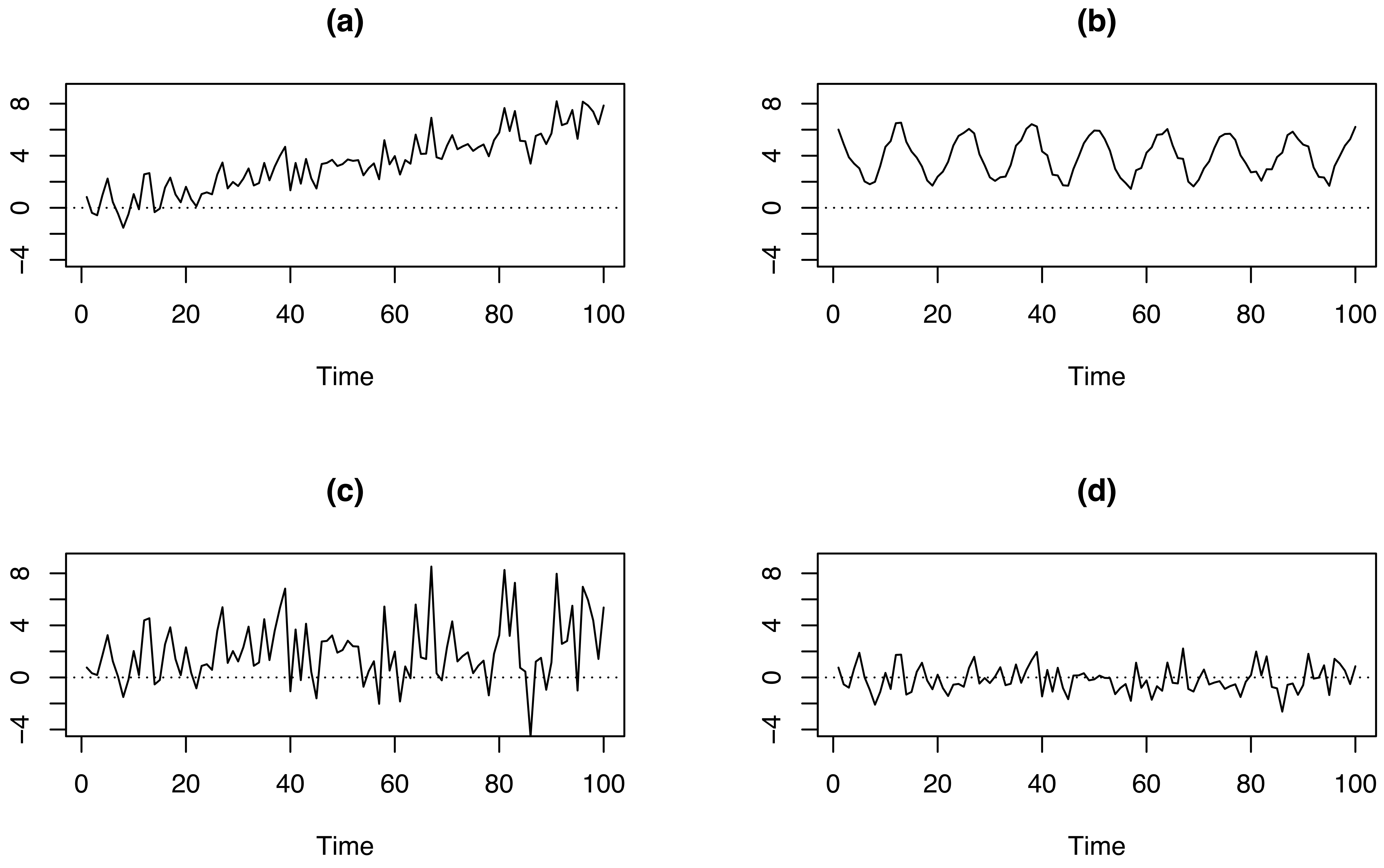
White Noise

Time series plots of White Noise:



White Noise

Time series plots of White Noise?

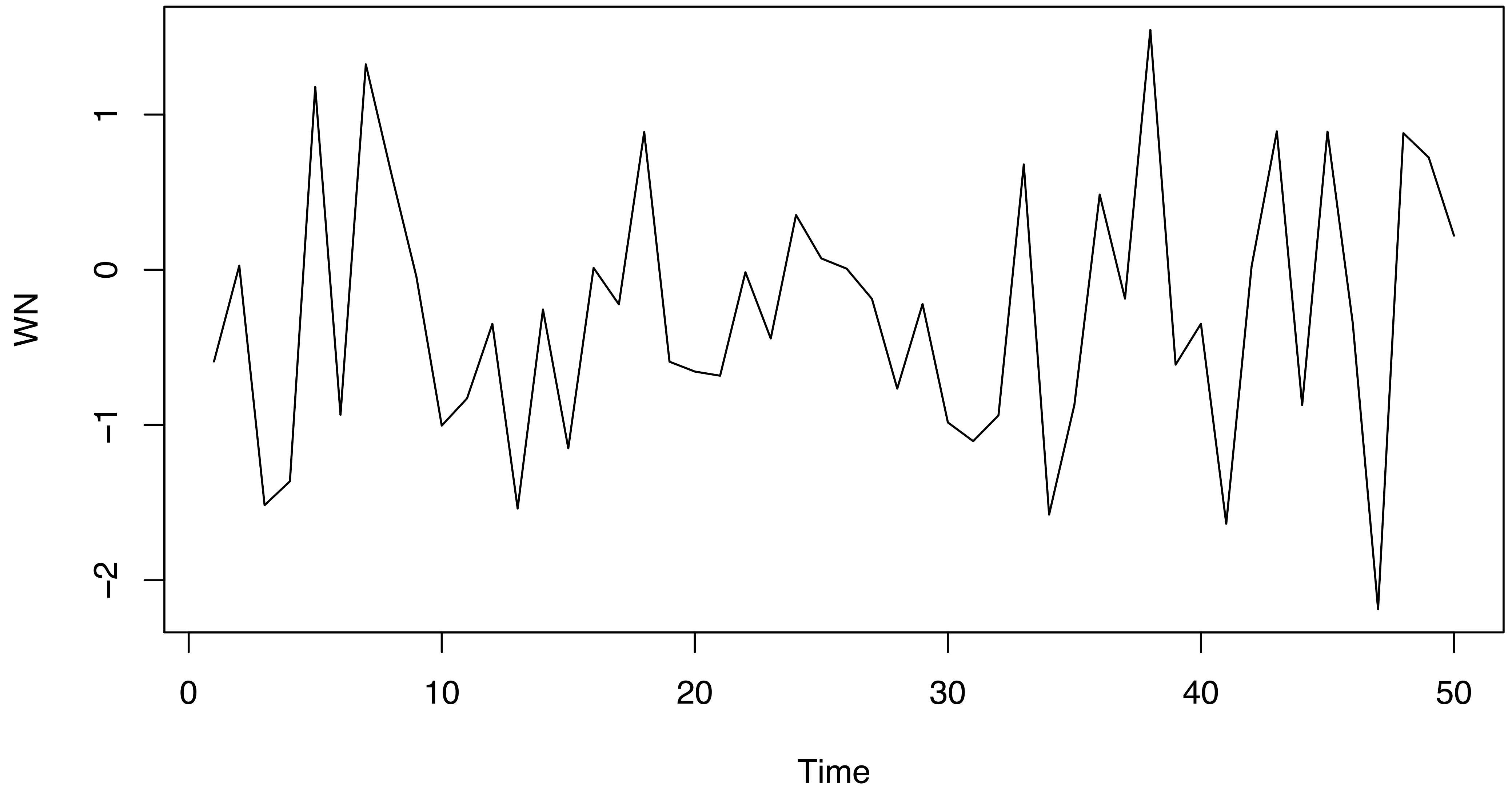


Simulating White Noise - I

```
> # Simulate n = 50 observations from the WN model
> WN_1 <- arima.sim(model = list(order = c(0, 0, 0)), n = 50)
> head(WN_1)

[1] -0.005052984  0.042669765  3.261154066  2.486431235
[4] 0.283119322  1.543525773
```

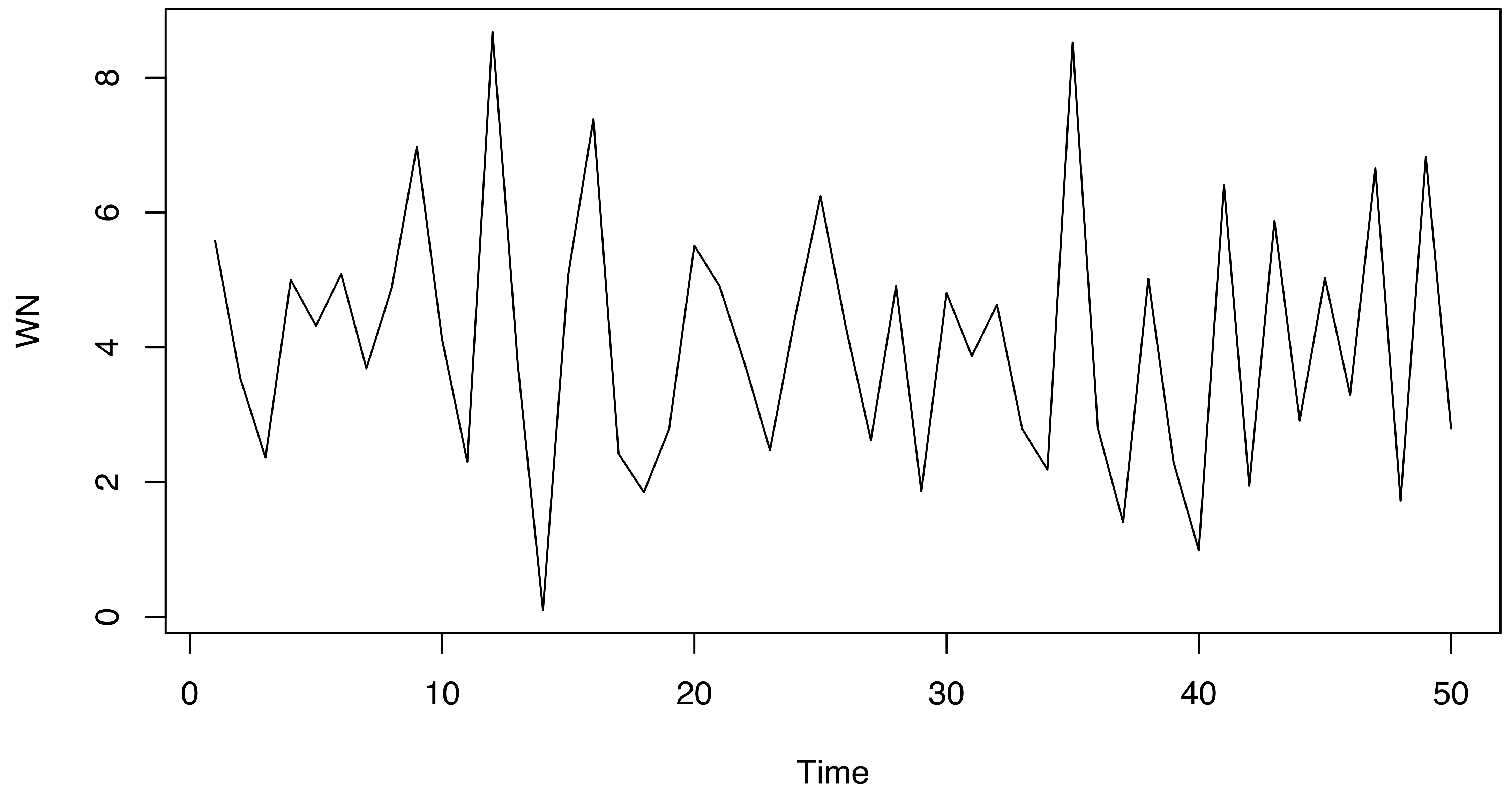
```
> ts.plot(WN_1)
```



Simulating White Noise - II

```
> # Simulate from the WN model with mean = 4, sd = 2
> WN_2 <- arima.sim(model = list(order = c(0, 0, 0)),
                    n = 50,
                    mean = 4,
                    sd = 2)
```

```
> ts.plot(WN_2)
```



Estimating White Noise

```
> # Fit the WN model with arima()
> arima(WN_2, order = c(0, 0, 0))

Coefficients:
      intercept
         4.0739
s.e.     0.2698

sigma^2 estimated as 3.639
```

```
> # Calculate the sample mean and sample variance of WN
> mean(WN_2)
[1] 4.0739

> var(WN_2)
[1] 3.713
```

Let’s practice!

The Random Walk (RW) Model

Random Walk

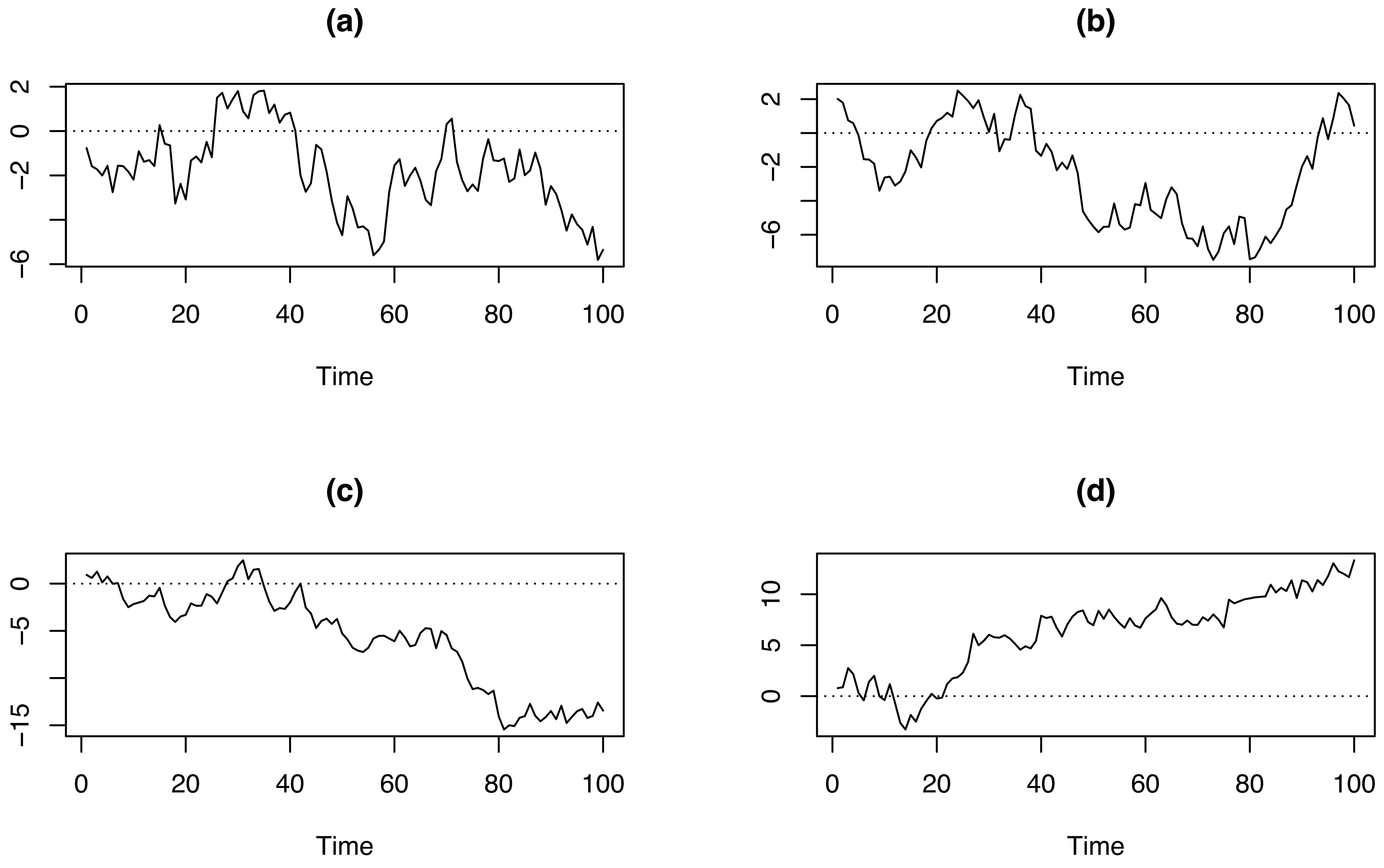
Random Walk (RW) is a simple example of a non-stationary process.

A random walk has:

- No specified mean or variance
- Strong dependence over time
- Its changes or increments are white noise (WN)

Random Walk

Time series plots of Random Walk:



Random Walk

The random walk recursion:

$$Today = Yesterday + Noise$$

More formally:

$$Y_t = Y_{t-1} + \epsilon_t$$

where ϵ_t is mean zero white noise (WN)

- Simulation requires an initial point Y_0
- Only one parameter, the WN variance σ_ϵ^2

Random Walk - I

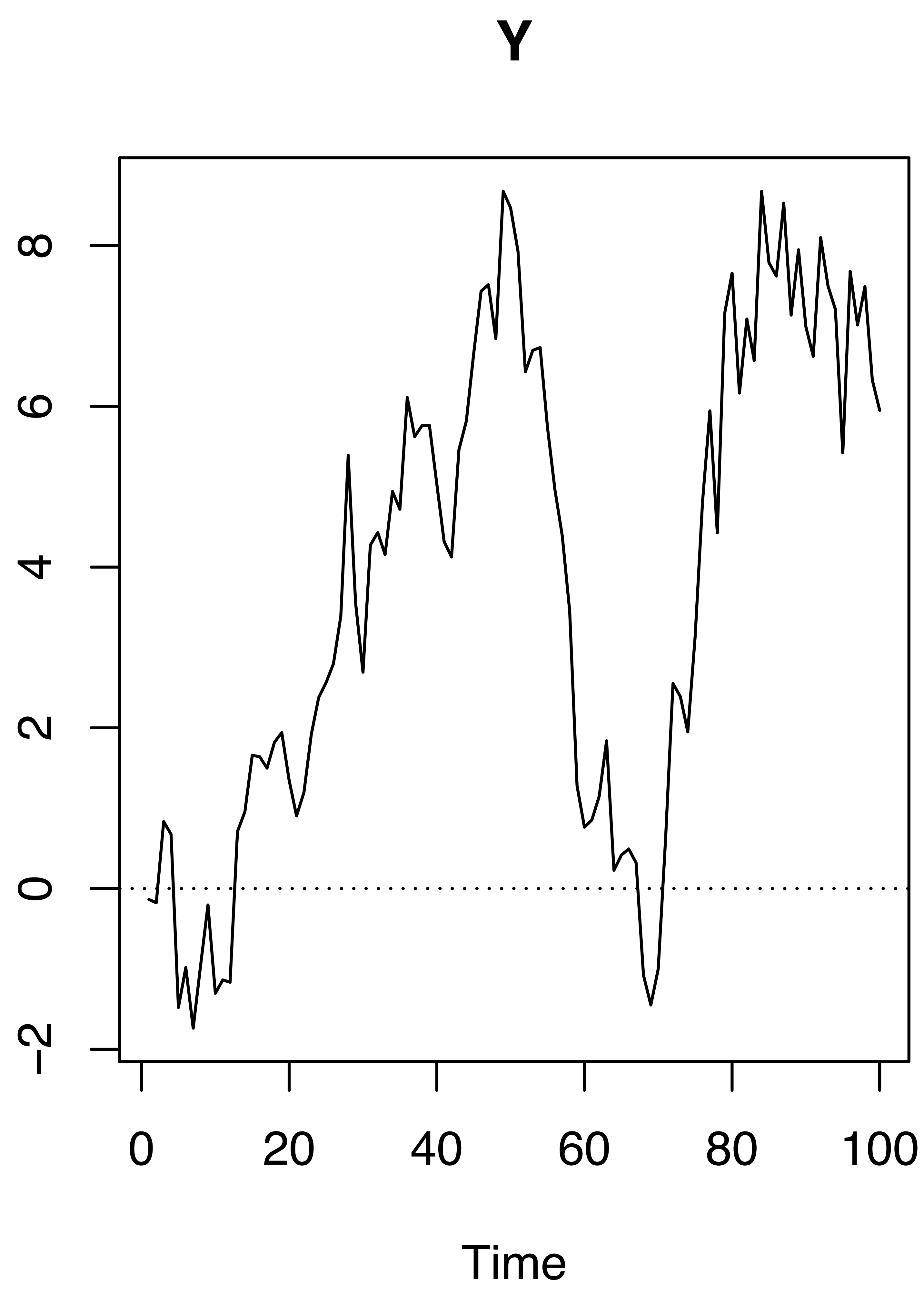
The random walk process:

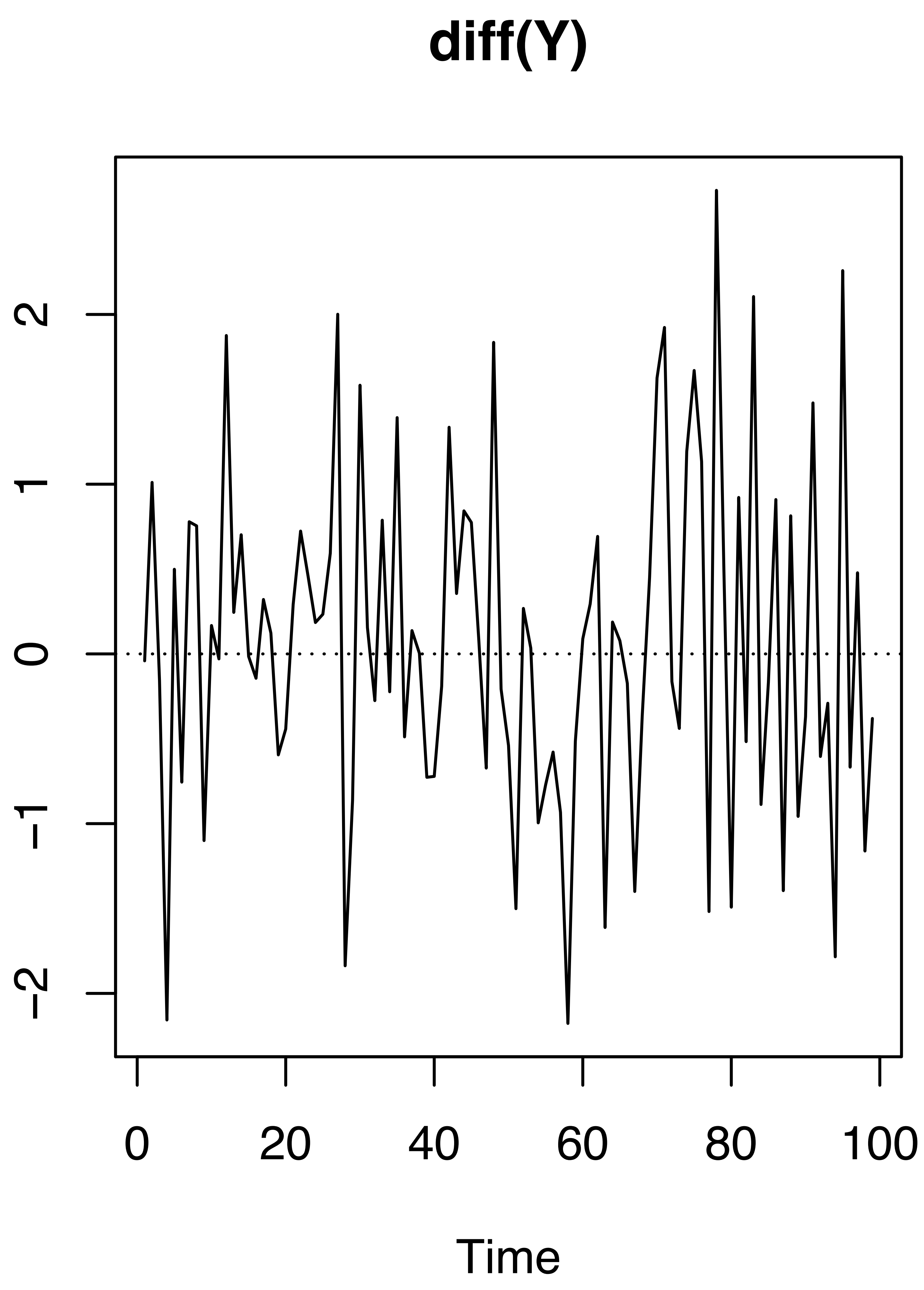
$$Y_t = Y_{t-1} + \epsilon_t$$

where ϵ_t is mean zero WN

As $Y_t - Y_{t-1} = \epsilon_t \longrightarrow$ `diff(Y)` is WN

Random Walk - II





Random Walk with Drift - I

The random walk with a drift:

$$Today = Constant + Yesterday + Noise$$

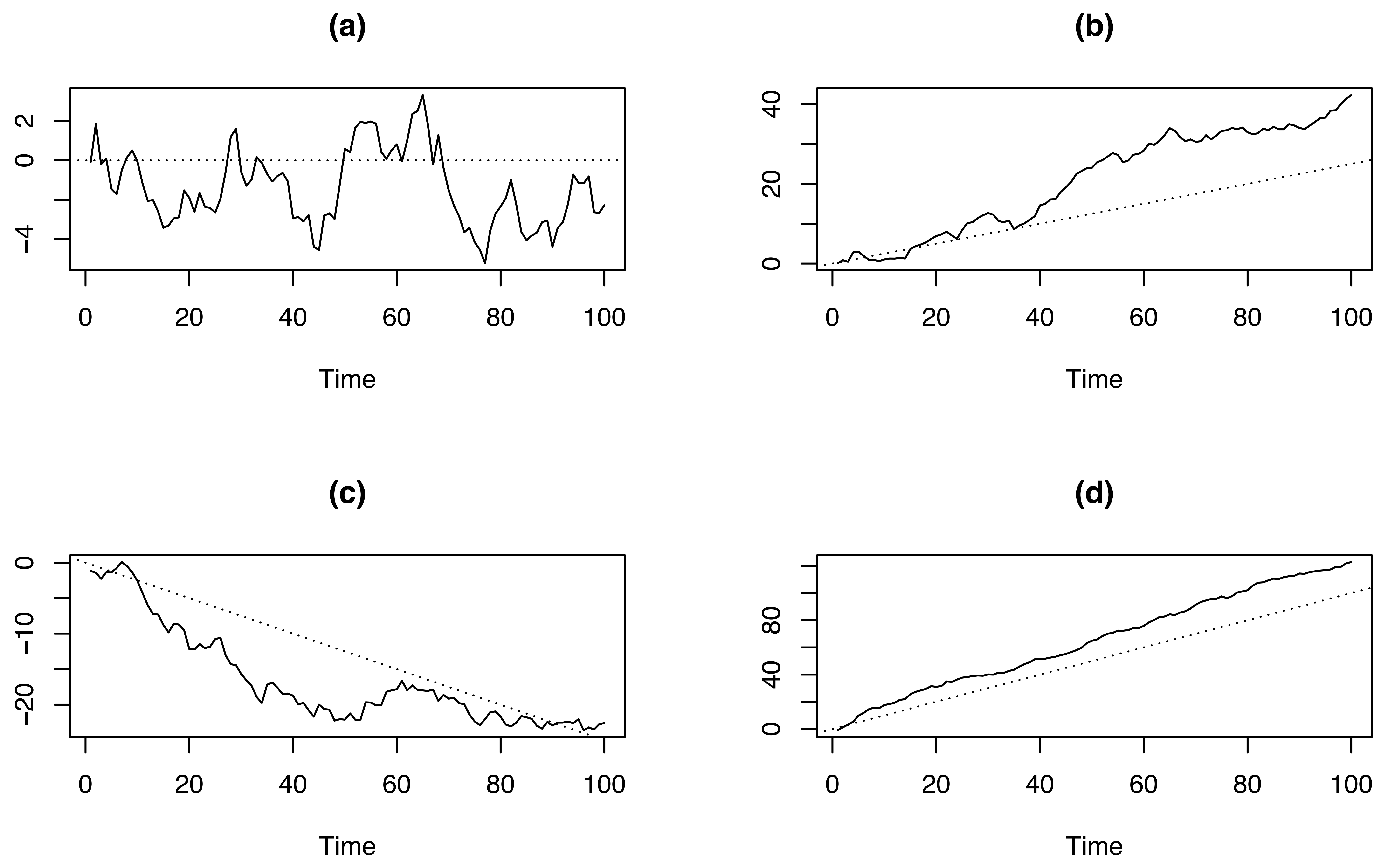
More formally: $Y_t = c + Y_{t-1} + \epsilon_t$

where ϵ_t is mean zero white noise (WN)

- Two parameters, constant c, and WN variance σ_ϵ^2
- $Y_t - Y_{t-1} = \ ?$ $\longrightarrow$ WN with mean c!

Random Walk with Drift - II

Time series plots of Random Walk with drift:



Let's practice!

Stationary Processes

Stationarity

- Stationary models are parsimonious
- Stationary processes have distributional stability over time

Observed time series:

- Fluctuate randomly
- But behave similarly from one time period to the next

Weak Stationarity - I

Weak stationary: mean, variance, covariance constant over time.

$Y_1, Y_2, \ldots$ is a *weakly stationary* process if:

- Mean μ of Y_t is same (constant) for all t
- Variance σ^2 of Y_t is same (constant) for all t
- And....

Weak Stationarity - II

- Covariance of Y_t and Y_s is same (constant) for all $|t-s|=h$, for all h.

For example, if the process is weakly stationary,

$$Cov(Y_2, Y_5) = Cov(Y_7, Y_{10})$$

since each pair is separated by three units of time.

Stationarity: Why?

A stationary process can be modeled with **fewer parameters.**

For example, we do not need a different expectation for each Y_t; rather they all have a common expectation, μ.

- Estimate μ accurately by $\overline{y}$

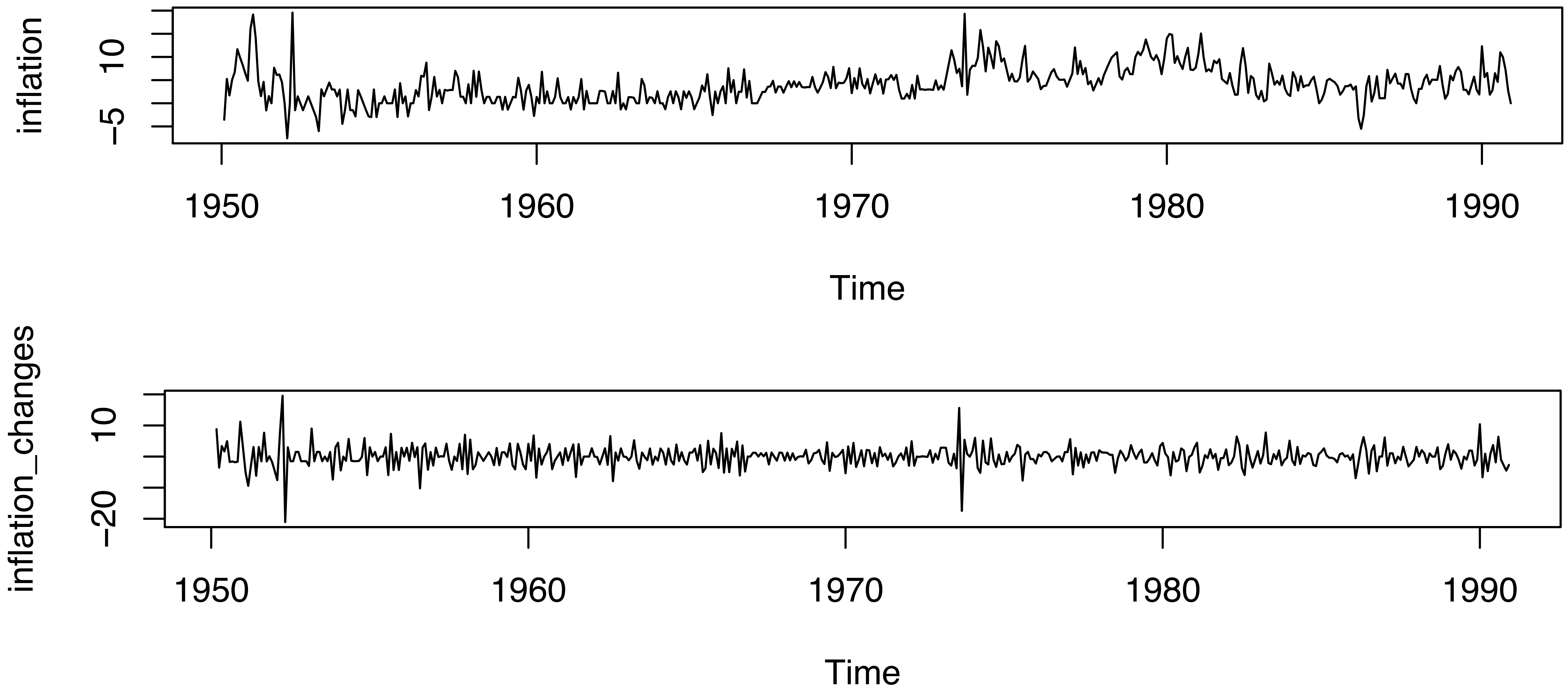
Stationarity: When?

Many financial time series do not exhibit stationarity, however:

- The **changes** in the series are often approximately stationary
- A stationary series should show random oscillation around some fixed level; a phenomenon called **mean-reversion**

Stationarity Example

Inflation rates and *changes* in inflation rates:



Let's practice!