The Simple Moving Average Model

The Simple Moving Average Model

The simple moving average (MA) model:

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

More formally: $Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$

where ϵ_t is mean zero white noise (WN).

Three parameters:

- The mean μ
- The slope θ
- The WN variance

MA Processes - I

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

$$Y_t = \mu + \epsilon_t + \theta \epsilon_{t-1}$$

- If slope θ is zero then:

$$Y_t = \mu + \epsilon_t$$

And Y_t is White Noise (μ, σ_ϵ^2)

MA Processes - II

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

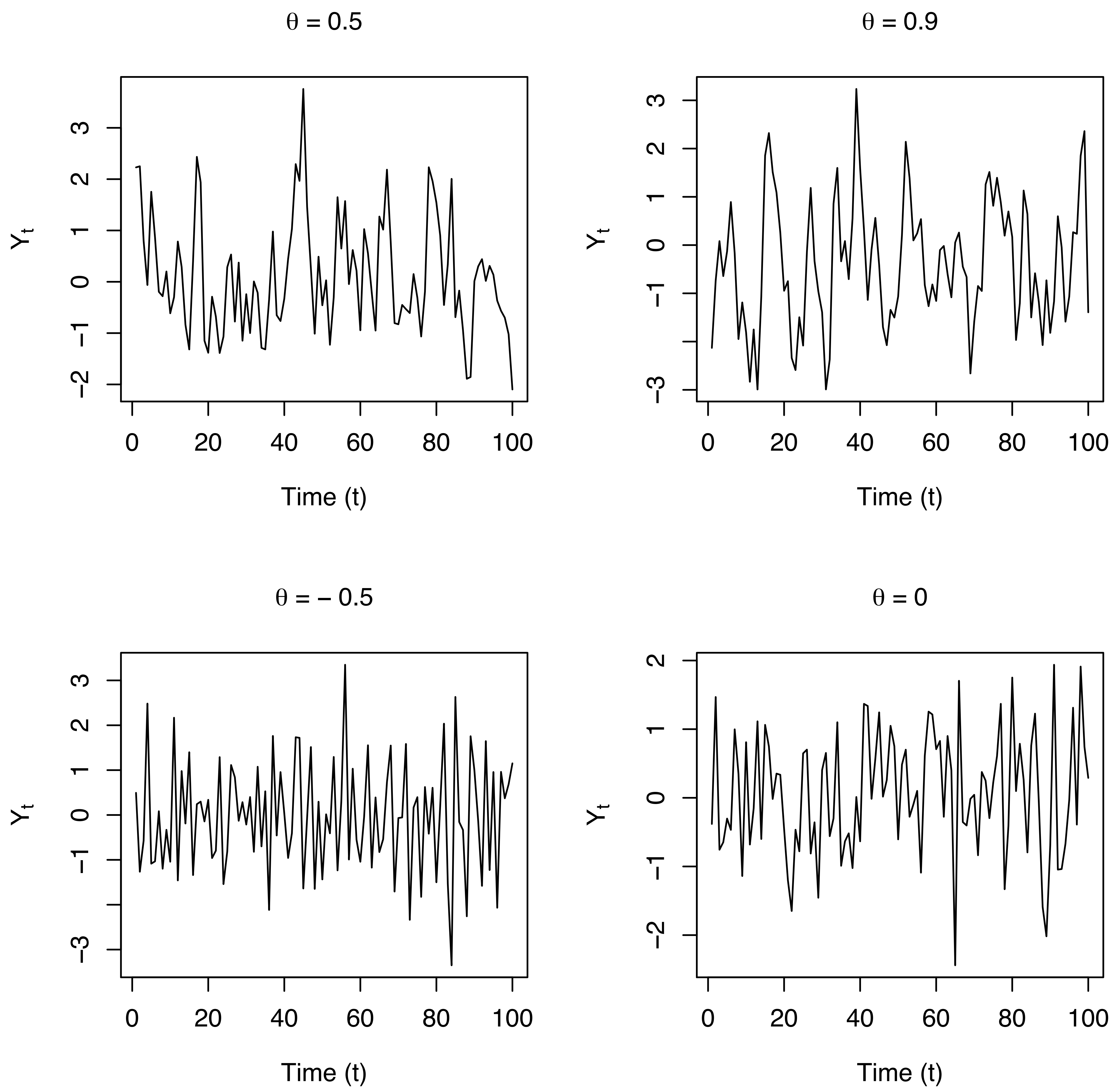
$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

- If slope θ is **not** zero then Y_t depends on both ϵ_t and ϵ_{t-1}

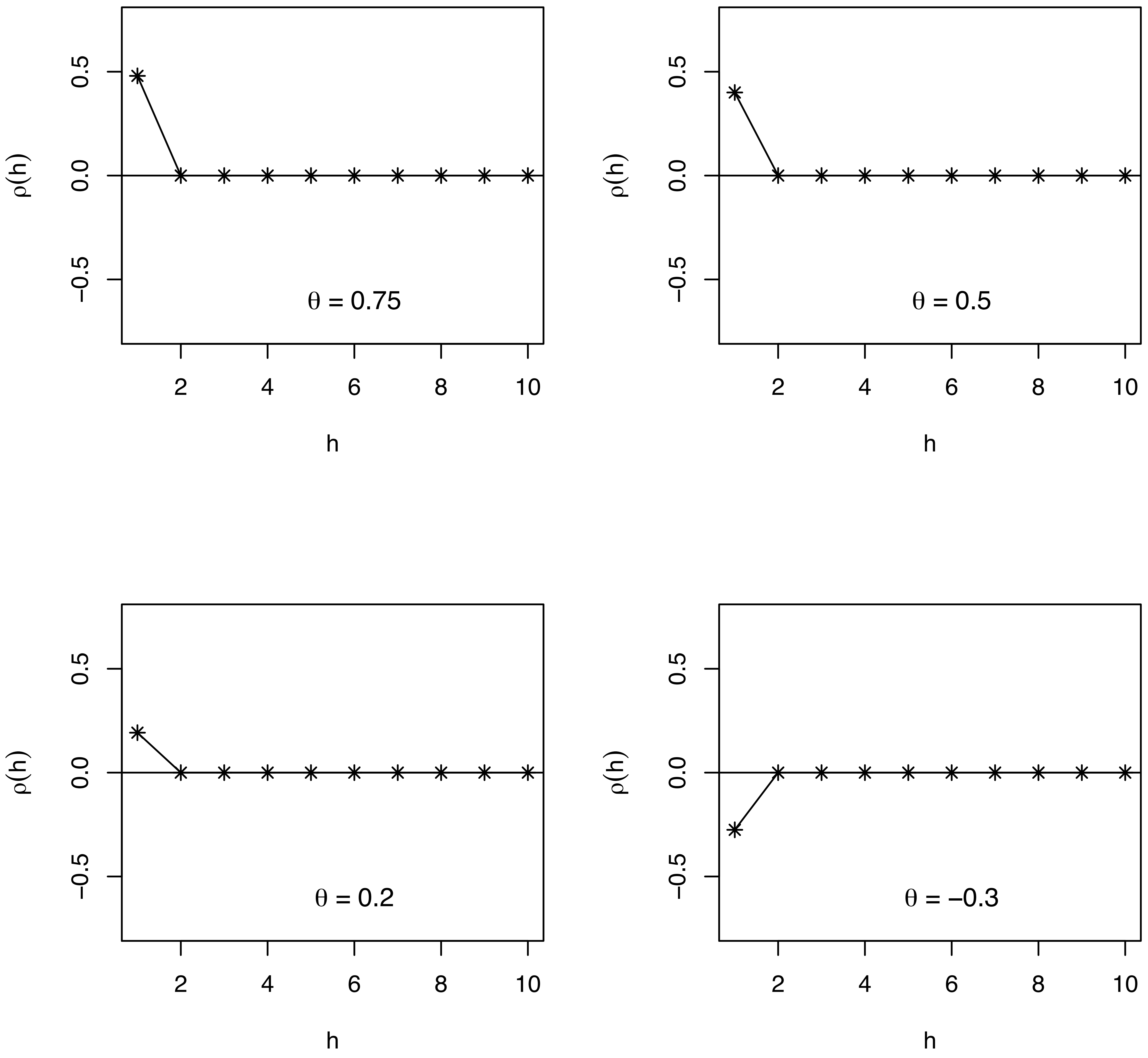
  And the process $\{Y_t\}$ is autocorrelated

- Large values of θ lead to greater autocorrelation
- Negative values of θ result in oscillatory time series

MA Examples



Autocorrelations



Only lag 1 autocorrelation non-zero for the MA model.

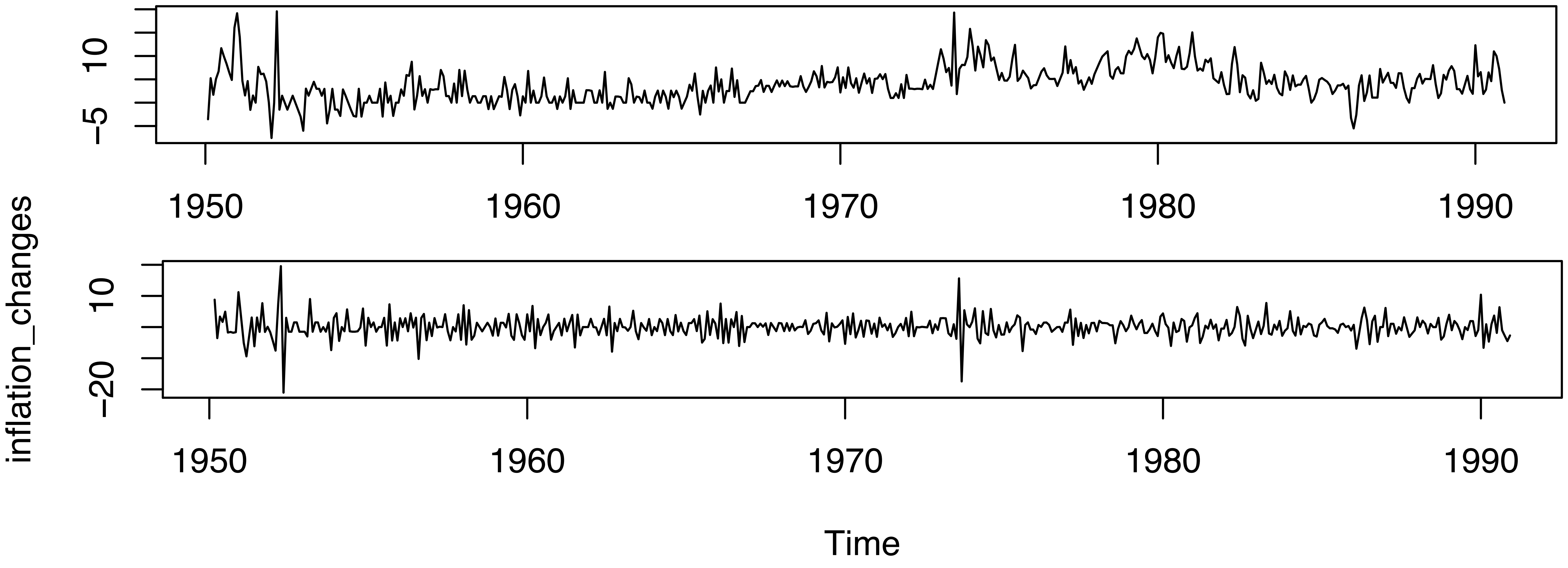
Let’s practice!

MA Model Estimation and Forecasting

MA Processes: Changes in Inflation Rate - I

- One-month US inflation rate (in percent, annual rate)
- Monthly observations from 1950 through 1990

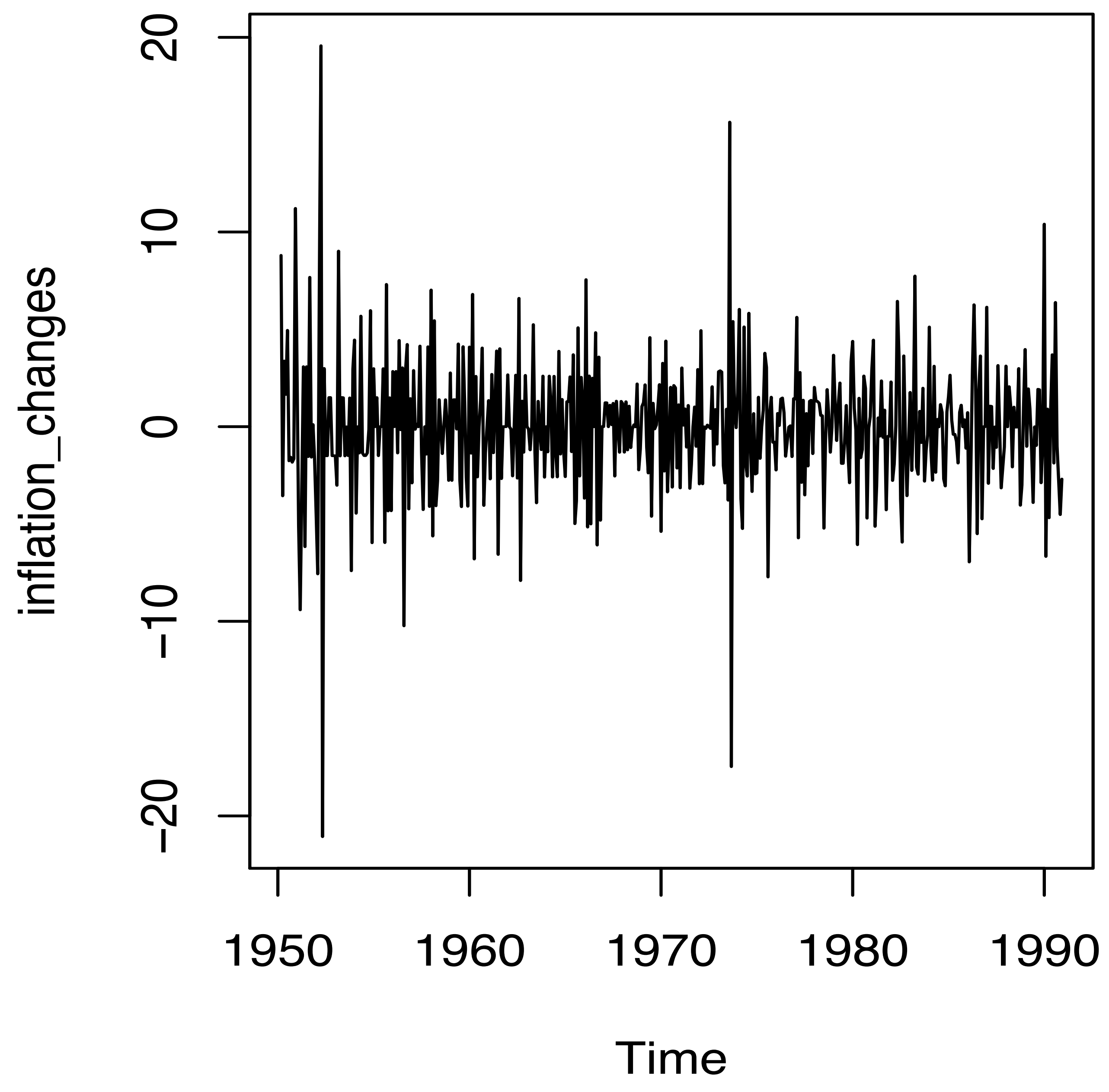
```
> data(Mishkin, package = "Ecdat")
> inflation <- as.ts(Mishkin[, 1])
> inflation_changes <- diff(inflation)
> ts.plot(inflation) ; ts.plot(inflation_changes)
```

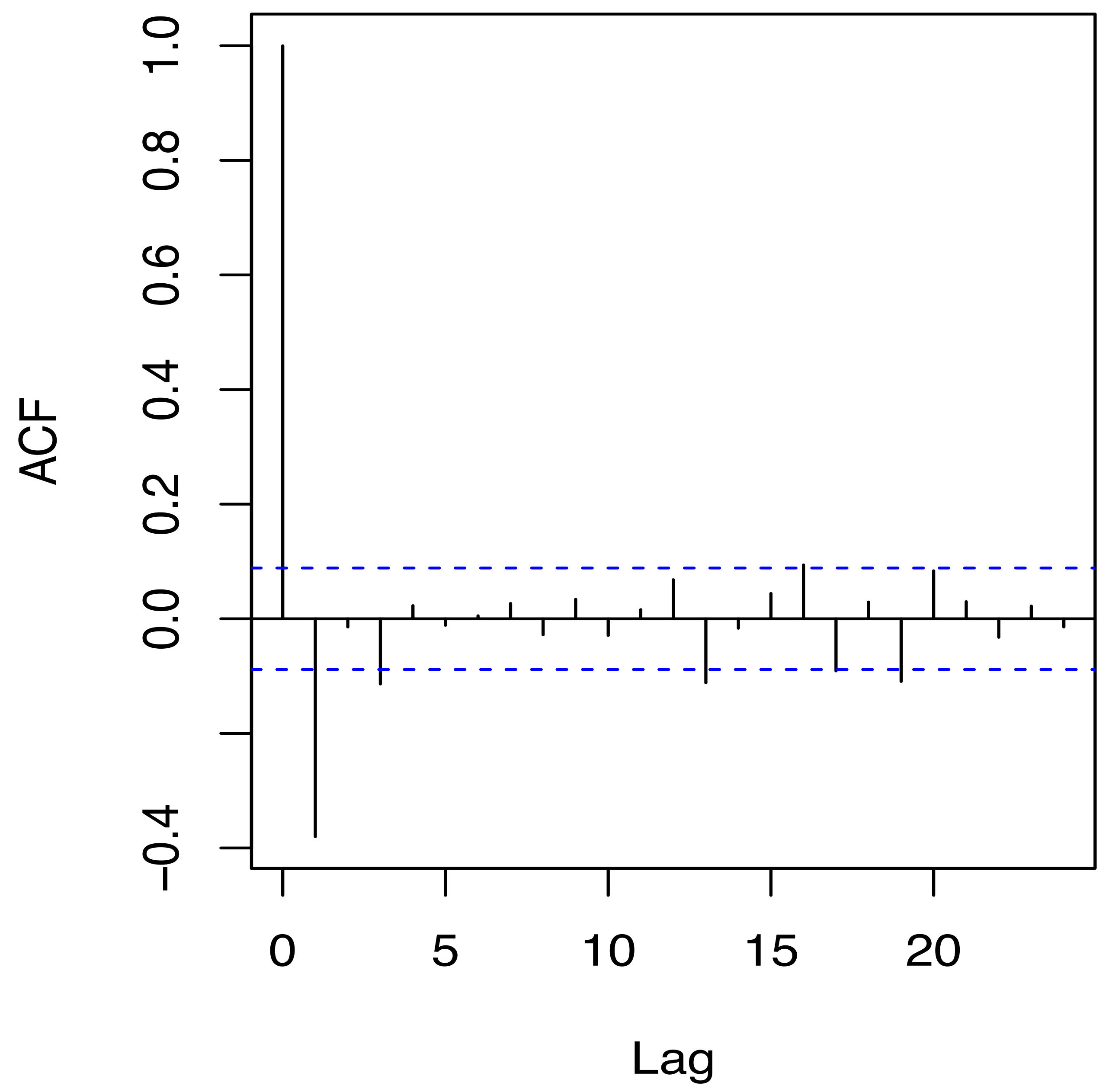


MA Processes: Changes in Inflation Rate - II

- `Inflation_changes`: changes in one-month US inflation rate
- Plot the series and its sample ACF:

```
> ts.plot(inflation_changes)
> acf(inflation_changes, lag.max = 24)
```





MA Processes: Changes in Inflation Rate - III

$$Today = Mean + Noise + Slope * (Yesterday's Noise)$$

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

$$\epsilon_t \sim WhiteNoise(0, \sigma_\epsilon^2)$$

```
> MA_inflation_changes <- arima(inflation_changes, order = c(0, 0, 1))
> print(MA_inflation_changes)

Coefficients:
          ma1  intercept
      -0.7932     0.0010
s.e.   0.0355     0.0281
sigma^2 estimated as 8.882
```

ma1 $= \hat{\theta}$, intercept $= \hat{\mu}$, sigma^2 $= \hat{\sigma}_\epsilon^2$

MA Processes: Fitted Values - I

- MA fitted values: $\widehat{Today} = \widehat{Mean} + \widehat{Slope} * \widehat{Yesterday's Noise}$

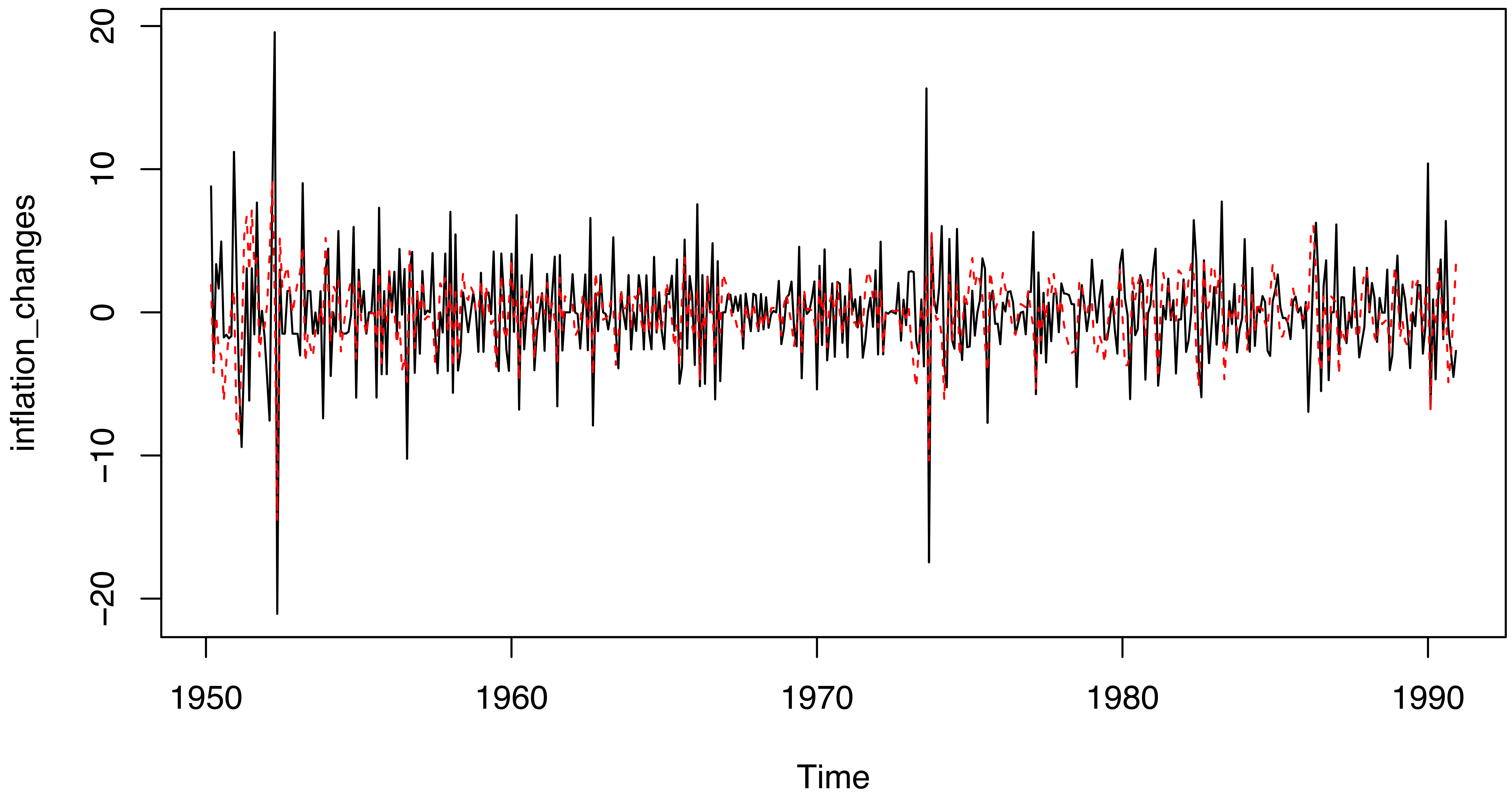
$$\hat{Y}_t = \hat{\mu} + \hat{\theta}\hat{\epsilon_{t-1}}$$

- Residuals= $Today - \widehat{Today}$

$$\hat{\epsilon}_t = Y_t - \hat{Y}_t$$

MA Processes: Fitted Values - II

```
> ts.plot(inflation_changes)
> MA_inflation_changes_fitted <- inflation_changes - residuals(MA_inflation_changes)
> points(MA_inflation_changes_fitted, type = "l", col = "red", lty = 2)
```



Forecasting

- 1-step ahead forecasts:

```
> predict(MA_inflation_changes)
$pred
          Jan
1991 4.831632

$se
          Jan
1991 2.980203
```

- h-step ahead forecasts:

```
> predict(MA_inflation_changes, n.ahead = 6)
$pred
           Jan      Feb      Mar      Apr      May      Jun
1991  4.831632 0.001049 0.001049 0.001049 0.001049 0.001049

$se
           Jan      Feb      Mar      Apr      May      Jun
1991  2.980203 3.803826 3.803826 3.803826 3.803826 3.803826
```

Let's practice!

Compare the AR and MA models

MA and AR processes

- MA model:

$$Today = Mean + Noise + Slope * (Yesterday'sNoise)$$

$$Y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

- AR model:

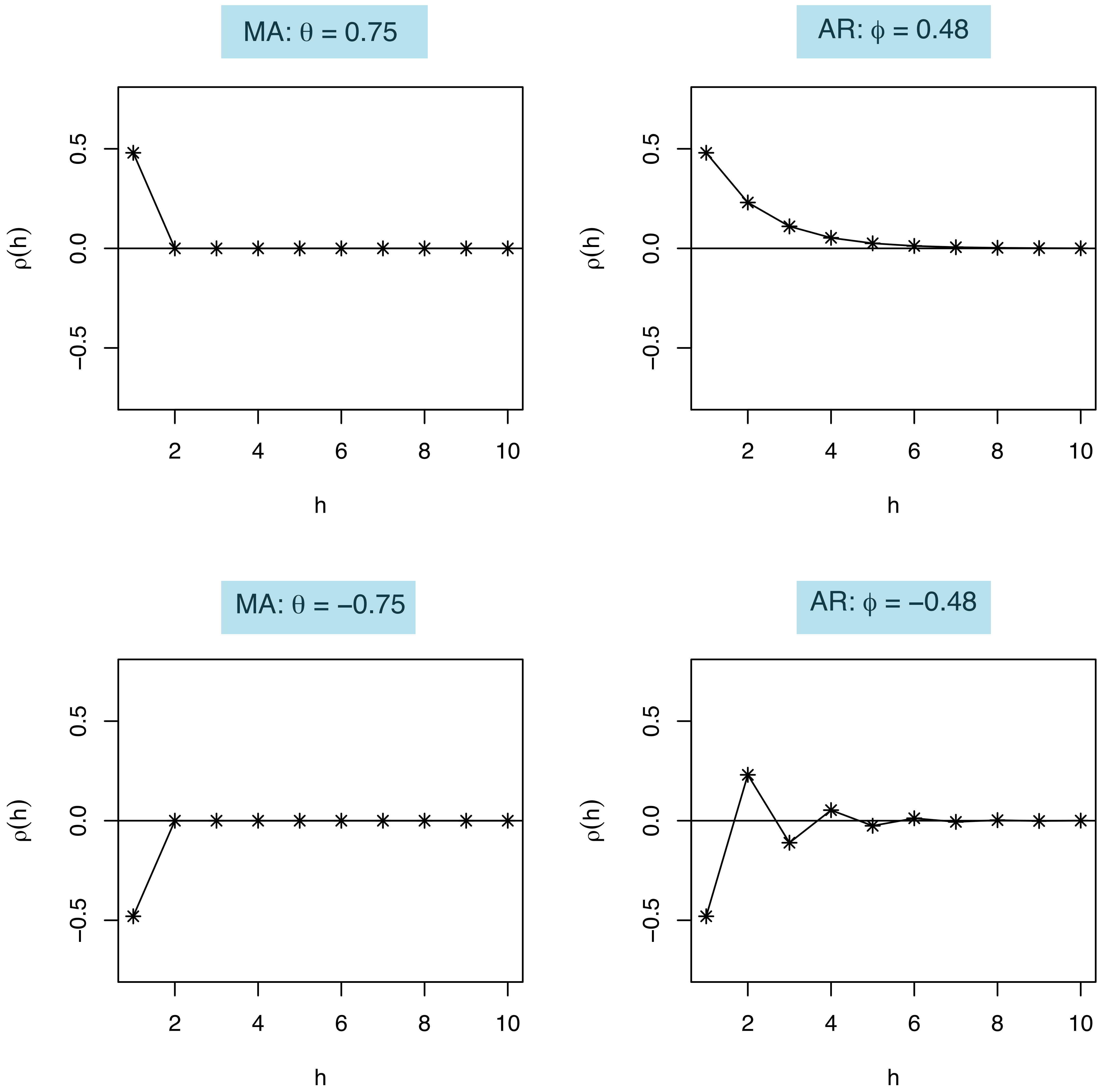
$$(Today - Mean) = Slope * (Yesterday - Mean) + Noise$$

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

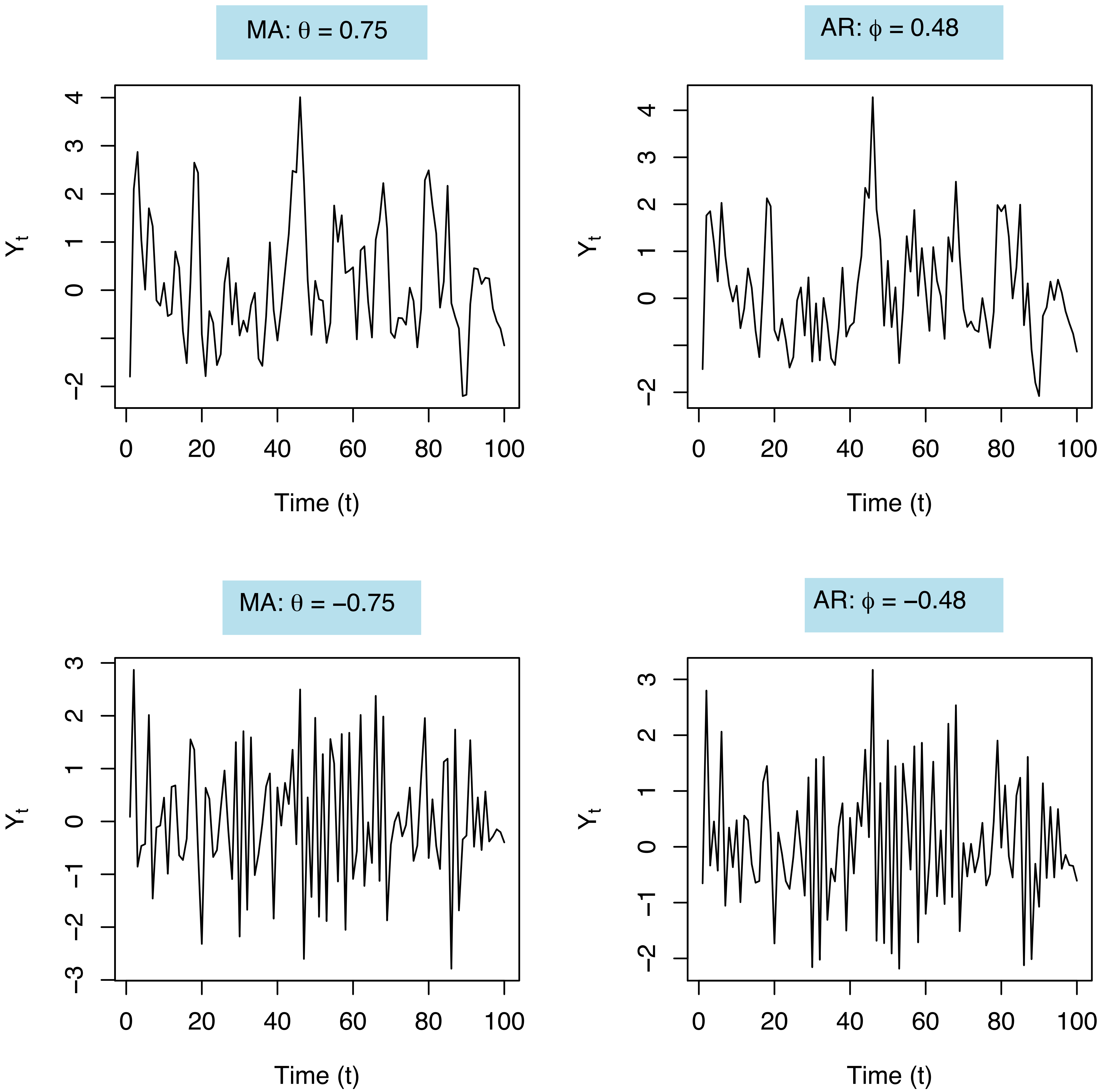
- Where:

$$\epsilon_t \sim WhiteNoise(0, \sigma_\epsilon^2)$$

MA and AR Processes: Autocorrelations

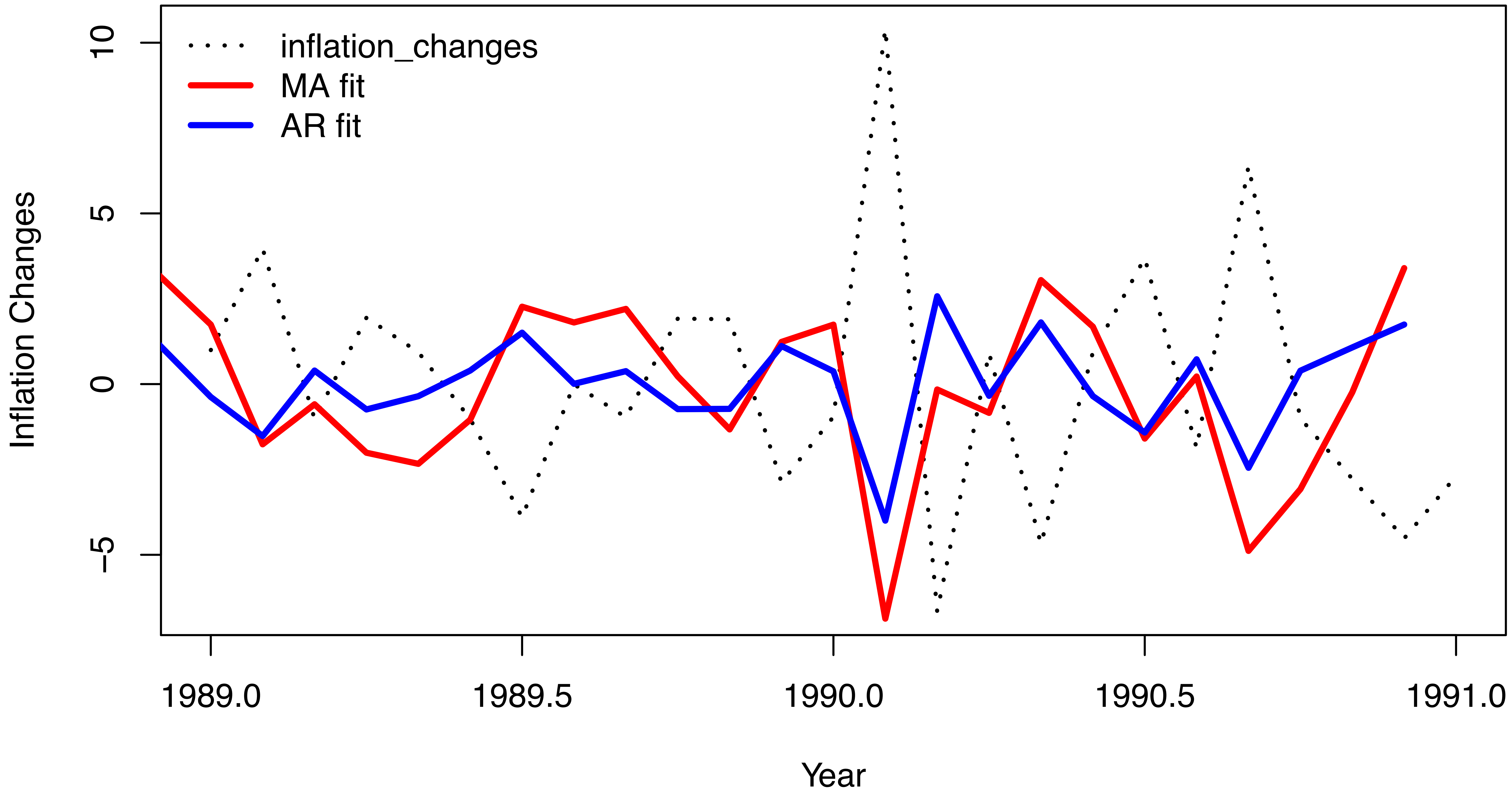


MA and AR Processes: Simulations



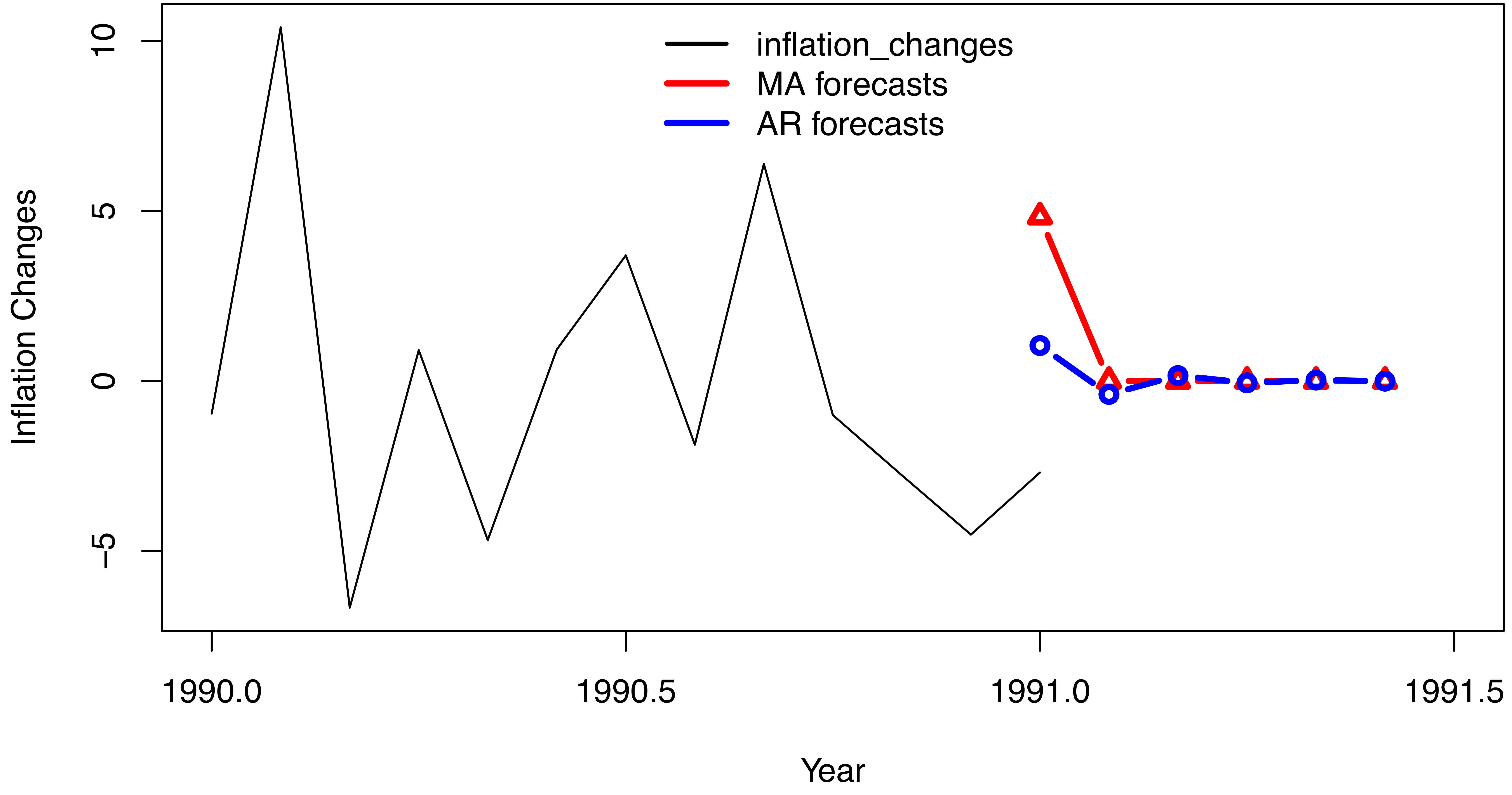
MA and AR Processes: Fitted Values

- Changes in one-month US inflation rate



MA and AR Processes: Forecasts

- Changes in one-month US inflation rate



Forecasting

Akaike Information Criterion

Bayesian Information Criterion

```
> MA_inflation_changes <- arima(inflation_changes, order = c(0,0,1))
          ma1  intercept
      -0.7932     0.0010
s.e.   0.0355     0.0281
sigma^2 estimated as 8.882: log likelihood = -1230.85, aic = 2467.7
> AIC(MA_inflation_changes)
   2467.703
> BIC(MA_inflation_changes)
   2480.286
```

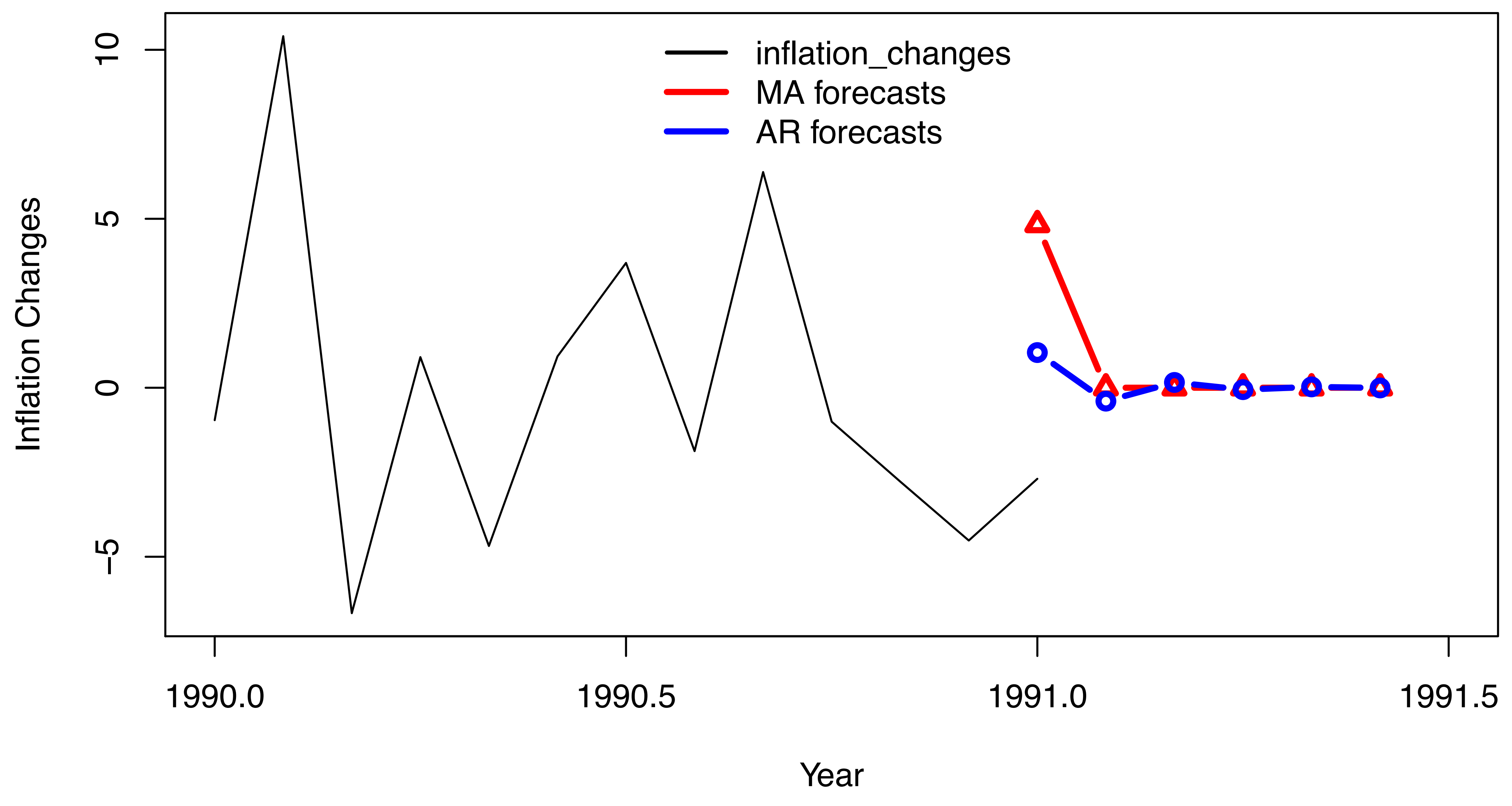
```
> AR_inflation_changes <- arima(inflation_changes, order = c(1,0,0))
          ar1  intercept
      -0.3849     0.0038
s.e.   0.0420     0.1051
sigma^2 estimated as 10.37: log likelihood = -1268.34, aic = 2542.68
> AIC(AR_inflation_changes)
   2542.679
> BIC(AR_inflation_changes)
   2555.262
```

Let's practice!

Congratulations!

What you've learned

- Manipulating `ts` objects, including `log()` and `diff()`
- Time series models: white noise, random walk, autoregression, simple moving average
- Time series simulation (`arima.sim`), fitting (`arima`), and forecasting (`predict`)



Thank you!