Behavior and Breakdown of Higher-Order Fermi-Pasta-Ulam-Tsingou Recurrences

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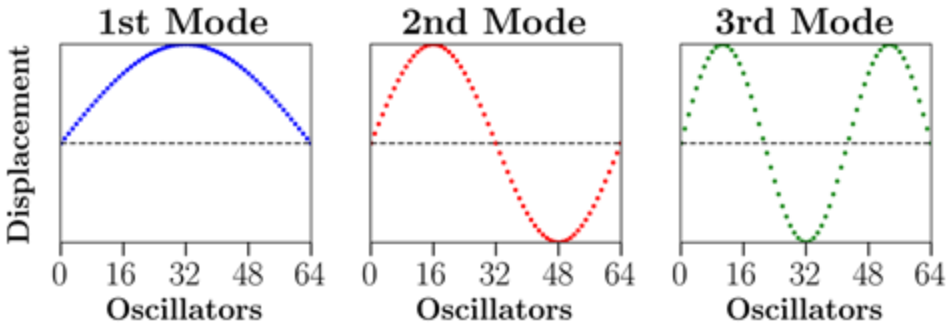
Fermi-Pasta-Ulam-Tsingou (FPUT) Lattice

Source: Porter, Mason A., et al. "Fermi, Pasta, Ulam and the Birth of Experimental Mathematics." *American Scientist* 97.3 (2009): 214-221, Figure 2

- One dimensional chain of masses connected by ***nonlinear*** springs
  - Linear springs: $F_n = k((\Delta x_{n+1} - \Delta x_n) - (\Delta x_n - \Delta x_{n-1}))$
  - Nonlinear springs: $F_n = k_j((\Delta x_{n+1} - \Delta x_n)^j - (\Delta x_n - \Delta x_{n-1})^j)$

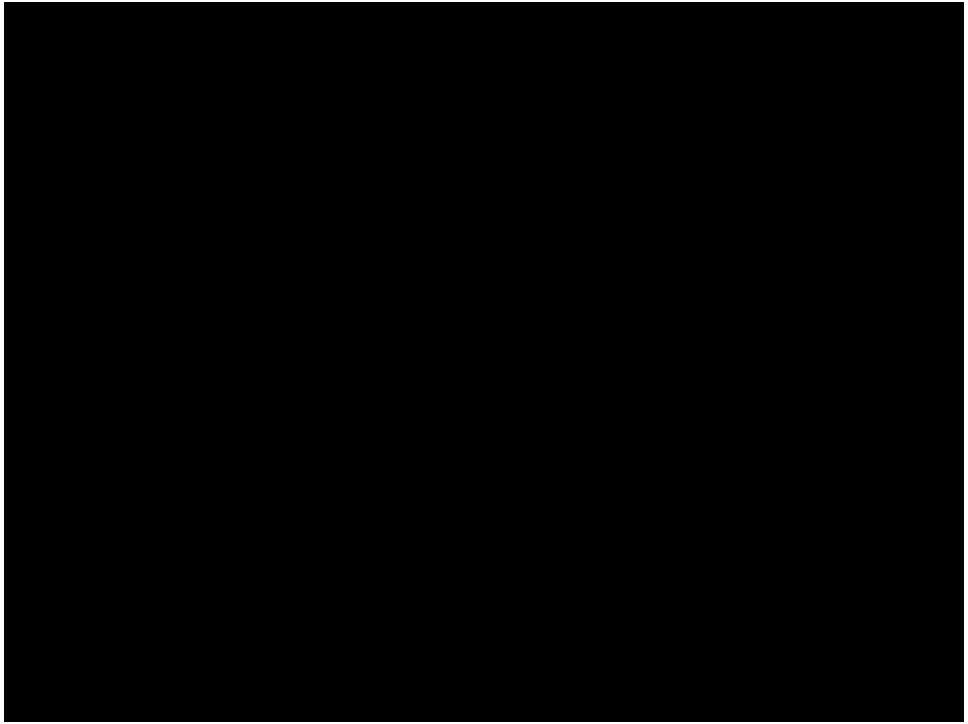
Original Study and Expectation

- Equipartition Theorem: Over time, energy would be shared equally among the *normal modes*.



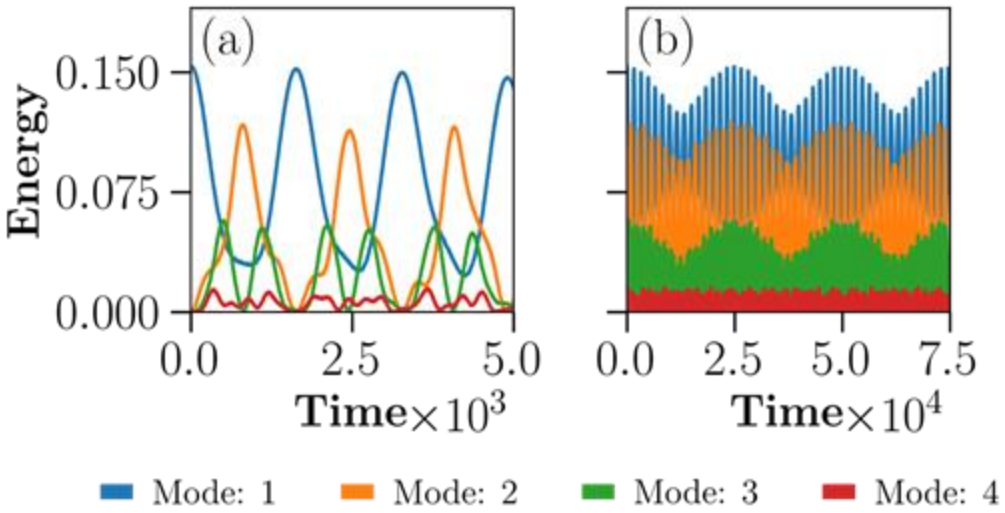
- Fermi, Pasta, Ulam, and Tsingou wanted to study the rate of thermalization - how long until equipartition is achieved.

Simulation of Lattice



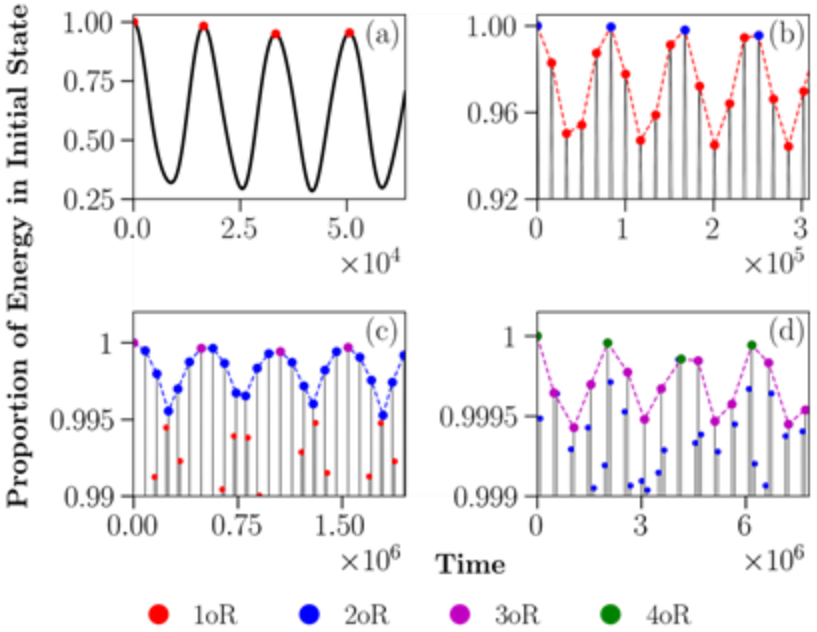
Previous Observations:

- For FPUT's initial conditions, energy was shared among only the lowest normal modes
- Remarkable near recurrences to the initial state occured (a)
- Longer time runs showed that these recurrences modulated and thus there were super-recurrences (b)



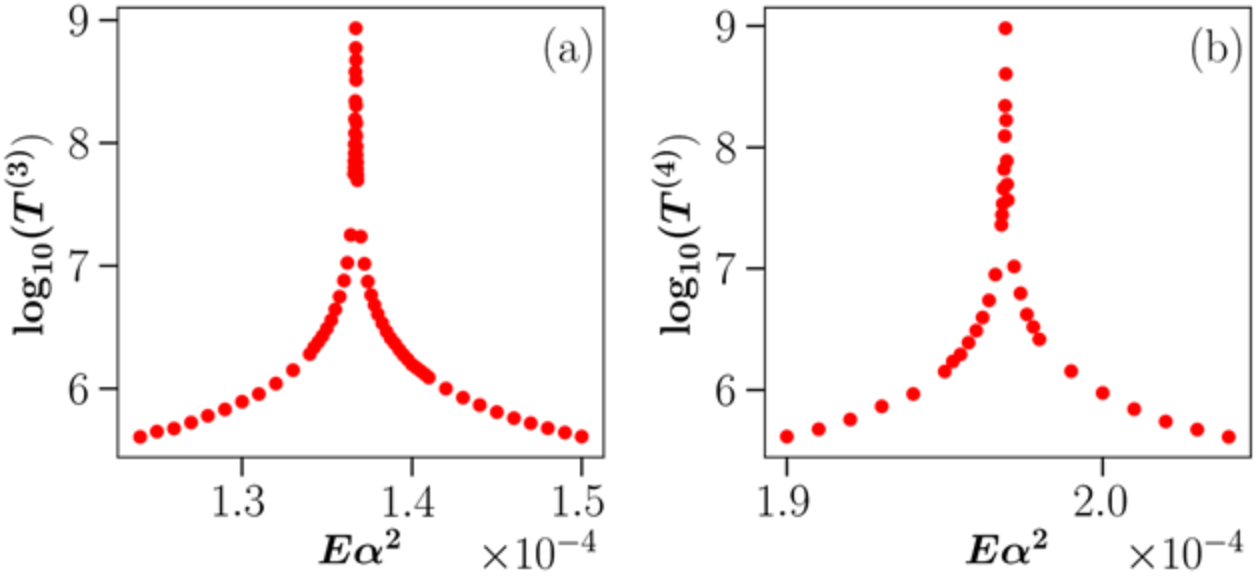
Higher-Order Recurrences (HoR)

- While FPUT recurrences and super-recurrences have been studied, higher order recurrences have not
- Our calculations show existence of even higher-order recurrences (super-super-recurrences, etc)



HoR Period Scaling

- Period scaling of these Higher-Order recurrences is nontrivial! Singularities!



FPUT Models: Alpha versus Beta Lattice

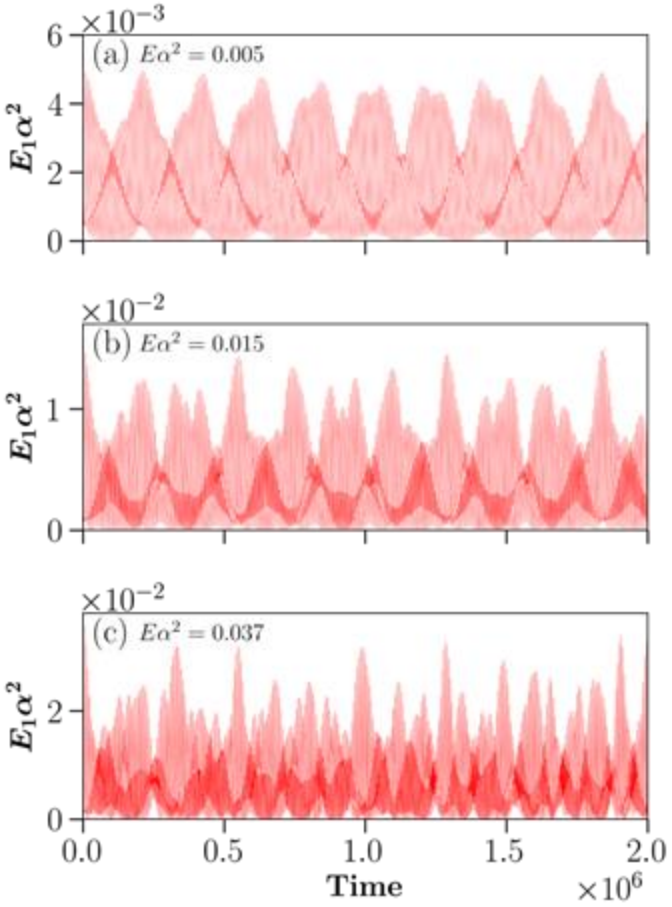
Alpha model:

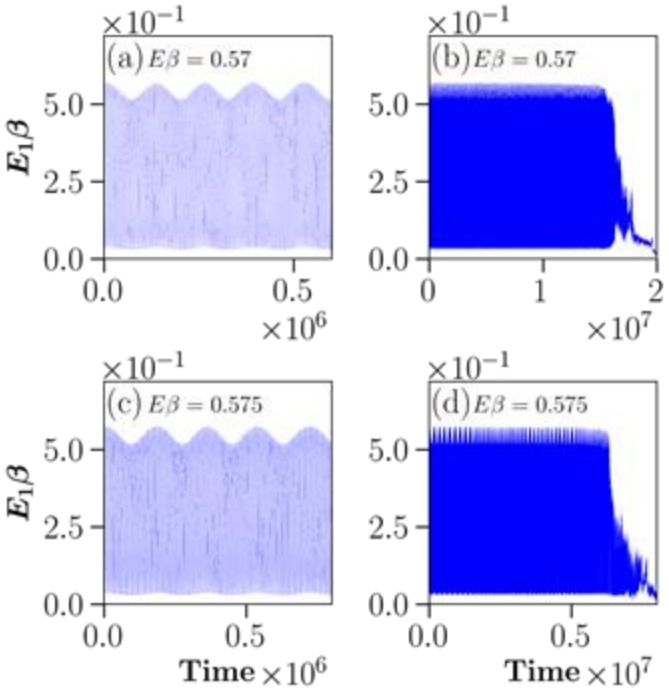
- Linear and Quadratic Force
- Exhibits HoR
- Singularies in 3oR and higher

Beta model:

- Linear and Cubic Force
- Exhibits HoR
- Singularies in 2oR and higher

Super-recurrence breakdown mechanisms

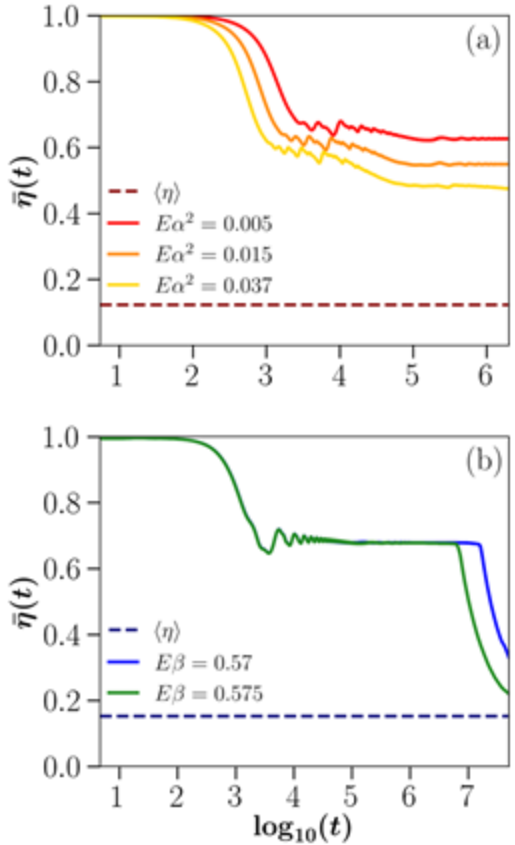




- Alpha model (red) super-recurrene breakdown is through a deformation as energy increases
- Beta model (blue) super-recurrences breakdown abruptly, sooner with more energy

Breakdown and Thermilization Relationship?

- A variety of entropy shown in figure is used as an indicator of equipartition.
- Figure (a) shows alpha model, figure (b) shows beta model.
- Alpha model super-recurrences breakdown without thermalizing. Beta model super-recurrences breakdown as the system starts to approach equilibrium.



Questions?