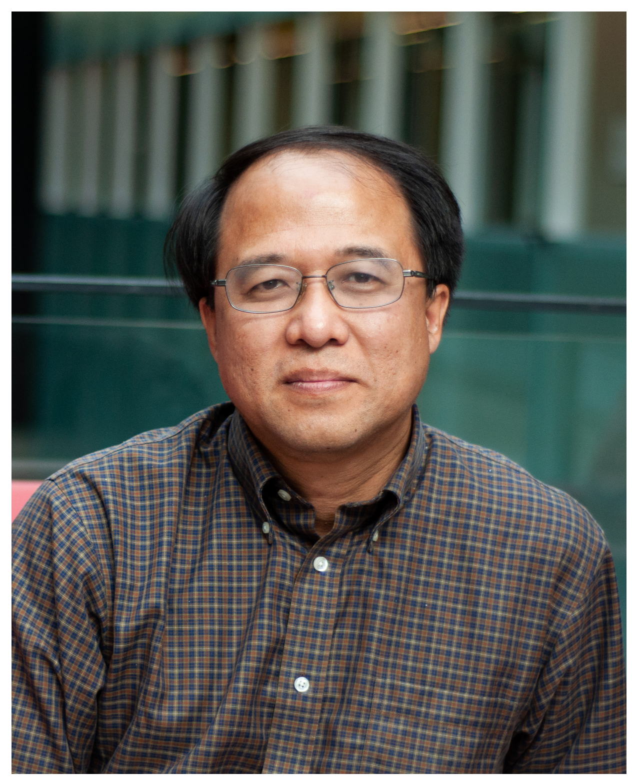
EXACT EMERGENT HIGHER FORM SYMMETRIES

Sal Pace (MIT)

in collaboration with Xiao-Gang Wen (MIT)



SP & X-G Wen: arXiv:2301.05261

HIGHER-FORM SYMMETRIES

Recently our understanding of symmetry has been revolutionized through modern generalizations

	Ordinary symmetry (0-form symmetry)	p-form symmetry
Symmetry charge	0-dimensional objects (particles)	p-dimensional objects (i.e., loops for $p = 1$)
Symmetry operator	Acts on entire lattice	Acts on codimension p subspace of lattice

Nussinov & Ortiz, PNAS 106.40 (2009): 16944-16949.
Gaiotto, Kapustin, Seiberg, & Willett, JHEP, 2015, 172 (2015)
McGreevy, arXiv:2204.03045 (2022)
Córdova, et. al., arXiv:2205.09545 (2022)

IN CONDENSED MATTER:

Microsopic Scale

$G_{\mathrm{UV}} \rightarrow$ Only 0-form symmetries

$G_{\mathrm{mid-IR}}$

Could include higher-form symmetries

low-energies or long-distances

G_{IR}

EMERGENT HIGHER-FORM SYMMETRY

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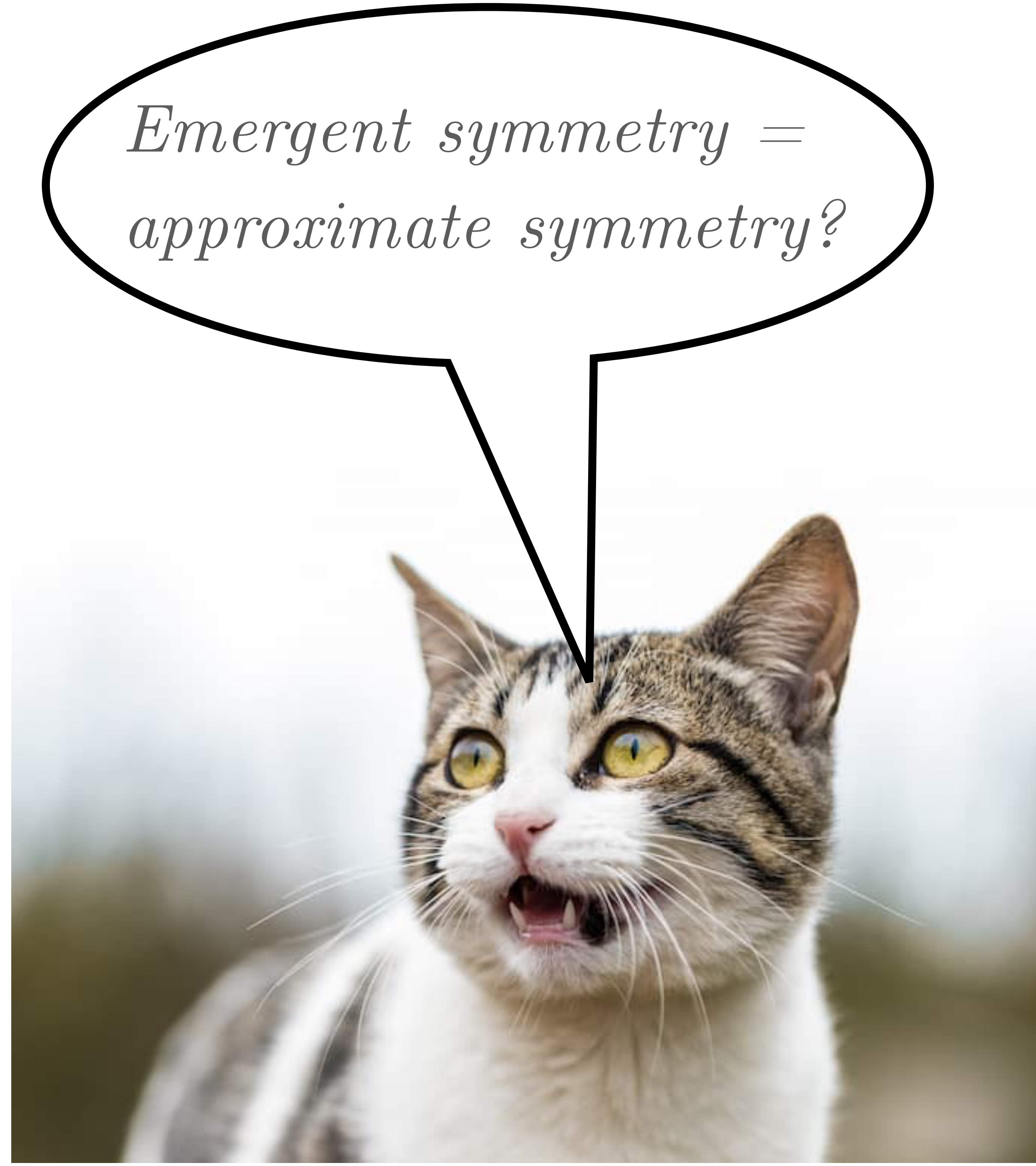

EMERGENT = APPROXIMATE?

Emergent 0-form symmetries are generally approximate since they can be explicitly broken by irrelevant operators

- Explicitly breaking a spontaneously broken 0-form symmetry: lifts GSD or gaps Goldstone modes

What about emergent higher-form symmetry?

EMERGENT = APPROXIMATE?

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➤ Explicitly breaking a spontaneously broken 0-form symmetry: lifts GSD or gaps Goldstone modes

What about emergent higher-form symmetry?

Emergent higher-form symmetries are exact symmetries

SP *& X-G Wen. arXiv:2301.05261*

Foerster, Nielsen, Ninomiya. Phys. Lett B 94.2 (1980): 135-140

Hastings, Wen. PRB 72.4 (2005): 045141

Iqbal, McGreevy. SciPost Phys. 13.5 (2022): 114

Córdova, Ohmori, Rudelius. JHEP 2022.11 (2022): 1-38

Cian, Hafezi, Barkeshli. arXiv:2209.14302

Cheng, Seiberg. arXiv:2211.12543

REST OF THE TALK

1. Why emergent higher form symmetries are exact symmetries — a condensed matter point of view

2. What are the physical consequences of exact emergent higher-form symmetries in condensed matter

$2+1$D $\mathbb{Z}_2$ GAUGE THEORY

Qubits on links of square lattice

$$H = -\sum_s \prod_{l \in \delta s} Z_l - \sum_p \prod_{l \in \partial p} X_l - h_x \sum_l X_l$$

Let $0 < h_x \ll 1$ and $\tilde{X}_l$ create e anyons at ∂l with energy cost Δ_e.

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$$W(\gamma) = \prod_{l \in \gamma} \tilde{X}_l \qquad \epsilon_\gamma \propto h_x^{|\gamma|}$$

$$H_{e\text{ free}} = -\sum_p W(\partial p) - \sum_{\text{loops } \gamma} \epsilon_\gamma W(\gamma)$$

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$2+1\text{D}\ \mathbb{Z}_2$ GAUGE THEORY

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$\mathcal{O}(\mathrm{e}^{-L})$

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$2+1$D $\mathbb{Z}_2$ GAUGE THEORY

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- $L \to \infty$: Exact emergent $\mathbb{Z}_2$ 1-form symmetry:

$$\tilde{X}_l \to s_l \tilde{X}_l \qquad \prod_{l\in\partial p} s_l = 1$$

THE GENERAL LESSON

If there are gapped topological excitations which are the endpoints of "thick" strings.

- Below their gap, there are only closed loops

$\implies$ emergent string flux conservation

$\implies$ emergent 1-form symmetry

Emergent conservation law cannot be violated by low-energy *local* operators.

- The low-energy effective theory will have an exact 1-form symmetry $\implies$ exact emergent symmetry

EXACT EMERGENT SYMMETRY

At zero temperature

emergent p-form symmetries are exact when $p \geq 1$

At finite temperature T

- Time direction compactified with radius $1/T$.
- At length scales $\gg 1/T$, a p-form symmetry can act like a $(p-1)$-form symmetry:

emergent p-form symmetries are exact when $p \geq 2$

PHYSICAL CONSEQUENCES I

Explicitly breaking a spontaneously broken higher-form symmetry: still exact GSD/gapless Goldstone modes because emergent symmetry is exact

- abelian topological orders,
- photons in $U(1)$ QSLs

Gaiotto, Kapustin, Seiberg, & Willett, JHEP, 2015, 172 (2015)

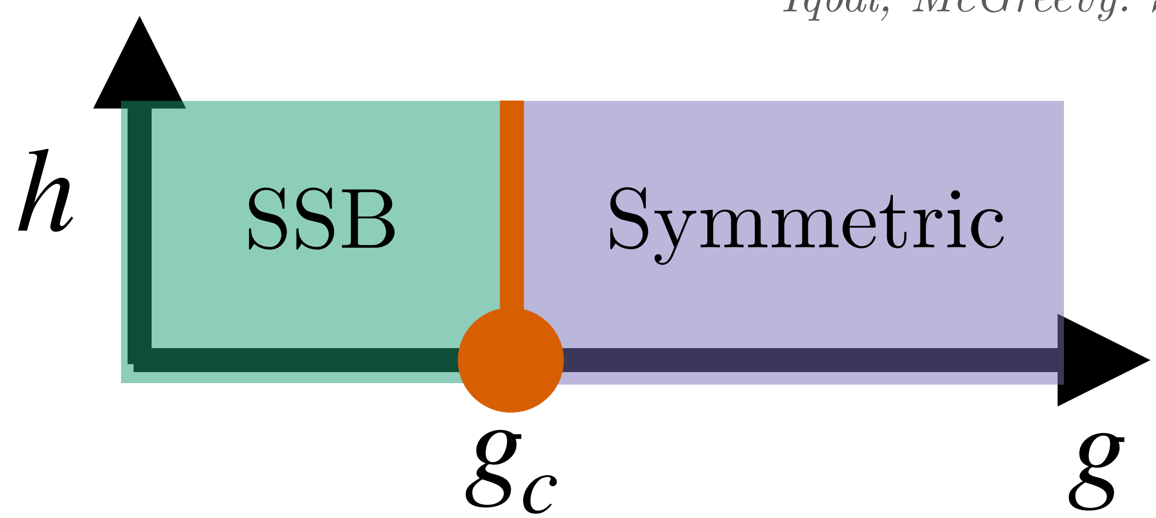
Long-range entangled topological phases are:

1. robust because characterized by exact emergent symmetries
2. typically only stable at finite temperature in $d > 3$ since one needs p-form SSB with $p \geq 2$

PHYSICAL CONSEQUENCES II

Exact emergent higher-form symmetries without SSB can:

1. characterize phase transitions: *Somoza, Serna, Nahum. PRX 11.4 (2021): 041008. Iqbal, McGreevy. SciPost Phys. 13.5 (2022): 114*



2. form an SPT phase: ***SP** & Wen, PRB 107, 075112 (2023). Moy, et al. SciPost Phys. 14.2 (2023): 023. Verresen, et al. arXiv:2211.01376 (2022).*
   - Boundary has exact emergent higher-form symmetry
   - *At low-energies*, turning on background gauge fields, there's anomaly inflow and a bulk topological response

PHYSICAL CONSEQUENCES III

Exact emergent higher-form symmetry can be anomalous

- The 't Hooft anomaly protects the phase's characteristic features
- Examples: Ising ferromagnet, bosonic superfluid

Exact emergent higher-form symmetry can couple nontrivially to 0-form symmetries, forming a higher-group structure

- Higher-group symmetry can also be anomalous
- Example: (anti)ferromagnetic superconductors

Tomáš. JHEP 2021.4 (2021): 1-40.

TAKE AWAY MESSAGE AND OUTLOOK

Emergent higher-form symmetries are **exact** symmetries

Generalized symmetries in condensed matter physics:

- Unifying perspective on different phases of matter
- Characterize new phases of matter
- Put constraints on strongly correlated phases
- Potential classification scheme of equilibrium phases: generalized Landau paradigm

__SP__ & X-G Wen, arXiv:2301.05261

EVIDENCE FROM NUMERICS

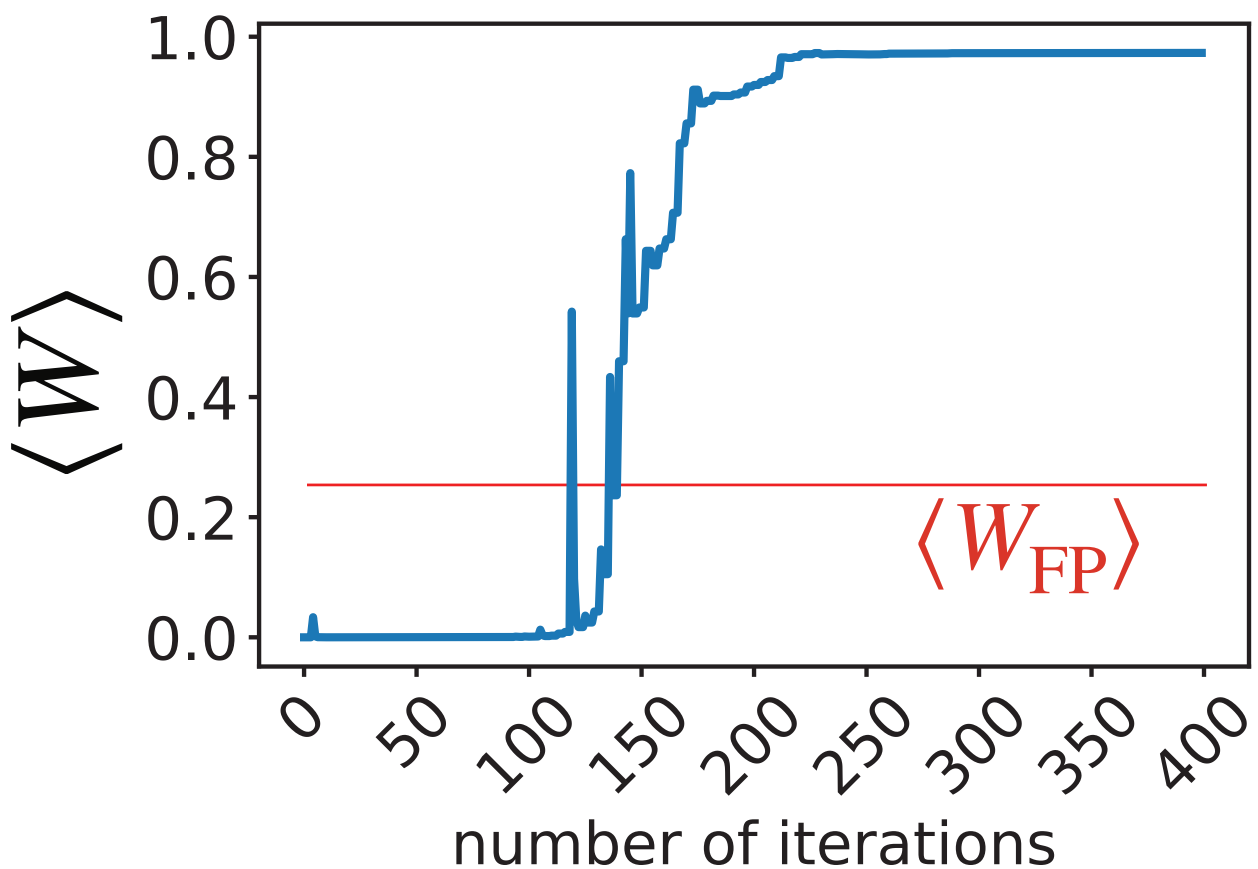
Precise form of dressed operators is not generally unique, depending on microscopic details

- ➤ Can be numerically constructed as a *matrix product operator* [Cian, Hafezi, & Barkeshli, arXiv:2209.14302]

Toric code with $h_x = 0.15$, $h_z = 0.05$ and $\Delta_e = \Delta_m = 2$

$$W_{\mathrm{FP}} = \prod_{e\in\gamma} X_e \implies \langle W_{\mathrm{FP}}\rangle \sim \mathrm{e}^{-|\gamma|/\xi}$$

$$W = \text{“}\prod_{e\in\gamma} \tilde{X}_e\text{”} \implies \langle W\rangle = 1$$



EMERGENCE WITHOUT SSB

$$H = -\frac{\Delta_e}{2}\sum_s \prod_{e\in\delta s} Z_e - \frac{\Delta_m}{2}\sum_p \prod_{e\in\partial p} X_e - h_x \sum_e X_e - h_z \sum_e Z_e$$

When $h_x \gg \Delta_e$, e anyons condense (Higgs phase)

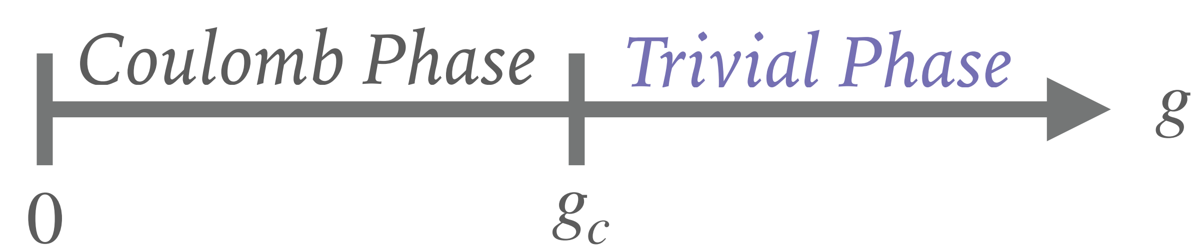
- No e free low-energy subspace $\Longrightarrow$ no emergent conservation law ($\mathbb{Z}_N$ electric flux) $\Longrightarrow$ no emergent $\mathbb{Z}_N^{(1)}$ symmetry
- There can be an m free low-energy subspace $\Longrightarrow$ emergent conservation law ($\mathbb{Z}_N$ magnetic flux) $\Longrightarrow$ emergent $\mathbb{Z}_N^{(1)}$ symmetry

***SP** & X-G Wen, arXiv:2301.05261*

TWISTED $U(1)$ GAUGE THEORY

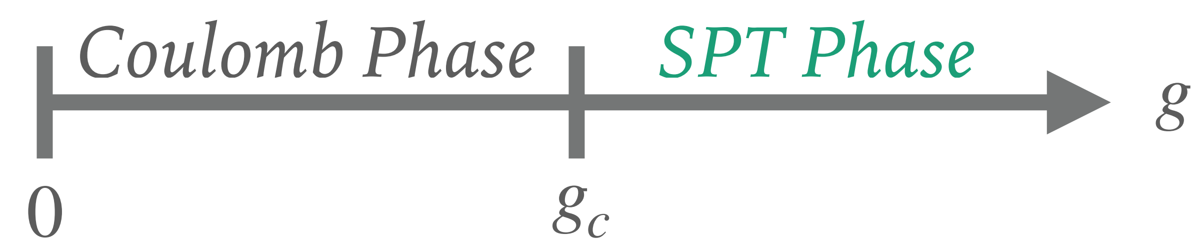
$U(1)$ gauge theory:

- $g > g_c$: monopoles condense



Twisted $U(1)$ gauge theory

- $g > g_c$: dyons condense



$(q_e, q_m) = (2{,}1)$ dyon $\rightarrow \mathbb{Z}_2^{(1)}$ SPT

Boundary: $\nu = 1/2$ bosonic FHQ

$(q_e^{1,2}, q_m^{2,1}) = (2{,}1)$ dyons $\rightarrow \mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(1)}$ SPT

Boundary: $\mathbb{Z}_2$ topological order

Need magnetic monopoles (dyons), why care?

- Quantum spin ice on a breathing pyrochlore lattice

SP & Wen, PRB 107, 075112 (2023)

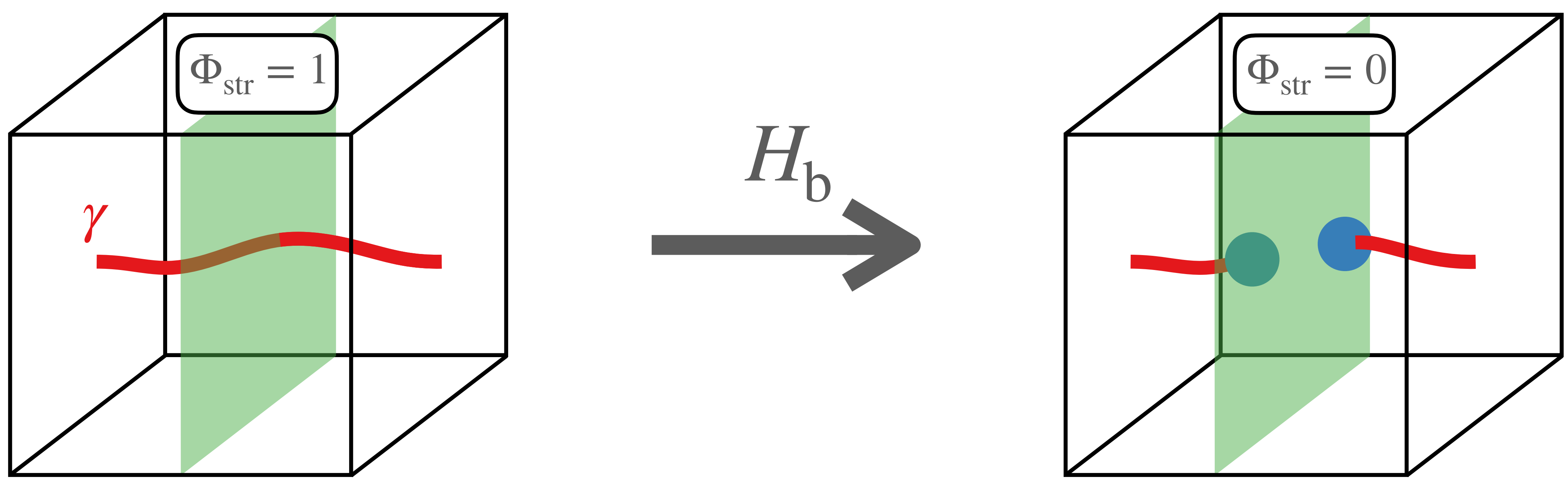
SP, Castelnovo, & Laumann, PRL 130, 076701 (2023)

BRIEF EXPLANATION

Consider $H = H_0 + H_\mathrm{b}$ in 3d where Hilbert space $\mathscr{H} = \bigotimes \mathscr{H}_i$,

H_0 has a 1-form symmetry, and H_b is a local perturbation.

Acting $b^\dagger(\gamma)$:



General H_b violates the flux conservation law of the 1-form symmetry, hence explicitly breaking H_0's 1-form symmetry