THE EMERGENT FINE STRUCTURE CONSTANT IN QUANTUM SPIN ICE IS LARGE

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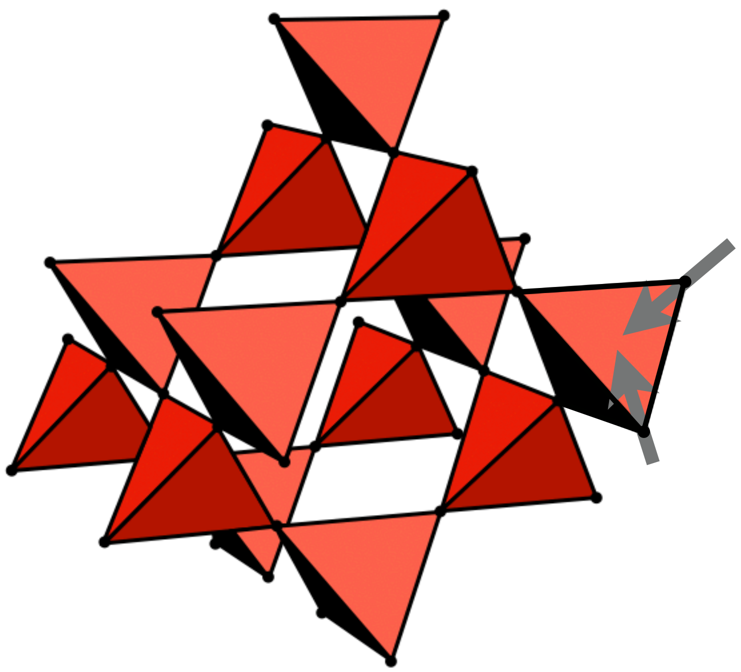
EMERGENT QED IN QUANTUM SPIN ICE



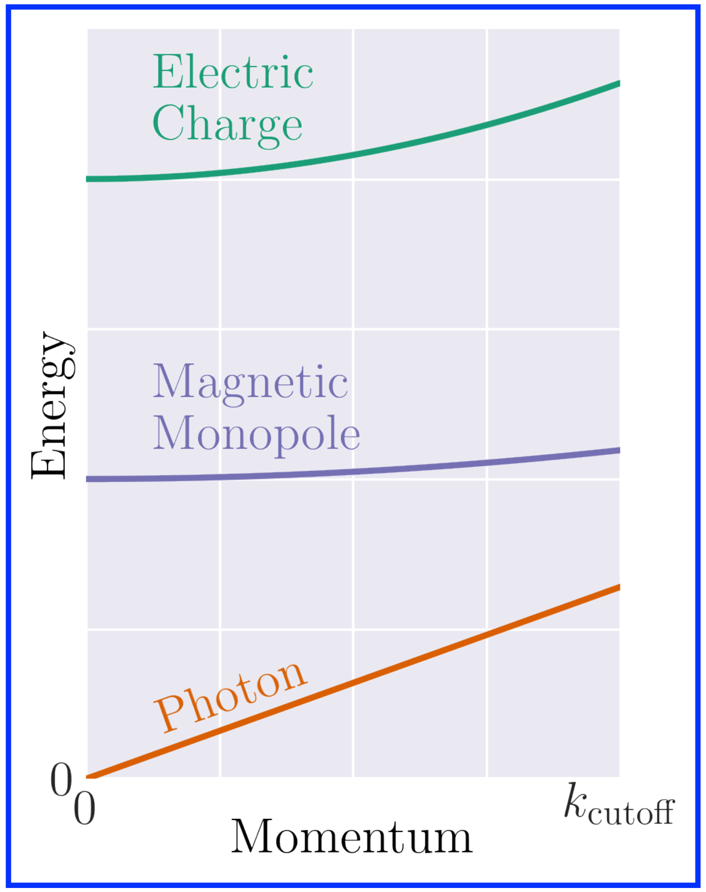
Pyrochlore Lattice

Short Distances (High Energy)

Long Distances (Low Energy)

$3+1$D Compact

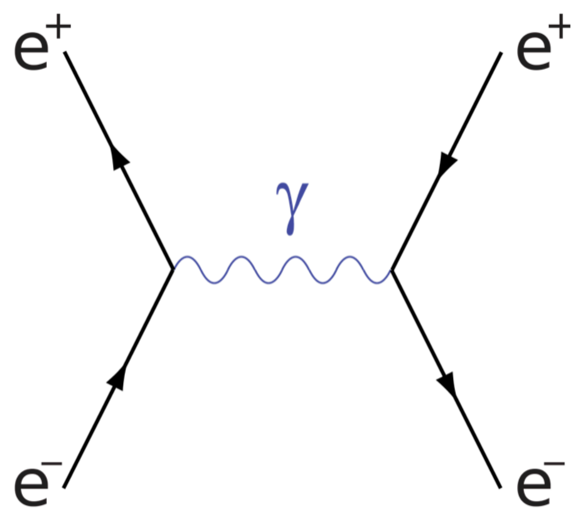
$U(1)$ Gauge Theory



Moessner, R. and S. L. Sondhi. *Physical Review B* 68.18 (2003): 184512.
Hermele, Michael, Matthew PA Fisher, and Leon Balents. *Physical Review B* 69.6 (2004): 064404.
Banerjee, Argha, et al. *Physical review letters* 100.4 (2008): 047208.
Shannon, Nic, et al. *Physical review letters* 108.6 (2012): 067204.
Kato, Yasuyuki, and Shigeki Onoda. *Physical review letters* 115.7 (2015): 077202.
Huang, Chun-Jiong, et al. *Physical review letters* 120.16 (2018): 167202.

FINE STRUCTURE CONSTANT

- Important dimensionless constant in $3+1$D QED: $\alpha = \dfrac{e^2}{4\pi\epsilon_o \hbar c}$



$\sim \quad \alpha$

Largest Stable

Atomic Number $\approx 1/\alpha$

<u>*QED of our universe*</u>

$\alpha_{QED} \sim 1/137$

<u>*QED in QSI*</u>

$\alpha_{QSI} \sim ???$

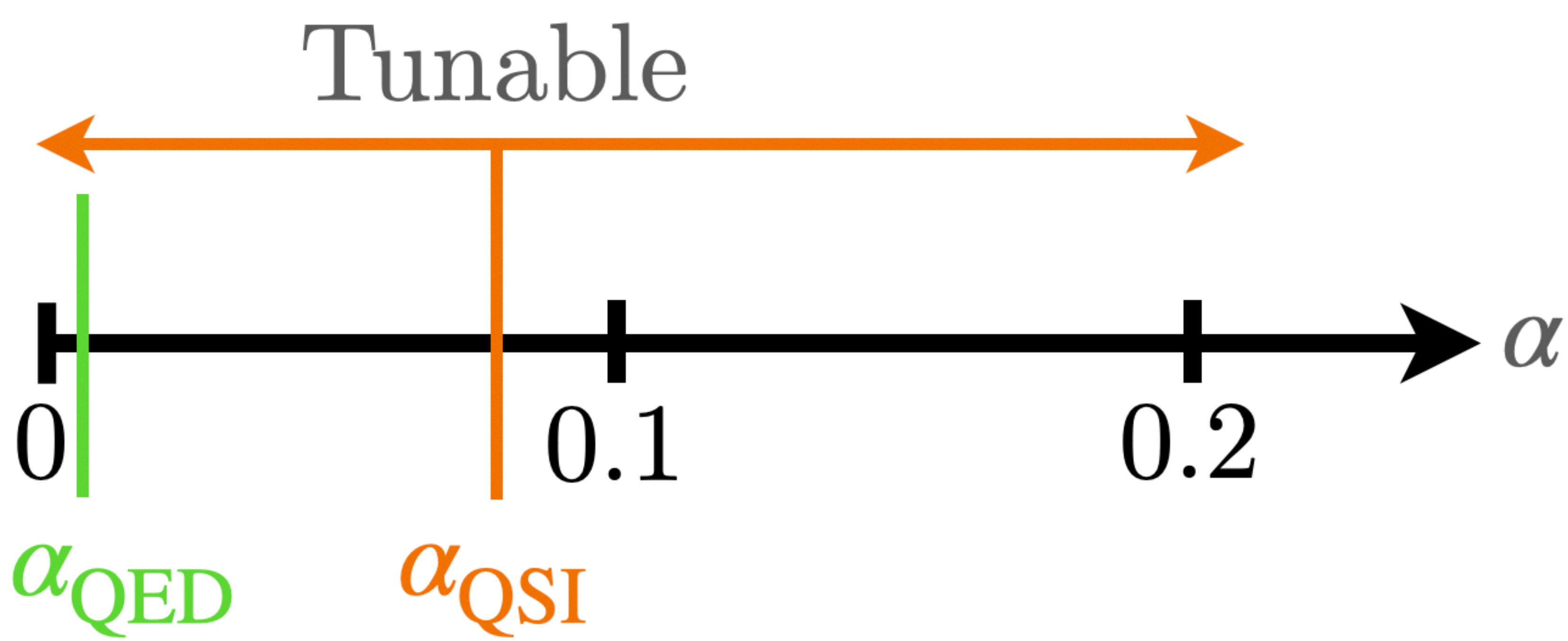
FINE STRUCTURE CONSTANT

"When I die, my first question to the devil will be: What is the meaning of the fine structure constant?"

Wolfgang Pauli

SUMMARY OF RESULTS

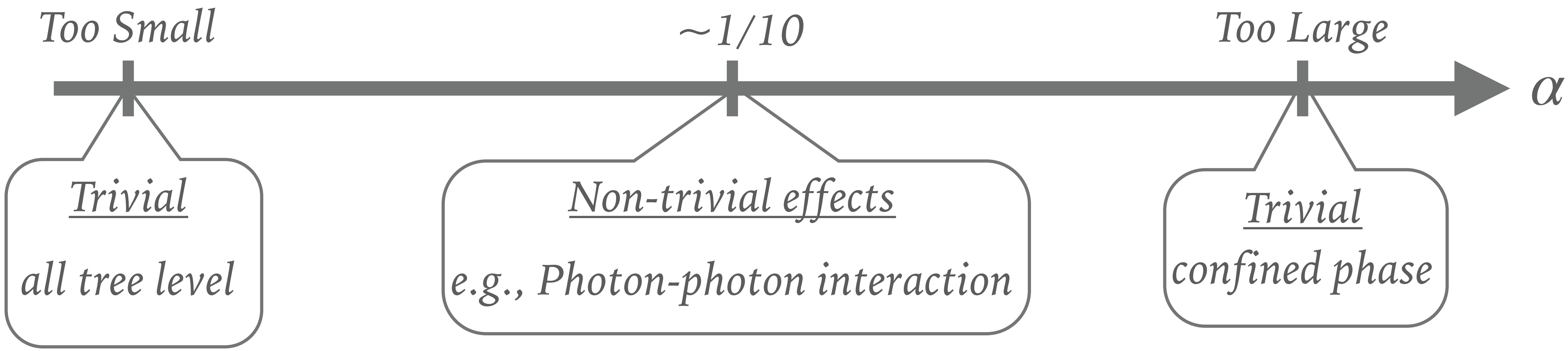
- Use exact diagonalization to measure fine structure constant in the deconfined emergent QED phase of QSI



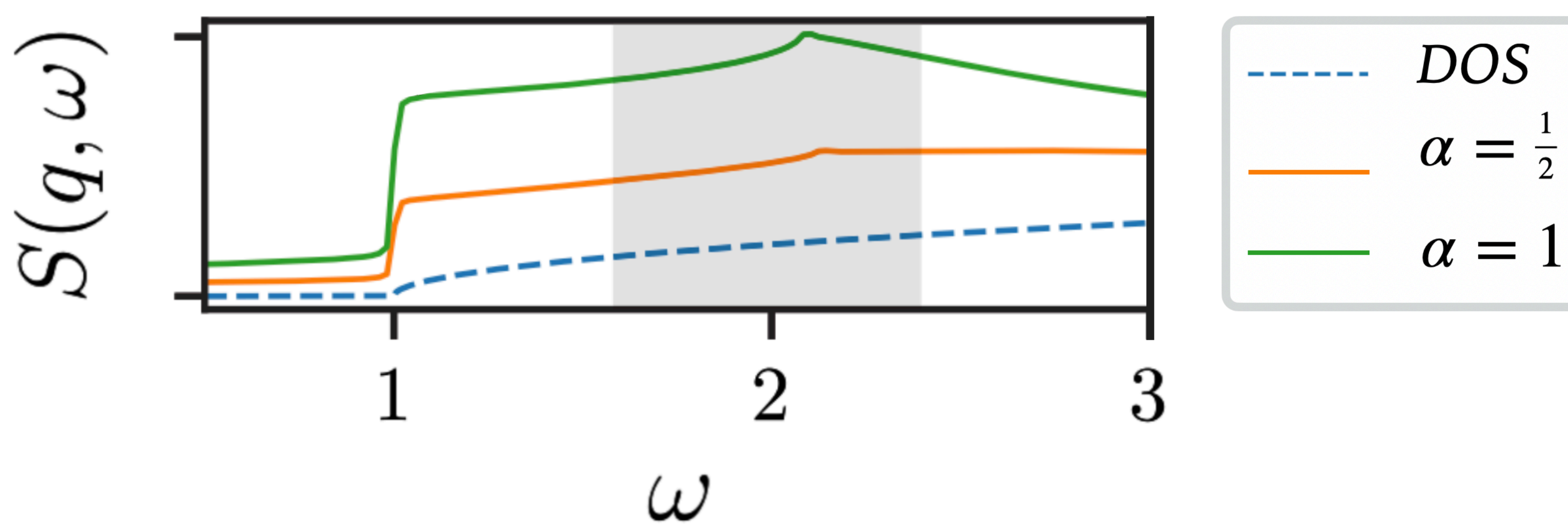
- α is order 0.1 (not 1/100, not 1) in any QSI material that realizes a deconfined QED phase

IMPLICATIONS OF α IN QSI

- α on a scale of 0.1 is a sweet spot



- Neutron scattering cross section enhancement



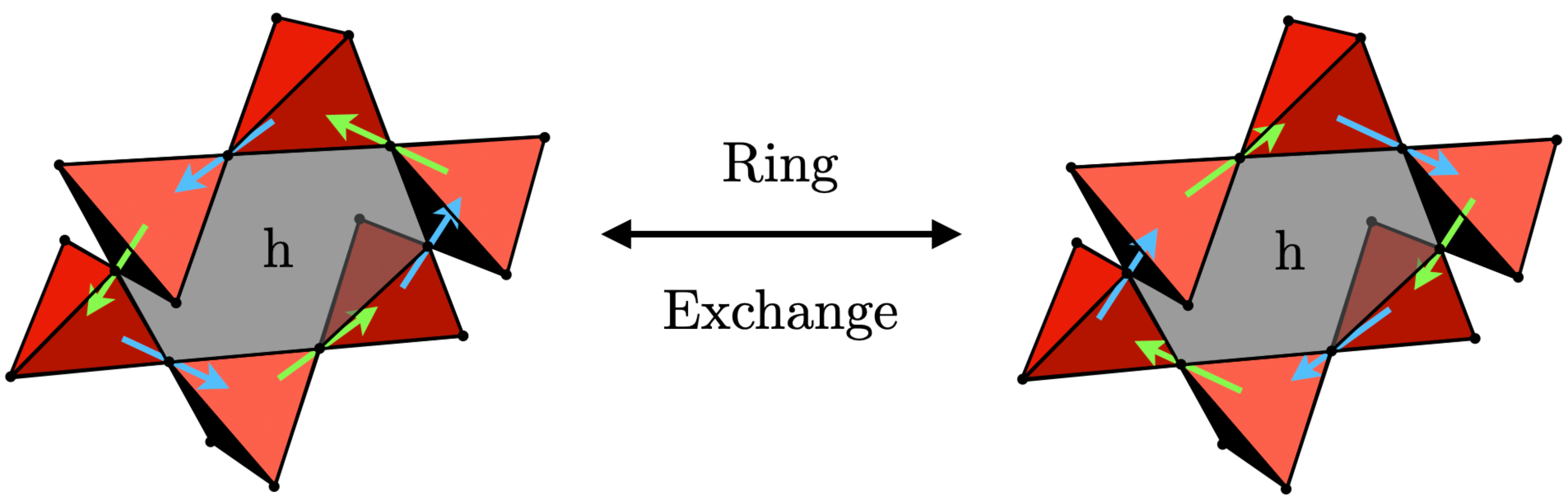
Morampudi, Siddhardh C., Frank Wilczek, and Chris R. Laumann. *Physical review letters* 124.9 (2020): 097204.

QSI MICROSCOPIC HAMILTONIAN

$$\hat{H}_{\mathrm{QSI}} = J_{zz} \sum_{\langle i,j \rangle} \hat{S}_i^z \hat{S}_j^z - g \sum_{\mathrm{h}} \left(\hat{W}_{\mathrm{h}} + \hat{W}_{\mathrm{h}}^{\dagger} \right)$$

Classical Spin Ice *Ring Exchange*

- $J_{zz} \gg g > 0$
- Define operator: $\hat{W}_{\mathrm{h}} = \hat{S}^{+}_{\mathrm{h},1} \hat{S}^{-}_{\mathrm{h},2} \hat{S}^{+}_{\mathrm{h},3} \hat{S}^{-}_{\mathrm{h},4} \hat{S}^{+}_{\mathrm{h},5} \hat{S}^{-}_{\mathrm{h},6}$



CHALLENGES OF MEASURING α

- Is it possible to get a theoretical estimate of α_{QSI}?
    - Difficult because of spin 1/2 constraint

Frustrates Electric Field
and truncates Hilbert space

- Numerically? Need independent measurement of c and e
    - Can measure c from linear photon dispersion
    - What about e?

Benton, Owen, Olga Sikora, and Nic Shannon. *Physical Review B* 86.7 (2012): 075154.

NUMERICALLY MEASURING e

- Measure from excited electric charges interacting?
  - Huge finite size effect because $V_{\text{coulomb}} \sim 1/r$

- Measure e without electric charges???

Gap out electric charges

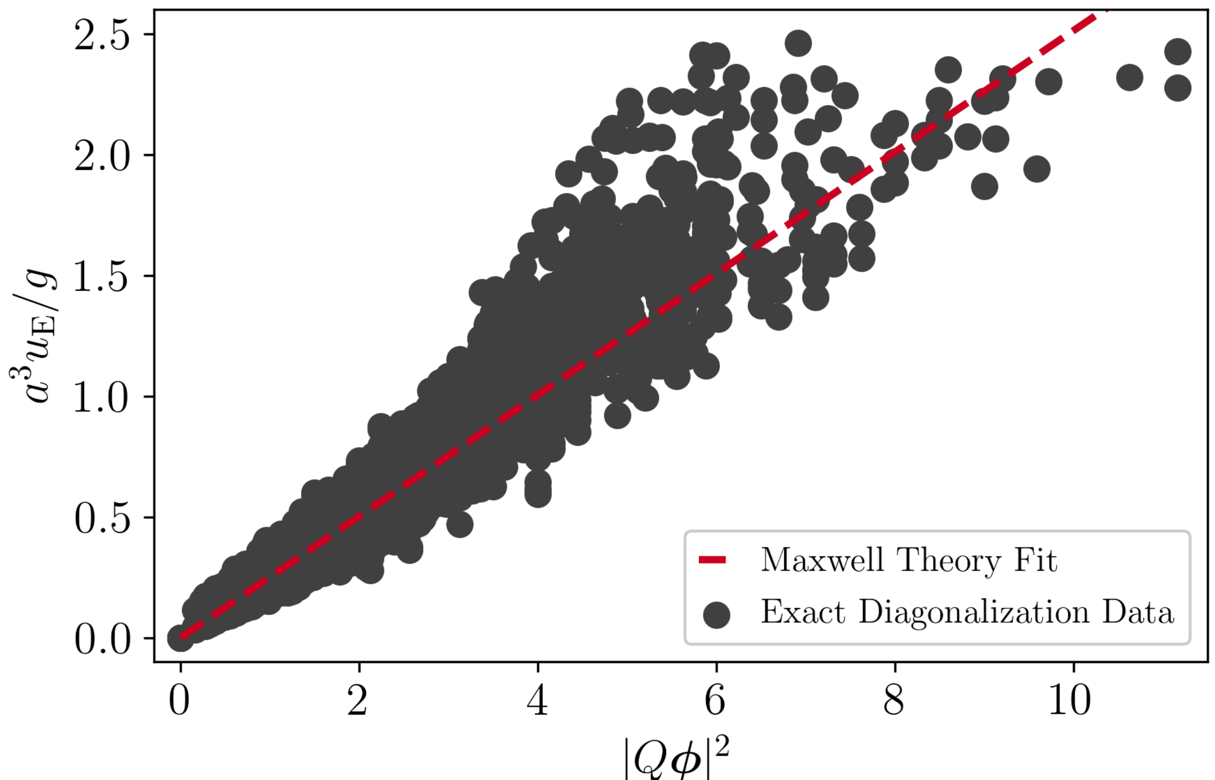
Use periodic boundary conditions

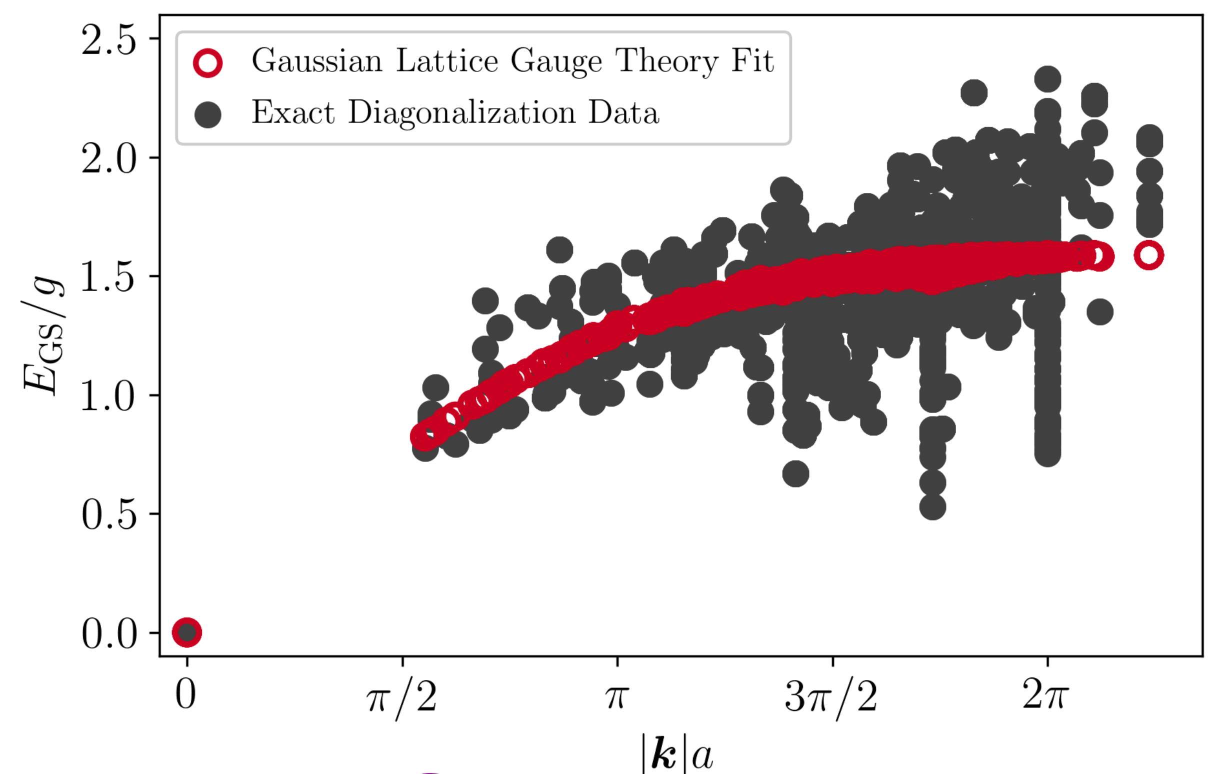
$\longrightarrow$

Conserved quantity corresponding to elementary units of electric flux threaded through the system

See preprint: arXiv:2009.04499

EMERGENT FINE STRUCTURE CONSTANT





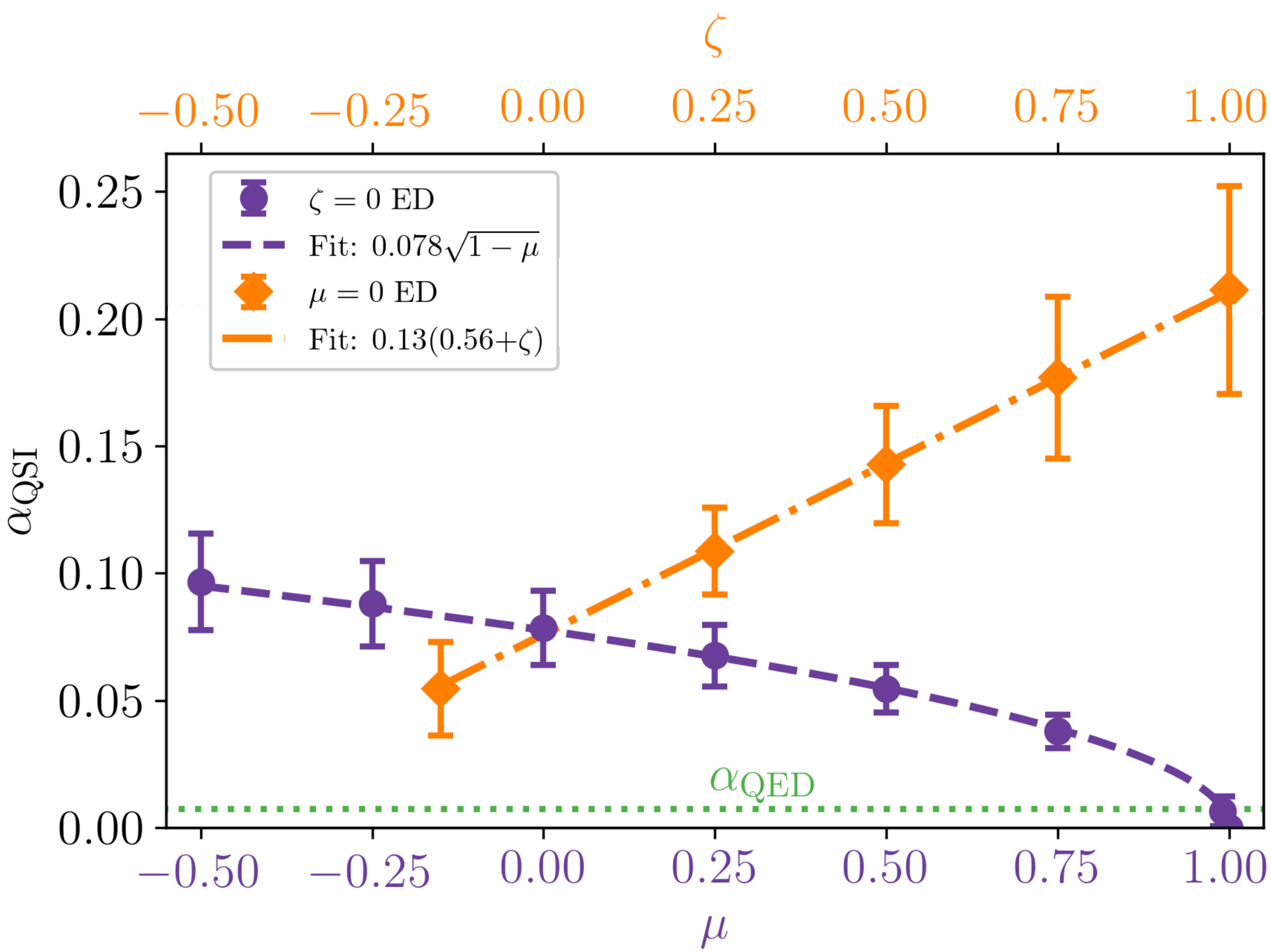
$$\frac{e_{\mathrm{QSI}}^2}{\epsilon_{\mathrm{QSI}}} = 0.5ag$$

$$c_{\mathrm{QSI}} = 0.51ag/\hbar$$

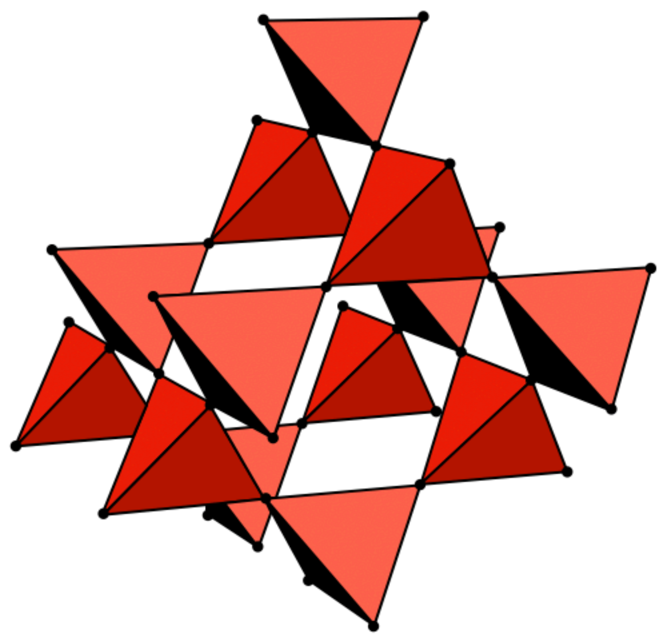
$$\alpha_{\mathrm{QSI}} = 0.08$$

PERTURBING THE MODEL

$$\hat{H}_{\mathrm{QSI}} = J_{zz} \sum_{\langle i,j \rangle} \hat{S}_i^z \hat{S}_j^z - g \sum_{\mathrm{h}} \left(\hat{W}_{\mathrm{h}} + \hat{W}_{\mathrm{h}}^{\dagger} \right) + \hat{H}_{\mathrm{p}}$$

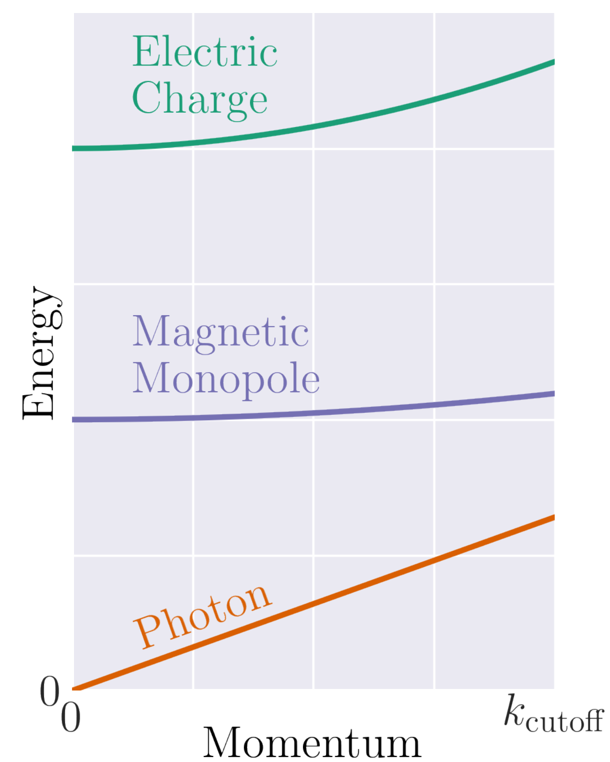


RECAP



Long Distance

Low Temperature



➤ $\alpha \sim 0.1$

Thank you for your attention!

Questions?

arXiv:2009.04499

QSI BALLPARK NUMBERS

- Lattice spacing: $a \sim 10$ Å
- Ring exchange energy: $g \sim 10$ μeV

Parameters	QSI	QED
c	10 m/s	3×10^8 m/s
e^2/ϵ	10^{-33} J m	2.9×10^{-27} J m
α	$1/10$	$1/137$

Gardner, Jason S., Michel JP Gingras, and John E. Greedan. *Reviews of Modern Physics* 82.1 (2010): 53.
Savary, Lucile, and Leon Balents. *Physical review letters* 108.3 (2012): 037202.

GAUSSIAN PHOTON DISPERSION

- Start from Gaussian Theory: $H = \frac{U}{2}\sum_{\langle \mathbf{rr}' \rangle} E_{\mathbf{rr}'}^2 + \frac{K}{2}\sum_h \left(\nabla \times A_{\mathbf{rr}'}\right)$
- Write:

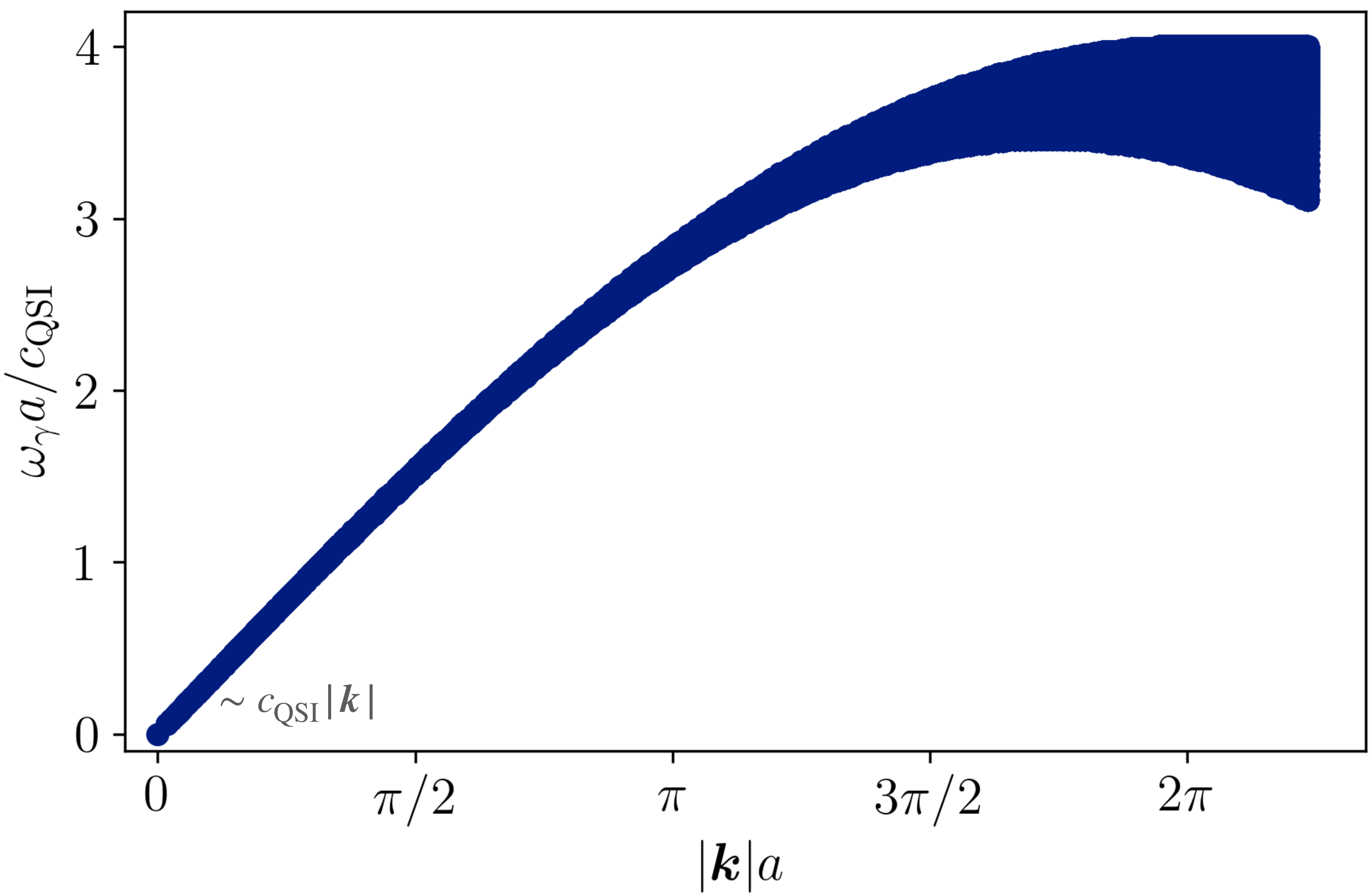
$$A_{(r,n)} = \frac{1}{\sqrt{N}}\sum_{\boldsymbol{k},\alpha}\sqrt{\frac{U}{2\omega_\alpha(\boldsymbol{k})}}\left[e^{-i\boldsymbol{k}\cdot\left(\boldsymbol{r}+\boldsymbol{e}_n/2\right)}\xi_{n\alpha}(\boldsymbol{k})a_\alpha(\boldsymbol{k}) + e^{i\boldsymbol{k}\cdot\left(\boldsymbol{r}+\boldsymbol{e}_n/2\right)}\xi^*_{\alpha n}(\boldsymbol{k})a^\dagger_\alpha(\boldsymbol{k})\right]$$

$$E_{(\boldsymbol{r},n)} = i\frac{1}{\sqrt{N}}\sum_{\boldsymbol{k},\alpha}\sqrt{\frac{\omega_\alpha(\boldsymbol{k})}{2U}}\left[e^{-i\boldsymbol{k}\cdot\left(\boldsymbol{r}+e_n/2\right)}\xi_{n\alpha}(\boldsymbol{k})a_\alpha(\boldsymbol{k}) - e^{i\boldsymbol{k}\cdot\left(\boldsymbol{r}+e_n/2\right)}\xi^*_{\alpha n}(\boldsymbol{k})a^\dagger_\alpha(\boldsymbol{k})\right]$$

  - ξ is spin-1 polarization tensor
  - $a_\alpha(k)$ destroys photon of momentum k
  - ω is the photon dispersion

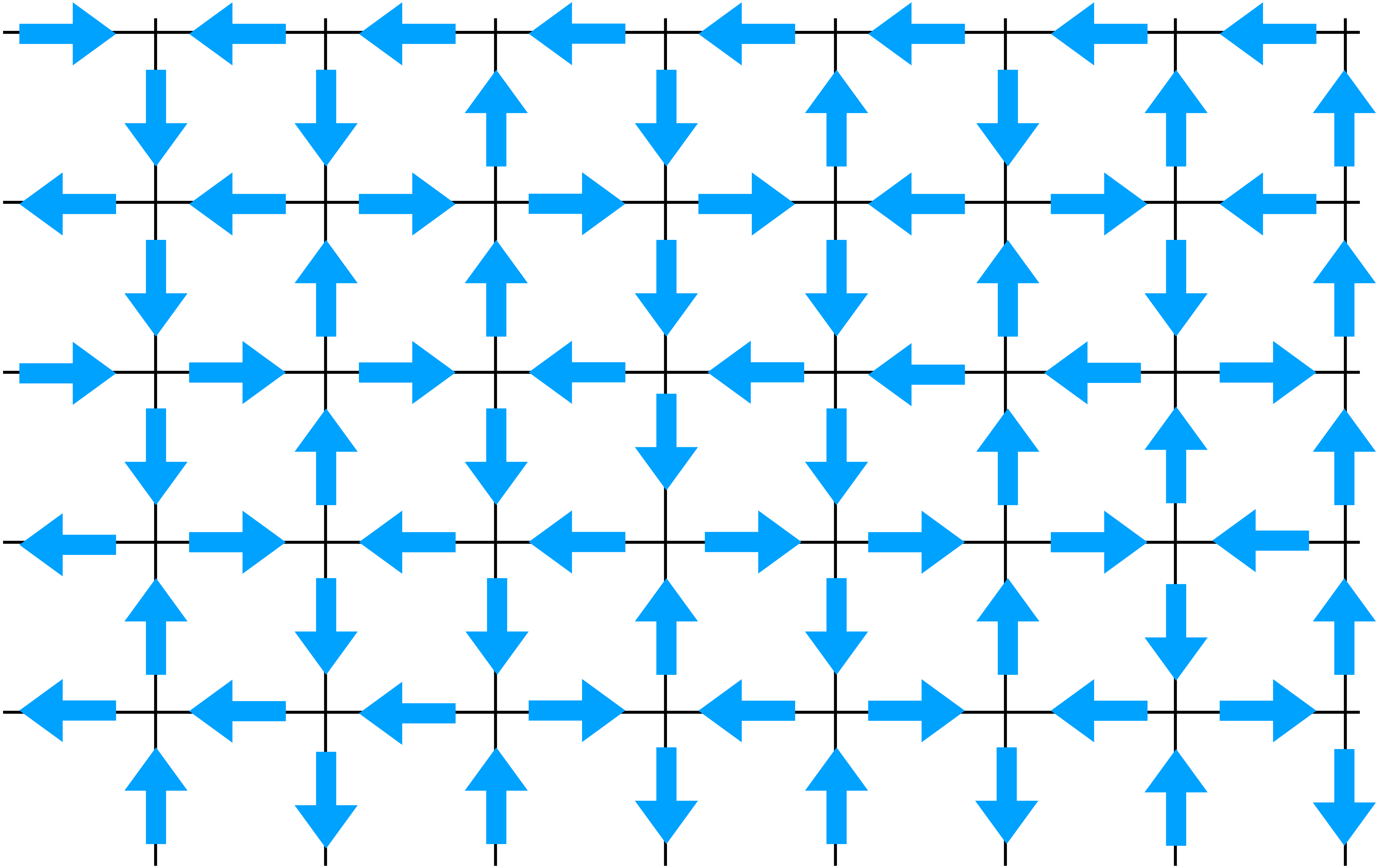
QSI GAUSSIAN PHOTON DISPERSION

$$\omega(k) = \frac{2c_{\mathrm{QSI}}}{a}\sqrt{3 - \cos\left(\frac{k_1 a}{2}\right)\cos\left(\frac{k_2 a}{2}\right) - \cos\left(\frac{k_1 a}{2}\right)\cos\left(\frac{k_3 a}{2}\right) - \cos\left(\frac{k_2 a}{2}\right)\cos\left(\frac{k_3 a}{2}\right)}$$



Benton, Owen, Olga Sikora, and Nic Shannon. *Physical Review B* 86.7 (2012): 075154.

ELECTRIC FLUX SECTORS



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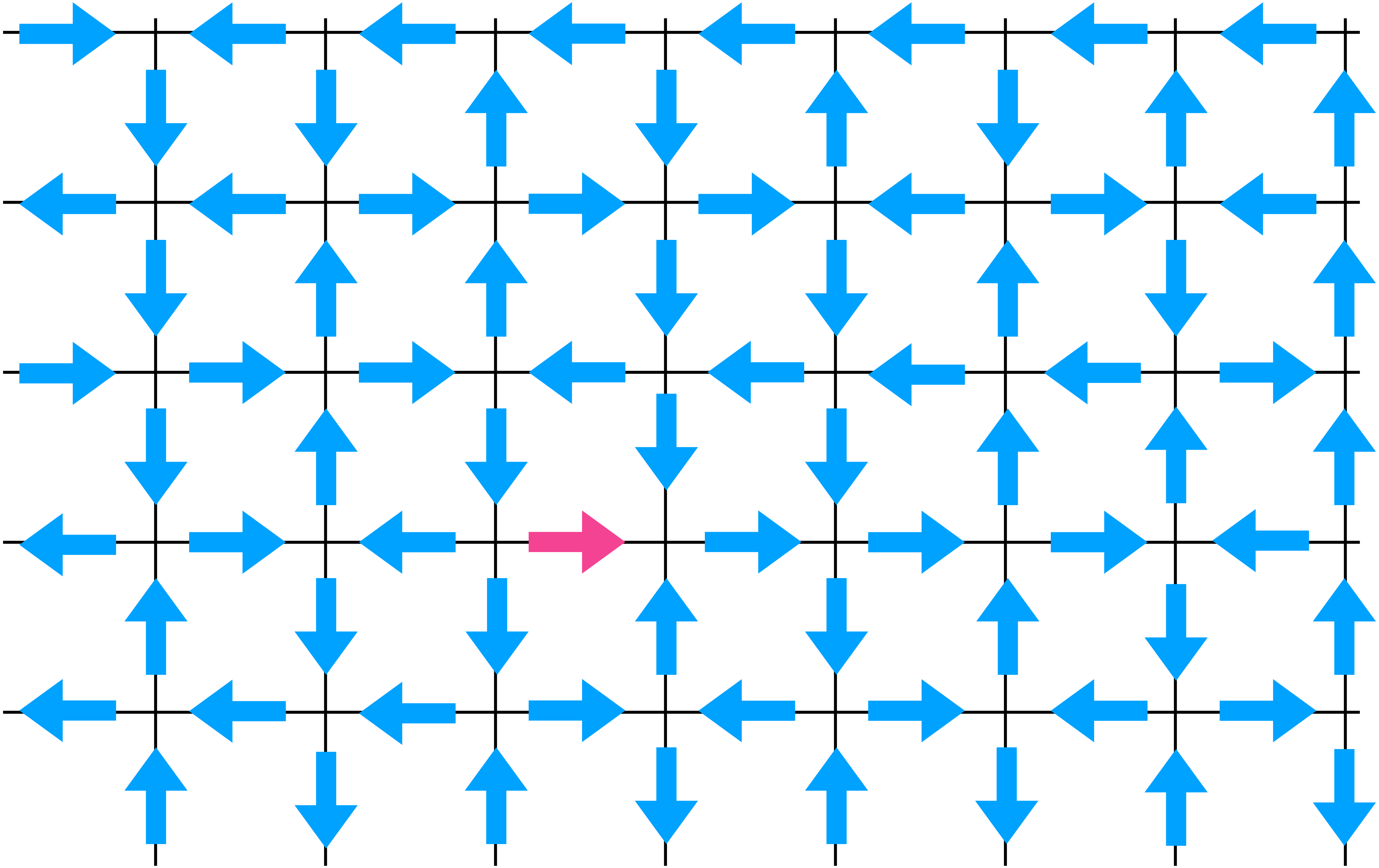

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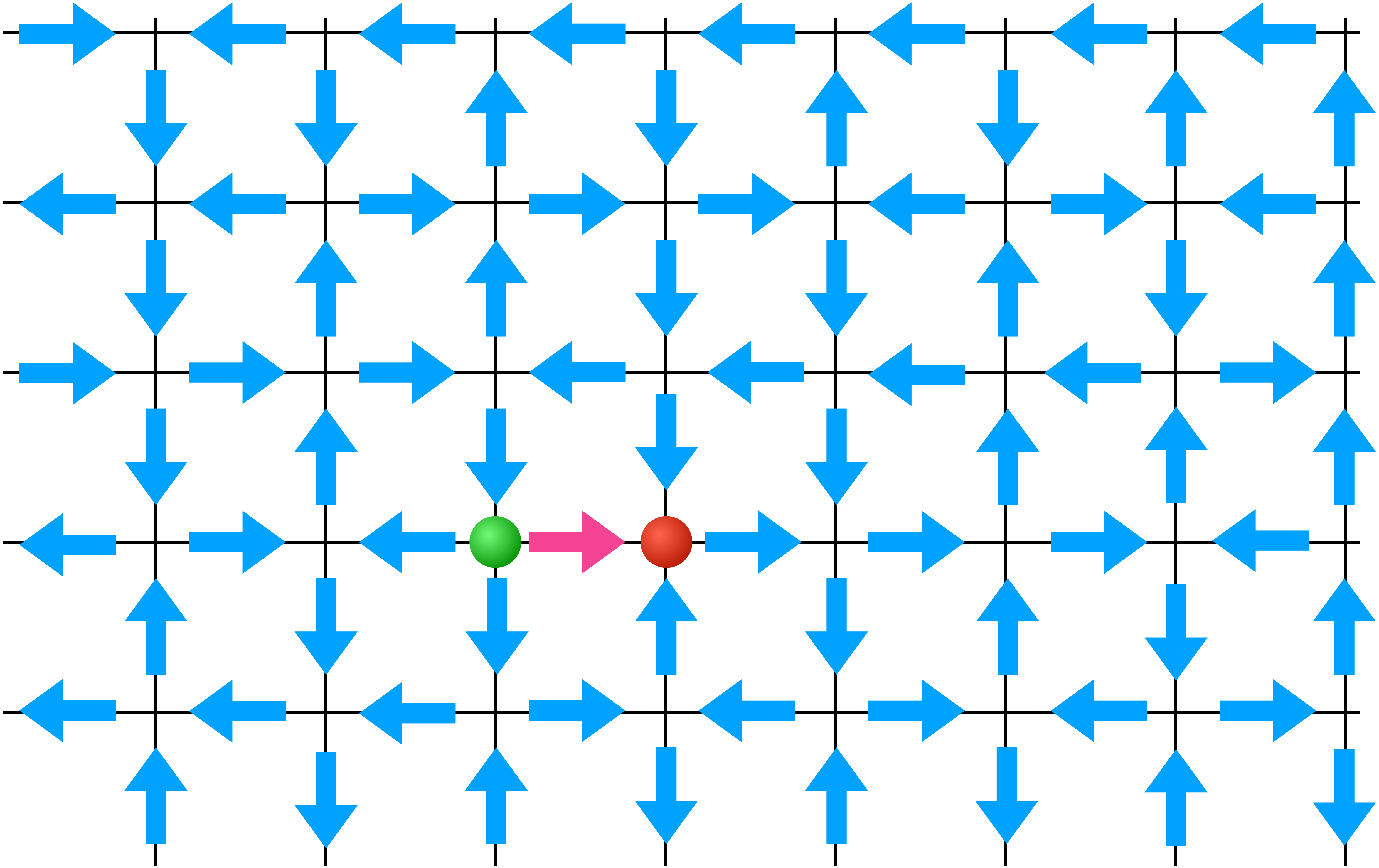

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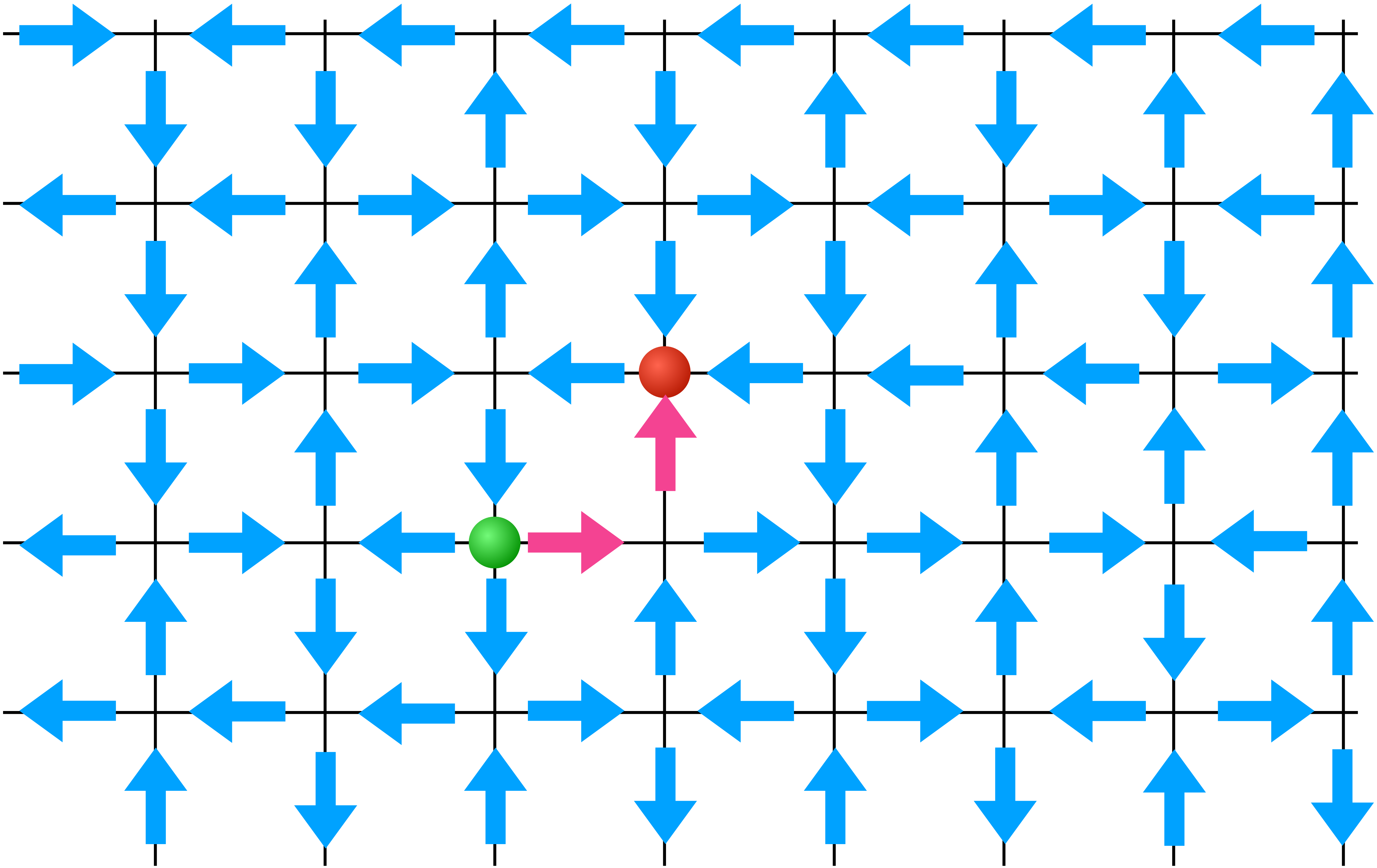

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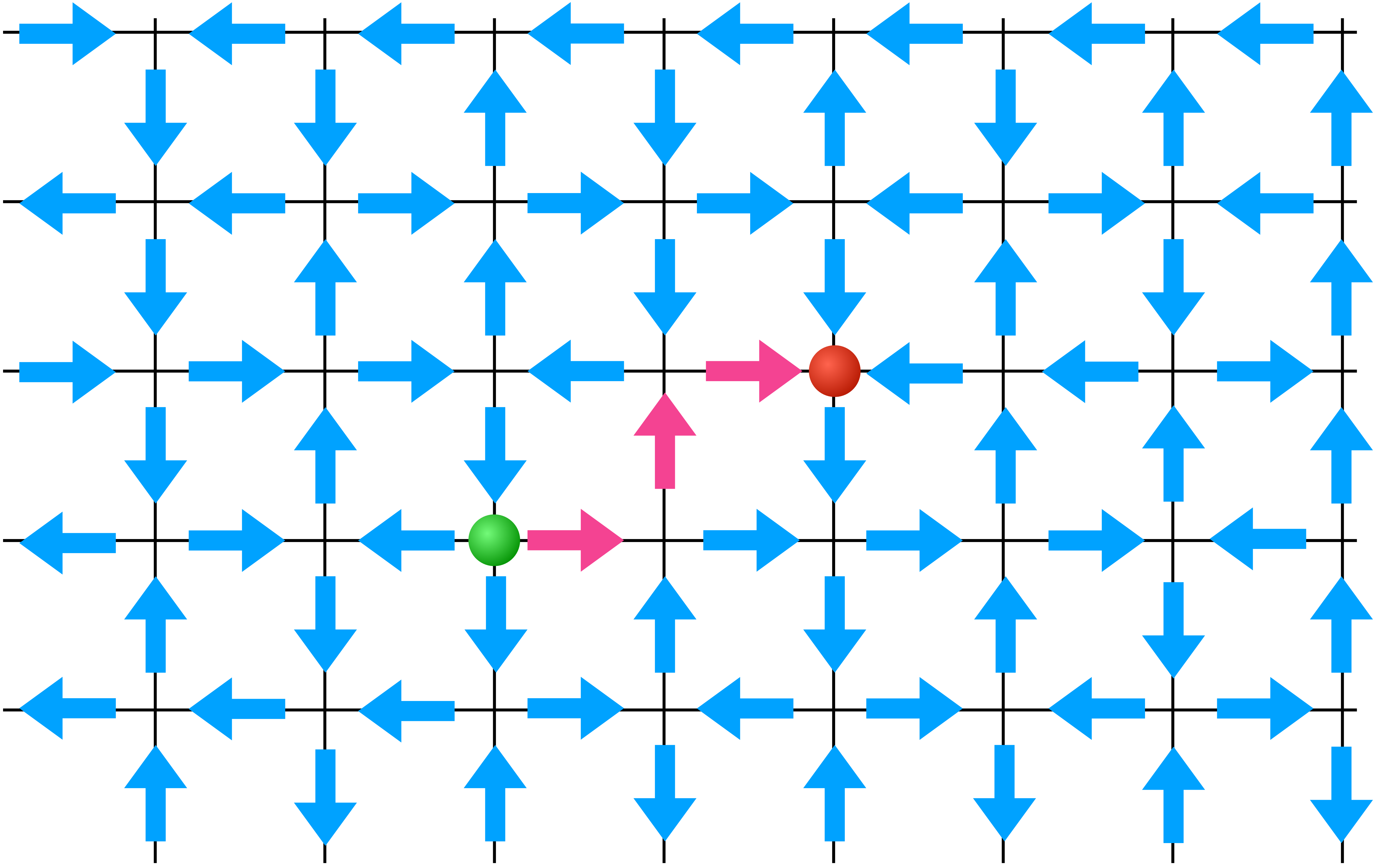

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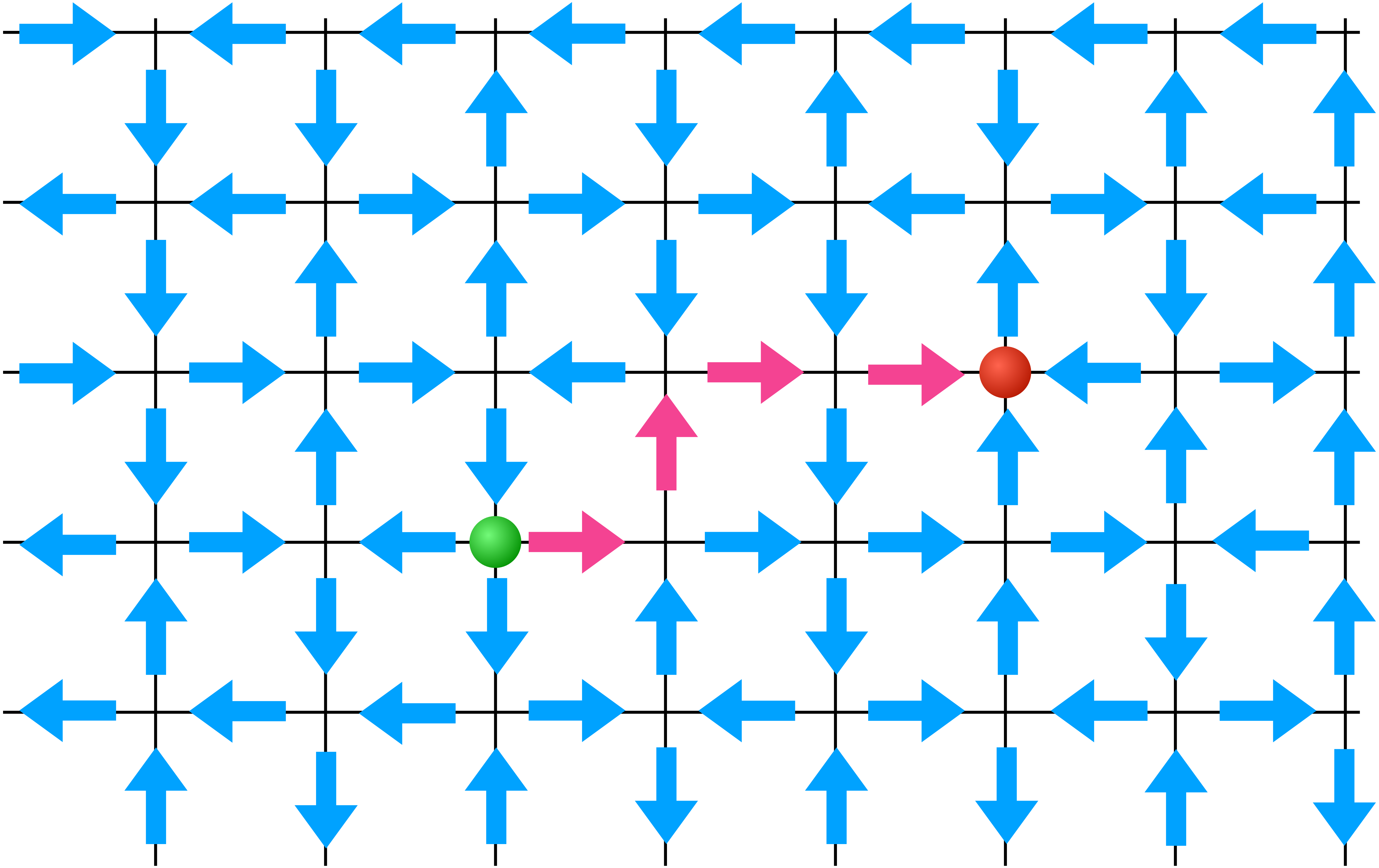

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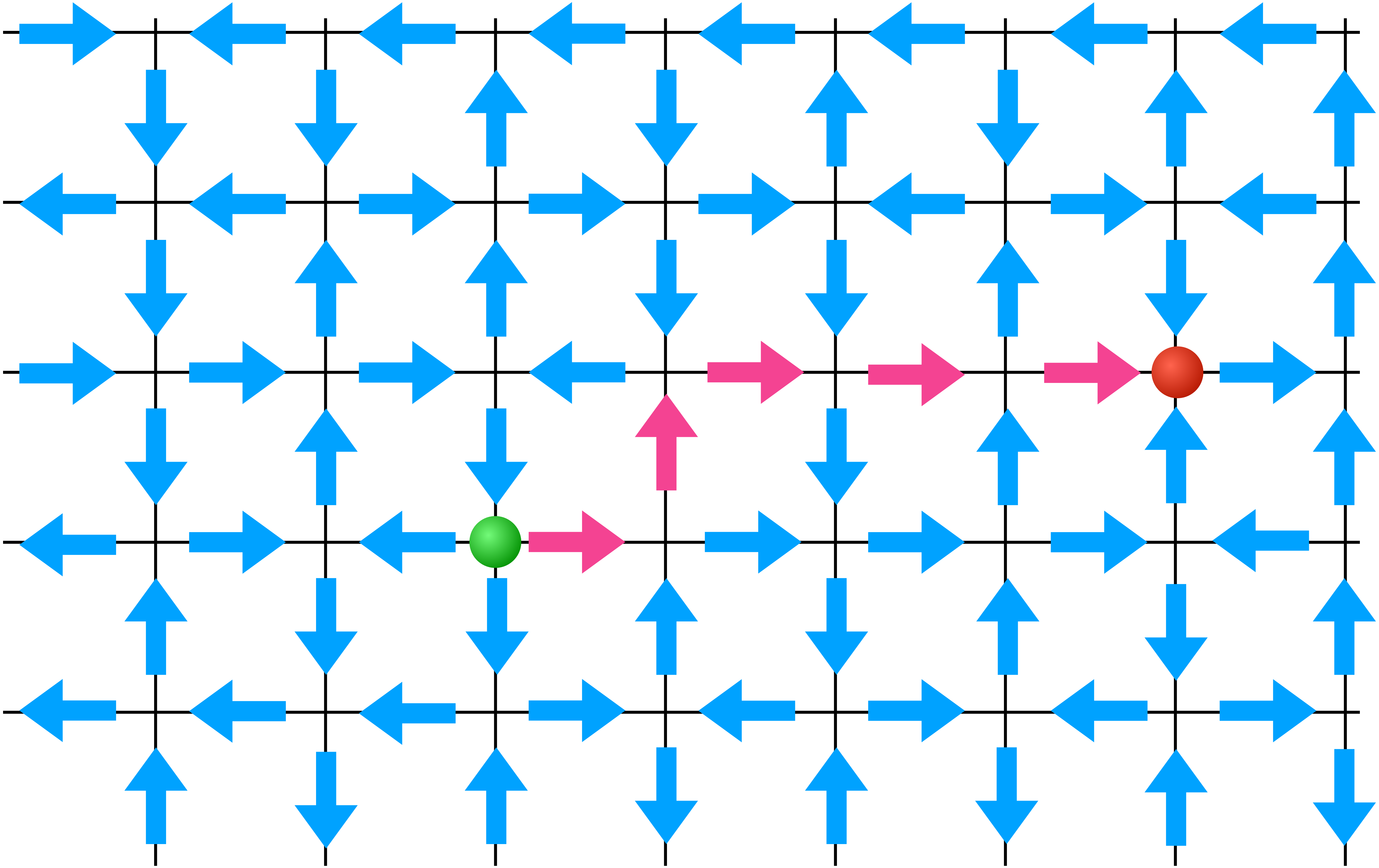

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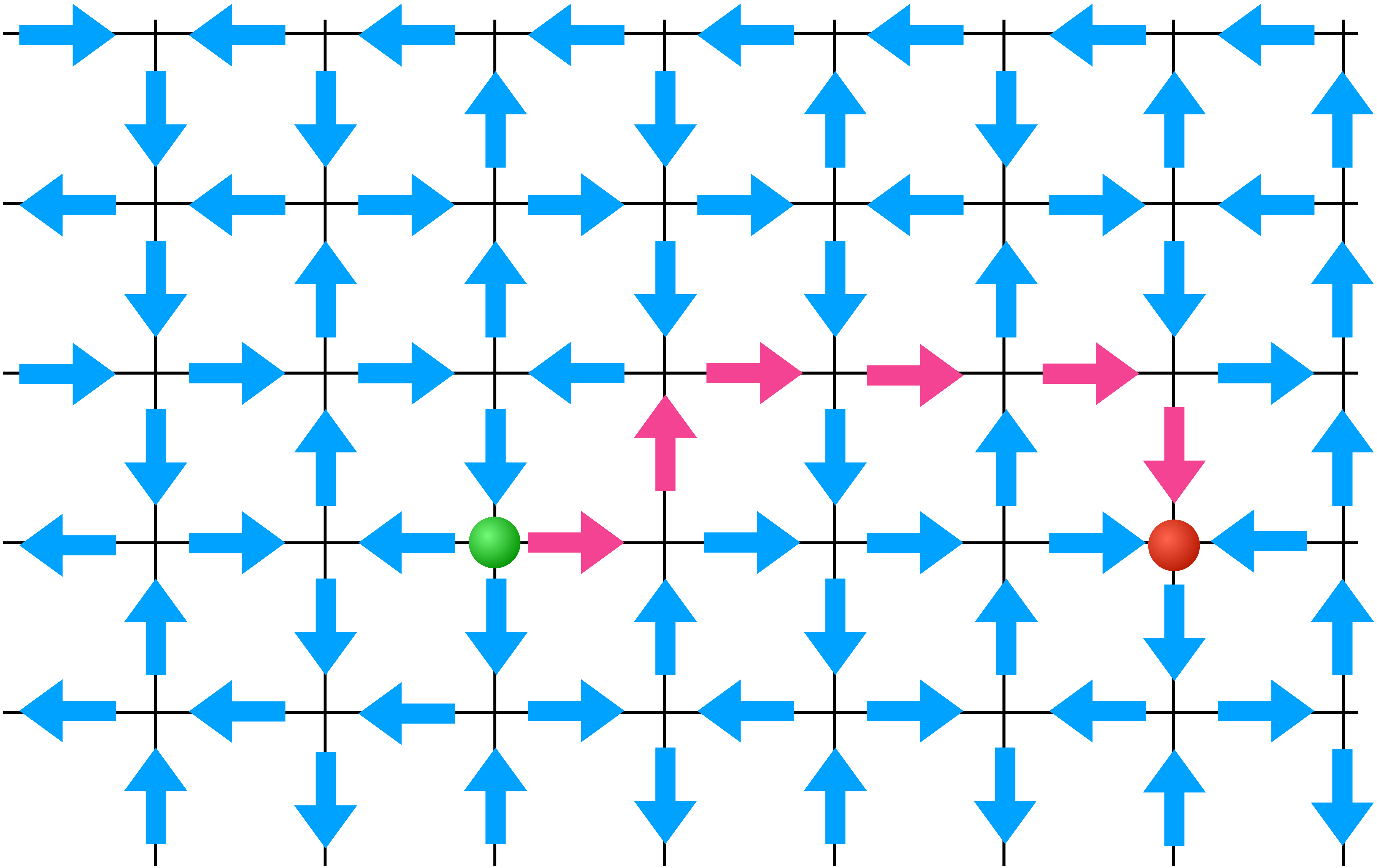

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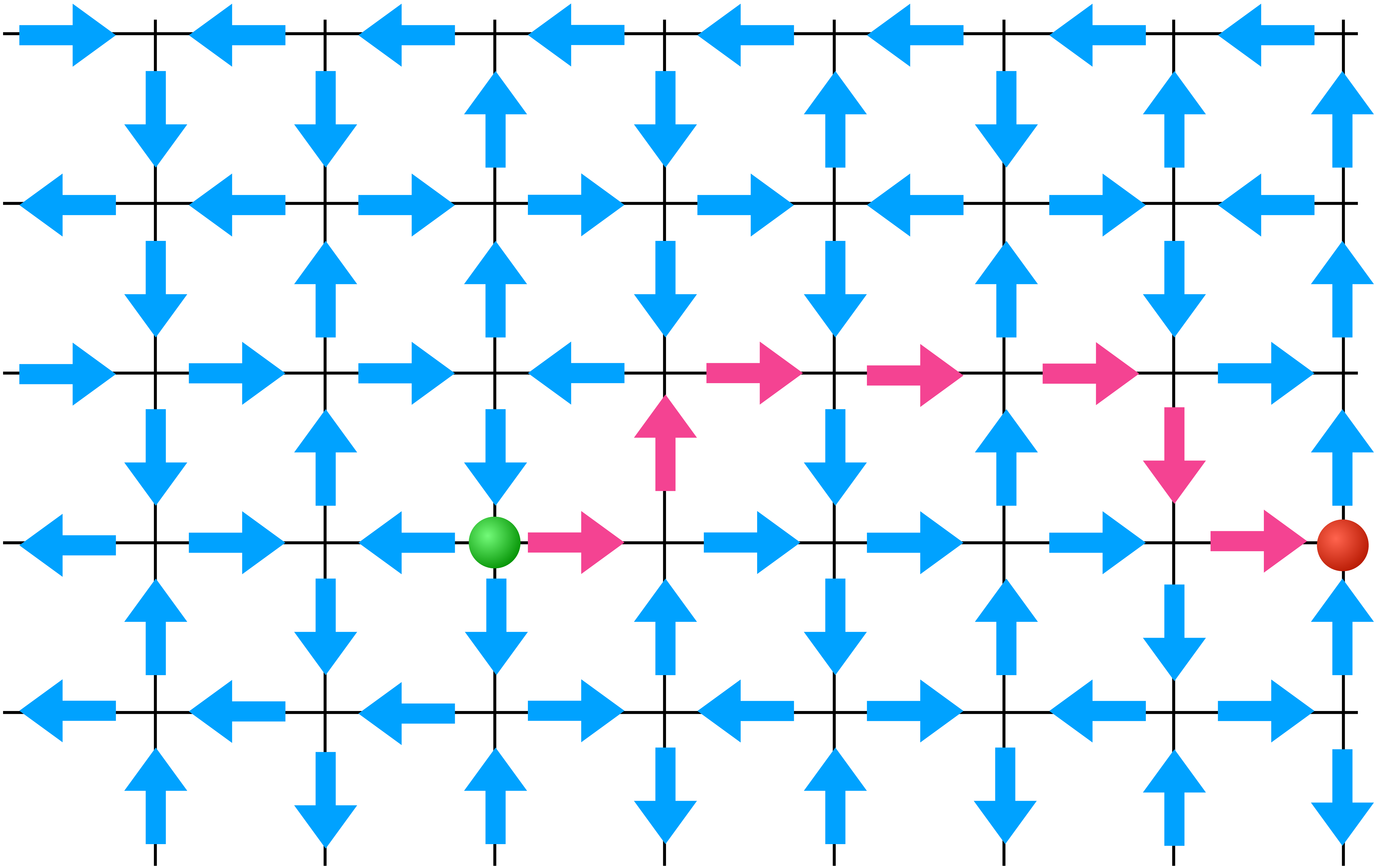

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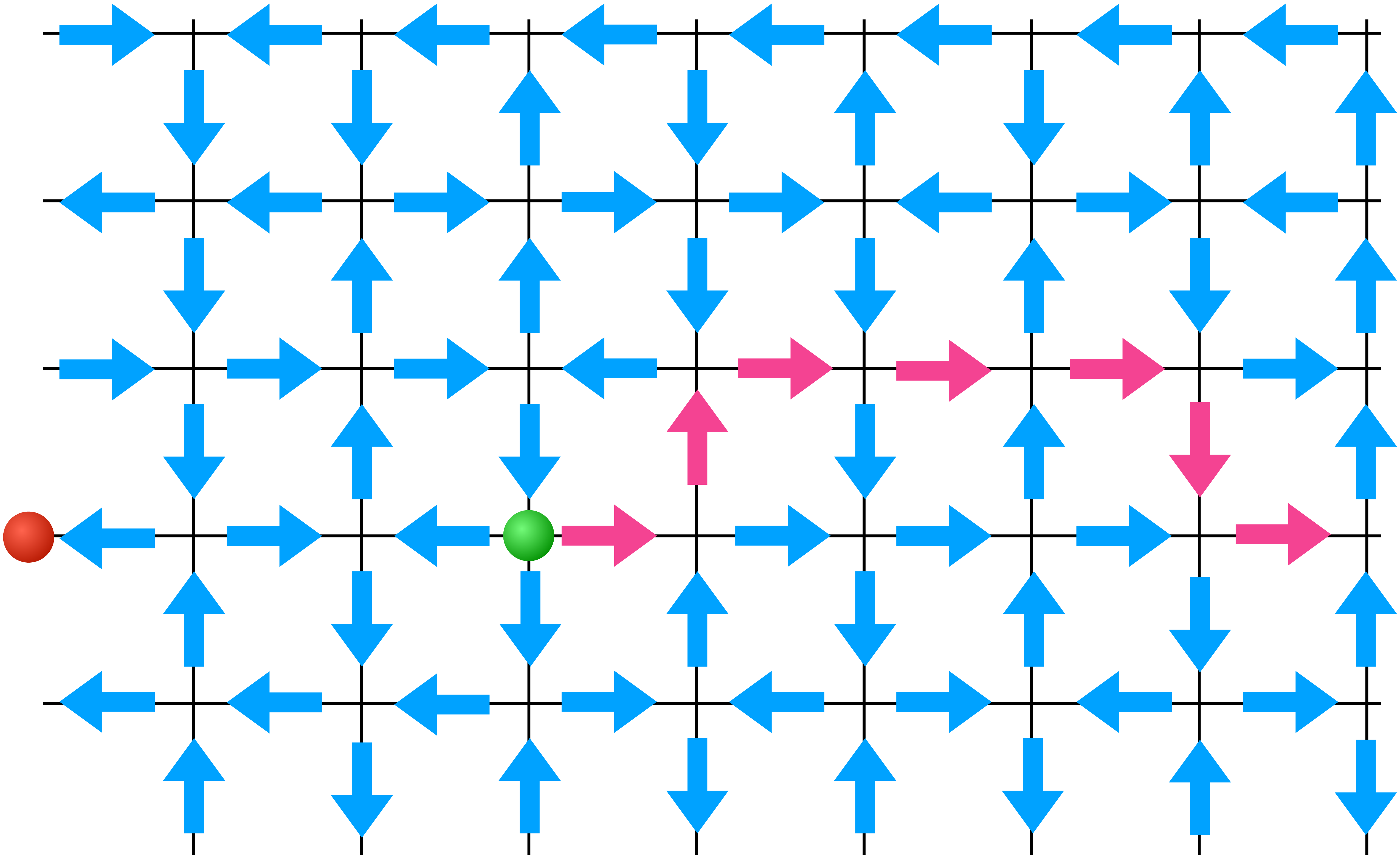

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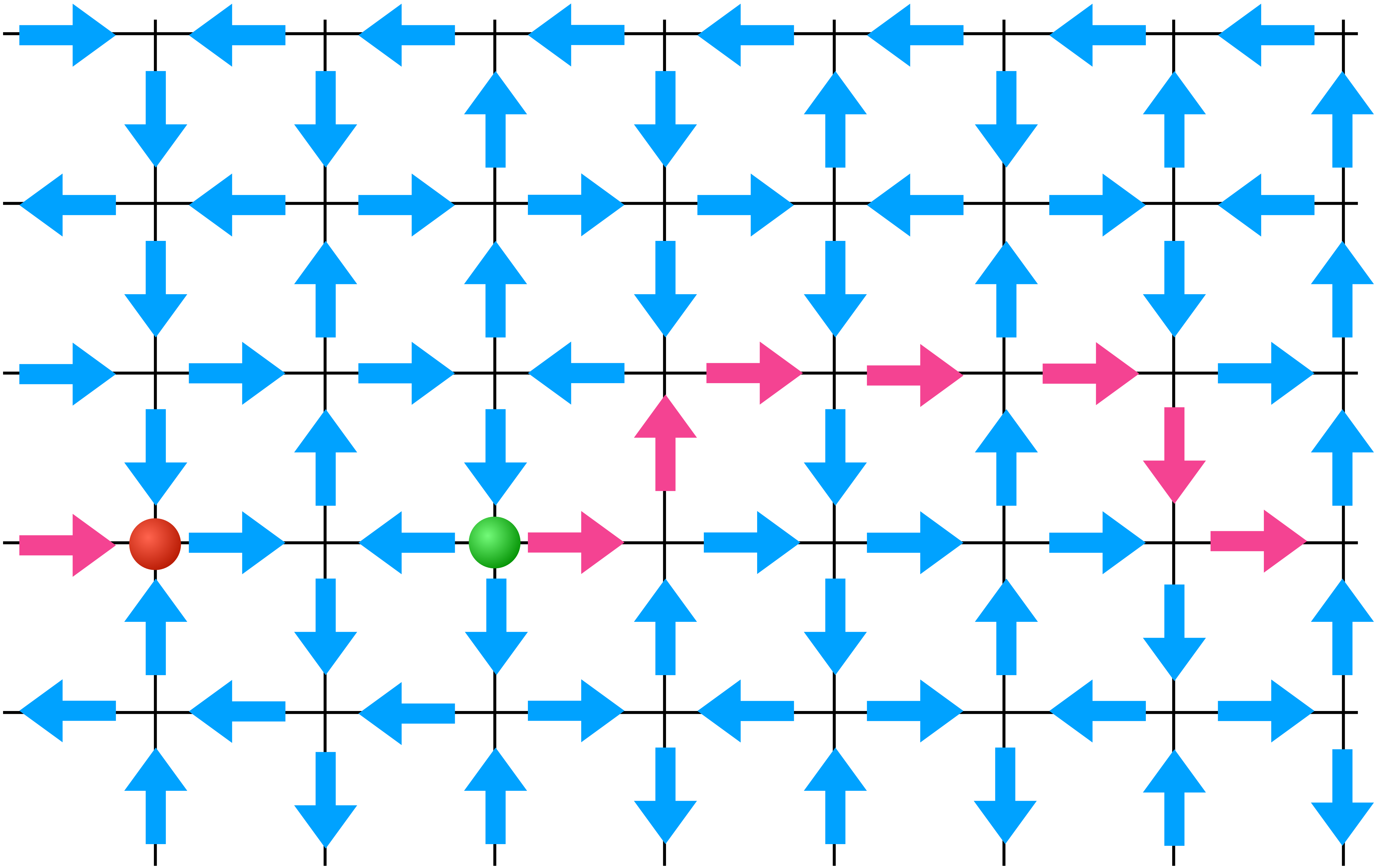

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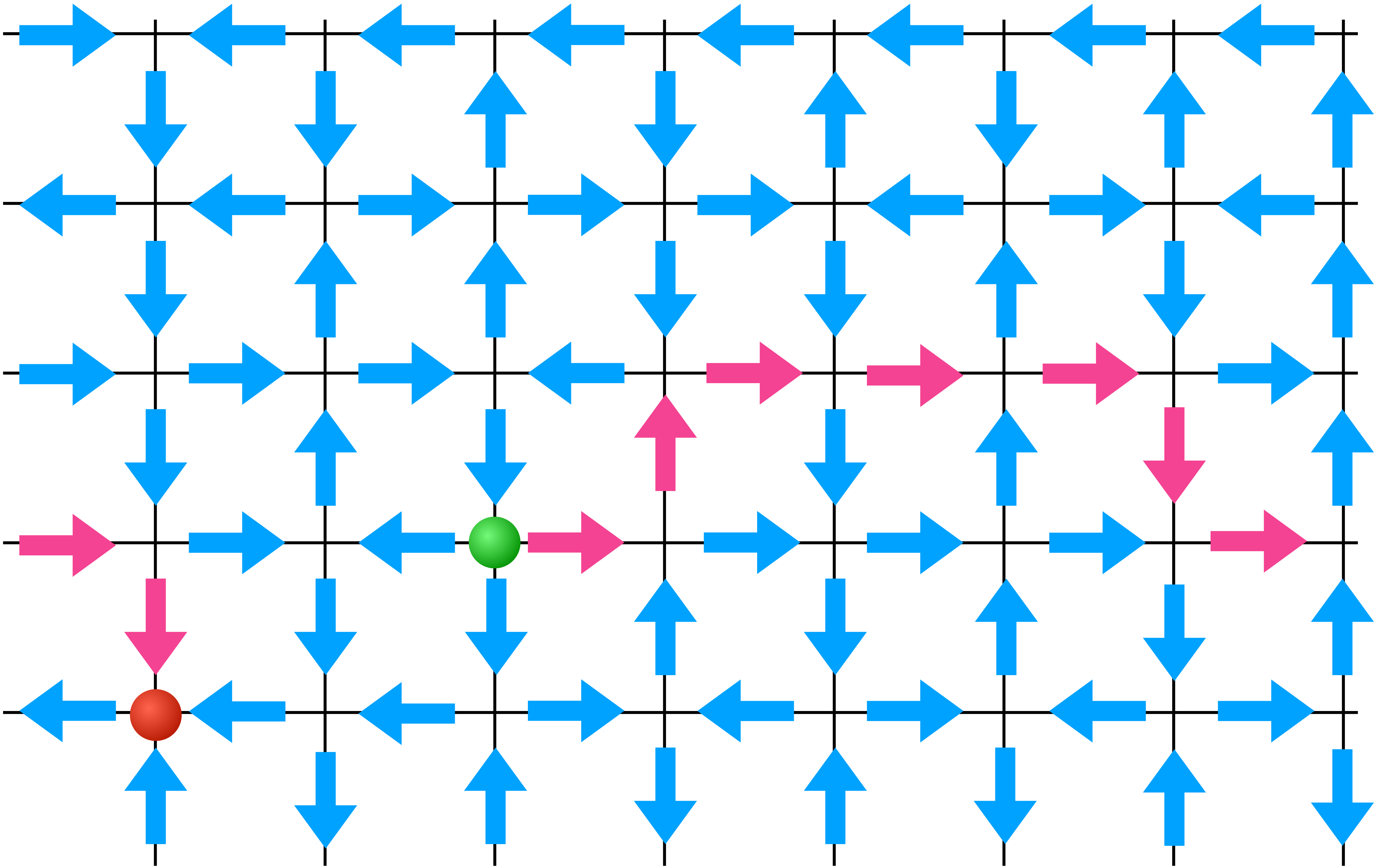

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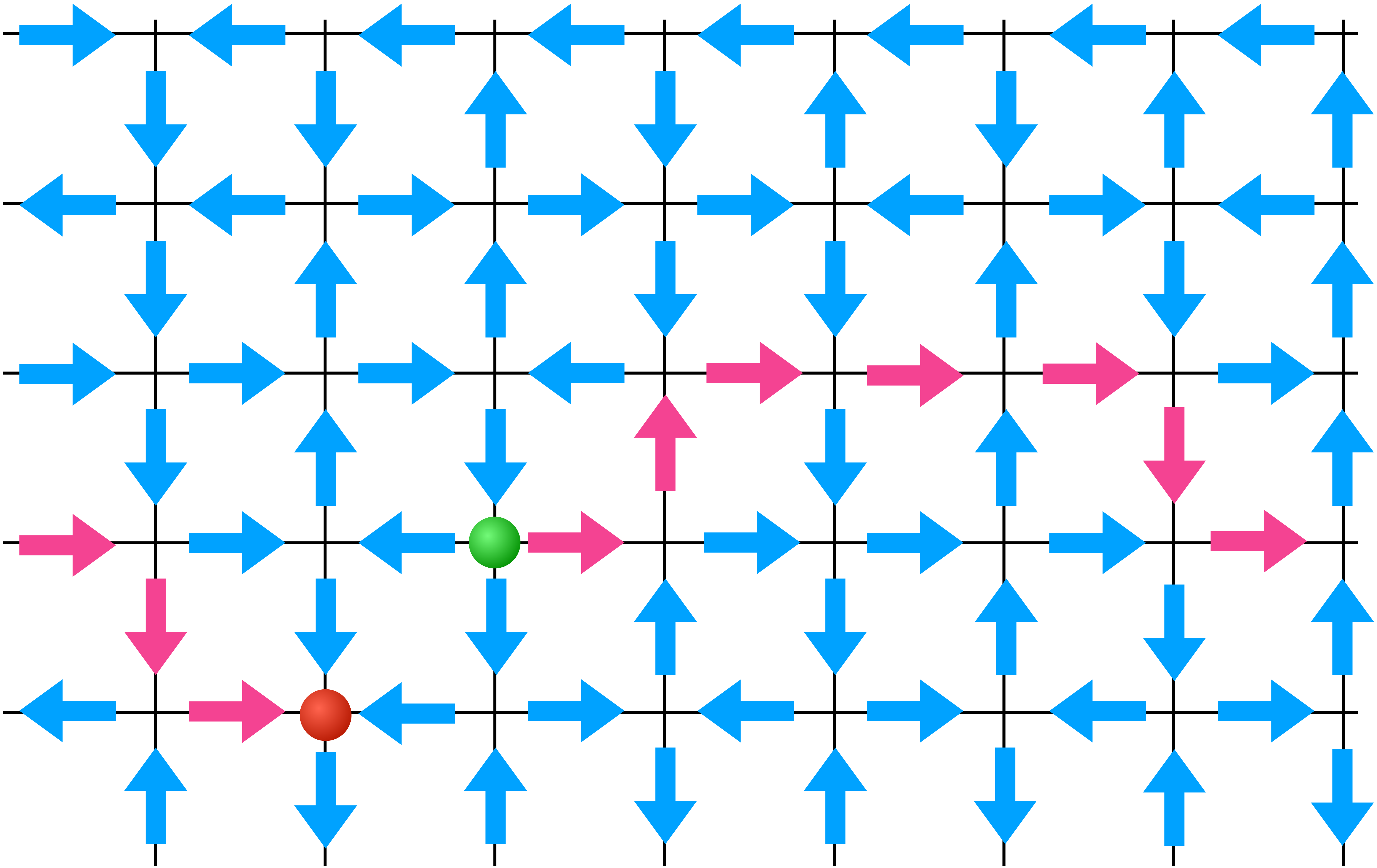

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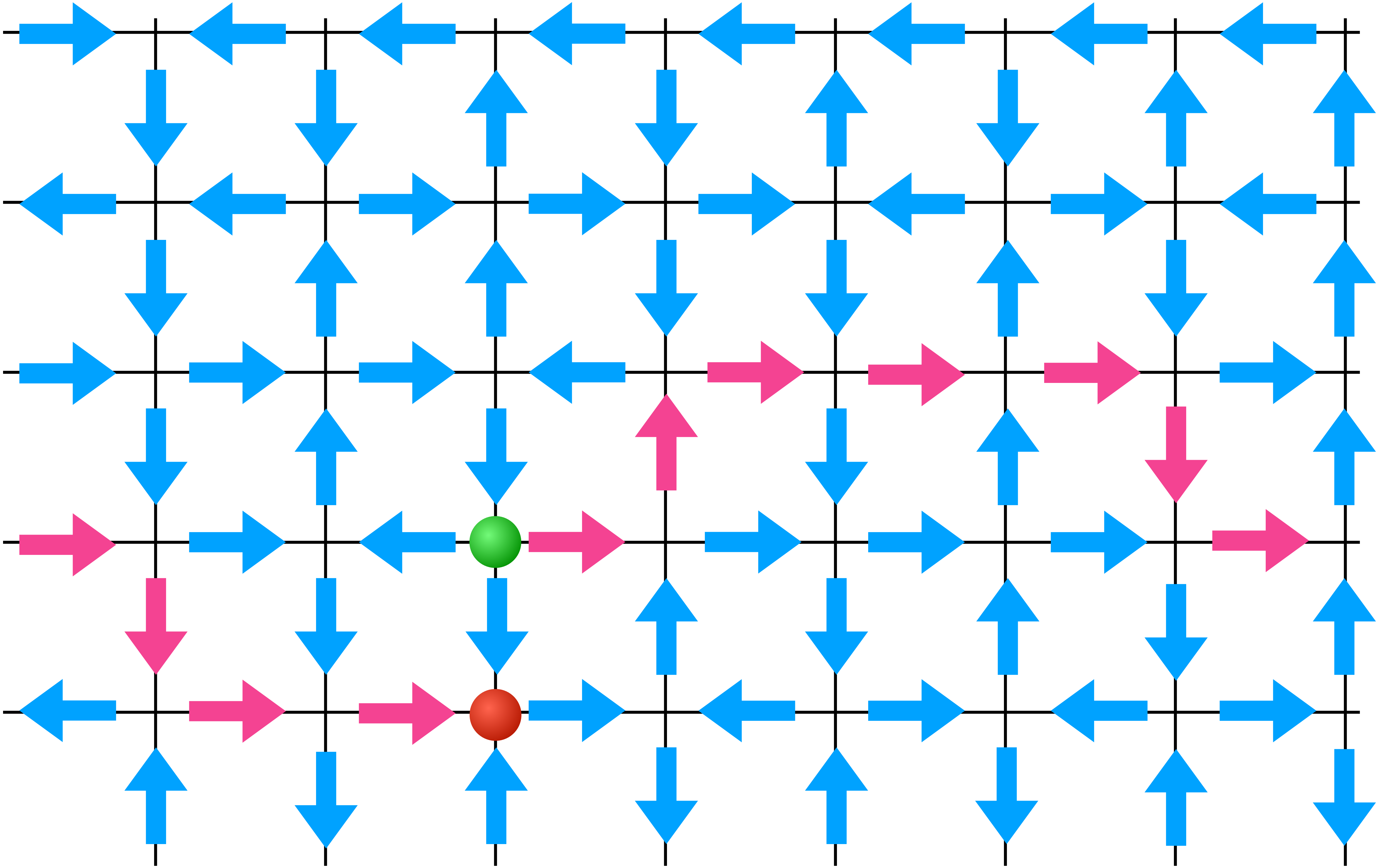

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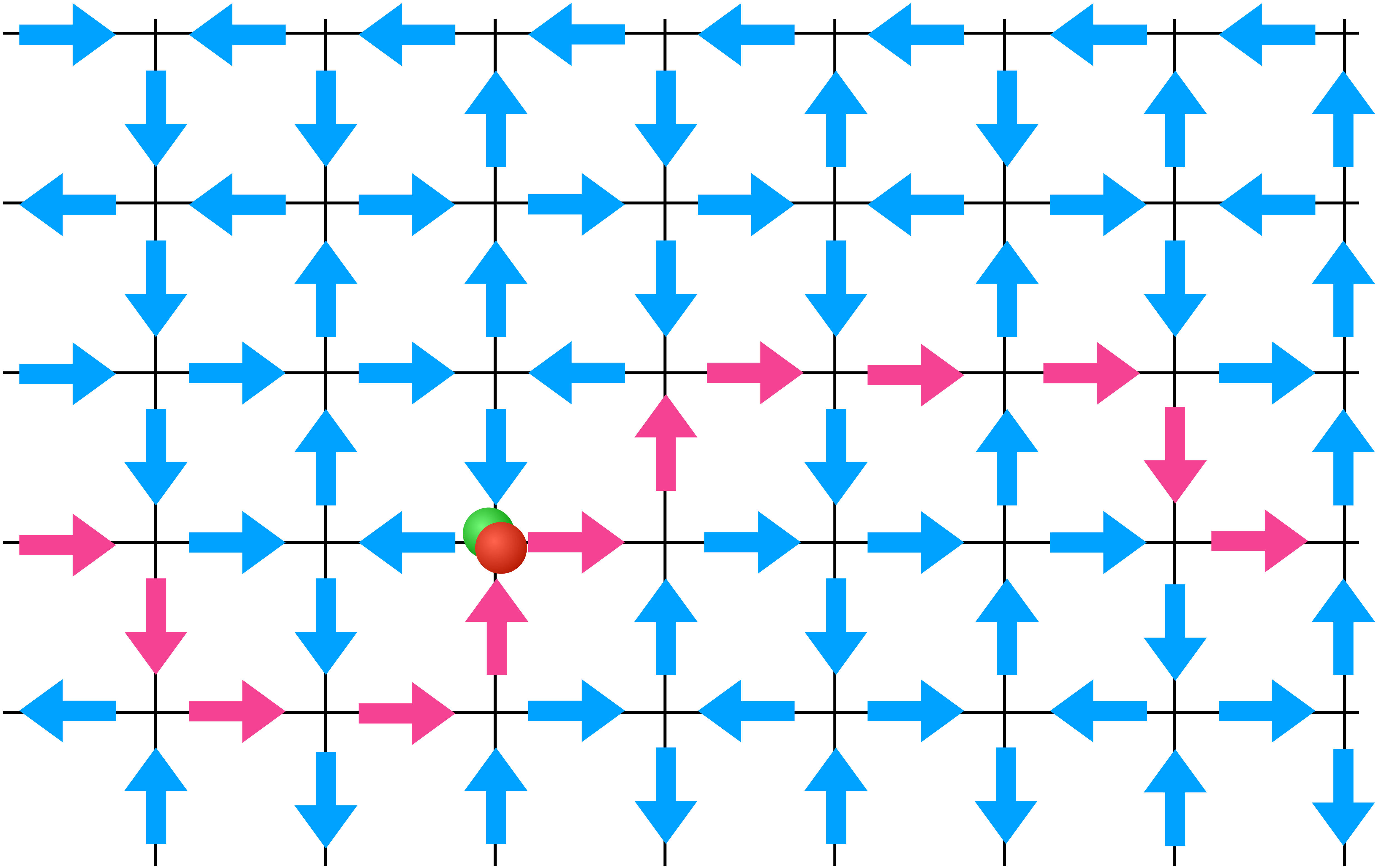

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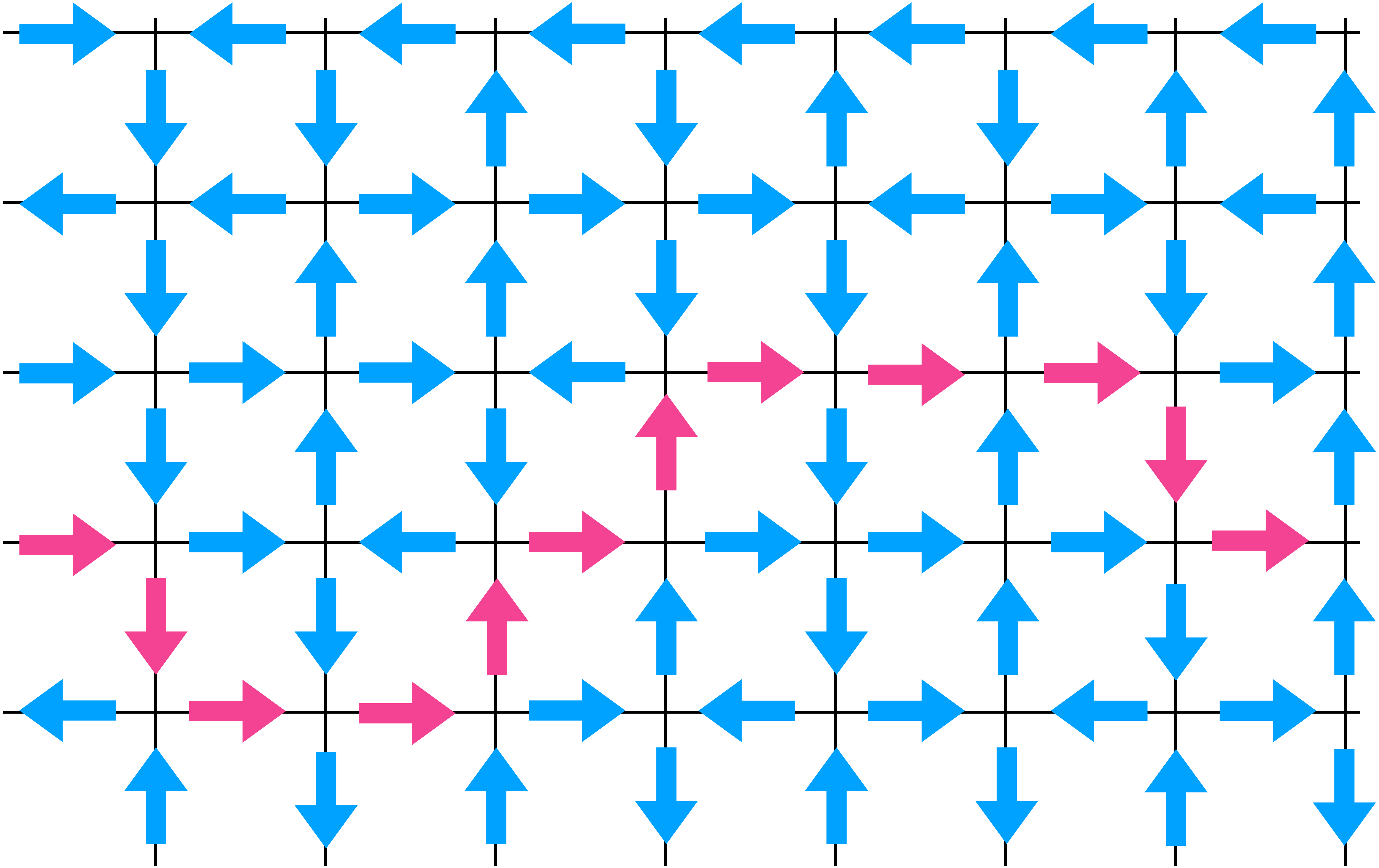

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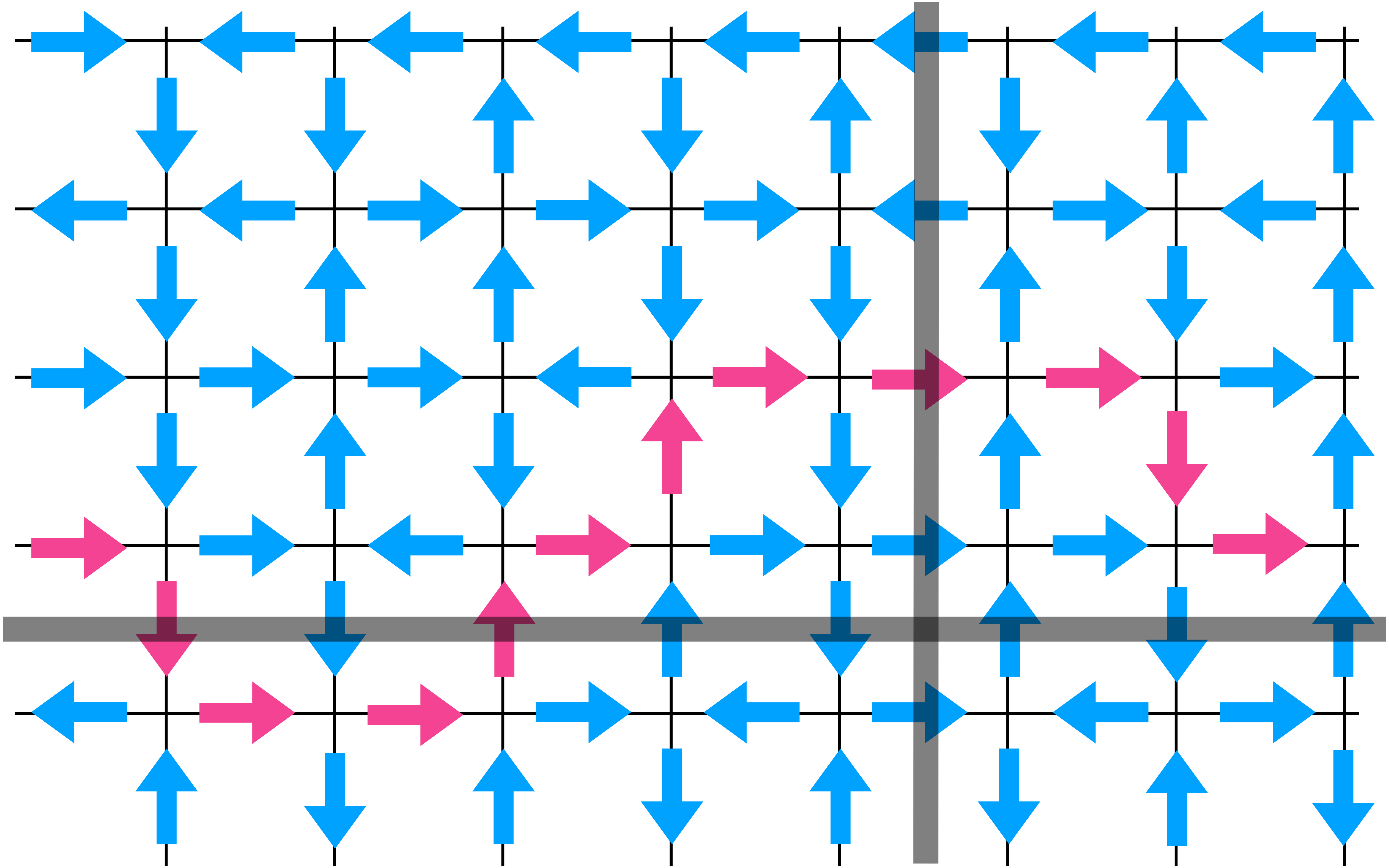

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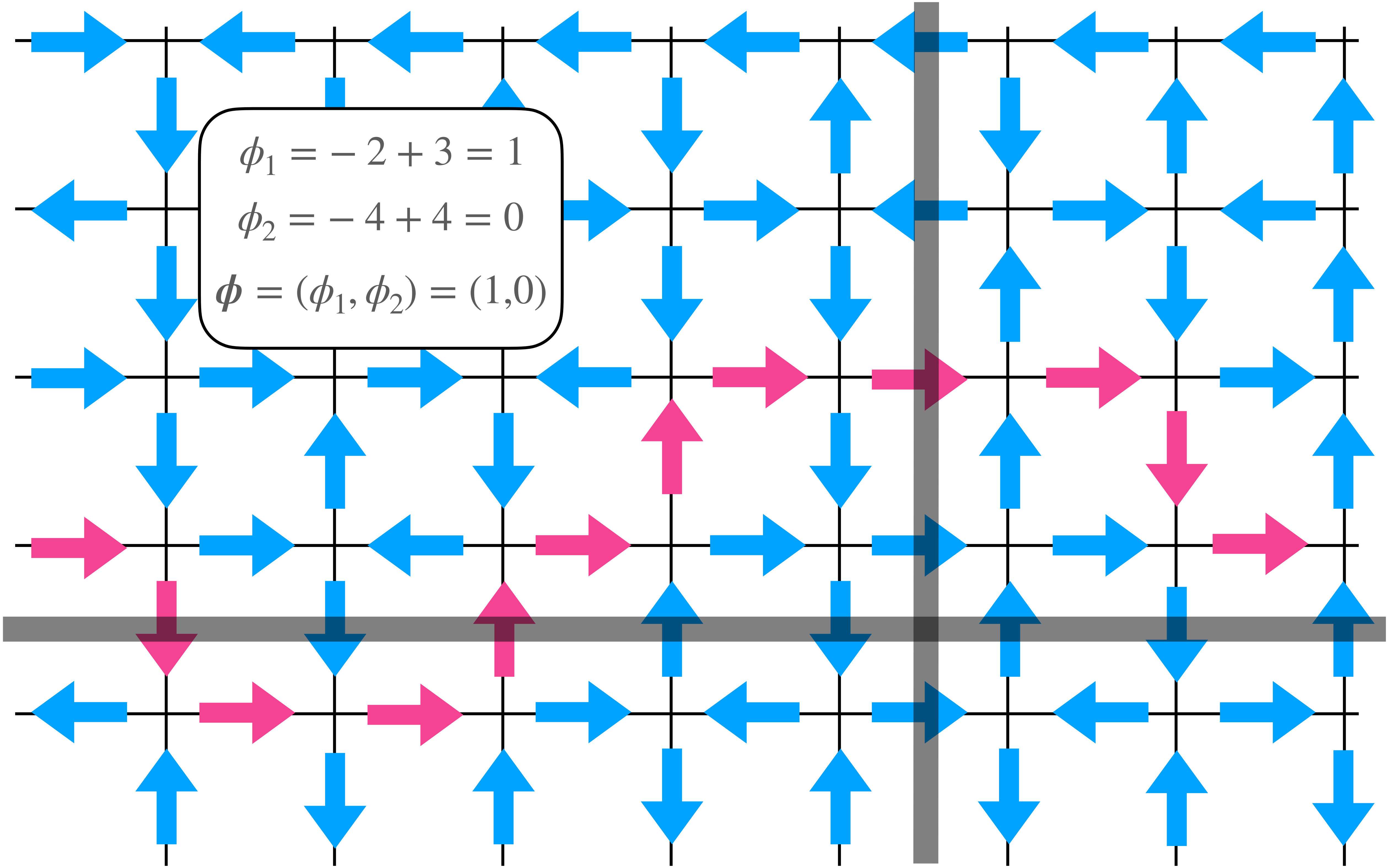
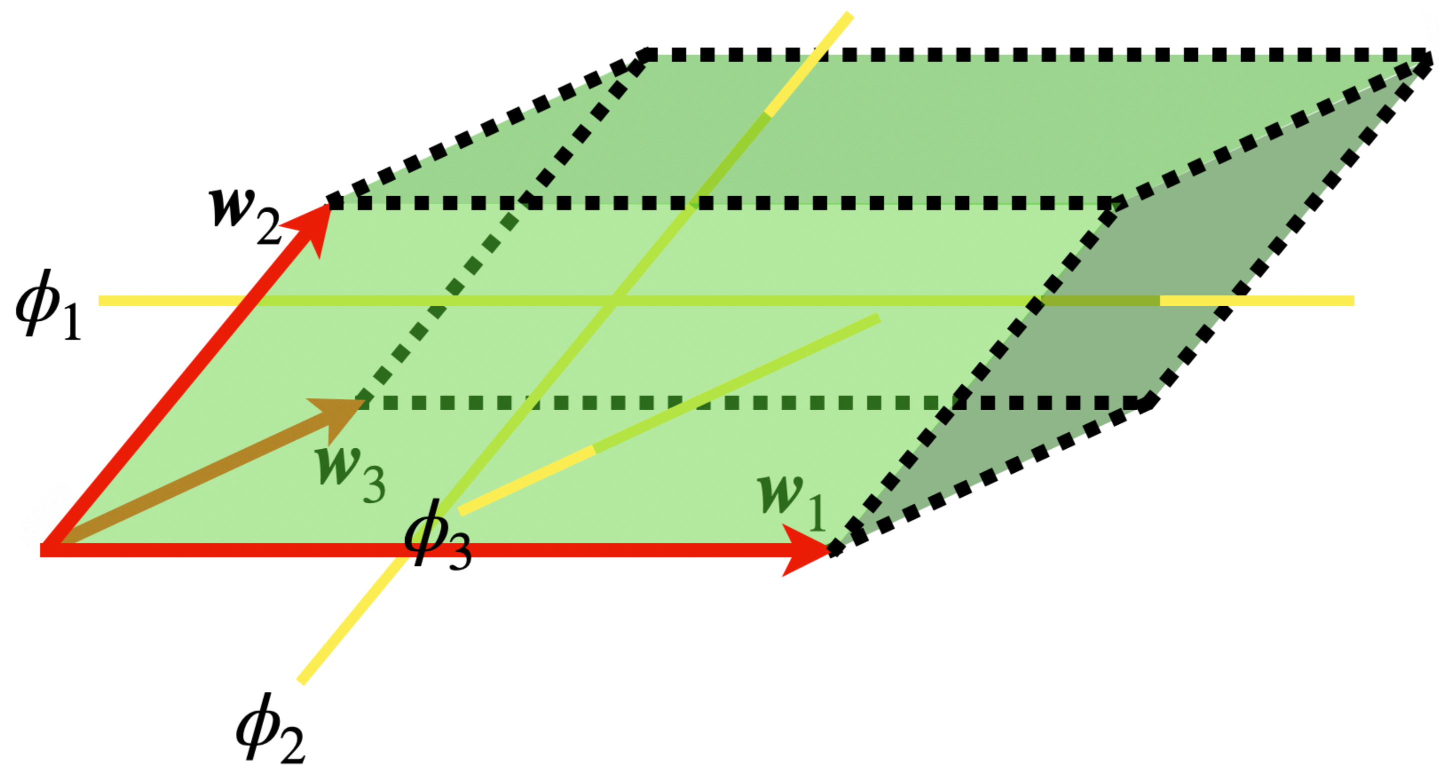


ELECTRIC FIELD ENERGY DENSITY

Electric Flux Sectors: $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$



$$\boldsymbol{E} = \frac{Q\boldsymbol{\phi}}{a^2}\frac{e_{\mathrm{QSI}}}{\epsilon_{\mathrm{QSI}}} \longrightarrow \boxed{u_E = \frac{1}{2}\frac{|Q\boldsymbol{\phi}|^2}{a^4}\frac{e_{\mathrm{QSI}}^2}{\epsilon_{\mathrm{QSI}}}}$$

NUMERICAL PLAN OF ATTACK

Perform ED on systems with up to 96 spins

- Project to constrained Hilbert space satisfying ice rules

- Periodic Boundary Conditions allows additional projections

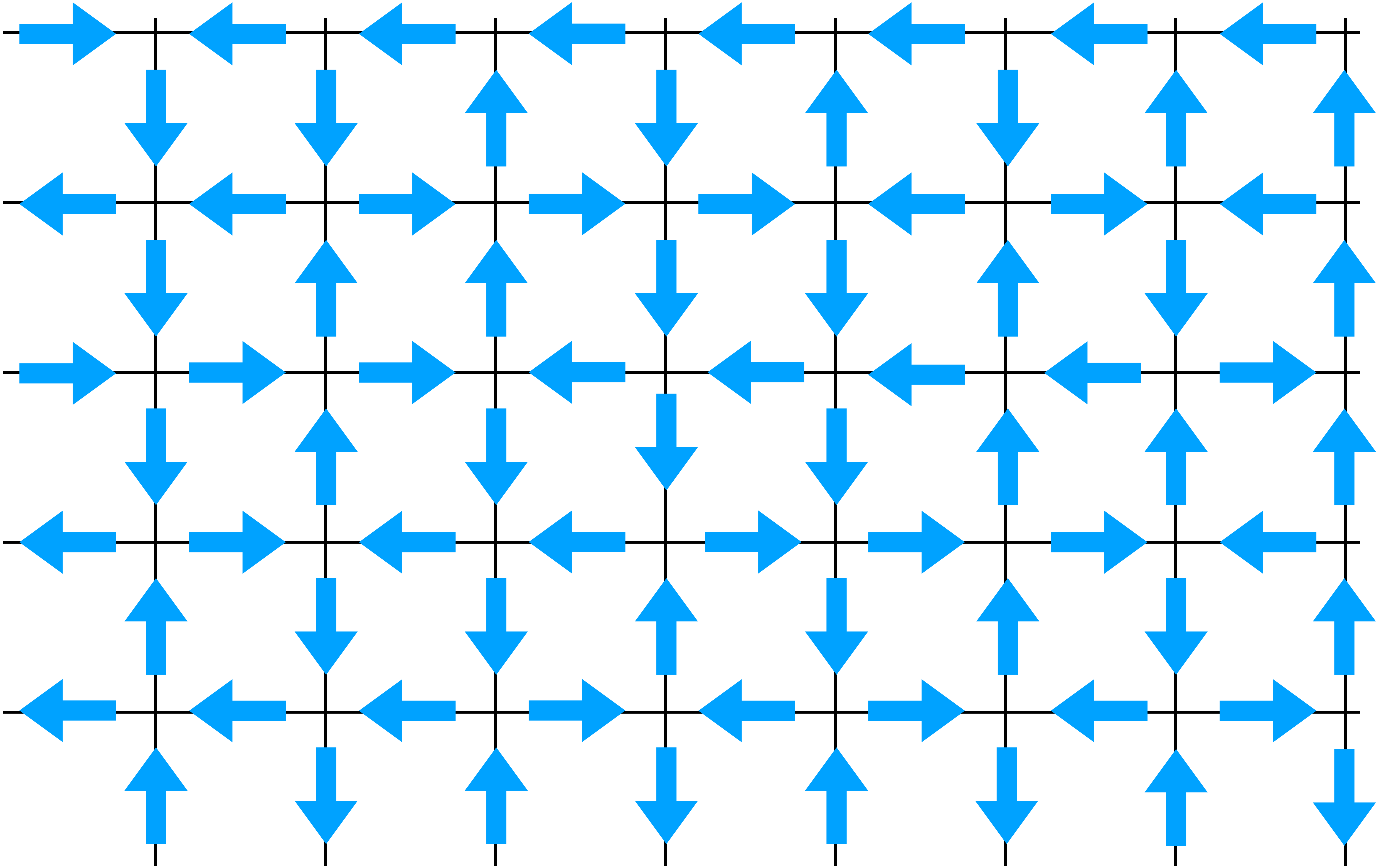
  A) Electric flux sectors

  B) Momentum sectors

	Before Projections	After Projections
$\dim \mathscr{H}$	$2^{96} \approx 8 \times 10^{28}$	$2^{22.7} \approx 7 \times 10^{6}$

- Use low-energy spectra to fit c using ground state dispersion and e using electric field energy density

EMERGENT ELECTRODYNAMICS



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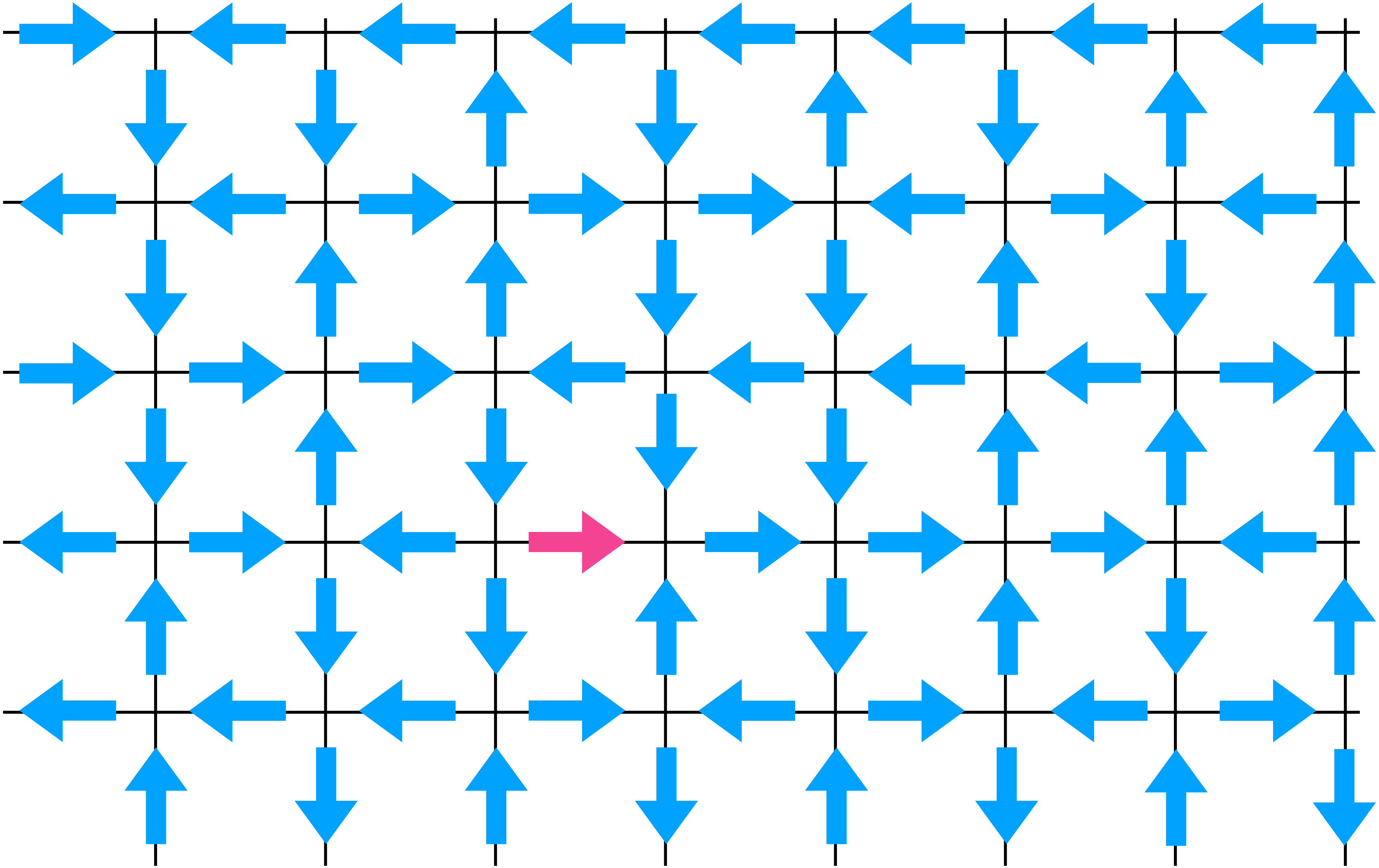

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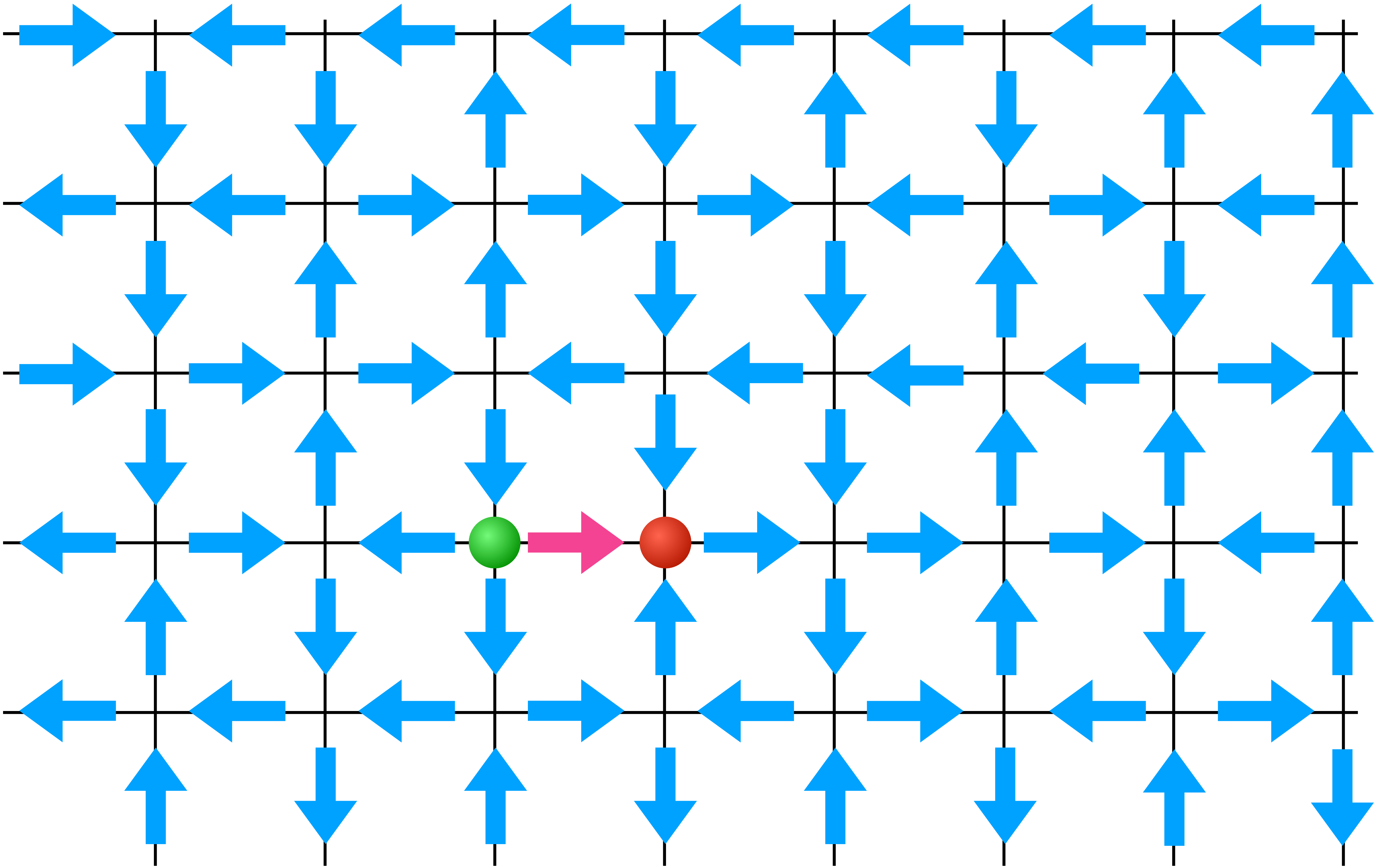

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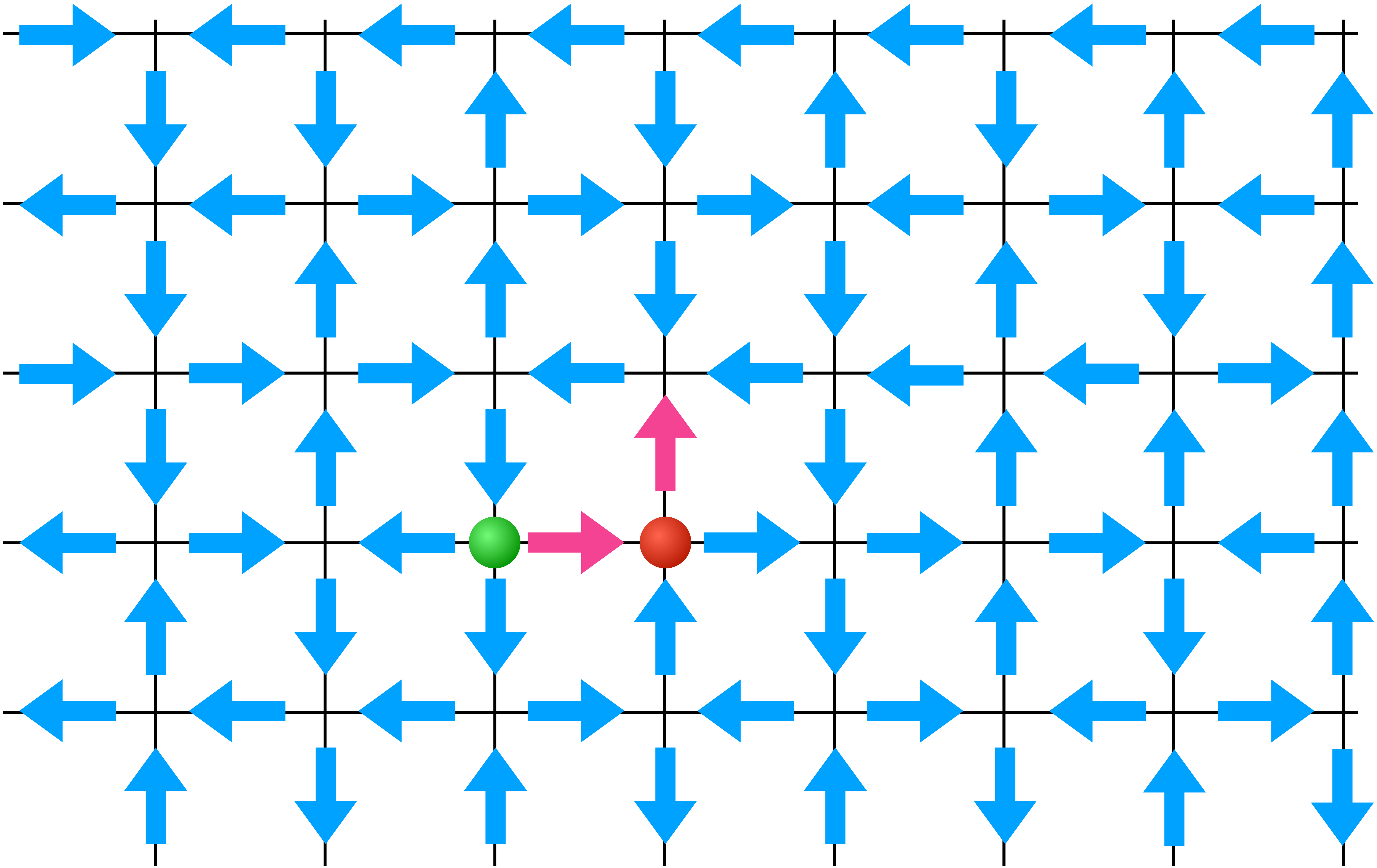

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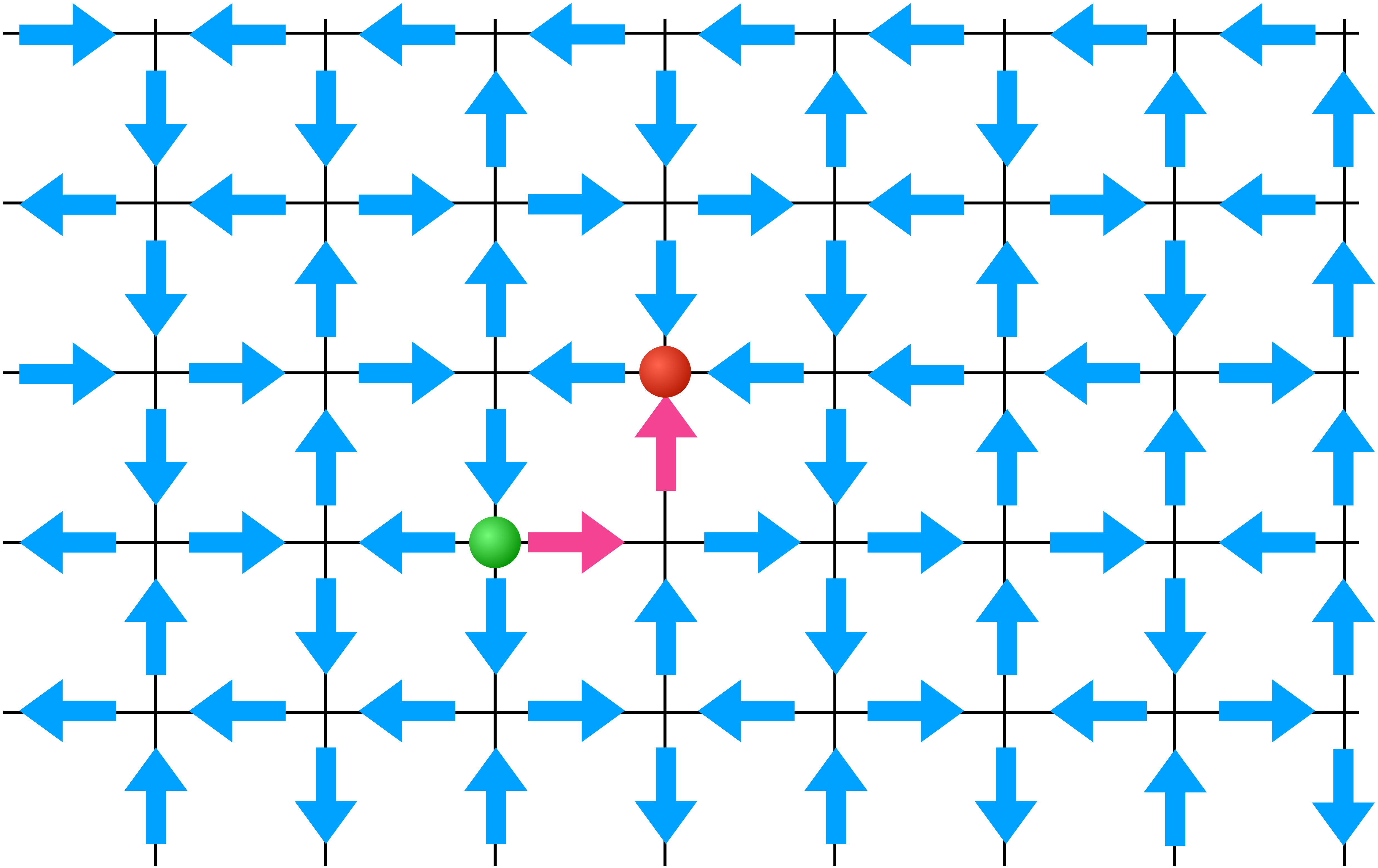

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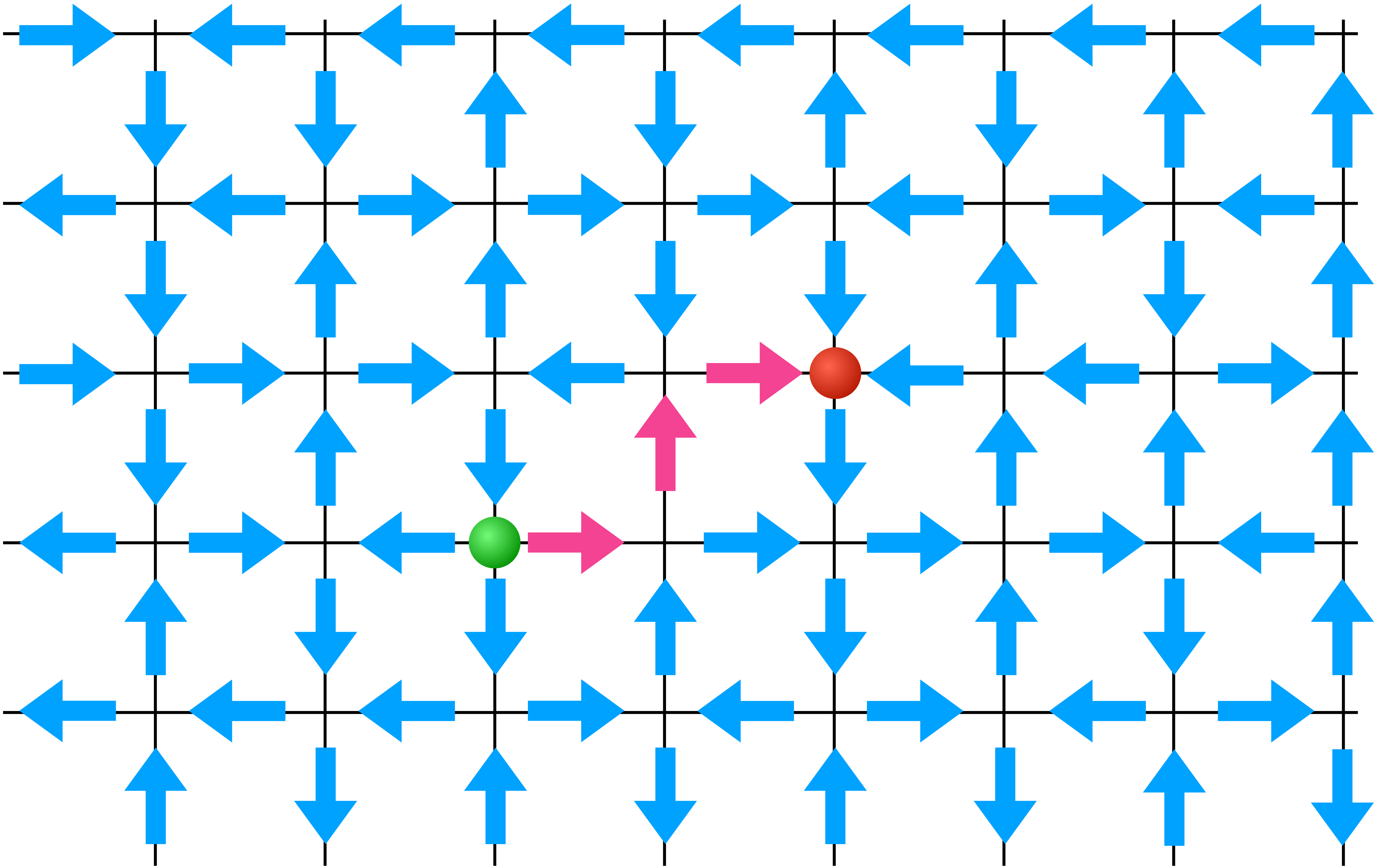

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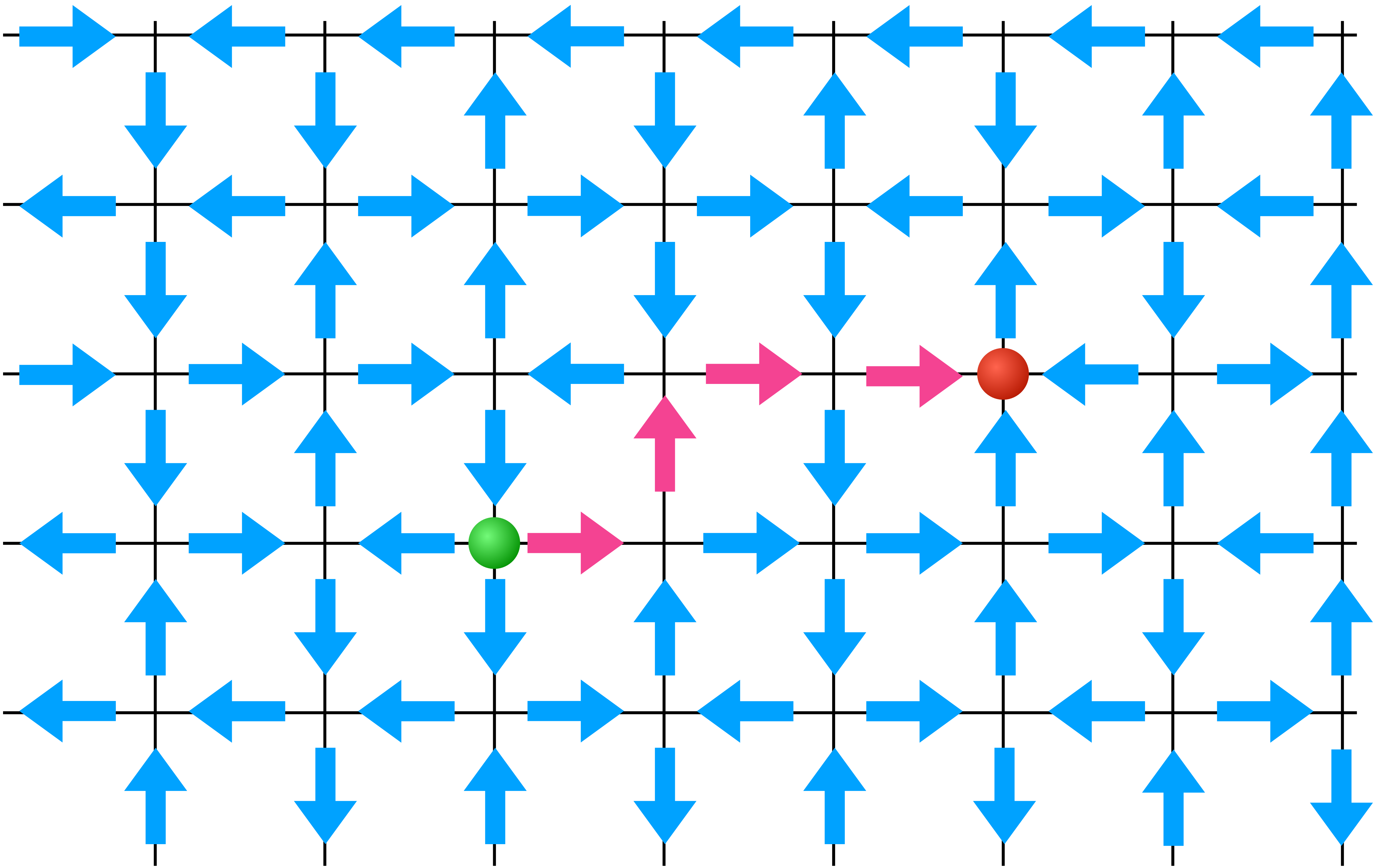

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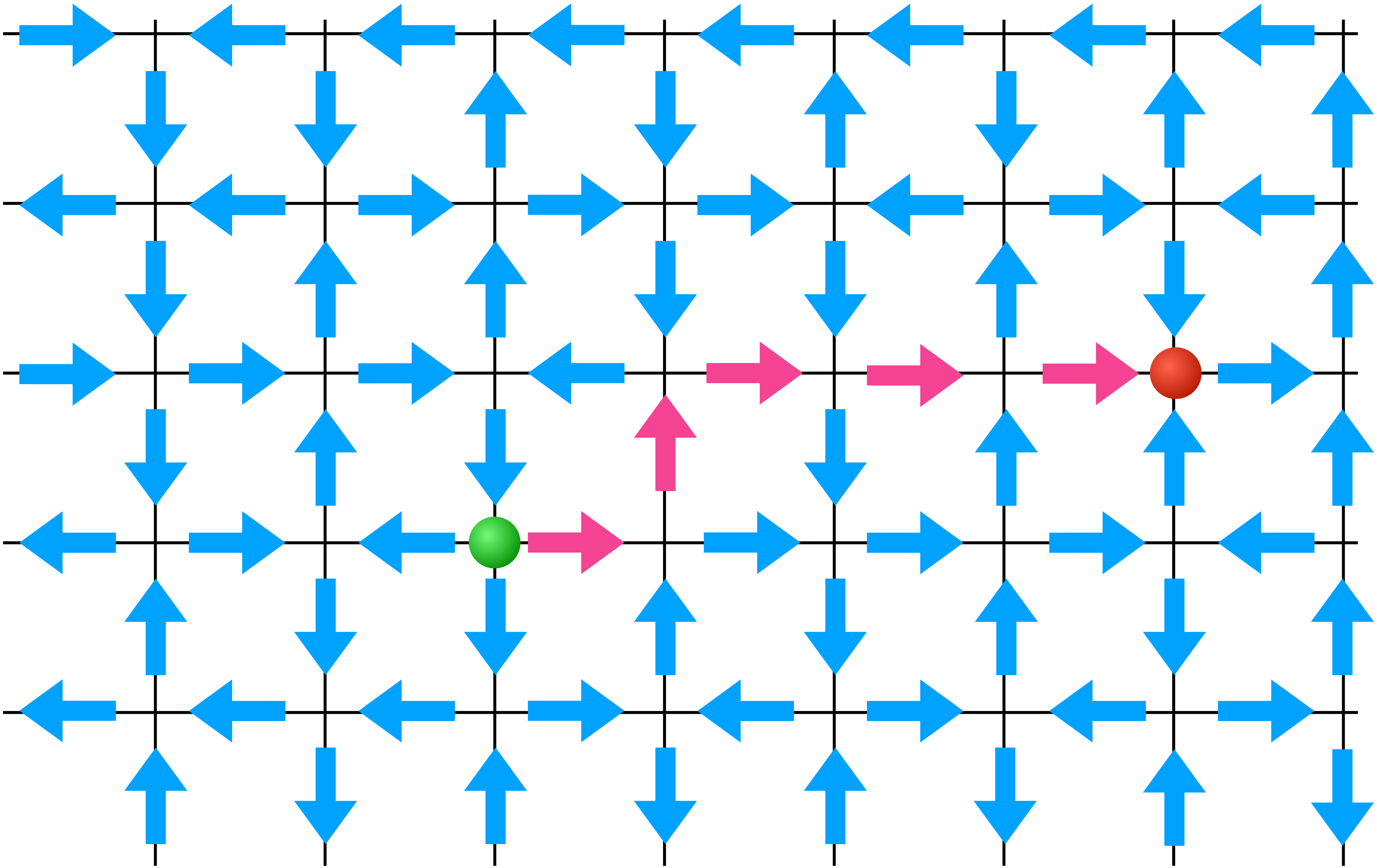

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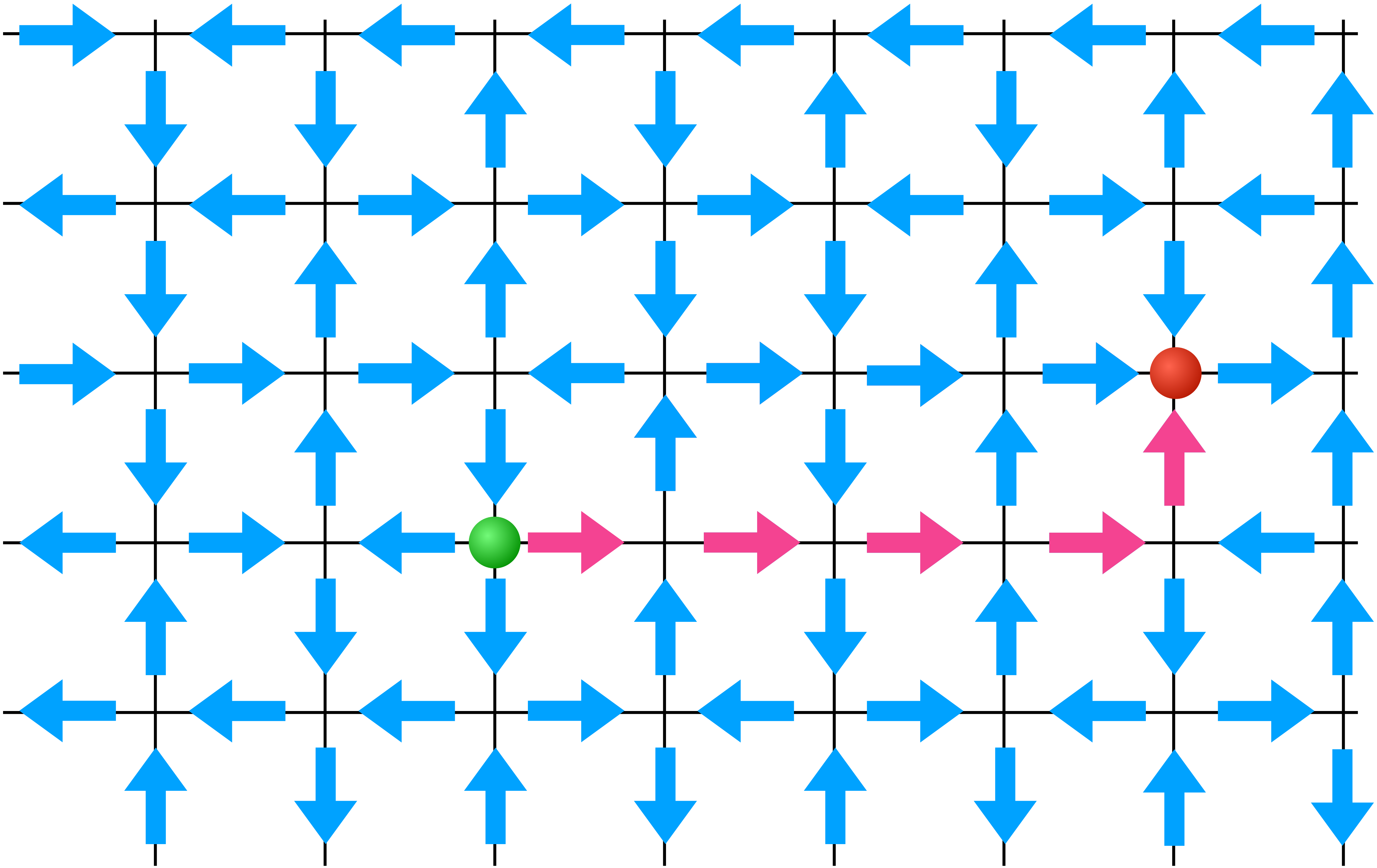

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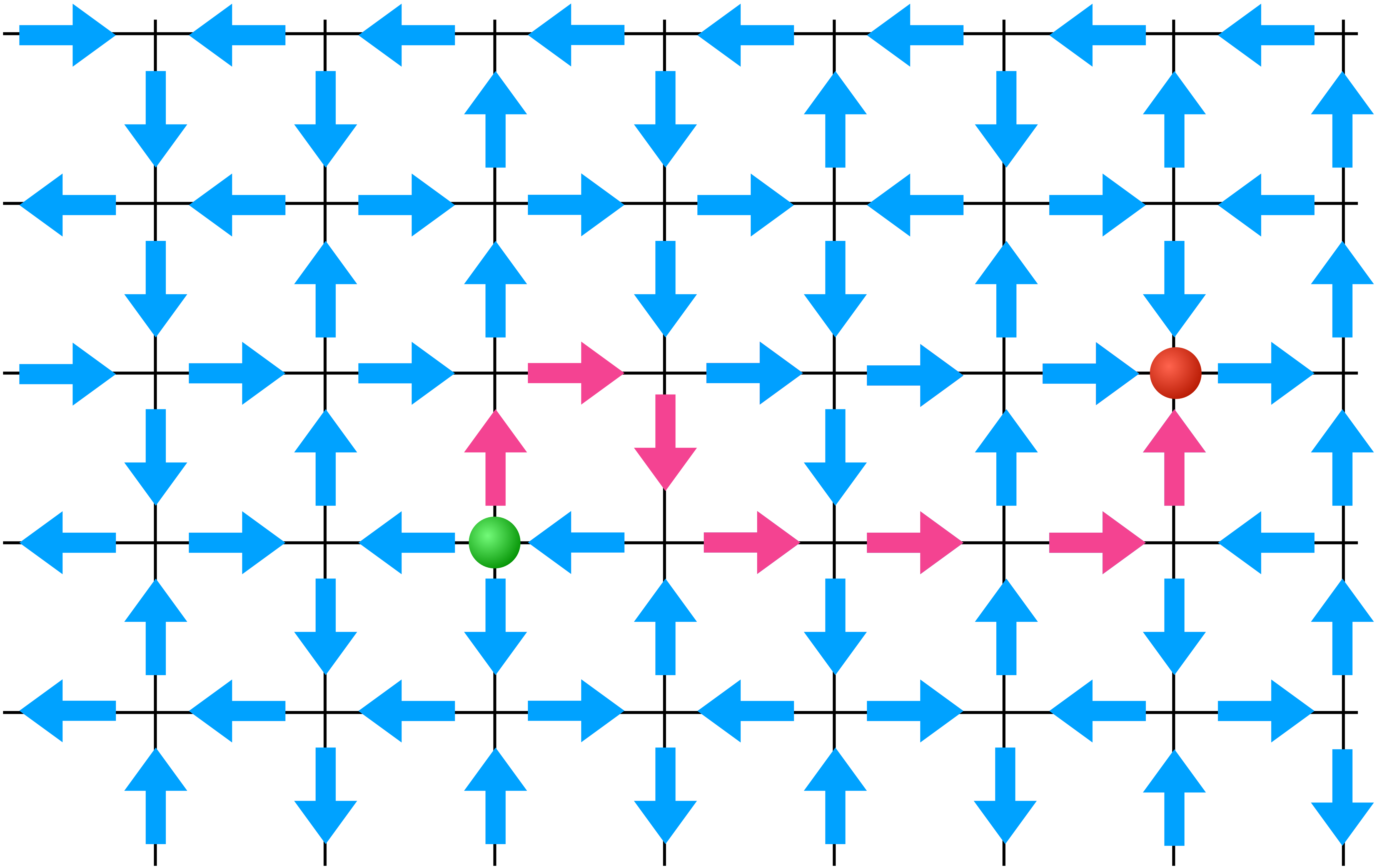

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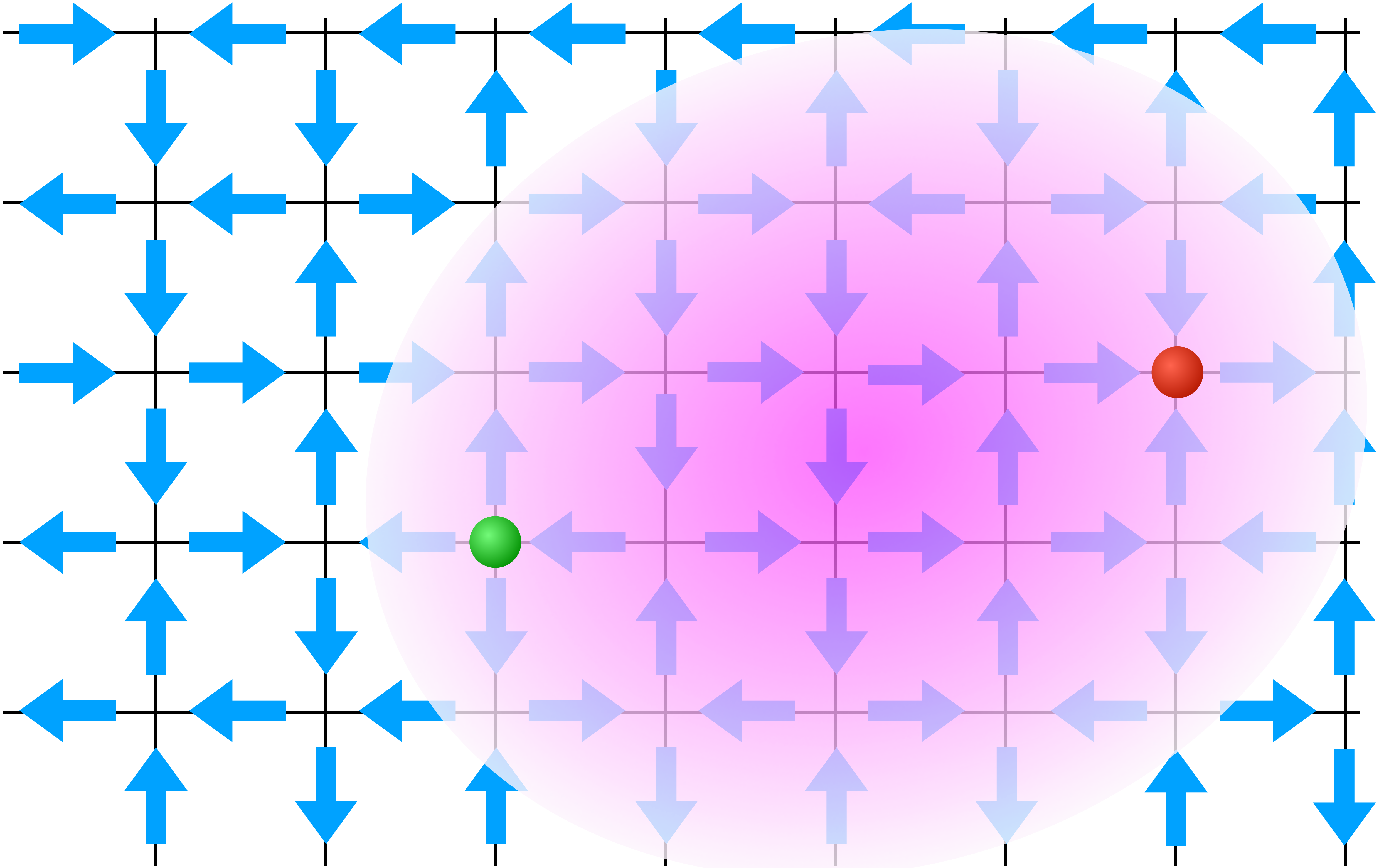

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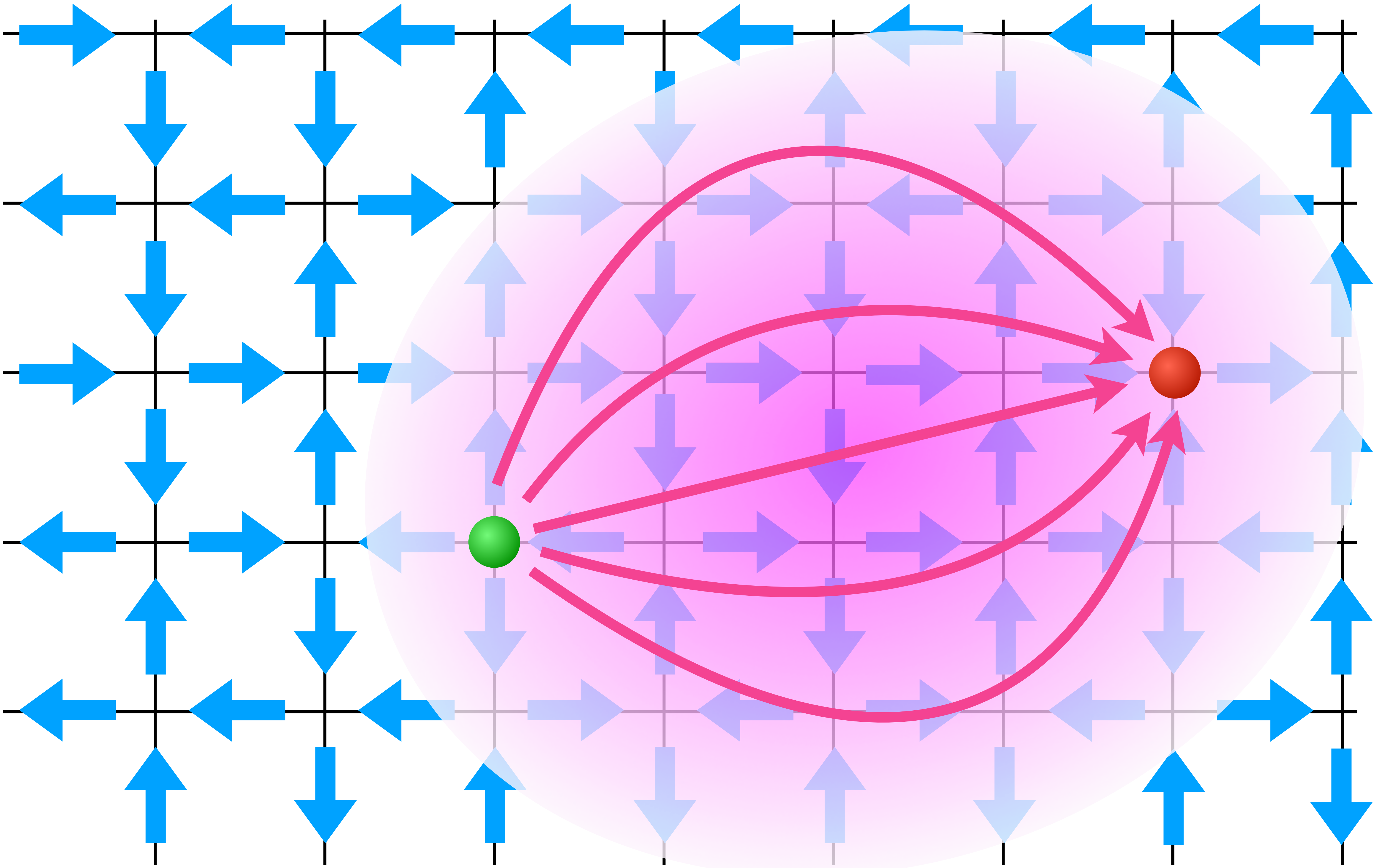


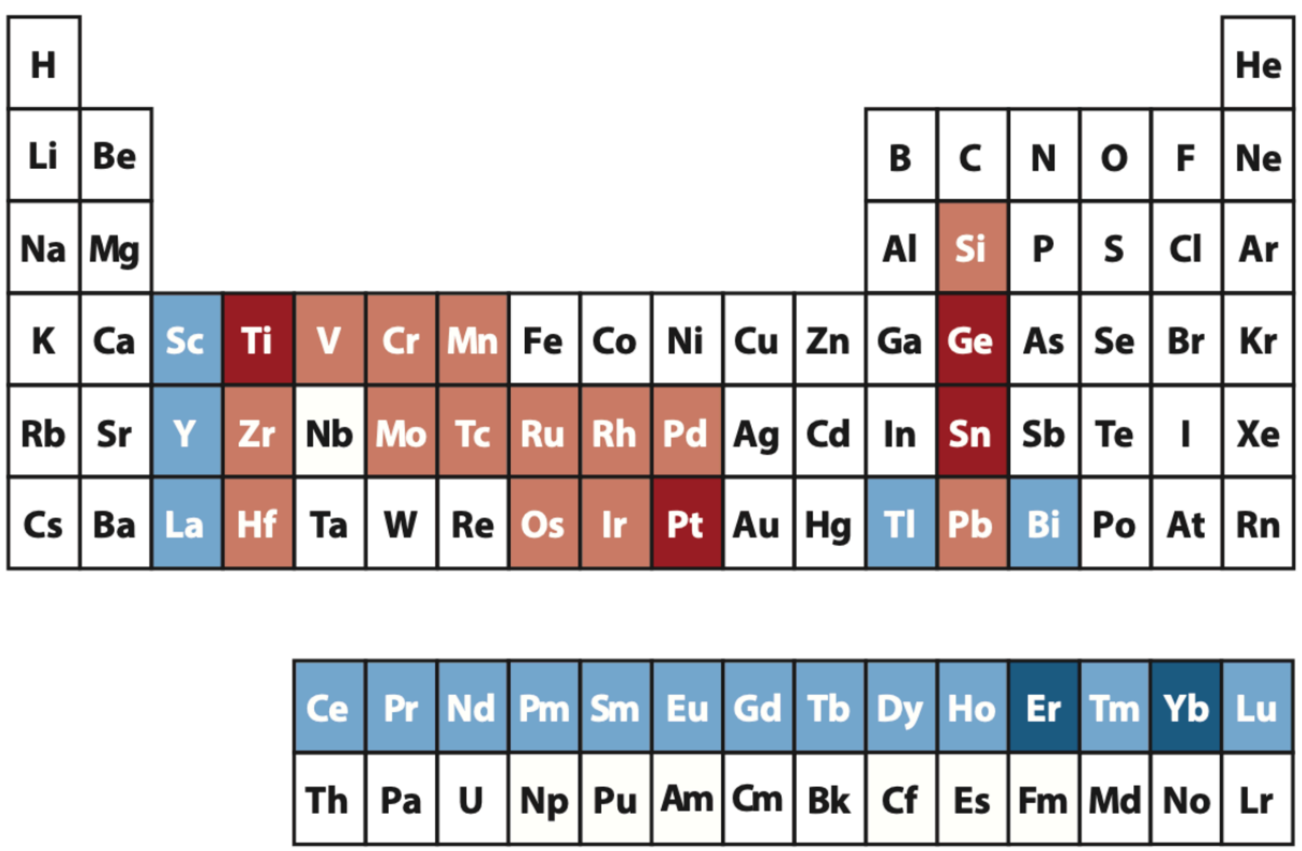
Castelnovo, Claudio, Roderich Moessner, and Shivaji L. Sondhi. *Nature* 451.7174 (2008): 42-45.

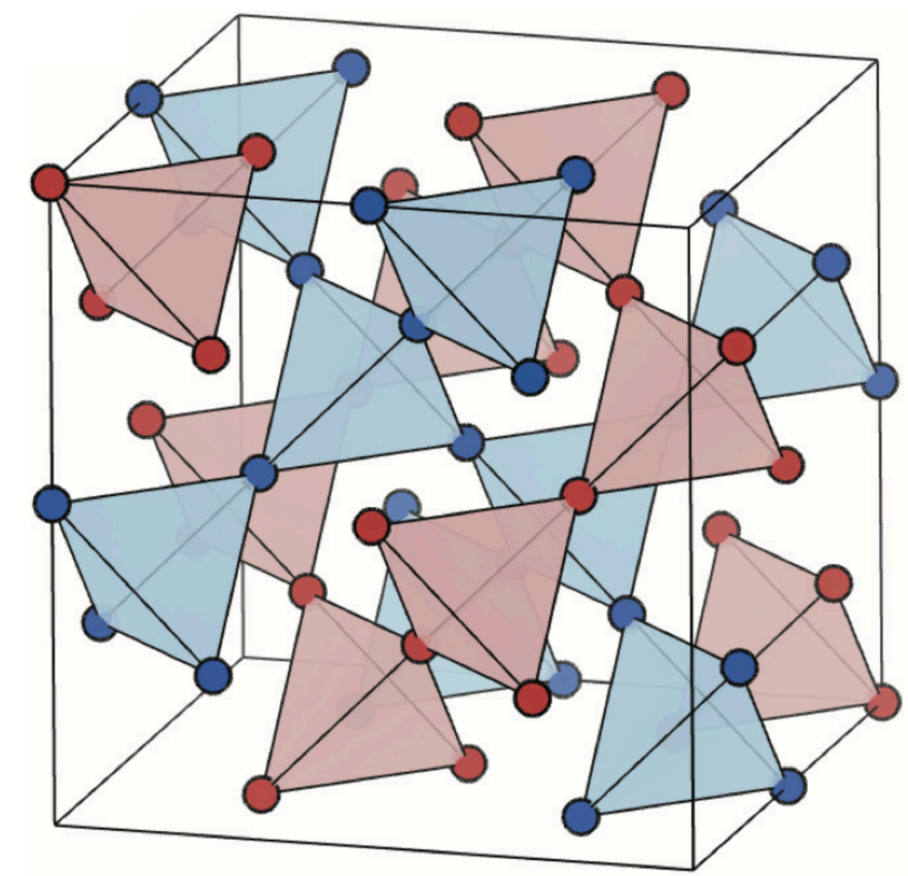
RARE-EARTH PYROCHLORE MAGNETS

$$A_2^{3+}B_2^{4+}O_7$$

Magnetic rare-earth ion

Non-magnetic





Hallas, Alannah M., Jonathan Gaudet, and Bruce D. Gaulin. *Annual Review of Condensed Matter Physics* 9 (2018): 105-124.

SPIN ICE MATERIALS

$$A_2^{3+}B_2^{4+}O_7$$

<u>Classical spin ice</u>

$Ho_2Ti_2O_7$

$Dy_2Ti_2O_7$

- Well established
- Order at low T

<u>Candidate quantum spin ice</u>

$Yb_2Ti_2O_7$ $Pr_2Sn_2O_7$ $Pr_2Hf_2O_7$

$Tb_2Ti_2O_7$ $Pr_2Zr_2O_7$

- Active area of experimental research
- No long-range order even at $T = 0$

Gardner, Jason S., Michel JP Gingras, and John E. Greedan. *Reviews of Modern Physics* 82.1 (2010): 53.
Hallas, Alannah M., Jonathan Gaudet, and Bruce D. Gaulin. *Annual Review of Condensed Matter Physics* 9 (2018): 105-124.
Rau, Jeffrey G., and Michel JP Gingras. *Annual Review of Condensed Matter Physics* (2019).

QUANTUM SPIN ICE

- Most general quantum Hamiltonian:

$$H = \sum_{\langle i,j \rangle} [J_{zz} S_i^z S_j^z - J_{\pm}\left(S_i^+ S_j^- + S_i^- S_j^+\right)$$

$$+J_{\pm\pm}\left[\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-\right]$$

$$+J_{z\pm}\left[S_i^z\left(\left(\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-\right) + i \leftrightarrow j\right)\right]]$$

- Experiments on $Yb_2Ti_2O_7$ found that in meV:

$$J_{zz} = 0.17 \pm 0.04, \quad J_{\pm} = 0.05 \pm 0.01$$

$$J_{\pm\pm} = 0.05 \pm 0.01 \quad J_{z\pm} = -0.14 \pm 0.01$$

EXPERIMENTAL CONSEQUENCES

1. Cherenkov Radiation
    - Theoretical study whose results where in terms of the fine structure constant
    - Since the speed of light is being tuned too, the threshold for Cherenkov radiation is also moved
2. Dynamical Structure Factor
    - Shows sharp lines from excitonic bound states and a continuum.
    - We now have all the information to know the spacing between bound states in numerics and neutron scattering experiment.

EXPERIMENTAL CONSEQUENCES

1. Cherenkov Radiation
    - Theoretical study whose results where in terms of the fine structure constant
    - Since the speed of light is being tuned too, the threshold for Cherenkov radiation is also moved
2. Dynamical Structure Factor
    - Shows sharp lines from excitonic bound states and a continuum.
    - We now have all the information to know the spacing between bound states in numerics and neutron scattering experiment.

Morampudi, Siddhardh C., Frank Wilczek, and Chris R. Laumann. *Physical Review Letters* 124.9 (2020): 097204.

XXZ MODEL TO QSI

- Setting $J_{\pm\pm} = J_{z\pm} = 0$, XXZ model:
  - $H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z + J_{\pm} \sum_{\langle i,j \rangle} \left(S_i^+ S_j^- + \mathrm{h.c.} \right)$
- Study quantum fluctuations within spin ice manifold (Set spinon gap to infinity)
  - For $J_{zz} \gg J_{\pm\pm}$, third order perturbation
  - $H_{eff} = P \left[H_{zz} + H_{\pm} - H_{\pm} \frac{1-P}{H_{zz}} H_{\pm} + H_{\pm} \frac{1-P}{H_{zz}} H_{\pm} \frac{1-P}{H_{zz}} H_{\pm} \right] P$

$$H_{eff} = -\frac{3J_{\pm}^3}{2J_{zz}^2} \sum_h \left(S_{h,1}^+ S_{h,2}^- S_{h,3}^+ S_{h,4}^- S_{h,5}^+ S_{h,6}^- + \mathrm{h.c.} \right)$$

- $$\equiv -g \sum_h \left(S_{h,1}^+ S_{h,2}^- S_{h,3}^+ S_{h,4}^- S_{h,5}^+ S_{h,6}^- + \mathrm{h.c.} \right)$$

EMERGENT COMPACT QED

- Introduce quantum rotor variables: $S_i^{\pm} = e^{\pm i\phi_i}$
- Introduce oriented link variables
  - $A_i = \pm\, \phi_i$
- Hamiltonian becomes $H = -2g \sum_h \cos(\mathrm{curl} A)$
- Consider Hamiltonian
  - $H = \frac{U}{2} \sum_r E_r^2 - K \sum_h \cos(\mathrm{curl} A)$
  - When $U \gg K$, gives same low-energy physics as above.

EMERGENT COMPACT QED: COMMENTS

- $H = \dfrac{U}{2} \sum_r E_r^2 - K \sum_h \cos(\mathrm{curl} A)$ is a compact $U(1)$ LGT.
- H is invariant under gauge transformation $A_{ij} \to A_{ij} + g_j - g_i$
- Canonically conjugate $[A, E] = i$
- magnetic monopoles
- Because $E_{ij} = \pm 1/2$, LGT is frustrated an non-trivial in $U \gg K$ limit

RECALL FINE STRUCTURE CONSTANT

- $\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi - 2i\sqrt{\pi\alpha}\phi^\dagger\overleftrightarrow{\partial_\mu}\phi A^\mu - 4\pi\alpha A_\mu A^\mu\phi^\dagger\phi$
  - Scalar QED Lagrangian
- α gives coupling strength between photon field and scalar boson field.
- Electron-positron to electron positron scatter leading order term is proportional to α
  - In the QED of our universe, $\alpha = 1/137$

FULL DISPERSION

- Effective theory near the RK point, with $U = 1 - \mu$

$$H_{RK} = \frac{U}{2}\sum_{\langle ij\rangle} E_{ij}^2 + \frac{K}{2}\sum_h (\nabla_h \times A)^2 + \frac{W}{2}\sum_h (\nabla_h \times E)^2$$

- Diagonalize to find the dispersion

$$\omega_{1,2}(\boldsymbol{k}) = \frac{2}{a}\sqrt{c^2\zeta(\boldsymbol{k}) + V\zeta^2(\boldsymbol{k})}$$

$$\zeta(\boldsymbol{k}) = 3 - \cos\left(\frac{k_1 a}{2}\right)\cos\left(\frac{k_2 a}{2}\right) - \cos\left(\frac{k_1 a}{2}\right)\cos\left(\frac{k_3 a}{2}\right) - \cos\left(\frac{k_2 a}{2}\right)\cos\left(\frac{k_3 a}{2}\right)$$

$$c = \sqrt{UK}a \quad \textit{and} \quad V = UW$$