Spacetime Symmetry-enriched SymTFT

- Based on 2409.18113 and 2507.02036 w Öner Aksoy and Ho Tat Lam

- Plan

  1) Symmetry topological field theory (SymTFT) 101
  2) SymTFTs with discrete-translation sym.
  3) Symmetry-enriched SymTFTs

  (Note: will omit many references during talk)

Background

Power of Symmetries

1) Selection rules
2) 't Hooft anomalies (Constraints on RG flows)
3) characterize IR phases

⋮

Modern perspective (Note: no subsystem sym here)

$$\text{Symmetry} \longleftrightarrow \text{topological defect}$$

→ e.g., $U(1)$ sym in $d+1$D. $\partial_\mu j^\mu = 0 \Rightarrow d * j = 0$

$$e^{i\theta \int_{\Sigma_d} * j} \longrightarrow e^{i\theta \int_{\Sigma_d + \partial B} * j} = e^{i\theta \int_{\Sigma_d} * j} e^{i\theta \int_B d * j} = e^{i\theta \int_{\Sigma_d} * j}$$

→ generalized syms: higher-form and non-invertible syms

Consider $d+1$ D QFT T w (generalized) sym S
(How to think of top defects?)

SymTFT: theoretical tool to represent S in a T-independent way

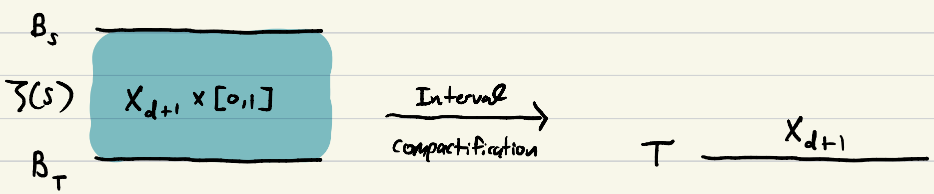
"Seperates Kinematics from dynamics"

depend only on S.
1) A $d+2$D TFT $\zeta(S)$ — the SymTFT of S.
2) A topological boundary B_S of $\zeta(S)$ w S sym

$\rightarrow$ Top def. of $\zeta(S)$ encode S (twisted) sym sectors

3) Boundary B_T of $\zeta(S)$ encoding the dynamics of T.

Sandwich



($\zeta(S)$ only needs to be topological in Interval direction)
(often ignore B_T boundary for sym only)

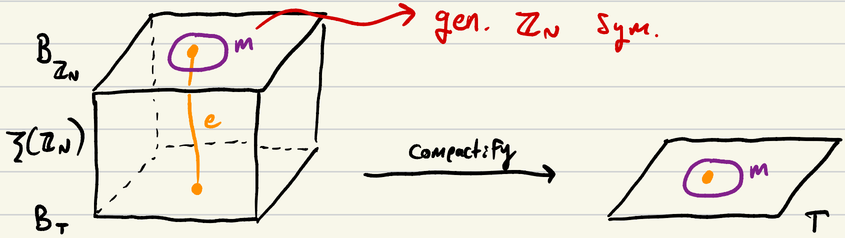
Example: $\mathbb{Z}_N$ sym in 1+1 D.

SymTFT is 2+1 D $\mathbb{Z}_N$ gauge theory $S = \frac{iN}{2\pi} \int a \wedge db$

$\rightarrow$ TDLs $e(\gamma) = e^{i\int_\gamma a}$ and $m(\gamma) = e^{i\int_\gamma b}$

$$e^N = m^N = 1 \qquad \langle e(\gamma_1)\, m(\gamma_2) \rangle = e^{\frac{2\pi i}{N} \mathrm{link}(\gamma_1, \gamma_2)}$$

→ $B_{\mathbb{Z}_N}$ = Dirichlet boudary condition : $a_0 = a_x = 0$. (e-cond bdy.)



→ TDL $e^q m^f$ ⇒ f-twisted, q sym charge

anomaly free

Finite G SymTFT = G gauge thy (Dirichlet BC has G sym)

SymTFT applications

1) discrete gauging (changing sym. boundary)

2) classify gapped IR phases (S-sym TFTs)

3) 't Hooft anomalies and anomaly matching.

⋮

(SymTFT mostly for internal syms)

Spacetime SymTFT?

Hard to get genuine spacetime SymTFT
⟶ spacetime sym defects? gauging spacetime sym ⇒ q gravity?

(Recent progress for continuous spacetime sym 2509:07965,
Apruzzi, Bondi, Etxebarria, Lam, Schafer-Nameki)

Can encode interplays between spacetime and int syms
(Focus on invertible sym)

(Note: Coleman-Mandula theorem in $\gtrsim 2+1$D)

1) group extensions $1 \to G_{int} \to G \to G_s \to 1$

2) Mixed anomalies

<u>Anomaly inflow theory</u> $\xrightarrow[G_{int}]{\text{gauging}}$ <u>Sym TFT</u>

G_{int} SPT — G_{int} DW thy

G SPT — G_{int} DW thy enriched by G_s.

(Spacetime Sym.-enriched SymTFT)

Discrete spatial translations

Two tyes of interplays in 1+1D

1) transl. grp act on $G_{int} \Rightarrow G_{int}$ is a modulated sym

e.g., $\mathcal{L} = (\partial_t\phi)^2 - \kappa(\partial_x^2\phi)^2$ (1+1D Lifshitz theory)

$\phi(x,t) \to \phi(x,t) + \alpha + \beta x$ (dipole sym)

$\Rightarrow T_\ell \triangleright (\alpha,\beta) = (\alpha+\beta\ell, \beta)$

$\to$ disc. transl $\mathbb{Z} \Rightarrow G = G_{int} \rtimes \mathbb{Z}$

Inflow thy — SymTFT

<u>Trivial SPT</u> $\xrightarrow[G_{int}]{\text{gauge}}$ <u>$\mathbb{Z}$-enriched G_{int} gauge-thy</u>

$G_{int} \rtimes \mathbb{Z}$ sym — $\mathrm{Rep}(G)^{(1)} \rtimes \mathbb{Z}$ sym

(transl. = anyon auto.)

2) mixed 't Hooft anomalies between $\mathbb{Z}$ and G_{int}

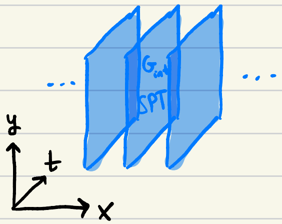
e.g., $G_{int} = SO(3)$ and the Lieb-Schultz-Mattis (LSM) thrm

$\Rightarrow$ called LSM anomalies. $\left\{ \begin{array}{l} \text{gauge } G_{int} \rightarrow \text{break transl.} \\ \text{No trivial sym gapped phase} \end{array} \right.$

Inflow thy

SymTFT

<u>$\mathbb{Z}$-inv G_{int}-SPT defect network</u> $\xrightarrow[G_{int}]{\text{gauge}}$ <u>$\mathbb{Z}$-enriched G_{int} gauge-thy</u>



gauged SPT defect network inserted.

(transl. act on magnetic defects)

In both cases, Sym TFT = foliated TFT.

$\rightarrow$ not top. in transl. direction (still top in int comp dir)

Example: $\mathbb{Z}_N$ dipole Sym

$G_{int} = \mathbb{Z}_N \times \mathbb{Z}_N$ with $T_1 \triangleright (\alpha, \beta) = (\alpha + \beta, \beta)$

Sym TFT: x-transl enriched $\mathbb{Z}_N \times \mathbb{Z}_N$ gauge thy

$\rightarrow$ TDLs (anyons) m_1, m_2, e_1, e_2

$$e_i^N = m_i^N = 1 \qquad \langle e_i(\gamma_1) m_j(\gamma_2) \rangle = \delta_{ij}\, e^{\frac{2\pi i}{N} \mathrm{link}(\gamma_1, \gamma_2)}$$

$\rightarrow T_x$ action on TDLs

$m_1 \longrightarrow m_1 \qquad m_2 \longrightarrow m_1 m_2 \qquad$ (Sym defects)

$e_1 \longrightarrow e_1 e_2^{-1} \qquad e_2 \longrightarrow e_2 \qquad$ (Sym charges)

→ Lagrangian realization

$$\mathcal{L} = \frac{iNa}{2\pi}\Big[A_t(\partial_x^2 B_y - \partial_y B_{xx}) + A_{xx}(\partial_t B_y - \partial_y B_t) + A_y(\partial_t B_{xx} - \partial_x^2 B_t)\Big]$$

- lattice spacing a (UV cutoff)
- Gauge transformation $\delta B_\bullet = \begin{cases} \partial_\bullet f & \bullet = t, y \\ \partial_x^2 f & \bullet = xx \end{cases}$ Same for A
- TDLs

$$M_1(\gamma) = e^{ia\int_\gamma \left(B_{xx}\, dx + \sum_{w=y,t} \partial_x B_w\, dw\right)}$$ Same for $e_2^{-1}(\gamma)$

$$M_2(\gamma) = e^{i\int_\gamma \left(x B_{xx}\, dx + \sum_{w=y,t} (x\partial_x B_w - B_w) dw\right)}$$ Same for $e_1(\gamma)$

$\Rightarrow$ T_x action by $x \to x+a$.

- Foliated TFT (not topological in x-direction)
  1) (on lattice) # Vacua on $L_x \times L_y$ spatial torus $(N \gcd(L_x/a, N))^2$
  2) Dual presentation (Ebisu, Honda, Nakanishi arXiv: 2401.10677)

$$S = \frac{iN}{2\pi}\int_{X_3} \left(A' \wedge dB' + \tilde{A} \wedge d\tilde{B} + A' \wedge \tilde{A} \wedge \frac{dx}{a}\right)$$

→ Topological BC (six, let's look @ two)

1) e_1, e_2 cond. boundary (Dirichlet BC $A_t = A_{xx} = 0$)

   - TDLs: M_1 and M_2 $\Rightarrow$ $\mathbb{Z}_N$ dipole sym.

2) m_1, e_2 cond. boundary

($B.C.$ $\partial_x A_t = \partial_t \tilde{A}_x$, $A_{xx} = \partial_x \tilde{A}_x$, same for B)

- TDLs: e_1 and m_2 $\Rightarrow$ Uniform $\mathbb{Z}_N \times \mathbb{Z}_N$ Sym.
- $\mathbb{Z}_N \times \mathbb{Z}_N$ Sym has an LSM anomaly with transl.

$\Rightarrow$ gauging by $\langle m_1, e_2 \rangle \rightarrow \langle e_1, m_2 \rangle$ breaks transl.

$\Rightarrow$ the magnetic Lagrangian subgroups (SPT phases)

$$\langle e_1 e_2^{-n}, m_2 m_1^{n} \rangle , \quad n \in \{0, 1, \dots, N-1\}$$

are not translation invariant

Apps of transl-enriched SymTFT

1) Classify modulated SPTs : T-invariant $H^2(G, U(1))$ classes

2) Relation of modulated Sym and LSM anomaly via gauging

General aspects of Sym-enriched SymTFTs

2+1D Q-enriched bosonic TO as a SymTFT
(assume Q is anomaly free)

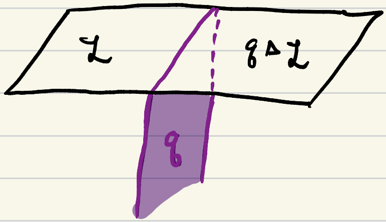
$\rightarrow$ data: Q-action on TDLs, Sym. fractionalization, ...
(1410.4540, Barkeshli, Bonderson, Cheng, Wang)

Add topological boundary $\mathcal{B}$ with anyon $\mathcal{L}$ condensed
(i.e., Sym boundary)

$\rightarrow$ Generally modifies Q.

1) Q explicitly brkn to $Q_{\mathcal{B}} = \{ g \in Q \mid g \triangleright \mathcal{L} \simeq \mathcal{L} \}$

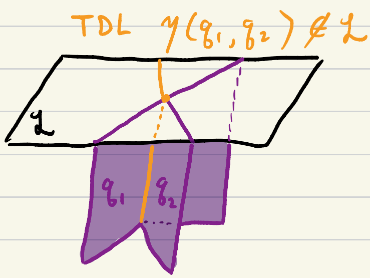
(Sym defect picture)



2) Q_B extended by boundary Sym A_B.

$$1 \longrightarrow A_B \longrightarrow G_B \longrightarrow Q_B \longrightarrow 1$$

(Sym defect picture)



G_B describes invertible sym for the Sym-enriched Sym TFT

→ G_B ext. class trivial if $\mathcal{L}$ carries no frac Q_B sym charge (such $\mathcal{L}$ cond all $\eta(g_1, g_2)$.)

→ For Symmetric Lagrangian algebras ($g \triangleright \mathcal{L} \simeq \mathcal{L}$ w no frac. charge)

$$G_B = A_B \rtimes Q$$

Applications

1) Changing BC $B \to B'$ changes $G_B \to G_{B'}$
⇒ dual sym from gauging

2) classify Q-enriched gapped IR phases using Symmetric Lagrangian algs.

$\Rightarrow$ Detect mixed anomalies : no Q-enriched SPTs the no G_B SPTs. (sufficient but not necesary cond for anomalies of G_B)

Summary

Discussed Sym-enriched SymTFTs & utility for spacetime Sym

Example with discrete translations — foliated SymTFT
$\rightarrow$ See paper for parity and time-reversal

Open questions : higher dim? non-inv Sym? time-translations?