An SPT-LSM theorem for weak SPTs with non-invertible symmetry

Sal Pace

tl;dr

We find a new class of entangled weak SPTs characterized by projective non-invertible symmetries on the lattice

- Projective invertible symmetries cannot have SPTs, but non-invertibility provides a loophole
- Projectiveness implies an SPT-LSM theorem

SP, Ho Tat Lam, and Ömer Aksoy, arXiv:2409.18113

Quantum phases $\iff$ Generalized Symmetry

A fundamental problem in CMT is to understand quantum phases

- How do we diagnose/classify phases of matter?
- (Generalized) symmetries can often answer this

Build-a-phase recipe

(1) Choose your generalized symmetries adjectives

$$a_1\text{–}a_2\text{–}a_3\text{–}\cdots \text{ Symmetry}$$

- e.g., n-form, (non-)invertible, subsystem, …

(2) Specify SSB and SPT (i.e., condensation pattern).

Ordered phases *Topological insulators*

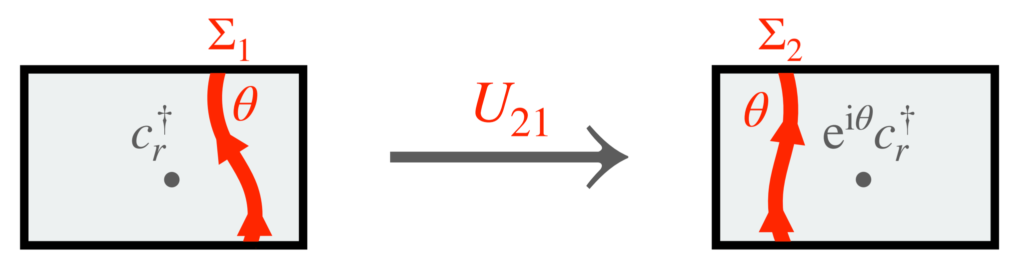
Topological order *Higgs* *Fracton* *Coulomb phases*

Phases we have yet to name!

SPTs and defects

An SPT phase is a gapped quantum phase protected by a symmetry with a unique ground state on all closed spaces

- Characterized by their bulk response to static probes
- Symmetry defects



A projective non-invertible SPT

1d lattice with a qubit and $\mathbb{Z}_4$ qudit on each site $j \sim j+L$

$$H = -\sum_j \sigma_j^x C_{j+1}\sigma_{j+1}^x + \frac{1}{4}\sum_j (Z_j - Z_j^\dagger)\,\sigma_j^z\,(Z_{j+1} - Z_{j+1}^\dagger)$$

- σ^x, σ^z act on qubits, X, Z act on $\mathbb{Z}_4$ qudits

Stabilizer code and has a unique gapped ground state. It is a weak $\mathbb{Z}_2 \times \mathsf{Rep}(D_8)$ SPT

$$U = \prod_j X_j^2, \quad R_1 = \prod_j \sigma_j^z, \quad R_2 = \prod_j Z_j^2, \quad R_{\mathsf{E}} = \frac{(1+R_1)\,(1+R_2)}{2}\prod_j Z_j^{\prod_{k=1}^{j-1}\sigma_k^z}$$

$$T\,|\mathrm{GS}\rangle = +\,|\mathrm{GS}\rangle \qquad U\,|\mathrm{GS}\rangle = +\,|\mathrm{GS}\rangle \qquad R_1\,|\mathrm{GS}\rangle = +\,|\mathrm{GS}\rangle$$

$$R_2\,|\mathrm{GS}\rangle = \begin{cases} +\,|\mathrm{GS}\rangle, & L \text{ even} \\ -\,|\mathrm{GS}\rangle, & L \text{ odd}\end{cases} \qquad R_{\mathsf{E}}\,|\mathrm{GS}\rangle = \begin{cases} +2\,|\mathrm{GS}\rangle, & L \text{ even} \\ 0, & L \text{ odd}\end{cases}$$

- Translation defects carry $\mathsf{Rep}(D_8)$ symmetry charge in $|\mathrm{GS}\rangle$
- Enforced by projective algebra $U\,R_{\mathsf{E}} = (-1)^L R_{\mathsf{E}}\,U$

The general $\mathsf{Rep}(G) \times Z(G)$ story

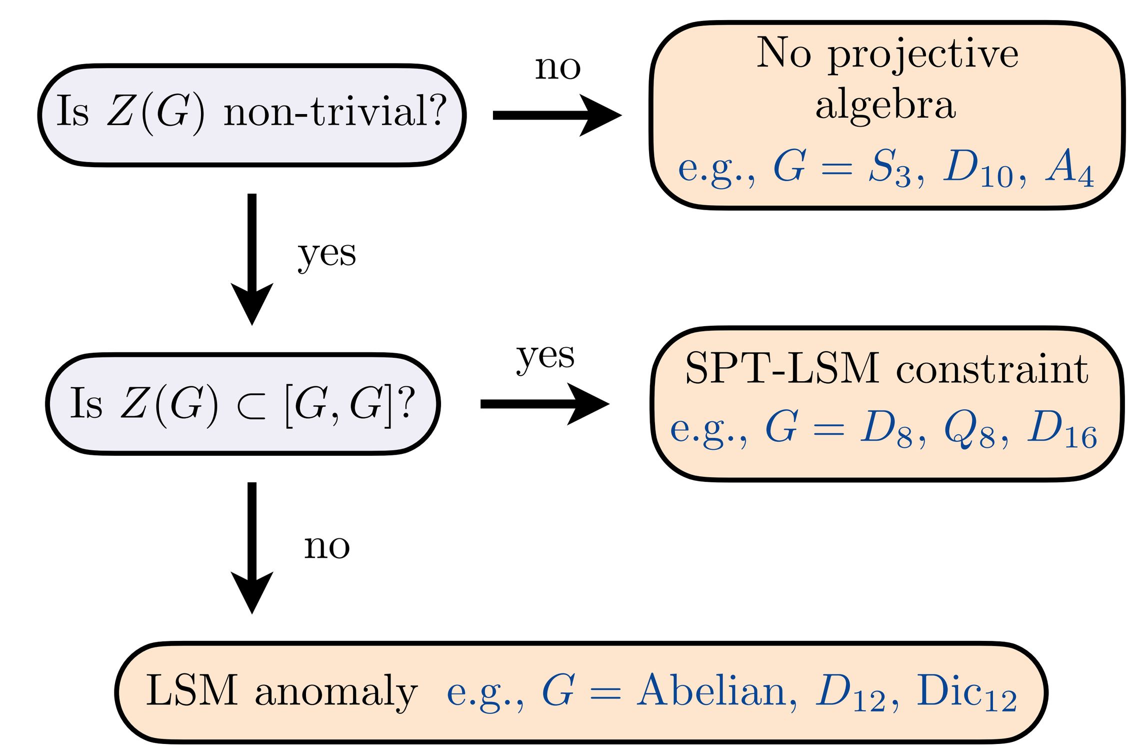
Projective $\mathbb{Z}_2 \times \mathsf{Rep}(D_8)$ symmetry is a special case of a more general projective $Z(G) \times \mathsf{Rep}(G)$ symmetry

$$U_{z_1}U_{z_2} = U_{z_1 z_2} \quad \text{and} \quad R_{\Gamma_a} \times R_{\Gamma_b} = \sum_c N_{ab}^c R_{\Gamma_c}$$

- Projectivity arises through the relation

$$R_\Gamma U_z = (\mathrm{e}^{\mathrm{i}\phi_\Gamma(z)})^L\, U_z R_\Gamma \quad \text{with } \mathrm{e}^{\mathrm{i}\phi_\Gamma(z)} = \mathrm{Tr}[\Gamma(z)]\,/\,d_\Gamma$$

Any SPT states must have translation defects dressed by $\mathsf{Rep}(G)$ symmetry charge $\Longrightarrow$ SPT-LSM constraint



A SymTFT perspective

What is the SymTFT for the projective $Z(G) \times \mathsf{Rep}(G)$?

- Symmetry-enriched $Z(G) \times G$ gauge theory with translations implementing an anyon automorphism
- T action can forbid magnetic Lagrangian algebras (LSM theorem) **SP**, Aksoy, and Lam arXiv:25XX.XXXXX