Generalized symmetries in ordered phases: bridging the ordinary and the exotic

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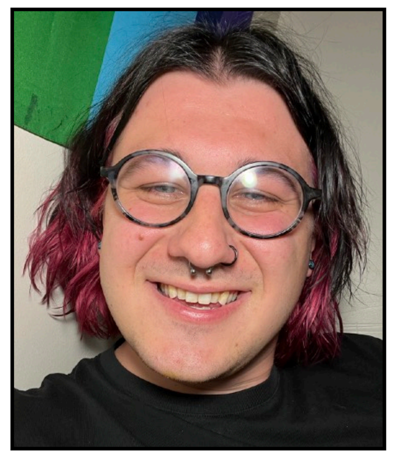
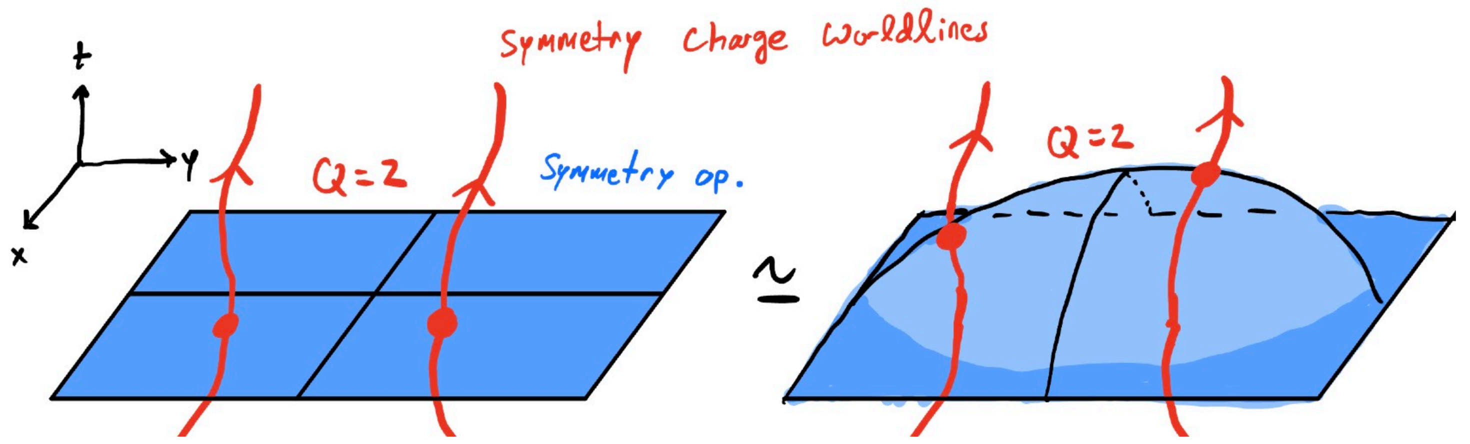


tl;dr

Generalized symmetries emerge at low energies in ordered phases. Thus, the most exotic symmetries appear in the most ordinary settings, and provide valuable insights into their phases and transitions.

The symmetry renassiance

- Global Symmetries → Topological operators



- The central dogma for generalized symmetries:

Topological operators ↔ Global Symmetries

	Symmetry Operator	Fusion Rule
Ordinary	Codimension 1	$U_a U_b = U_{a\cdot b}$
Higher-form	Codimension >1	———
Non-invertible	———	$U_a U_a^\dagger \neq 1$

- Why call these symmeties?

1. There is an operator U_a with $U_a H = H U_a$
2. Symmetry charges can condense $\Longrightarrow$ SSB phases
3. Can have 't Hooft anomalies $\Longrightarrow$ SPT phases

If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

Exact emergent symmetries

- Are generalized symmetries relevant to cond-mat?

Microscopics: only ordinary symmetries
At low-energy: ordinary & generalized symmetries

- How good are emergent symmetries?

Ordinary	Higher-form
Approximate since broken by local (irrelevant) opeartors	*Exact since only broken by nonlocal operators*

- Emergent higher-form symmetries can:

1. Sponteanously break $\Longrightarrow$ give rise to topological order and emergent photons
2. Characterize phase transitions
3. Be anomalous $\Longrightarrow$ characterize SPT phases

*See complete story at: **SDP** & X-G Wen arXiv:2301:05261*

SSB and homotopy defects

- Sponteanously breaking invertible 0-form symmetry

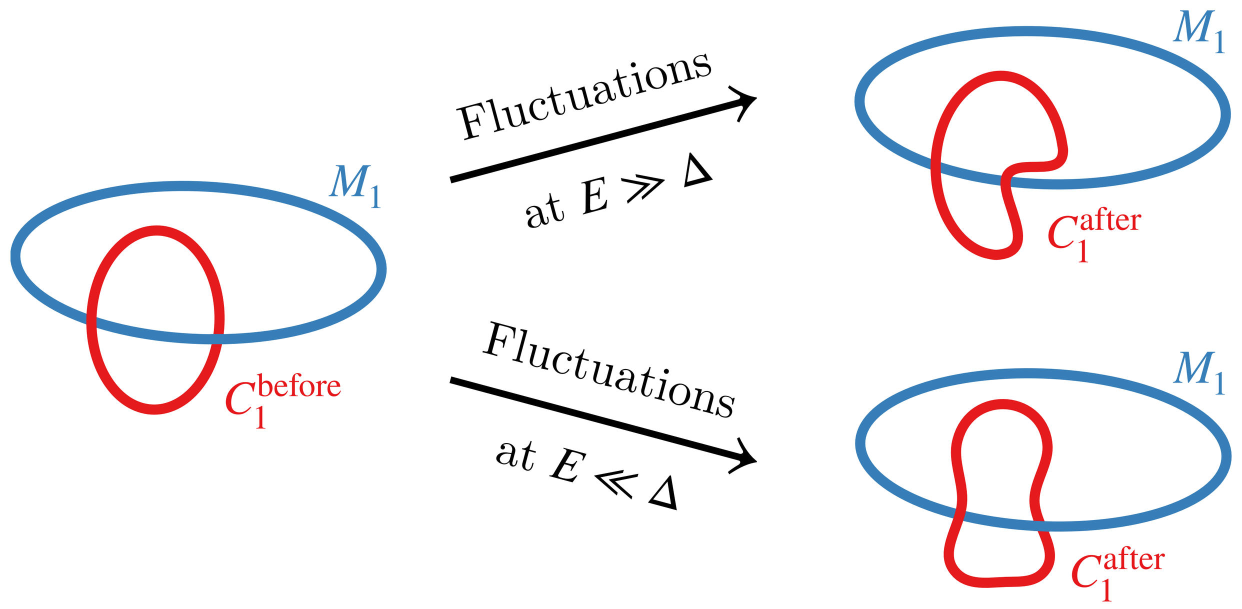
$$G \xrightarrow{\text{ssb}} H$$

produces two types of excitations:

1. Gapless Goldstone modes when G is continuous
2. Gapped homotopy defects classified by free homotopy classes $[C_k, G/H]$, where C_k is a k -submanifold. When $C_k \simeq S^k$, classification based on homotopy groups $\pi_k(G/H)$

Emergent generalized symmetries in ordered phases

- Homotopy defects are detected by topological opeartors at low energies



Examples:

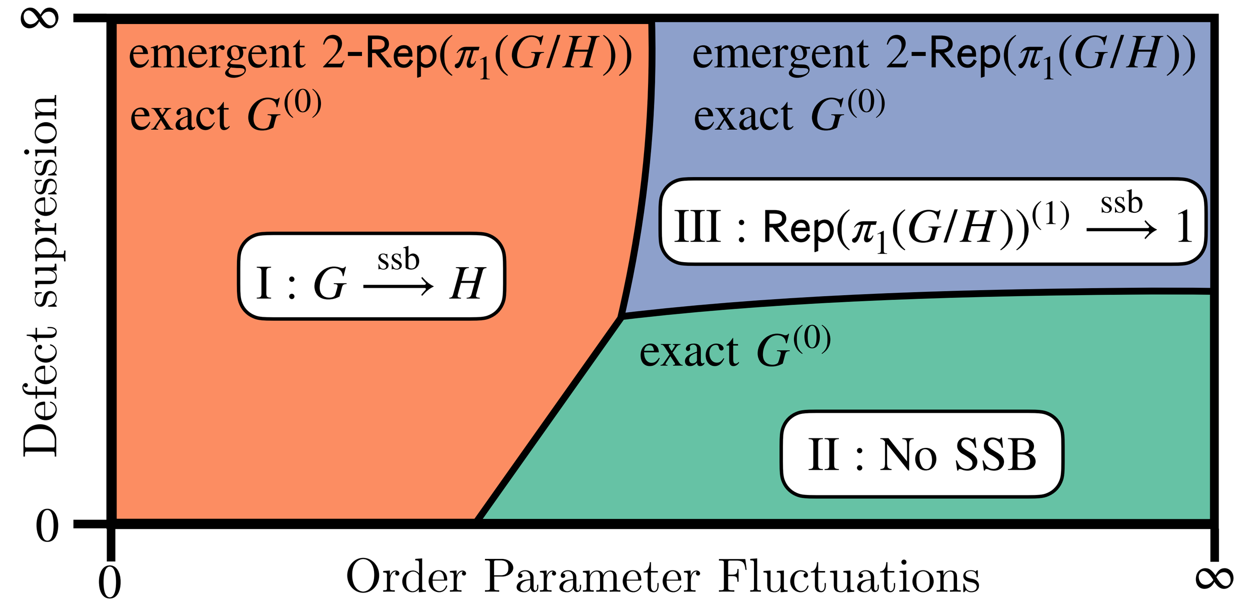
1. $e^{i\int_{M_1} ds_i \frac{\epsilon_{ij}\partial_j\theta}{2\pi}}$ detects vorticies in a 2d superfluid
2. $e^{i\int_{M_2} dA_i \frac{\epsilon_{ijk}\vec{n}\cdot(\partial_j\vec{n}\times\partial_k\vec{n})}{8\pi}}$ detects skyrmions in a 2d magnet

- Homotopy defects are charged objects under this emergent symmetry

1. p-dimensional defect $\Longrightarrow$ p-form symmetry
2. $\pi_k(G/H)$ defects $\Longrightarrow$ symmetry group: $\mathrm{Hom}(\pi_k(G/H), U(1))$
3. $[S^1, G/H]$ defects $\Longrightarrow$ symmetry category: d-$\mathrm{Rep}(\pi_1(G/H))$

- This emergent symmetry can:

1. have a mixed 't Hooft anomaly with G
2. sponteanously break $\Longrightarrow$ nontrivial disordered phases



*See complete story at: **SDP** arXiv:2307:?????*