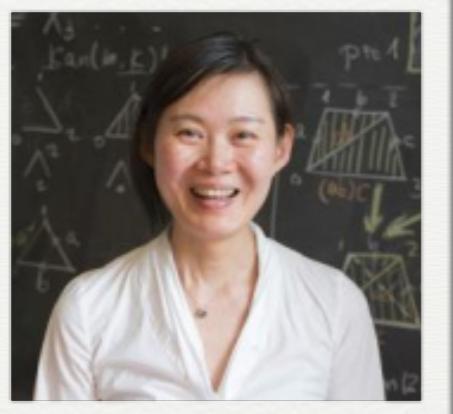


GENERALIZED SYMMETRIES AND QUANTUM DISORDERING

Sal Pace (MIT)

SP, arXiv:2308.05730

SP, C Zhu, A Beaudry, and X-G Wen, arXiv:2310.08554



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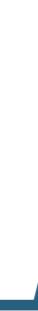
A SYMMETRY RENAISSANCE

Our understanding of **symmetry** has been revolutionized
through modern generalizations

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Our understanding of **symmetry** has been revolutionized through modern generalizations

Higher-form symmetry



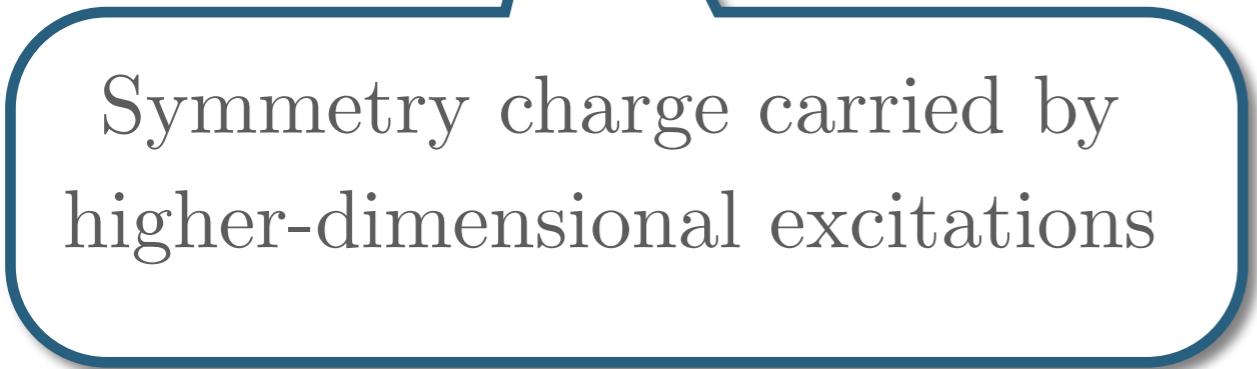
Symmetry charge carried by
higher-dimensional excitations

A SYMMETRY RENAISSANCE

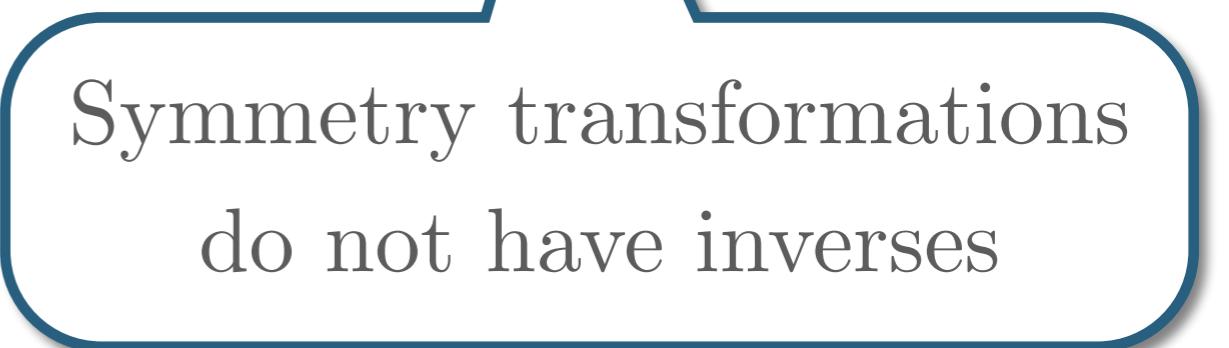
Our understanding of **symmetry** has been revolutionized through modern generalizations

Higher-form symmetry

Non-invertible symmetry



Symmetry charge carried by higher-dimensional excitations



Symmetry transformations do not have inverses

A SYMMETRY RENAISSANCE

Our understanding of **symmetry** has been revolutionized through modern generalizations

Higher-form symmetry

Non-invertible symmetry

Dipole symmetry

Higher-group symmetry

Subsystem symmetry

Biform symmetry

Harmonic symmetry

Framed symmetry

Modulated symmetry

Bundle symmetry

Lower-form symmetry

A SYMMETRY RENAISSANCE

Our understanding of **symmetry** has been revolutionized through modern generalizations



Higher symmetry

Subsystem symmetry

Harmonic symmetry

Bundle symmetry

Geometry

No symmetry

Symmetry

Symmetry

Symmetry

Trained symmetry

Module symmetry

Lower-form symmetry

BUILD-A-PHASE WORKSHOP, INC.

A recipe for exploring and characterizing phases of matter

- (1) Choose your **generalized symmetries** adjectives

$a_1-a_2-a_3-\dots$ Symmetry

- (2) Specify **SSB** pattern

- (3) Specify residual symmetries **SPT** pattern

BUILD-A-PHASE WORKSHOP, INC.

A recipe for exploring and characterizing phases of matter

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3 + 1D Example:

a_1 a_2 a_3 a_4

Finite non-invertible 1-form subsystem symmetry

- **SSB** gives non-abelian fractons

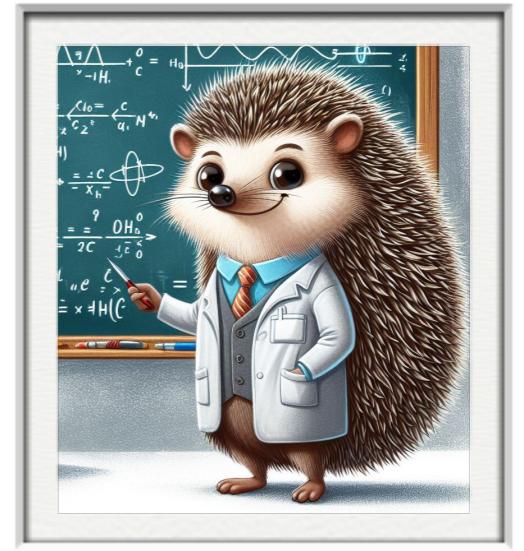
SP, arXiv:2308.05730

TL;DR

SP, C Zhu, A Beaudry, and X-G Wen, arXiv:2310.08554

Generalized symmetries (non-invertible + higher-form) exist in ordinary ordered phases

- Superfluids, nematics, magnets, etc
- Related to topological defects (*hedgehogs, vortices, skyrmions, etc*)



Spontaneously breaking them drives a transition into an exotic disordered phase of matter with

- Symmetry enriched (non-)abelian topological orders
- Interesting gapless modes (*e.g., emergent photons*)

ORDERED PHASES

A phase where an ordinary internal symmetry G is spontaneously broken

- Universal features determined by the SSB pattern

$$G \xrightarrow{\text{ssb}} H \subset G$$

- Order parameter takes values in the coset space

$$\mathcal{M} = G/H = \{gH : g \in G\}$$

Topological defects are gapped excitations characterized by the topology of \mathcal{M}

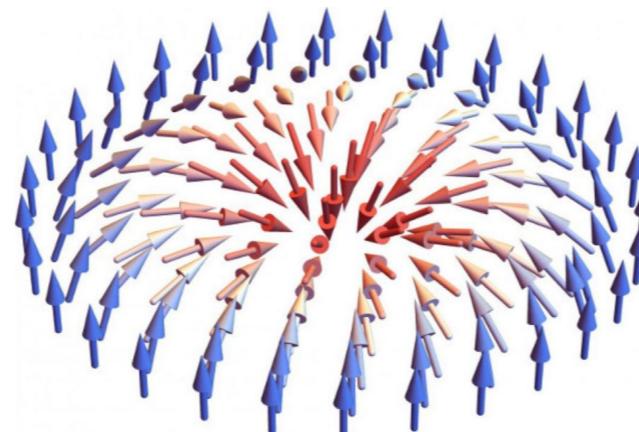
- Can be dynamical or non-dynamical (i.e., frozen in place)

SYMMETRIES IN ORDERED PHASES

When **topological defects** are non-dynamical, their topological invariants are conserved \implies a corresponding **symmetry**

Example 1: $SO(3) \xrightarrow{\text{ssb}} SO(2)$ in 2 + 1D

- $\mathcal{M} = S^2$ and $\pi_2(\mathcal{M}) \simeq \mathbb{Z}$ labels **skyrmions**



- **Skyrmion** number conservation \implies **$U(1)$ symmetry**

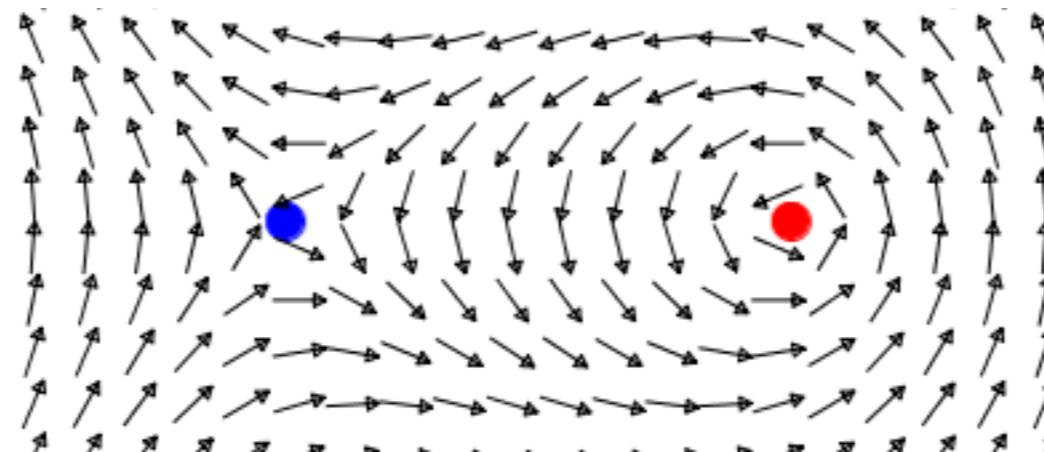
$$\mathcal{S}_\pi = U(1)$$

SYMMETRIES IN ORDERED PHASES

When **topological defects** are non-dynamical, their topological invariants are conserved \implies a corresponding **symmetry**

Example 2: $U(1) \xrightarrow{\text{ssb}} 1$ in 2 + 1D

- $\mathcal{M} = S^1$ and $\pi_1(\mathcal{M}) \simeq \mathbb{Z}$ labels **vortices**



- Winding number flux conservation $\implies U(1)$ 1-form symmetry

[Gaiotto, *et al* (2015)]

$$\mathcal{S}_\pi = U(1)^{(1)}$$

SYMMETRIES IN ORDERED PHASES

When **topological defects** are non-dynamical, their topological invariants are conserved \implies a corresponding **symmetry**

Example 3: $SO(3) \xrightarrow{\text{ssb}} \mathbb{Z}_2 \times \mathbb{Z}_2$ in 2 + 1D

- $\mathcal{M} = SU(2)/Q_8$ and conjugacy classes of $\pi_1(\mathcal{M}) \simeq Q_8$ labels vortices

$$[Q_8] = \{[1], [-1], [\pm i\sigma^x], [\pm i\sigma^y], [\pm i\sigma^z]\}$$

- $[Q_8]$ flux conservation \implies **Rep(Q_8)** 1-form symmetry
[SP, arXiv:2308.05730]

$$\mathcal{S}_\pi = \text{2-Rep}(Q_8)$$

THE ABSTRACT NONSENSE

In $d + 1$ D, **topological defects** are classified by the homotopy d -type of \mathcal{M} :

1. $\pi_1(\mathcal{M}), \pi_2(\mathcal{M}), \dots, \pi_d(\mathcal{M})$
2. $\pi_1(\mathcal{M})$ action on $\pi_k(\mathcal{M})$
3. Postnikov k -invariants (special cocycles)

Homotopy d -type data captured by a d -group $\mathbb{G}_\pi^{(d)}$:

$$\mathcal{M}_{\leq d} \simeq B\mathbb{G}_\pi^{(d)}$$

► S_π symmetry defects = $\mathbb{G}_\pi^{(d)}$ electric defects

$$\mathcal{S}_\pi = d\text{-Rep}(\mathbb{G}_\pi^{(d)})$$

SPONTANEOUSLY BREAKING \mathcal{S}_π

\mathcal{S}_π is not spontaneously broken in the ordered phase

- Topological defects are confined (*i.e.*, *area law*)
- e.g., superfluid vortices are log confined

\mathcal{S}_π can spontaneously break, driving a transition out of the ordered phase

- Topological defects will deconfine (*i.e.*, *perimeter law*)
- Typically leads to an exotic phase of matter

SPONTANEOUSLY BREAKING \mathcal{S}_π

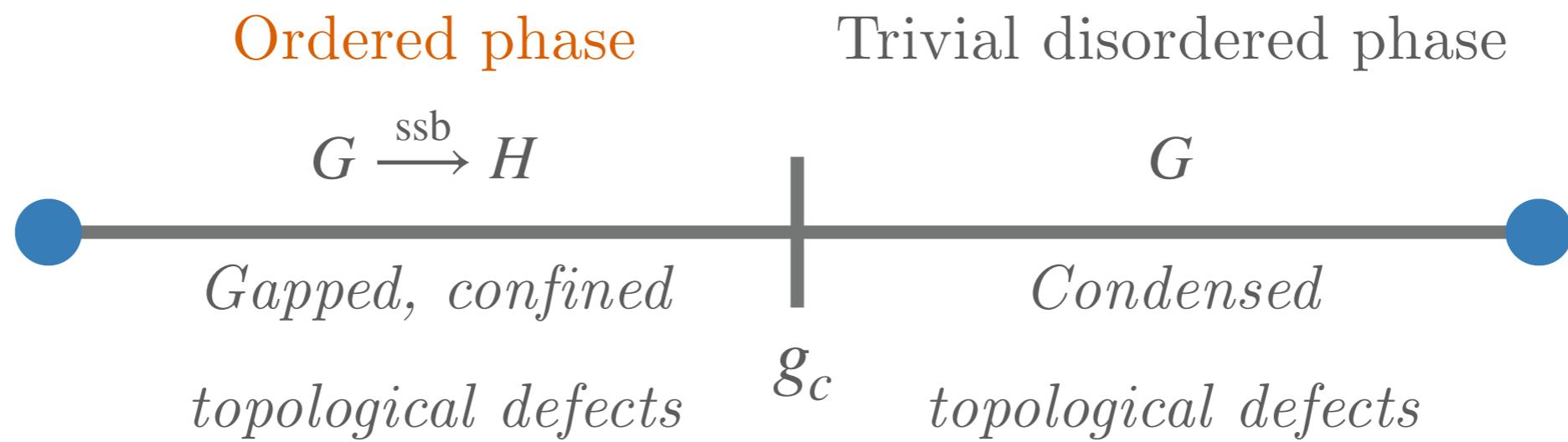
What happens to the microscopic G symmetry when spontaneously breaking \mathcal{S}_π ?

- Ordered states ($G \xrightarrow{\text{ssb}} H$) want to confine **topological defects**
- \mathcal{S}_π SSB states have a \mathcal{S}_π symmetry charge condensate that wants to deconfine **topological defects**
- The latter contradicts the former:

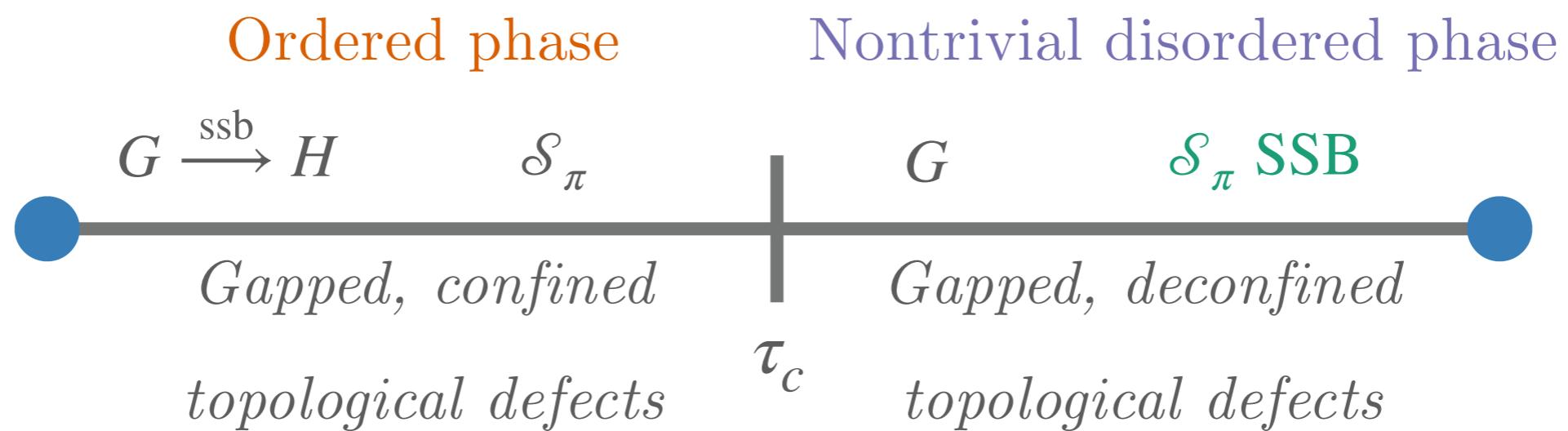
Spontaneously breaking \mathcal{S}_π must restore G

TWO KINDS OF DISORDERINGS

With dynamical **defects** \implies without \mathcal{S}_π :



With non-dynamical **defects** \implies with \mathcal{S}_π :



THE POWER OF SYMMETRY

\mathcal{S}_π is a non-perturbative tool to identify and classify exotic disordered phases neighboring ordered phases [SP, arXiv:2308.05730]

d	Ordered phase $G \xrightarrow{\text{ssb}} H$	Nontrivial disordered phases
3	$U(1) \xrightarrow{\text{ssb}} 1$	none
3	$U(1) \times U(1) \xrightarrow{\text{ssb}} 1$	$U(1)^{(1)} \xrightarrow{\text{ssb}} \mathbb{Z}_n^{(1)}$
2	$SO(3) \xrightarrow{\text{ssb}} 1$	$\mathbb{Z}_2^{(1)} \xrightarrow{\text{ssb}} 1$
2	$SO(3) \xrightarrow{\text{ssb}} \mathbb{Z}_2 \times \mathbb{Z}_2$	$\text{Rep}(Q_8)^{(1)} \xrightarrow{\text{ssb}} H^{(1)}$ ($H = 1, \mathbb{Z}_2, \mathbb{Z}_4$)

- Finite $\mathbb{G}_\pi^{(d)}$: nontrivial disordered phase = deconfined phase of $\mathbb{G}_\pi^{(d)}$ gauge theory
- Nonfinite $\mathbb{G}_\pi^{(d)}$: nontrivial disordered phase = gapless phase

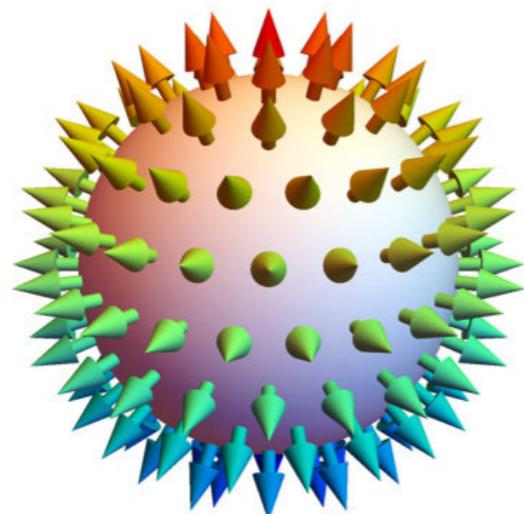
DISORDERING A 3 + 1D AFM

Consider SSB pattern $SO(3) \xrightarrow{\text{ssb}} SO(2)$ in 3 + 1D

Two types of **topological defects**

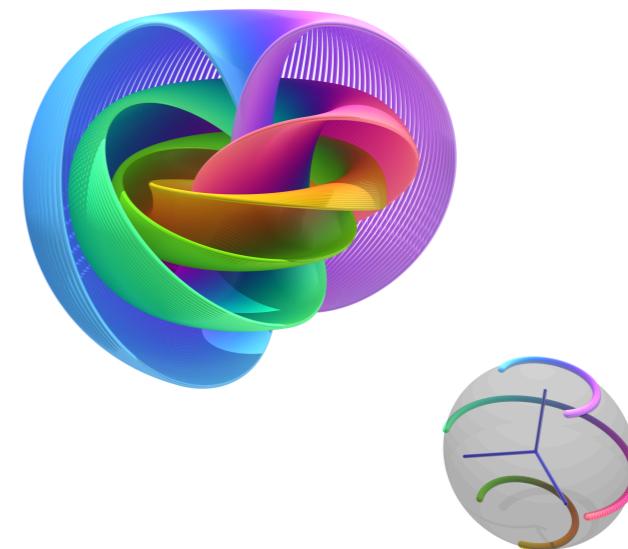
(1) Hedgehogs

- classified by $\pi_2(S^2) = \mathbb{Z}$
- Particles in space



(2) Hopfions

- classified by $\pi_3(S^2) = \mathbb{Z}$
- Textures in space



DISORDERING A 3 + 1D AFM

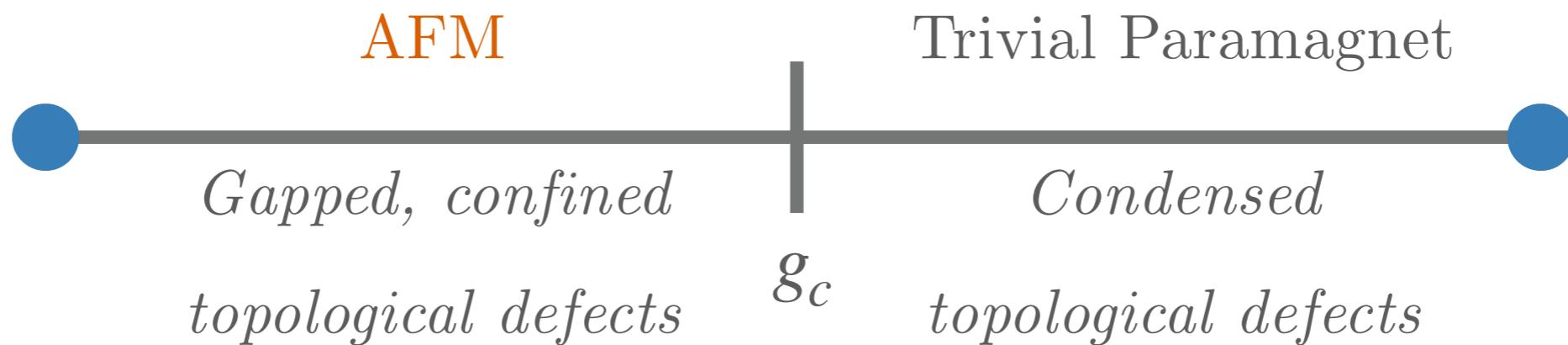
Consider SSB pattern $SO(3) \xrightarrow{\text{ssb}} SO(2)$ in 3 + 1D

Three types of disordering transitions

[SP, C Zhu, A Beaudry, and X-G Wen, arXiv:2310.08554]

(1) Dynamical hedgehogs and dynamical hopfions

- \mathcal{S}_π is trivial



DISORDERING A 3 + 1D AFM

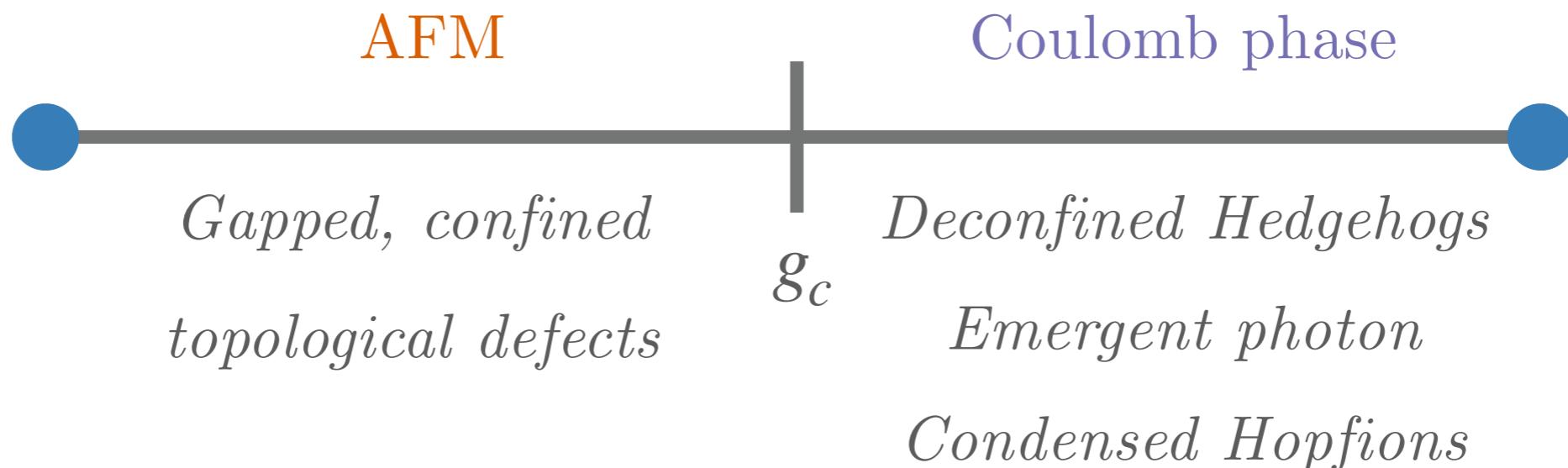
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Three types of disordering transitions

[SP, C Zhu, A Beaudry, and X-G Wen, arXiv:2310.08554]

(2) Non-dynamical hedgehogs and dynamical hopfions

- \mathcal{S}_π describes a **$U(1)$ 1-form symmetry**



DISORDERING A 3 + 1D AFM

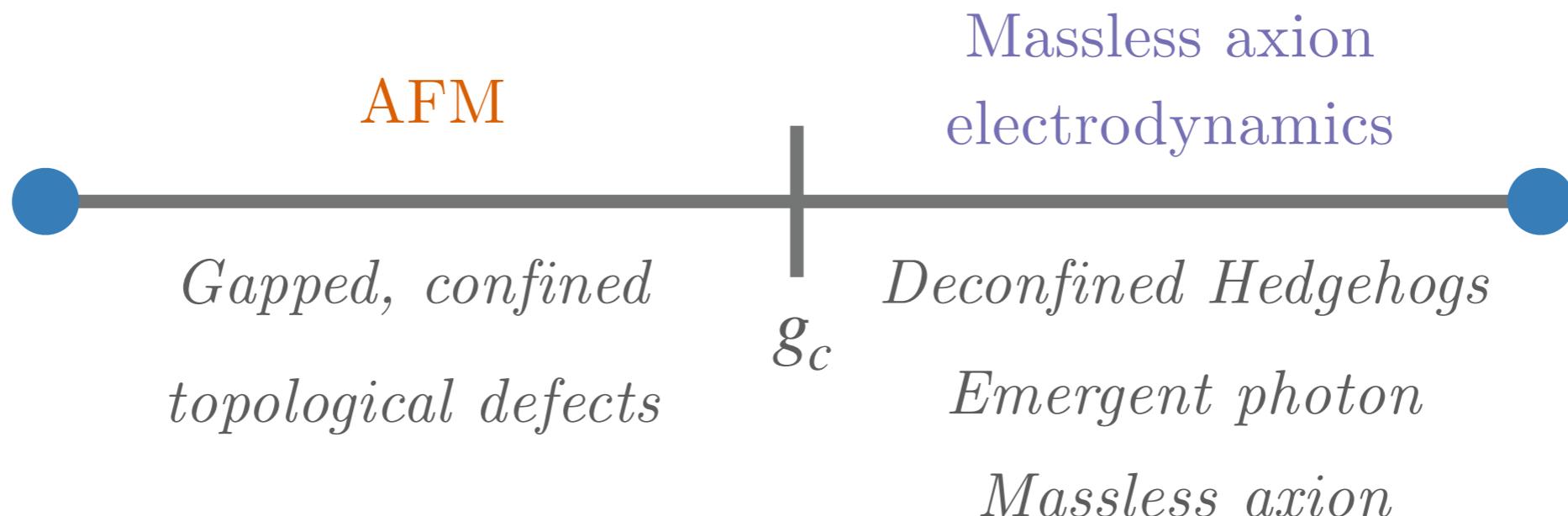
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Three types of disordering transitions

[SP, C Zhu, A Beaudry, and X-G Wen, arXiv:2310.08554]

(3) Non-dynamical hedgehogs and non-dynamical hopfions

- \mathcal{S}_π describes a **$U(1)$ 1-form** and **non-invertible 0-form symmetries**



THANKS FOR LISTENING

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SP, arXiv:2308.05730

SP, C Zhu, A Beaudry, and X-G Wen, arXiv:2310.08554

- (1) Generalized symmetries (non-invertible + higher-form)
exist in ordinary ordered phases
- (2) SSBing them drives a transition into an exotic disordered
phase of matter \implies classification of disordered phases

