Defect-free quantum disordering: a generalized symmetries perspective

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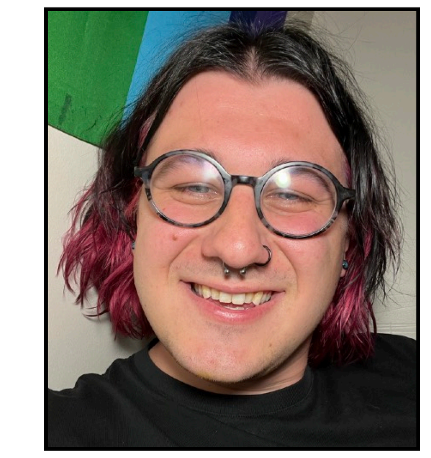


SCAN ME

Summary

1. Generalized symmetries $\mathcal{S}_\pi$ exist in ordered phases without defects (e.g., vortices, hedgehogs, etc)
   - $\mathcal{S}_\pi$ is emergent in defect-suppresed ordered phases
2. Spontaneously breaking $\mathcal{S}_\pi$ drives a phase transition into a nontrivial disordered phase.
3. Proposal: defect-free disordered phases classified by spontaneous symmetry breaking patterns of $\mathcal{S}_\pi$

S Pace, arXiv:2308.05730

S Pace, C Zhu, A Beaudry, X-G Wen arXiv:2310.08554

Example 1: Defect-free $O(3)$ model

$$S = \frac{1}{2}\int \mathrm{d}t\, \mathrm{d}^d x\, (\partial_\mu \vec{n})^2 \qquad \vec{n}\cdot\vec{n} = 1 \qquad SO(3) \xrightarrow{\text{ssb}} SO(2)$$

2 + 1D: Defect-free condition

$$\partial_\mu J^\mu = 0 \qquad J^\mu = \frac{\epsilon^{\mu\nu\rho}}{8\pi}\vec{n}\cdot(\partial_\nu \vec{n}\times\partial_\rho\vec{n})$$

Skyrmion number $N = \int \mathrm{d}^2\vec{x}\; J_0 \in \mathbb{Z}$ is conserved

$\longrightarrow$ $\mathcal{S}_\pi = U(1)$ symmetry

Defect-free disordered phase:

$\longrightarrow$ Spontaneously break $\mathcal{S}_\pi$: Skyrmion superfluid

3 + 1D: Defect-free condition

$$\partial_\mu J^{\mu\nu} = 0 \qquad J^{\mu\nu} = \frac{\epsilon^{\mu\nu\rho\sigma}}{8\pi}\vec{n}\cdot(\partial_\rho \vec{n}\times\partial_\sigma\vec{n})$$

"Skyrmion flux" $N(\Sigma) = \int_\Sigma \mathrm{d}S\; \hat{n}_k J_{0k} \in \mathbb{Z}$ is conserved

$\longrightarrow$ $\mathcal{S}_\pi = U(1)$ 1-form symmetry

Defect-free disordered phase:

$\longrightarrow$ Spontaneously break $\mathcal{S}_\pi$: Coulomb phase

Generalized symmetries 101

	Symmetry Operator	Fusion Rule
Ordinary	d-dimensional	$U_a U_b = U_{a\cdot b}$
p-form	$(d-p)$-dimensional	*unspecified*
Non-invertible	*unspecified*	$U_a U_b = \sum_c N_{ab}^c U_c$

Mathematical description?

Ordinary symmetries $\Longrightarrow$ group G

Generalized symmetries $\Longrightarrow$ higher category $\mathscr{C}$

Why call these symmetries?

1. Symmetry operator U_a obeys $U_a H = H U_a$
2. There exist symmetry defects (i.e., twist defects)
3. Characterize phases of matter (e.g., SPT and SSB)

If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

Example 2: Defect-free nematic

Consider 2 + 1D magnet with SSB pattern:

$$SO(3) \xrightarrow{\text{ssb}} \mathbb{Z}_2\times\mathbb{Z}_2$$

$\longrightarrow$ Phase supports vortices classified by

$$[Q_8] = \{[1],[-1],[\pm i\sigma^x],[\pm i\sigma^y],[\pm i\sigma^z]\}$$

Without vortices, there is a conserved flux whose quantum numbers are $[Q_8]$

$\longrightarrow$ $\mathcal{S}_\pi = \mathrm{Rep}(Q_8)$ 1-form non-invertible symmetry

$\longrightarrow$ Symmetry operators labeled by representations of Q_8

Defect-free disordered phase:

$\longrightarrow$ Spontaneously break $\mathcal{S}_\pi$: Q_8 quantum double phase

$\longrightarrow$ Nonabelian topological order

The general story

Consider an ordered phase in $d+1$D with spontaneous symmetry breaking pattern

$$G \xrightarrow{\text{ssb}} H$$

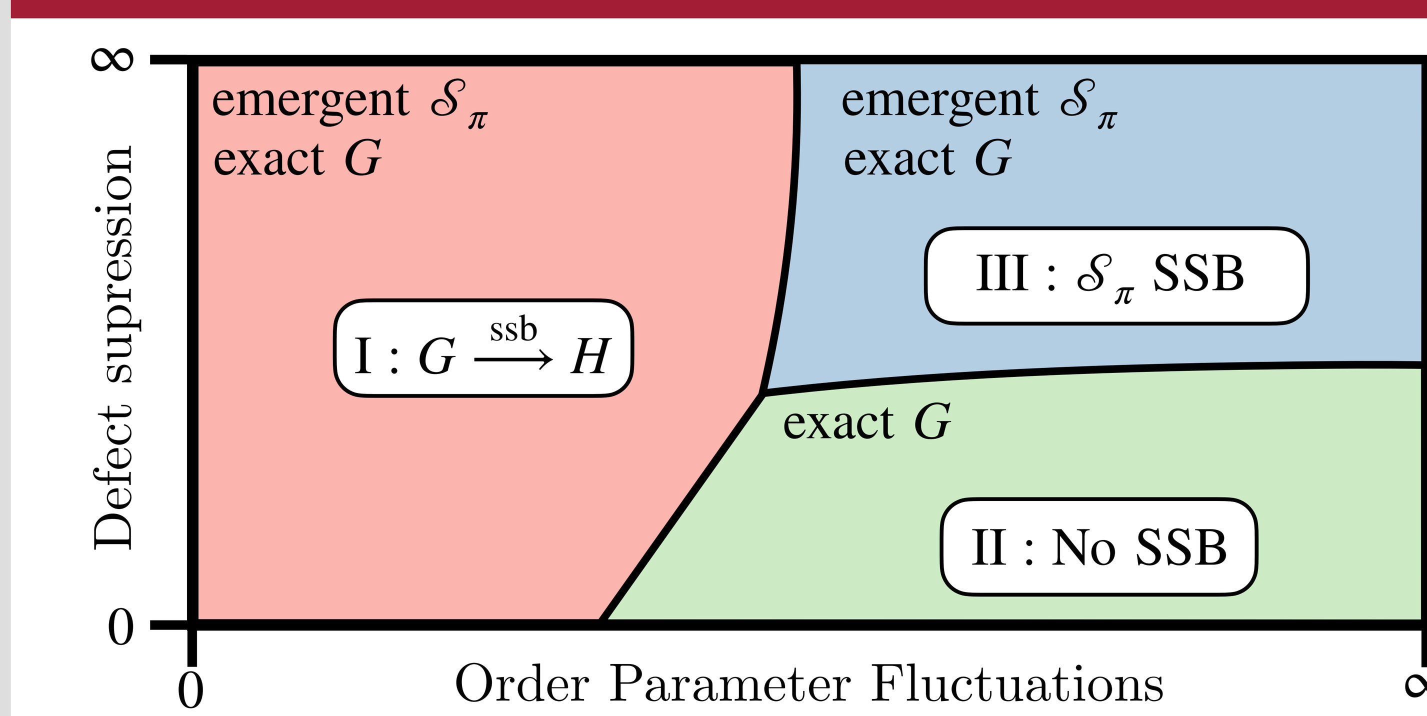
- Lessons from examples 1 and 2:
  1. Defect-free condition $\Longrightarrow$ symmetry $\mathcal{S}_\pi$
  2. $\mathcal{S}_\pi$ symmetry numbers = defect's topological numbers
- Defects are classified by the homotopy d-type of G/H:
  1. $\pi_1(G/H)$, $\pi_2(G/H)$, $\cdots$, $\pi_d(G/H)$
  2. $\pi_1(G/H)$ action on $\pi_k(G/H)$
  3. Postnikov k-invariants (special cocycles)
- Homotopy d-type data captured by a d-group $\mathbb{G}_\pi^{(d)}$:

$$(G/H)_{\leq d} \simeq B\mathbb{G}_\pi^{(d)}$$

$\Longrightarrow$ G/H defects classification = $\mathbb{G}_\pi^{(d)}$ Magnetic defects:

$$\mathcal{S}_\pi = d\text{-Rep}(\mathbb{G}_\pi^{(d)})$$

Schematic phase diagram



I: Ordered phase (defects confined)

II: Trivial disordered phase (defects proliferated)

III: Nontrivial disordered phase (defects deconfined)

$\Longrightarrow$ $\mathcal{S}_\pi$ SSB patterns classify these disordered phases