GENERALIZED SYMMETRIES AND QUANTUM DISORDERING

Sal Pace (MIT)

SP, arXiv:2308.05730

SP, C Zhu, A Beaudry, and X-G Wen, arXiv:2310.08554

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A SYMMETRY RENAISSANCE

Our understanding of symmetry has been revolutionized through modern generalizations

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Higher-form symmetry

Symmetry charge carried by higher-dimensional excitations

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Higher-form symmetry

Symmetry charge carried by higher-dimensional excitations

Non-invertible symmetry

Symmetry transformations do not have inverses

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Higher-form symmetry

Non-invertible symmetry

Dipole symmetry

Higher-group symmetry

Subsystem symmetry

Biform symmetry

Harmonic symmetry

Framed symmetry

Modulated symmetry

Bundle symmetry

Lower-form symmetry

A SYMMETRY RENAISSANCE

Our understanding of symmetry has been revolutionized through modern generalizations

Bundle symmetry

Lower-form symmetry

BUILD-A-PHASE WORKSHOP, INC.

A recipe for exploring and characterizing phases of matter

(1) Choose your generalized symmetries adjectives

$$\mathrm{a}_1\text{–}\mathrm{a}_2\text{–}\mathrm{a}_3\text{–}\cdots\ \text{Symmetry}$$

(2) Specify SSB pattern

(3) Specify residual symmetries SPT pattern

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(3) Specify residual symmetries SPT pattern

$3+1$D Example:

a_1 a_2 a_3 a_4

Finite non-invertible 1-form subsystem symmetry

➤ SSB gives non-abelian fractons

TL;DR

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Generalized symmetries (non-invertible + higher-form) exist in ordinary ordered phases

- Superfluids, nematics, magnets, etc
- Related to topological defects (*hedgehogs, vortices, skyrmions, etc*)

Spontaneously breaking them drives a transition into an exotic disordered phase of matter with

- Symmetry enriched (non-)abelian topological orders
- Interesting gapless modes (*e.g., emergent photons*)

ORDERED PHASES

A phase where an ordinary internal symmetry G is spontaneously broken

- Universal features determined by the SSB pattern

$$G \xrightarrow{\text{ssb}} H \subset G$$

- Order parameter takes values in the coset space

$$\mathscr{M} = G/H = \{gH : g \in G\}$$

Topological defects are gapped excitations characterized by the topology of $\mathscr{M}$

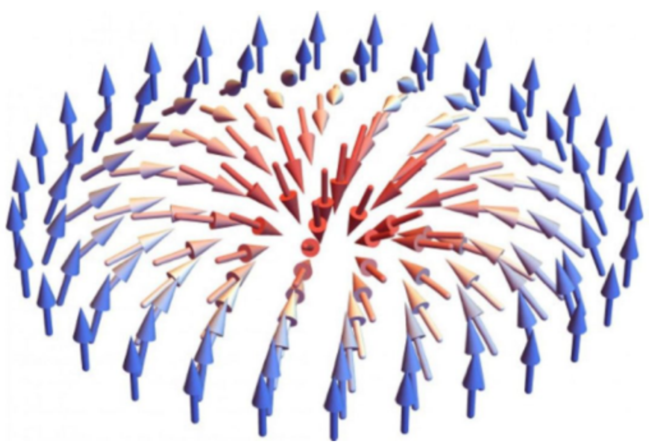
- Can be dynamical or non-dynamical (i.e., frozen in place)

SYMMETRIES IN ORDERED PHASES

When topological defects are non-dynamical, their topological invariants are conserved $\Longrightarrow$ a corresponding symmetry

$$\text{Example 1: } SO(3) \xrightarrow{\text{ssb}} SO(2) \text{ in } 2+1\text{D}$$

- $\mathscr{M} = S^2$ and $\pi_2(\mathscr{M}) \simeq \mathbb{Z}$ labels skyrmions



- Skyrmion number conservation $\Longrightarrow$ $U(1)$ symmetry

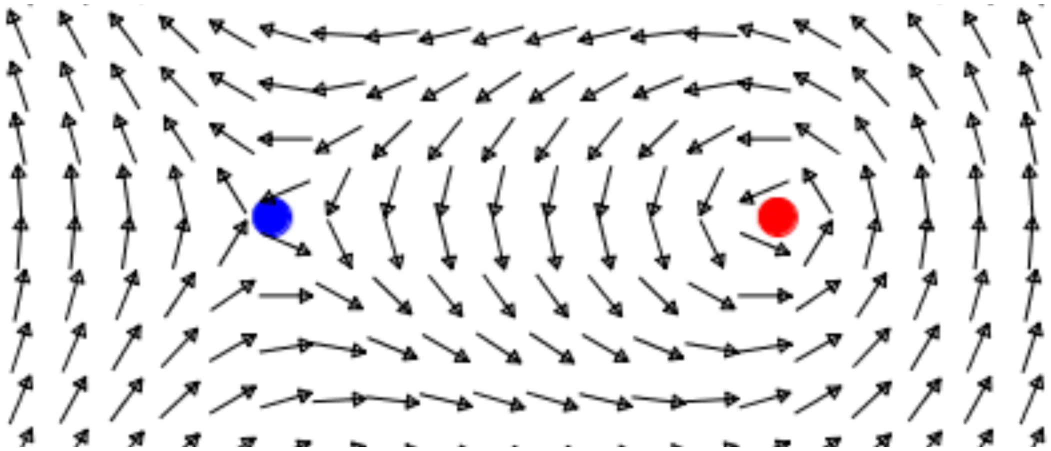
$$\mathcal{S}_\pi = U(1)$$

SYMMETRIES IN ORDERED PHASES

When topological defects are non-dynamical, their topological invariants are conserved $\implies$ a corresponding symmetry

$$\text{Example 2: } U(1) \xrightarrow{\text{ssb}} 1 \text{ in } 2+1\text{D}$$

- $\mathscr{M} = S^1$ and $\pi_1(\mathscr{M}) \simeq \mathbb{Z}$ labels vortices



- Winding number flux conservation $\implies$ $U(1)$ 1-form symmetry

  [Gaiotto, *et al* (2015)]

$$\mathcal{S}_\pi = U(1)^{(1)}$$

SYMMETRIES IN ORDERED PHASES

When topological defects are non-dynamical, their topological invariants are conserved $\Longrightarrow$ a corresponding symmetry

$$\text{Example 3: } SO(3) \xrightarrow{\text{ssb}} \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ in } 2+1\text{D}$$

- $\mathscr{M} = SU(2)/Q_8$ and conjugacy classes of $\pi_1(\mathscr{M}) \simeq Q_8$ labels vortices

$$[Q_8] = \{[1], [-1], [\pm \mathrm{i}\sigma^x], [\pm \mathrm{i}\sigma^y], [\pm \mathrm{i}\sigma^z]\}$$

- $[Q_8]$ flux conservation $\Longrightarrow$ $\mathrm{Rep}(Q_8)$ 1-form symmetry

  [**SP**, arXiv:2308.05730]

$$\mathcal{S}_\pi = 2\text{-}\mathrm{Rep}(Q_8)$$

THE ABSTRACT NONSENSE

In $d + 1$D, topological defects are classified by the homotopy d-type of $\mathscr{M}$:

1. $\pi_1(\mathscr{M}), \pi_2(\mathscr{M}), \cdots, \pi_d(\mathscr{M})$
2. $\pi_1(\mathscr{M})$ action on $\pi_k(\mathscr{M})$
3. Postnikov k-invariants (special cocycles)

Homotopy d-type data captured by a d-group $\mathbb{G}_\pi^{(d)}$:

$$\mathscr{M}_{\leq d} \simeq B\mathbb{G}_\pi^{(d)}$$

➤ S_π symmetry defects $= \mathbb{G}_\pi^{(d)}$ electric defects

$$\mathcal{S}_\pi = d\text{-Rep}(\mathbb{G}_\pi^{(d)})$$

SPONTANEOUSLY BREAKING $\mathcal{S}_\pi$

$\mathcal{S}_\pi$ is not spontaneously broken in the ordered phase

- Topological defects are confined (*i.e., area law*)
- e.g., superfluid vortices are log confined

$\mathcal{S}_\pi$ can spontaneously break, driving a transition out of the ordered phase

- Topological defects will deconfine (*i.e., perimeter law*)
- Typically leads to an exotic phase of matter

SPONTANEOUSLY BREAKING $\mathcal{S}_\pi$

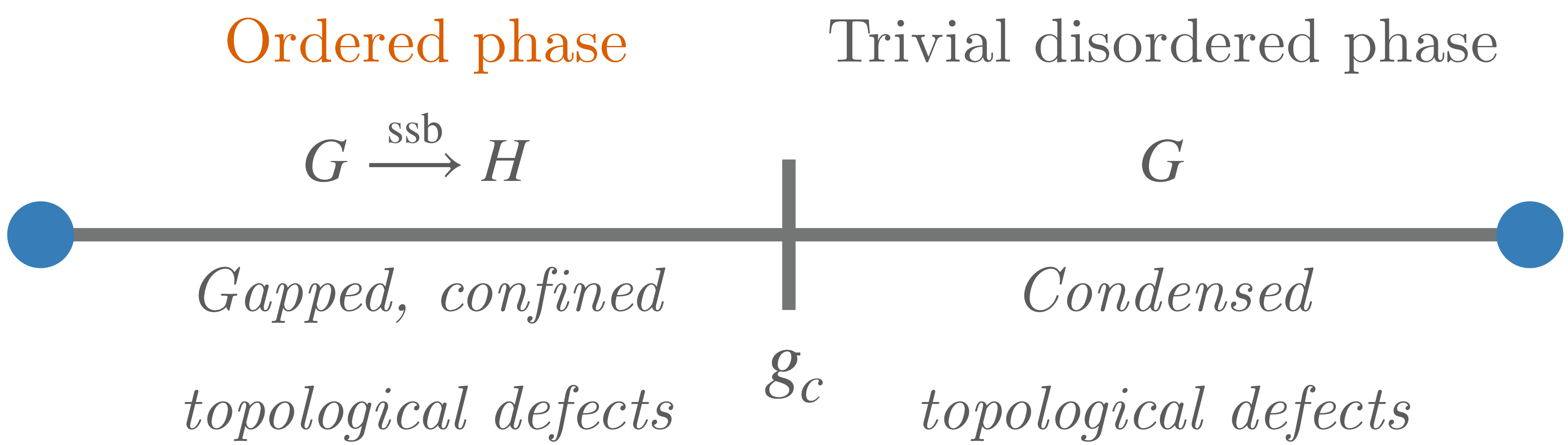
What happens to the microscopic G symmetry when spontaneously breaking $\mathcal{S}_\pi$?

- Ordered states ($G \xrightarrow{\text{ssb}} H$) want to confine topological defects
- $\mathcal{S}_\pi$ SSB states have a $\mathcal{S}_\pi$ symmetry charge condensate that wants to deconfine topological defects
- The latter contradicts the former:

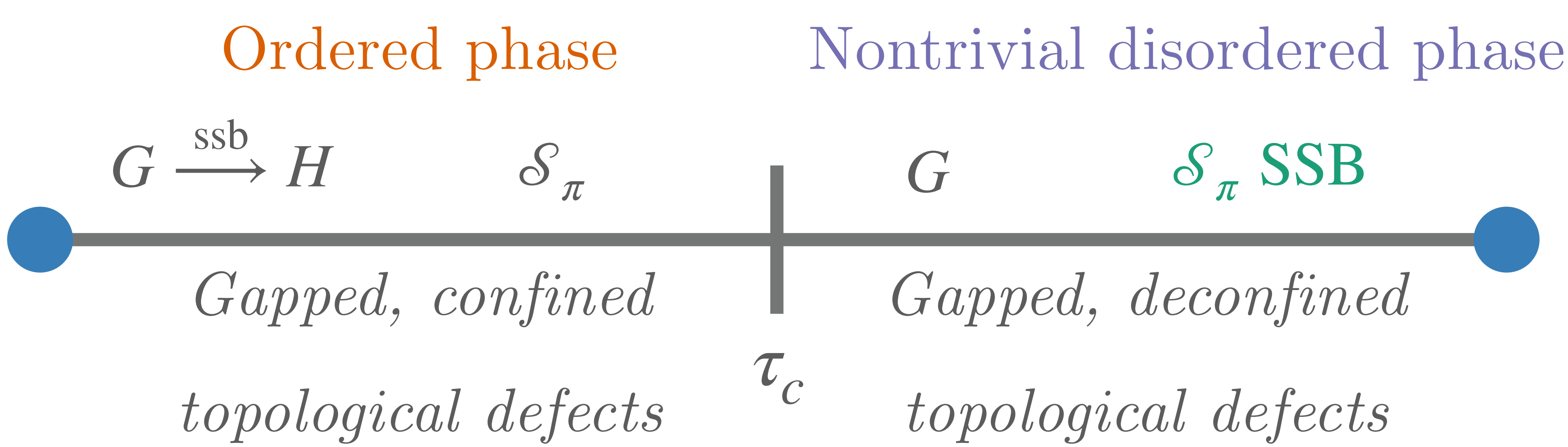
$$\boxed{\text{Spontaneously breaking } \mathcal{S}_\pi \text{ must restore } G}$$

TWO KINDS OF DISORDERINGS

With dynamical defects $\Longrightarrow$ without $\mathcal{S}_\pi$:



With non-dynamical defects $\Longrightarrow$ with $\mathcal{S}_\pi$:



THE POWER OF SYMMETRY

$\mathcal{S}_\pi$ is a non-perturbative tool to identify and classify exotic disordered phases neighboring ordered phases [**SP**, arXiv:2308.05730]

d	Ordered phase $G \xrightarrow{\text{ssb}} H$	Nontrivial disordered phases
3	$U(1) \xrightarrow{\text{ssb}} 1$	none
3	$U(1) \times U(1) \xrightarrow{\text{ssb}} 1$	$U(1)^{(1)} \xrightarrow{\text{ssb}} \mathbb{Z}_n^{(1)}$
2	$SO(3) \xrightarrow{\text{ssb}} 1$	$\mathbb{Z}_2^{(1)} \xrightarrow{\text{ssb}} 1$
2	$SO(3) \xrightarrow{\text{ssb}} \mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathrm{Rep}(Q_8)^{(1)} \xrightarrow{\text{ssb}} H^{(1)} \quad (H = 1, \mathbb{Z}_2, \mathbb{Z}_4)$

- Finite $\mathbb{G}_\pi^{(d)}$: nontrivial disordered phase = deconfined phase of $\mathbb{G}_\pi^{(d)}$ gauge theory

- Nonfinite $\mathbb{G}_\pi^{(d)}$: nontrivial disordered phase = gapless phase

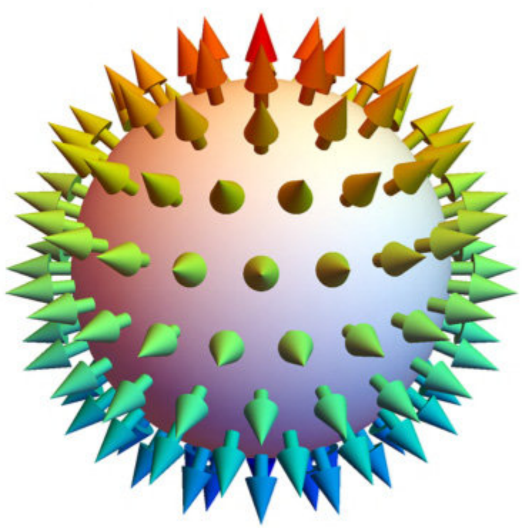
DISORDERING A $3+1$D AFM

Consider SSB pattern $SO(3) \xrightarrow{\text{ssb}} SO(2)$ in $3+1$D

Two types of topological defects

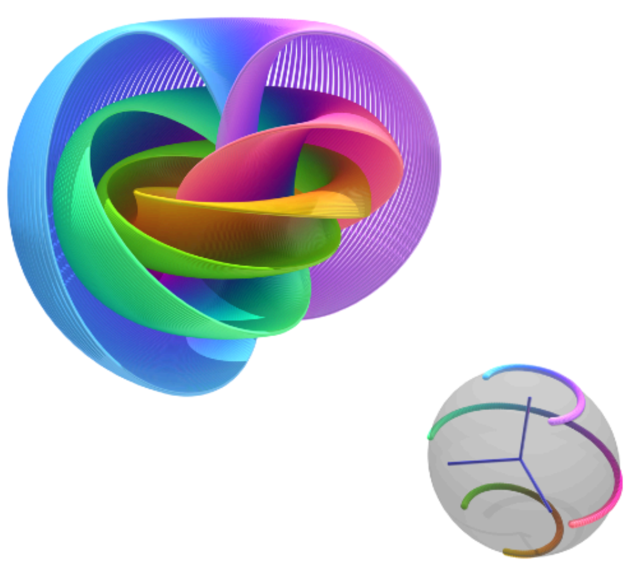
(1) Hedgehogs

- classified by $\pi_2(S^2) = \mathbb{Z}$
- Particles in space



(2) Hopfions

- classified by $\pi_3(S^2) = \mathbb{Z}$
- Textures in space



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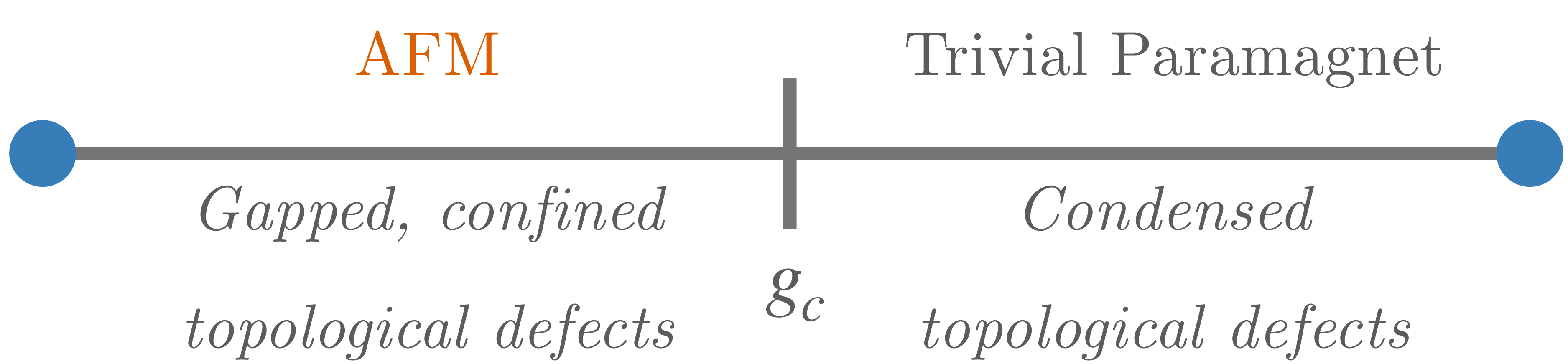
Three types of disordering transitions

[**SP**, C Zhu, A Beaudry, and X-G Wen, arXiv:2310.08554]

(1) Dynamical hedgehogs and dynamical hopfions

➤ $\mathcal{S}_\pi$ is trivial



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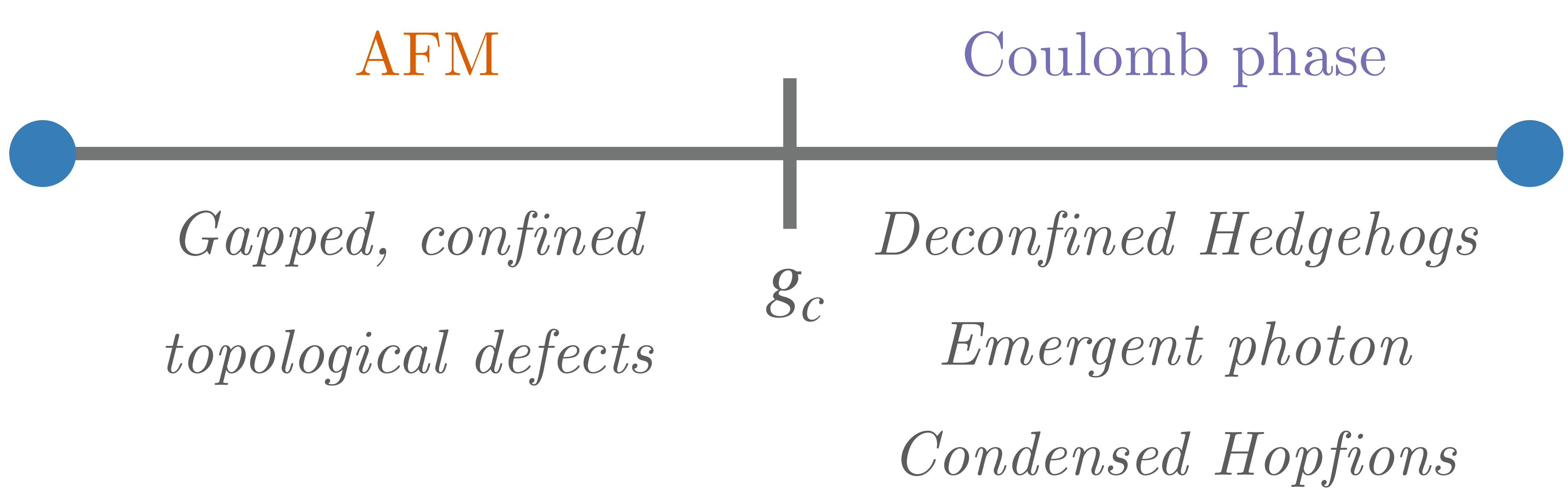
(2) Non-dynamical hedgehogs and dynamical hopfions

➤ $\mathcal{S}_\pi$ describes a $U(1)$ 1-form symmetry



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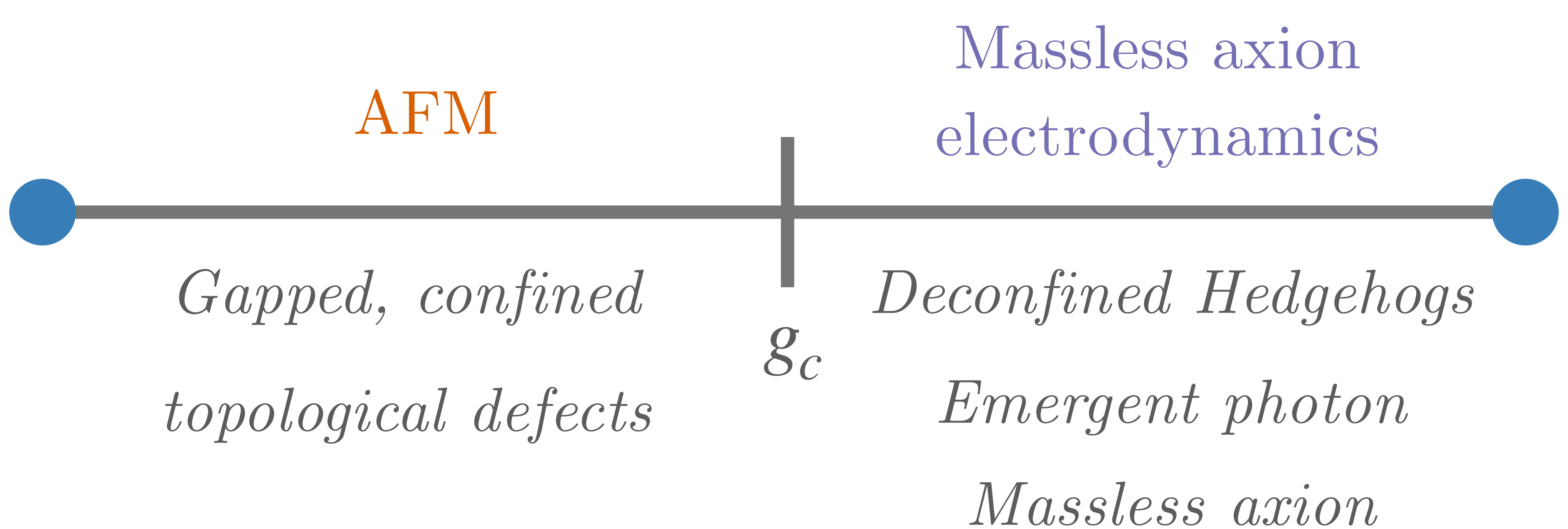
(3) Non-dynamical hedgehogs and non-dynamical hopfions

- $\mathscr{S}_\pi$ describes a $U(1)$ 1-form and non-invertible 0-form symmetries



THANKS FOR LISTENING

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(1) Generalized symmetries (non-invertible + higher-form) exist in ordinary ordered phases

(2) SSBing them drives a transition into an exotic disordered phase of matter $\Longrightarrow$ classification of disordered phases