Introduction to Financial Engineering (HW2)

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- Please follow the guidelines for assignments given in the Module Handbook.
- All programs should be written in R (compilable without errors or warnings).
- You should submit a write-up (.pdf) of the program as well as the source code (.r).
- File names should be as `yoursurname_yourname_HW2.extension`
- You should submit via moddle.
- Deadline: 24th October 2025 at 10am.

1. Run the following code corresponding three quarterly time series data and answer the questions below.

   (a) Plot both `Tbill` and `Del.Tbill`. Use both time series and ACF plots, ADF and KPSS test to show that the series are or are not stationary. What kind of heteroskedasticity can you see in `Del.Tbill` series?

   (b) Fit an ARMA(1,0)/GARCH(1,0) model using `garchFit` to the series believed to be stationary? What are the estimates for the parameters in the model?

   (c) What is plotted by `acf(res)`? What, if anything, does the plot tell you about the fit of the model?

   (d) What is plotted by `acf(res^2)`? What, if anything, does the plot tell you about the fit of the model?

   (e) What is plotted by `acf(res_std^2)`? What, if anything, does the plot tell you about the fit of the model?

   (f) Is there anything noteworthy in the plot produced by the `plot(res_std)`?

   (g) Now fit an ARMA/GARCH to the series `diff(log(Tbill))`. Do you see any advantages of working with the difference of the logarithms of the T-bill rate?

```r
data(Tbrate,package="Ecdat")
library(tseries)
library(fGarch)
#   r = the 91-day treasury bill rate
#   y = the log of real GDP
#   pi = the inflation rate

Tbill=Tbrate[,1]
Del.Tbill=diff(Tbill)

garch.model=garchFit(formula=~arma(1,0)+garch(1,0),Tbill)
summary(garch.model)
garch.model@fit$matcoef

res=residuals(garch.model)
res_std=res/garch.model@sigma.t
par(mfrow=c(2,3))
plot(res)
acf(res)
acf(res^2)
plot(res_std)
acf(res_std)
acf(res_std^2)
```

2. On Black Monday, the return on the S&P 500 was -22.8%. Ouch! This exercise attempts to answer the question, what was the conditional probability of a return this small on Black Monday? Conditional means given the information avaliable the previous trading day. Run the following code:

```
library(Ecdat)
library(fGarch)
data(SP500,package="Ecdat")
returnBM=SP500$r500[1805]
x=SP500$r500[(1804-2*253+1):1804]
plot(c(x,returnBM),type='l')
results=garchFit(~arma(1,0)+garch(1,1),data=x,cond.dist="std")
dfhat=as.numeric(results@fit$par[6])
forecast=predict(results,n.ahead=1)
```

The SP500 returns are in the data set `SP500` in the `Ecdat` package. The returns are in the variable `r500`. Black Monday is the 1850th return in the data set. This code fits an AR(1)/GARCH(1,1) model to the last two years of data before Black Monday. The conditional distribution of the white noise is the t-distribution. From the plot you can see that the Black Monday was highly unusual. The parameter estimates are in `results@fit$par` and the sixth parameter are the degrees of freedom of the t-distribution. The `predict` function is used to predict one step ahead. The object `forecast` will contain, mean forecast, mean error and standard deviation, which is the conditional standard deviation of the return on Black Monday.

   (a) Use the information above to calculate the conditional probability of a return less or equal to -0.228 on Black Monday.

   (b) Compute and plot the standarized residuals. Also plot the ACF of the residuals and their squares. Do they indicate an adequate model fit?

   (c) Would an AR(1)/ARCH(1) model provide an adequate fit?

   (d) Does an AR(1) model provide an adequate fit?

3. Download the data set `data_HW_3.RData` form Moodle. Follow the step-by-step model selection to fit an ARMA(p,q)/GARCH(P,Q) model to it. (Hint: q=0 and all other orders are less or equal to 2.)