EE 5143 Homework 3

Work on this particular homework should not use a computer unless specified in the problem. (Computers are always ok though for checking your answers.)

1. Find the Jordan forms for the matrices

$$A_1 = \begin{bmatrix} 0 & 4 & 3 \\ 0 & 20 & 16 \\ 0 & -25 & -20 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix}$$

2. Find the characteristic polynomials and the minimal polynomials of the following matrices:

$$\begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix} \qquad \begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{bmatrix} \qquad \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 1 & 0 \\ 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_1 \end{bmatrix}$$

3. For the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

use the spectrum of $\mathbf{A}$ to compute $\mathbf{A}^{103}$ and $e^{\mathbf{A}t}$. You should only use a computer at 3 steps if needed: factoring the characteristic polynomial if needed, finding the β values once you've set up your system of equations, and to simplify the final evaluation of $h(\mathbf{A})$.

4. **WORKBOOK PROBLEM 3.** For the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 22 & 12 & -8 & -8 \\ 44 & 30 & -16 & -18 \\ -6 & -4 & 2 & 4 \end{bmatrix}$$

- a. Find the characteristic polynomial of $\mathbf{A}$ by computing the determinant of the eigenvalue problem. You may use a computer to simplify the polynomial once you've obtained it from the determinant formula.
- b. Find the eigenvalues of $\mathbf{A}$ and write the factorization of the characteristic polynomial in terms of the eigenvalues. You may use a computer for help factoring.
- c. Compute generalized eigenvectors of $\mathbf{A}$
- d. Use parts a, b and c to find the Jordan form of $\mathbf{A}$.
- e. Show that the Jordan form $\hat{\mathbf{A}}$ of $\mathbf{A}$ satisfies the similarity transformation by providing the appropriate $\mathbf{Q}$ matrix and verifying the appropriate product. You may use a computer to compute the actual matrix multiplication.

- f. Compute $e^{\hat{\mathbf{A}}\mathbf{t}}$. While you can't use a computer for this part, you can save yourself a lot of time and effort by using the properties of functions for matrices in Jordan forms, and specifically the pattern demonstrated in equation 3.48 in your book (3rd edititition) for a Jordan block of size n.
- g. Use parts e and f to compute $e^{\mathbf{A}\mathbf{t}}$ using the similarity transformation