-

2.3 Conditional Statement (cont)

- Each of the following expressions is an equivalent form of the conditional statement $\boxed{p \to q}$:
  - p implies q
  - If p then q
  - q, if p
  - p only if q
  - p is sufficient for q
  - q is necessary for p
  - q unless $\sim p$
- $\boxed{p \to q \equiv \sim p \vee q}$

  equivalent to

Q1

Q1a Set

Given the universal set $U = \{x: x \in Z^+, 10 \le x \le 20\}$ containing the sets A and B defined as follows:
$A = \{x: x \in U, x$ is a prime number or x is a perfect square$\}$.
$B = \{ x: x \in U, x$ is a multiple of 4 or x is a number ending in 5$\}$.

(i) Write all the elements in sets U, A, and B. (3 marks)

(ii) Solve

(1) $(\overline{A \cup B})$, (2 marks)

(2) $A \oplus B$. (2 marks)

(i) $U = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$
$A = \{11, 13, 16, 17, 19\}$
$B = \{12, 15, 16, 20\}$

(ii) (1) $A \cup B = \{11, 13, 16, 17, 19\} \cup \{12, 15, 16, 20\}$
$= \{11, 12, 13, 15, 16, 17, 19, 20\}$

$(\overline{A \cup B}) = \{10, 14, 18\}$

(2) $A \oplus B = (A - B) \cup (B - A)$
$= \{11, 13, 17, 19\} \cup \{12, 15, 20\}$
$= \{11, 12, 13, 15, 17, 19, 20\}$

Given the universal set is $\{p, q, r, s, t, u, v, w\}$. Let $A = \{p, q, r, s\}$, $B = \{r, t, v\}$, and $C = \{p, s, t, u\}$. Find the elements of the following sets:

i)	$B \cap C$	ii)	$A \cup C$
iii)	$\overline{C}$	iv)	$A \cap B \cap C$
v)	$(A \cup B) \cap (A \cap C)$	vi)	$\overline{(A \cup B)}$
vii)	$B - C$	viii)	$B \oplus C$

i) $B \cap C = \{t\}$

ii) $A \cup C = \{p, q, r, s, t, u\}$

iii) $\overline{C} = \{q, r, v, w\}$

iv) $A \cap B \cap C = \{\}$ or $\emptyset$

v) $(A \cup B) \cap (A \cap C) = \{p, q, r, s, t, v\} \cap \{p, s\}$

$= \{p, s\}$

vi) $(A \cup B) = \{p, q, r, s, t, v\}$

$\overline{(A \cup B)} = \{v, w\}$

vii) $B - C = \{r, v\}$

viii) $B \oplus C = (B - C) \cap (C - B)$

$= \{p, r, s, u, v\}$

Consider the following subsets of a standard English language dictionary:

$A = \{x$: x is a word that appears before *dog*$\}$

$B = \{x$: x is a word that appears after *cat*$\}$

$C = \{x$: x is word containing two identical consecutive letters$\}$

i) Decide which of the following statements are true:

a) $C \subseteq A \cup B$ b) *aardvark* $\in \overline{B} \cap C$

c) *moose* $\in B \oplus C$ d) $A \cap B = \emptyset$

ii) Describe in words the elements of the following sets:

a) $A \cap B \cap C$ b) $(A \cup B) \cap \overline{C}$

Q1b recursive/explicit

Given the sequence defined by the recurrence relation $a_{n+1} = 3a_n + 2^n$ with the initial term $a_1 = 1$. Compute the second, third, fourth, and fifth terms of the sequence and calculate the sum of the first five terms of the given sequence. (5 marks)

$a_1 = 1$

$a_2 = a_{1+1} = 3a_1 + 2^1 = 3(1) + 2 = 5$

$a_3 = a_{2+1} = 3a_2 + 2^2 = 3(5) + 4 = 19$

$a_4 = a_{3+1} = 3a_3 + 2^3 = 3(19) + 8 = 65$

$a_5 = a_{4+1} = 3a_4 + 2^4 = 3(65) + 16 = 211$

Sum of first five terms

$= 1 + 5 + 19 + 65 + 211$

$= 301$

Write out the first four terms (begin with $n = 1$) of the sequence whose general term is given as follows.

i) $a_n = 5^n$ ii) $c_1 = 2.5, c_n = c_{n-1} + 1.5$

i) $a_n = 5^n$

$a_1 = 5^1 = 5$

$a_2 = 5^2 = 25$

$a_3 = 5^3 = 125$

$a_4 = 5^4 = 625$

ii) $c_1 = 2.5$, $c_n = c_{n-1} + 1.5$

$c_2 = 2.5 + 1.5 = 4.0$

$c_3 = 4.0 + 1.5 = 5.5$

$c_4 = 5.5 + 1.5 = 7.0$

Write a formula for the nth term of the sequence. Identify your formula as recursive or explicit.

i) 1, 3, 5, 7, … ii) 1, -1, 1,-1, 1, -1, …

Q1c GCD/ present in form sa+tb /LCM

Given $a = 144$ and $b = 60$, use the Euclidean algorithm to solve the greatest common divisor (GCD) of a and b and present it in the form of $sa + tb$, where $s,t \in \mathbb{Z}$. Hence, compute the least common multiple (LCM) of 144 and 60. (5 marks)

GCD / LCM

$a = 144$, $b = 60$

$GCD(144, 60) = 12$

$144 = 2(60) + 24$ $24 = 144 - 2(60)$

$60 = 2(24) + 12$ $12 = 60 - 2(24)$

$24 = 2(12) + 0$

$12 = 60 - 2(24)$

$12 = 60 - 2(144 - 2(60))$

$12 = 60 - 2(144) + 4(60)$

$12 = -2(144) + 5(60)$

$LCM = \frac{144 \times 60}{12}$

$= 720$

For the following pairs of integers m and n,

i) find the greatest common divisor of the m and n. Hence deduce the least common multiple of m and n.

ii) rewrite m and n in the form of $am + bn$, $a, b \in Z$.

a) $m = 27$, $n = 72$

b) $m = 3510$, $n = 672$

Q1d function

Injective: 1 to 1
Surjective: range/y都有用到 → onto
Bijective: 上面两个都是就是

Consider the function $f : \mathbb{R} \to \mathbb{R}$. Examine whether the function $f(x) = x^2 + 2x$ is one-to-one and onto. Is f a bijective function? Justify your answers. (8 marks)
Let $A = \{-1, 0, 1, 2\}$ and $f: A \to Z$ be given by $f(x) = \left\lfloor \frac{x^2+1}{3} \right\rfloor$. i) Find the range of f. ii) Determine whether the function f is injective, surjective or bijective. Justify your answer.
Given $f(x) = 2x - 1$, a function from $X = \{1, 2, 3\}$ to $Y = \{1, 2, 3, 4, 5\}$. Find the domain and range of the function f. Hence determine whether the function is bijective and explain your answer.

Q2

Q2 Chapter 3 study all

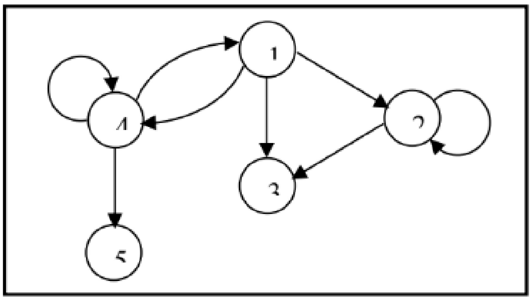
Consider a set $E = \{1, 3, 4, 6, 8, 9\}$ and define a relation R on E by $x\,R\,y$ if and only if x is a factor of y and $x \neq y$.

(i) Provide the ordered pairs that belong to the relation R. (2 marks)

(ii) Compute the in-degree and out-degree of each vertex. (3 marks)

(iii) Identify the domain and range of the relation R. (2 marks)

Refer to the following digraph of a relation R.

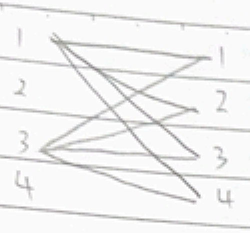


i) Give the in-degree and the out-degree of each vertex.
ii) List the ordered pairs belonging to the relation R.
iii) Represent R in the matrix form.

Let R be the relation on $\{1, 2, 3, 4\}$ given by $u\,R\,v$ if and only if $u + 2v$ is odd. Represent R in each of the following ways:

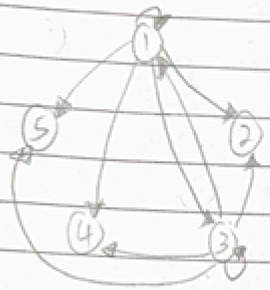
i) as a set of ordered pairs; ii) in graphical form;
iii) in matrix form.

Give the in-degree and out-degree of each vertex.



i) $R = \{(1,1), (1,2), (1,3), (1,4), (3,1), (3,2), (3,3), (3,4)\}$

ii)



iii) $M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

	1	2	3	4	5
In	2	2	2	2	2
Out	5	0	5	0	0

Matrix

Determine whether the given relation on $A = \{1, 2, 3, 4\}$ is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Explain your answers.

i) $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$

ii) $R = \{(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)\}$

iii) $R = \varnothing$

iv) $R = \{(1, 2), (1, 3), (3, 1), (1, 1), (3, 3), (3, 2), (1, 4), (4, 2), (3, 4)\}$

f

Let set $A = \{1, 2, 3, 4\}$ and the relation R on the set A is represented by the matrix

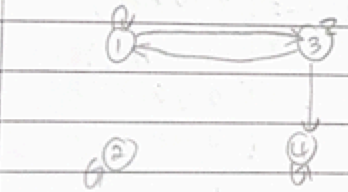
$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(i) Examine whether the relation R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Give a counterexample if the answer is "No". (11 marks)

(ii) Apply Warshall's algorithm to obtain the matrix of the transitive closure of R. (7 marks)

Matrix

2.



i) Is reflexive

Not irreflexive since $(1,1) \in R$

Not symmetric since $(3,4) \in R$ but $(4,3) \notin R$

Not asymmetric since $(1,1) \in R$

Not antisymmetric since $(1,3)$ and $(3,1) \in R$

Not transitive since $(1,3)$ and $(3,4) \in R$ but $(1,4) \notin R$

ii) $W_R = M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$C_1 = 1, 2$

$R_1 = 1, 3$

$(1,1), (2,1), (1,3), (2,3)$

$W_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$C_2 = 2$

$R_2 = 1, 2, 3$

$(2,1), (2,2), (2,3)$

$W_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$C_3 = 1, 2, 3$

$R_3 = 1, 3, 4$

$(1,1), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,3), (3,4)$

$W_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$C_4 : 1, 2, 3, 4$

$R_4 = 4$

$(1,4), (2,4), (3,4), (4,4)$

$M_R^{\infty} = W_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

*Equivalence = reflexive && symmetric && transitive

Determine whether the following relation R on the set A is an equivalence relation. If yes, find A/R.

i) $A = \{a, b, c, d\}$, $R = \{(a, a), (a, b), (b, a), (b, b), (c, c), (c, d), (d, c), (d, d)\}$

ii) $A = \{1, 2, 3, 4\}$, $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 3), (1, 3), (4, 1), (4, 4)\}$

iii) $A = \{2, 3, 5, 6, 8\}$, $x \mathrel{R} y$ if and only if $3|(x - y)$.

iv) $A = \{1, 2, 3, 4, 5\}$, $x \mathrel{R} y \Leftrightarrow x \equiv y \pmod 2$.

The following arrays describe a relation R on a set $A = \{1, 2, 3, 4\}$:
VERT = [1, 2, 6, 4]
TAIL = [1, 2, 2, 4, 4, 3, 4, 1]
HEAD = [2, 2, 3, 3, 4, 4, 1, 3]
NEXT = [8, 3, 0, 5, 7, 0, 0, 0]
Compute both the digraph of R and the matrix $\mathbf{M}_R$.

Compute

Let $A = \{a, b, c, d, e\}$ and let the equivalence relations R and S on A be given by

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{M}_S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

i) Compute

a) $\mathbf{M}_{R \circ R}$ b) $\mathbf{M}_{S \circ R}$

c) $\mathbf{M}_{R \circ S}$ d) $\mathbf{M}_{S \circ S}$

ii) Compute the partition of A corresponding to $R \cap S$.

Compute

i) a) $M_{R \circ R} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

iii) $M_{R \circ S} = M_S \circ M_R$

$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

) $M_{S \circ R} = M_R \circ M_S$

$= \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

d) $M_{S \circ S} = M_S \circ M_S$

$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

ii) $M_{R \cap S} = M_R \cap M_S$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cap \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Q3

Q3a PDNF PCNF using truth table

Let $A = (p \wedge (q \vee \sim r)) \rightarrow (\sim p \vee r)$.

(i) Complete the truth table given below. Identify whether the expression A is a tautology, contradiction or contingency. (4 marks)

p	q	r	$\sim p$	$\sim r$	$q \vee \sim r$	$p \wedge (q \vee \sim r)$	$\sim p \vee r$	$(p \wedge (q \vee \sim r)) \rightarrow (\sim p \vee r)$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

(ii) Write the Principal Disjunctive Normal Form (PDNF) of A and Principal Conjunctive Normal Form (PCNF) of A. (4 marks)

PDNF & PCNF

$(p \wedge (q \vee \sim r)) \rightarrow (\sim p \vee r)$

p	q	r	$\sim p$	$\sim r$	$q \vee \sim r$	$\sim p \vee r$	$p \wedge (q \vee \sim r)$	$p \wedge (q \vee \sim r) \rightarrow (\sim p \vee r)$
T	T	T	F	F	T	T	T	T
T	T	F	F	T	T	F	T	T
T	F	T	F	F	F	T	F	T
T	F	F	F	T	T	F	T	F
F	T	T	T	F	T	T	F	T
F	T	F	T	T	T	T	F	T
F	F	T	T	F	F	T	F	T
F	F	F	T	T	T	T	F	T

contingency.

PDNF A = $pqr + pq\bar{r} + p\bar{q}r + \bar{p}qr + \bar{p}q\bar{r} + \bar{p}\bar{q}r + \bar{p}\bar{q}\bar{r}$

PCNF A = $p\bar{q}\bar{r}$

i. $q \wedge (p \vee \sim q)$

$= (q \wedge p) \vee (q \wedge \sim q)$

$= (p \wedge q) \vee C$

$= (p \wedge q)$

PDNF $A = (p \wedge q)$

PDNF $\sim A = (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

PCNF $A = (p \vee \sim q) \wedge (\sim p \vee q) \wedge (p \vee q)$

PCNF $\sim A$ = $(\sim p \vee \sim q)$

Without constructing truth tables, obtain the principal disjunctive normal form of A, the principal disjunctive normal form of $\sim A$, the principal conjunctive normal form of A, and the principal conjunctive normal form of $\sim A$, if the normal form exists, for each expression A below:

i) $q \wedge (p \vee \sim q)$ ii) $p \rightarrow (p \wedge (q \rightarrow p))$

iii) $(q \rightarrow p) \wedge (\sim p \wedge q)$

2. i. $q \wedge (p \vee \sim q)$

$= (q \wedge p) \vee (q \wedge \sim q)$

$= (p \wedge q) \vee C$

$= (p \wedge q)$

PDNF $A = (p \wedge q)$

PDNF $\sim A = (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

PCNF $A = (p \vee \sim q) \wedge (\sim p \vee q) \wedge (p \vee q)$

PCNF $\sim A = (\sim p \vee \sim q)$

ii) $p \to (p \wedge (q \to p))$

$= \sim p \vee (p \wedge (\sim q \vee p))$

$= \sim p \vee [(p \wedge \sim q) \vee (p \wedge p)]$

$= \sim p \vee [(p \wedge \sim q) \vee p]$

$= \sim p \vee [(p \wedge \sim q) \vee (p \wedge (q \vee \sim q)]$

$= \sim p \vee [(p \wedge \sim q \wedge p \wedge (q \vee \sim q)]$

$= \sim p \vee [(p \wedge (q \vee \sim q)]$

$= \sim p \vee p$

$= T$

PDNF A : $pq + p\bar{q} + \bar{p}q + \bar{p}\bar{q}$.

PDNF $\sim A = \emptyset$ (contradiction)

PCNF A $= \emptyset$

PCNF $\sim A = (p+q)(p+\bar{q})(\bar{p}+q)(\bar{p}+\bar{q})$

iii) $(q \to p) \wedge (\sim p \wedge q)$

$= (\sim q \vee p) \wedge (\sim p \wedge q)$

$= (\sim q \vee p) \wedge \sim p \wedge q$

$= (\sim p \wedge q \wedge \sim q) \vee (\sim p \wedge p \wedge q)$

$= (\sim p \wedge C) \vee (C \wedge q)$

$= C \vee C$

$= C$

PDNF A: $\emptyset$

PDNF $\sim A$: $pq + p\bar{q} + \bar{p}q + \bar{p}\bar{q}$

PCNF A : $(p+q)(p+\bar{q})(\bar{p}+q)(\bar{p}+\bar{q})$

PCNF $\sim A$: $\emptyset$

Q3b demonstrate expression xxxx to the simplest form using laws of algebra of propositions

Demonstrate the expression $((a \to b) \land (d \lor \sim a)) \lor (\sim b \land a)$ to the simplest form by using the laws of algebra of propositions. (7 marks)

$$((a \to b) \land (d \lor \sim a)) \lor (\sim b \land a)$$
$$= ((\sim a \lor b) \land (d \lor \sim a)) \lor (\sim b \land a)$$
$$\lor a) = (\sim a \lor b) \land (d \lor \sim a) \lor (\sim b \land a)$$
$$= (\sim a \lor b) \lor (\sim b \land a) \land (d \lor \sim a)$$
$$= (\sim(a \land \sim b)) \lor (a \land \sim b) \land (d \lor \sim a)$$
$$= T \land (d \lor \sim a)$$
$$= \sim a \lor d$$

Express the following using $\land$, $\lor$, and $\sim$ only. Simplify your expressions.

i) $(p \to q) \lor \sim p$ ii) $(p \to q) \lor p$

iii) $(p \leftrightarrow \sim q) \lor q$ iv) $p \land \sim(q \to p)$

2 i) $(p \to q) \lor \sim p$
$$= (\sim p \lor q) \lor \sim p$$
$$= \sim p \lor q \lor \sim p$$
$$= \sim p \lor q$$

ii) $(p \to q) \lor p$
$$= (\sim p \lor q) \lor p$$
$$= (p \lor \sim p) \lor q$$
$$= T \lor q$$
$$= T$$

iii) $(p \leftrightarrow \sim q) \lor q$
$$= [(\sim p \lor q) \land (q \lor p)] \lor q$$
$$= (p \lor (\sim p \lor q)) \land (q \lor p)$$
$$= (p \lor \sim p \lor q) \land (q \lor p)$$
$$= T \land (q \lor p)$$
$$= (p \lor q)$$

iv) $p \land \sim(q \to p)$
$$= p \land \sim(\sim q \lor p)$$
$$= p \land (q \land \sim p)$$
$$= p \land \sim p \land q$$
$$= C$$

Q3c prefix/infix/postfix (similar to past year) & Binary Search Tree

The following arithmetic expression is given in the prefix form. Calculate the value and write the expression in fully parenthesised form. Hence, show its binary tree and write the expression in postfix form.

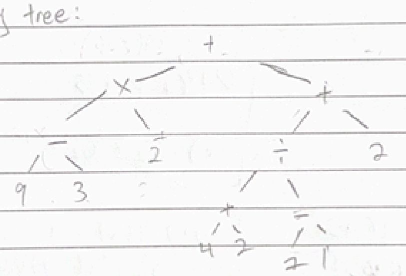
$+ \ \times \ - \ 9 \ 3 \ 2 \ + \ \div \ + \ 4 \ 2 \ - \ 7 \ 1 \ 2$ (10 marks)

Prefix / infix / Postfix

1. Parenthesised form : $((9-3) \times 2) + (((4+2) \div (7-1)) + 2)$

Post fix : $9 3 - 2 \times \ 4 2 + 7 1 - \div 2 + +$

Binary tree:

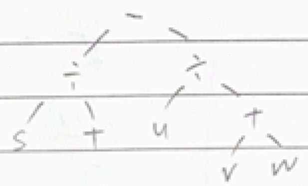


Draw a binary tree to represent the following expression.

$$(s \div t) - (u \div (v + w))$$

Write the expression in infix and postfix notations.

2.



Infix : $s \div t - u \div v + w$

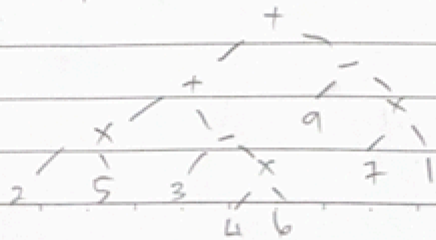
Postfix : $s t \div u \div v w + -$

For the following expression in Polish form, draw its binary tree, and write the corresponding fully parenthesised expression:

$+ + \times 2 5 - 3 \times 4 6 - 9 \times 7 1$

Evaluate the expression.

3. Parenthesised expression : $((2 \times 5) + (3 - (4 \times 6))) + (9 - (7 \times 1))$



Q4

Q4a Combination

A library has 20 volunteers, consisting of 8 women and 12 men. Four volunteers are randomly selected to attend a special training session. What is the probability that:

(i) All four selected volunteers are men? (3 marks)

(ii) Two men and two women are selected? (3 marks)

i) 12C4 = 495
ii) 8C2 * 12C2 = 1848

An examination paper has two parts, Part A and Part B. There are 8 questions in Part A and 10 questions in Part B. A candidate is required to do 5 questions from Part A (which must include either Question 1 or Question 2 but not both) and any 7 from Part B. In how many ways can he complete the paper?

2C1 *6C4 *10C7 = 3600

It is required to seat 4 men and 4 women in a row so that the men and the women are occupying the **alternate places**. How many such arrangements are possible?

4 x 4 x 3 x 3 x 2 x 2 x 1 x 1 = 576 (MWMWMWMW)
4 x 4 x 3 x 3 x 2 x 2 x 1 x 1 = 576 (WMWMWMWM)

576 + 576 = 1162

Find the number of different arrangements of the six letters in the word 'ELEVEN' in which
(a) all three letters 'E' are consecutive,
(b) the first letter is 'E' and the last letter is 'N'.

a)

E	E	E			
	E	E	E		
		E	E	E	
			E	E	E

3P3*4 = 24 *4是可能性的数量

b) condition fixed:
4P4/2! =12 (上-空位数量 **P** 下-可以放的东西的数量/ 可以放的东西重复的数量！)

EEEAAAOO
6行5空位
5P5/3!/2! *6

Q4b Contingency table

Q4c probability distribution discrete

An IT company is hiring for two positions: a Software Developer and a Data Analyst. The probability that an applicant is proficient in Python (event T) is 0.6, and the probability that an applicant is proficient in SQL (event S) is 0.5. The probability that an applicant is proficient in both Python and SQL is 0.3.

(i) What is the probability that an applicant is proficient in either Python or SQL or both? (2 marks)

(ii) If an applicant is known to be proficient in Python, what is the probability that they are also proficient in SQL? (2 marks)

(iii) Are the events of being proficient in Python and being proficient in SQL independent? Provide your answer with calculations. (3 marks)

Let X denote the number of auto accidents that occur in a city during a week. The following table lists the probability distribution of X.

x	0	1	2	3	4	5	6
$P[X=x]$	0.12	0.16	0.22	0.18	0.14	0.12	0.06

Determine the probability that the number of auto accidents that will occur during a given week in this city is
(a) exactly 4 (b) at least 3 (c) less than 3 (d) 3 to 5

x	0	1	2	3	4	5	6
$P[X=x]$	0.12	0.16	0.22	0.18	0.14	0.12	0.06

The Webster Mail Order Company sells expensive stereos by mail. The following table lists the frequency distribution of the number of orders received per day by this company during the past 100 days.

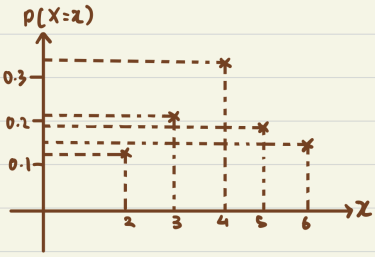
Number of orders received per day	2	3	4	5	6
Number of days	12	21	34	19	14

(a) Construct a probability distribution table for the number of orders received per day. Draw a graph of the probability distribution.

(b) Let X denote the number of orders received on any given day. Find the following probabilities.
(i) $P(X = 3)$ (ii) $P(X \geq 3)$ (iii) $P(2 \leq X \leq 4)$ (iv) $P(X < 4)$

(100 days)

x	2	3	4	5	6
$P[X=x]$	0.12	0.21	0.34	0.19	0.14



i) $P(X=3)=0.21$

ii) $P(X\geq 3) = 1-P(X=2)$
$= 1-0.12 = 0.88$

iii) $P(2\leq X \leq 4)$
$=0.12+0.21+0.34=0.67$

iv) $P(X<4)$
$=0.12+0.21=0.33$

c) The continuous random variable Y has the probability density function given by

$$f(y) = \begin{cases} c(2-y), & \text{for } 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

where c is a constant.

(i) Compute the value of c. (3 marks)

(ii) Solve the mean and the standard deviation of Y. (9 marks)

[Total: 25 marks]

Discrete :

$E(X) : \sum x P(X=x)$

$Var(X) = \sum x^2 [P(X=x)] - [\sum x P(X=x)]^2$

Continuous :

$E(X) = \int x \ f(x) \ dx$

$Var(X) = \int x^2 f(x) \ dx - [\int x \ f(x) \ dx]^2$

E是mean，Var是variance

Standard deviation $= \sqrt{Var(X)}$

A continuous random variable X has the density function

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 \leq x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Show that $P(0 < X < 2) = 1$.

(b) Find $P(X < 1.2)$.

The weekly demand for Pepsi, in thousands of litres from a local chain of efficiency stores, is a continuous random variable X having the probability density

$$f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the mean and variance of X.

Rules

Use some valid argument forms to deduce the conclusion from the premises for the arguments below.

i)
$p \vee q$
$p \rightarrow r$
$\therefore r \vee q$

ii)
$p \rightarrow q$
$\sim q$
$\sim r$
$\therefore \sim(p \vee r)$

iii)
$p \vee q$
$\sim p \vee r$
$\sim r$
$\therefore q$

iv)
$p \rightarrow q$
$r \rightarrow \sim q$
r
$\therefore \sim p$

Use valid forms to deduce the conclusions from the premises, giving a reason for each step.

i)
$\sim p \vee q \rightarrow r$
$s \vee \sim q$
$\sim t$
$p \rightarrow t$
$\sim p \wedge r \rightarrow \sim s$

ii)
$\sim p \leftrightarrow q$
$q \rightarrow r$
$\sim r$

Construct a proof for each of the following arguments, giving all necessary additional assertions. Specify the rules of inferences used at each step.

i) If today is Monday, then I have a Mathematics test or programming test. If my programming lecturer is sick, then I will not have a programming test. Today is Monday and my programming lecturer is sick. Therefore…

ii) If Carolyn gets the supervisor's position and works hard, then she will get a raise. If she gets the raise, then she will buy a new car. She has not purchased a new car. Therefore …

---

b) Demonstrate the expression $((a \rightarrow b) \wedge (d \vee \sim a)) \vee (\sim b \wedge a)$ to the simplest form by using the laws of algebra of propositions. (7 marks)

$((a \rightarrow b) \wedge (d \vee \sim a)) \vee (\sim b \wedge a)$
$= ((\sim a \vee b) \wedge (d \vee \sim a)) \vee (\sim b \wedge a)$
$= (\sim a \vee b) \wedge (\sim a \vee d) \vee (a \wedge \sim b)$
$= (\sim a \vee b) \vee (a \wedge \sim b) \wedge (\sim a \vee d)$
$= \sim(a \wedge \sim b) \vee (a \wedge \sim b) \wedge (\sim a \vee d)$
$= T \wedge (\sim a \vee d)$
$= \sim a \vee d$

$F \wedge p = F$
$F \vee p = p$

$T \wedge p = p$
$T \vee p = T$

4C

- Discrete
  - $E(X) = \sum x \, P(X=x)$
  - $Var(X) = \underbrace{\sum x^2 \, P(X=x)}_{E(X^2)} - \left[E(X)\right]^2$
- Continous
  - $E(X) = \int x \, f(x) \, dx$
  - $Var(X) = E(X^2) - \left[E(X)\right]^2$

    $= \int x^2 f(x) \, dx - \left[E(X)\right]^2$

$P(X=x) = f(x) dx.$

1a) $a_1 = 1$

$a_2 = 3a_1 + 2^1 = 5$

$a_3 = 3a_2 + 2^2 = 19$

$a_4 = 3a_3 + 2^3 = 65$

$a_5 = 3a_4 + 2^4 = 211$

$1 + 5 + 19 + 65 + 211 = 301$

b.) (i) $U = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{11, 13, 16, 17, 19\}$

$B = \{12, 15, 16, 20\}$

(ii) (1) $\overline{(A \cup B)} = \overline{\{11, 12, 13, 15, 16, 17, 19, 20\}}$

$= \{10, 14, 18\}$

(2) $A \oplus B = \{11, 12, 13, 15, 17, 19, 20\}$

c) $\gcd(144, 60)$

$= \gcd(60, 24)$

$= \gcd(24, 12)$

$= 12$

$144 = 2(60) + 24$

$60 = 2(24) + 12$

$24 = 2(12) + 0$

gcd $= 12$

lcm $= \frac{144 \times 60}{12} = 720$

$12 = 60 - 2(24)$

$= 60 - 2[144 - 2(60)]$

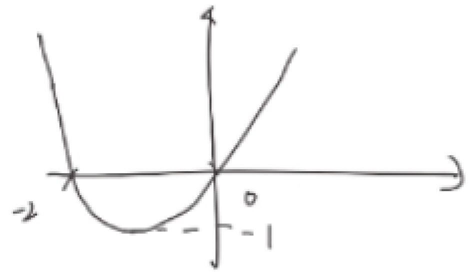
$= 5(60) - 2(144)$

d) $f: \mathbb{R} \to \mathbb{R}$

$f(x) = x^2 + 2x$

$0: x(x+2$

$f(-1) = (-1)^2 + 2(-1)$



Not one to one, $f(-2) = f(0) = 0$

Not onto since Range $= \{[-1, 0)\} \neq \mathbb{R}$ = Codomain.

$\therefore$ Not Bijective.

2a)

i) $R = \{(1,3), (1,4), (1,6), (1,8), (1,9), (3,6), (3,9), (4,8)\}$

ii)

	1	3	4	6	8	9	
In	0	1	1	2	2	2	8
Out	5	2	1	0	0	0	8

iii) Domain = $\{1, 3, 4\}$

Range = $\{3, 4, 6, 8, 9\}$

b) (i) Reflexive

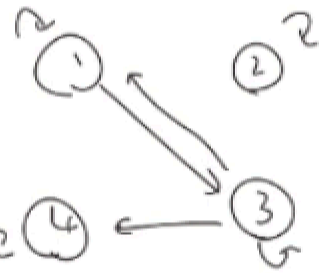
Not Irreflexive, $(1,1) \in R$

Not Symmetric, $(3,4) \in R$, $(4,3) \notin R$

Not Antisymmetric, $(1,3), (3,1) \in R$

Not Assymmetric, $(1,1) \in R$

Not Transitive, $(1,3), (3,4) \in R$
$(1,4) \notin R$



$$W_0 = M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C1 : 1, 3
R1 : 1, 3

$W_1 = W_0$

C2 : 2
R2 : 2

$W_2 = W_0$

C3 : 1, 3
R3 : 1, 3, 4
(1, 4)

$$W_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C_4 : 1, 3, 4
R_4 : 4

$$M_{R^\infty} = W_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

p	q	r	$\sim p$	$\sim r$	$q \vee \sim r$	$p \wedge (q \vee \sim r)$	$\sim p \vee r$	$(p \wedge (q \vee \sim r)) \to (\sim p \vee r)$
T	T	T	F	F	T	T	T	T
T	T	F	F	T	T	T	F	F
T	F	T	F	F	F	F	T	T
T	F	F	F	T	T	T	F	F
F	T	T	T	F	T	F	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	F	F	F	T	T
F	F	F	T	T	T	F	T	T

Contingency

(ii) PDNF of A: $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge r)$

$\vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$

$= (pqr) + (p\bar{q}r) + (\bar{p}qr) + (\bar{p}q\bar{r})$

$+ (\bar{p}\bar{q}r) + (\bar{p}\bar{q}\bar{r})$

PCNF of A: $(\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee r)$

PCNF of A : $(\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee r)$

b.) $((a \to b) \wedge (d \vee \sim a)) \vee (\sim b \wedge a)$

$\equiv ((\sim a \vee b) \wedge (d \vee \sim a)) \vee (\sim b \wedge a)$

$\equiv (\sim a \vee (b \wedge d)) \vee (\sim b \wedge a)$

$\equiv (\sim a \vee (\sim b \wedge a)) \vee (b \wedge d)$

$\equiv [(\sim a \vee \sim b) \wedge (\sim a \vee a)] \vee (b \wedge d)$

$\equiv (\sim a \vee \sim b) \vee (b \wedge d)$

$\equiv (\sim a \vee \sim b \vee b) \wedge (\sim a \vee \sim b \vee d)$

$\equiv (\sim a \vee \sim b \vee d)$

c.)

$+ \times - 9\ 3\ 2 + \div + 4\ 2\ -7\ 1\ 2$

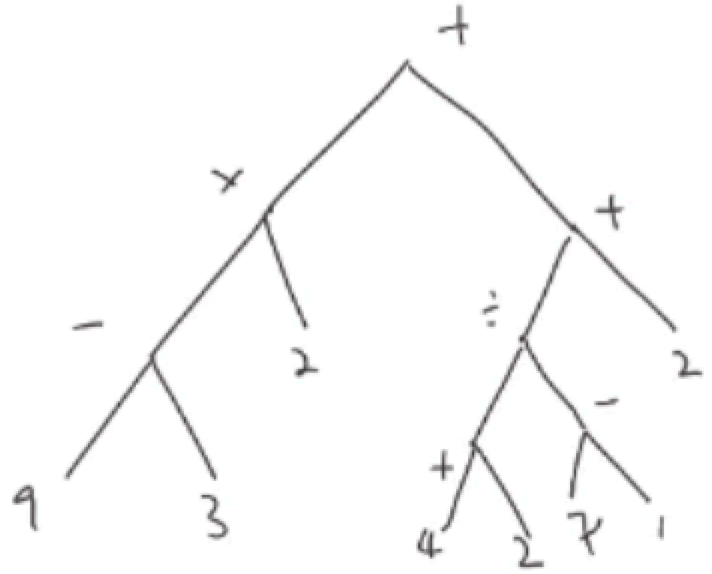
$= + \times 6\ 2 + \div 6\ 6\ 2$

$= + 12 + 1\ 2$

$= + 12\ 3$

$= 15$

$((9-3)\times 2)+(((4+2)\div(7-1))+2)$



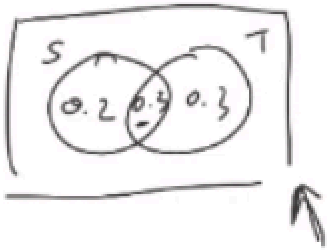
Postfix: 9 3 − 2 × 4 2 + 7 1 − ÷ 2 + +

Q4

$P(T) = 0.6$

$P(S) = 0.5$

$P(S \cap T) = 0.3$



(i) $0.2 + 0.3 + 0.3 = 0.8$

$0.6 + 0.5 - 0.3 = 0.8$

(ii) $P(S \mid T) = \dfrac{P(S \cap T)}{P(T)}$

$= \dfrac{0.3}{0.6}$

$= 0.5$ $\qquad \dfrac{3}{6}$

(iii) $P(S) \times P(T) = 0.5(0.6) = 0.3 = P(S \cap T)$

Independent

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

b) (i)

$$\underbrace{\_\ \_\ \_\ \_}_{^{12}C_4} = 495$$

(ii)

$$\underbrace{\_\ \_}_{^{14}C_2} \times \underbrace{\_\ \_}_{^{8}C_2} = \cancel{2} \; 2548$$

b) (i)

$$\underbrace{\_\ \_\ \_\ \_}_{{}^{20}C_4} = 4845$$

$$\underbrace{\_\ \_\ \_\ \_}_{{}^{12}C_4} = 495$$

Probability $= \frac{495}{4845} = \frac{33}{323}$

(ii)

$$\underbrace{\_\ \_}_{{}^{12}C_2} \times \underbrace{\_\ \_}_{{}^{8}C_2} = 1848$$

$$\frac{1848}{4845} = 0.3814$$

c) (i) $\int_0^2 c(2-y)\,dy = 1$

$c\int_0^2 (2-y)\,dy = 1$

$C = \frac{1}{2}$

(ii) $E(Y) = \int_0^2 \frac{1}{2}y(2-y)\,dy$

$= \frac{2}{3}.$

$Var(Y) = \int_0^2 \frac{1}{2}y^2(2-y)\,dy - E(Y)^2$

$= \frac{2}{3} - \frac{4}{9}$

$= \frac{2}{9}$

$SD = \sqrt{Var} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$