[December 2024] BMMS1653 Discrete Structures: Assignment (50%)

Programme: RSD2 **(Group** 5 **)**
Leader : Lim Jun Wei
Member : Chia Ming Yi
Member : Ong Yi Xin

Marks: ______ /50

Answer **ALL** questions. All relevant and detailed workings must be shown.

Question 1

Let $A = \{18, 20, 26, 32, 38\}$. A relation R on set A is defined by xRy if and only if $x \equiv y \pmod 4$.

(a) Write R as a set of ordered pairs. (2 marks)
(b) Find the in-degree and out-degree of each vertex. (2 marks)
(c) Determine whether relation R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give a counterexample if your answer is "No". (9 marks)

(d) Another relation S on set A whose matrix is given by $M_S = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$. Find the matrix of $R \circ S^{-1}$. (4 marks)

Question 2

Let $f(x) = \left\lceil \frac{x^2+x-2}{x+1} \right\rceil$ be a function from $A = \{1, 2, 3, 4, 5, 6\}$ to $B = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

(a) Determine whether f is one-to-one, onto and everywhere defined. Explain your answers. (6 marks)
(b) Is f bijective? Explain your answer. (2 marks)

Question 3

Without constructing truth table, obtain the principal conjunctive normal form (PCNF) of $A \equiv [(p \vee (q \rightarrow {\sim}r)) \vee {\sim}q] \rightarrow {\sim}(p \vee {\sim}r)$. (8 marks)

Question 4

Show the scope of each quantifier in the following expression $(\exists y)[{\sim}P(x,y) \rightarrow (\forall x)(Q(x) \vee {\sim}R(x,y))]$. Determine whether the given expression is a statement or not. If not, state the reason. (4 marks)

Question 5

The following arithmetic expression is given in Polish form.

$+ \; - \; \times \; 5 \; 6 \; + \; 9 \; \div \; 8 \; 2 \; - \; 7 \; \times \; 4 \; 5$

(a) Calculate the value represented by the expression. (3 marks)
(b) Write the corresponding fully parenthesised expression and draw its binary tree. (6 marks)
(c) Write the expression in postfix and infix form. (4 marks)

[End of Question Paper]

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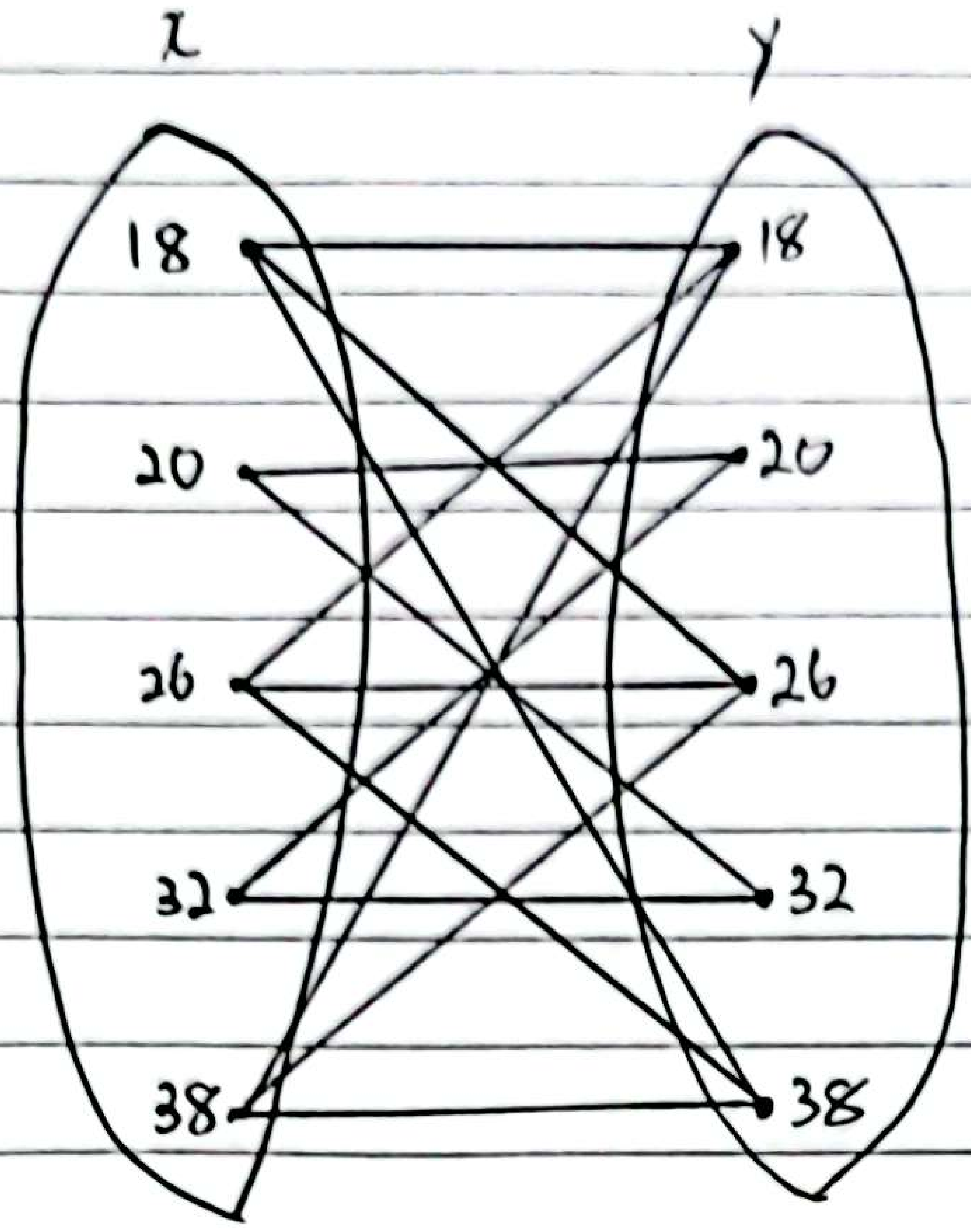
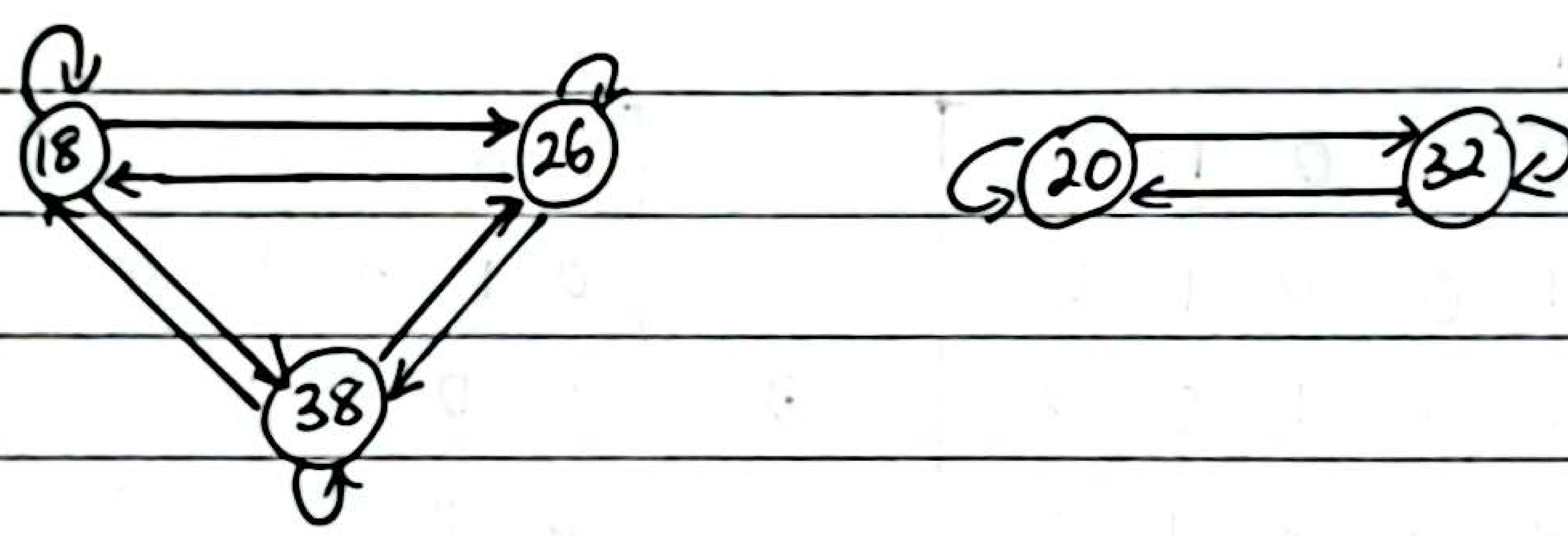
Question 1



a) R = { (18,18), (18,26), (18,38), (20,20), (20,32), (26,18), (26,26), (26,38), (32,20), (32,32), (38,18), (38,26), (38,38) }

b)



	18	20	26	32	38
In-degree	3	3	3	2	2
Out-degree	3	3	3	2	2

c)
- Relation R is reflexive
- Relation R is not irreflexive, $\because$ $18 R 18$
- Relation R is symmetric.
- Relation R is not asymmetric, $\because$ $18 R 18$ or $18 R 26$ but $26 R 18$.
- Relation R is not antisymmetric, $\because$ $18 R 26$ and $26 R 18$ but $18 \neq 26$.
- Relation R is transitive.

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d)

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$M_{S^{-1}} = (M_S)^T$$

$$= \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$M_{R \circ S^{-1}} = M_{S^{-1}} \odot M_R$$

$$= \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

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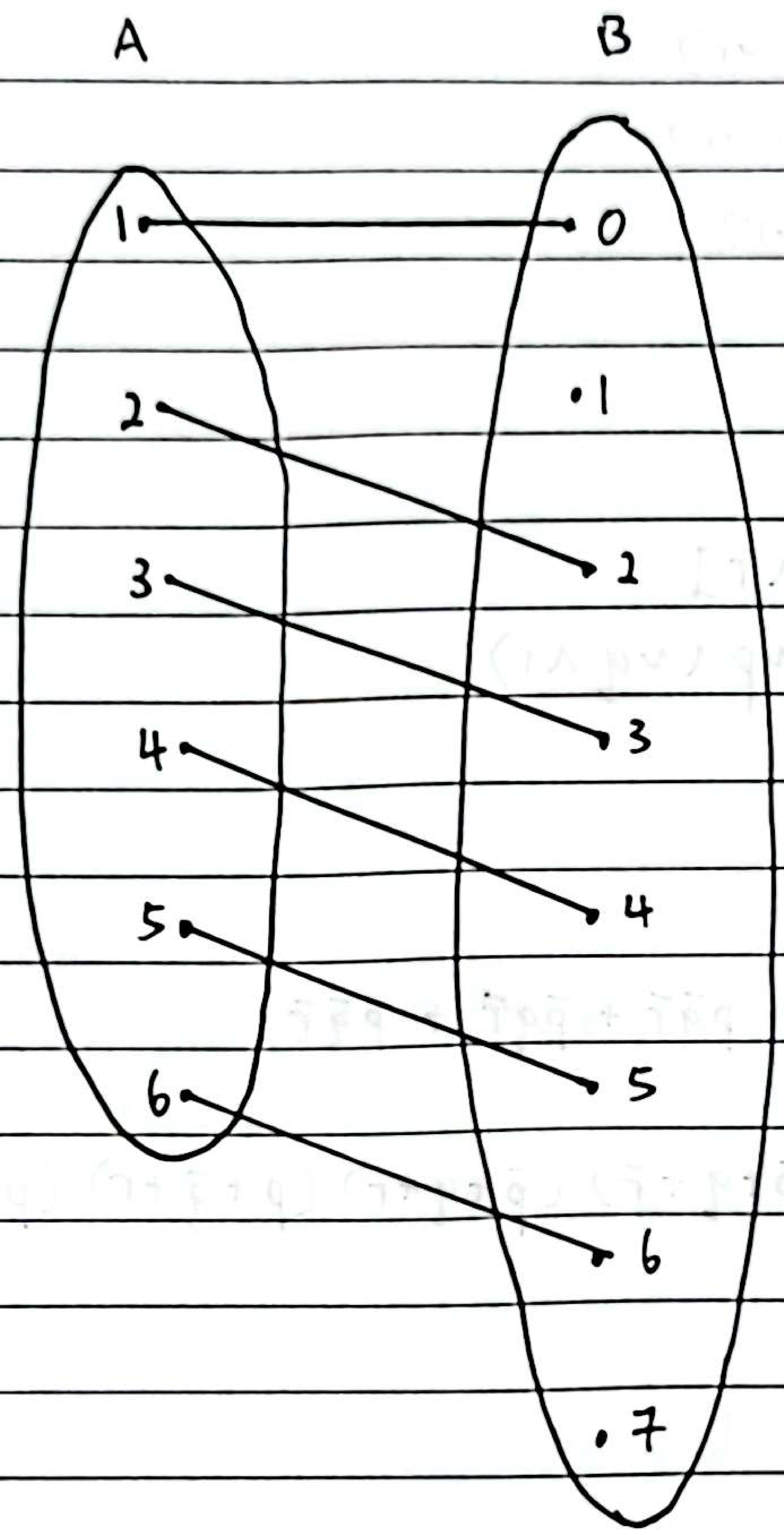
Question 2

a)



a) $\because f(1)=0, f(2)=2, f(3)=3, f(4)=4, f(5)=5, f(6)=6$, $\therefore f$ is one to one.

$\because \text{Ran}(f) = \{0,2,3,4,5,6\} \neq B$, $\therefore f$ is not onto.

$\because \text{Dom}(f) = \{1,2,3,4,5,6\} = A$, $\therefore f$ is everywhere defined.

b) f is not bijective because $\text{Ran}(f) = \{0,2,3,4,5,6\} \neq B$ (not surjective) although f is injective.

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Question 3

$[(p \vee (q \rightarrow \sim r)) \vee \sim q] \rightarrow \sim (p \vee \sim r)$

$= [(p \vee (\sim q \vee \sim r)) \vee \sim q] \rightarrow \sim (p \vee \sim r)$

$= [p \vee \sim q \vee \sim r \vee \sim q] \rightarrow \sim (p \vee \sim r)$

$= [p \vee \sim q \vee \sim r] \rightarrow \sim (p \vee \sim r)$

$= \sim (p \vee \sim q \vee \sim r) \vee (\sim p \wedge r)$

$= (\sim p \wedge q \wedge r) \vee (\sim p \wedge r)$

$= (\sim p \wedge q \wedge r) \vee [\sim p \wedge (q \vee \sim q) \wedge r]$

$= (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r)$

$= (\sim p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r)$

PDNF A $= \bar{p}qr + \bar{p}\bar{q}r$

PDNF ~A $= pqr + pq\bar{r} + p\bar{q}r + p\bar{q}\bar{r} + \bar{p}q\bar{r} + \bar{p}\bar{q}\bar{r}$

PCNF A $= (\bar{p}+\bar{q}+\bar{r})(\bar{p}+\bar{q}+r)(\bar{p}+q+\bar{r})(\bar{p}+q+r)(p+\bar{q}+r)(p+q+r)$

PCNF ~A $= (p+\bar{q}+\bar{r})(p+q+\bar{r})$

Question 4

$\exists y : \sim P(x,y) \rightarrow (\forall x)(Q(x) \vee \sim R(x,y))$

$\forall x : Q(x) \vee \sim R(x,y)$

The given expression is not a statement because the variable x in $\sim P(x,y)$ is a free variable.

Question 5

a) Infix form :

$[(5\times6)-(9+(8\div2))]+[7-(4\times5)]$

$= [30-(9+4)]+[7-20]$

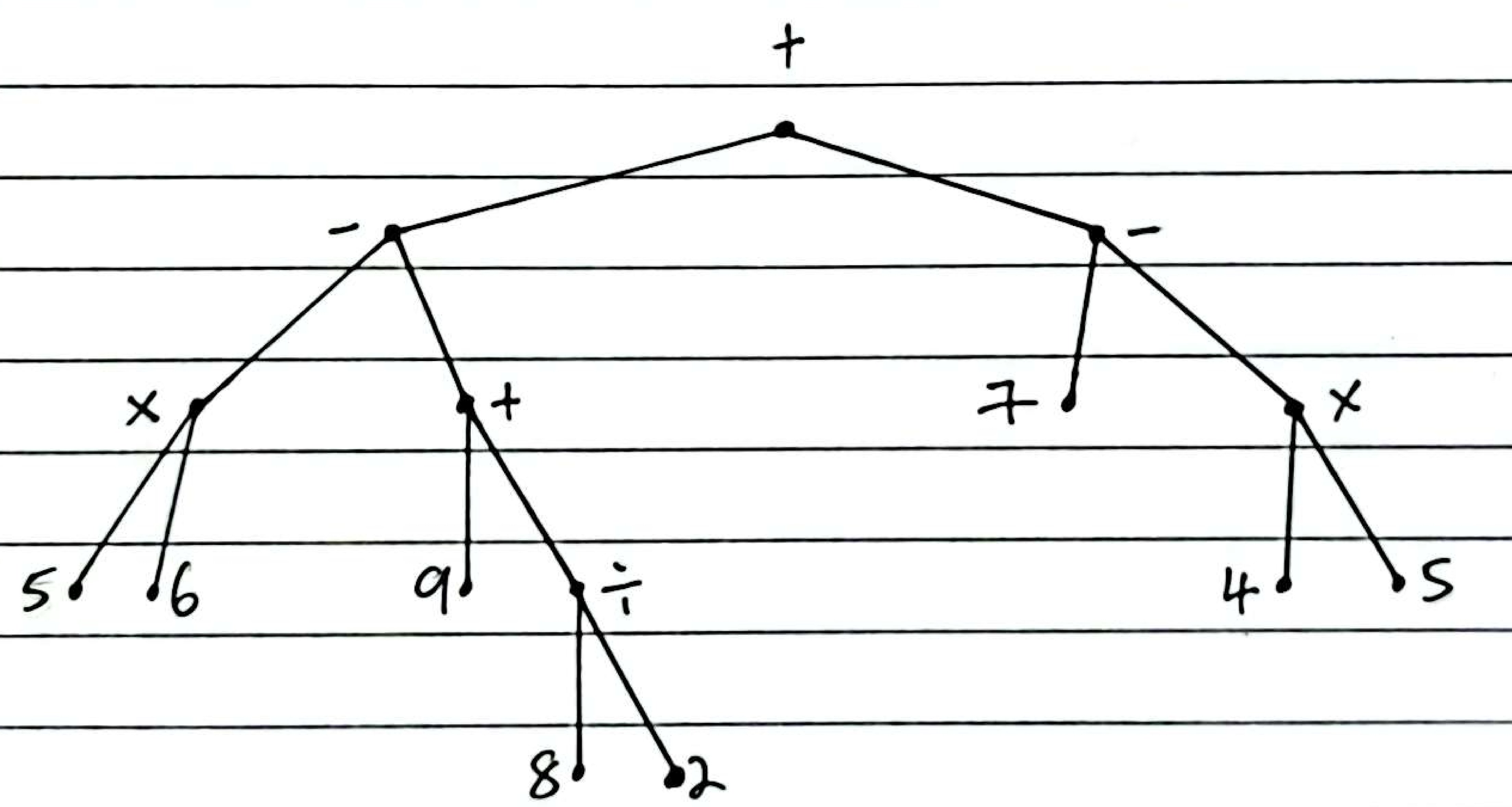
$= [30-13]+[-13]$

$= 17-13$

$= 4$

b) Corresponding fully parenthesised expression :

$[(5\times6)-(9+(8\div2))]+[7-(4\times5)]$



c) Postfix form :

$5\,6\times9\,8\,2\div+-7\,4\,5\times-+$

Infix form :

$[(5\times6)-(9(8\div2))]+[7-(4\times5)]$