Practice Questions on Counting (Week 1)

The following questions are taken from "Rosen Ch 6 Counting.pdf", Chapters 6.1 and 6.3.

Tutorial Questions

1. (Chp6.1, Q3) A multiple-choice test contains 10 questions. There are four possible answers for each question.
    a. In how many ways can a student answer the questions on the test if the student answers every question?
    b. In how many ways can a student answer the questions on the test if the student can leave answers blank?

2. (Chp6.1, Q23) How many positive integers between 100 and 999 inclusive:
    a. are divisible by 7?
    b. are odd?
    c. have the same three decimal digits?
    d. are not divisible by 4?
    e. are divisible by 3 or 4?
    f. are not divisible by either 3 or 4?
    g. are divisible by 3 but not by 4?
    h. are divisible by 3 and 4?

3. (Chp6.1, Q29) How many license plates can be made using either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters?

4. (Chp6.1, Q45) How many ways are there to seat six people around a circular table where two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?

5. (Chp6.1, Q47) In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if:
    a. the bride must be next to the groom?
    b. the bride is not next to the groom?
    c. the bride is positioned somewhere to the left of the groom?

6. (Chp6.3, Q3) How many permutations of $\{a, b, c, d, e, f, g\}$ end with a?

7. (Chp6.3, Q13) A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

8. (Chp6.3, Q19) A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes
    a. are there in total?
    b. contain exactly two heads?
    c. contain at most three tails?
    d. contain the same number of heads and tails?

9. (Chp6.3, Q23) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? *Hint*: First position the men and then consider possible positions for the women.

10. (Chp6.3, Q27) A club has 25 members.
    a. How many ways are there to choose four members of the club to serve on an executive committee?
    b. How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?

11. 19 SMU students are going to Cathay Cinema
    a. There are 22 seats in each row. The students can pick any seat in row A. How many ways are there for these 19 students to occupy the row?
    b. Andrew, Brandon, and Cheryl decide to join the group of 19 students. Brandon prefers to sit at either end of the row A. How many ways are there to arrange the students on the same row of 22 seats so that Brandon's preference is satisfied?

12. There are 10 men and 10 women.
    a. How many ways are there to arrange these people in a row?
    b. How many ways are there to seat these 20 people, if there are two different circular tables? One table is blue, and the other table is red. Each table has 5 men and 5 women alternating. Two seatings for a table are considered the same when everyone on that table has the same two neighbors without regard to whether they are right or left neighbors.

Extra Practice Questions

13. (Chp6.1, Q5) Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?

14. (Chp6.1, Q24) How many positive integers between 1000 and 9999 inclusive
    a. are divisible by 9?
    b. are even?
    c. have distinct digits? (e.g. 123 and 231 do <u>not</u> have distinct digits)
    d. are not divisible by 3?
    e. are divisible by 5 or 7?
    f. are not divisible by either 5 or 7?
    g. are divisible by 5 but not by 7?
    h. are divisible by 5 and 7?

15. (Chp6.1, Q30) How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

16. (Chp6.1, Q44) How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

17. (Chp6.1, Q46) In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if
    a. the bride must be in the picture?
    b. both the bride and groom must be in the picture?
    c. exactly one of the bride and the groom is in the picture?

18. (Chp6.3, Q7) Find the number of 5-permutations of a set with nine elements.

19. (Chp6.3, Q15) In how many ways can a set of five letters be selected from the English alphabet?

20. (Chp6.3, Q18) A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes
    a. are there in total?
    b. contain exactly three heads?
    c. contain at least three heads?
    d. contain the same number of heads and tails?

21. (Chp6.3, Q24) How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other?

22. (Chp6.3, 33) Suppose a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

23. In a class of Computational Thinking in year 2020, there are 20 SIS students and 10 non-SIS students.
    a. How many ways are there to choose a single group of three (3) members with at least one non-SIS student?
    b. How many ways are there to form 10 groups of 3 members for doing projects with at least one non-SIS student in each group?

24. Singapore's national football team is one of the 11 teams competing at the football tournament in the 2019 Southeast Asian Games.
    a. In the qualification round, the 11 teams are divided into two groups: Group A (5 teams) and Group B (6 teams) respectively. Every team will compete once with each of the other teams in the same group.
        i. How many ways are there to divide the 11 teams into the two groups?
        ii. How many matches are there in the qualification round?

    b. Singapore football team has 21 members, including 3 goalkeepers, 8 defenders, 6 midfielders and 4 forwards. The coach applies the 4-4-2 formation, which has 4 defenders, 4 midfielders, 2 forwards and 1 goalkeeper, to form the playing team of 11 players. Except for the goalkeeper, the other positions are equally split into left and right sides. For example, there are 2 left midfielders and 2 right midfielders. How many ways are there to form the playing team, if the left side and right sides are not considered the same position?

    c. Before its first match begins, the playing team of 11 players huddles in a circle. How many ways are there to create the circle, such that the goalkeeper will not be standing immediately next to any forward? Left and right neighbors are considered different.

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