Practice Questions on Greedy Heuristics (Week 10)

Tutorial Questions

1. Suppose we are solving the Traveling Salesman Problem (TSP) for 20 cities. How many possible routes need to be examined if:

   a) the distance between two cities are symmetric, e.g., A to B is the same as B to A
   b) the distances between two cities are different in different directions, e.g., A to B is shorter or longer than B to A.

2. The following table shows the distances between 7 cities. Work out a proposed route and the distance of the proposed round trip.

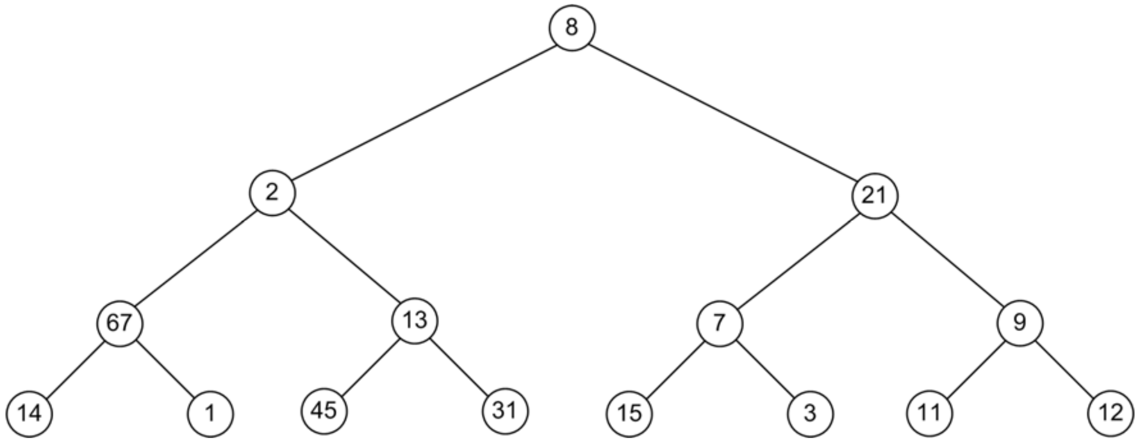
   a) Using the first greedy algorithm in the slide with A with the starting city.
   b) Using the second greedy algorithm in the slide starting with the closest pair of cities.

	A	B	C	D	E	F	G
A	-	205	86	76	259	278	144
B	205	-	246	250	112	81	191
C	86	246	-	17	325	326	230
D	76	250	17	-	323	329	220
E	259	112	325	323	-	79	173
F	278	81	326	329	79	-	230
G	144	191	230	220	173	230	-

3. Following is a greedy algorithm for finding the path from the root to a leaf in a binary tree with the highest sum of values of its nodes.

   "Starting from the root, go to the child of the current node with the higher value and record the edge; repeat until we reach a leaf"

   a) Perform the above algorithm on the tree below.
   b) Provide another example tree to show that the above algorithm may not find the path with the highest sum of values for that tree.



4. You work as a cashier in a convenience store. Suppose the coin denominations are 20 cents, 10 cents, 5 cents, and 1 cent. Your job is to give change to customers, in minimal number of coins. For example, if the change is 35 cents, the minimal number is 3 coins, consisting of 20-cent, 10-cent, and 5-cent coins.

   a) Design a brute force algorithm to work out the change in as few coins as possible.
   b) Design a greedy algorithm to work out the change.
   c) For the above coin denominations, is there any change amount for which greedy fails to find the optimal solution?

5. You have a bag that can carry no more than 15 kg. You have the following items to put into the bag:
   - item A has weight of 12 kg and value of $10
   - item B has weight of 5 kg and value of $8
   - item C has weight of 4 kg and value of $2
   - item D has weight of 3 kg and value of $5
   - item E has weight of 2 kg and value of $6

   You want to optimize the sum of values of items to carry in the bag, without going over the 15 kg limit.

   a) Design a brute force algorithm to determine which items to be put into the bag.
   b) Design a greedy algorithm to determine the items.
   c) For the above five items, is there any weight limit for which greedy fails to find the optimal solution?

Extra Practice Questions

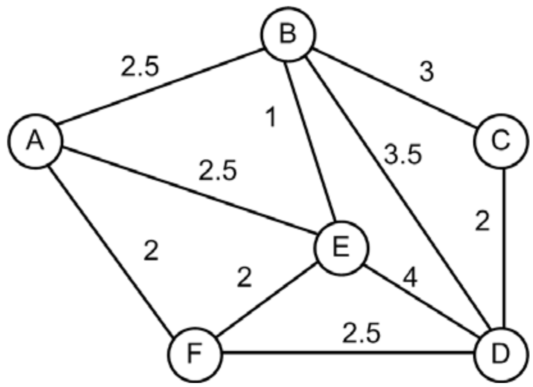
6. What would be the answer to the Q1a and Q1b in the Tutorial Questions above, if the tour must cover all cities but there is no need to return to the starting point?

7. What are the complexities of both the greedy algorithms (greedy 1 and greedy 2) for the Travelling Salesman Problem covered in your lecture slides?

8. Following is a greedy algorithm for finding the shortest path between the pair of nodes (**s**, **t**) in a graph.

   "Starting from **s**, go to the node closest to the current node that has not yet been visited and record the edge; repeat until we reach **t**".

   a) Perform the above algorithm on the following graph with the pair of nodes (A, D). (i.e. starting from A, ending at D.)
   b) Provide another example pair of nodes (other than A, D) to show that the above algorithm may not find the shortest path for this pair.



9. Suppose that in the Kingdom of Shrek, the coin denominations are 1-dollar, 60-cent, 35-cent, 5-cent, and 1-cent. You are a shopkeeper. Each time a customer pays for purchase, your objective is to return the change (if any) with as few coins as possible.

   You adopt the greedy strategy to always return the largest coin denomination possible. For example, if the change is $2.65, then:
   - first you return 1-dollar coin, remainder: $1.65
   - then you return 1-dollar coin, remainder: $0.65
   - then you return 60-cent coin, remainder: $0.05
   - then you return 5-cent coin, remainder: $0.

   Therefore the solution is 4 coins, which happens to be optimal.
   a) Provide another change amount that the Greedy Strategy will be successful to discover the optimal solution.
   b) Provide a change amount whereby the Greedy Strategy will fail to discover the optimal solution. In other words, there exists another solution that would return fewer coins than the greedy solution.

10. You are the manager of the Singapore ski team for the Winter Olympics, and your team consists of 5 members of heights h1, h2, h3… h5, and you are provided with 5 skis of length l1, l2, l3… l5. There are hence 5! (or 120) unique combinations of skiers to skis. To be able to ski as quickly as possible, a skier should get skis the length of which matches his height. Hence, to ensure an Olympic medal, you need to assign skis to each skier to minimize the sum of the absolute differences between the height of the skier and the length of his ski.

    Describe clearly a greedy algorithm to get a reasonably good ski/skier assignment. What is the worst-case complexity of your algorithm?

~End