Practice Questions on Graph (Week 9)

Tutorial Questions

1. Scout Airways flies from Singapore to all the capitals in ASEAN countries. However, there is a return flight to Singapore only if the capital's population is larger than Singapore's. Otherwise, to return to Singapore, one needs to first fly to another capital with a return flight to Singapore. There is always a flight from any capital city to another capital city (other than Singapore) with the next largest population size (e.g., there is a flight from Bandar Seri Begawan (pop: 0.1M) to Naypyidaw (pop: 0.9M)). The following table lists the population sizes of capitals in South East Asia.

Country	*Capital*	*Population*
Singapore	Singapore	5.2M
Indonesia	Jakarta	10.2M
Malaysia	Kuala Lumpur	1.6M
Thailand	Bangkok	8.3M
Philippines	Manila	1.7M
Brunei	Bandar Seri Begawan	0.1M
Vietnam	Hanoi	6.6M
Cambodia	Phnom Penh	1.5M
Laos	Vientiane	6.5M
Myanmar	Naypyidaw	0.9M

   Draw a directed graph, where the vertices are the capital cities, and an edge exists from one city to another city only if there is a flight from the first to the second city.

2. A graph has 10 vertices, *A* to *J*.
   - *A* connects to *B* and *I*.
   - *B* connects to *D* and *H*.
   - *C* connects to *D*.
   - *D* connects to *H*.
   - *E* connects to *F* and *G*.
   - *F* connects to *A*.
   - *G* connects to *J*.
   - *H* connects to *E*.
   - *I* connects to *B* and *J*.
   - *J* connects to *A*.

   a) Represent this graph in adjacency list and adjacency matrix respectively.
   b) Which vertices have the highest in-degree?
   c) Which vertices have the highest out-degree?
   d) What is the sequence of vertices visited by breadth-first search beginning from *A*?
   e) What is the sequence of vertices visited by depth-first search beginning from *A*?

3. Here are the names of 8 courses in a particular degree program, and their pre-requisites:
   - IS200, PMSB and DM have no pre-requisites
   - To take OOAD, the student must have passed IS200
   - To take SE, the student must have passed OOAD
   - To take EI, student must have passed OOAD, DM and PMSB
   - To take AA, the student must have passed IS200 and EI
   - To take EWS, the student must have passed SE and EI

a) Draw a directed graph to represent these courses and their pre-requisites
b) Represent the graph you have drawn in the form of an adjacency matrix:

From \ To	IS200	DM	OOAD	SE	EI	AA	EWS	PMSB
IS200								
DM								
OOAD								
SE								
EI								
AA								
EWS								
PMSB								

c) What will be the order in which the courses are visited if a Depth First Search is performed starting from IS200 given that the order of the vertices in the graph goes like this: IS200, DM, OOAD, SE, EI, AA, EWS, PMSB (as shown in the table above).
d) Assuming that a student can only take 1 course per term, suggest ONE valid topological order in which all these courses can be taken over 8 terms.

4. G is a graph with the following adjacency list:

A	B E F
B	C D F
C	D
D	F
E	D F
F	C

a) Which node is visited right after visiting node F if we perform depth-first-search on G, starting from A, based on the adjacency list above?
b) Invert the direction of one edge in G so that we get a DAG. Find a topological ordering in the resulting DAG.

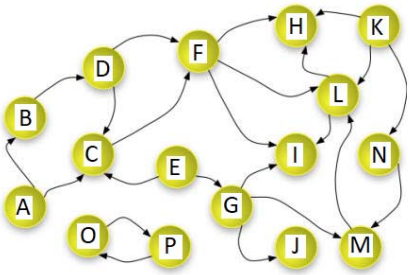
5. A graph has six vertices, labeled A to F. There are exactly two paths from A to F. The first path is (A, B, C, F). The second path is (A, D, E, F). All the edges in this graph can be found in either of this path.

a) If you run topological sorting algorithm, what is the output topological ordering?
b) Could there be more than one possible outputs? What affects which output you get?

Extra Practice Questions

6. You are attending a friend's birthday party. Suppose that you want to track who knows whom in the birthday party, what kind of graph: directed or undirected, will you use? Can it have cycles?

7. Refer to the graph below:



(Diagram adapted from www.openarchives.org/ore/0.2/datamodel-images/WebGraphBase.jpg)

a) Represent this graph in adjacency list and adjacency matrix respectively.
b) Which vertices have the highest in-degree?
c) Which vertices have the highest out-degree?
d) What is the sequence of vertices visited by **BFS** beginning from *A*?
e) What is the sequence of vertices visited by **DFS** beginning from *A*?

8. G is a weighted graph with the following adjacency list.

A	(B, 2)	(E, 1)	(F, 5)
B	(A, 2)	(D, 3)	(F, 3)
C	(D, 4)		
D	(B, 3)	(C, 4)	(F, 3)
E	(A, 1)		
F	(A, 5)	(B, 3)	(D, 3)

A simple cycle is a cycle that goes through each of its nodes exactly one. Add one edge in to G so that the resulting graph contains at least one simple cycle going through all the nodes.

9. You are considering taking a certificate program in marketing. There are three levels of courses.
   - The first-level courses are 1A, 1B, and 1C. There are no prerequisites for first-level courses.
   - The second-level courses are 2A and 2B. The prerequisites for 2A are 1A and 1B. The prerequisites for 2B are 1B and 1C.
   - The third-level courses are 3A, 3B, and 3C. 2A is the prerequisite for all third-level courses. In addition, to take 3B, you first need to take 2B. To take 3C, you first need to take 1C.

   You need to complete the program in eight terms, taking one course per term. You have to propose the sequence of courses that you will follow over the eight terms. Figure out one valid sequence of courses.

10. A graph has **n** vertices, with labels 1, 2, ..., **n**. There is an edge from every vertex with label **i** to another vertex with label **j** if **i** < **j**.
    a) How many edges exist in this graph?
    b) Is there any cycle in this graph?
    c) How many topological orderings exist in this graph?

~End