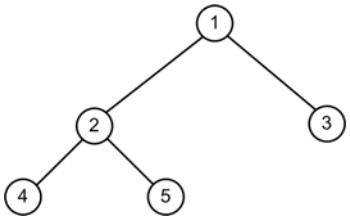
Answer Key for Greedy Heuristics (Week 10)

Tutorial Questions

1. See: https://youtu.be/gk1FTQRe3BI
   Number of routes:
   a) 19! / 2
   b) 19!

2. See https://youtu.be/NqHyaE1EUI4
   Routes:
   a) A → D → C → G → E → F → B → A, cost = 861.
      Mirror is also OK: A → B → F → E → G → C → D → A. same cost
   b) C → A → G → E → F → B → D → C, cost = 830
      Mirrors and any of the other 6 "rotations" are OK.
      e.g. of rotation: A → G → E → F → B → D → C → A

3. See https://youtu.be/6gdLZ2cDfWg
   Tree:
   a) 8 + 21 + 9 + 12 = 50
   b) Another example:



   Greedy path: 1 + 3 = 4
   Optimal path: 1 + 2 + 5 = 8

4. See https://youtu.be/dh5dxYVWn1k
   Coin change:
   a) Try all combinations of different number of coins (combinations of 1 coin, combinations of 2 coins, and so on).
   b) Repeatedly deduct the largest coin denomination.
   c) No

5. See https://youtu.be/5x2yQojkcHU
   Bag:
   a) Try all possible subsets of items:
      Only 1 item {A}, {B}, {C}, {D}, {E}. There are $^5C_1$ such subsets.
      Only 2 items {A,B}, {A,C}, {A,D}, {A,E}, {B,C}, … there are $^5C_2$ combinations.
      Only 3 items {A,B,C}, {A,B,D}, {A,B,E}, {A,C,D}, … there are $^5C_3$ combinations.
      Only 4 items {A,B,C,D}, {A,B,C,E} … there are $^5C_4$ combinations.
      Only 5 items {A,B,C,D,E} there is only 1 combination

b) Repeatedly put in item with greatest ratio of value over weight until you breach the 15kg limit. (another possible greedy algo will be to repeatedly put in item with smallest weight until you breach the weight limit)
c) Yes, e.g., 7 kg limit

Extra Practice Questions

6. Number of routes:
   a) 20! / 2. No concept of identical "rotations".
   b) 20!

7. Complexity:
   a) Greedy 1 is $O(n^2)$
   b) Greedy 2 is $O(n^3)$

8. Shortest path:
   a) Greedy: A, F, E, B, C, D
   b) Pair (C, E). Greedy is C, D, F, E, with weight 6.5. Optimal is C, B, E, with weight 4.

9. Change amount:
   a) $1.95
   b) $2.80.
   Greedy algo will result in: 2x$1, 1x60c, 4x5c. total of 7 coins.
   A better solution will be: 2x$1, 2x35c, 2x5c. total of only 6 coins.

10. Greedy algorithm 1:
   Step 1: Mark all skiers as unallocated, mark all skis as unallocated.
   Step 2: For each possible pair of skier/ski: (e.g. (h1,l1), (h1,l2), (h1,l3), (h1,l4), (h1,l5), (h2,l1) …)
   Calculate |hx-ly|
   Step 3: Sort |hx-ly| and each pair of skier/ski in ascending order.
   Step 4: Repeat until all skiers are allocated a ski:
   - Consider the skier/ski pair from the top of the sorted list.
   - If both the skier and ski are still unallocated, mark both as allocated, and this will form one pair in your final set. Remove this pair from the sorted list.
   - If either the skier or ski is already allocated, remove this pair from the sorted list.

   Greedy algorithm 2 (not as good):
   Step 1: Mark all skiers as unallocated, mark all skis as unallocated.
   Step 2: Randomly select 1 unallocated skier. For this skier:
   - Calculate |h-lx| for each unallocated ski.
   - Match the ski with the smallest |h-lx| value to this skier.
   - Mark the matched skier and ski as allocated.

   Repeat step 2 above until there is no more unallocated skier.

~End