School of
Information Systems

SMU
SINGAPORE MANAGEMENT UNIVERSITY

IS-COR1702: COMPUTATIONAL THINKING
WEEK 1B: COUNTING (COMBINATORICS)

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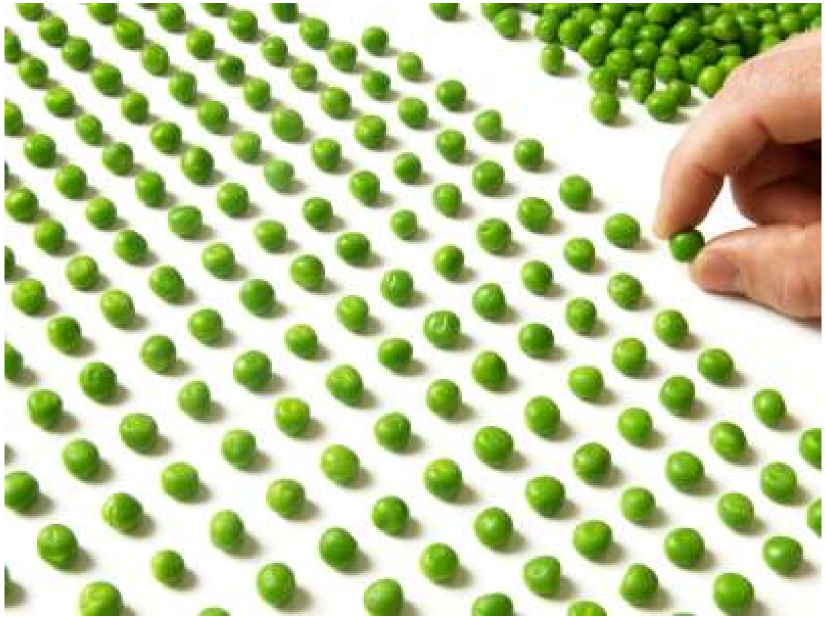
Counting

Basic principles in combinatorics



- Product Rule
- Sum Rule
- Permutation
- Combination

Reference

- Kenneth H. Rosen, "Discrete Mathematics and Its Applications", 7th Edition, Mc-Graw Hill, 2012.
  - Chapter 6 Counting
    - 6.1 The Basics of Counting
    - 6.3 Permutations and Combinations
  - Copyrighted scanned copy available on eLearn
    - "Rosen - Ch 6 Counting.pdf"

- Easy to read and understand:
  - https://www.mathsisfun.com/combinatorics/combinations-permutations.html

Product Rule

If there are m ways to do one thing,
and n ways to do another thing, then
there are m x n ways of doing both things.

http://en.wikipedia.org/wiki/Rule_of_product

There are 2 floors in SIS with seminar rooms.
Each floor has 4 seminar rooms.
How many seminar rooms are there in SIS?

- Two things to do:
  - Pick a floor - 2 ways
  - Pick a room in a floor - 4 ways
- Both things have to be done together
- Based on Product Rule, Total: 2 x 4 = 8 ways

A binary string consists only of 0's and 1's.
How many possible strings of length n may exist?

- A binary string of length n has n binary characters (or bits).
- Binary strings of length 3
  - 000, 001, 010, 011, 100, 101, 110, 111
- Each bit can be either 0 or 1, i.e., two possibilities.
- For 3 bits, we have three things to do together:
  - pick first bit: 2 ways
  - pick second bit: 2 ways
  - pick third bit: 2 ways
- Based on Product Rule, Total = 2 x 2 x 2 = 2^3
- For n bits, 2^n possibilities

Sum Rule

If there are `m` ways to do one thing,
and `n` ways to do another thing, and
both things cannot be done at the same time,
then there are `m` + `n` ways of doing only one of
the things.

http://en.wikipedia.org/wiki/Rule_of_sum

You can go to one of the 4 seminar rooms in level 2, or the 4 seminar rooms in level 3.
How many possible seminar rooms can you go to in SIS?

- There are 4 rooms in level 2.
- There are 4 rooms in level 3.
- We can go to either a room in level 2 or a room in level 3, but not both.
- Based on Sum Rule, Total = 4 + 4 = 8.

You can choose an FYP project from the 20 projects offered by industry, 10 projects offered by professors, or propose your own project.
How many possible projects do you have?

- You can choose only one project.
  - 20 industry project
  - 10 professor project
  - 1 own project
- Total = 20 + 10 + 1 = 31 possible projects

Product Rule + Sum Rule

- More complex counting problems require both rules

Inclusion-Exclusion Principle

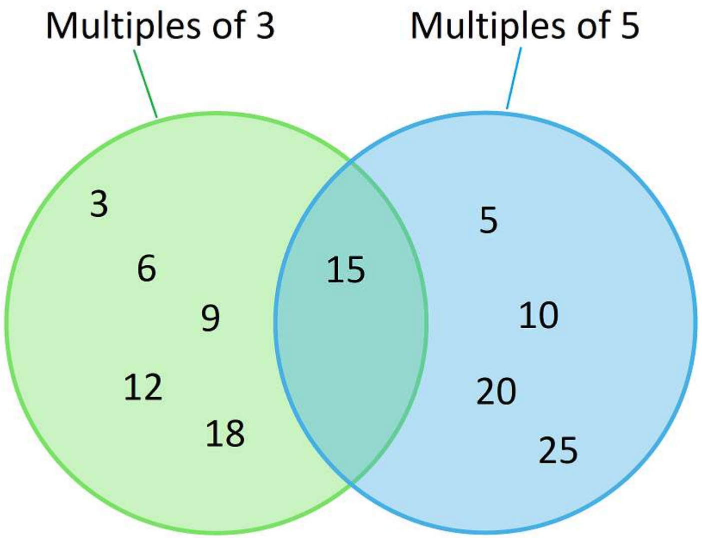
If two things can be done at the same time, the sum rule will lead to double counting.
In this case, we add the number of ways to do each task, and then subtract the number of ways to do both tasks.

http://en.wikipedia.org/wiki/Inclusion%E2%80%93exclusion_principle

How many integers from 1 to 1000 are multiples of 3 or multiples of 5?

- Multiples of 3: {3, 6, 9, 12, 15, ..., 999}
  - no of integers = 1000/3 = 333
- Multiples of 5: {5, 10, 15, ..., 1000}
  - no of integers = 1000/5 = 200
- Note that multiples of both 3 and 5 (e.g., 15, 45) are counted twice
- Multiples of both 3 and 5: {15, 30, 45, ... }
  - no of integers = 1000/15 = 66
- Inclusion-Exclusion Principle: Total = 333 + 200 - 66 = 467

How many integers from 1 to 1000 are multiples of 3 or multiples of 5?



- 333 integers in green circle (of which 66 are in overlapping area)
- 200 integers in blue circle (of which 66 are in overlapping area)
- 66 integers in overlapping area (double counted)
- Total number of integers in green and blue area = 333 + 200 - 66 = 467

In-class Exercises

Every car registered in Singapore has a license plate number, which begins with a letter 'S', followed by two other letters, 4 digits, and another letter (all upper case) How many possible license plate numbers are there?

It turns out that all plate numbers beginning with SH are for taxis only. How many possible license plate numbers for private car owners?

In-class Exercises

You are supposed to create a new password. Min 6 characters and max 8 characters. Alphanumeric characters (letters & numbers). Case insensitive. The password has to contain at least one digit. How many possible passwords are there?

If passwords may contain lower case letters and digits, how many 6-character passwords start with a lower case letter ‘a’ or ends with a lower case letter ‘z’?

Permutation

A permutation of a set of objects is an ordered arrangement of these objects.

- Given a set S = {A, B, C}, its permutations are:
  - A, B, C
  - A, C, B
  - B, A, C
  - B, C, A
  - C, A, B
  - C, B, A

- Given a set T = {1, 2, 3}, its permutations are:
  - 1, 2, 3
  - 1, 3, 2
  - 2, 1, 3
  - 2, 3, 1
  - 3, 1, 2
  - 3, 2, 1

The number of permutations of a set of length n is $n!$

r-Permutation

An r-permutation of a set of n objects is an ordered arrangement of r of the n objects.

- Given a set S = {A, B, C}, its 2-permutations are:
  - A, B
  - A, C
  - B, A
  - B, C
  - C, A
  - C, B
- Given a set T = {1, 2, 3}, its 2-permutations are:
  - 1, 2
  - 1, 3
  - 2, 1
  - 2, 3
  - 3, 1
  - 3, 2

The number of r-permutations of a set of length n is:

$${}^{n}P_{r} = n! / (n - r)!$$

150 countries compete in the Olympics. How many ways are there to select first-place, second place, and third-place winners?

- Reason it out:
  - first-place: 150 choices
  - second-place: 149 choices
    - first-place cannot also be second place
  - third-place: 148 choices
- Total = 150 x 149 x 148 = 3,307,800 ways

- Use the formula:
- $^{150}P_3 = 150! / (150 - 3)!$
- $= 150! / 147!$
- $= 148 \times 149 \times 150$
- = 3,307,800 ways

Combination

A combination of a set of objects is an <u>unordered selection</u> of these objects.

- Given a set S = {A, B, C}, its combination is:
  - {A, B, C}
- Given a set T = {1, 2, 3}, its combination is:
  - {1, 2, 3}

The number of combinations of a set of length `n` is `1`.

r-Combination

An r-combination of a set of n objects is an unordered selection of r objects out of the r objects.

- Given a set S = {A, B, C}, its 2-combinations are
  - A, B
  - A, C
  - B, C

- Given a set T = {1, 2, 3}, its 2-combinations are:
  - 1, 2
  - 1, 3
  - 2, 3

The number of r-combinations of a set of length n is:

$$ {}^{n}C_{r} = \frac{n!}{r!(n - r)!} $$

150 countries compete in the Olympics. How many ways to select the top 3 winners if we do not care about their specific ranking?

- No of 3-permutations is 150 x 149 x 148 = 3,307,800
- These 3-permutations contain all ways to order a selection of 3 countries. No of ways to order 3 countries is 3! = 6.
- No of ways to select 3 countries is 3,307,800/6 = 551,300.

- Use the formula:
- $^{150}C_3 = 150! / 3!\,(150 - 3)!$
- $= 150! / 3!\,147!$
- $= 148 \times 149 \times 150 / 6$
- = 551,300 ways

In-class Exercises

How many permutations of the letters A to H if the three letters "ABC" has to occur consecutively? What if they must appear together, but not necessarily consecutively?

You are inviting 3 other guests (so there are 4 diners including yourself) to have dinner at home. Your dining table can seat 6. How many different seating arrangements can you have?

In-class Exercises

We would like to form a 7-member committee, consisting of 3 faculty members and 4 students. There are 40 faculty members and 300 students.
How many different ways can we form the committee?

How many ways are there for 3 men and 3 women to stand in a line so that no two women stand next to each other?

Summary

- Basic principles of counting
  - Product Rule: m ways to do one thing; n ways to do another thing; m x n ways of doing both things.
  - Sum Rule: if only one thing can be done, m + n ways
    - Inclusion-Exclusion Principle: if there are cases when both things are done together, we need to subtract the number of ways to do both from the sum.
  - Permutation: number of ways to get an ordered arrangement from a set
  - Combination: number of ways to get an unordered selection from a set

Road Map

Algorithm Design and Analysis

- Week 1: Introduction, Counting, Python Programming
- Week 2: Python Programming
- Week 3: Complexity
- Week 4: Iteration, Decomposition
- Week 5: Recursion

Have you installed Python / PythonLabs? Refer to instructions on eLearn (Announcement)

Fundamental Data Structures

(Weeks 6 - 10)

Computational Intractability & Heuristic Reasoning

(Weeks 11 - 13)