浙江大学 2014－2015 学年冬季学期

《高级数据结构与算法分析》课程期末考试试卷

课程号：__21120490__，开课学院：软件学院、计算机学院、竺可桢学院

考试试卷：√A 卷、B 卷（请在选定项上打√）

考试形式：√闭、开卷（请在选定项上打√），允许带___无____入场

考试日期：__2015__年__1__月__28__日，考试时间：__120__分钟

诚信考试，沉着应考，杜绝违纪。

考生姓名：_____学号：_____所属院系：_____

题序	一	二	三						四	总 分
得分										
评卷人										

Answer Sheet

Part I (20)				
1.	2.	3.	4.	5.
6.	7.	8.	9.	10.

Part II (18)	
1. ①_____ ②_____ ③_____	2. ①_____ ②_____ ③_____

Part III (47)	
1. After inserting 27:	1. After deleting 7 and 18:

2. AVL insertions for 88, 70, 61, 96, 120, 90, 65

2. Splay insertions for 88, 70, 61, 96, 120, 90, 65

3.

4.

5.

6.	7.

Part IV (15)

(1)

L	1	2	3	4	5	6	7	8	9	10	11
R											

(2)

(3)

NOTE: Please write your answers on the answer sheet.

注意：请将答案填写在答题纸上。

I. Please fill in the blanks (the answer for each blank is unique). (2 points each)

1. For an AVL tree of depth D (depth(root)=0), if **all** the balance factors of non-leaf nodes are 0, the total number of nodes in the tree must be_____.
 a. 2^D-1 b. 2^{D+1}-1 c. 2^{D-1}+1 d. 2^D+1

2. The worst case time complexity for finding an element in a splay tree with N nodes is _____.
 a. O(1) b. O(logN) c. O(N) d. O(NlogN)

3. Which of the following statements concerning a B+ tree of order M is true?
 a. The root always has between 2 and M children
 b. Not all leaves are at the same depth
 c. Leaves and nonleaf nodes have some key values in common
 d. All nonleaf nodes have between ⌈M/2⌉ and M children

4. Which one of the following statements is true?
 a. The children in a binomial tree are arranged in increasing rank
 b. A recursive implementation of skew heaps can fail because of lack of stack space
 c. Npl(NULL) = 0
 d. The result of merging two skew heaps can’t be a leftist heap

5. Given a binomial queue with 19 nodes, how many trees are left after calling DeleteMin three times?
 a. 1 b. 2 c. 4 d. 16

6. Which of the following statements about Inverted File Index is FALSE?
 a. Inverted File contains a list of pointers to all occurrences of term in the text
 b. Inverted File Index is used for accelerate the speed of information retrieval
 c. Inverted File Index is a kind of data structure based on binary search tree
 d. Index is a mechanism for locating a given term in a text

7. Among the following problems, _____ is NP-complete.
 a. Euler circuit problem b. Bin packing problem
 c. Halting problem
 d. Deciding the non-existence of a Hamiltonian circuit

8. To pack the given 9 items of sizes 0.2, 0.9, 0.3, 0.1, 0.7, 0.5, 0.6, 0.8, 0.4 into the bins with unit capacity by the *on-line* algorithms, the next-fit uses _____ bins.
 a. 5 b. 6 c. 7 d. 8

9. Which one of the following statements is FALSE?

a. Closest points problem can be polynomially reduced to Hamiltonian circuit problem

b. Satisfiability problem can be polynomially reduced to Hamiltonian circuit problem

c. All-pairs longest paths problem is an NP problem

d. Hamiltonian circuit problem can be polynomially reduced to Euler circuit problem

10. When solving a problem with input size N by divide and conquer, if at each stage the problem is divided into 8 sub-problems of equal size N/3, and the conquer step takes $O(N^2 \log N)$ to form the solution from the sub-solutions, then the overall time complexity is ___.

a. $O(N^2 \log N)$ b. $O(N^2 \log^2 N)$ c. $O(N^3 \log N)$ d. $O(N^{\log 8/\log 3})$

II. Given the function descriptions of the following two (pseudo-code) programs, please fill in the blank lines. (18 points)

1. The function is to merge two binomial queues H1 and H2, and return H1 as the result. (9 points)

```
BinQueue  Merge( BinQueue H1, BinQueue H2 )
{   BinTree T1, T2, Carry = NULL;
    int i, j;
    H1->CurrentSize += H2-> CurrentSize;
    for ( i=0, j=1; j<= H1->CurrentSize; ①_____ ) {
       T1 = H1->TheTrees[i]; T2 = H2->TheTrees[i];
       switch( 4*!!Carry + 2*!!T2 + !!T1 ) {
        case 0: case 1: break;
        case 2: H1->TheTree[i] = T2; H2->TheTrees[i] = NULL; break;
        case 3: Carry = CombineTrees( T1, T2 );
                H1->TheTrees[i] = H2->TheTrees[i] = NULL; break;
        case 4: H1->TheTrees[i] = Carry; Carry = NULL; break;
        case 5: Carry = CombineTrees( T1, Carry ); H1->TheTrees[i] = NULL; break;
        case 6: Carry = CombineTrees( T2, Carry ); H2->TheTrees[i] = NULL; break;
        case 7: H1->TheTrees[i] = T2;  Carry = ②_____;
                ③_____; break;
       } /* end switch */
    } /* end for-loop */
    return H1;
}
```

2. The function FKS is to find the K-th smallest element from the array A[L]…A[R] where all the elements are distinct. The function **Partition(A, L, R, Pivot)** is to break the elements in A[L]…A[R] into two parts, such that all the elements to the left are smaller than **pivot** and all these to the right are larger than **pivot**, and the position of **pivot** is returned. (9 points)

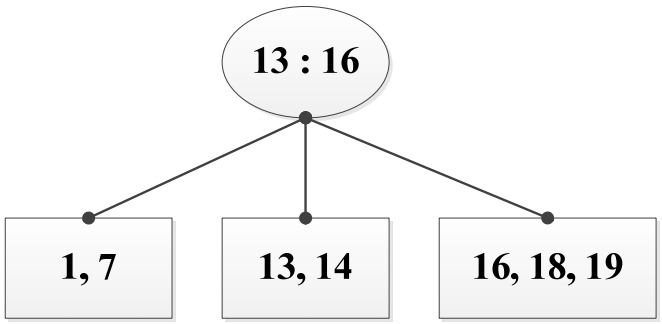
```
int FKS ( int A[], int L, int R, int K )
{  if ( L == R )  return A[L];
   Pivot = A[L];
   P = Partition( A, L, R, Pivot );
   N = P – L + 1;
   if ( K == N )
      return ①_____________________;
   else if ( K < N )
      return ②_____________________;
   else
      return ③_____________________;
}
```

III. Please write or draw your answers for the following problems on the answer sheet. (47 points)

1. Givena B+ tree of order 3 as shown in the figure. Please show:

(a) the result of inserting 27 with splitting strategy; (2 points) and

(b) the results of deleting 7 and then 18 from the tree obtain from (a). (4 points)



2-3 tree for problem 1

2. Please draw the results of inserting {88, 70, 61, 96, 120, 90, 65} into an initially empty AVL tree, and an initially empty splay tree. (10 points)

3. Given some characters with frequencies 3, 4, 6, 8, 12, 13, 15, 18, 25, 40 respectively. Please draw the Huffman tree for the characters. (8 points)

4. Supposed that there are three operations on a stack S:

(1) Push(S,e): push element e to stack S;

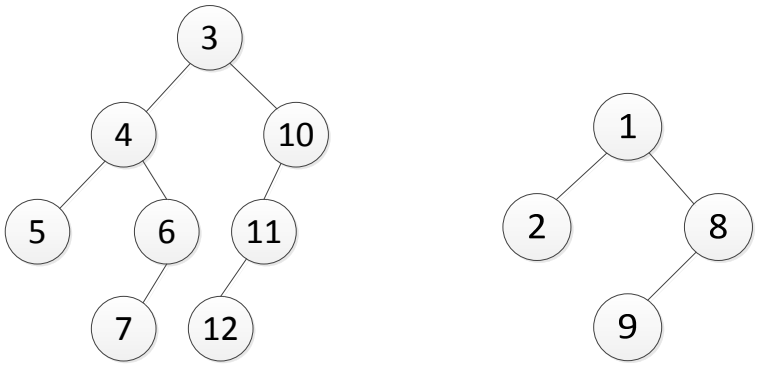
(2) Pop(S): pop an element from S; and

(3) MultiPop(S, k): consecutively pop k elements from S.

What is the amortized time bound for n consecutive operations of Push, Pop, or MultiPop? Please explain your answer. (7 points)

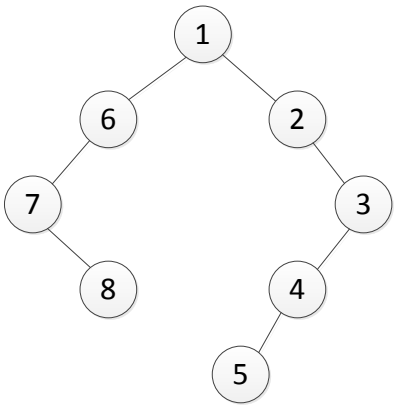
5. In a turnpike reconstruction problem, the distance set is given as {1, 2, 2, 3, 3, 3, 5, 5, 6, 8}. Please reconstruct the coordinates of the points. (6 points)

6. Please draw the result of merging two given leftist heaps. (6 points)



Leftist heaps for problem 6

7. Please draw the result of DeleteMin from the given skew heap. (4 points)



Skew heap for problem 7

IV. Rod-cutting Problem (15 points)

Given a rod of total length N inches and a table of selling prices P_L for lengths L = 1, 2, …… , M. You are asked to find the maximum revenue R_N obtainable by cutting up the rod and selling the pieces. For example, based on the following table of prices, if we are to sell an 8-inch rod, the optimal solution is to cut it into two pieces of lengths 2 and 6, which produces revenue $R_8 = P_2 + P_6 = 5+17 = 22$. And if we are to sell a 3-inch rod, the best way is not to cut it at all.

Length L	1	2	3	4	5	6	7	8	9	10
Price P_L	1	5	8	9	10	17	17	20	23	28

Now please

(1) list all the maximum revenues from R_1 to R_{11}; (4 points)

(2) describe the algorithm for finding R_N for any positive N;(8 points) and

(3) analyze the time complexity of your algorithm. (3 points)