浙江大学 2008－2009 学年冬季学期

《高级数据结构与算法分析》课程期末考试试卷

开课学院：软件学院、计算机学院、竺可桢学院 ，考试形式：**闭卷**，允许带 无 入场

考试时间：2009 年 1 月 6 日， 所需时间： 120 分钟

考生姓名：_____学号：_____专业：_____教师：_____

题序	一	二	三	四	总 分
得分					
评卷人					

Answer Sheet

Part I （2 分/空）

1. b	2. bc	3. b		4. bc
5. b	6. b	7A. b	7B. a	8. b

Part II （3 分/空）

1.

① `L[i][j] = L[i - 1][j - 1] + 1`

② `L[i][j] = Max(L[i-1][j], L[i][j-1])`

③ `L[M][N]`

2.

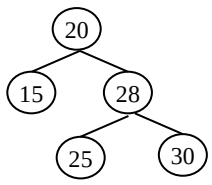
① `K->Right`

② `K1->Left`

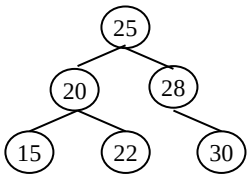
③ `K2`

Part III

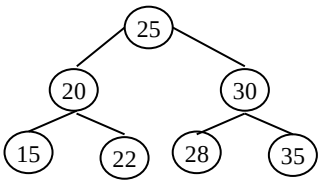
1. insert 28 （2 分）



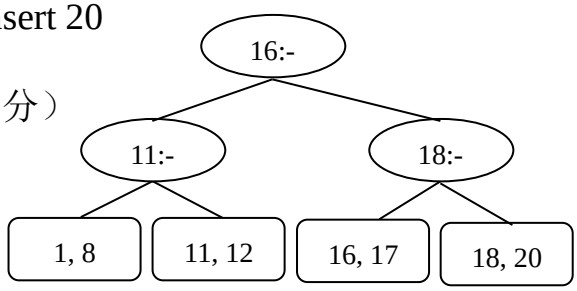
1. insert 22 （2 分）



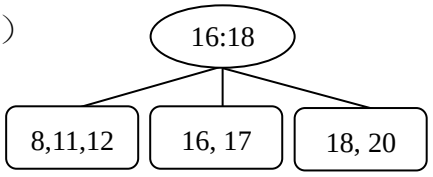
1. insert 35 （2 分）



2. insert 20 （2 分）



2. delete 1 （2 分）



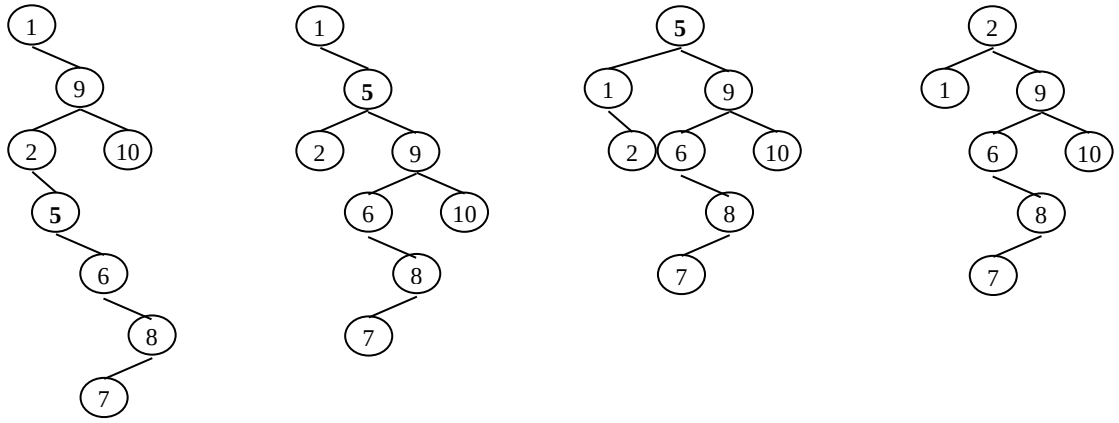
3.

A	B	C	D	E
01	001	1	0000	0001

Minimum length of the encoded text = 2*3+3*2+1*5+4*1+4*1 = 25

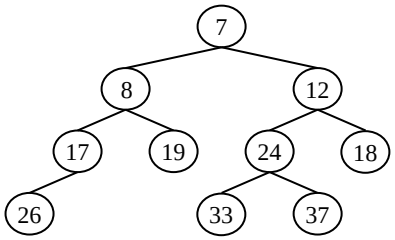
（6 分）答案不唯一

4.



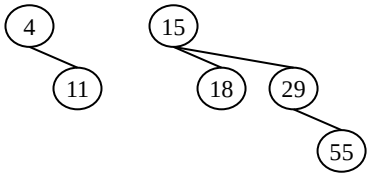
（8 分）只要是 search tree 就给 2 分

5.



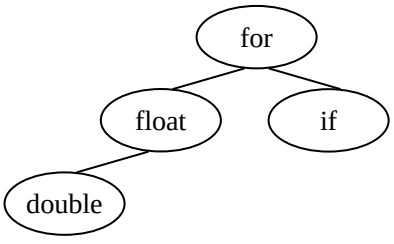
（9 分）只要是 min heap 就给 2 分

6.



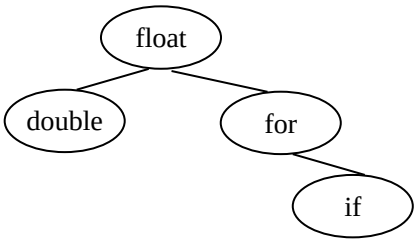
（5 分）答案不唯一

7. The binary search tree obtained from greedy method: （3 分）



total access time = 1.90 （1 分）

7. The optimal binary search tree （5 分）



total access time = 1.80 （1 分）

Part IV

a. （6分） Weighted, undirected graph. Each house is a vertex. Each road is an edge, and the length of the road is the weight of the edge.

b. （8 分） Step 1: find all-pairs shortest path length by dynamic programming;
Step 2: for each house i, find the shortest path with the maximum length L[i];
Step 3: find the smallest number L[min] from L[0]~L[N-1], that house 'min' is the house for the center.

c. （2分） The all-pairs shortest path algorithm is $O(N^3)$;
Finding the maximum length L[i] is O(N) for each i, and hence for all i is $O(N^2)$;
Finding the smallest number is O(N).
Therefore the total time complexity is $O(N^3)$.

NOTE: Please write your answers on the answer sheet.

注意：请将答案填写在答题纸上。

I. Please fill in the blanks (There could be multiple answers for one blank). (2 points for each blank) *Note: Zero point for a blank selection since there is at least one answer for each problem.*

1. An AVL tree of height 5 has the minimum size of ____ nodes. (The height of a leaf is defined to be 0.)
   a. 12 b. 20 c. 31 d. 33

2. For a B-tree of order M, _____ is(are) correct.
   a. The root must have between ⌈M/2⌉ and M children
   b. Each nonroot leaf has between ⌈M/2⌉ and M data
   c. All the actual data are stored at the leaves
   d. Each interior node contains M smallest key values in the subtrees

3. A Huffman tree that has N leaf nodes must have ____ nodes in total.
   a. cannot be determined b. 2N-1 c. 2N d. 2N+1

4. For the following kinds of problems, ____ is(are) NP.
   a. If we cannot prove a solution is true in polynomial time
   b. If we can solve the problem on a nondeterministic machine in polynomial time
   c. If the problem can be polynomially reduced to the Satisfiability problem
   d. If the problem is not decidable

5. For a skew heap,_____ is(are) true.
   a. Skew heap has an average cost of O(logN) per operation
   b. No tests are required to determine when to swap children
   c. The right path length of a skew heap of N nodes is no more than log(N+1)
   d. None of the above

6. To implement a binomial queue,_____ is(are) true.
   a. The roots of binomial trees are stored in a linked list
   b. Left-child-next-sibling structure is used to represent each binomial tree
   c. The subtrees of a binomial tree are linked in increasing sizes
   d. None of the above

7. To solve a problem with input size N by divide and conquer algorithm, if the conquer step takes O(1) extra work to form the solution from the sub-solutions, then among the following four dividing methods, ___ is the best while ___ is the worst.
   a. divide into 3 sub-problems of equal complexity N/2
   b. divide into 2 sub-problems of equal complexity N/3
   c. divide into 3 sub-problems of equal complexity N/3
   d. divide into 2 sub-problems of equal complexity N/2

8. The problem of "4 queens" is to place 4 queens on a 4×4 chessboard such that no two queens attack. If the problem is to be solved by backtracking method, we need to check _____ edges of the game tree after the 1st queen is placed at the 2nd column.

a. 3 b. 5 c. 13 d. 17

II. Given the function descriptions of the following two (pseudo-code) programs, please fill in the blank lines. (18 points)

1. The function is to find the length of the longest common subsequence of A[1]…A[M] and B[1]…B[N]. (9 points)

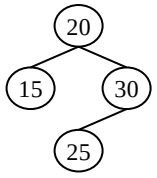
```
int Max( int x, int y ) { return (x>y)?x:y; }
int  LCSLength( ElementType A[],  int M,  ElementType B[],  int N )
{
    TwoDimArray L[M+1][N+1];
    // L[i][j] stores the length of the longest common subsequence of
    //A[1]…A[i] and B[1]…B[j]
    int  i, j;  // index of A and B

    for ( i = 1; i <= M; i ++ )  L[ i ][ 0 ] = 0;
    for ( j = 1; j <= N; j ++ )  L[ 0 ][ j ] = 0;
    for ( i = 1; i <= M; i ++ )
        for ( j = 1; j <= N; j ++ )
            if ( A[ i ] == B[ j ] )
                ① _____;
            else
                ② _____;
    return  ③ _____;
}
```

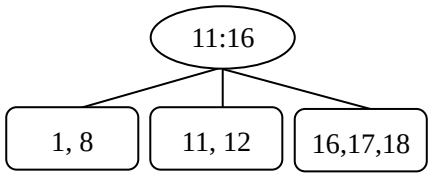
2. The function is to do the right—left double rotation for an AVL tree, where K is the trouble finder. (9 points)

```
static Position  DoubleRotateWithRight( Position  K )
{   Position  K1, K2;
    K1 = K->Right;
    K2 = K1->Left;
    ①_____ = K2->Left;
    ②_____ = K2->Right;
    K2->Left = K;
    K2->Right = K1;
    K1->Height = Max( Height(K1->Left), Height(K1->Right) ) + 1;
    K->Height = Max( Height(K->Left), Height(K->Right) ) + 1;
    K2->Height = Max( K1->Height, K->Height ) + 1;
    return  ③_____;
}
```

III. Please write or draw your answers for the following problems on the answer sheet. (48 points)

1. Please show the results of inserting 28, 22, and 35 into the given AVL tree (6 points).



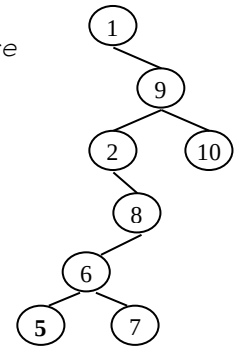


2. Given a B-tree of order 3. Please show the results of inserting 20 and then deleting 1. (4 points)

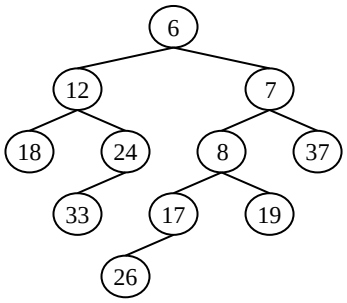
3. Given a text string which consists of five distinct characters A, B, C, D, and E, with frequencies 3, 2, 5, 1, and 1, respectively. Please give the Huffman codes for the characters. What is the minimum length of the encoded text? (6 points)

4. Please show each rotation of deleting 5 from the given splay tree (8 points).

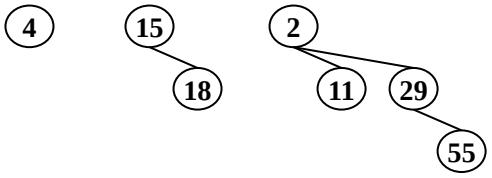
Note: *More detailed steps you show, more partial credits you may receive.*



5. Please show the result of deleting the minimum number from the given leftist heap (9 points).



6. Please show the result of deleting the minimum number from the given binomial queue (5 points).



7. Show the binary search tree obtained from greedy method, and the optimal binary search tree for the following words, where the frequency of occurrence is in parentheses: double (0.25), float (0.30), for (0.35), if (0.10). What are the total access times of the two binary search trees, respectively? (10 points)

IV. People are planning to build an emergency center in a village. Given N houses in the village and the length of the roads connecting them, you are supposed to find the house for the emergency center such that the distance between the center and the furthest house is minimized.

a. Which data structure is the best for representing the map of the village, and how? (6 points)

b. Please describe your algorithm by nature language or pseudo-code. Make sure that your algorithm has the time complexity no more than O(N^3). (8 points)

c. Please specify the time complexity of your algorithm. (2 points)