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Optical Flow

Optical Flow Vectors

- Vector field of pixel movements
- Can be used to detect moving objects
- Visual Correspondance
    - Points that correspond to the same thing in 3D space, across two different times
- Vector between two visual correspondances → optical flow vector
- Field of optical flow vectors → vector field
- Pencil = Geometric term for all lines through point
    - Optical flow vectors lie on pencils
- Solving Problem of *Ego Motion*
    - Ego Motion = How the system itself is moving

Benefits of Measuring Optical Flow

- Allows you to find translations between points
- **Inertial sensors**
    - Measure velocity/acceleration/rotation to estimate location/motion
    - Much less accurate than optical flow → Smallest unit is a pixel

Handling Rotation

- Cyclotortion with velocity γ
- *How does this change the optical flow?*
    - Flow becomes **tangential** to concentric circles centered on the axis of rotation
- Some rotation around vertical axis with velocity β

- *How does this change the optical flow?*
  - Flow becomes **tangential** to concentric **hyperbolas** that limit to x-axis
- What about rotation around x-axis with velocity α?
  - Flow becomes **tangential** to concentric **hyperbolas** that limit to the y-axis
- Rotation around multiple axis
  - $\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$ is the rotation vector
  - Sum vectors to get final outcome

Rotation and translation

- Rotational Vector → $\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$ Translational vector → $\begin{bmatrix} u \\ v \\ w \end{bmatrix}$
- (X,Y,Z) → (x,y) where $x = \frac{fX}{Z}, y = \frac{fY}{Z}$
- $(\frac{dx}{dt}, \frac{dy}{dt}) = (u, v)$
- $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = - \begin{bmatrix} u \\ v \\ w \end{bmatrix} \therefore \dot{x} = u, \dot{y} = v, \dot{z} = w$
- $\frac{\dot{x} = \dot{x}Z - x\dot{z}}{Z^2}$
- Flow equations for only translation
  - $u = \frac{W}{Z}(x - \frac{U}{W})$
  - $v = \frac{W}{Z}(y - \frac{V}{W})$
  - $(\frac{U}{W}, \frac{V}{W})$ is the point where all vectors meet
- Optical flow will be the same for multiples of velocity equal to multiples of distance
  - Causes ambiguity → Depth required to find exact translation
- Flow equations for only rotation

- $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = -\vec{\omega}x \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

- $\dot{x} = \alpha xy - \beta(1 + x^2) + \gamma y$

- $\dot{y} = \alpha(1 + y^2) - \beta xy + \gamma x$

- Flow with unrestricted movement (rotation and translation)
  - Sum of translational and rotational equations
  - $u = \frac{W}{Z}(x - \frac{U}{W}) + \alpha xy - \beta(1 + x^2) + \gamma y$

  - $v = \frac{W}{Z}(y - \frac{V}{W}) + \dot{y} = \alpha(1 + y^2) - \beta xy + \gamma x$

- $\star$*Note*: Rotation is separate from scene
  - Rotational flow will be the same across scenes
  - Independent of distance
  - On the other hand, translational flow is dependent on depth (Z)

- Concluding Remarks
  - Translation → Easy pattern, only expansion vectors
  - Rotation → Easy pattern, only circles/ hyperbolas
  - Translation + Rotation → Not easy. Need to separate effects of translation and rotation
  - Depends on FOV → Larger FOV = Easier separation
  - Imagine spherical eye model (like an insect)
    - Spherical eye moving through space → Flow expanding along geodesics (*think longitude*) with the transational axis as their axis on front side, and will contract on back side
    - Spherical eye rotating → Flow will be tangential to concentric circles around sphere (*think longitude*)
  - Spherical eyes have focus of contraction and focus of expansion
  - Allows them to separate the problem and have better optical flow abilities