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Epipolar Geometry

Three Questions

1. **Correspondence Geometry**
   - Given an image point x in the first view, how does this constrain the position of the corresponding point x' in the second image?
2. **Camera Geometry (motion)**
   - Given a set of corresponding image points $x_i \leftrightarrow x'_i, i = 1, ..., n$
   - What are the cameras P and P' for the two views?
   - What is the geometric transformation between the two views?
3. **Scene Geometry (structure)**
   - Given corresponding image points $x_i \leftrightarrow x'_i$ and cameras P, P'
   - What is the position of the point X in space?

Geometry

- Epipolar plane π is the plane formed by P, P', X
- Plane passes through camera planes and points x, x' lie on the edges of the plane
- Epipoles - Intersection of baseline with image plane
  - Projection of projection center in other image
  - Vanishing point of camera motion direction
- Baseline - Line formed by π and the epipoles $\rightarrow$ Translation between cameras
- Cross Produt

$$a \times b = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} b = [a_x]b$$

- Triple Scalar Product → 3 points are coplanar

$$(a \times b) \cdot c = 0$$

- Equations

$$P' = RP + t$$
$$p = MP, M = [I|0]$$
$$p' = M'P', M' = [R|t]$$
$$\text{P is the point in space, p,p' are points in image}$$
$$\vec{O'p'}, \vec{OO'}, \vec{Op} \text{ are coplanar} \implies p' \cdot [t \times (Rp)] = 0, \begin{cases} p = (u, v, 1)^T \\ p' = (u', v', 1)^T \end{cases}$$
$$p' \cdot [t \times (Rp)] = 0 \implies p' \cdot [(t \times R)p] = 0 \implies p'([t_x]R)p = 0$$
$$\underbrace{E = [t_x]R}_{\text{Essential Matrix}} \implies \boxed{p'Ep = 0}$$

- Essential Matrix
  - E can be used to find rotation and translation
  - Computed from point correspondences