Simulation Of Traffic Flow Using a Stochastic Automaton Model On 4x4 Grid System

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Abstract

A stochastic discrete automaton model based on kinetic model theory was applied for traffic simulation in a 4x4 grid system. The factors that were adjusted to understand the macroeffects on the system were:Individual car maximum velocity, individual probability breakdown, total number of cars. The increased maximum velocity showed greater average distance covered by cars up to a threshold maximum velocity of 240km/h. This was closely related to dispersion time of traffic jams at the crossroads, which showed linear behaviour up to 40 cars per road. The effect of breakdown probability on average distance travelled by a car was found to be significant, while when the number of cars was increased greatly (4 to 192) the distance travelled decreased by only 7%. It was found that the greatest impact on overall traffic flow was the behaviour of cars at the central crossroad and T-junctions. Traffic flow system management was investigated through changing some two direction roads into one direction two lane roads, either inwards or outwards from the central crossroads. Inward one direction roads were found to increase central crossroad traffic flow by 50% while reducing the traffic flow at the subsequent outer T-junction. The outwards one way roads away from the central crossroad halved the central crossroad traffic while making the subsequent outside T-junction the busiest junction in the system with an increase of 10% relative to other junctions .

Contents

1 Introduction

Traffic Modelling was first introduced in the 1950s, initially by Lighthill and Whitham by treating vehicles as particles in a fluid [1]. Significant progress has been made through the use of different types of models to represent various parameters of traffic configurations[5]. There are two main methods of traffic simulation, one is the 'Hydrodynamic' model which ignores the motion of individual vehicles and models traffic flow as movement of fluid[2]. In this model traffic flow is simulated by solving evolution equations of macroscopic parameters and the motion of cars is dependent on overall behaviour of the system. The second method is analogous to kinetic theory[6] which treats the motion of individual cars independently [3]. The behaviour of vehicles is dependent on microscopic dynamics and the behaviour is mainly affected by the action of vehicles around that said vehicle.

This paper takes the kinetic theory approach to simulating traffic flow based on Nagel and Schreckenberg algorithm [4] and applying it in to 4 by 4 grid, similar to grid based cities such as New York[7]. The main parameters in the model are : Number of Cars, Maximum Velocity on road, Probability of Breakdown of the cars. These parameters are varied to investigate the subsequent effect on traffic. In addition to this, this paper investigates the time taken for traffic jams to disperse dependent on a varied number of cars. Finally, traffic manipulation through road direction is investigated, where some two way roads are turned into single one direction two lane carriageways.

2 The Model

The grid that the cars drive on consists of four quadrants with a central crossroad and four additional outer T-junctions along the road intersections. The position of cars is given by x and y coordinates x:[-100,100] y:[0,100]. No car can leave the grid. If the simulation is left to run forever the cars will drive indefinitely. All cars drive on the left side of the road. Maximum road velocity, number of cars , breakdown probability can be adjusted, further to this the initial starting velocity and position of each car can be changed.

The motion of car is treated independently and as follows:

1. Each car wants to drive at the speed limit and so if velocity of car (V) is below the speed limit $Vmax$; V is increased by 1

2. Cars drive in either positive/negative x−direction or positive/negative y−direction and they do not want to hit the car in front. If a car is at position i and the car in front is at site $i + d$ then if $V \geq d$, V is set equal to d −1. This stops the cars from accelerating and hitting the car in front

3. Unexpected events cause slowing down. This is a random component that is intended to model external influences, such as what is happening on the other carriageway, or just irrational behaviour by drivers. If $V > 0$ then, with a probability P, V is reduced to $V - 1$. There is also constant standing chance of a total car stop where V is set to 0, this has 1% of occurring. All of this happens after V has been updated by the first two rules.

4. Each vehicle is moved forward V places, using the new value of V

When approaching the crossroads or the T-junctions:

1. If a car has approached the junction, it has to stop $(V = 0)$ at the ”traffic light position” a position just before the center of cross roads.

2. When the car has stopped it has to decide where to go, this is done randomly

3. When the car has decided its direction and it is the cars turn to drive it will check if the position it wants to go to is occupied. If it is occupied the car waits at its current position and ends its turn of movement. If the position is unoccupied the car goes to that position.

These rules govern the motion of cars and the parameters are changed to investigate the effect of traffic flow of the overall system. Each run of the simulations is done 20,000 times equivalent to 5.56 hours of driving. In my calculations each unit cell of the grid is equivalent to 7.5m and Vmax is usually set to 5, equivalent to 120 km/h . Hence our model represents large intersection on a highway rather than an urban location.

Please note that the use of the word ”breakdown”. For breakdown probability if occurred will reduce the speed to V-1 rather than come to a full stop.

Figure 1: Shows the simulation in action where the y and x axis are the position of each car. The title represents the set parameters and colour coding is used to represent the speed of each car. Mild traffic can be seen at the crossroads. It is also important to note that the starting position of each simulation is set as cars split equally on the 4 main roads driving into the centre. Colour coding for car speeds: Purple = 0 Dark Blue= 1 Grey =2 Dark Green=3 Light Green=4 Yellow=5

3 The Dependence Of Maximum Speed Limit On Traffic Flow

The maximum speed was varied from 80km/h to 270km/h and the average distance of each car travelled measured. The motion of cars is random, hence the average distance of travel by a car is a good measure of traffic flow because the distribution of cars at the separate positions of the grid will be independent of maximum velocity due to symmetry of the grid. The traffic flow though crossroads positions was also measured in section 6 of the paper and can be used as evidence for the symmetry argument,this will be highlighted later on. There were 192 cars in the simulation with a breakdown probability of 0.25 per time frame for a total driving time of 5.6 hours.

Directly from the live plot of the grid it was evident that the cars covered the straight distances much quicker, but there was a longer waiting time at the crossroads because more cars arrived. Hence the question to ask, is it beneficial to increase the speed limit to allow more traffic flow?

The answer to this question depends closely on the time taken for the dispersion of traffic at the crossroads by the time the next car arrives. Hence it is expected that the average distance travelled by a car should increase as maximum speed is increased up to a threshold. This is exactly what was observed:

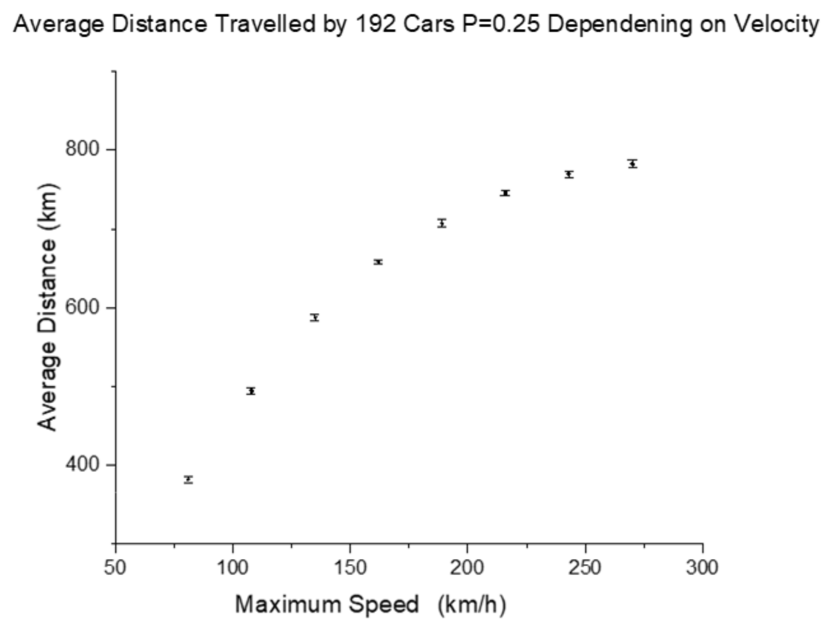


Figure 2: Average distance travelled depending on maximum Velocity by 192 Cars, breakdown probability=025. The Error is the standard deviation of all individual distances travelled by each car. Note the logarithmic behaviour and a threshold maximum average distance around 760km.

It can be seen from Figure 2 that the average distance travelled increased as the maximum speed limit was increased with a threshold around 240km/h. A logarithmic trend seems appropriate and after 240km/h the average distance travelled increase flattened and approached the limit of 760km. This is a useful result to manage traffic flow, but the dangers of increased speed should be considered with addition to fuel expense. Dispersion of central traffic is key to this relationship and so is the length of the road up to the crossroads.

4 Dispersion of Central Crossroad Traffic Jams

Traffic simulation even for a simple 4x4 grid set up has underlying complex interconnected factors. An analytical link between detailed micro dynamics which build up to macro effects of the whole system are investigated through the parameters changed. The effect of changing the speed limit is closely related to dispersion of traffic at the crossroads.

A forced traffic jam is created at the central crossroad by setting an equal number of cars at the four different roads leading to the centre. The cars all drive towards the centre and start with random positions at that said road with random velocities below or at the speed limit generated with constant probability. The traffic jam occurs as expected and the time taken for a traffic jam to disperse is measured. Definition of dispersion is taken to be when each road on the crossroads has less than three cars at the queue. Cars drive when their road has the “green” traffic light (i.e one road drives at a time). The number of cars at each road is changed and the time taken for traffic dispersion is measured, while probability of breakdown is set at 0.25 for all cars.

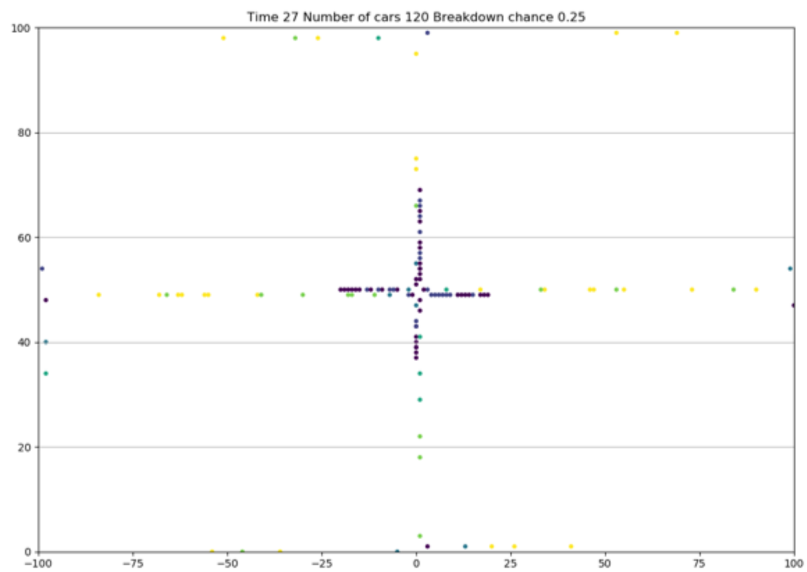


Figure 3: A diagram indicating the set up of a forced traffic jam at the central crossroads

Figure 4 shows an interesting behaviour of a linear increase in dispersion time as the number of cars is increased. A waiting time of 211 seconds or 3.5 minutes at 30 cars on each road seems appropriate and realistic with competent drivers and no traffic at the roads leaving the crossroads. In addition, the dispersion time could be predicted very accurately with a gradient of 1.72±0.04 (i.e. Time for dispersion= 1.72*Number of Cars). However, interesting behaviour arises after about 160 cars where there is a sudden jump at dispersion time. The explanation to this behavioural pattern can only be explained through observing the live traffic simulation. This is because after 160 cars the initial cars leaving the central crossroad have enough time to go around one of the quadrants and return to the central crossroad fuelling a traffic jam in a cyclical behaviour. In this environment with a crossroad of length 750m y-axis and 1500m x-axis a constant traffic jam will form where the dispersion time will not increase linearly but exponentially if more than 40 cars are stuck on each road. This could occur because of an incident which would result in a constant influx of cars. This links with the previous findings that when the speed limit is higher than 240km/h this decreases the time for the cars to drive around the quadrant and return to a crossroad, without increasing overall distance travelled

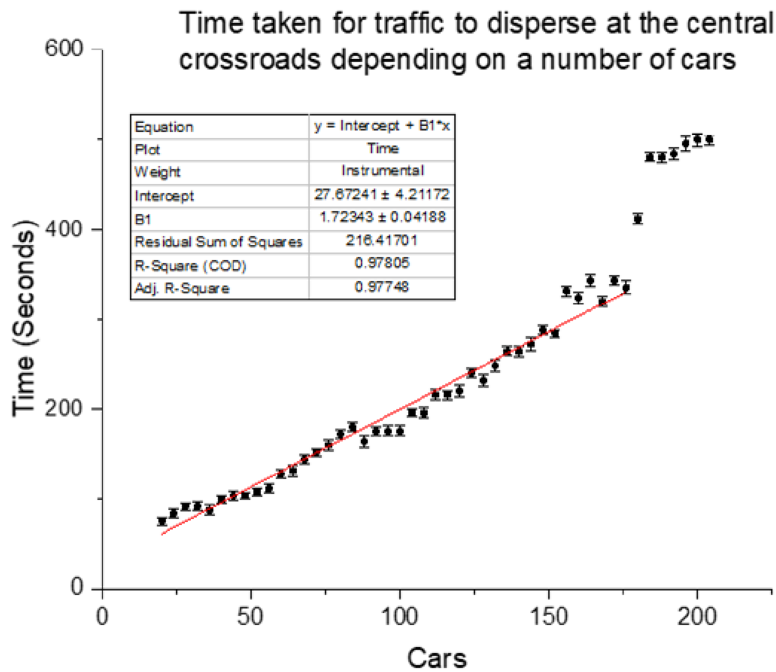


Figure 4: Time taken for traffic to disperse at central crossroads depending on number of cars. The Error is the standard deviation of 5 repeated runs. Note the linear behaviour up to a 160 cars

because the saved time is spent at the traffic light. This is because the dispersion time is more than the time taken for the initial car to return to the traffic jam. This scenario with a simple grid and with the same breakdown probability of P=0.25 for all cars seems unrealistic, but the dispersion relation could be useful in real life application. An example of a real life application of the dispersion relation is if a queue of 40 cars is formed on the crossroads. Here using this information, the navigation system can suggest a longer but less congested route.

5 The Effect of Changing the Number of Cars and Probability of Breakdown

Real life traffic behaviour has random events which cause congestion. These events can range from driver mistakes or random accidents in the environment. This can be exasperated with an increased number of drivers. The breakdown probability hardcoded into the simulation introduces these random congestion effects. The breakdown probability can be adjusted to any number and if the breakdown occurred reduces the speed of the car to V-1, there is also a constant background chance of total stop (V=0), such as if a child runs into a road. The simulation investigated the effects of the breakdown behaviour and the results were as expected. The breakdown chance was changed from 0 to 0.95 with 160 cars and a total driving time of 5.6 hours.

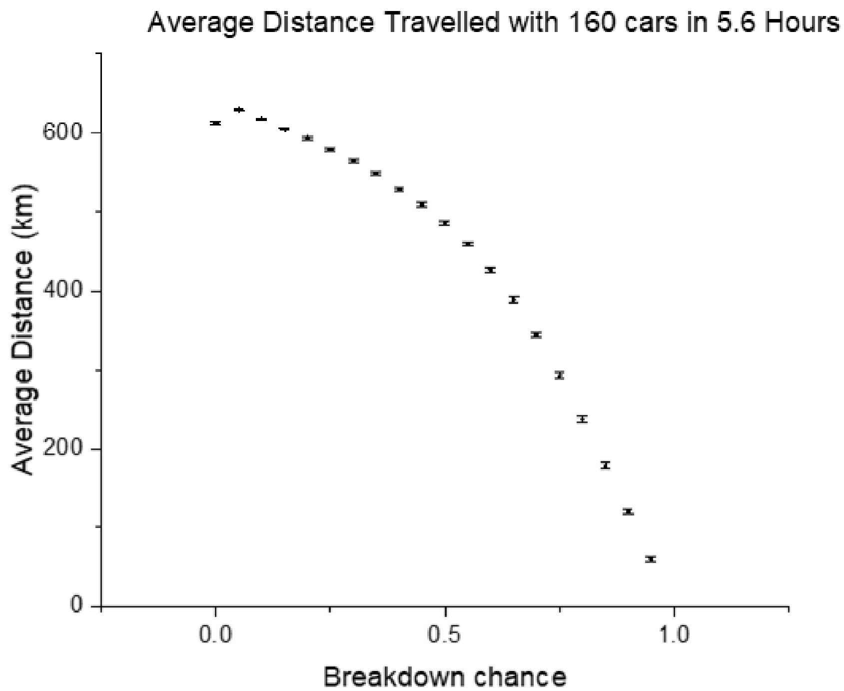
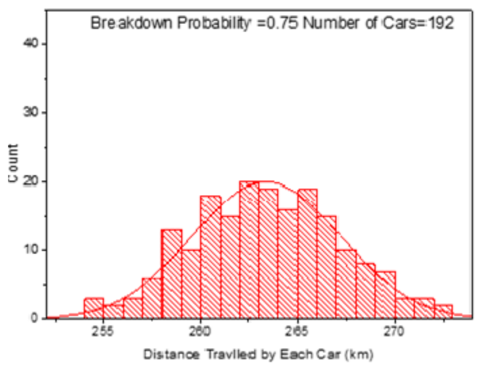


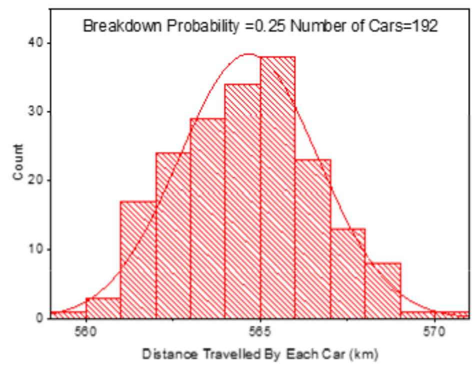
Figure 5: Average distance travelled with 160 cars in 5.6 hour driving time, dependent on varying Breakdown chance. The error was calculated from standard deviation of total distances of each car

Figure 5 shows negative quadratic behaviour with the breakdown probability having significant impact for larger values when $p>0.25$. This can be explained by the fact that if a car is more likely to have a breakdown on the road a traffic jam is much more likely to form. The average distance travelled by each car was reduced from 606km at $P=0.15$ to 485km at $P=0.5$ reinforcing the impact of breakdown probability on traffic flow. Nonetheless, these numbers can be considered unrealistic as the breakdown probability is never this high except for instances in cold and icy conditions parallel to bad conditions of cars, such as in poverty stricken countries. Another finding to consider is the distribution of total distance travelled of the cars at the different breakdown probabilities.

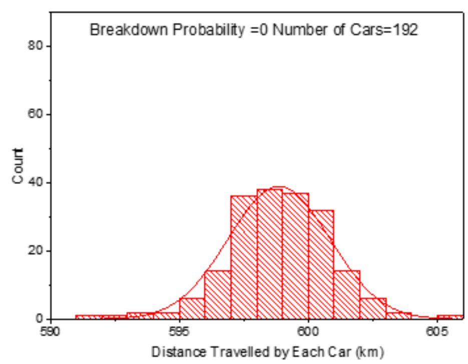
From Figure 6 it can be inferred that at different breakdown probabilities the distribution of distance travelled remained roughly the same for all cars with standard deviation (std)=2 for $P=0$ and $P=0.25$, implying that most cars experienced similar driving conditions. When $P>0.5$ there was a greater std in the distribution of total travelled distance. For example at Breakdown probability with P=0.75 the standard deviation was 3.8. This behaviour can be explained, as hotspots of traffic jams can exist through random interconnected events. An example is where three cars breakdown simultaneously at different intervals, hence there will be a queue of cars behind those said cars. A linear flow of cars has now segmented into three hotspots of traffic jams, hence increasing the distribution of traffic flow of cars, subsequently increasing the standard deviation.



(a) Mean=263.4 Standard Deviation=3.8



(b) Mean=564.7 Standard Deviation=2.0



(c) Mean=598.9 Standard Deviation=2.0

Figure 6: Distribution of distances travelled for each car with breakdown probabilities: 0, 0.25,0.75

For the next investigation the number of cars was increased from 4 to 192 and the corresponding average distance travelled measured, with the error of the average distance being the standard deviation of the distribution.

Figure 7 showed a similar behaviour to the breakdown distribution of a negative quadratic effect. However, note that an increase of 192 cars results in a fall from 600km to 565km in the total average distance travelled. This is minimal compared to the fact that the roads were nearly completely full. This is an interesting discovery conveying how the major disruption to traffic flow is not a sudden increase in cars but rather a abrupt event which hinders smooth traffic distribution such as an accident on the road.

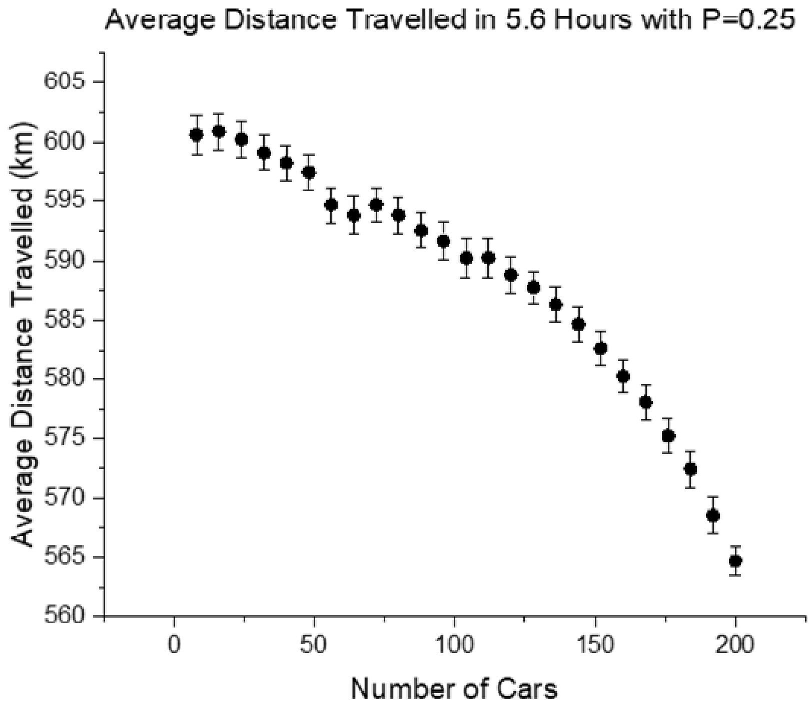
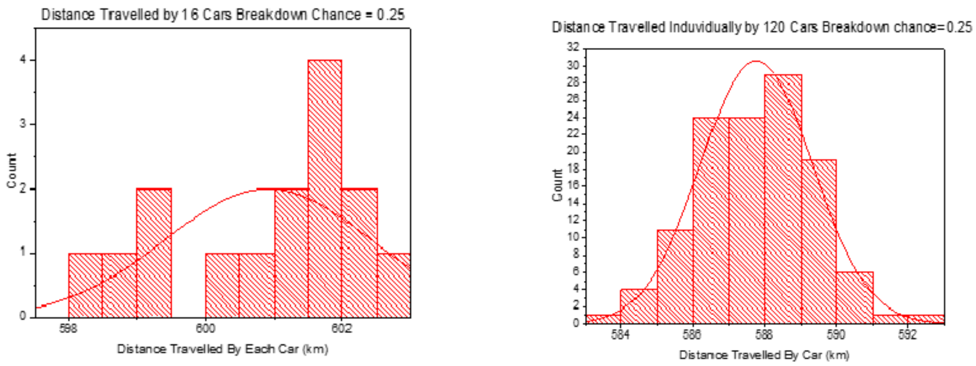


Figure 7: Average Distance travelled in 5.6 Hours with P=0.25 dependent on the number of cars. The error is calculated from standard deviation of individual total distances. Note the number of cars has a minor effect on distance travelled

From Figure 8 it can be seen that the distribution of individual speeds did not change by a significant amount with a standard deviation remaining at 1.5 with 120 and 16 cars. It can be observed that the number of cars has a less significant impact on traffic flow.



(a) Mean=263.4 Standard Deviation=3.8 (b) Mean=564.7 Standard Deviation=2.0

Figure 8: Distribution of distances travelled for each car with 16 and 120 Cars

The effect of a single parameter measured is difficult to visualize as it is closely correlated to the other factors. A better, understanding of the situation can be done through a 3D Surface plot of breakdown probability, number of cars and the effect on average distance travelled.

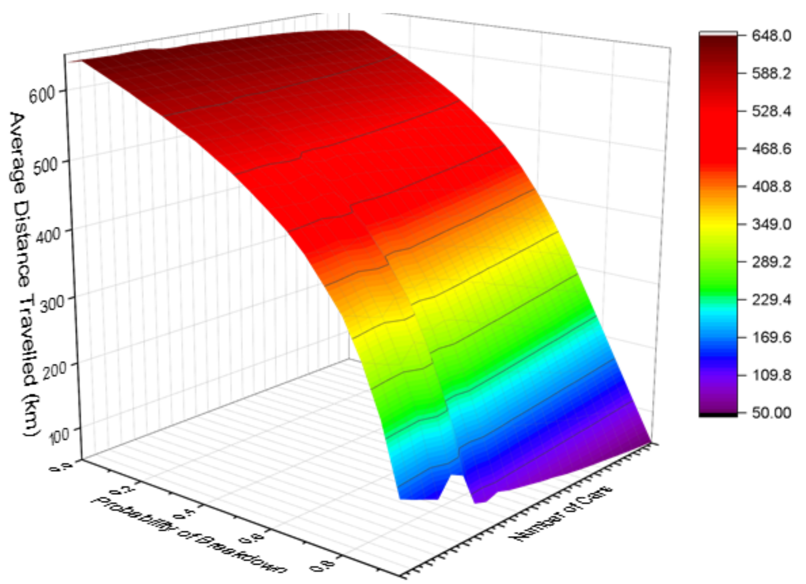


Figure 9: a 3D surface plot of probability of breakdown and number of cars against the average distance travelled. Note the significant impact of breakdown probability compared to the number of cars

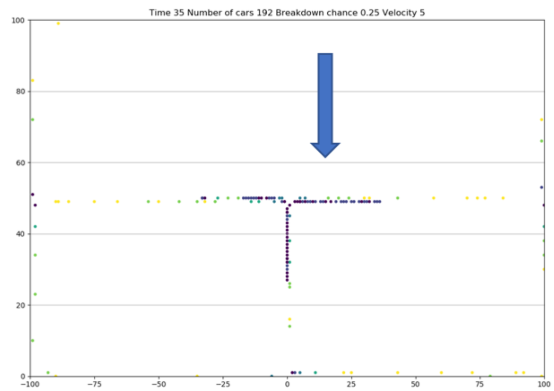
From Figure 9 it can be inferred that the number of cars has a minor effect on traffic flow. Major reduction on average distance travelled arises from the breakdown of cars causing a pile up of cars and hence a traffic jam. The maximum velocity of cars is a useful tool in managing the flow of cars, but the main factor affecting the macro dynamics of the system is the behaviour of cars at the traffic lights. The linear relationship of traffic dispersion dependent on the number of cars had a threshold at 40 cars per road. Hence we arrive at a conclusion that management of a traffic flow system is a management of crossroads. If the threshold of the number of cars at a crossroad is below the required limit, which is calculated to be at about 40 cars per road at the central crossroad, then we arrive at a stable system with constant traffic flow and a linear dispersion of traffic jams. When the threshold has been overcome we arrive at cyclical effects where we develop constant road traffic jams and the addition of cars fuel the stoppage of movement. A correct analysis to consider, is the management of the system to maintain the number of cars below that said threshold. In the current 4 quadrant design there is one clear method in the management of the traffic flow system-the control of the direction of the roads. Each road has two lanes and cars can drive in opposite directions , turning some roads into one way carriage ways will allows for management of traffic and further dispersion of traffic due to the cars having several options of routes if the desired route is not available.

6 Management Of Traffic Flow Through One Way Roads

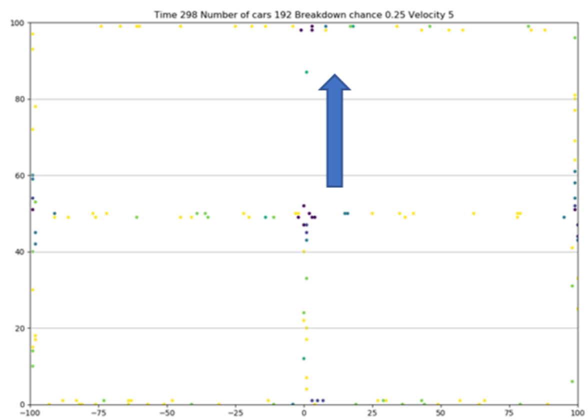
The traffic flow of the 4x4 system is dependent on the traffic flow at the junctions. A steady efficient movement of cars and linear dispersion of traffic can be seen when there are less than 40 cars at the central crossroads. Hence the next proposition for the management of traffic flow is change of the 2-direction roads into 1-direction roads with two lanes in the same direction. This would subsequently allow for cars to have an option of which lane to take from the central crossroads if turning into a one way road. For example, if from the central crossroad, the upper middle lane is turned into a one way road going upwards, at the crossroads the cars coming from the left side would take the left lane and the cars coming from the right would take the right lane.

Furthermore, a car from any initial direction would be able take the adjacent unoccupied lane if the desired lane is occupied. In addition to this, turning one of the central crossroads into a one direction road heading away from the crossroads would change the corresponding outer T-junction into 2 direction carriage ways. Consequently, if the central upper road can only go upwards the corresponding upper T-junction will become a simple 2 direction road.

The roads changed are the four roads heading to the central crossroad and they can be one direction carriageways either only heading towards or away from the central cross road, hence there are eight combinations of road systems in total. An example diagram can be seen below on Figure 10. For the eight combinations the average distance travelled and the number of cars going through each traffic light was counted. This was done for 192 cars with probability of breakdown at 0.25 for a total driving time of 2.78 hours



(a) Inwards



(b) Outwards

Figure 10: Two possibilities of direction in a one way road. Note in (b) the upper road is turned into a one direction, two lane road heading outwards from the central crossroad. Hence there are 3 lanes feeding the central crossroads and 4 lanes feeding the outer T-junction.

Table 1: Table to show number of counts at the central crossroads, outer T-junctions and the total average distance travelled by a car for each system. Note the distance travelled is in 1 cell unit

	Central Crossroads	Upper T-junction	Lower T-junction	Left T-junction	Right T-junction	Average Distance travelled per Car
No Change (Reference)	29735	21105	20720	20900	20894	37549
Central Upper Road Driving Only Up	14415	16582	13627	14889	14295	29232
Central Upper Road Driving Only Down	23770	9010	16371	14649	14670	29205
Central Left Road Driving Only Right	24960	15416	15185	9307	16983	38334
Central Left Road Driving Only Left	14515	15787	14567	16787	13678	38204
Central Right Road Driving Only Left	25164	13482	14527	9111	14565	38401
Central Right Road Driving Only Right	15770	13498	15678	13999	17670	38300
Central Lower Road Driving Only Down	15643	14564	17876	14565	15643	29124
Central Lower Road Driving Only Up	25464	14543	8909	15464	14556	28987

Firstly it is important to notice how the average distance travelled (1 cell units) decreases massively when the central lower (Lower:29124,28987) and upper roads (Upper:29232,29205) are changed into one direction roads . While, when the left (Left:38334,38204) or the right (Right: 38401,38300) road become one way, the average distance travelled increases slightly . This is due to the fact that forcing these roads into one direction roads there is more traffic flow on that road, because the probability of that road being unoccupied is very high as there are two lanes. For example, if a car is at a crossroad and wants to drive left but the position is occupied it is more likely to drive on the one direction two lane carriageway afterwards because of an additional existing lane heading towards the same direction. Furthermore, the roads on the y-axis i.e. upper and lower roads are half the length the roads on the x-axis, hence the average total distance travelled is reduced .

The second change to address is the reduction of cars entering the central crossroad and T-junctions on all road system combinations. The reference case had most hits at the central crossroads at 29,735 while all the T-junctions had roughly same count at about 21,000 each. However, when any of the roads were turned into one way roads, the crossroad and T-junction counts were reduced in general. This is because, when the roads becomes one way, some T-junctions become obsolete for cars driving on the outside of the grid. Let us generalise, our roads change into two categories. One road going inward towards the central crossroad and one road going outwards away from the central crossroad. Hence, if the chosen road is one way and outwards from the central crossroads, the subsequent outer T-junction becomes a simple road for the cars driving on the outer grid. Imagine, you are a car driving on the outer rim and you approach a T-junction which has a one direction road feeding it from the central crossroad, you no longer can drive into the central crossroad, but have to drive straight forward. Overall, the lack of choice and the following change of T-junctions into simple roads reduces the overall counts and could explain the slight increase of total distance travelled.

Now let us discuss the management of traffic flow, which is the most important and desired outcome for changing the roads. When a chosen road is turned into an outward one direction road away from the central crossroads the traffic at the central crossroads is reduced. It is effectively halved for all cases compared to the reference case: 14415 (Central Up Only Up), 14515 (Central Left Only Left) ,15770 (Central Right Only Right), 15643 (Central Lower Only Lower). Meanwhile the outer T-junction on the subsequent road becomes the busiest junction. This pattern is repeated for all one direction roads heading away from the the central crossroads. The increase of counts at the junction is relatively small compared to other junctions, with an increase of around 10%. When the road is turned into a one direction road heading inward towards the central crossroad, the changes are the most drastic. When both lanes are driving towards the centre there are 5 lanes in total feeding to the crossroad, hence the traffic at the centre increases and is approximately 50% greater than all the other junctions except the outer T-junction where the cars are originating from. For example, if the central lower road has two lanes driving up towards the central crossroads the number of counts at the lower T-junction is greatly reduced to 8909, because now there is only a flow of 2 channels into that junction instead of the usual 3.

Overall, the desired results was achieved where the traffic flow was controlled through one way roads and the subsequent motion of cars was channelled into the desired crossroads and junctions. Let us imagine a complicated grid based city where due to uneven proportion of the spread of population there are more cars origination from the same area and driving towards the same location. It is very much the case in real life due to socio-economic segmentation of the general population into different regions. This would cause daily repeated traffic in specific quadrants, due to the same route of the commute, hence these results could be used for efficient management of traffic flow through the changing of key roads into one direction two lanes carriageways. These results would also be expected in the simulation if the cars did not have random direction of movement, but rather determined locations to drive to and can be a future improvement to the model. Furthermore, if the busy quadrants are known and are simulated in the model, a machine learning algorithm could be used to decide which roads are needed to be changed to reduce the hotspots of traffic. Instead of doing this systematically as was done in this paper. To add to this, a machine learning algorithm would be much more efficient than a human due to the exponential increase in complication when the map is increased to more than a 4x4 grid.

7 Conclusion

The kinetic theory model for simulation of traffic clearly showed useful results. The dependence of maximum velocity and the average total distance travelled by each car was investigated and was found to have logarithmic behaviour up to a threshold velocity of 240km/h. This was closely related to the dispersion time of traffic at the central crossroads, where a traffic jam was forced and the subsequent traffic dispersion time measured with varying number of cars in the traffic jam. The results showed that the dispersion time increased linearly up to 160 cars, afterwards there was a significant step upwards. This was because after 160 cars (40 cars per road) the dispersion time was long enough for the cars to go round the outside road and return to the central crossroads fuelling traffic in a cyclic behaviour.

The increased average total distance travelled by a car as maximum velocity increased up to a threshold and stagnation afterwards can be explained by the fact that cars travel greater distances overall as they can cover straight roads faster, but the total distance travelled is dependent on dispersion time of traffic i.e. after 240km/h the average distance travelled did not increase greatly because the cars were stuck in a traffic jam. The dependency of breakdown probability and the number of cars in the system were investigated with the average distance travelled being the measured variable. The 3D surface plot showed that the number of cars had minimal impact on traffic flow as the average distance travelled decreased from 600km to 565km when the number of cars was increased from 4 to 192. Meanwhile, the breakdown probability had a significant impact on traffic flow. When the breakdown probability increased there was greater chance for traffic jams to form and the average distance travelled seems to decrease in quadratic behaviour.

The effect of turning the roads into two lane carriage ways was investigated. Through this method, the traffic flow was controlled and two categories of two lane one direction roads were identified- Outwards or Inwards from the central crossroads . The inward one direction roads increased traffic flow significantly in the central crossroad by 50%, while the outer T-junction had reduced traffic flow. The one direction roads heading outward from the central crossroad reduced traffic flow in the central crossroads by half and forced the subsequent outer T-junction to become the busiest hotspot in the entire system. Overall, the microscopic parameters were changed, but it was found that the greatest impact on the macro effect of the system is the behaviour of cars at the crossroads and T-junction. Various analysis showed the ability to estimate the traffic congestion at the junctions, and one direction roads were investigated as a tool to control traffic flow. These results could be implemented and be potentially useful in major grid based cities. Further improvements, in the model could consist of cars having a determined origin and target location to drive to instead of random choice. Furthermore, for a more complex system a machine learning algorithm could be implemented.It would use data to choose which roads to turn into one way two lane roads to reduce traffic at a desired quadrant instead of systematically changing the roads as was done in this paper.

8 References

1. M. Lighthill, G. Whitham (1955) *“On kinematic waves. A theory of traffic flow on long crowded roads”* London, The Royal society DOI: https://doi.org/10.1098/rspa.1955.0089

2. I. Bonzani (2000) *“Hydrodynamic models of traffic flow: Drivers’ behaviour and nonlinear diffusion”* Torino, Mathematical and Computer Modelling, Volume 31, Issues 6–7, Pages 1-8, DOI: https://doi.org/10.1016/S0895-7177(00)00042-X.

3. I. Bonzani, L.M. Gramani Cumin,(2008) *”Modelling and simulations of multilane traffic flow by kinetic theory methods”* Torino, Computers and Mathematics with Applications, Volume 56, Issue 9,Pages 2418-2428, DOI: https://doi.org/10.1016/j.camwa.2008.05.033.

4. K. Nagel, M.Schreckenberg, (1992) *”A cellular automaton model for freeway traffic”* Journal de Physique I, vol 2, pages 2221-2229 DOI: https://doi.org/10.1051/jp1:1992277f

5. W. Leutzbach, (1988) *”Introduction to the Theory of the Traffic Flow”* Berlin

6. I Prigogine, R Herman(1971) *”Kinetic Theory of vehicular travel”* New York Elsevier Publishing

7. Koeppel, Gerard (2015) *” City on a Grid: How New York Became New York. ”* Boston, Da Capo Press