"Optimal Bundling Strategies for Cable Companies"

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Table of contents

How can cable tv and internet providers maximize profits?

Unique product and cost structures require cable providers to investigate complex pricing policies

- Multiproduct monopolist with substitutable products
- Capacity constraint creates a nearly vertical cost curve
- Bundling, Usage Based Pricing, 3-Part Tariffs

Figure : Aggregate Demand and Marginal Cost Curves for Internet Providers

Literature Review

- Adams and Yellen (1976)
  - Provided examples illustrating cases when pure or mixed bundling could outperform separate pricing models
- McAfee, McMillan, and Whinston (1989)
  - Develop reservation price model for a multiproduct monopolist
  - Consumers purchase at most 1 unit of either good and value them independently
  - MMW prove that mixed bundling is always weakly better than pure bundling and investigate necessary and sufficient conditions for mixed bundling to outperform separate pricing as well
- Venkatesh and Kamakura (2003)
  - Extend MMW to consider special cases of multiproduct monopolists selling complements and substitutes

Basic Model Specification

- Multiproduct monopolist sells two substitutes and bundle at prices p_1, p_2, p_b
- Consumers' valuations (v_1, v_2) for the goods fall on continuous uniform distributions from 0 to 1
- Consumers purchase at most one unit and have marginal utility from good i:

$$mu_i(v_i) = \begin{cases} v_i & : q_j = 0 \\ \beta * v_i & : q_j = 1 \end{cases}$$

- Monopolist seeks to maximize profits while constrainting internet consumption below some cap, c.

Figure : 5 Consumer Types

Figure : Internet Users

$\beta = 0$ Page 1

- $\beta = 0$ implies perfect crowdout so that no one will purchase both goods
- Without a capacity constraint, max profit $= \frac{2}{3\sqrt{3}} \approx .385$ and is achieved when $p_1 = p_2 = \frac{1}{\sqrt{3}} \approx .577$
- This means a cap will only bind if it is $\leq \frac{1}{3}$
- Green area $= \frac{1}{2} - p_1 + p_2 + \frac{p_1^2}{2} - p_1 p_2$
- Red area $= \frac{1}{2} + p_1 - p_2 - \frac{p_1^2}{2}$

Figure : $p_1 = .5, p_2 = .5$

- Revenue $= \frac{p_1}{2} + \frac{p_2}{2} - p_1^2 - p_2^2 + 2p_1 p_2 - \frac{3}{2} p_1^2 p_2 + \frac{p_1^3}{2}$

$\beta = 0$ Page 2

- We want to maximize $\frac{p_1}{2} + \frac{p_2}{2} - p_1^2 - p_2^2 + 2p_1p_2 - \frac{3}{2}p_1^2p_2 + \frac{p_1^3}{2} + \lambda(c - (\frac{1}{2} - p_1 + p_2 + \frac{p_1^2}{2} - p_1p_2))$ where $c \in (0, \frac{1}{3})$
- So, the optimal p_1 and p_2 are implicitly defined by the solution to the system:
  - $(1 - p_1)\left[\frac{1}{2} - 2p_1 + 2p_2 - 3p_1p_2 + \frac{3}{2}p_1^2\right] = (p_1 - p_2 - 1)\left[\frac{1}{2} - 2p_2 + 2p_1 - \frac{3}{2}p_1^2\right]$
  - $p_2 = \frac{c - \frac{1}{2} + p_1 - \frac{p_1^2}{2}}{1 - p_1}$

Figure : P_1

Figure : P_2

Figure : Profit

$\beta = 1$ Page 1

- $\beta = 1$ implies that there is no crowd out effect and marginal utilities are constant
- In absence of a bundle, optimal pricing strategy is $p_1 = p_2 = \frac{1}{2}$, profit $= \frac{1}{2}$, and cap would bind at $c = \frac{1}{2}$
- With a bundle, optimal pricing strategy is $p_1 = p_2 = \frac{1}{27}(2\sqrt{61} - 1) \approx .54$, $p_b = \frac{11}{54} + \frac{5}{54}\sqrt{61} \approx .926$
- Then, the maximum revenue is $\frac{182}{2187} + \frac{122}{2187}\sqrt{61} \approx .51891$ and the cap would bind at $c \approx .530083$

Figure : No Bundle

Figure : Bundle

$\beta = 1$ Page 2

- TV consumer area (red)$= (1 - p_2)(p_b - p_2)$
- Bundle consumer area (blue)$=$
  $(p_1 + p_2 - p_b)(2 - p_1 - p_2) + (1 - p_1)(1 - p_2)$
- Internet consumer area (green) $=$
  $(1 - p_1)(p_b - p_1)$
- $R(p_1, p_2, p_b) = 2p_b(p_1 + p_2 - p_1^2 - p_1^2) + p_b - p_1^2 - p_2^2 + p_1^3 + p_2^3 + p_b^2(p_1 + p_2) - p_1 p_2 p_b$
- Our goal is to maximize:

$$\pi(p_1, p_2, p_b, \lambda) = R(p_1, p_2, p_b) + \lambda \left[c - 1 - p_2 + p_b + p_1 p_2 - p_2 p_b + p_2^2 \right]$$

where $c \in (0, .530084)$

Figure : Bundle

$\beta = 1$ Page 3

We can use the first order conditions to establish a system of 4 equations and 4 unknowns to find the following as function of c:

Figure : Profit

Figure : P_1

Figure : P_2

Figure : P_b

General Model Page 1

- The region of consumers who purchase the bundle can take on many different forms depending on the value of β and the relative bundle price
- "Skinny" Cone:
  $\beta > \frac{1}{2}, \beta(p_1 + p_2) \leq p_b < (1-\beta)p_1 + \beta p_2 + 2\beta - 1$
- "Fat" Cone: $\beta > \frac{1}{2}, p_b < \beta(p_1 + p_2)$
- $\beta = 0$ Case: $\beta > \frac{1}{2}, p_b > (1-\beta)p_1 + \beta p_2 + 2\beta - 1$

Figure : "Skinny"

Figure : "Fat"

Figure : $\beta = 0$

General Model Page 2

- Reverse "Skinny" Cone:
  $\beta < \frac{1}{2}, \beta(p_1 + p_2) > p_b \geq (1-\beta)p_1 + \beta p_2 + 2\beta - 1$
- Reverse "Fat" Cone: $\beta < \frac{1}{2}, p_b < (1-\beta)p_1 + \beta p_2 + 2\beta - 1$
- $\beta = 0$ Case: $\beta < \frac{1}{2}, p_b > \beta(p_1 + p_2)$

Figure : "Skinny"

Figure : "Fat"

Figure : $\beta = 0$

General Model Page 3

- Parallel Regions: $\beta = \frac{1}{2}, \beta(p_1 + p_2) > p_b$
- $\beta = 0$ Case: $\beta = \frac{1}{2}, p_b \geq \beta(p_1 + p_2)$

- As β changes, the monopolist may prefer to offer the bundle to different consumers
- When solving for general solutions in terms of β and c, one must be careful in dealing with the number of choices for the monopolist

Figure : Parallel

Figure : $\beta = 0$

General Model Page 4

Numerical results from maximizing profits when $\beta = .8$ leave many questions unanswered

Figure : Profit

Figure : P_1

Figure : P_2

Figure : P_b

Next Steps and Future Goals

- Investigate how demand theoretically shifts as monopolists increase internet prices
- Reach analytic conclusions for a general $\beta > \frac{1}{2}$ and $\beta < \frac{1}{2}$
  - Or possibly, numeric conclusions after further exhausting possibilities
- Anticipating a decline in β, predict dynamic pricing strategy for monopolists
- Introducing a more complex consumption funtion to allow for non-linear pricing strategy (3 part tariffs)
- Empirical testing, matching model predictions with data, looking for demand shifts