Preventing floodings in the Ebro basin

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Problem presented by

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Problem statement ??demanar opinio JQ?? Under heavy rain, some rivers use to have a discharge large enough to flood into nearby urban areas. A way to cope with this problem is to designate some fields adjacent to the river as *floodable*: At a cost, it is allowed to divert part of the flow into these fields by way of large gates which exist alongside the river. The area of the gate opening can be modified remotely. One wants to know the best control strategy for opening the gates, given the hydrologic profile of the flood which is gathered upstream some hours before it reaches the control point.

Report prepared by ??
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1 Introduction

According to the EU directive [1]

> 'flood' means the temporary covering by water of land not normally covered by water. This shall include floods from rivers, mountain torrents, Mediterranean ephemeral water courses, and floods from the sea in coastal areas, and may exclude floods from sewerage systems

The initial data is the forecasted flood hydrogram (the graph of the discharge as a function of time) computed from observations at a point upstream of the control point.

We assume that the water height at the control point is a increasing function of the discharge Q at that point. Three parameters are relevant:

- A threshold q_{min} that corresponds to the level of the gates: For $q < q_{\mathrm{min}}$ the system is not controllable;
- The *typical maximum flooding* discharge $q_{\mathrm{mao}} > q_{\mathrm{min}}$ is the maximum discharge for a recurrent flood, one that happens every two or three years.
- The maximum admisible discharge $q_{\mathrm{max}} > q_{\mathrm{mao}}$, that makes the water to spill over the floodable areas.

2 Cutting the flood

There are four stages and corresponding strategies when a flood is imminent:

1. If the maximum forecast discharge $q_{\mathrm{i}} < q_{\mathrm{mao}}$, the gates are not opened.
2. $q_{\mathrm{i}} > q_{\mathrm{mao}}$: In this case we use the flooding areas to cut the upmost part of the hydrogram to force a plateau at level q_{f} (the computed maximum after the cutting) and we have $q_{\mathrm{f}} < q_{\mathrm{max}}$. In this case the areas will we drained into the river once the level becomes less than q_{min} again.
3. Similar to the previous stage, but now we forecast that, even using the toal capacity W, we will have $q_{\mathrm{f}} > q_{\mathrm{max}}$. In this case we can avoid overflooding the areas, but want to delay this moment as far as possible.
4. The hydrogram is not unimodal and $q_{\mathrm{i}} > q_{\mathrm{mao}}$, but $q_{\mathrm{f}} < q_{\mathrm{max}}$. This is similar to the stage 2.

5. Like stage 4 but now $q_{\mathrm{f}} > q_{\max}$. This is similar to the stage 3.

The forecast discharge $q(t)$ will be converted to $\tilde{q}(t)$ by the control actions such that:

- $q(t) = \tilde{q}(t)$ except for $t \in I = [a, b]$ (the controlling period)
- If for all t, $q(t) < q_{\min}$, then $q(t) = \tilde{q}(t)$ (No gate opening below this level)
- For all t: $\tilde{q}(t) \leq q(t)$, for all t;
- Denote by W the total capacity of the floodable areas, Then

$$\int (q(t) - \tilde{q}(t))\, dt \leq W. \tag{1}$$

- If possible, ensure that $q_{\mathrm{f}} = \max \tilde{q} < q_{\max}$

Define $R = Q - \tilde{Q}$. The previous conditions imply that R is positive, has support on I and $\int R\, dt \leq V$.

Let $I = (a, b)$ the control period. It is clear that to minimize $\max \tilde{q}$ this function must be constant $\tilde{q}(t) = q_{\mathrm{f}}$ on I, for otherwise we could replace it for a constant equal to its mean value on I. Thus (1) could be written as:

$$\int_a^b q\, dt \leq q_{\mathrm{f}}(b - a) + W, \tag{2}$$

and the problem is to find a, b and fulfiling (2) with $q_{\mathrm{f}} < q_{\max}$. Add $Q(a) = Q(b) = q$ to (2) and we are set. To cut the most of the water we may impose equality on condition (2).

2.1 The case of monotonic hydrograms

When $q(t)$ has a single maximum at the time instant $t_0 \leq t_m \leq t_1$ is it possible. Pick a reference instant t_0 before the start of the flood and define for all $0 < y \leq q_{\mathrm{i}}$

$$V(y) = \int_{t_0}^{t_1} [q(t) - y]_+ \, dt \leq W,$$

where $[x]_+ = \max\{x, 0\}$. This expression could be written on the interval (a, b) defined by the condition $q(t) \geq q_{\mathrm{f}}$, as an iterated integral as

$$V(y) = \int_a^b \left(\int_y^{q(t)} dq \right) dt = \int_y^{q_{\mathrm{i}}} \left(\int_{t_-(q)}^{t_+(q)} dt \right) dq = \int_y^{q_{\mathrm{i}}} (t_+(q) - t_-(q))\, dq,$$

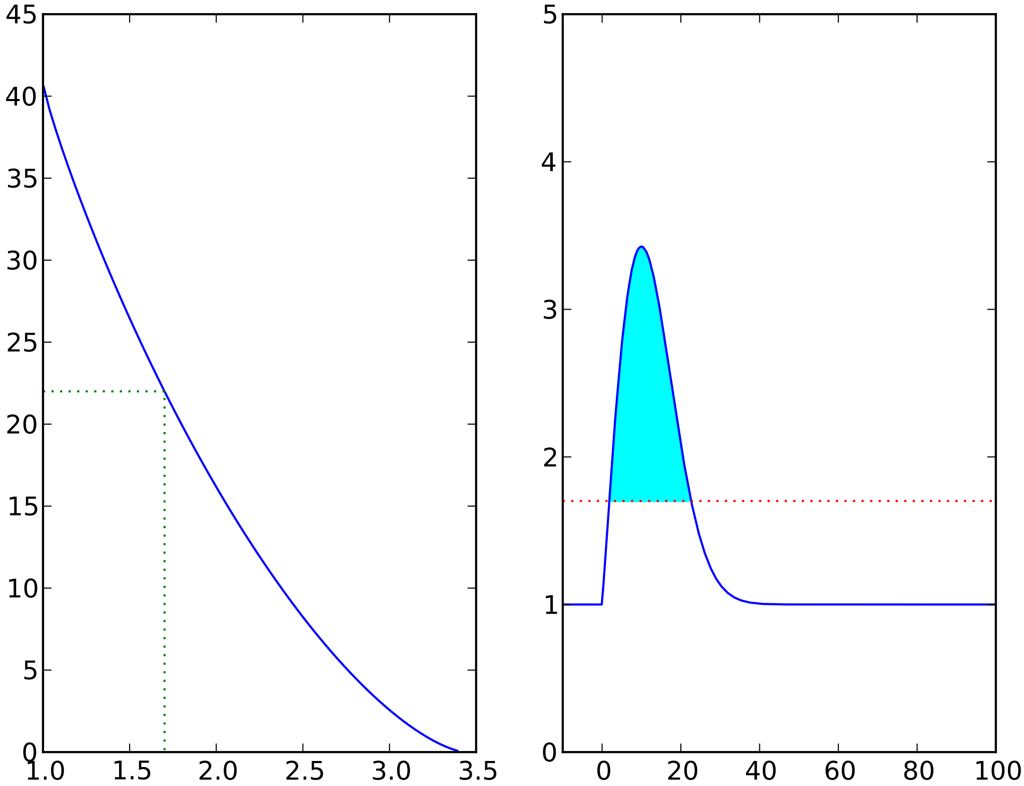

Figure 1: **Left**: $V(q)$ for the test function with the cut volume $W = 22.0$ corresponding to $q = q_\mathrm{f}$. **Right**: The cut volume is the part of the graph of the test function above $q = q_\mathrm{f}$.

by inverting the ordering of the integration and where $t_-(q)$ (respectively $t_+(q)$) is the unique $t < t_\mathrm{m}$ (respectively $t > t_\mathrm{m}$) with $q(t) = q$. Since $V'(y) = t_-(y) - t_+(y) \leq 0$, $V \geq 0$ is decreasing with $V(q_\mathrm{i}) = 0$ and it is easy to find the point q_f such that $V(q_\mathrm{f}) = W$ by bisection or inverse interpolation.

2.2 A numerical experiment

??Describe the experiment??

3 Incorporating cost

Let us now modify the assumption that the impact of flooding the urban areas is constant by making it depend on the number of residents affected by the flooding. Assume we have geographic data to describe $C(h)$, the number of residents that live below heigh h. Let C_{f} be the cost of flooding the fields, then the total cost will be

$$C = \begin{cases} C(h) & \text{no flooding} \\ C_{\mathrm{f}} + C(h(q_*)) & \text{when flooding} \end{cases}$$

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Acknowledgements

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References

[1] *Directive 2007/60/EC of the European Parliament and of the Council of 23 October 2007 on the assessment and management of flood risks* `http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2007:288:0027:0034:EN:PDF`

??More references here??