Preventing floodings in the Ebro basin

May 16, 2011

Problem presented by

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Problem statement ??demanar opinio JQ?? Under heavy rain, some rivers use to have a discharge large enough to flood into nearby urban areas. A way to cope with this problem is to designate some fields adjacent to the river as *floodable*: At a cost, it is allowed to divert part of the flow into these fields by way of large gates which exist alongside the river. The area of the gate opening can be modified remotely. One want to know the best control strategy for opening the gates, given the hydrologic profile of the flood which is gathered upstream some hours before it reaches the control point.

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1 Introduction

The initial data is the forecasted flood hydrogram (the graph of the discharge as a function of time) computed from observations at a point upstream of the control point. Two parameters are relevant: the *maximal flow* and the *total volume* of the flood.

We assume that the water height at the control point is a increasing function of the discharge Q at that point.

Let us considere three states related to the flow Q:

- The *steady state* corresponds to $Q < Q_{\mathrm{min}}$, where Q_{min} is the flow that brings the water at the control point high enough for the gates to open.
- A *typical flood* state corresponds to $Q_{\mathrm{min}} < Q < Q_{\mathrm{mao}}$. The flow Q_{mao} is the maximum flow for a recurrent flood, one that happens every two or three years.
- The *high flood* state happens when $Q_{\mathrm{mao}} < Q < Q_{\mathrm{max}}$, where Q_{max} is the flow that makes the water level spill over the floodable areas.
- Finally, when $Q > Q_{\mathrm{max}}$

For this the following control strategies are defined:

- If the maximum forecast flow Q_{i} is less than Q_{mao}, the gates are not opened.
- When $Q_{\mathrm{i}} > Q_{\mathrm{mao}}$ and $Q_{\mathrm{f}} < Q_{\mathrm{max}}$. In this case the goal is to minimize the ??. To compute this, one cuts from the hydrogram an area equivalent to the capacity of the floodable areas, to get a maxim plateau of Q_{f} flow.

2 Cutting the flood

We want to ensure $Q_{\mathrm{f}} < Q_{\mathrm{max}}$ whenever possible, where Q_{max} is the *peak flow* that results of gates opening.

- $Q(t) = \tilde{Q}(t)$ except for $t \in [t_0, t_1]$ (the controlling period)
- If $Q(t) < Q_{\mathrm{min}}$, then $Q(t) = \tilde{Q}(t)$ (No gate opening below this level)
- $\tilde{Q}(t) \leq Q(t)$, for all t;

- Denote by V the total capacity of the floodable areas, Then
$$\int (Q(t) - \tilde{Q}(t))\, dt \leq V. \tag{1}$$
- If possible $Q_{\mathrm{f}} = \sup \tilde{Q} < Q_{\max}$

Define $R = Q - \tilde{Q}$. The previous conditions imply that R is positive, has compact support and $\int R\, dt \leq V$. Condition ?? implies that $R = 0$.

An obvious choice for $\tilde{Q}$ is to pick $I = [a, b]$ such that $\tilde{Q}(t) = Q(t) - Q_{\mathrm{set}}$ for $t \in I$ and $Q(t) = \tilde{Q}(t)$ elsewhere. The capacity condition (1) reads as
$$\int_a^b Q\, dt \leq V + Q_{\mathrm{set}}(b - a)$$
and $Q_{\max} = Q_{\mathrm{set}}$ in this case.

Let us call *action support* the interval $I = [a, b]$ where Q and $\tilde{Q}$ differ. To cut the most of the water we impose equality on condition (1) giving
$$\int_a^b Q\, dt = \int_a^b \tilde{Q}\, dt + V$$
it is also clear that to minimize $\max \tilde{Q}$ this function must be constant $\tilde{Q}(t) = q$, for otherwise we could replace it for a constant equal to its mean value on I. Therefore (1) reads:
$$\int_a^b Q\, dt = q(b - a) + V, \tag{2}$$
and the problem is to find a, b and fulfiling (2) and minimizing $Q_{\max}$. Add $Q(a) = Q(b) = q$ to (2) and we are set.

2.1 The case of monotonic hydrograms

When Q has a single maximum at the time instant $a \leq t_m \leq b$ is it possible. Consider
$$W(t) = \int_{-\infty}^{t} Q\, d\tau.$$
Notice that $W(t)$ is increasing and $Q(t)$ is increasing on the subinterval $[a, t_m]$ and decreasing on $[t_m, b]$ so we can now take $W_+(q) = W(t_+^{-1}(q))$ defined on $[Q(a), Q_{\max}]$ and $W_-(q) = W(t_-^{-1}(q))$ in the interval $[Q(b), Q_{\max}]$. Now
$$f(q) = W_-(q) - W_+(q) = \int_{t_+}^{t_-} Q\, d\tau = V$$

and $f(q)$ is a decreasing function whose range goes to zero. By binary bisection it is very easy to find the point q_* such that $f(q_*) = V$.

3 Incorporating cost

Let us now modify the assumption that the impact of flooding the urban areas is constant by making it depend on the number of residents affected by the flooding. Assume we have geographic data to describe $C(h)$, the number of residents that live below heigh h. Let C_{f} be the cost of flooding the fields, then the total cost will be

$$C = \begin{cases} C(h) & \text{no flooding} \\ C_{\mathrm{f}} + C(h(q_*)) & \text{when flooding} \end{cases}$$

Acknowledgements

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References

[1] *Directive 2007/60/EC of the European Parliament and of the Council of 23 October 2007 on the assessment and management of flood risks* `http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2007:288:0027:0034:EN:PDF`

??More references here??