Cost modelling of the two main strategies in a flooding

Main problem: there are two possible strategies

We have two possible strategies to follow when scenario $q_{max} < q_f$ occurs and thus we can not avoid the flood: we can either choose to open the gate when the flow q achives q_{max} or open it when q_f is achived. Each strategy has its advantages:

- On the one hand, if we open the gate at q_{max}, we are going to be delaying the flood as much as we can, but when it arribes, the effects will be quite important.
- On the other hand, if we wait to open the gates until q achieves q_f, the flood will arrive sooner, but its effects will be reduced, because the intensity of the flow is going to be lower.

Designing a cost function: a possible solution

Thus, we want to design a cost function in order to measure the "cost" of each strategy. This cost function will take on account the number of affected people depending on the heigh of the water in the flood and also the time that the strategy is are able to delay it. The cost function that we are going to consider is defined as equation 0.1 shows. The cost that we are assuming from an initial time t_i to a certain time t is given by:

$$C(t) = \int_{t_i}^{t} \frac{\rho(h(q(t)))}{t} dt, \tag{0.1}$$

where ρ is the density of population that lives below a certain heigh h (where the heigh depends on q, both q and h known). We have divided the density by t in order to "decrease the cost" when increasing the time, since we want to minimize the effects of the flood but also give time to the population to leave the most affected zones. If we want to give more importance to the delay of the flood we can change the modelling cost and use some other t^r, $r > 1$.

Finding the strategy that minimizes the cost function

We want then to compute the cost function for each one of the two main strategies to follow in a flooding scenario. Hence, the most valid strategy will be the one that gives the minimum final cost at the ending time t_f.

Strategy A: opening the gates when q_f is achieved

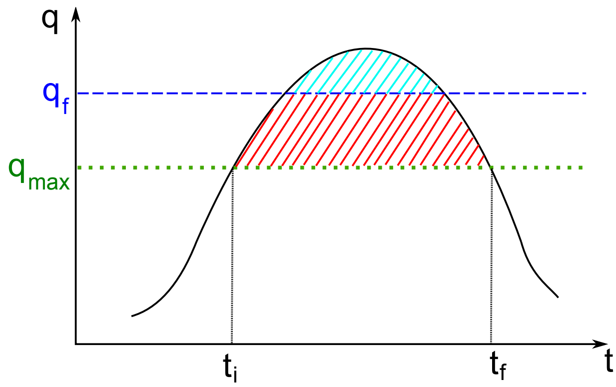

Figure 0.1: Flow diagram for strategy A

Figure 0.1 shows that with strategy A we are going to be able to eliminate the flow marked in light blue. The dangerous flow that we are not going to be able to stop and that is going to be penalized by the cost function is marked in red. To be able to compute the cost function, we need the population density of the affected zone. However, we can easily find an upper bound for the cost of this strategy:

$$\begin{aligned} C(t_f) &= \int_{t_i}^{t_f} \frac{\rho(h(q(t)))}{t} dt \overset{q(t)\leq q_f(t)\ \forall t}{\leq} \int_{t_i}^{t_f} \frac{\rho(h(q_f(t)))}{t} dt \\ &= \int_{t_i}^{t_f} \frac{\rho_{q_f}}{t} dt = \rho_q \cdot \ln(\frac{t_f}{t_i}), \end{aligned}$$

where we have denoted by ρ_{q_f} the density of population that lives below $h(q_f)$.

Thus, we can bound (this may be useful if we could compare it with B... but we are not able to do so... we need ρ in order to be able to say something) the cost of strategy A as

$$C_A \leq \rho_q \cdot \ln(\frac{t_f}{t_i}), \tag{0.2}$$

where ρ_{q_f}, t_i and t_f are all known.

Strategy B: opening the gates when q_{max} is achieved

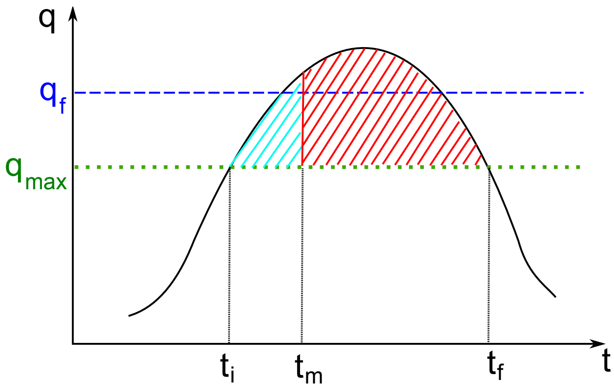

Figure 0.2: Flow diagram for strategy B

With strategy B we find that despite reaching higher flow values, the time in which q_{max} is achieved is delayed, and thus there are no "human" costs until a bigger time than the one used in strategy A. However, we are just able to delay the flow into a known time t_m, where the auxiliar area is full and we can not take more water from the river. Thus, the cost of strategy B is

$$C(t_f) = \int_{t_i}^{t_f} \frac{\rho(h(q(t)))}{t} dt = \int_{t_m}^{t_f} \frac{\rho(h(q(t)))}{t} dt,$$

where we must recall that all the data required in the integral is known, and thus, in a particular case it is straight forward to compute the cost.

Hence,

$$C_B = \int_{t_m}^{t_f} \frac{\rho(h(q(t)))}{t} dt, \tag{0.3}$$

and then depending on the scenario that is placed we can compute both C_A and C_B and have a reference of which one of the strategies will bring up more advantages.