Preventing floodings in the Ebro basin

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Problem presented by

Joseba Quevedo
Sistemes Avançats de Control, UPC

Problem statement ??demanar opinio JQ?? During the raining season, rivers use to have a flow large enough to flood into the nearby urban areas. To cope with this problem, some fields adjacent to the river basin are designated as *floodable*: At a cost, it is allowed to divert part of the flow into these fields by way of large gates which exist alongside the river. The area of the gate opening can be modified remotely. One want to know the better control strategy for opening the gates, given the hydrologic profile of the avenue which gathered upstream some hours before it reaches the control point.

Report prepared by ??
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Study Group contributors Abel Gargallo Peir, Yi Ming, Manuel Quezada de Luna, Jordi Saludes, Adri Simon, Jeff Springer

1 Introduction

The initial data is the forecasted avenue hydrogram (the graph of the flow as a function of time) computed from observations at a point upstream of the control point. Two parameters are relevant: the *maximal flow* and the *total volume* of the avenue.

We assume that the water height at the control point is a increasing function of the flow Q at that point.

Let us considere three states related to the flow Q:

- The *steady state* corresponds to $Q < Q_{\text{min}}$, where Q_{min} is the flow that brings the water at the control point high enough for the gates to open.
- A *typical avenue* state corresponds to $Q_{\text{min}} < Q < Q_{\text{mao}}$. The flow Q_{mao} is the maximum flow for a recurrent avenue, one that happens every two or three years.
- The *high avenue* state happens when $Q_{\text{mao}} < Q < Q_{\text{max}}$, where Q_{max} is the flow that makes the water level spill over the floodable areas.
- Finally, when $Q > Q_{\text{max}}$

For this the following control strategies are defined:

- If the maximum forecast flow Q_{i} is less than Q_{mao}, the gates are not opened.
- When $Q_{\text{i}} > Q_{\text{mao}}$ and $Q_{\text{f}} < Q_{\text{max}}$. In this case the goal is to minimize the ??. To compute this, one cuts from the hydrogram an area equivalent to the capacity of the floodable areas, to get a maxim plateau of Q_{f} flow.

2 Cutting the avenue

We want to ensure $Q_{\text{f}} < Q_{\text{max}}$ whenever possible, where Q_{max} is the *peak flow* that results of gates opening.

- $Q(t) = \tilde{Q}(t)$ except for $t \in [t_0, t_1]$ (the controlling period)
- If $Q(t) < Q_{\text{min}}$, then $Q(t) = \tilde{Q}(t)$ (No gate opening below this level)

- $\tilde{Q}(t) \leq Q(t)$, for all t;
- Denote by V the total capacity of the floodable areas, Then

$$\int (Q(t) - \tilde{Q}(t))\, dt \leq V. \tag{1}$$

- If possible $Q_{\mathrm{f}} = \sup \tilde{Q} < Q_{\mathrm{max}}$

Define $R = Q - \tilde{Q}$. The previous conditions imply that R is positive, has compact support and $\int R\, dt \leq V$. Condition ?? implies that $R = 0$.

An obvious choice for $\tilde{Q}$ is to pick $I = [a, b]$ such that $\tilde{Q}(t) = Q(t) - Q_{\mathrm{set}}$ for $t \in I$ and $Q(t) = \tilde{Q}(t)$ elsewhere. The capacity condition (1) reads as

$$\int_a^b Q\, dt \leq V + Q_{\mathrm{set}}(b - a)$$

and $Q_{\mathrm{max}} = Q_{\mathrm{set}}$ in this case.

Let us call *action support* the interval $I = [a, b]$ where Q and $\tilde{Q}$ differ. To cut the most of the water we impose equality on condition (1) giving

$$\int_a^b Q\, dt = \int_a^b \tilde{Q}\, dt + V$$

it is also clear that to minimize $\max \tilde{Q}$ this function must be constant $\tilde{Q}(t) = q$, for otherwise we could replace it for a constant equal to its mean value on I. Therefore (1) reads:

$$\int_a^b Q\, dt = q(b - a) + V, \tag{2}$$

and the problem is to find a, b and fulfiling (2) and minimizing Q_{max}. Add $Q(a) = Q(b) = q$ to (2) and we are set.

2.1 The case of monotonic hydrograms

When Q has a single maximum at the time instant $a \leq t_m \leq b$ is it possible. Consider

$$W(t) = \int_{-\infty}^{t} Q\, d\tau.$$

Notice that $W(t)$ is increasing and $Q(t)$ is increasing on the subinterval $[a, t_m]$ and decreasing on $[t_m, b]$ so we can now take $W_+(q) = W(t_+^{-1}(q))$ defined on $[Q(a), Q_{\mathrm{max}}]$ and $W_-(q) = W(t_-^{-1}(q))$ in the interval $[Q(b), Q_{\mathrm{max}}]$. Now

$$f(q) = W_-(q) - W_+(q) = \int_{t_+}^{t_-} Q\, d\tau = V$$

and $f(q)$ is a decreasing function whose range goes to zero. By binary bisection it is very easy to find the point q_* such that $f(q_*) = V$.

3 Incorporating cost

Let us now modify the assumption that the impact of flooding the urban areas is constant by making it depend on the number of residents affected by the flooding. Assume we have geographic data to describe $C(h)$, the number of residents that live below heigh h. Let C_f be the cost of flooding the fields, then the total cost will be

$$C = \begin{cases} C(h) & \text{no flooding} \\ C_\mathrm{f} + C(h(q_*)) & \text{when flooding} \end{cases}$$

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References