PCFG

PCFG

- Probabilistic / Stochastic Context Free Grammars:
  - Simple probabilistic models capable of handling recursion
  - A CFG with probabilities attached to rules
  - Rule probabilities → how likely is it that a particular rewrite rule is used?

Formal Definition of PCFG

- A PCFG consists of
  - A set of terminals $\{w_k\}$, k = 1,….,V

    $\{w_k\}$ = { child, teddy, bear, played…}
  - A set of non-terminals $\{N^i\}$, i = 1,…,n

    $\{N_i\}$ = { NP, VP, DT…}
  - A designated start symbol N^1
  - A set of rules $\{N^i \rightarrow \zeta^j\}$, where ζ^j is a sequence of terminals & non-terminals

    NP $\rightarrow$ DT NN
  - A corresponding set of rule probabilities

Rule Probabilities

- Rule probabilities are such that

$$\forall i \quad \sum_{\mathrm{i}} \mathrm{P}(\mathrm{N}^{i} \rightarrow \zeta^{j}) = 1$$

E.g., P(NP $\rightarrow$ DT NN) = 0.2

P(NP $\rightarrow$ NN) = 0.5

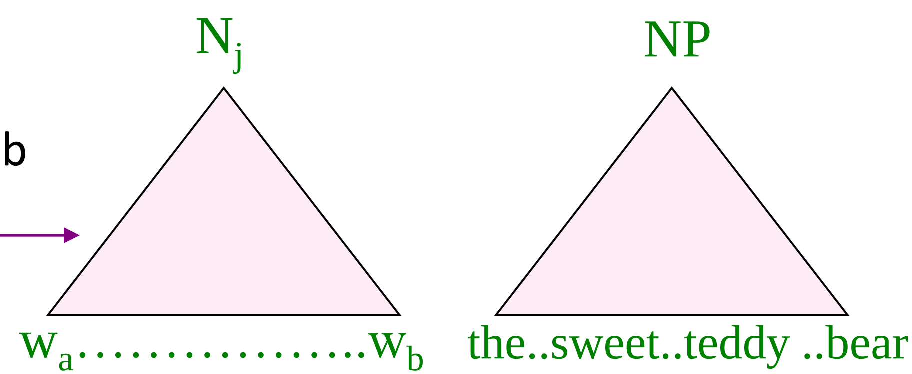
P(NP $\rightarrow$ NP PP) = 0.3

- P(NP $\rightarrow$ DT NN) = 0.2
  - Means 20 % of the training data parses use the rule NP $\rightarrow$ DT NN

Probability of a sentence

- Notation :
  - w_{ab} – subsequence $w_a \ldots w_b$
  - N_j dominates $w_a \ldots w_b$

    or yield(N_j) = $w_a \ldots w_b$



- Probability of a sentence = $P(w_{1m})$

$$P(w_{1m}) = \sum_t P(w_{1m}, t)$$

Where t is a parse tree of the sentence

$$= \sum_t P(t) P(w_{1m} \mid t)$$

$$= \sum_{t: yield(t) = w_{1m}} P(t) \qquad \text{Q } P(w_{1m} \mid t) = 1$$

If t is a parse tree for the sentence w_{1m}, this will be 1 !!

Probability of a parse tree

- *Domination* :We say N_j dominates from k to l, symbolized as $N^j_{k,l}$, if $W_{k,l}$ is derived from N_j
- P (tree |sentence) = P (tree | $S_{1,l}$)

  where $S_{1,l}$ means that the start symbol S dominates the word sequence $W_{1,l}$
- P (t |s) approximately equals joint probability of constituent non-terminals dominating the sentence fragments (next slide).

Assumptions of the PCFG model

- Place invariance :

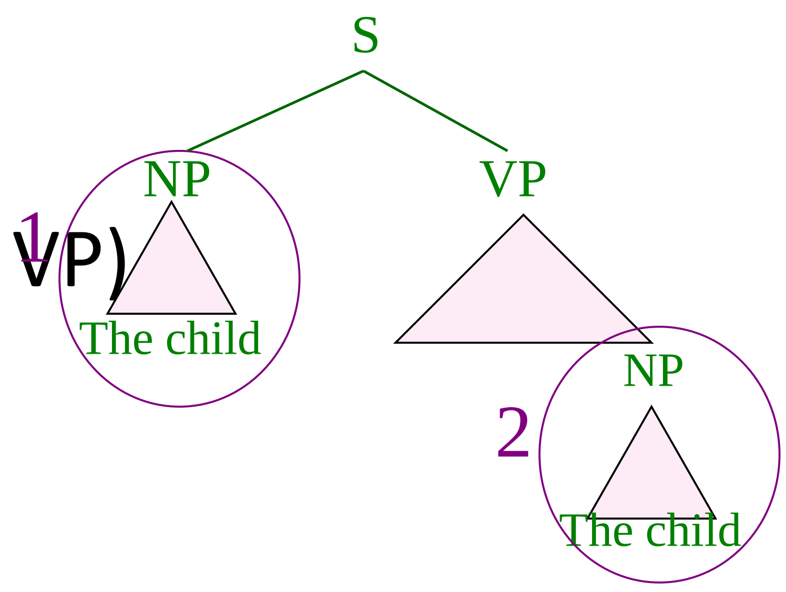
  P(NP $\rightarrow$ DT NN) is same in locations 1 and 2

- Context-free :

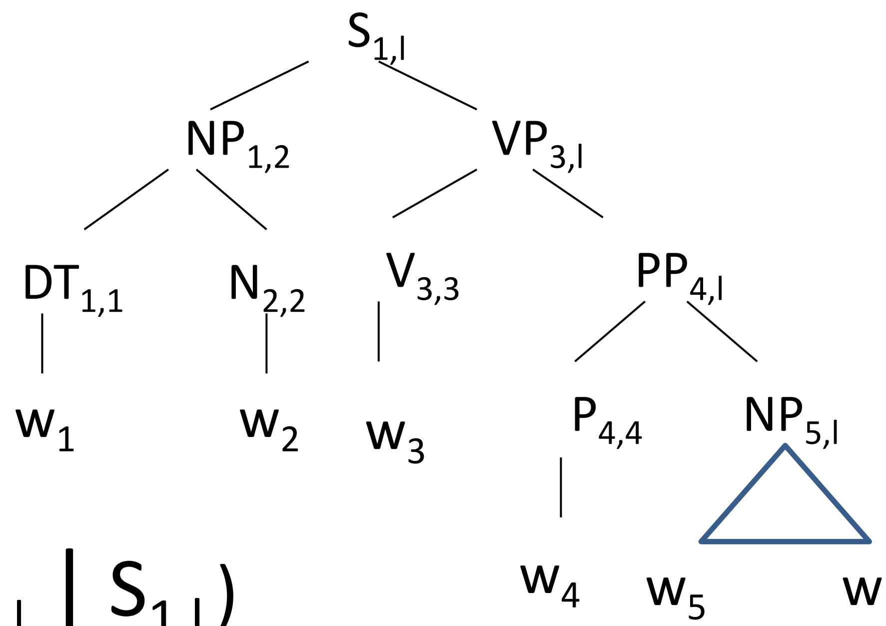
  P(NP $\rightarrow$ DT NN | anything outside "The child")
  = P(NP $\rightarrow$ DT NN)

- Ancestor free : At 2,

  P(NP $\rightarrow$ DT NN|its ancestor is VP)
  = P(NP $\rightarrow$DT NN)



Probability of a parse tree (cont.)



$P(t|s) = P(t \mid S_{1,l})$

$= P(NP_{1,2}, DT_{1,1}, w_1,$
$\quad NP_{2,2}, w_2,$
$\quad VP_{3,l}, V_{3,3}, w_3,$
$\quad PP_{4,l}, P_{4,4}, w_4, NP_{5,l}, w_{5\ldots l} \mid S_{1,l})$

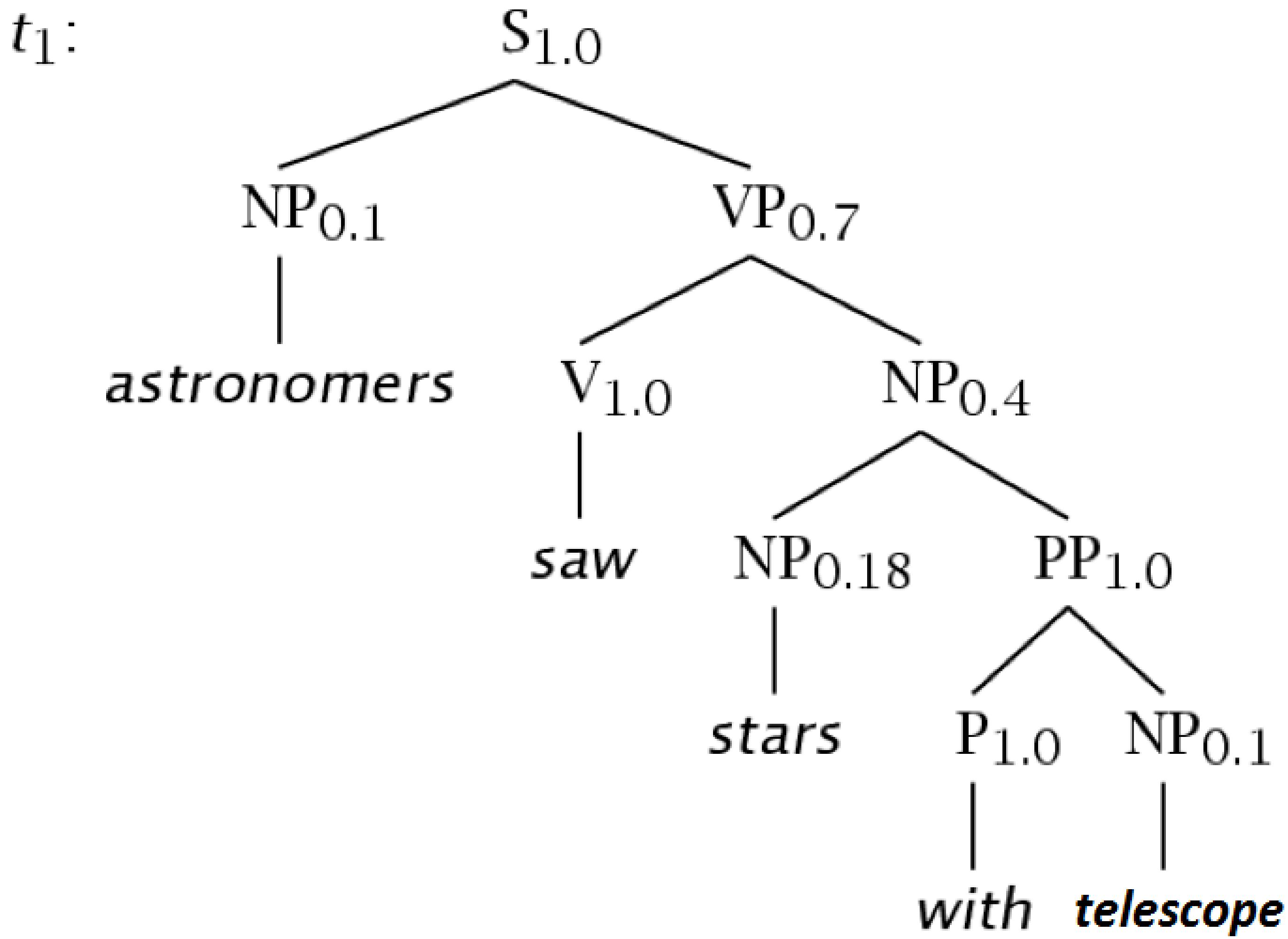
$= P(NP_{1,2}, VP_{3,l} \mid S_{1,l}) * P(DT_{1,1}, N_{2,2} \mid NP_{1,2}) * D(w_1 \mid DT_{1,1}) * P(w_2 \mid N_{2,2}) * P(V_{3,3}, PP_{4,l} \mid VP_{3,l}) * P(w_3 \mid V_{3,3}) * P(P_{4,4}, NP_{5,l} \mid PP_{4,l}) * P(w_4|P_{4,4}) * P(w_{5\ldots l} \mid NP_{5,l})$

(Using Chain Rule, Context Freeness and Ancestor Freeness)

A Simple PCFG (in CNF)

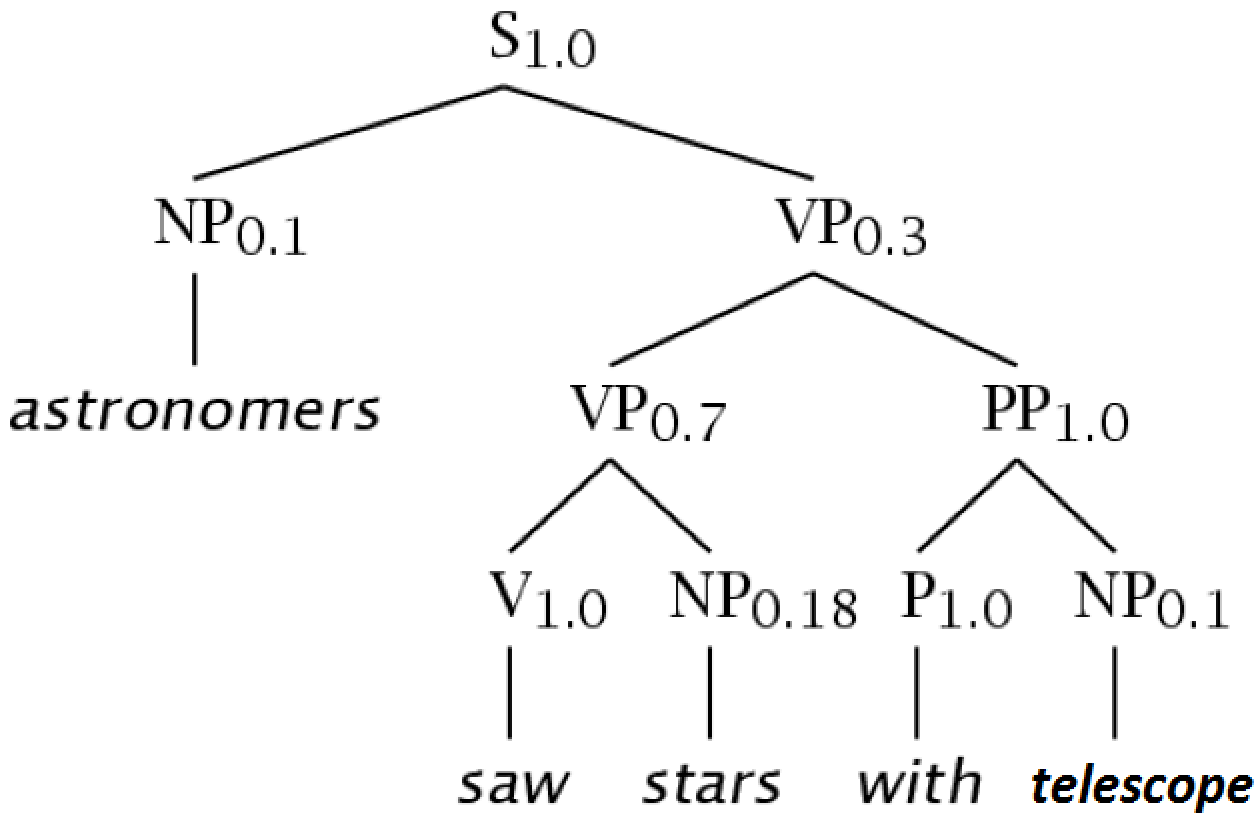
S	→	NP VP	1.0	NP	→	NP PP	0.4
VP	→	V NP	0.7	NP	→	*astronomers*	0.1
VP	→	VP PP	0.3	NP	→	*ears*	0.18
PP	→	P NP	1.0	NP	→	*saw*	0.04
P	→	*with*	1.0	NP	→	*stars*	0.18
V	→	*saw*	1.0	NP	→	*telescope*	0.1

Example Trees



Example Trees

t_2:



Probability of trees and strings

- $P(t)$: The probability of tree is the product of the probabilities of the rules used to generate it.
- $P(w_{1n})$: The probability of the string is the sum of the probabilities of the trees which have that string as their yield

Tree and String probabilities

W_{15}=astronomers saw star with telescope

$P(t_1) = 1.0 * 0.1 * 0.7 * 1.0 * 0.4 *0.18*$
$1.0 *1.0 *0.1$
$= 0.000504$

$P(t_2) = 1.0 * 0.1 * 0.3 * 0.7 * 1.0 * 0.18 *$
$1.0 * 1.0 *0.1$
$= 0.000378$

$P(w_{15}) = P(t_1) + P(t_2)$
$= 0.000504 + 0.000378$
$= 0.000882$

Features of PCFGs

Features of PCFGs

- As the number of possible trees for a given input grows, a PCFG gives some idea of the plausibility of a particular parse
- But the probability estimates are based purely on structural factors, and do not factor in lexical co-occurrence. Thus, PCFG does not give a very good idea of the plausibility of the sentence.
- Real text tends to have grammatical mistakes. PCFG avoids this problem by ruling out nothing, but by giving implausible sentences a low probability
- In practice, a PCFG is a worse language model for English than an n-gram model
- All else being equal, the probability of a smaller tree is greater than a larger tree

Important Questions?

- Let W_{1m} be a sentence, G a grammar, t a parse tree
  1. What is the most likely parse of sentence?

     $$\text{argmax}_t P(t/w_{1m}, G)$$

  2. What is the probability of a sentence?

     $$P(w_{1m}/G)$$

  2. How to learn the rule probabilities in the grammar G?

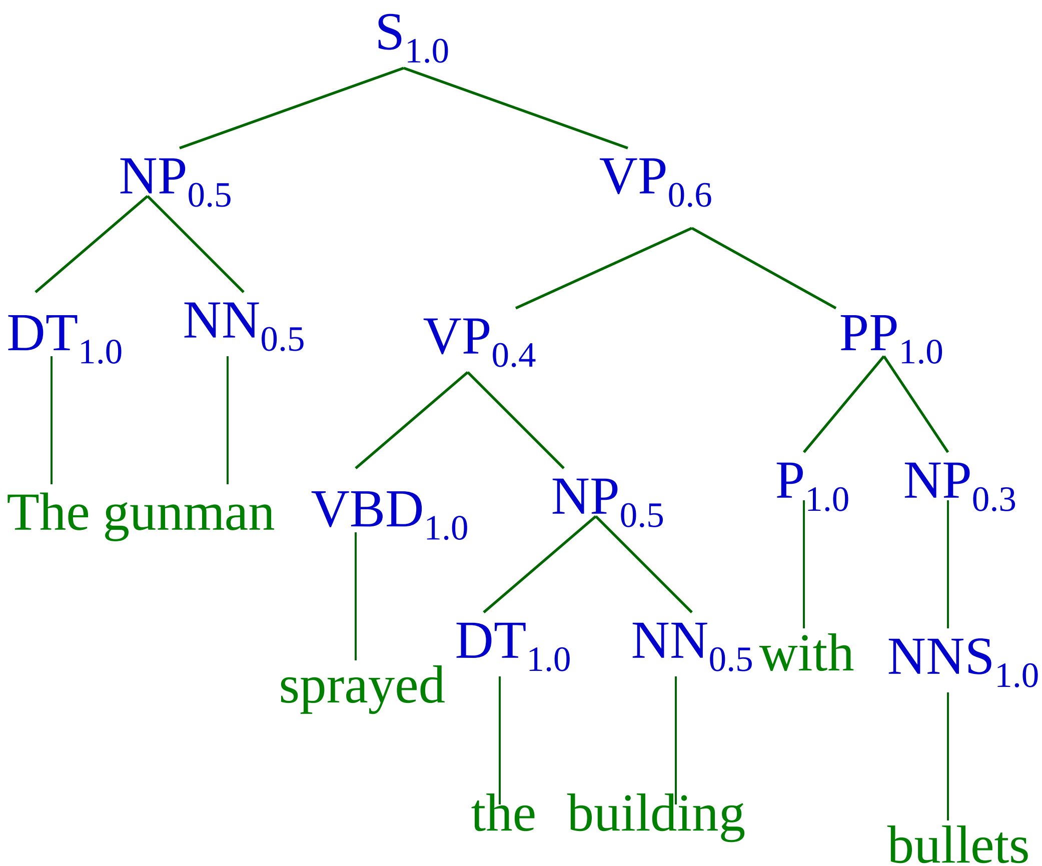
Example PCFG Rules & Probabilities

- S → NP VP 1.0
- NP → DT NN 0.5
- NP → NNS 0.3
- NP → NP PP 0.2
- PP → P NP 1.0
- VP → VP PP 0.6
- VP → VBD NP 0.4

- DT → the 1.0
- NN → gunman 0.5
- NN → building 0.5
- VBD → sprayed 1.0
- NNS → bullets 1.0

Example Parse t_1`

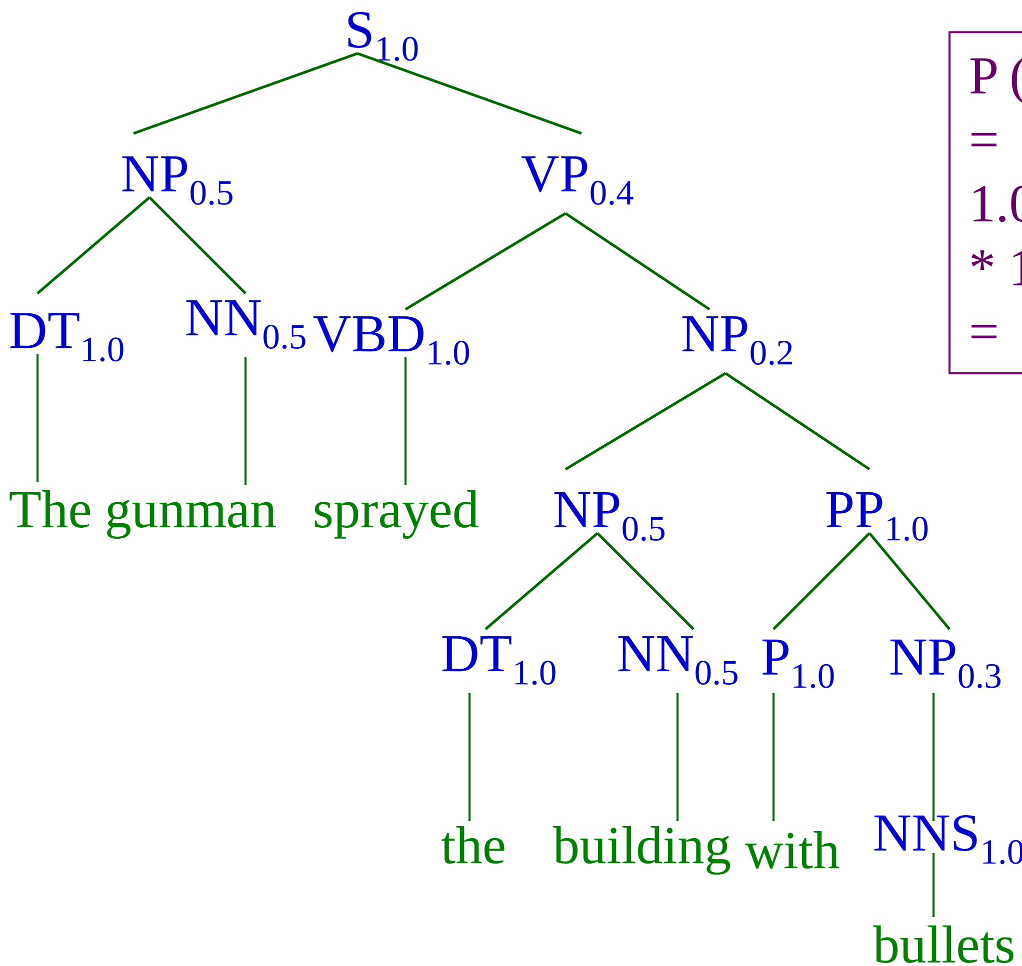
- The gunman sprayed the building with bullets.



$$P(t_1) = 1.0 * 0.5 * 1.0 * 0.5 * 0.6 * 0.4 * 1.0 * 0.5 * 1.0 * 0.5 * 1.0 * 1.0 * 0.3 * 1.0 = 0.00225$$

Another Parse t_2

- The gunman sprayed the building with bullets.



P (t_2)
$= 1.0 * 0.5 * 1.0 * 0.5 * 0.4 * 1.0 * 0.2 * 0.5 * 1.0 * 0.5 * 1.0 * 1.0 * 0.3 * 1.0$
$= 0.0015$

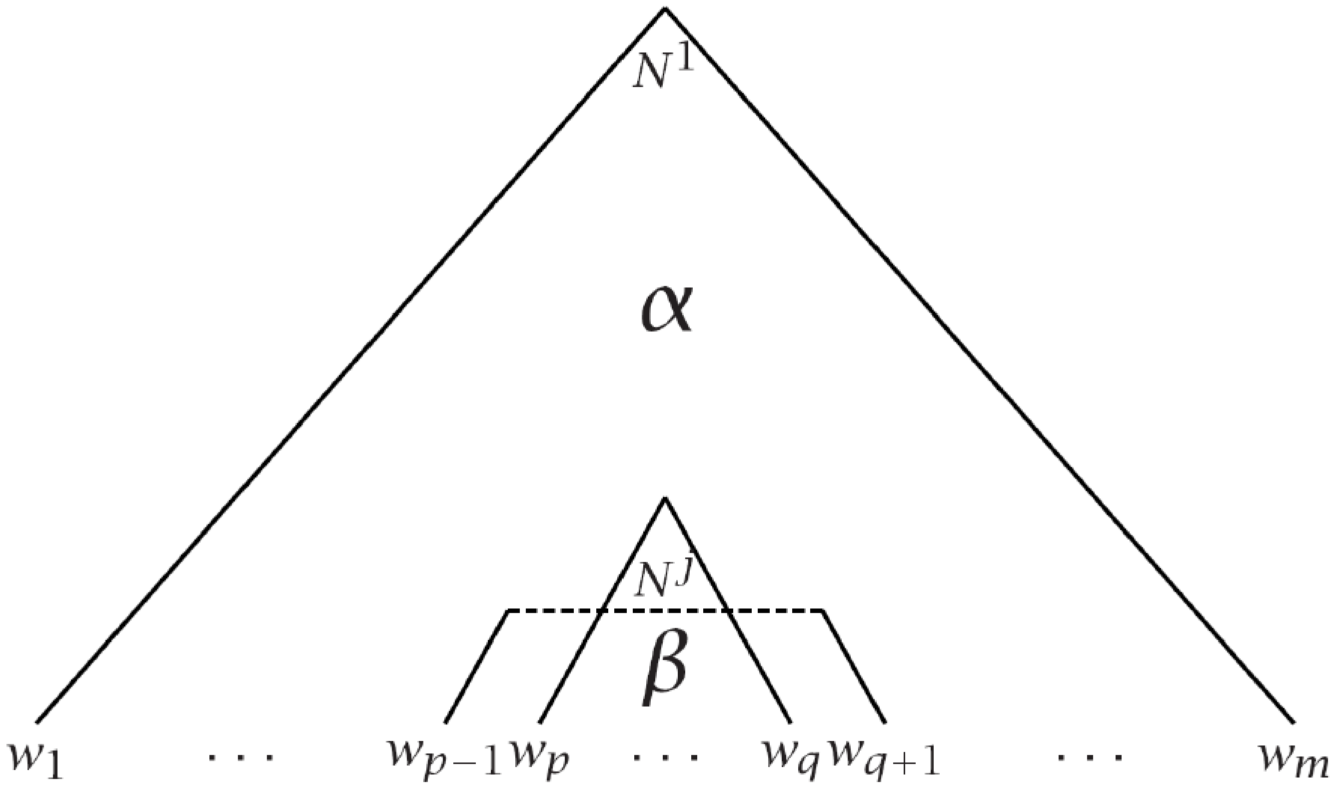
PCFGs - Inside-outside probabilities

Probability of a Sentence

$$\mathbf{P(w_{1m}/G)}$$

- In general, simply summing the probabilities of all possible parse trees is not an efficient way to calculate the sentence probability
- We use **inside algorithm**, a dynamic programming algorithm based on concept of inside –outside probabilities.

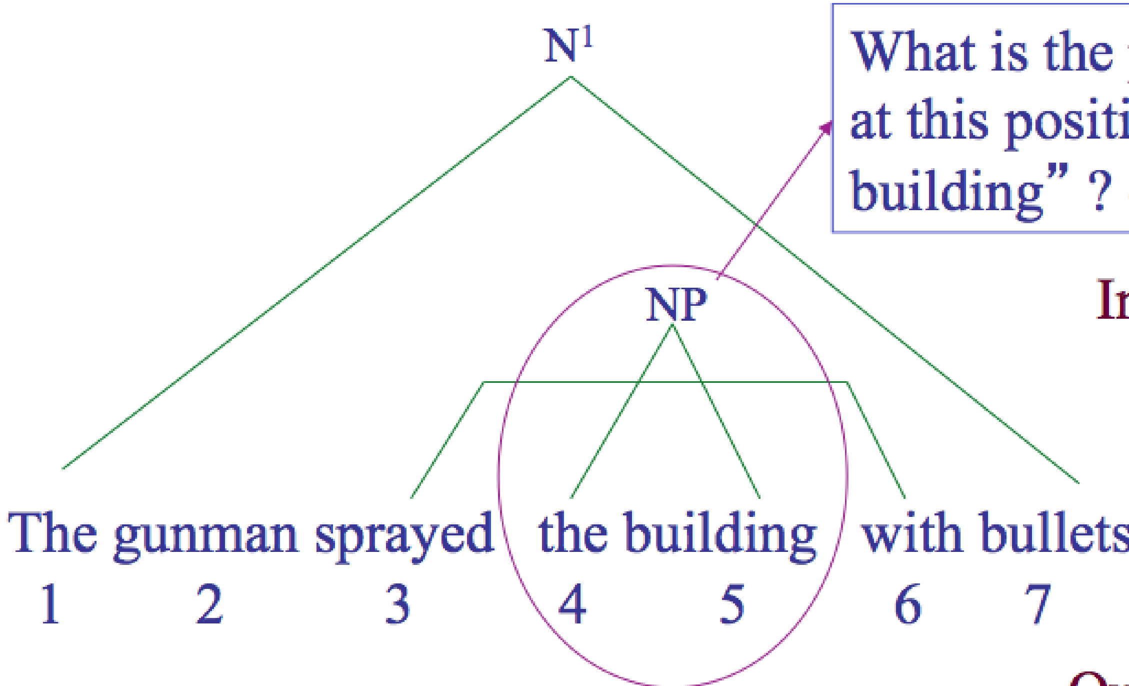
Inside and Outside Probabilities



Outside: $\alpha_j(p,q) = P(w_{1(p-1)}, N^j{}_{pq}, w_{(q+1)m} | G)$

Inside: $\beta_j(p,q) = P(w_{pq} | N^j{}_{pq}, G)$

Inside-outside probabilities



What is the probability that an NP at this position will derive "the building" ? - $\beta_{NP}(4,5)$

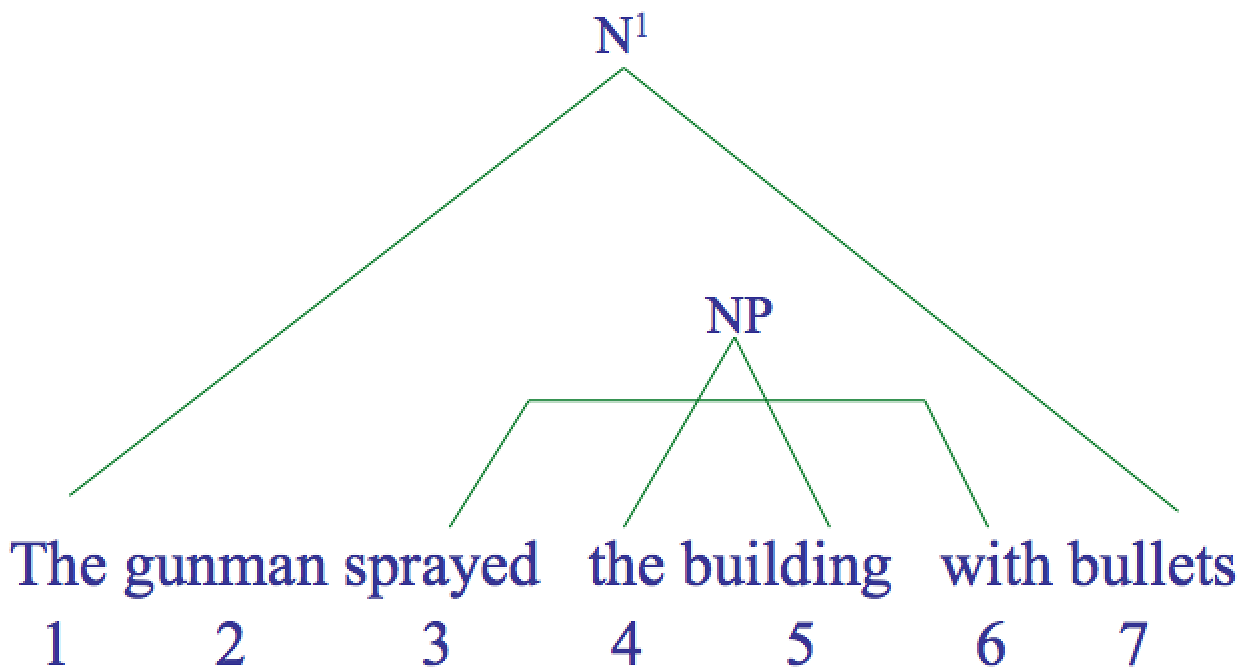
Inside Probabilities

Outside Probabilities

What is the probability of starting from N^1 and deriving "The gunman sprayed", a NP and "with bullets" ? - $\alpha_{NP}(4,5)$

Inside-outside probabilities

$$\alpha_{NP}(4,5) \text{ for "the building"}$$
$$= P(\text{The gunman sprayed}, NP_{4,5}, \text{with bullets} \mid G)$$
$$\beta_{NP}(4,5) \text{ for "the building"} = P(\text{the building} \mid NP_{4,5}, G)$$



Probability of a string

- **Inside Probability**

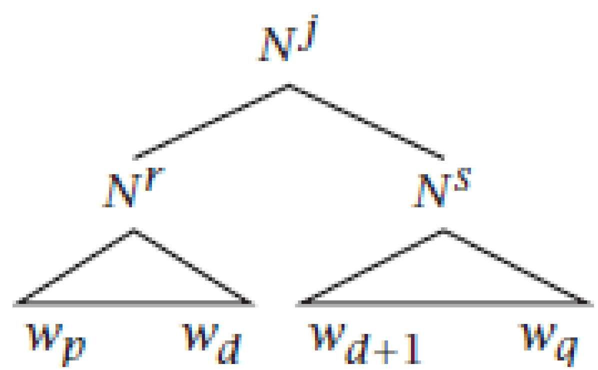
$$P(w_{1m}/G) = P(N^1 \longrightarrow w_{1m} / G)$$
$$= P(w_{1m}/ N^1_{1m}, G)$$
$$= \beta_1 (1, m)$$

- The internal probability of a substring is calculated by induction on the length of the string subsequence.
- **Base case: We want to find** $\beta_j (k, k)$ ***(the probability of a rule :*** $N^j \longrightarrow w_k$

$$\beta_j (k, k) = P(w_k / N^j_{kk} , G)$$
$$= P(N^j \longrightarrow w_k / G)$$

Induction Step

- **Induction: We want to find $\beta_j(p, q)$ *for $p < q$.*** *As this is the inductive* step using a Chomsky Normal Form grammar, the first rule must be of the form $N^j \longrightarrow N^r N^s$*, so we can proceed by induction, dividing the* string in two, in various places, and summing the result:



- These inside probabilities can be calculated bottom up.

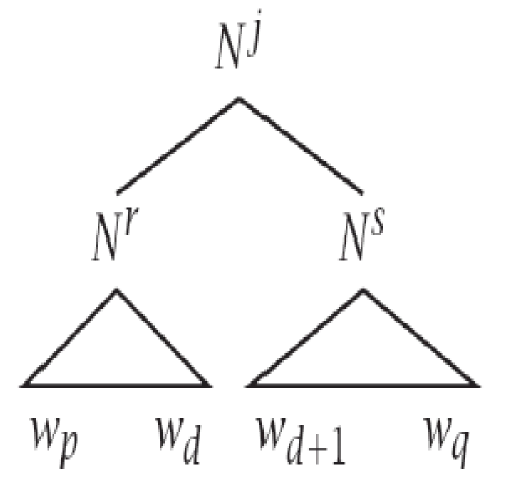
For all j,

$$
\begin{aligned}
\beta_j(p,q) &= P(w_{pq} \mid N^j_{pq}, G) \\
&= \sum_{r,s} \sum_{d=p}^{q-1} P(w_{pd}, N^r_{pd}, w_{(d+1)q}, N^s_{(d+1)q} \mid N^j_{pq}, G) \\
&= \sum_{r,s} \sum_{d=p}^{q-1} P(N^r_{pd}, N^s_{(d+1)q} \mid N^j_{pq}, G) \\
&\qquad P(w_{pd} \mid N^j_{pq}, N^r_{pd}, N^s_{(d+1)q}, G) \\
&\qquad P(w_{(d+1)q} \mid N^j_{pq}, N^r_{pd}, N^s_{(d+1)q}, w_{pd}, G) \\
&= \sum_{r,s} \sum_{d=p}^{q-1} P(N^r_{pd}, N^s_{(d+1)q} \mid N^j_{pq}, G) \\
&\qquad P(w_{pd} \mid N^r_{pd}, G) P(w_{(d+1)q} \mid N^s_{(d+1)q}, G) \\
&= \sum_{r,s} \sum_{d=p}^{q-1} P(N^j \rightarrow N^r N^s) \beta_r(p,d) \beta_s(d+1,q)
\end{aligned}
$$

Inside Probabilities: Induction Step

Assuming Chomsky Normal Form, the first rule must be of the form $N^j \rightarrow N^r N^s$

$$\beta_j(p,q) = \sum_{r,s} \sum_{d=p}^{q-1} P(N^j \rightarrow N^r N^s)\beta_r(p,d)\beta_s(d+1,q)$$



•Thus, we find all ways that a certain constituent can be built out of smaller constituents by varying what the labels of the two smaller constituents are and which words each spans.

- **Consider different splits of the words - indicated by d**
  E.g., “the huge” and “building” or “the” and “huge building”
- **Consider different non-terminals to be used in the rule:**
  E.g., NP →DT NN,
  NP → DT NNS

Calculation of Inside Probabilities

Given the PCFG:

$S \rightarrow NP\ VP$ 1.0	$NP \rightarrow$ astronomers 0.1
$PP \rightarrow P\ NP$ 1.0	$NP \rightarrow$ telescope 0.18
$VP \rightarrow V\ NP$ 0.7	$NP \rightarrow$ saw 0.04
$VP \rightarrow VP\ PP$ 0.3	$NP \rightarrow$ stars 0.18
$NP \rightarrow NP\ PP$ 0.4	$V \rightarrow$ saw 1.0
$P \rightarrow$ with 1.0	

Q) Find the probability of the following sentence using inside probability?

Sentence: Astronomers saw stars with telescope

Calculation of Inside Probabilities

	1	2	3	4	5
1	$\beta_{NP} = 0.1$		$\beta_S = 0.0126$		$\beta_S = 0.0015876$
2		$\beta_{NP} = 0.04$ $\beta_V = 1.0$	$\beta_{VP} = 0.126$		$\beta_{VP} = 0.015876$
3			$\beta_{NP} = 0.18$		$\beta_{NP} = 0.01296$
4				$\beta_P = 1.0$	$\beta_{PP} = 0.18$
5					$\beta_{NP} = 0.18$
	astronomers	*saw*	*stars*	*with*	telescope

Outside Probabilities

Outside Probabilities

Base case :

$$\beta_1(1,m) = 1$$

$$\beta_j(1,m) = 0 \text{ if } j \neq 1$$

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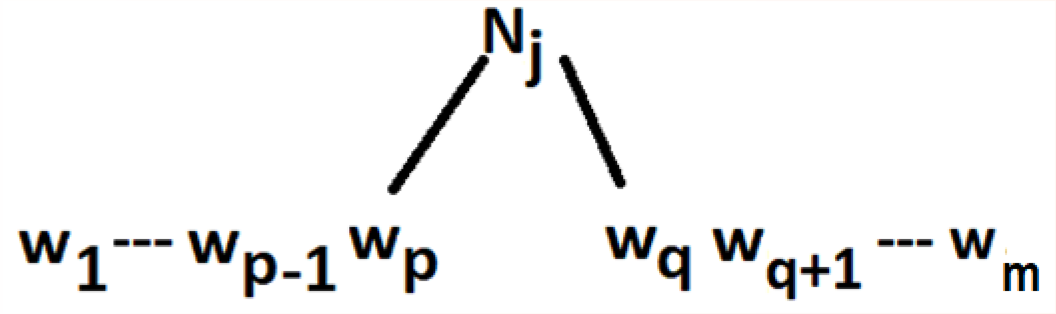
Inductive Case: Compute outside probabilities in top down manner



Outside Probabilities

Base case :

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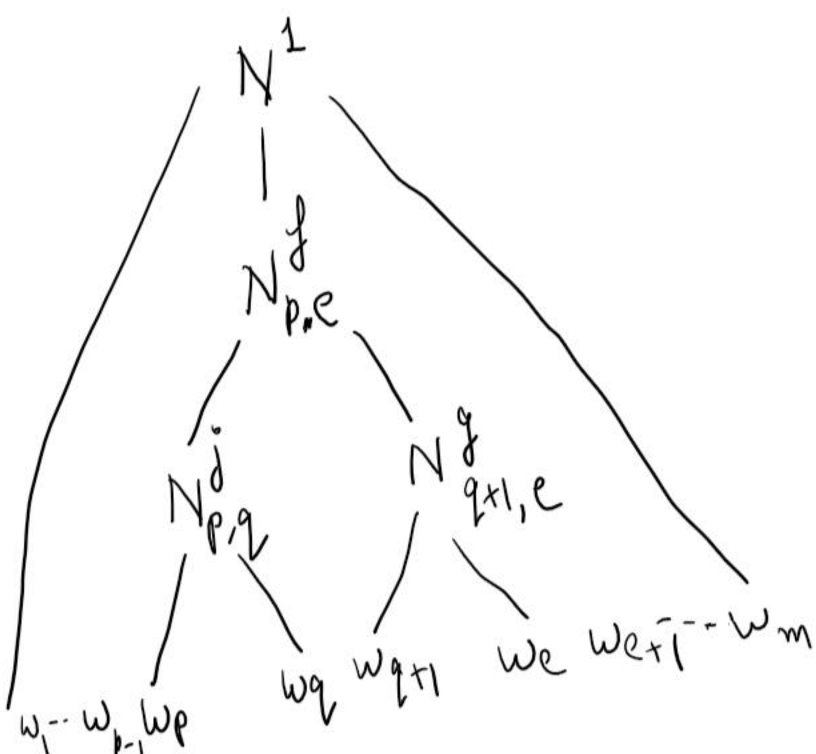
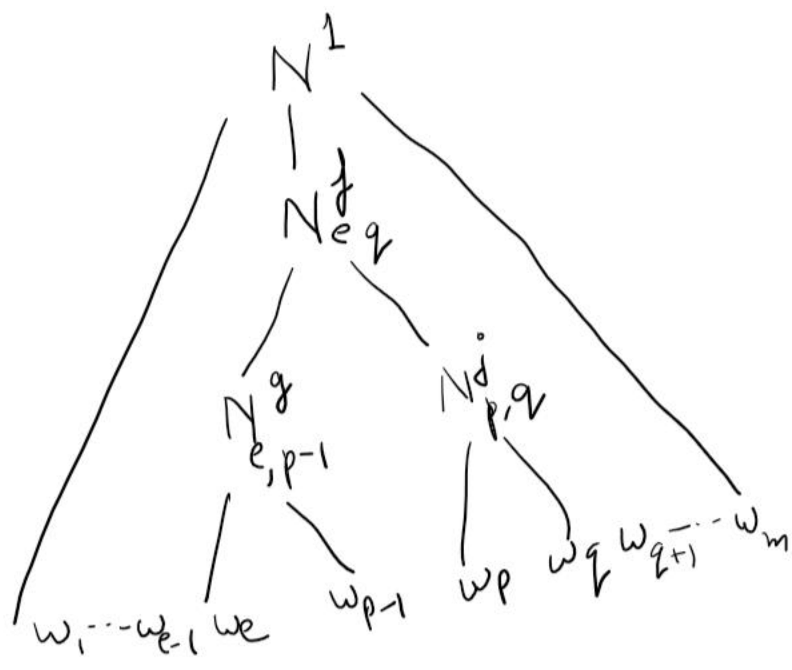




Outside probability

$$\alpha_j(p,q) = \sum_{f,g} \sum_{e=q+1}^{m} \alpha_f(p,e)\, P(N^f \to N^j N^g)\, \beta_g(q+1,e) +$$

$$\sum_{f,g} \sum_{e=1}^{p-1} \alpha_f(e,q)\, P(N^f \to N^g N^j)\, \beta_g(e,p-1)$$

$$\alpha_j(p,q)\,\beta_j(p,q) = P(w_{1(p-1)}, N^j_{p,q}, w_{q+1,m} \mid G) * P(w_{pq} \mid N^j_{p,q}\, G)$$

$$= P(w_{1m}, N^j_{p,q} \mid G)$$

$$P(w_{1m}, N_{pq} \mid G) = \sum \alpha_j(p,q)\,\beta_j(p,q)$$

$$= P(N_1 \to w_{1m},\ N_{pq} \to w_{pq} \mid G)$$

Problem: Consider the following PCFG:

S → N V 1.0	N → she 0.2
V → V NP 0.7	N → pizza 0.2
NP → N P 1.0	V → eats 0.3
P → PP N 1.0	PP → without 1.0
N → N P 0.4	N → anchovies 0.2

Sentence: She eats pizza without anchovies

Use the inside-outside probabilities to estimate the probability of the sentence?

Triangular table:

She	Eats	Pizza	Without	anchovies
N X11	S X12	S X13	Φ X14	S X15
	V X22	V X23	X24	V X25
		N X33	X34	NP, N X35
			PP X44	P X45
				N X55

X12= X11 X22 = N V = S

X23= X22 X33 = V N= V

X34= X33 X44 = N PP = Φ

X45= X44 X55 = PP N = P

X13=X11 X23, X12 X33 = N V, S N =S, Φ = S

CONTD…

X24 = X22 X34, X23 X44 = V Φ, V PP = Φ

X35 =X33 X45, X34 X55 = N P, Φ N = NP, N

X14= X11 X24, X12 X34, X13 X44 = Φ

X25=X22 X35, X23 X45, X24X55

= V NP, V N, V P, Φ N

= V

X15 = X11 X25, X12 X35, X13 X45, X14 X55

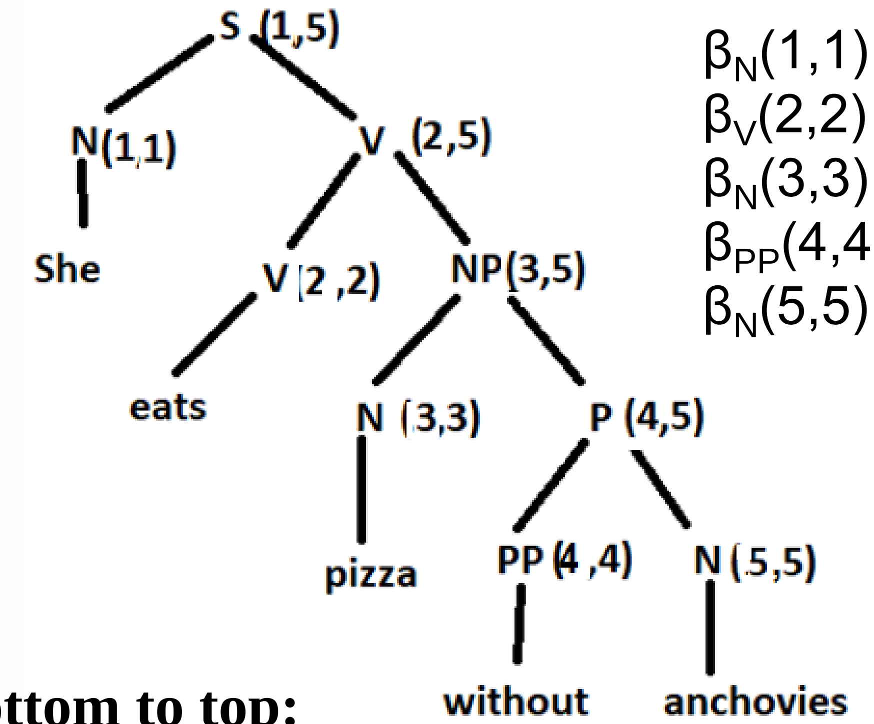
= N V, S NP, S N, S P

= S

Calculation of inside probabilities

Inside: $\beta_j(p,q) = P(w_{pq} | N^j{}_{pq}, G)$

Parse Tree:



$\beta_N(1,1) = P(N \rightarrow \text{She}) = 0.2$
$\beta_V(2,2) = P(V \rightarrow \text{eats}) = 0.3$
$\beta_N(3,3) = P(N \rightarrow \text{pizza}) = 0.2$
$\beta_{PP}(4,4) = P(PP \rightarrow \text{without}) = 1.0$
$\beta_N(5,5) = P(N \rightarrow \text{anchovies}) = 0.2$

Start from bottom to top:

$\beta_P(4,5) = P(P \rightarrow PP\ \ N) * \beta_{PP}(4,4) * \beta_N(5,5) = 1*1*0.2 = 0.2$
$\beta_{NP}(3,5) = P(NP \rightarrow N\ \ P) * \beta_N(3,3) * \beta_P(4,5) = 1*0.2*0.2 = 0.04$
$\beta_V(2,5) = P(V \rightarrow V\ \ NP) * \beta_V(2,2) * \beta_{NP}(3,5) = 0.7*0.3*0.04=0.0084$
$\beta_S(1,5) = P(S \rightarrow N\ \ V)*\beta_N(1,1)*\beta_V(2,5) = 1*0.2*0.0084 = 0.00168$

Calculation of outside probabilities

Outside: $\alpha_j(p,q) = P(w_{1(p-1)}, N^j{}_{pq}, w_{(q+1)m}|G)$

- **Outside probabilities** (**α**) are computed in top down manner. For a given rule used in building the table, the outside probability for each child is updated using the outside probability of its parent non terminal and inside probability of its siblings.

$\alpha_S(1,5) = 1$

$\alpha_V(2,5) = P(S \rightarrow N\ V) * \beta_N(1,1) * \alpha_S(1,5) = 1*0.2*1 = 0.2$

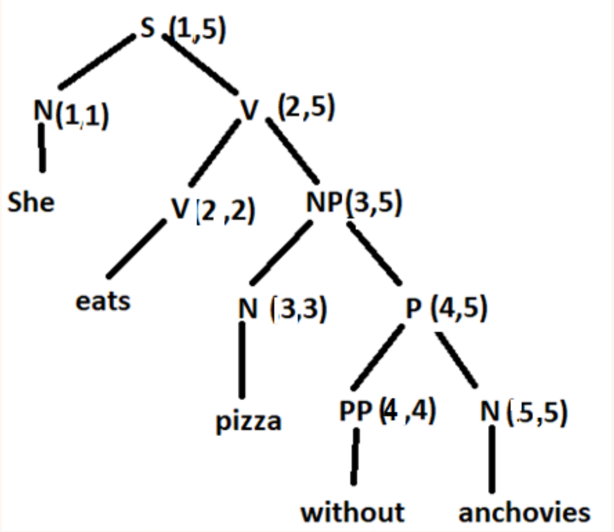
$\alpha_{NP}(3,5) = P(V \rightarrow V\ NP) * \beta_V(2,2) * \alpha_V(2,5)$

$= 0.7*0.3*0.2 \quad = \quad 0.042$

$\alpha_P(4,5) = P(NP \rightarrow N\ P) * \beta_N(3,3) * \alpha_{NP}(3,5)$

$= 1*0.2*0.042$

$= 0.0084$



Triangular table:

$\beta_N(1,1)=0.2$				$\alpha_S(1,5) = 1$
	$\beta_V(2,2)=0.3$			$\alpha_V(2,5) = 0.2$
		$\beta_N(3,3)=0.2$		$\alpha_{NP}(3,5)=0.042$
			$\beta_{PP}(4,4)=1$	$\alpha_P(4,5)= 0.0084$
				$\beta_N(5,5)=0.2$
She	**Eats**	**Pizza**	**Without**	**anchovies**

Top Down Bottom Up Parsing

for

Structurally ambiguous sentences

Top Down Bottom Up Chart Parsing for Structurally Ambiguous Sentences

- Sentence “I saw a boy with a telescope”
- Grammar:

S	→	NP VP
NP	→	ART N | ART N PP | PRON
VP	→	V NP PP | V NP
PP	→	P NP
ART	→	a | an | the
N	→	boy | telescope
PRON	→	I
V	→	saw
P	→	with

Chart Parsing

I	saw	a	boy	with	a	telescope

1 2 3 4 5 6 7 8

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12}VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12}N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12}N_{23}PP_{3?}$

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

NP

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

NP

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$

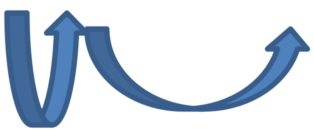
$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

$NP_{35} \rightarrow ART_{34} \bullet N_{45}$

$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$



NP

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

$NP_{35} \rightarrow ART_{34} \bullet N_{45}$

$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$

$NP_{35} \rightarrow ART_{34} N_{45} \bullet$

$NP_{3?} \rightarrow ART_{34} N_{45} \bullet PP_{5?}$

$PP_{5?} \rightarrow \bullet P_{56} NP_{6?}$

NP

Chart Parsing

	I	saw	a	boy	with	a	telescope
1	2	3	4	5	6	7	8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

$NP_{35} \rightarrow ART_{34} \bullet N_{45}$

$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$

$NP_{35} \rightarrow ART_{34} N_{45} \bullet$

$NP_{3?} \rightarrow ART_{34} N_{45} \bullet PP_{5?}$

$PP_{5?} \rightarrow \bullet P_{56} NP_{6?}$

NP

NP

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

$NP_{35} \rightarrow ART_{34} \bullet N_{45}$

$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$

$NP_{35} \rightarrow ART_{34} N_{45} \bullet$

$NP_{3?} \rightarrow ART_{34} N_{45} \bullet PP_{5?}$

$PP_{5?} \rightarrow \bullet P_{56} NP_{6?}$

$PP_{5?} \rightarrow P_{56} \bullet NP_{6?}$

$NP_{67} \rightarrow \bullet PRON_{67}$

$NP_{68} \rightarrow \bullet ART_{67} N_{78}$

$NP_{6?} \rightarrow \bullet ART_{67} N_{78} PP_{8?}$

NP

NP

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

$NP_{35} \rightarrow ART_{34} \bullet N_{45}$

$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$

$NP_{35} \rightarrow ART_{34} N_{45} \bullet$

$NP_{3?} \rightarrow ART_{34} N_{45} \bullet PP_{5?}$

$PP_{5?} \rightarrow \bullet P_{56} NP_{6?}$

$PP_{5?} \rightarrow P_{56} \bullet NP_{6?}$

$NP_{67} \rightarrow \bullet PRON_{67}$

$NP_{68} \rightarrow \bullet ART_{67} N_{78}$

$NP_{6?} \rightarrow \bullet ART_{67} N_{78} PP_{8?}$

$NP_{68} \rightarrow ART_{67} \bullet N_{78}$

$NP_{6?} \rightarrow ART_{67} \bullet N_{78} PP_{8?}$

NP

NP

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12}VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12}N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12}N_{23}PP_{3?}$

$NP_{12} \rightarrow PRON_{12}\bullet$

$S_{1?} \rightarrow NP_{12}\bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23}NP_{3?}PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23}NP_{3?}$

$VP_{2?} \rightarrow V_{23}\bullet NP_{3?}PP_{??}$

$VP_{2?} \rightarrow V_{23}\bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34}N_{45}PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34}N_{45}$

$NP_{35} \rightarrow ART_{34}\bullet N_{45}$

$NP_{3?} \rightarrow ART_{34}\bullet N_{45}PP_{5?}$

$NP_{35} \rightarrow ART_{34}N_{45}\bullet$

$NP_{3?} \rightarrow ART_{34}N_{45}\bullet PP_{5?}$

$PP_{5?} \rightarrow \bullet P_{56}NP_{6?}$

$PP_{5?} \rightarrow P_{56}\bullet NP_{6?}$

$NP_{67} \rightarrow \bullet PRON_{67}$

$NP_{68} \rightarrow \bullet ART_{67}N_{78}$

$NP_{6?} \rightarrow \bullet ART_{67}N_{78}PP_{8?}$

$NP_{68} \rightarrow ART_{67}\bullet N_{78}$

$NP_{6?} \rightarrow ART_{67}\bullet N_{78}PP_{8?}$

$NP_{68} \rightarrow ART_{67}N_{78}\bullet$

$PP_{5?} \rightarrow P_{56}NP_{6?}\bullet$

NP

NP

NP

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

$NP_{35} \rightarrow ART_{34} \bullet N_{45}$

$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$

$NP_{35} \rightarrow ART_{34} N_{45} \bullet$

$NP_{3?} \rightarrow ART_{34} N_{45} \bullet PP_{5?}$

$PP_{5?} \rightarrow \bullet P_{56} NP_{6?}$

$PP_{5?} \rightarrow P_{56} \bullet NP_{6?}$

$NP_{67} \rightarrow \bullet PRON_{67}$

$NP_{68} \rightarrow \bullet ART_{67} N_{78}$

$NP_{6?} \rightarrow \bullet ART_{67} N_{78} PP_{8?}$

$NP_{68} \rightarrow ART_{67} \bullet N_{78}$

$NP_{6?} \rightarrow ART_{67} \bullet N_{78} PP_{8?}$

$NP_{68} \rightarrow ART_{67} N_{78} \bullet$

$PP_{5?} \rightarrow P_{56} NP_{6?} \bullet$

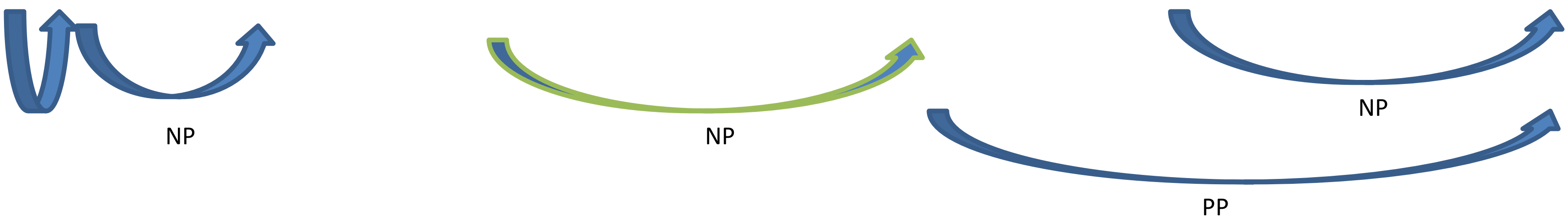


Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$

$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$

$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$

$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

$NP_{35} \rightarrow ART_{34} \bullet N_{45}$

$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$

$NP_{35} \rightarrow ART_{34} N_{45} \bullet$

$NP_{3?} \rightarrow ART_{34} N_{45} \bullet PP_{5?}$

$PP_{5?} \rightarrow \bullet P_{56} NP_{6?}$

$PP_{5?} \rightarrow P_{56} \bullet NP_{6?}$

$NP_{67} \rightarrow \bullet PRON_{67}$

$NP_{68} \rightarrow \bullet ART_{67} N_{78}$

$NP_{6?} \rightarrow \bullet ART_{67} N_{78} PP_{8?}$

$NP_{68} \rightarrow ART_{67} \bullet N_{78}$

$NP_{6?} \rightarrow ART_{67} \bullet N_{78} PP_{8?}$

$NP_{68} \rightarrow ART_{67} N_{78} \bullet$

$PP_{5?} \rightarrow P_{56} NP_{6?} \bullet$

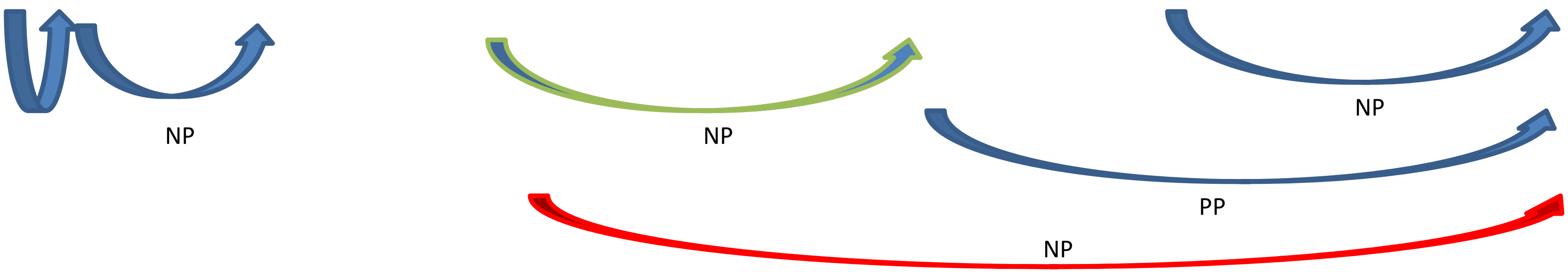


Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$

$NP_{12} \rightarrow \bullet PRON_{12}$

$NP_{13} \rightarrow \bullet ART_{12} N_{23}$

$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$

$NP_{12} \rightarrow PRON_{12} \bullet$

$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$

$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$

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$NP_{35} \rightarrow \bullet ART_{34} N_{45}$

$NP_{35} \rightarrow ART_{34} \bullet N_{45}$

$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$

$NP_{35} \rightarrow ART_{34} N_{45} \bullet$

$NP_{3?} \rightarrow ART_{34} N_{45} \bullet PP_{5?}$

$PP_{5?} \rightarrow \bullet P_{56} NP_{6?}$

$PP_{5?} \rightarrow P_{56} \bullet NP_{6?}$

$NP_{67} \rightarrow \bullet PRON_{67}$

$NP_{68} \rightarrow \bullet ART_{67} N_{78}$

$NP_{6?} \rightarrow \bullet ART_{67} N_{78} PP_{8?}$

$NP_{68} \rightarrow ART_{67} \bullet N_{78}$

$NP_{6?} \rightarrow ART_{67} \bullet N_{78} PP_{8?}$

$NP_{68} \rightarrow ART_{67} N_{78} \bullet$

$PP_{5?} \rightarrow P_{56} NP_{6?} \bullet$

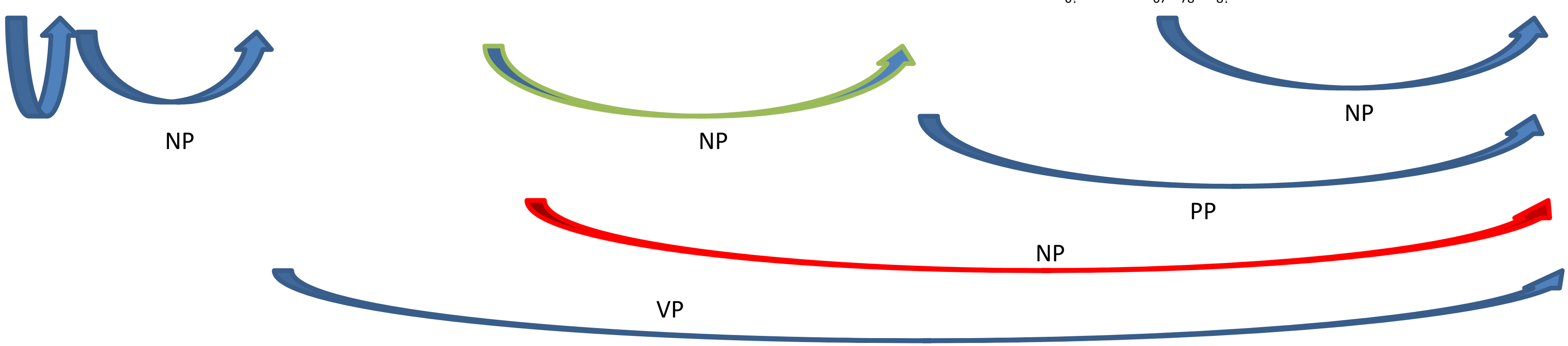

Chart Parsing

I saw a boy with a telescope

1 2 3 4 5 6 7 8

$S_{1?} \rightarrow \bullet NP_{12} VP_{2?}$	$NP_{12} \rightarrow PRON_{12} \bullet$	$VP_{2?} \rightarrow V_{23} \bullet NP_{3?} PP_{??}$	$NP_{35} \rightarrow ART_{34} \bullet N_{45}$	$NP_{35} \rightarrow ART_{34} N_{45} \bullet$	$PP_{5?} \rightarrow P_{56} \bullet NP_{6?}$	$NP_{68} \rightarrow ART_{67} \bullet N_{78}$	$NP_{68} \rightarrow ART_{67} N_{78} \bullet$
$NP_{12} \rightarrow \bullet PRON_{12}$	$S_{1?} \rightarrow NP_{12} \bullet VP_{2?}$	$VP_{2?} \rightarrow V_{23} \bullet NP_{3?}$	$NP_{3?} \rightarrow ART_{34} \bullet N_{45} PP_{5?}$	$NP_{3?} \rightarrow ART_{34} N_{45} \bullet PP_{5?}$	$NP_{67} \rightarrow \bullet PRON_{67}$	$NP_{6?} \rightarrow ART_{67} \bullet N_{78} PP_{8?}$	$PP_{5?} \rightarrow P_{56} NP_{6?} \bullet$
$NP_{13} \rightarrow \bullet ART_{12} N_{23}$	$VP_{2?} \rightarrow \bullet V_{23} NP_{3?} PP_{??}$	$NP_{3?} \rightarrow \bullet ART_{34} N_{45} PP_{5?}$		$PP_{5?} \rightarrow \bullet P_{56} NP_{6?}$	$NP_{68} \rightarrow \bullet ART_{67} N_{78}$		
$NP_{1?} \rightarrow \bullet ART_{12} N_{23} PP_{3?}$	$VP_{2?} \rightarrow \bullet V_{23} NP_{3?}$	$NP_{35} \rightarrow \bullet ART_{34} N_{45}$			$NP_{6?} \rightarrow \bullet ART_{67} N_{78} PP_{8?}$