Ex. The inter arrival and the service time distribution of a queing system are given below. Estimate the average queue length and the average waiting time of the customers in the queue, by using Monte Carlo Simulation technique. Simulate for a period of time en compassing ten arrivals.

Inter arrival time		Service time	
Minutes	Probability	Minutes	Probability
2	0.15	1	0.10
4	0.23	3	0.22
6	0.35	5	0.35
8	0.17	7	0.23
10	0.10	9	0.10

Use the following random numbers :

i) For inter-arrival
48, 55, 91, 40, 93, 01, 83, 63, 47, 52.

ii) For service time :
47, 36, 57, 04, 79, 55, 10, 13, 57, 09.

Solution: Calculate cummulative probability and assign range of 2-digit random numbers 00 to 99 for interarrival time and service time.

Interarrival time.				Service time.			
Minutes	Prob.	Cum. Prob.	Random No.	Minutes	Prob.	Cummul. prob.	Random No.
2	0.15	0.15	00 - 14	1	0.10	0.10	00 - 09
4	0.23	0.38	15 - 37	3	0.22	0.32	09 - 31
6	0.35	0.73	38 - 72	5	0.35	0.67	32 - 66
8	0.17	0.90	73 - 89	7	0.23	0.90	67 - 89
10	0.10	1.00	90 - 99.	9	0.10	1.00	90 - 99.

Determine interarrival time & service time using given a set of random numbers for 10 arrivals.

Simmulation Run	1	2	3	4	5	6	7	8	9	10
Random No.	48	55	91	40	93	01	83	63	47	52
Inter arrival time (min.)	6	6	10	6	10	2	8	6	6	6

Simmulation Run	1	2	3	4	5	6	7	8	9	10
Random No.	47	36	57	04	79	55	10	13	57	09
Service time (min.)	5	5	5	1	7	5	3	3	5	1

Prepare a table to estimate queue length and waiting ~~length~~ time of customers for 10 arrivals.

Simulation Run	Inter Arrival time	Service time	Arrive time IN min.	Service Begins	Service Ends	waiting time of custom	System Idle time
1	6	5	6	6	11	0	6
2	6	5	12	12	17	0	1
3	10	5	22	22	27	0	5
4	6	1	28	28	29	0	1
5	10	7	38	38	45	0	9
6	2	5	40	45	50	5	—
7	8	3	48	50	53	2	—
8	6	3	54	54	57	0	1
9	6	5	60	60	65	0	3
10	6	1	66	66	67	0	1
						$\Sigma = 7$	$\Sigma = 27$

No. of waiting customers = 2

Avg. que length = 2/10 = 0.2 ✓

Avg. waiting time = $\frac{7}{10}$ = 0.7 min ✓

Avg waiting ... = 10 ... = $\frac{10}{...}$ = $\frac{1}{6}$ person/min

Problem:

The occurence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous day, the rain distribution is given by:

Event	Probability
No rain	0.50
1 cm rain	0.25
2 cm rain	0.15
3 cm rain	0.05
4 cm rain	0.03
5 cm rain	0.02

If it did not rain the rain distribution is given by:

Event	Probability
No rain	0.75
1cm. rain	0.15
2cm rain	0.06
3 cm rain	0.04

Simulate the city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random numbers:

67 63 39 55 29 78 70 06 78 76

for simulation. Assume that for the first day of the simulation it had not rained the day before.

Sol^n: The Ran. No's 00-99 are allocated in proportion to the probabilites associated with each event. If it rained on the previous day, the rain distribution and the random numb allocation are given below:

Event	Probability	Cum. Prob.	Random Number interval
No rain	0.50	0.50	00 - 49.
1cm rain	0.25	0.75	50 - 74
2cm rain	0.15	0.90	75 - 89
3cm rain	0.05	0.95	90 - 94
4 cm rain	0.03	0.98	95 - 97

Similarly, if it did not rain the previous day, the necessary distribution and the random number allocation is given below :

Event	Probability	Cum. Prob.	Random number interval.
No rain	0.75	0.75	00 - 74
1cm rain	0.15	0.90	75 - 89
2cm rain	0.06	0.96	90 - 95
3cm rain	0.04	1.00	96 - 99.

Let us now simulate the rainfall for 10 days using the given random numbers. For the first day it is assumed that it had not rained the day before :.

Day	Random Numbers	Event
1	67	No rain
2	63	No rain
3	39	No rain
4	55	No rain
5	29	No rain
6	78	1cm rain
7	70	1cm rain
8	06	No rain.
9	78	1cm rain
10	76	2 cm rain.

Hence during the simulated period, it did not rain on 6 days out of 10. The total rain fall during the period was 5 cm.

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