Simulation

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Before we start…

- Probability sum never exceeds one
- Simulation uses past data for solving the problem.
- Simulation uses random numbers for solving the problem.
- Random number is a number in a sequence of numbers whose probability of occurrence is the same as that of any other number in that sequence. (Random number set or random number chart is supplied for solving the problem.)
- Simulation is approximation, but any problem can be solved having past data

You should know….

1. What do understand by simulation ? Explain in brief.
2. State the various advantages of simulation.
3. State the various limitations of simulation
4. Explain Monte-Carlo simulation technique and state the various steps involved in it.

SIMULATION

- Simulation is the process of designing a model of a real system and conducting experiments with the model for the purpose of understanding behavior for the operation of the system.
- A duplication of the original system
- To understand the implementation of the system
- The method can be used to for solving variety of situations such as inventory problem, queuing problem, financial decisions etc.

ADVANTAGES OF SIMULATION

- Simulation is a widely accepted technique of operations research due to the following reasons:

* It is straightforward and flexible.

* It can be used to analyze large and complex real world situations that cannot be solved by conventional quantitative analysis models.

* It is the only method sometimes available.

* It studies the interactive effect of individual components or variables in order to determine which ones are important.

- Simulation model, once constructed, may be used over and again to analyze all kinds of different situations.
- It is the valuable and convenient method of breaking down a complicated system into subsystems and their study. Each of these subsystems works individually or jointly with others.

LIMITATIONS OF SIMULATION

* Since simulation model mostly deals with uncertainties, the results of simulation are only reliable approximations involving statistical errors, optimum results cannot be produced by simulation.
* In many situations, it is not possible to identify all the variables, which affect the behaviour of the system.
* In very large and complex problems, it is very difficult to make the computer program in view of the large number of variables and the involved inter–relationship among them.
* For problems requiring the use of computer, simulation may be comparatively costlier and time consuming in many cases.
* Each solution model is unique and its solutions and inferences are not usually transferable to other problems, which can be solved by other techniques.

MONTE-CARLO SIMULATION

The Monte-Carlo method is a simulation technique in which statistical distribution functions are created using a series of random numbers.

The Monte-Carlo simulation procedure can be summarized in the following steps:

Step 1: Clearly define the problem:

(*a) Identify the objectives of the problem.*

(*b) Identify the main factors, which have the greatest effect on the objective of the problem.*

MONTE-CARLO SIMULATION

Step 2: Construct an approximate model:

(*a) Specify the variables and parameters of the mode.*

(*b) Formulate the appropriate decision rules, i.e. state the conditions under which the experiment* is to be performed.

(*c) Identify the type of distribution that will be used. Models use either theoretical distributions* or empirical distributions to state the patterns of the occurrence associated with the variables.

(*d) Specify the manner in which time will change.*

While solving simulation problem…

1. The past data summary may be given in the form of probabilities of occurrence for which sum of all probabilities is equal to one or frequencies.
2. In case past data is given in terms of frequencies then convert frequencies into probabilities just dividing individual frequency by the sum of frequencies.
3. Probability or frequency is for the variable, normally on which the output or problem solution depends.
4. Find the cumulative probability for the variable, where we will get the sum of probability as one.

While solving simulation problem…

5. Allocate random numbers

   We will be using two digit random numbers i.e. 00 to 99 (total 100). The probability values will be two digits after the decimal such as 0.24, 0.30 i.e. 0.01 to 1.00 (total 100).

   while allocating random numbers with cumulative probability values, it is required to match 100 random numbers to 100 probability values for example

Prob.	**0.01**	**0.34**	**0.50**	**0.67**	**0.87**	**1.00**
Random Number	00	33	49	66	86	99

While solving simulation problem…

6. The total number of random numbers provided indicates number of trials expected.
7. Depending on the output expected design suitable or appropriate tabular arrangement so that it can be calculated by fixing the variable using random numbers.

Simulation 1

Here the variable is demand/week. It is needed to calculate the average demand which can be calculated by dividing the sum of demands by the number of weeks.

As the data is provided in the form of frequencies, let us convert it into corresponding probability as below

For example 2 /50=0.04

Demand/week	**0**	**5**	**10**	**15**	**20**	**25**
Frequency	2	11	8	21	5	3
Probabilty	0.04	0.22	0.16	0.42	0.10	0.06

Simulation 1

Following is the past data for 50 weeks for a confectionary item. By using the random numbers provided below, calculate the average demand.

Demand/week	**0**	**5**	**10**	**15**	**20**	**25**
Frequency	2	11	8	21	5	3

Random numbers:

35, 52, 13, 90, 23, 73, 34, 57, 35, 83

Simulation 1

Demand/ week	Proba- bility	Cumulative probability	Random numbers allocated
0	0.04	0.04	00-03
5	0.22	0.26	04-25
10	0.16	0.42	26-41
15	0.42	0.84	42-83
20	0.10	0.94	84-93
25	0.06	1.00	94-99

Simulation 1

Week	Random No.	Demand
1	35	10
2	52	15
3	13	5
4	90	20
5	23	5
6	73	15
7	34	10
8	57	15
9	35	10
10	83	15
Total demand		**120**

Simulation 1

Average demand= sum of demands/no. of weeks

= 120/10= **12**

Note:

During week 1 the random number is 35. Random number 35 is in the range 26-41, for which the corresponding demand/week is 10

Similarly, During week 3 the random number is 13. Random number 13 is in the range 04-25, for which the corresponding demand/week is 5

In this way all the corresponding demands are calculated.

Simulation 2

The bakery shop keeps stock of a popular brand of cake. Previous experience indicates the daily demand as below

Daily demand	**0**	**15**	**25**	**35**	**45**	**50**
Probability	0.01	0.15	0.20	0.50	0.12	0.02

Consider the following sequence of random numbers:

21, 27, 47, 54, 60, 39, 43, 91, 25, 20

Simulate the demand for the next 10 days. Find out the stock situation if the owner of the bakery shop decides to make 30 cakes every day. Calculate average demand, total unsold and shortage cakes as per production policy.

Simulation 2

Let us first allocate the random numbers

Demand/ week	Proba-bility	Cumulative probability	Random numbers allocated
0	0.01	0.01	00
15	0.15	0.16	01-15
25	0.20	0.36	16-35
35	0.50	0.86	36-85
45	0.12	0.98	86-97
50	0.02	1.00	98-99

Simulation 2

Day	Random No.	Demand	Daily production 30 cakes	
			Unsold cakes	Shortage
1	21	25	5	-
2	27	25	5	-
3	47	35	-	5
4	54	35	-	5
5	60	35	-	5
6	39	35	-	5
7	43	35	-	5
8	91	45	-	15
9	25	25	5	-
10	20	25	5	-
	Total	320	20	40

Simulation 1

Average demand= sum of demands/no. of weeks

= 320/10= **32 cakes**

Total unsold cakes= 20 cakes

Total shortage of cakes= 40 cakes

Note:

In the same problem if the cost of manufacturing cake is given as Rs 100 and selling price is Rs 150 and further it is assumed that unsold cake on the same day is loss then it is possible to calculate loss or profit and average profit. Also given the cost of loss of sale due to shortage as Rs 10

Profit per cake sold = Rs 50

Loss per unsold cake = Rs 100

Simulation 1

The table can be modified further to calculate

Profit/loss= 50XNo. Of cakes sold-100XNo. Of unsold cakes-10X No. of shortage cakes

Day 1 – Demand =25 cakes

Profit/loss= 50X25-100X5 = Rs 750

Day 8 – Demand = 45 Cakes

Profit/loss= 50X30-10X15 = Rs 1350

Average profit= Total profit/No. of days

= 11600/10 = Rs 1160

Simulation 2

Day	R. No.	Demand	Daily production 30 cakes		Profit/Loss
			Unsold cakes	Shortage	
1	21	25	5	-	750
2	27	25	5	-	750
3	47	35	-	5	1450
4	54	35	-	5	1450
5	60	35	-	5	1450
6	39	35	-	5	1450
7	43	35	-	5	1450
8	91	45	-	15	1350
9	25	25	5	-	750
10	20	25	5	-	750
	Total	320	20	40	11600

Simulation 3

An automobile company manufactures around 150 bikes . The daily production varies from 146 to 154. The finished bikes are transported in a truck accommodating 150 bikes. Simulate average number of bikes waiting for shipment and average number of empty spaces in truck.

Use following random numbers

80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57

Production/ day	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**
Probability	0.04	0.09	0.12	0.14	0.11	0.10	0.20	0.12	0.08

Simulation 3

Let us first allocate the random numbers

Production/day	**Probability**	**Cumulative probability**	**Random numbers allocated**
146	0.04	0.04	00-03
147	0.09	0.13	04-12
148	0.12	0.25	13-24
149	0.14	0.39	25-38
150	0.11	0.50	39-49
151	0.10	0.60	50-59
152	0.20	0.80	60-79
153	0.12	0.92	80-91
154	0.08	1.00	92-99

Day	R. No.	Production	Unshipped bikes	Empty space in truck
1	80	153	3	-
2	81	153	3	-
3	76	152	2	-
4	75	152	2	-
5	64	152	2	-
6	43	150	-	-
7	18	148	-	2
8	26	149	-	1
9	10	147	-	3
10	12	147	-	3
11	65	152	2	-
12	68	152	2	-
13	69	152	2	-
14	61	152	2	-
15	57	151	1	-
		Total	21	9

Simulation 3

Average number of bikes waiting for shipment = 21/15

= 1.4 bikes/day

Average empty spaces in truck= 9/15

= 0.6 bikes/day

Simulation 4

The following is the arrival and service pattern observed at a petrol station. Calculate the average waiting time per customer and idle time for the pump. Assume that petrol pump opens at 8.00 am

Random Nos. for arrival

78, 26,94, 08, 46, 63, 18, 35, 59, 12, 97, 82

Random Nos. for service

44, 21, 73, 96, 63, 35, 57, 31, 84, 24, 05, 37

Arrivals		**Services**	
Inter arrival time (min)	Prob.	Service time (min)	Prob.
2	0.22	4	0.28
4	0.30	6	0.40
6	0.24	8	0.22
8	0.14	10	0.10
10	0.10		

Simulation 4

Let us first allocate the random numbers

Arrival time (min)	Prob.	Cum Prob.	R. No. allocated	Service time (min)	Prob.	Cum Prob.	R. No. allocated
2	0.22	**0.22**	**00-21**	4	0.28	**0.28**	**00-27**
4	0.30	**0.52**	**22-51**	6	0.40	**0.68**	**28-67**
6	0.24	**0.76**	**52-75**	8	0.22	**0.90**	**68-89**
8	0.14	**0.90**	**76-89**	10	0.10	**1.00**	**90-99**
10	0.10	**1.00**	**90-99**				

Customer No.	R. No.	Arrival time	Entry in queue	Service starts	R. No.	Service time	Service ends	Customer waiting time	Pump idle time
1	78	8	8.08	8.08	44	6	8.14	-	8
2	26	4	8.12	8.14	21	4	8.18	2	-
3	94	10	8.22	8.22	73	8	8.30	-	4
4	08	2	8.24	8.30	96	10	8.40	6	-
5	46	4	8.28	8.40	63	6	8.46	12	-
6	63	6	8.34	8.46	35	6	8.52	12	-
7	18	2	8.36	8.52	57	6	8.58	16	-
8	35	4	8.40	8.58	31	6	9.04	18	-
9	59	6	8.46	9.04	84	8	9.12	18	-
10	12	2	8.48	9.12	24	4	9.16	24	-
11	97	10	8.58	9.16	05	4	9.20	18	-
12	82	8	9.06	9.20	37	6	9.26	14	-
								140	12

Simulation 4

Average waiting time per customer = 140/12 = 11.67 mins

Pump idle time = 12 mins