

## Assignment Problem (A.P.)

- Assignment cost/cost - Minimization Model.
- Allocation model.

Prob. 1 Reduced matrix method / Hungarian Method / + Hospital

A machine tool company decides to make four subassemblies through four contractors. Each contractor is to receive only one subassembly. The cost of each subassembly is determined by the bids submitted by each contractor as shown below in rupees. Assign the different subassemblies to contractors so as to minimize total cost.

		Contractors			
		1	2	3	4
Subassemblies	1	15	13	14	17
	2	11	12	15	13
	3	13	12	10	11
	4	15	17	14	16

Before we solve

- + Assignment matrix should be a square matrix
- + Balanced A.P.
- + Minimization Type. (cost, time, distance travelled)

Let us solve

1. Select the least cost from the row and subtract from entire row. so that at least one zero will be available in each row.

2. Examine the column, for at least one zero.

If not, then select the least cost/element from the respective column and subtract from the remaining element.

3. Assign wherever there is zero, such that it requires to cancel minimum number of zeros.

- Assigned zero
- Cancelled zero / marked zero

4. If each row as well each column consists of atleast and at most one assigned zero, then calculate total assignment cost by considering the values in original matrix of assigned zero.

### Solution

Subtracting minimum element/cost from each row from entire row we get -  
Contractor

	1	2	3	4
1	15	13	14	17
2	11	12	15	19
3	13	12	10	11
4	15	17	14	16

Sub assemblies

	1	2	3	4
1	2	0	1	4
2	0	1	4	2
3	3	2	0	1
4	1	3	0	2



Subtracting the least from 4<sup>th</sup> Column we get-

	1	2	3	4
1	2	0	1	3
2	0	1	4	1
3	3	2	X	0
4	1	3	0	1

As each row as well as each column consists of at least and at most assignment, the obtained solution is optimum.

Referring to the original cost matrix  
Minimum assignment cost -

$$Z_{\min} = [13 + 11 + 11 + 14] = \text{Rs } \underline{\underline{49}}$$

The optimum assignment policy is.

Subassembly	Contractor
1	- 2
2	- 1
3	- 4
4	- 3.

Prob 2

4

Four different jobs are to be done on four different machines. The matrix below gives time to complete ~~an~~ job on each machine. Assign jobs to different machines so that total time to complete all jobs can be minimized.

	A	B	C	D
1	5	7	11	6
2	8	5	9	6
3	4	7	10	7
4	10	4	8	3

Jobs

+ Square  
Mini

Solution

Substracting the least cost/time element from each row

	A	B	C	D
1	0	2	6	1
2	3	0	4	1
3	0	3	6	3
4	7	1	5	0

Substracting the least element for column n 3 we get -

	A	B	C	D
1	0	2	2	1
2	3	0	X	1
3	X	3	2	3
4	7	1	1	0

— I.B.F.S

(Initial Basic feasible  
solution)

As each row and each column does not contains at least one assigned zero, the obtained IBFS solution is not optimum.

### check for optimality

1. Mark <sup>(✓)</sup> the row(s) which does not contains assigned zero.
  2. Inspect the marked row for marked / canceled zero and mark <sup>(✓)</sup> the corresponding column.
  3. Inspect the marked column for assigned zero and mark <sup>(✓)</sup> the corresponding row.
  4. Repeat 1, 2, & 3 as required.
  5. Draw lines through marked column(s) and unmarked row(s).
  6. If the number of lines < order of matrix solution is not optimum. then improve the solution.
- 

	A	B	C	D	
A	0	2	2	1	✓
B	3	0	X	1	
C	X	3	2	3	
D	7	1	1	0	✓
	✓				

As 3 (no. of lines) < order of matrix 4.

## Improving the solution

1. Select the least from uncovered elements.
  2. Add at intersection and subtract from remaining uncovered
- 

The least from uncovered elements is 1, adding at intersection and subtracting from remaining uncovered we get -

1	0	1	1	∞
2	4	0	∞	1
3	∞	2	1	2
4	8	1	1	0

- S.B.F.S.  
(Second Basic Feasible Solution)

check for optimality -

	A	B	C	D	
1	0	1	1	∞	✓
2	4	0	∞	1	
3	∞	2	1	2	✓
4	8	1	1	0	✓

As no of lines (3) < Order of matrix (4)

S.B.F.S. is not optimum.

## Improving The solution

Selecting the least(1) from uncovered elements, adding it at intersection and subtracting from remaining uncovered we get -

	A	B	C	D
1	0	⊗	⊗	⊗
2	5	0	⊗	2
3	⊗	1	0	2
4	8	⊗	⊗	0

T.B.F.S

(Third Basic feasible  
solution)

As each row & column consists of an assignment  
Optimum solution is

$$Z_{\min} = [5 + 5 + 10 + 3] = 23 \checkmark$$

Optimum assignment is

Jobs	Machines
1	- A
2	- B
3	- C
4	- D

- A) Unbalanced assignment problem.
- B) Maximization assignment problem.
- C) Unbalanced & maximization A.P.
- D) Restricted A.P.
- E) Travelling salesman Problem (T.S.P.)

## Unbalanced minimization Problem

Prob 3

A company has four machines on which to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is as below. What are the jobs assignments which will minimize the cost:

		Machine			
		W	X	Y	Z
Job	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

Solution

Balancing the given matrix by adding dummy row  
we get -

	W	X	Y	Z
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22
dummy	0	0	0	0

Subtracting the least from each row, we get -

	w	x	y	z	
A	0	6	10	14	✓
B	0	5	9	11	✓
C	0	5	9	12	✓
Dummy	0	0	0	0	

1 B.F.S

Improving the solution

Subtracting the least (5) from remaining uncovered and adding at intersection we will get -

	w	x	y	z	
A	0	1	5	9	✓
B	0	0	4	6	✓
C	0	0	4	7	✓
D	5	0	0	0	
	✓	✓			

S.B.F.S.

As no of lines (3) < order of matrix (4)

S.B.F.S. nor optimum.

Improving the solution

Subtracting the least (4) -----

	w	x	y	z	
A	0	1	0	5	
B	0	0	0	2	
C	0	0	0	3	
Dummy	9	4	0	0	

T.B.F.S.

The optimum cost -

$$Z_{\min} = [18 + 13 + 19 + 0] = \underline{50}$$

The optimum assignment policy is.

Job

A - W

B - X

C - Y

#### Prob 4 - Maximization Assignment problem

A department has four subordinates and four tasks to be performed. The subordinates differ in efficiency. The estimates of the profit in rupees each subordinate would earn is given in the matrix below. How should the tasks be allocated so as to maximize the total earnings?

Task

	A	B	C	D	
Subordinate	1	5	40	20	5
2	25	35	30	25	
3	15	25	20	10	
4	15	5	30	15	

#### Solution -

Maximization & Balanced.

For converting maximization to minimization, subtracting each element from the highest element from the matrix.

The highest element is 40, subtracting each element from it, we get -

	A	B	C	D
1	35	0	20	35
2	15	5	10	15
3	25	15	20	30
4	25	35	10	25

Subtracting the least from each row we get -

	A	B	C	D
1	35	0	20	35
2	10	0	5	10
3	10	0	5	15
4	15	25	0	15

Subtracting the least element from Col. A & D we get -

	A	B	C	D
1	25	0	20	25
2	0	0	5	0
3	0	0	5	5
4	5	25	0	5

Referring to original matrix.

$$Z_{\max} = [40 + 25 + 15 + 30] = 110$$

Optimum assignment -

Subordinate	Task
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1	- B
---	-----

2	- D
---	-----

3	- C
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## Prob 5 - Maximization and Unbalanced

Solve the following assignment problem for maximization

		Task			
		A	B	C	D
Subordinate	1	5	40	20	5
	2	25	35	30	25
	3	15	25	20	10

### Solution

Balancing by introducing dummy row, we get -

	A	B	C	D
1	5	40	20	5
2	25	35	30	25
3	15	25	20	10
Dummy	0	0	0	0

Subtracting all the elements from the highest element (40), so as to convert maximization to minimization we get -

	A	B	C	D
1	35	0	20	35
2	15	5	10	15
3	25	15	20	30
Dummy	40	40	40	40

Subtracting the least from each row, we get -

A	B	C	D	
35	0	20	35	✓
10	✗	5	10	✓
10	✗	5	15	✓
0	✗	✗	✗	

- I.B.F.S

Improving The solution we get -

30	0	15	30	✓
5	✗	0	5	✓
5	✗	✗	10	✓
0	5	✗	✗	

S.B.F.S.

Improving The solution  
Task

	A	B	C	D
1	25	0	15	25
2	✗	✗	0	✗
3	0	✗	✗	5
Dummy	✗	10	5	0

$$Z_{\max} = [40 + 30 + 15] = 85 \quad \checkmark$$

Optimum Assignment -

Subordinate      Task

1	-	B
2	-	C

Prob6 - Restricted A.P.

four new machines  $M_1, M_2, M_3$  and  $M_4$  are to be installed in a machine shop. There are five vacant places A, B, C, D & E available. Because of limited space, machine  $M_2$  cannot be placed at C and  $M_3$  cannot be placed at A. The following is the corresponding cost matrix. find the optimum assignment schedule.

	A	B	C	D	E
$M_1$	4	6	10	5	6
$M_2$	7	4	-	5	4
$M_3$	-	6	9	8	2
$M_4$	9	3	7	2	3

Solution:-

Balancing AP by introducing dummy row, we get -

(Minimization & restricted)

$M_1$	4	6	10	5	6
$M_2$	7	4	oo	5	4
$M_3$	oo	6	9	6	2
$M_4$	9	3	7	2	3
Dummy	0	0	0	0	0

Subtracting ~~repeat~~ elements we get -

	A	B	C	D	E
M <sub>1</sub>	0	2	6	1	2
M <sub>2</sub>	3	0	0	1	0
M <sub>3</sub>	0	4	7	4	0
M <sub>4</sub>	7	1	5	0	1
Dummy	0	0	0	0	0

$$Z_{\min} = (4 + 4 + 2 + 2) = 12$$

Machine                          Place

M<sub>1</sub> — A

M<sub>2</sub> — B

M<sub>3</sub> — E

M<sub>4</sub> — C

Theory questions to be prepared.

1. Write mathematical formulation for assignment problem.
2. State and explain similarities and differences in between assignment problem and transportation problem.
3. Write short note - Travelling salesman problem.

### Prob7 :- Travelling Salesman Problem (TSP)

A salesman wants to visit - A, B, C, D and E. He doesn't want to visit any city twice before completing his tour of all the cities and wishes to return to the point of starting journey. Cost of going from one city to another is shown below. Find the least cost route.

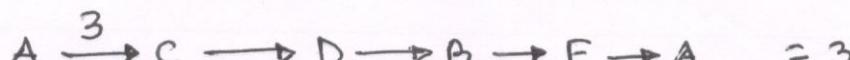
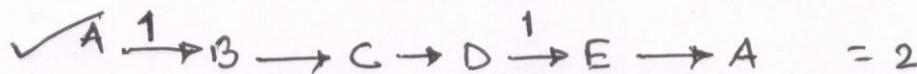
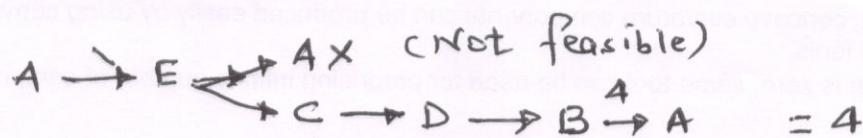
	A	B	C	D	E
A	-	2	5	7	1
B	6	-	3	8	2
C	8	7	-	4	7
D	12	4	6	-	5
E	1	3	2	8	-

	A	B	C	D	E
A	-	1	4	6	0
B	4	-	1	6	0
C	4	3	-	0	3
D	8	0	2	-	1
E	0	2	1	7	-

	A	B	C	D	E
A	-	1	3	6	0
B	4	-	0	6	0
C	4	3	-	0	3
D	8	0	1	-	1
E	0	2	0	7	-

Reduced cost matrix with zero in each row

Reduced cost matrix with zero in each column



The resulting feasible solution is

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow A$$

and it involves cost of

$$(2+3+4+5+1) = \underline{\text{Rs } 15}$$