

Soln: X company has taken the construction of 3rd floor for rent with a view to locate one of their zonal offices. There are 5 main rooms in this floor to be assigned to 5 managers. Each room has its own advantages and disadvantages. The rooms are of different sizes & shapes. Each of the 5 managers were asked to rank their room preference amongst the rooms 301-305. These preferences were given in tables. Most of the managers did not list all the rooms since they were not satisfied with some of these rooms. Assuming that these preferences can be certified by numbers. Find at which manager should be assigned to which room so that their total preference ranking is minimum.

Table 1
Managers

M ₁	M ₂	M ₃	M ₄	M ₅
302	302	303	302	301
303	304	301	305	302
304	305	304	304	304
	301	305	303	
		302		

Solⁿ:

Ranking Matrix

	M ₁	M ₂	M ₃	M ₄	M ₅
301	∞	4	2	∞	1
302	1	1	5	1	2
303	2	∞	1	4	∞
304	3	2	3	3	3
305	∞	3	4	2	∞

✓	∞	3	1	∞	1
✓	1	0	4	0	1
✓	1	∞	1	3	∞
✓	2	1		1	1
✓	∞		2	1	∞

Managers
M₁
M₂
M₃
M₄
M₅

Room No.
302
304
303
305
301

OR Assignment

	301	302	303	304	305
M ₁	∞	①	2	3	∞
M ₂	4	①	∞	2	3
M ₃	2	5	①	3	4
M ₄	∞	①	4	3	2
M ₅	①	2	∞	3	∞

∞	0	1	2	∞
3	0	∞	①	2
1	4	0	2	3
∞	0	3	2	1
0	1	∞	2	∞

	301	302	303	304	305	Maxed. Room
✓	∞	①	2	∞	✓	M ₁ → 302
✓	3	0	∞	①	2	M ₂ → 304
✓	1	4	0	2	3	M ₃ → 303
✓	∞	①	4	3	2	M ₄ → 305
✓	①	2	∞	3	∞	M ₅ → 301

Multiple optimal solution

A company has four jobs, the following matrix show the cost of assigning each job to each machine.

		m/c			
		M ₁	M ₂	M ₃	M ₄
Jobs	W ₁	40	50	60	65
	W ₂	30	38	46	48
	W ₃	25	33	41	43
	W ₄	39	45	51	59

How should the tasks be assigned to the m/c's to minimize total cost?

Row op.	0	10	20	25
	0	8	16	18
	0	8	16	18
	0	6	12	20
	0	6	12	20

Coln op.	0	4	8	7
0	0	2	4	0
0	0	2	4	0
0	0	2	4	0
0	0	2	4	0

Reduced matrix 1	0	2	6	7
	0	0	2	0
	0	0	2	0
	0	0	2	0
	2	0	0	4

Cancelled once, as it is
Cancelled twice add 2
Not cancelled, subtr 2

0	2	6	7
0	0	2	0
0	0	2	0
2	0	0	4

W ₁	M ₁	40
W ₂	M ₂	38
W ₃	M ₄	43
W ₄	M ₃	51
		172

W ₁	M ₁	40
W ₂	M ₄	48
W ₃	M ₂	33
W ₄	M ₃	51
		172

An airlines that operates 7 days a week has the following time-table. The crew must have a minimum layover time of 5 hrs between flights. Obtain pairing of flights that minimises lay-over time away from home. For any given pairing the crew will be based at a city that will result in smaller layover time. For each pairing also mention the town where the crew will be based.

Flight No.	Delhi - Jaipur		Flight No.	Jaipur - Delhi	
	Departure	Arrival		Departure	Arrival
101	7.00 am	8.00 am	201	8.00 am	9.15 am
102	8.00 am	9.00 am	202	8.00 am	9.45 am
103	1.30 pm	2.30 pm	203	12.00 noon	1.15 pm
104	6.00 pm	7.30 pm	204	5.30 pm	6.45 pm

Min Layover time = 5 hrs

1 Unit = 15 min

• Layover time with Delhi as HQ.

⇒ Leaves Jaipur By Flight No.

	201	202	203	204
101	24x4 = 96	24x4 96	28x4 112	30x4 120
102	23x4 92	23x4 92	27x4 108	25x4 100
103	17x30 70	17x30 70	21x30 86	27 108
104	12x30 50	12x30 50	16x30 66	22 88

Reaches by Flight No.

* Layover time with Jaipur as HQ

→ Leaves Delhi By Flight No.

	201	202	203	204
101	21x45 87	85	71	49
102	22x45 91	89	75	53
103	28x45 113	111	97	75
104	8x45 35	33	115	93

Reaches Delhi by Flight No.

Minimum Layover time matrix

	201	202	203	204
101	87*	85*	71*	38°
102	91*	89*	75*	34°
103	70°	70°	86°	75*
104	35*	33*	66°	88°

• With Delhi as HQ.
* with Jaipur as HQ
with both Delhi & Jaipur as HQ.

Row opn:

49	47	33	0
57	55	41	0
0	0	16	5
2	0	33	15

Coln opn:

✓ 49	47	17	0
57	55	25	0
0	0	0	5
✓ 2	0	17	15

Reduce Matrix -1

Cancelled once
add 15
Cancelled twice
add 2
Not cancelled
subtract 2

✓ 47	47	15	0
55	55	23	0
0	2	0	7
0	0	19	15

Reduce Matrix -2

✓ 32	47	0	0
✓ 53	55	21	0
✓ 10	4	0	9
0	0	17	15

Flight No.	Flight No.	Min key over time.	HQ
101	203	71	Jaipur.
102	204	34	Delhi
103	201	70	Delhi
104	202	33	Jaipur.
		208	

On solving the problem it is assumed that a plane flying from Delhi for Jaipur must come back to Delhi at the immediate next opportunity.

It is further assumed that each plane will make only one forward & one return trip and thus there must be four planes for four forward & return flights.

The problem is to determine the optimum routing.

A solicitors firm employs typists on hourly piece rate basis for their work. There are five typists and their charges and speeds are different. According to an earlier understanding only one job is given to one typist and the typist is paid for full hour even if he works for a fraction of an hour. Find the least cost allocation for the following data:

Typist	Rate/hour (₹)	No. of pages typed/hour
A	5	12
B	6	14
C	3	8
D	4	10
E	4	11

Job	Number of page
P	199
Q	175
R	145
S	298
T	178

Solⁿ:

Step 1: Formulation of problem

	Job				
	P	Q	R	S	T
A	85	75	65	125	75
B	90	78	66	132	78
C	75	66	57	114	69
D	80	72	60	120	72
E	76	64	56	112	68

Typist

$$AP = \frac{199}{12} = 16.5 \approx 17 \times 5 = 85 \text{ ₹}$$

$$CR = \frac{145}{8} = 18.12 \approx 19 \times 3 = 57 \text{ ₹}$$

$$\begin{bmatrix} 85 & 75 & 65 & 125 & 75 \\ 90 & 78 & 66 & 132 & 78 \\ 75 & 66 & 57 & 114 & 69 \\ 80 & 72 & 60 & 120 & 72 \\ 76 & 64 & 56 & 112 & 68 \end{bmatrix} \rightarrow \begin{bmatrix} 20 & 10 & 0 & 60 & 10 \\ 24 & 12 & 0 & 66 & 12 \\ 18 & 9 & 0 & 57 & 12 \\ 20 & 12 & 0 & 60 & 12 \\ 20 & 8 & 0 & 56 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 4 & 10 \\ 6 & 3 & 9 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 3 \\ 3 & 1 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 0 & 4 & 0 \\ 6 & 4 & 10 & 10 & 2 \\ 10 & 1 & 0 & 1 & 2 \\ 2 & 4 & 0 & 4 & 2 \\ 2 & 1 & 0 & 0 & 2 \end{bmatrix}$$

Solⁿ I

	P	Q	R	S	T
A	1	0	1	2	10
B	4	1	0	7	1
C	0	0	2	0	2
D	10	1	0	1	1
E	3	0	3	10	4

Solⁿ II

	P	Q	R	S	T
A	1	0	1	2	10
B	4	1	0	7	1
C	0	0	2	0	2
D	10	1	0	1	1
E	3	0	3	10	4

	<u>Job</u>			
typist	<u>Solⁿ I</u>	Cost	Sol ⁿ	Cost
A	T	75	T	75
B	R	66	R	66
C	Q	66	S	114
D	P	80	P	80
E	S	112	Q	<u>64</u>
		<u>399</u>		<u>399</u>