Replacement Analysis

- Asset (fixed) -

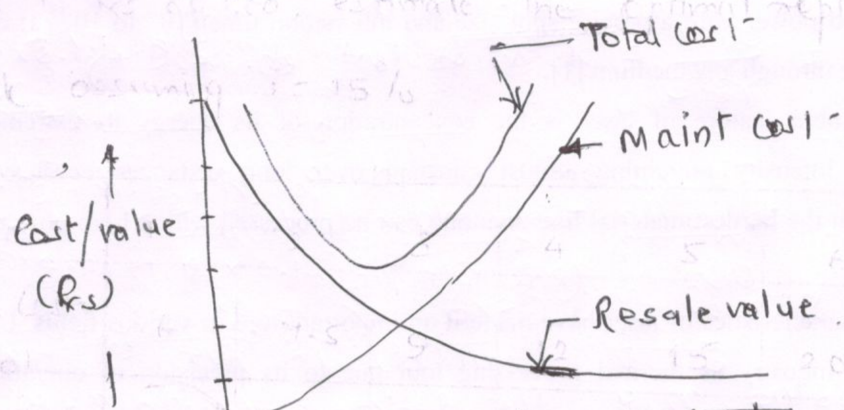
Capital Cost - Purchasing cost.

Resale value - Scrap value - Book on value.

Running Cost / Maintenance cost / Operating cost.

- Capital cost | Purchasing cost - One time payment at the start.
- Resale value - At the end of the period.
- Running | Operating | Maint - Spread over the life of an asset.

- Maint. cost increases with the life of an asset -
- Resale value decreases with the life of an asset.



It is assumed that

(i) Maint. cost incurs at the start of the year | Period and

(ii) Resale value ~~recei~~ will be at the end of the year.

Ex:- A car $C = 12$ lakh. $S = 2$ lakhs. life = 10 year (Sold)

$$\underline{AAC} = \frac{12-2}{10} = \frac{10}{10} = 1 \text{ lakh}$$

Why replacement? Tech. Dev. / Replacement of existing assets / Retrofitting

Let

C = Capital Cost / Purchasing cost. $\boxed{\text{Case 1st}}$

Replacement of items which detoriates with time

S = Salvage value

$f(t)$ = Maint cost

n = Number of years.

AAC = Average annual cost

$$\text{Total cost} = C - S + \Sigma f(t)$$

$$\boxed{AAC = \frac{1}{n}\left[C - S + \Sigma f(t)\right]}$$

$\boxed{\text{Ex 1}}$

The cost of a machine is Rs 6100 and its scrap value is only Rs 100. The maintenance costs are found from experience to be

Year	1	2	3	4	5	6	7	8
Maintenance cost in Rs	100	250	400	600	900	1250	1600	2000.

When should machine be replaced?

$C = Rs\ 6{,}100 \qquad S = Rs\ 100$

Year (n) (1)	Maint cost f(t) (2)	Total Maint. Cost $\Sigma f(t)$ (3)	(C − S) (4)	Total Cost 5 = (3 + 4)	AAC
1	100	100	6000	6,100	6100
2	250	350	6000	6350	3175
3	400	750	6000	6,750	2250
4	600	1350	6000	7,350	1837
5	900	2250	6000	8250	1650
6	1250	3500	6000	9,500	$\boxed{1583}$
7	1600	5100	6000	11,100	1586
8	2000	7100	6000	13,100	1638

As it is observed that Average Annual cost is the least at the ~~end of~~ sixth year, hence replace the machine at the end of the sixth year.

Example 2

A machine owner finds from his past records that the costs per year of maintaining a machine whose purchase price is Rs 6000 are as given below. Determine the age at which the machine is due for replacement.

Year (n)	1	2	3	4	5	6	7	8
Maint Cost (Rs) f(t)	1000	1200	1400	1800	2300	2800	3400	4000
Resale price (S)	3000	1500	750	375	200	200	200	200

$C = Rs\ 6,000$

Year (n) ①	Maint cost f(t) ②	Resale Price (S) ③	(C-S) ④	Total Maint Cost $\Sigma f(t)$ ⑤	Total Cost (6 = 4+5)	Average annual Cost (AAC) $7 = \frac{1}{n}[C-S+\Sigma f(t)]$
1	1000	3000	3000	1000	4000	4000
2	1200	1500	4500	2200	6700	3,350
3	1400	750	5250	3600	8850	2,950
4	1800	375	5625	5400	11,025	2,756.25
5	2300	200	5800	7700	13,500	$\boxed{2700}$
6	2800	200	5800	10,500	16,300	2716.67
7	3400	200	5800	13,900	19,700	2814.28
8	4000	200	5800	17,900	23,700	2962.5

As the AAC is the least during 5th year, replace the machine at the end of the fifth year.

Case 2nd

Replacement of items which deteriorates with time and money value changes with time.

Re 1 —— 1 Year —— i —— $(1+i)$

$(1+i)$ —— 2nd year —— i —— $(1+i)+i(1+i) = (1+i)^2$

$(1+i)^2$ —— 3rd Year —— i —— $(1+i)^2+i(1+i)^2 = (1+i)^3$

⋮

At the end of n year —— $(1+i)^n$.

$(1+i)^{-n}$ —— Present worth factor (pwf) or present value.

$(1+i)^{-n}$ – Compound amount factor

$r = \frac{1}{(1+i)}$ — Present worth factor.

r — Discount rate / depreciation value.

We know that

Total Cost $= C - S + \Sigma f(t)$

Assuming $f(t)$ - at the begining of the period & Resale at the end.

Total Cost $= \left[C + \Sigma f(t) \cdot r^{n-1} - S r^n \right]$

$$AAe = \frac{1}{\Sigma r^{n}} \left[C + \Sigma f(t) r^{n-1} - S r^n \right]$$

$$AAe = \frac{1}{\Sigma r^{n-1}} \left[C + \Sigma f(t) r^{n-1} \right]$$ — if resale is not given.

$\boxed{Ex}$ — A machine costs Rs 5000. If the money is worth 10% per year, find out the replacement age considering the following data of running cost -

Year (n)	1	2	3	4	5	6	7	8	9	10
Running Cost	800	800	800	800	800	1000	1200	1400	1600	1800

$$i = \frac{1}{1+i} = \frac{1}{1+0.10} = \frac{1}{1.1} = 0.9091$$

$C = Rs\ 5000$

Year (n) ①	$f(t)$ ②	r^{n-1} ③	$f(t) \cdot r^{n-1}$ ④	$\Sigma f(t) \cdot r^{n-1}$ ⑤	$C + \Sigma f(t) \cdot r^{n-1}$ ⑥	Σr^{n-1} ⑦	AAC ⑧ = 6 ÷ 7
1	800	1.0000	800	800	5800	1.000	5800
2	800	0.9091	727	1527	6527	1.9091	3419
3	800	0.8264	661	2188	7188	2.7355	2628
4	800	0.7513	601	2789	7789	3.4868	2234
5	800	0.6830	546	3335	8335	4.1698	1999
6	1000	0.6209	621	3956	8956	4.7907	1896
7	1200	0.5645	677	4633	9633	5.3552	1799
8	1400	0.5132	718	5351	10,351	5.8684	1764
9	1600	0.4665	746	6097	11,097	6.3349	$\boxed{1752}$
10	1800	0.4241	763	6860	11,860.	6.7590	1755

Replace at the end of 9th year.

Ex

A truck owner estimates that running cost and salvage value of truck for various years. as tabulated below. If the purchase price of truck is Rs 80,000, estimate the optimal replacement age of the truck assuming $i = 15\%$.

Year	1	2	3	4	5	6	7	8
Running Cost ('000)	6	7.5	9	12	15	20	25	30
Resale value ('000)	60	40	35	30	25	20	20	20

$$r = \frac{1}{1+i} = \frac{1}{1+0.15} = 0.8695.$$

Salvage value S_n	Running Cost $f(t)$	r^{n-1}	Discounted R. Cost $f(t)r^{n-1}$	$\Sigma f(t).r^{n-1}$	$C+\Sigma f(t)r^{n-1}$	r^n	$S_n.r^n$	$C+\Sigma f(t)r^{n-1}-S_nr^n$	Σr^n	AAC
2	3	4	5=3×4	6	7	8	9	10=7-9	11	12= 10÷11
60	6	1.0000	6.0000	6.0000	86.0000	0.8695	52.17	33.83	0.8695	38.9074
40	7.5	0.8695	6.5212	12.5212	92.5212	0.7560	30.24	62.2812	1.6255	38.3151
35	9	0.7560	6.804	19.3252	99.3252	0.6575	23.01	76.3152	2.283	33.4275
30	12	0.6575	7.884	27.2092	107.2092	0.5715	17.145	90.0642	2.8545	31.5516
25	15	0.5715	8.5725	35.7817	115.7817	0.4969	12.42	103.3617	3.3514	$\boxed{30.8413}$
20	20	0.4969	9.938	45.7197	125.7197	0.4321	8.642	117.0777	3.7835	30.9442
20	25	0.4321	10.8825	56.5222	136.5222	0.3757	7.514	129.0082	4.1592	31.0175
20	30	0.3757	11.2710	67.7932	147.7932	0.3269	6.538	141.2552	4.4861	31.4873

Replace at the end of 5th year.

Group Replacement:-

Following data is observed for a certain Type of light bulbs.

End of week	1	2	3	4	5	6
Probability of failure to date	0.09	0.25	0.49	0.85	0.97	1.00

Total number of bulbs are 1000. and if the bulb fails in service, it costs Rs 3 to replace but if all bulbs are replaced at time it can be done Re 0.70 for a bulb. It is proposed to replace all the bulbs at fixed interval whether or not they have burnt out.

1. What is the best interval between group replacement?
2. At what group replacement price per bulb would a policy of strictly individual replacement become preferable to the adopted policy.

Solution

Total number of bulbs in the system $= N_0 = 1000$

Let p_i be the probability that a new light bulb fails during i^{th} week of its life.

$p_1 = 0.09$

$p_2 = 0.25 - 0.09 = 0.16$

$p_3 = 0.49 - 0.25 = 0.24$

$p_4 = 0.85 - 0.49 = 0.36$

$p_5 = 0.97 - 0.85 = 0.12$

$p_6 = 1 - 0.97 = 0.03$

Since the sum of all probabilities is unity all probabilities higher than p_6 must be zero. i.e. $p_7 = p_8 = p_9 = etc = 0$. Thus all light bulbs are sure to burn out by 6th week.

Further we assume

1. That light bulb which fail during a week are replaced just before the end of that week.
2. The actual %age of failures during a week for a sub-population of bulbs with same age is the same as the expected %age of failure during the week for that sub-population.

Let N_i = No. of replacements made at the end of ith week.

$N_0 = N_0$ $\qquad = 1,000$

$N_1 = N_0 \cdot p_1$ $\qquad = 90$

$= 1000 \times 0.09$

$N_2 = N_0 \cdot p_2 + N_1 p_1$

$= 1000 \times 0.16 + 90 \times 0.09$ $\qquad = 168$

$= 160 + 8.1$

$N_3 = N_0 \cdot p_3 + N_1 \cdot p_2 + N_2 \cdot p_1$ $\qquad = 269$

$= (1000 \times 0.24) + (90 \times 0.16) + (168 \times 0.09)$

$= 240 + 14.4 + 15.12$

$N_4 = N_0 \cdot p_4 + N_1 \cdot p_3 + N_2 p_2 + N_3 \cdot p_1$

$= (1000 \times 0.36) + (90 \times 0.24) + (168 \times 0.16) + (269 \times 0.09)$ $\qquad = 432$

$= 360 + 21.6 + 26.88 + 24.21$

$N_5 = N_0 \cdot p_5 + N_1 \cdot p_4 + N_2 \cdot p_3 + N_3 \cdot p_2 + N_4 p_1$

$= (1000 \times 0.12) + (90 \times 0.36) + (168 \times 0.24) + (269 \times 0.16)$

$+ (432 \times 0.09)$

$N_6 = N_0 p_6 + N_1 \cdot p_5 + N_2 \cdot p_4 + N_3 p_3 + N_4 \cdot p_2 + N_5 \cdot p_1$

$= (1000 \times 0.03) + (90 \times 0.12) + (168 \times 0.36) + (269 \times 0.24)$

$+ (432 \times 0.16) + (274 \times 0.09)$

$= 30 + 10.8 + 60.48 + 64.56 + 69.12 + 24.66 \qquad = 260$

$N_7 = 0 + N_1 p_6 + N_2 p_5 + N_3 \cdot p_4 + N_4 p_3 + N_5 \cdot p_2 + N_6 p_1 \qquad = 291$

Thus we find that the number of bulbs failing each week increases till the 4th week, then decreases and again increases from 7th week. Thus N_i will continue to oscillate till the system attains a steady system.

Average (expected) life of light bulbs $= \sum_{i=1}^{6} i p_i$

$= (1 \times 0.09) + (2 \times 0.16) + (3 \times 0.24) + (4 \times 0.36) + (5 \times 0.12)$

$+ (6 \times 0.03) = 3.35$

$\therefore$ Avg. no. of failures/week $= \frac{1000}{3.35} = 299$

Cost of individual replacement of bulbs

$= Rs.\ (3 \times 299) = Rs\ 897$

Since the replacement of all 1000 bulbs in operation cost Re. 0.70 per bulb and replacement of individual bulb costs Rs 3 the total cost of replacement is.

End of week	Total cost of group replacement.	Avg cost/ week.
1	$1000 \times 0.70 + 90 \times 3 = 970$	970
2	$1000 \times 0.70 + 3(90+168) = 1474$	$\boxed{737.00}$
3	$1000 \times 0.70 + 3(90+168+269) = 2281$	760.33

As the average minimum cost is in the 2nd week it is optimal to have a group replacement after every two weeks.

Let Rs x be the group replacement price for bulbs.

$$897 < \frac{1000 \times x + 3(90+168)}{2}$$

$$x > \underline{1.02}$$