Find the optimal solution to the problem given in the following table.

Warehouse / Factory	W_1	W_2	W_3	W_4	Supply
F_1	14	25	45	5	6
F_2	65	25	35	55	8
F_3	35	3	65	15	16
Demand	4	7	6	13	30 / 30

1) Demand = Supply
2) I.B.F.S by VAM, N-W, LCM.
3) Test of optimality

Step 2 Initial basic feasible solution by VAM.

Warehouse / Factory	W_1	W_2	W_3	W_4	Supply	Column Penalties	R_i
F_1	14 (4)	25 *	45 *	5 (2)	6	9, 9, 40, 40 F	0
F_2	65 *	25 *	35 (6)	55 (2)	8	10, 20, 20, 20	50
F_3	35 *	3 (7)	65 *	15 (9)	16	12, 20, (50) F	10
Demand	4	7	6	13	30 / 30		
Row Penalties	21, (21) F	(22) F	10, 10, 10, 10	10, 10, 10, (50)			
K_j	+14	-7	-15	5			

Step 3 Test of optimality

Required no. of allocations to avoid degeneracy

$= m+n-1 = 3+4-1 = 6$.

Actual no. of allocations = 6.

Hence test of optimality.

Step 4: Optimization by MODI method.

a) Calculate R_i & K_j such that

$R_i + K_j = C_{ij}$ (stepping stone square).

b) Improvement Potential = $c_{ij} - (R_i + K_j)$. (for water cells).

$c_{ij} - (R_i + K_j)$.

$F_1W_2 \Rightarrow 25 - (0 - 7) = 32$

$F_1W_3 = 45 - (0 - 15) = 60$.

$F_2W_1 \Rightarrow 65 - (50 + 14) = 1$

$F_2W_2 \Rightarrow 25 - (50 - 7) = -18$

$F_3W_1 = 35 - (14 + 10) = 11$

$F_3W_3 \Rightarrow 65 - (10 - 15) = 70$.

As Improvement potential value is -ve for F_2W_2 location further improvement is possible by tracing a colose loop and F_2W_2 is the starting point of close loop.

F \ W	W_1	W_2	W_3	W_4	~~Supply~~ R_i
F_1	14 (4)	25 *	45 *	5 (2)	0
F_2	65 *	25 (2)	35 (6)	55 *	32
F_3	35 *	3 (5)	65 *	15 (11)	10
K_j	14	−7	3	5	

IP cell.

$c_{ij} - (R_i + K_j)$ — water cells

$F_1W_2 \Rightarrow 25 - (0 - 7) = 32$

$F_1W_3 \Rightarrow 45 - (3 + 0) = 42$

$F_2W_1 = 65 - (32 + 14) = 19$

$F_2W_4 = 55 - (32 + 5) = 18$.

$F_3W_1 = 35 - (10 - 14) = 11$

$F_3W_3 = 65 - (10 + 3) = 52$.

As IP is +ve for each water cell, above soln is optimal.

∴ Transportation Cost = $14 \times 4 + 5 \times 2 + 25 \times 2 + 35 \times 6 + 3 \times 5 + 15 \times 11$

$= 506$.