Project Crashing

The project duration can be reduced **by assigning more resources** to project activities. But, doing this would somehow increase our project cost!

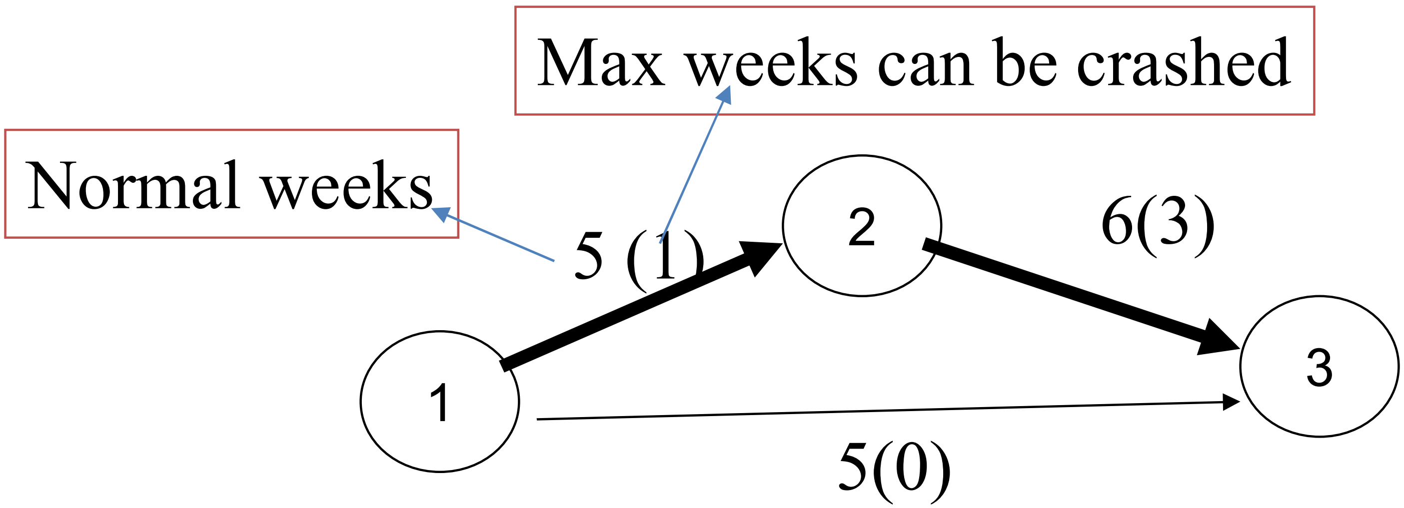
How do we strike a balance?

- ***Project crashing*** is a method for shortening project duration by reducing one or more ***critical activities*** to a time less than normal activity time.

Trade-off concept

- Here, we adopt the **"Trade-off"** concept
- We attempt to "**crash**" some "**critical**" events by allocating more resources to them, so that the time of one or more critical activities is reduced to a time that is less than the normal activity time.
- How to do that:
- **Question: What criteria should it be based on when deciding to crashing critical times?**

Example – crashing (1)



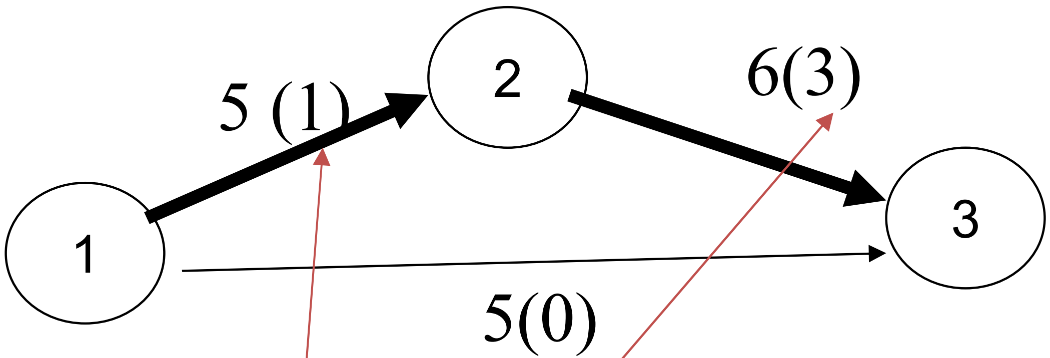
The critical path is 1-2-3, the completion time =11

How? Path: 1-2-3 = 5+6=11 weeks

Path: 1-3 = 5 weeks

Now, how many days can we “crash” it?

Example – crashing (1)



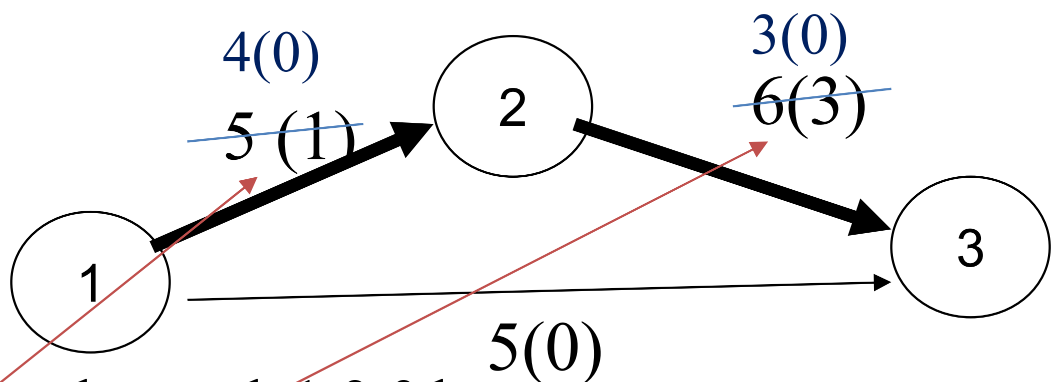
The maximum time that can be crashed for:

Path 1-2-3 = 1 + 3 = 4

Path 1-3 = 0

Should we use up all these 4 weeks?

Example – crashing (1)



If we used all 4 days, then path 1-2-3 has

(5-1) + (6-3) = 7 completion weeks

Now, we need to check if the completion time for path 1-3 has lesser than 7 weeks (why?)

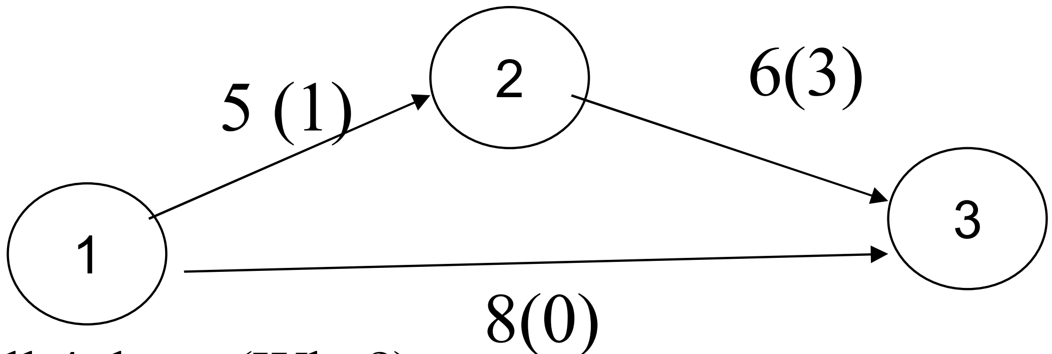
Now, path 1-3 has (5-0) = 5 weeks

Since path 1-3 still shorter than 7 weeks, we used up all 4 crashed weeks

Question: What if path 1-3 has, say 8 weeks completion time?

Example – crashing (1)

Such as



Now, we cannot use all 4 days (Why?)

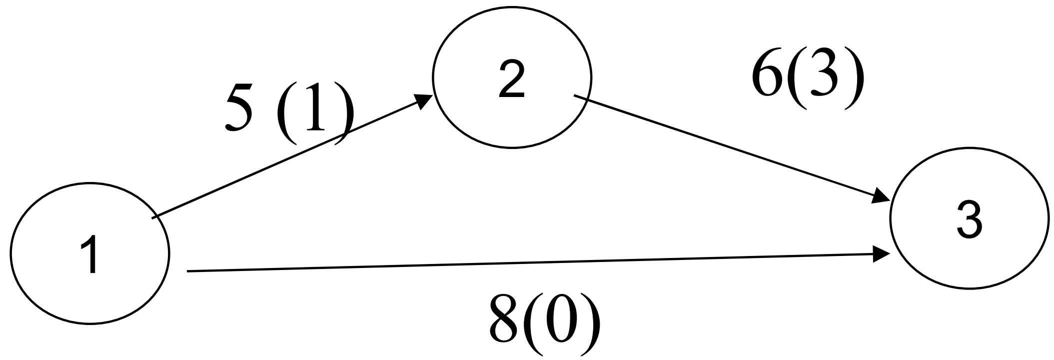
Because path 1-2-3 will not be critical path anymore as path 1-3 would now has longest hour to finish

Rule: When a path is a critical path, it will not stay as a critical path

So, we can only reduce the path 1-2-3 completion time to the same time as path 1-3. (HOW?)

Example – crashing (1)

Solution:



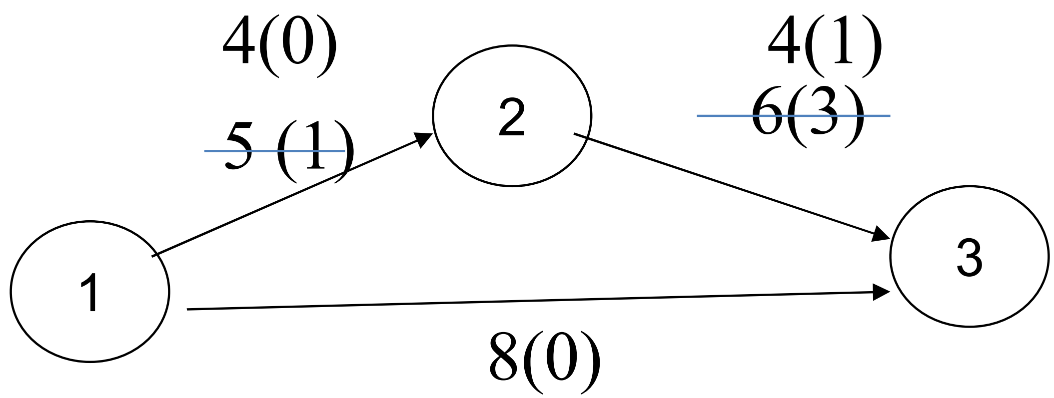
We can only reduce total time for path 1-2-3 = path 1-3,

that is 8 weeks

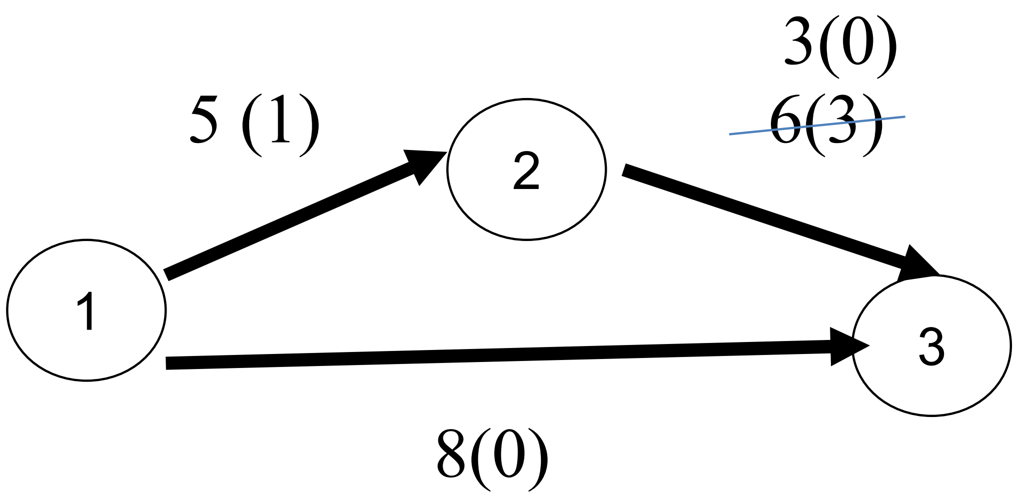
If the cost for path 1-2 and path 2-3 is the same then
We can random pick them to crash so that its completion
Time is 8 weeks

Example – crashing (1)

Solution:



OR



Now, paths 1-2-3 and 1-3 are both critical paths

Project Crashing and Time-Cost Trade-Off General Relationship of Time and Cost (2 of 2)

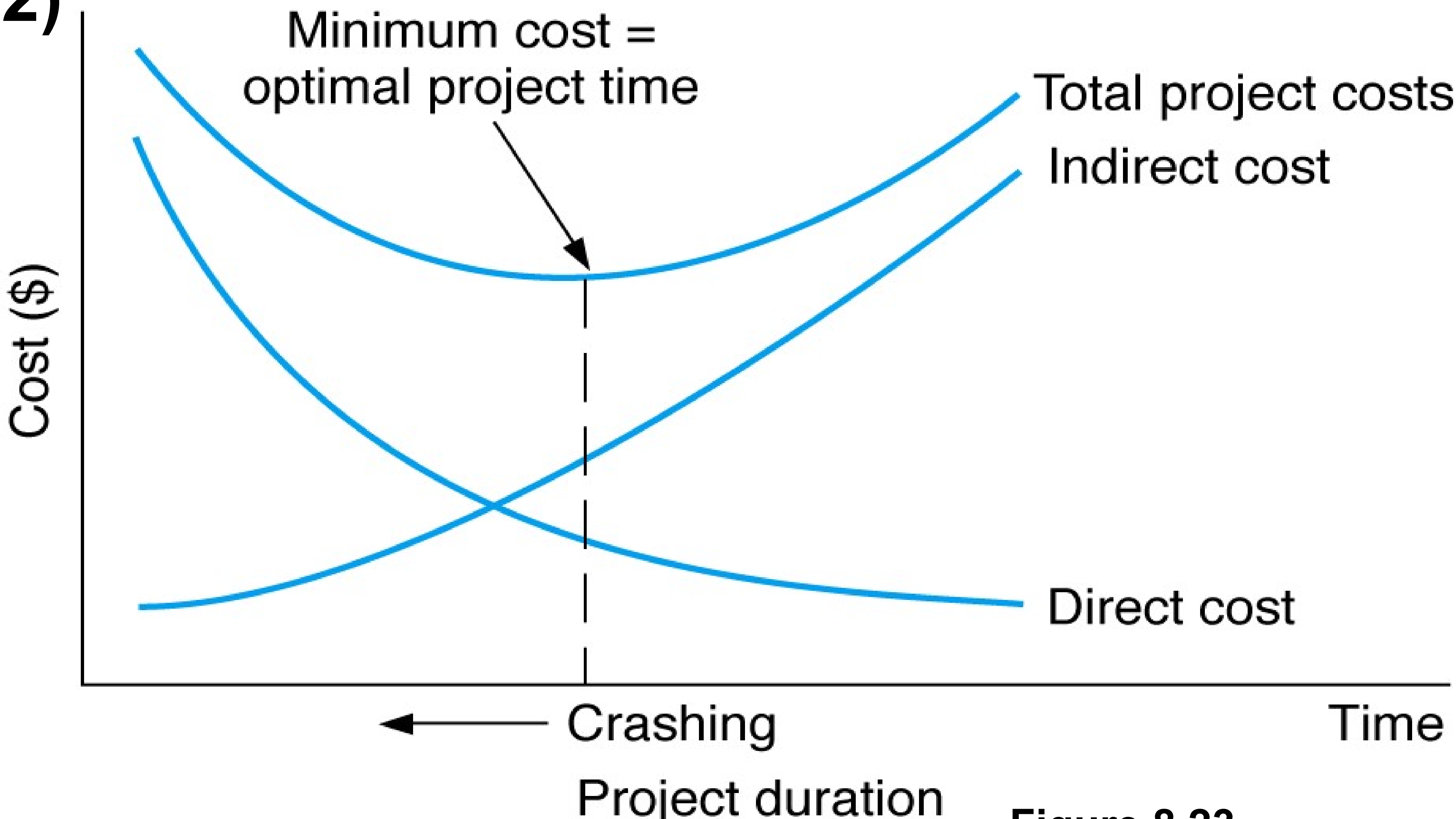
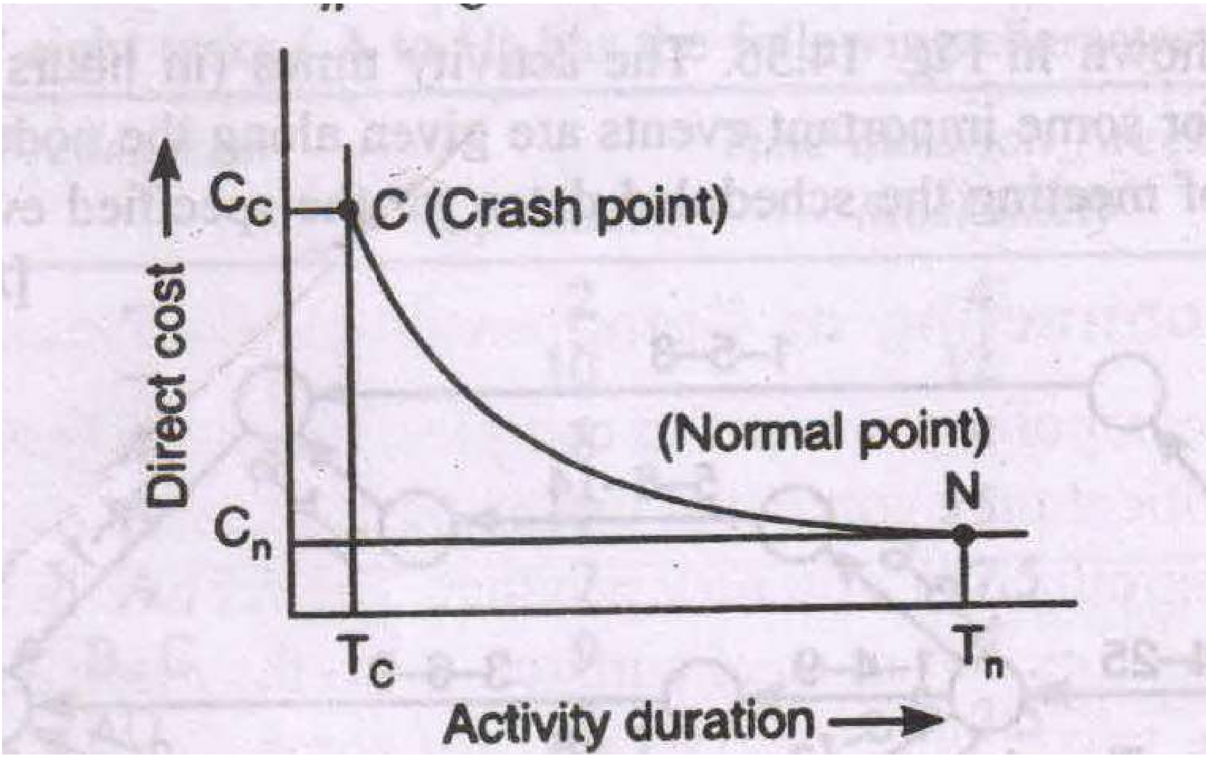
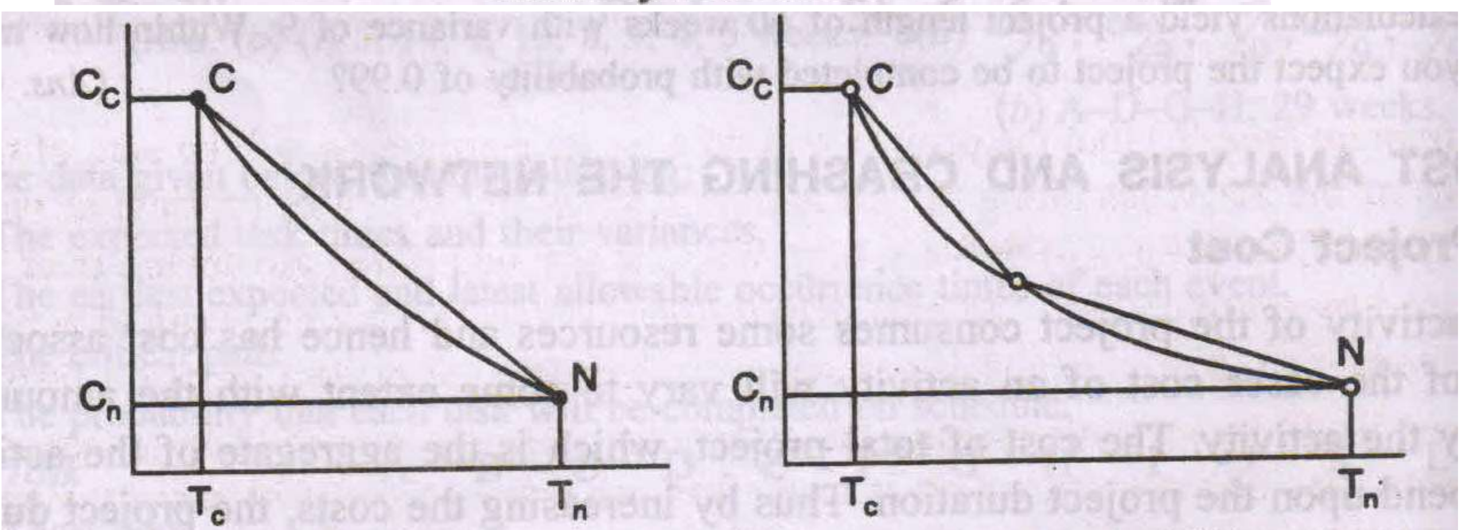


Figure 8.23
The Time-Cost Trade-Off

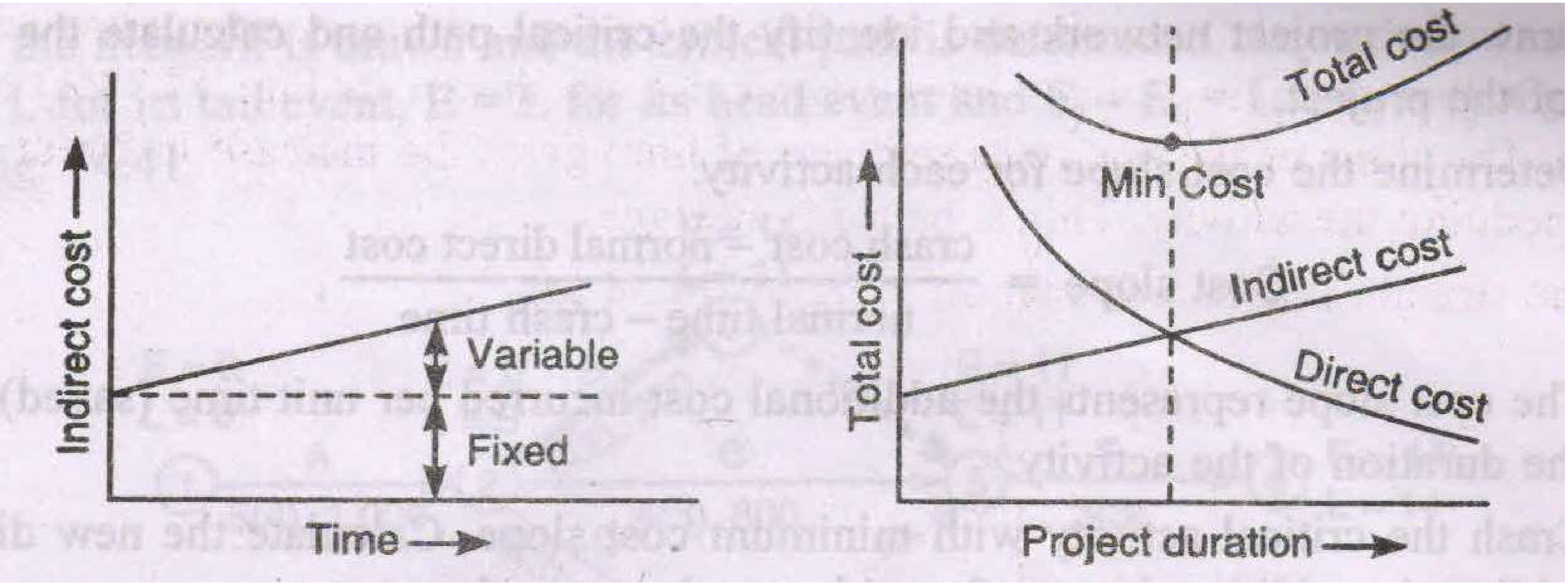


$$\text{cost slope} = \frac{C_C - C_n}{T_n - T_C} = \frac{\Delta C}{\Delta T}.$$



insignificant. As the activity is compressed, the direct cost goes on increasing. If it is compressed beyond C, the cost increases very rapidly for insignificant change in the activity duration. This point is called '*crash point*'. Crash time is thus the minimum activity duration to which an activity can be compressed by increasing the resources and hence by increasing the direct cost.

Indirect cost



Indirect Cost

Project indirect cost can further be sub-divided into two parts: fixed indirect cost and variable indirect cost. The fixed indirect cost is due to the general and administrative expenses, licence fee, insurance cost and taxes and does not depend upon the progress of the project. The variable indirect cost depends upon the time consumed by the project and consists of overhead expenditure, supervision, interest on capital and depreciation, penalty for delays (if any), etc. It is assumed that the indirect cost increases linearly with time as shown in figure 14.39.

Project Crashing and Time-Cost Trade-Off Example Problem (2 of 5)

Cost Slope (Crash cost & crash time have a linear relationship):

$$\frac{Total\ Crash\ Cost}{Total\ Crash\ Time} = \frac{\$2000}{5\ weeks} = \$400/wk$$

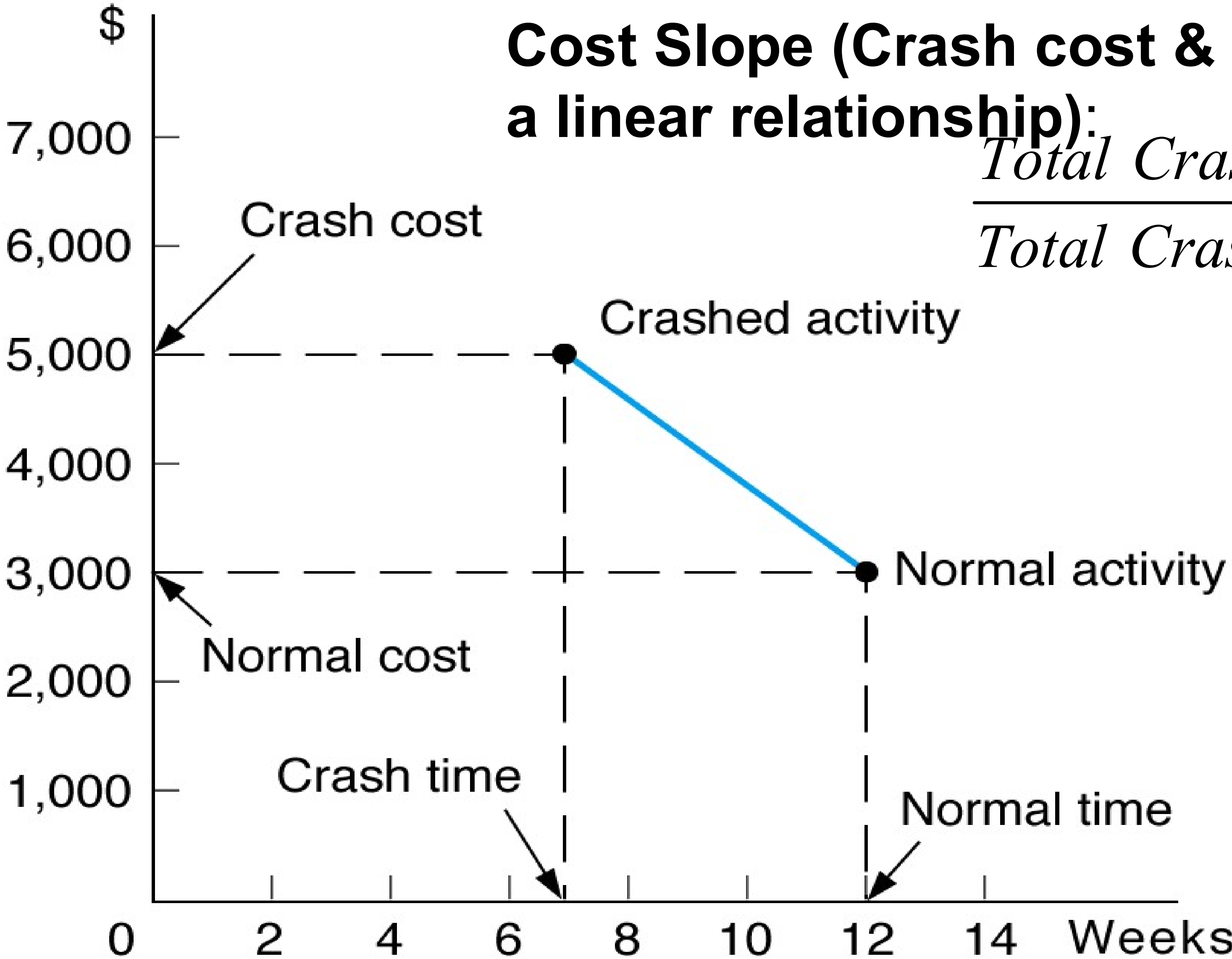


Figure 8.20

The *normal time* (t_n) for the completion of the project will be the sum of the normal time rations of the critical activities and the *normal direct cost* (C_n) of the project will be the sum of e normal cost of all the activities since each and every activity has to be executed to complete e project.

The *minimum time* (t_c) that the project will take for its completion will be sum of the crashed e durations of the activities along the critical path. If all the activities (critical as well as non-ical) are crashed, the cost will be very high without any additional advantage over and above e one obtained by crashing only the critical activities. Therefore, the non-critical activities need t be expedited since their crashing is not going to decrease the project duration further.

EXAMPLE 14.12-1

List of activities for erecting a canteen in a factory is given below with other relevant details. Job A must precede all others while job E must follow others. Apart from this, jobs can run concurrently.

Code	Job description	Normal Duration (days)	Normal Cost (Rs.)	Crash Duration (days)	Crash Cost (Rs.)
A	*Lay foundation and build walls*	*5*	*3,000*	*4*	*4,000*
B	*Tile roofing*	*6*	*1,200*	*2*	*2,000*
C	*Instal electricity*	*4*	*1,000*	*3*	*1,800*
D	*Instal plumbing*	*5*	*1,200*	*3*	*2,000*
E	*Connect services to finish*	*3*	*1,600*	*3*	*1,600*
			8,000		

(i) Draw the network and identify the critical path.

(ii) Crash the network fully to find out minimum duration.

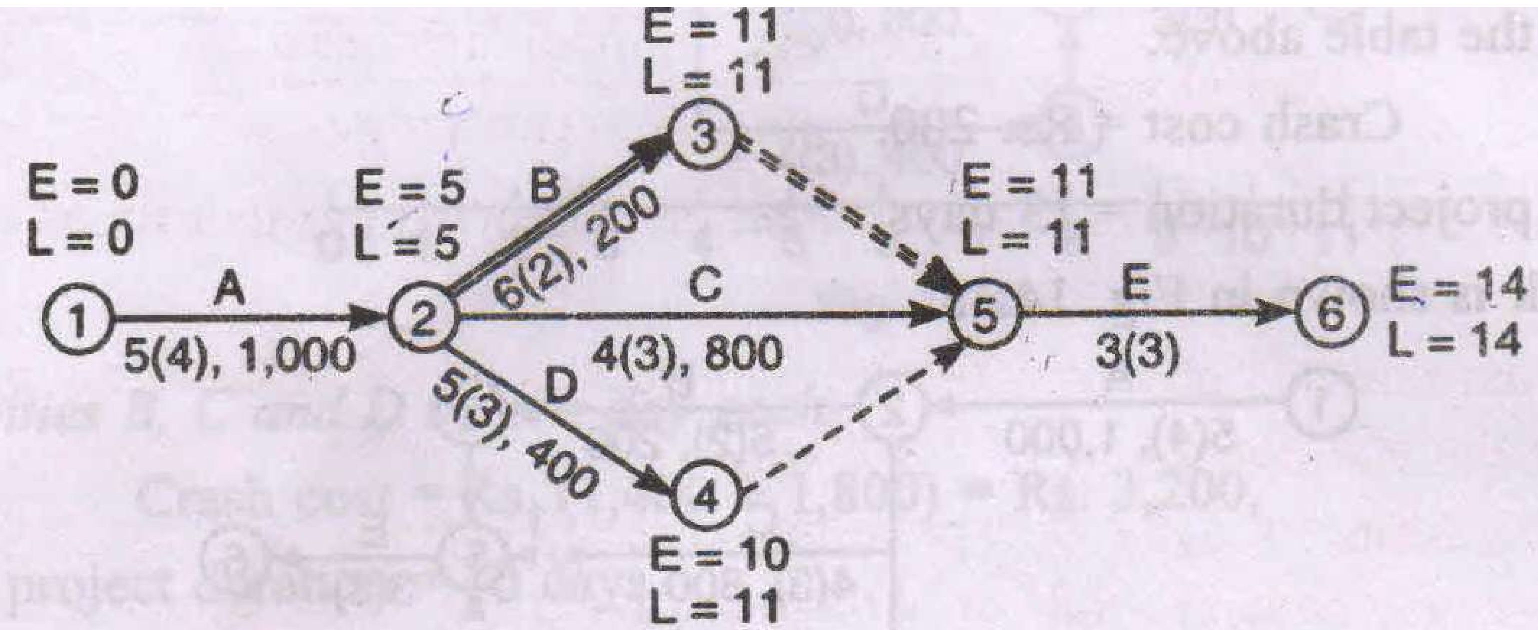
(iii) If indirect costs are Rs. 300 per day, determine time-cost trade off for the project.

Solution

First, the cost slope for each activity and the normal direct cost of the project is calculated. This is shown in the table below.

Job	:	A (1-2)	B (2-3)	C (2-5)	D (2-4)	E (5-6)
Cost slope (Rs./day)	:	1,000	200	800	400	—

The normal direct cost from the given table is = Rs. 8,000.



(*i*) Critical path is 1-2-3-5-6.

Normal duration = 14 days.

Normal cost = Rs. (8,000 + 300 × 14) = Rs. 12,200.

(*ii*) Represent the network on time-scaled diagram. This is shown in Fig. 14.42. Activities along the critical path are arranged along the horizontal line 1-2-3-5-6.

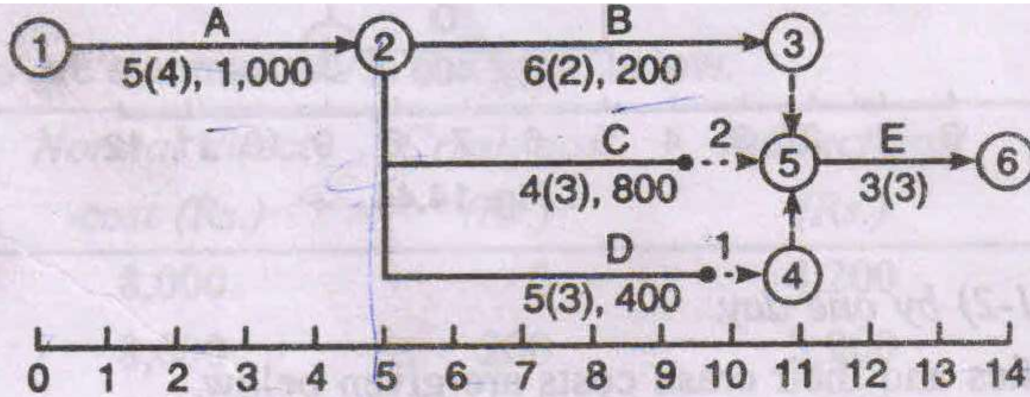


Fig. 14.42

Crash activity B (2–3), C (2–5) and D (2–4) by one day each.

The various alternative critical activities and their crash costs are given below.

Activity	*Cost (Rs.)*	*Activity*	*Cost (Rs.)*
A (1-2)	1,000	B (2-3)	200
		D (2-4)	Nil
		C (2-5)	Nil
			200

While crashing a critical activity, only those non-critical activities will require crashing which start from the tail event of the critical activity under consideration or earlier and finish at the head event of the critical activity or later than that. This rule may be kept in mind while crashing the activities.

$\therefore$ Crash cost = Rs. 200,

project duration = 13 days.

The network is shown in Fig. 14.43.

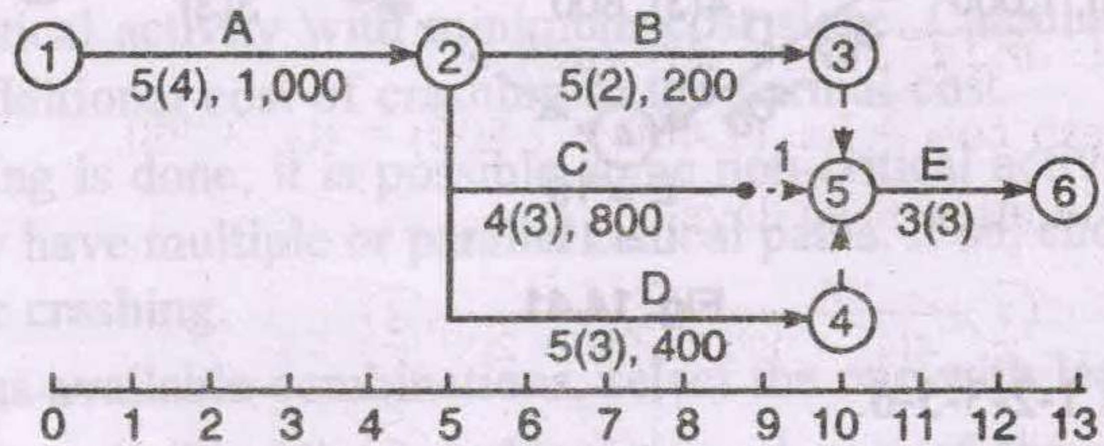


Fig. 14.43

Crash activities B, C and D by one day each.

The various activities and their crash costs are given below.

Activity	*Cost (Rs.)*	*Activity*	*Cost (Rs.)*
A (1-2)	1,000	B (2-3)	200
		D (2-4)	400
		C (2-5)	Nil
			600

$\therefore$ Crash cost = Rs. (200 + 600) = Rs. 800,

project duration = 12 days.

The network is shown in Fig. 14.44.

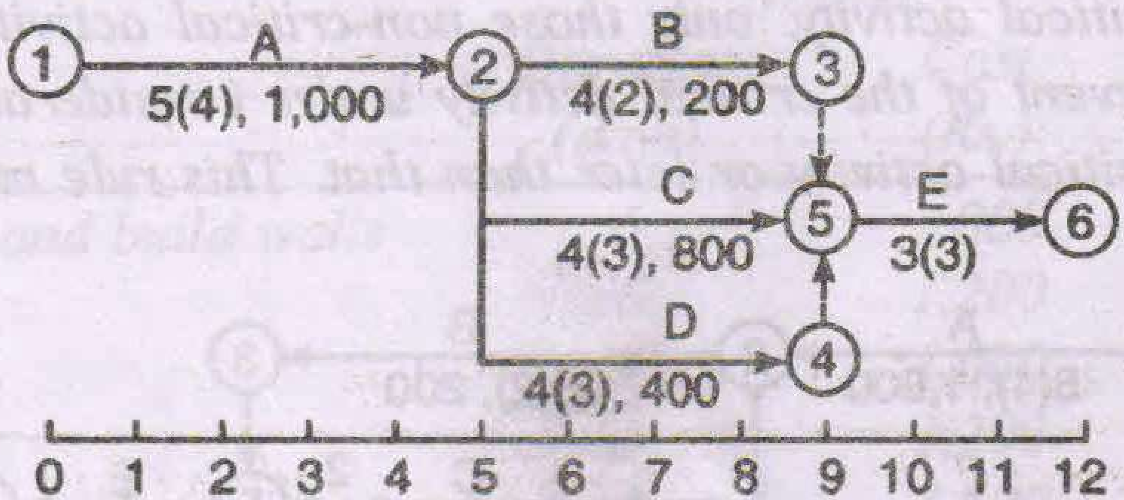


Fig. 14.44

Crash activity A (1-2) by one day.

The various activities and their crash costs are given below.

Activity	*Cost (Rs.)*	*Activity*	*Cost (Rs.)*
A (1-2)	1,000	B (2-3)	200
		D (2-4)	400
		C (2-5)	800
			1,400

$\therefore$ Crash cost = Rs. (1,000 + 800) = Rs. 1,800,

project duration = 11 days.

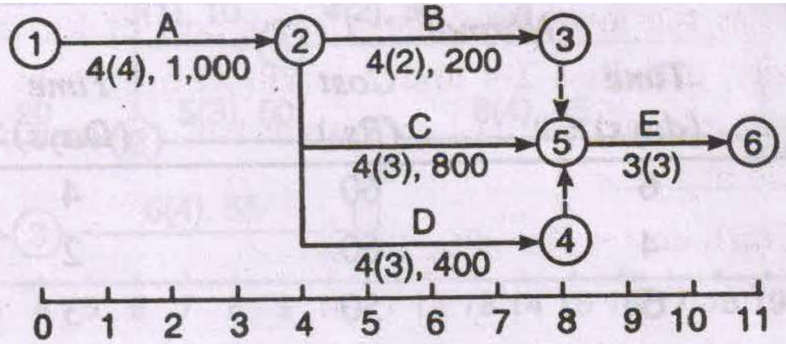


Fig. 14.45

Crash activities B, C and D by one day each.

$\therefore$ Crash cost = Rs. (1,400 + 1,800) = Rs. 3,200,

project duration = 10 days.

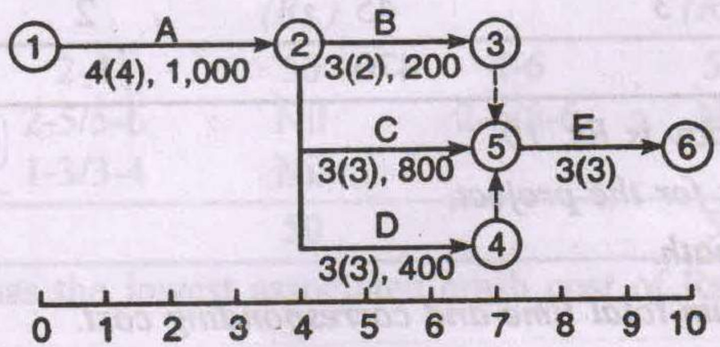


Fig. 14.46

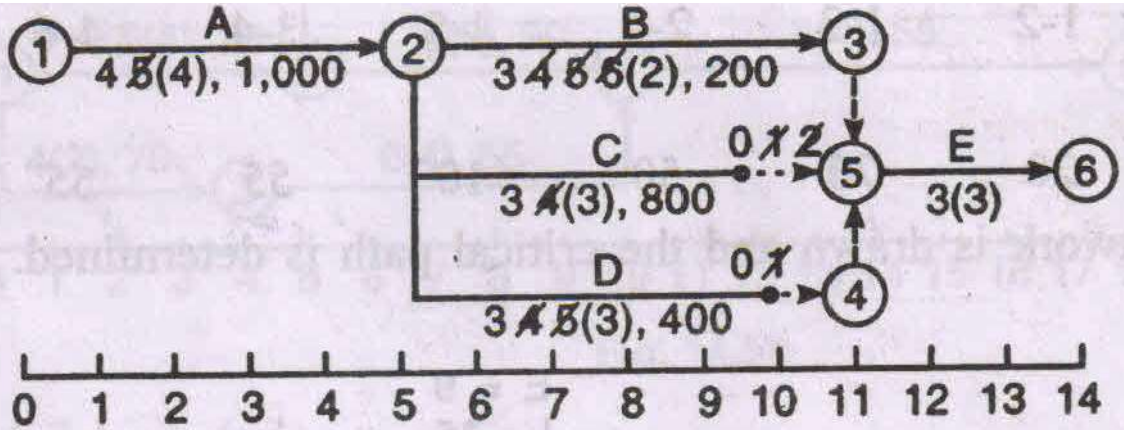


Fig. 14.47

The above results are summarised in the table below.

Duration (days)	*Normal direct cost (Rs.)*	*Crash cost (Rs.)*	*Indirect cost (Rs.)*	*Total cost (Rs.)*
14	8,000	0	4,200	12,200
13	8,000	200	3,900	12,100
12	8,000	800	3,600	12,400
11	8,000	1,800	3,300	13,100
10	8,000	3,200	3,000	14,200

(*iii*) Thus time-cost trade off exists at 13 days when B is crashed by 1 day and the total minimum cost is Rs. 12,100.

Activity	Normal Time (days)	Normal Cost (Rs.)	Crash Time (Days)	Crash Cost (Rs.)
1-2	6	60	4	100
1-3	4	60	2	200
2-4	5	50	3	150
2-5	3	45	1	65
3-4	6	90	4	200
4-6	8	80	4	300
5-6	4	40	2	100
6-7	3	45	2	80
		470		

The indirect cost per day is Rs. 10.

(i) Draw the network for the project.

(ii) Find the critical path.

(iii) Determine minimum total time and corresponding cost.

$\frac{60}{2} - 50$

Solution

First, the cost slope for each activity and the normal direct cost of the project is calcul[ated]. This is shown in the table below.

Activity :	1-2	1-3	2-4	2-5	3-4	4-6	5-6	6-7
Cost slope (Rs./day) :	20	70	50	10	55	55	30	35

(*i*) Next, the network is drawn and the critical path is determined. This is shown in [Fig.] 14.48.

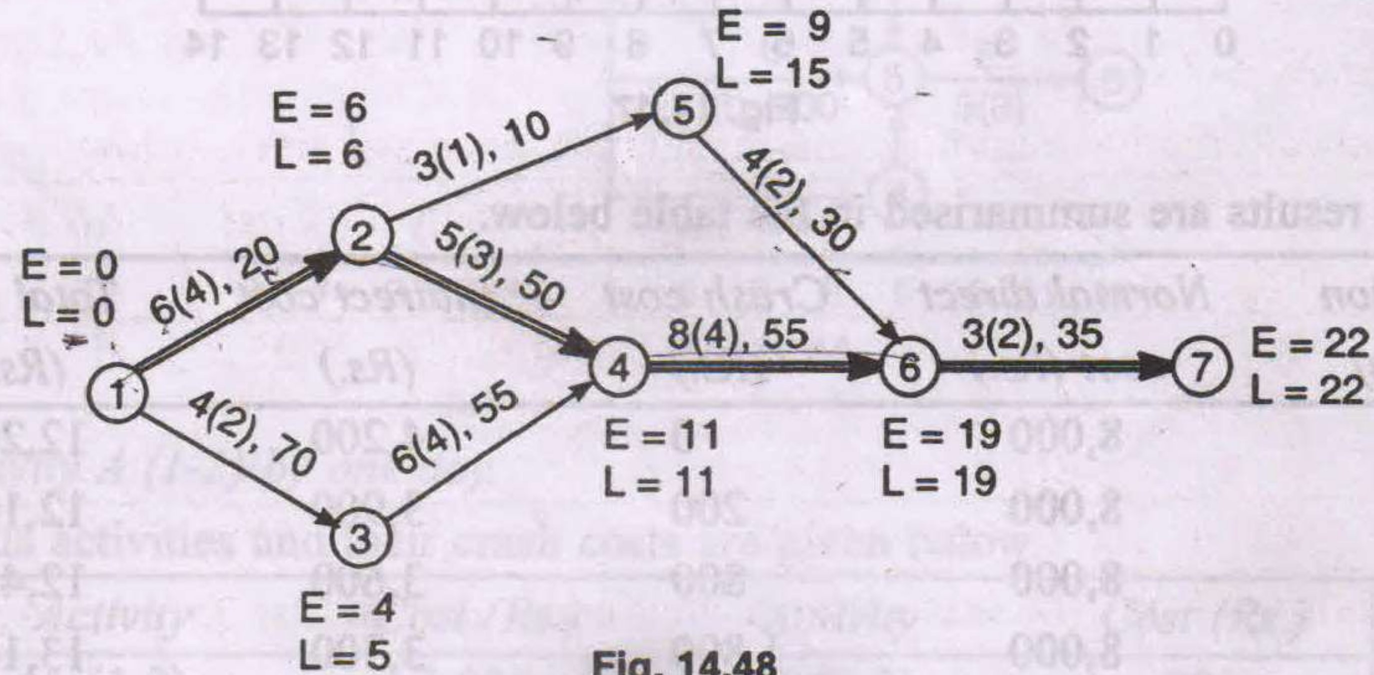


Fig. 14.48

(*ii*) The critical path is 1-2-4-6-7.

(*iii*) Normal duration = 22 days.

Normal cost = Rs. (470 + 22 × 10) = Rs. 690.

Now represent the network on time-scaled diagram. This is shown in Fig. 14.49.

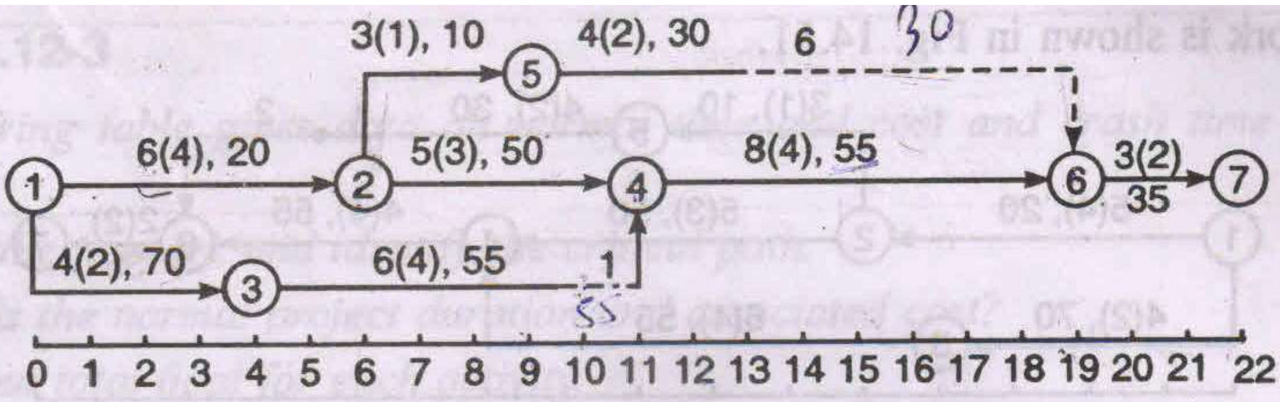


Fig. 14.49

Crash activity 1-2 by 1 day.

The various alternative activities and their crash costs are given below.

(a) Activity	*Cost (Rs.)*	*(b) Activity*	*Cost (Rs.)*	*(c) Activity*	*Cost (Rs.)*	*(d) Activity*	Cost *(Rs.)*
1-2	20	2-4	50	4-6	55	6-7	35
1-3/3-4	Nil	2-5/5-6	Nil	2-5/5-6	Nil		
		1-3/3-4	Nil				
	20		50		55		35

Since activity 1-2 has the lowest associated crash cost of Rs. 20 per day, it is crashed by ne day.

∴ Crash cost = Rs. 20,

project duration = 21 days.

The network is shown in Fig. 14.50.

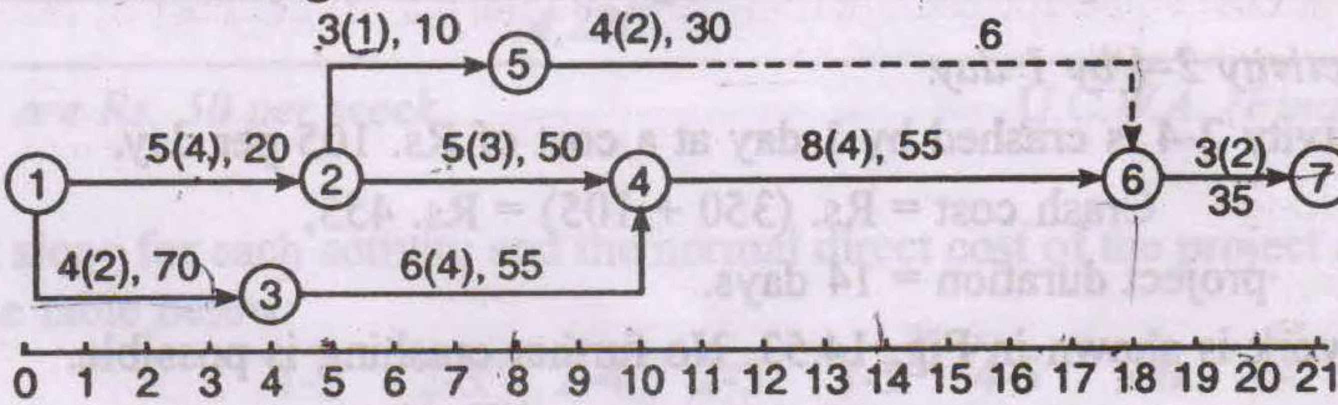


Fig. 14.50

Crash activity 6-7 by 1 day.

The various alternative activities and their crash costs are given below.

(a) Activity	Cost (Rs.)	(b) Activity	Cost (Rs.)	(c) Activity	Cost (Rs.)	(d) Activity	Cost (Rs.)
1-2	20	2-4	50	4-6	55	6-7	35
1-3/3-4	55	2-5/5-6	Nil	2-5/5-6	Nil		
		1-3/3-4	55				
	75		105		55		

Since activity 6-7 has the lowest associated crash cost of Rs. 35 per day, it is crashed by one day.

$\therefore$ Crash cost = Rs. (20 + 35) Rs. 55,

project duration = 20 days.

Crash activity 4-6 by 4 days.

As evident from the above table, next activity 4-6 has the lowest associated crash cost of Rs. 55 per day and as seen from Fig. 14.50 it can be crashed by 4 days.

$\therefore$ Crash cost = Rs. (55 + 4 × 55) = Rs. 275,

project duration = 16 days.

The network is shown in Fig. 14.51.

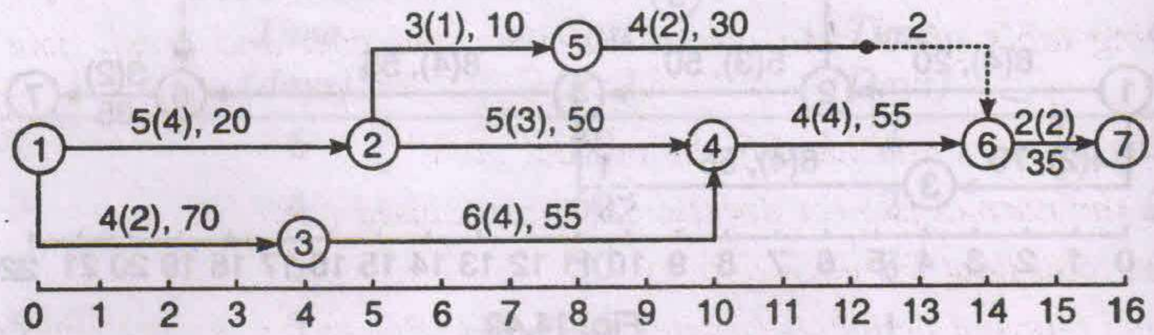


Fig. 14.51

Crash activity 1-2 by 1 day.

Next, activity 1-2 is crashed by 1 day at a cost of Rs. 75 per day.

$\therefore$ Crash cost = Rs. (275 + 75) = Rs. 350,

project duration = 15 days.

The network is shown in Fig. 14.52.

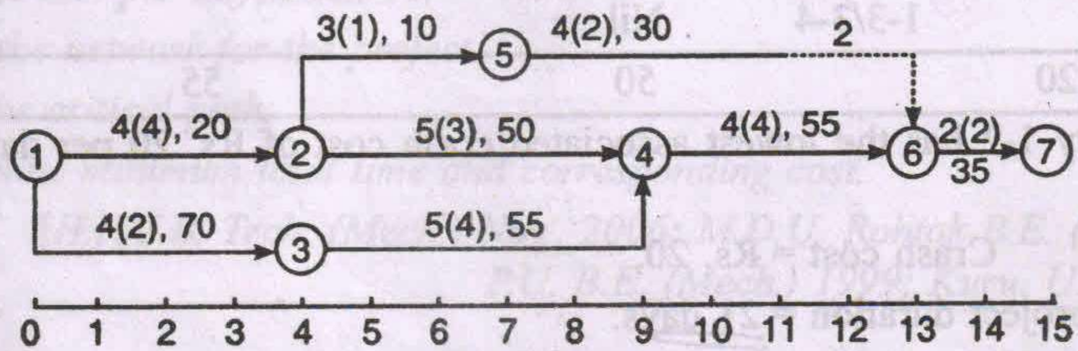


Fig. 14.52

Crash activity 2-4 by 1 day.

Now activity 2-4 is crashed by 1 day at a cost of Rs. 105 per day.

$\therefore$ Crash cost = Rs. (350 + 105) = Rs. 455,

project duration = 14 days.

The network is shown in Fig. 14.53. No further crashing is possible.

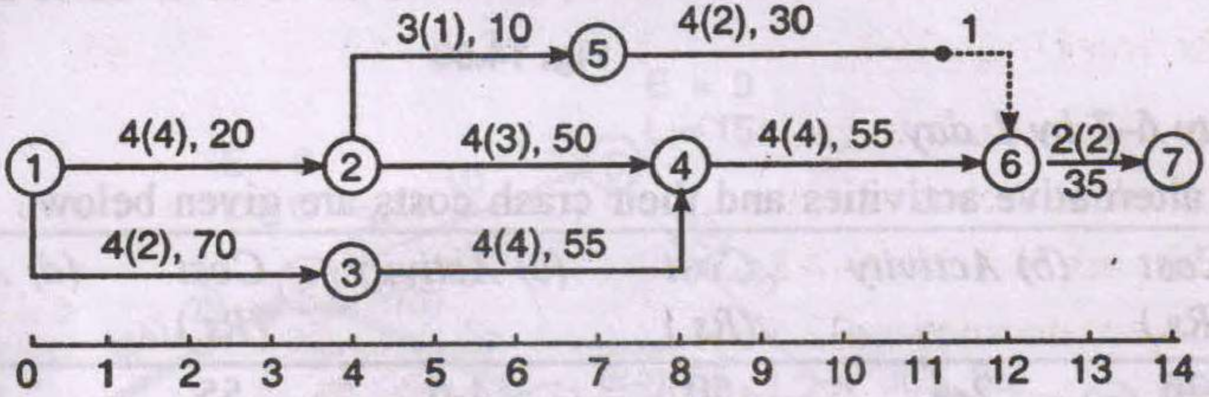


Fig. 14.53

The complete crashing of the network from 22 days to 14 days can be represented in a ngle diagram (Fig. 14.54).

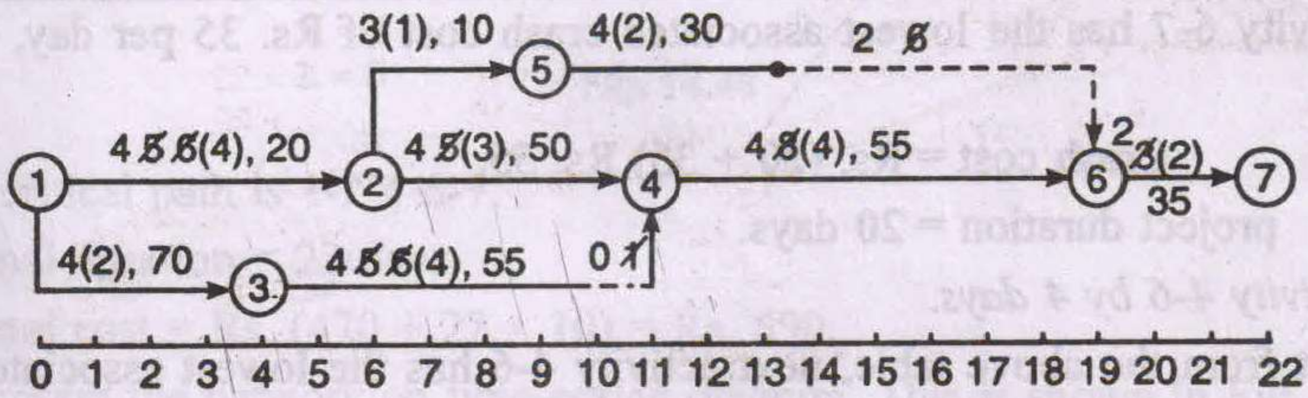


Fig. 14.54

$\therefore$ Minimum total time = 14 days,

corresponding cost = Rs. (470 + 14 × 10 + 455) = Rs. 1,065.

EXAMPLE 14.12-3

The following table gives data on normal time and cost and crash time and cost for a project.

(a) Draw the network and identify the critical path.

(b) What is the normal project duration and associated cost?

(c) Find out total float for each activity.

(d) Crash the relevant activities systematically and determine the optimum project time and cost.

Activity	*Normal*		*Crash*	
	Time (weeks)	*Cost (Rs.)*	*Time (weeks)*	*Cost (Rs.)*
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	810
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410
6-7	4	400	3	470
6-8	13	780	10	900
7-8	10	1,000	9	1,200
		4,220		

Indirect costs are Rs. 50 per week. [*I.C.W.A. (Final) Dec., 1988*]

Solution

First, the cost slope for each activity and the normal direct cost of the project are calculated. This is shown in the table below.

Activity	:	1-2	2-3	2-4	2-5	3-5	4-5	5-6	6-7	6-8	7-8
Cost slope (Rs./week)	:	100	–	80	45	50	–	45	70	40	200

Normal direct cost = Rs. 4,220.

Next, the network is drawn and the critical path is found. This is shown in Fig. 14.55.

(*a*) The critical path is 1-2-5-6-7-8.

(*b*) Normal project duration = 32 weeks,

normal project cost = Rs. (4,220 + 32 × 50) = Rs. 5,820.

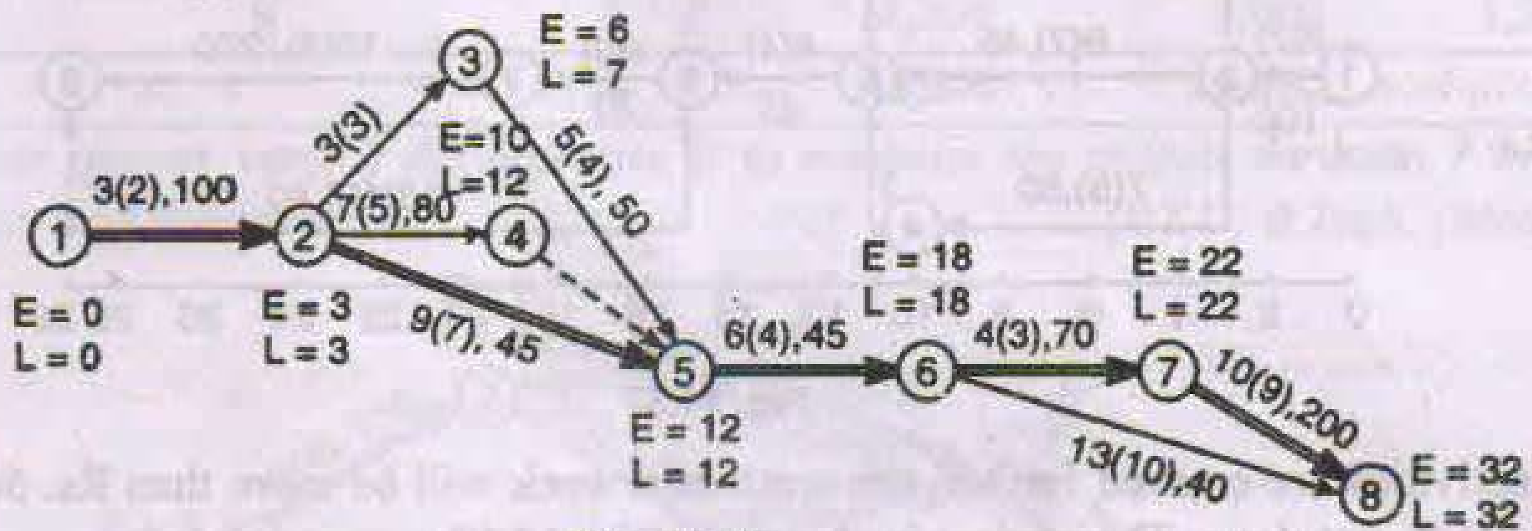


Fig. 14.55

(*c*) Total float for each activity is found in the table below.

(*d*) Since the indirect cost is Rs. 50/ week and the network is to be crashed only upto optimum time and cost, only those activities need to be crashed for which the total cost slope is ≤ Rs. 50/ week. The time-scaled diagram of the network is shown in Fig.14.56.

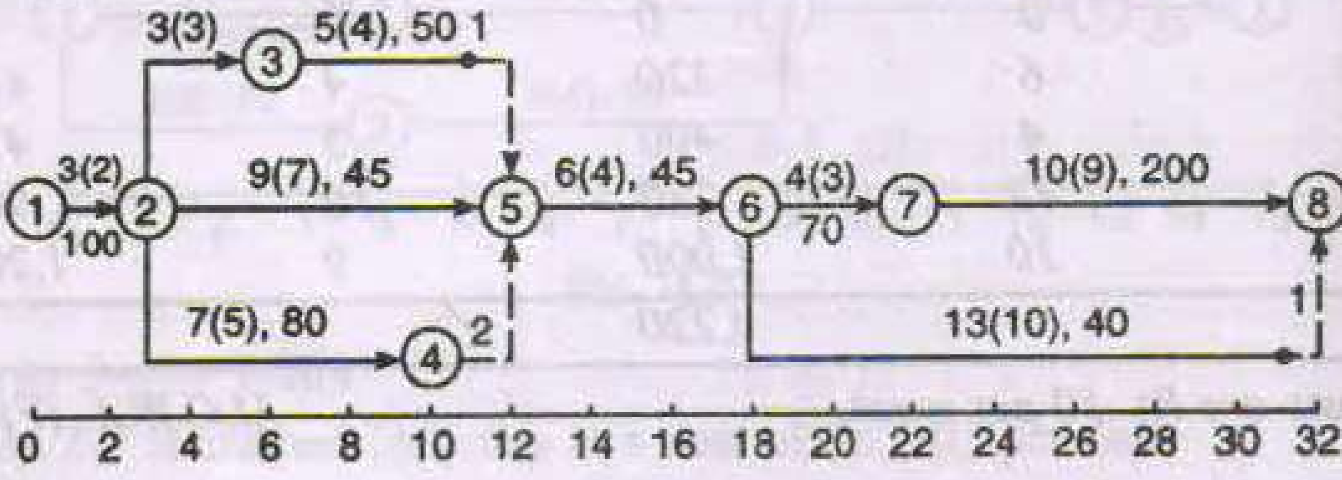


Fig. 14.56

Crash activity 5-6 by 2 days at a crash cost of Rs. 45/week.

Crash cost = Rs. (2 × 45) = Rs. 90,

project duration = 30 weeks.

Crash activity 2-5 by 1 day at a crash cost of Rs. 45/week.

Crash cost = Rs. (90 + 45) = Rs. 135,

project duration = 29 weeks.

project duration = 29 weeks.

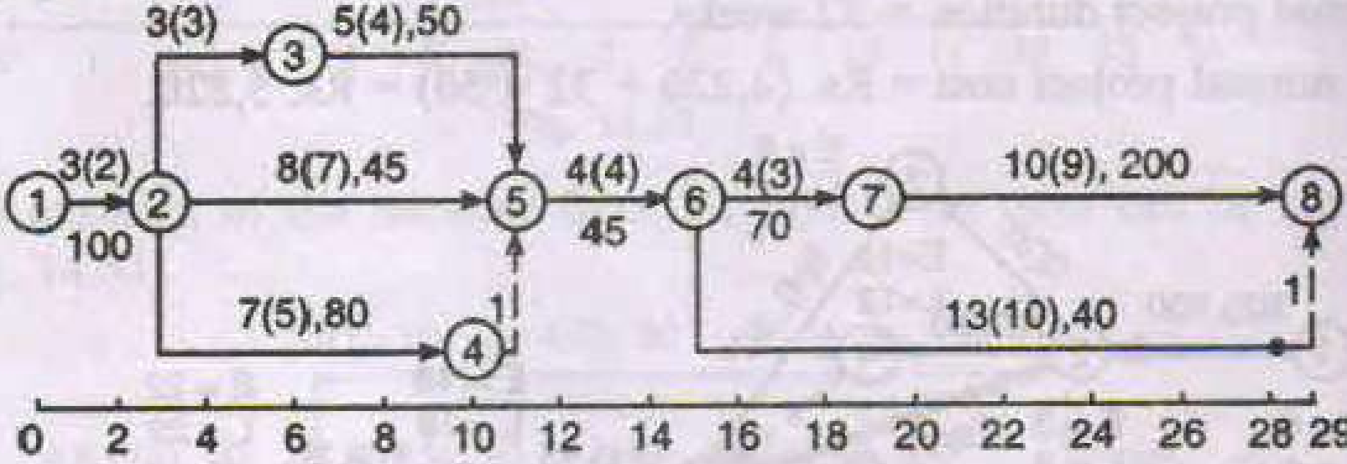


Fig. 14.57

If the activities are crashed further, the crash cost/week will be more than Rs. 50 and hence further crashing is not done. The project is shown in Fig. 14.57.

The crashing of the network from 32 weeks to 29 weeks can be represented in a single diagram (Fig. 14.58).

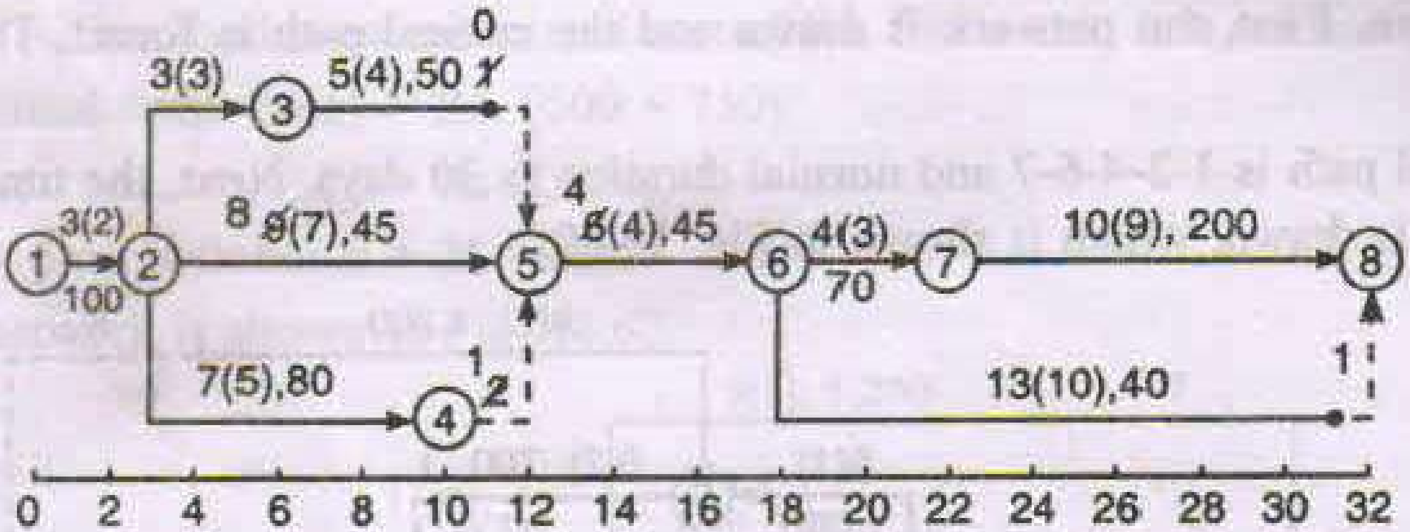


Fig. 14.58

The above results are summarised in the table below :

Duration (weeks)	*Normal direct cost (Rs.)*	*Crash cost (Rs.)*	*Indirect cost (Rs.)*	*Total cost (Rs.)*
32	4,220	–	1,600	5,820
31	4,220	45	1,550	5,815
30	4,220	90	1,500	5,810
29	4,220	135	1,450	5,805

∴ Optimum project duration = 29 weeks,

optimum total project cost = Rs. 5,805.