Ⓐ Check for optimality (IBFS-NW) ⑤

(i) $m+n-1$ = No. of allocations / Basic cells. – T.P. without degenracy
$3+4-1 = 6$ = No. of Basic cells.

(ii) All allocations are independent of each other. i.e. it is not possible to have close loop passing through basic cells.

				u_i
19 (5)	30 (2)	50 [10]	10 [20]	−30
70 [51]	30 −θ (6)	40 +θ (3)	60 [70]	−30
40 [−9]	8 [−52] +θ	70 −θ (4)	20 (14)	0
v_j: 49	60	70	20	

If $m+n-1$ = No. of allocations / Basic cells.
└ T.P. without degenracy

& $m+n-1 \neq$ No. of allocations / Basic
└ T.P. with dengenry.

Check for optimality

1. Assign u_i or v_j to the row or coloumn which has maximum number of allocated / basic cells and assign '0' to it.
2. Calculate remaining u_i and v_j by observing
(u_i+v_j) = value of basic cell.
3. Calculate net evaluations only for non basic cells by observing Net evaluations = Cell value − (u_i+v_j)
4. If all the net evaluations ≥ 0 – solution is optimum Otherwise improve the solution.

$50-(-30+70) = 10$ | $60-(-30+20) = 70$
$10-(-30+20) = 20$ | $40-(49+0) = -9$

As all net evaluations are not positive (≥0), the IBFS is not optimum.

Improving the solution

1) Select the most negative net evaluation and mark (✓) the corresponding cell; (such that it will not form a closed loop by considering previous basic cells.
2) Complete the close loop passing through basic cells.
3) Allocate '+Q' in marked cell. and −Q and +Q at each corner of the loop
4) Calculate min value of Q only considering only −θ.
5) Improve the solution accordingly.

Improving the solution

$$\text{Min}\left\{(8-Q),(4-Q)\right\}=0$$

$$\therefore Q=\underline{4}$$

19 (5)	30 (2)	50	10
70	30 (2)	40 (7)	60
40	8 (4)	70	20 (14)

S.B.F.S

$$TTC = \left[(19\times5)+(30\times2)+(30\times2)+(40\times7)+(8\times4)+(20\times14)\right]$$

$$= 95+60+60+280+32+280$$

Check for optimality

1) $m+n-1 = 3+4-1 = 6 =$ No. of allocations

2) All allocations are indepent of each other i.e. it is not possible to have close loop passing through basic cells.

19 (5)	30 $-\theta$ (2)	50 $\boxed{10}$	10 $\boxed{-32}$ $+\theta$ ✓	30
70 $\boxed{51}$	30 (2)	40 (7)	60 $\boxed{18}$	30
40 $\boxed{43}$	8 $+\theta$ (4)	70 $\boxed{52}$	20 $-\theta$ (14)	8
-11	0	10	12	

As all net evalvations are not positive (≥ 0), the solution is not optimum.

Improving the solution

$$\text{Min}\{(2-\theta), (14-\theta)\} = 0 \quad \therefore \theta = \underline{\underline{2}}$$

19 (5)	30	50	10 (2)
70	30 (2)	40 (7)	60
40	8 (6)	70	20 (12)

T.B.F.S.

$$T.T.C = Rs\left[(19\times5)+(10\times2)+(30\times2)+(40\times7)+(8\times6)+(20\times12)\right]$$

$$= (95+20+60+280+48+240) = Rs$$

$$= Rs\ 743$$

Check for optimality

1) $m+n-1 = 3+4-1 = 6 =$ No. of allocations

2) All allocations are independent of each other i.e. it is not possible to have a close loop passing through basic cells.

	W_1	W_2	W_3	W_4	
f_1	19 (5)	30 $\boxed{32}$	50 $\boxed{42}$	10 (2)	-10
f_2	70 $\boxed{19}$	30 (2)	40 (7)	60 $\boxed{18}$	22
f_3	40 $\boxed{11}$	8 (6)	70 $\boxed{52}$	20 (12)	0
	29	8	18	20	

As all net evaluations are ≥ 0, the obtained solutions ~~are~~ ~~pos~~ is optimum.

$\therefore$ Optimum Transportation Cost = Rs 743/-

The optimum delivery schedule is.

factory f_1 to Warehouse W_1 - 5 units

factory f_1 to Warehouse W_4 - 2 units

factory f_2 to Warehouse W_2 - 2 units

factory f_2 to Warehouse W_3 - 7 units

factory f_3 to Warehouse W_2 - 6 units

factory f_3 to Warehouse W_4 - 12 units.

⑧ Check for optimality (Row Minima Method) TTC – 1110

19 [-11]	30 [-20]	50 [-10]	10 (7)	−10
70 [60]	30 −θ (8)	40 +θ (1)	60 [70]	−30
40 (5)	8 [-52] ✓ +θ	70 −θ (6)	20 (7)	0
40	60	70	20	

I BFS

1) m+n−1 = 3+4−1 = 6 = No of Basic cells. – T.P. without degeneracy

2) All allocations are independent of each other i.e. it is not possible to have close loop passing through basic cells.

As all net evaluations are not positive (≥0), the solution is not optimum.

Improving the solution

Min {(8−θ), (6−θ)} = 0 ∴ θ = 6

19	30	50	10 (7)
70	30 (2)	40 (7)	60
40 (5)	8 (6)	70	20 (7)

SBFS

Total transportation cost –

TTC = Rs [(10×7) + (30×2) + (40×7) + (40×5) + (8×6) + (20×7)]

= Rs [70 + 60 + 280 + 200 + 48 + 140)

= Rs 798

Check for optimality

				u_i
19 $\boxed{-11}$ ✓ $+\theta$	30 $\boxed{32}$	50 $\boxed{42}$	10 $-\theta$ (7)	−10
70 $\boxed{8}$	30 (2)	40 (7)	60 $\boxed{18}$	22
40 $-\theta$ (5)	8 (6)	70 $\boxed{52}$	20 $+\theta$ (7)	0
40	8	18	20	

i) $m+n-1 = 3+4-1 =$ No of basic cells − T.P. without degeneracy

ii) All allocations are independent of each other i.e. it is not possible to have close loop passing through basic cells.

Improving the solution

$$\text{Min}\ \{(7-\theta),\ (5-\theta)\} = 0 \quad \therefore \theta = 5$$

19 (5)	30	50	10 (2)
70	30 (2)	40 (7)	60
40	8 (6)	70	20 (12)

TBFS

$$TTC = [(19\times5)+(10\times2)+(30\times2)+(40\times7)+(8\times6)+(20\times12)]$$

$$= Rs\ (95+20+60+280+48+240) = Rs\ 743$$

Same solution with same allocations found to be optimum by N-W Corner rule.

(c) Check for optimality (Matrix minima method TTC - Rs 814)

19 $\boxed{-11}$ ✓ +θ	30 $\boxed{32}$	50 $\boxed{50}$	10 −θ (7)	−10
70 (2)	30 $\boxed{-8}$	40 (7)	60 $\boxed{10}$	30
40 −θ (3)	8 (8)	70 $\boxed{60}$	20 +θ (7)	0
40	8	10	20	

IBFS

1) All m+n−1 = 3+4−1 = 6 = No of allocations

2) All allocations are independent of each other i.e. it is not possible to have close loop passing through basic cells.

— Solution not optimum.

$$\text{Min}\{(3-\theta), (7-\theta)\} = 0 \qquad \theta = 3$$

19 (3)	30	50	10 (4)
70 (2)	30	40 (7)	60
40	8 (8)	70	20 (10)

SBFS

TTC = Rs [(19×3) + (10×4) + (70×2) + (40×7) + (8×8) + (20×10)]

= Rs (57 + 40 + 140 + 280 + 64 + 200)

= Rs 781/-

Check for optimality

19 +θ (3)	30 $\boxed{32}$	50 $\boxed{61}$	10 −θ (4)	−10
70 −θ (2)	30 $\boxed{-19}$ ✓ +θ	40 (7)	60 $\boxed{-1}$	41
40 $\boxed{11}$	8 −θ (8)	70 $\boxed{71}$	20 +θ (10)	0
29	8	−1	20	

1) m+n−1 = 3+4−1 = 6 = No. of allocations. – T.P. without degeneracy
2) All allocations are independent of each other i.e. it is not possible to have close loop passing through basic cells.

As all net evaluations are not positive (≥0), the obtained solution is not optimum.

Improving the solution

$$\text{Min}\{(8-\theta), (4-\theta), (2-\theta)\} = 0 \therefore \theta = 2$$

19 (5)	30	50	10 (2)
70	30 (2)	40 (7)	60
40	8 (6)	70	20 (12)

$$TTC = [(19\times5)+(10\times2)+(30\times2)+(40\times7)+(8\times6)+(20\times12)]$$

$$= \text{Rs}\ (95+20+60+280+48+240)$$

$$= \underline{\text{Rs}\ 743/-}$$

Optimum

D) Check for optimality

(VAM & Coloumn Minima Method Rs 779/-)

19 (5)	30 $\boxed{32}$	50 $\boxed{60}$	10 (2)	10
70 $\boxed{19}$	30 $\boxed{-18}$ $+\theta$	40 (7)	60 $-\theta$ (2)	60
40 $\boxed{11}$	8 $-\theta$ (8)	70	20 $+\theta$ (10)	20
9	−12	−20	0	

IBFS

1) m+n−1 = No. of allocations = 6 = No of basic cells.
2) All allocations are independent of each other. i.e. it is not possible to have close loop passing through basic cells.

As all net evaluations are not positive (≥ 0), the solution is not optimum.

Improving the solution

$$\text{Min } \theta = \{(2-\theta), (8-\theta)\} = 0 \quad \therefore \theta = 2$$

19 (5)	30	50	10 (2)
70	30 (2)	40 (7)	60
40	8 (6)	70	20 (12)

$$TTC = [(19\times5) + (10\times2) + (30\times2) + (40\times7) + (8\times6) + (20\times12)]$$

$$= Rs\ 743/-$$

optimum