Assignment Problem (A.P.)

- Assignment cost / cost – Minimization Model.
- Allocation model. + Hospital

Reduced matrix method / Hungarian Method /

Prob. 1

A machine tool company decides to make four subassemblies through four contractors. Each contractor is to receive only one subassembly. The cost of each subassembly is determined by the bids submitted by each contractor as shown below in rupees. Assign the different subassemblies to contractors so as to minimize total cost.

Contractors

Subassemblies	1	2	3	4
1	15	13	14	17
2	11	12	15	13
3	13	12	10	11
4	15	17	14	16

Before we solve

+ Assignment matrix should be a square matrix
  + Balanced A.P.

+ Minimization Type. (cost, time, distance travelled)

Let us solve

1. Select the least cost from the row and substract from entire row. So that at least one zero will be available in each ~~col~~ row.

2. Examine the coloumn, for at least one zero.
If not, then select the least cost/element from the respective coloumn and substract from the remaining element.

3. Assign whererer there is zero, such that it requires to cancel minimum number of zeros.

$\boxed{0}$ – Assigned zero

$\boxed{\cancel{0}}$ – Cancelled zero/marked zero

4. If each row as well each coloumn consists of atleast and at most one assigned zero, then calculate Total assignment cost by considering the values in original matrix of assigned zero.

Solution

Substracting minimum element/cost from each row from entire row we get-

Contractor

Sub assemblies	1	2	3	4
1	15	13	14	17
2	11	12	15	13
3	13	12	10	11
4	15	17	14	16

	1	2	3	4
1	2	0	1	4
2	0	1	4	2
3	3	2	0	1
4	1	3	0	2

Substracting the least from 4th Coloumn we get-

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 2 & \boxed{0} & 1 & 3 \\ 2 & \boxed{0} & 1 & 4 & 1 \\ 3 & 3 & 2 & \cancel{0} & \boxed{0} \\ 4 & 1 & 3 & \boxed{0} & 1 \end{array}$$

As each row as well as each coloumn consists of atleast and atmost assignment, the obtained solution is optimum.

Refeering to the original cost matrix.

Minimum assignment cost-

$$Z_{min} = [13 + 11 + 11 + 14] = \text{Rs } \underline{49}$$

The optimum assignment policy is.

Subassembly		Contractor
1	–	2
2	–	1
3	–	4
4	–	3.

Prob 2

Four different jobs are to be done on four different machines. The matrix below gives time to complete ~~an~~ a job on each machine. Assign jobs to different machines so that total time to complete all jobs can be minimized.

Machines

Jobs	A	B	C	D
1	5	7	11	6
2	8	5	9	6
3	4	7	10	7
4	10	4	8	3

Square
Mini

Solution

Subtracting the least cost/time/element from each row

	A	B	C	D
1	0	2	6	1
2	3	0	4	1
3	0	3	6	3
4	7	1	5	0

Subtracting the least element for coloumn 3 we get -

	A	B	C	D
1	$\boxed{0}$	2	2	1
2	3	$\boxed{0}$	$\cancel{0}$	1
3	$\cancel{0}$	3	2	3
4	7	1	1	$\boxed{0}$

— I.B.F.S

(Initial Basic feasible Solution)

As each row and each coloumn doesnot contains atleast & atmost one assigned zero, the obtained IBFs/solution is not optimum.

Check for optimality:

1. Mark (✓) the row(s) which donot contains assigned zero.
2. Inspect the marked row for marked/canceled zero and mark (✓) the corresponding coloumn.
3. Inspect the marked coloumn for assigned zero and mark(✓) the corresponding row.
4. Repeat 1, 2, & 3 as required.
5. Draw lines through marked coloumn(s) and unmarked row(s).
6. If the number of lines < order of matrix solution is not optimum. then improve the solution.

	A	B	C	D	
1	$\boxed{0}$	2	2	1	✓
2	3	$\boxed{0}$	$\cancel{0}$	1	
3	$\cancel{0}$	3	2	3	✓
4	7	1	1	$\boxed{0}$	
	✓				

As 3 (no. of lines) < order of matrix 4.

Improving the solution

1. Select the least from uncovered elements.
2. Add at intersection and substract from remaining uncovered

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The least from uncovered elements is 1, adding at intersection and substracting from remaining uncovered we get –

1	$\boxed{0}$	1	1	$\cancel{0}$
2	4	$\boxed{0}$	$\cancel{0}$	1
3	$\cancel{0}$	2	1	2
4	8	1	1	$\boxed{0}$

– S.B.F.S. (Second Basic feasible Solution)

check for optimality –

	A	B	C	D
1	$\boxed{0}$	1	1	$\cancel{0}$
2	4	$\boxed{0}$	$\cancel{0}$	1
3	$\cancel{0}$	2	1	2
4	8	1	1	$\boxed{0}$

As no of lines (3) < Order of matrix (4)

S.B.F.S. is not optimum.

Improving The solution

Selecting The least(1) from uncovered elements, adding it at intersection and substracting from remaining uncovered we get -

	A	B	C	D
1	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$
2	5	$\boxed{0}$	$\cancel{0}$	2
3	$\cancel{0}$	1	$\boxed{0}$	2
4	8	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

T.B.F.S

(Third Basic feasible Solution)

As each row & coloumn consists of an assignment Optimum solution is

$$Z_{min} = [\, 5 + 5 + 10 + 3 \,] = 23$$

Optimum assignment is

Jobs		Machines
1	—	A
2	-	B
3	-	C
4	-	D

A) Unbalanced assignment problem.

B) Maximization assignment problem.

C) Unbalanced & maximization A.P.

D) Restricted A.P.

E) Travelling salesman Problem (T.S.P.)

Unbalanced minimization Problem

Prob 3

A company has four machines on which to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is as below. What are the jobs assignments which will minimize the cost:

Machine

Job		W	X	Y	Z
	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

Solution

Balancing the given matrix by adding dummy row we get-

	W	X	Y	Z
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22
dummy	0	0	0	0

Substracting the least from each row, we get-

	W	X	Y	Z	
A	$\boxed{0}$	6	10	14	✓
B	$\cancel{0}$	5	9	11	✓
C	$\cancel{0}$	5	9	12	✓
Dummy	$\cancel{0}$	$\boxed{0}$	0	0	
	✓				

IBFs

Improving the solution

Substracting the least (5) from remaining uncovered and adding at intersection we get-

	W	X	Y	Z	
A	$\boxed{0}$	1	5	9	✓
B	$\cancel{0}$	$\boxed{0}$	4	6	✓
C	$\cancel{0}$	$\cancel{0}$	4	7	✓
D	5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	
	✓	✓			

S.B.F.s.

As no of lines (3) < order of matrix (4)

SBF.s. not optimum.

Improving the solution

Substracting the least (4) .....

Machine

Job		W	X	Y	Z
	A	$\boxed{0}$	1	1	5
	B	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2
	C	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	3
	Dummy	9	4	$\cancel{0}$	$\boxed{0}$

T.B.F.s.

The optimum cost

$Z_{min} = [18 + 13 + 19 + \underline{0}] = \underline{50}$

The optimum assignment policy is.

Job

A − w

B − x

C − y

Prob 4 − Maximization Assignment problem

A department has four subordinates and four tasks to be performed. The subordinates differ in efficiency. The estimates of the profit in rupees each subordinate would earn is given the matrix below. How should the tasks be allocated so as to maximize the total earnings?

Task

Subordinate	A	B	C	D
1	5	40	20	5
2	25	35	30	25
3	15	25	20	10
4	15	5	30	15

Solution −

Maximization & Balanced.

For converting maximization to minimization, substracting each element from the highest element from the matrix.

The highest element is 40, substracting each element from it, we get

	A	B	C	D
1	35	0	20	35
2	15	5	10	15
3	25	15	20	30
4	25	35	10	25

Substracting the least from each row we get

	A	B	C	D
1	35	0	20	35
2	10	0	5	10
3	10	0	5	15
4	15	25	0	15

Substracting the least element from Col. A & D we get

	A	B	C	D
1	25	$\boxed{0}$	20	25
2	$\cancel{0}$	$\cancel{0}$	5	$\boxed{0}$
3	$\boxed{0}$	$\cancel{0}$	5	5
4	5	25	$\boxed{0}$	5

Referring to original matrix.

$$Z_{max} = [40 + 25 + 15 + 30] = 110$$

Optimum assignment

Subordinate		Task
1	–	B
2	–	D
3	–	A

Prob5 – Maximization and Unbalanced

Solve The following assignment problem for maximization

Task

Subordinate	A	B	C	D
1	5	40	20	5
2	25	35	30	25
3	15	25	20	10

Selution

Balancing by introducing dummy row, we get -

	A	B	C	D
1	5	40	20	5
2	25	35	30	25
3	15	25	20	10
Dummy	0	0	0	0

Substracting all the elements from The highest element (40), so as to convert maximization to minimization we get -

	A	B	C	D
1	35	0	20	35
2	15	5	10	15
3	25	15	20	30
Dummy	40	40	40	40

Substracting the least from each row, we get-

A	B	C	D
35	$\boxed{0}$	20	35
10	$\cancel{0}$	5	10
10	$\cancel{0}$	5	15
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$

– IBFS

Improving the solution we get-

30	$\boxed{0}$	15	30
5	$\cancel{0}$	$\boxed{0}$	5
5	$\cancel{0}$	$\cancel{0}$	10
$\boxed{0}$	5	$\cancel{0}$	$\cancel{0}$

S.B.F.S.

Improving the solution

Task

Subordinate	A	B	C	D
1	25	$\boxed{0}$	15	25
2	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$
3	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	5
Dummy	$\cancel{0}$	10	5	$\boxed{0}$

$$Z_{max} = [40 + 30 + 15] = 85$$

Optimum Assignment-

Subordinate		Task
1	-	B
2	-	C

Prob 6 - Restricted A.P.

Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D & E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The following is the corresponding cost matrix. Find the optimum assignment schedule.

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	–	5	4
M_3	–	6	9	6	2
M_4	9	3	7	2	3

Solution:-

Balancing AP by introducing dummy row, we get -

(Minimization & restricted)

M_1	4	6	10	5	6
M_2	7	4	∞	5	4
M_3	∞	6	9	6	2
M_4	9	3	7	2	3
Dummy	0	0	0	0	0

Subtracting ~~requi~~ elements we get-

	A	B	C	D	E
M1	$\boxed{0}$	2	6	1	2
M2	3	$\boxed{0}$	∞	1	$\cancel{0}$
M3	∞	4	7	4	$\boxed{0}$
M4	7	1	5	$\boxed{0}$	1
Dummy	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

$Z_{min} = (4+4+2+2) = 12$

Machine		Place
M_1	—	A
M_2	—	B
M_3	—	E
M_4	—	C

Theory Questions to be prepared.

1. Write mathematical formulation for assignment problem.
2. State and explain similarities and differences inbetween assignment problem. and transportation problem.
3. Write short note – Travelling salesman problem.

Prob7 :- Travelling Salesman Problem (TSP)

A salesman wants to visit A, B, C, D and E. He doesnot want to visit any city twice before completing his tour of all the cities and wishes to return to the point of starting journey. Cost of going from one city to another is shown below. Find the least cost route.

	A	B	C	D	E
A	–	2	5	7	1
B	6	–	3	8	2
C	8	7	–	4	7
D	12	4	6	–	5
E	1	3	2	8	–

	A	B	C	D	E
A	–	1	4	6	0
B	4	–	1	6	0
C	4	3	–	0	3
D	8	0	2	–	1
E	0	2	1	7	–

Reduced cost matrix with zero in each row

	A	B	C	D	E
A	–	1	3	6	0
B	4	–	0	6	0
C	4	3	–	0	3
D	8	0	1	–	1
E	0	2	0	7	–

Reduced cost matrix with zero in each column

$A \rightarrow E \rightarrow A$ × (Not feasible)

$ \rightarrow C \rightarrow D \rightarrow B \xrightarrow{4} A \quad = 4$

✓ $A \xrightarrow{1} B \rightarrow C \rightarrow D \xrightarrow{1} E \rightarrow A \quad = 2$

$A \xrightarrow{3} C \rightarrow D \rightarrow B \rightarrow E \rightarrow A \quad = 3$

The resulting feasible solution is

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow A.$$

and it involves cost of

$$(2+3+4+5+1) = \underline{Rs\ 15}$$