Games theory

- Theory of games
- Competitive strategies

J. von. Neumann - father of games theory.

- Based on 'minmax principle'.

You should know

1. What are the characteristics of game?
2. What do you understand by 'pure strategy' and 'mixed strategy'.
3. Explain the following terms.
   i) player ii) Game iii) play iv) strategy
   v) pure strategy vi) Mixed strategy vii) Optimal strategy
   viii) Two person zero sum game ix) Zero sum game.
   x) pay off.
4. What are the limitations of games theory.
5. Explain principles of dominance.

Rules for solving games theory.

1. Look for saddle point.
2. Reduce the game by dominance.
3. Solve for mixed strategy

Prob 1

Solve the game. find out the game value and optimum strategies for each player

		Player B 1	2
Player A	1	4	6
	2	3	5

<u>Solution</u>

		B 1	2	Max (min)
A	1	4	6	(4)
	2	3	5	3
min (max)		(4)	6	

As min (max) = max (min)

saddle point exists

∴ value of game = $v = 4$

Optimum strategies for player A = $\{1, 0\}$

and

Optimum strategies for player B = $\{1, 0\}$

Prob

Solve the game

		Player B				
		1	2	3	4	5
Player A	1	-4	-2	-2	3	1
	2	1	0	-1	0	0
	3	-6	-5	-2	-4	4
	4	3	1	-6	0	-8

Solution

		B					
		1	2	3	4	5	max (min)
A	1	-4	-2	-2	3	1	(-4)
	2	1	0	-1	0	0	(-1)
	3	-6	-5	-2	-4	4	-6
	4	3	1	-6	0	-8	-8
min (max)		3	1	(-1)	3	4	

As min (max) = max (min)

saddle point exists

$\therefore$ Game value $= v = -1$

Optimum strategy for player A = $\{0, 1, 0, 0\}$

Optimal strategy for player B = $\{0, 0, 1, 0, 0\}$

Dominance property | Principles of dominance

Dominance property for coloumn.

Every value in the dominating coloumn(s) must be less than or equal to the corresponding value of the dominated coloumn.

+ Avg. dominance for coloumns.

Dominance property for rows.

Every value in the dominating row(s) must be greater than or equal to the corresponding values of the dominated rows

+ Average dominance for rows.

+ Any number coloumn / row can dominate any number of coloumn / row.

+ While solving game donot reduce the game by dominance after 2x2.

$\boxed{\text{Prob 3}}$ Reduce the game by dominance and solve it.

B

A	1	2	3	4	5
1	1	3	2	7	4
2	3	4	1	5	6
3	6	5	7	6	5
4	2	0	6	3	1

Solution

B

A	1	2	3	4	5	max (min)
1	1	3	2	7	4	1
2	3	4	1	5	6	1
3	6	5	7	6	5	(5)
4	2	0	6	3	1	0
min(max)	6	(5)	7	7	6	

As min (max) = max (min)

Saddle point exists.

$\therefore$ game value $= v = 5$

Row 3 dominates row 4

B

A	1	2	3	4	5
1	1	3	2	7	4
2	3	4	1	5	6
3	6	5	7	6	5

Coloumn 1 and 2 dominates Coloumn 4

Coloumn 2 dominates Coloumn 5

$$
\begin{array}{cc|ccc}
 & & 1 & 2 & 3 \\
\hline
 & 1 & 1 & 2 & 2 \\
A & 2 & 3 & 4 & 1 \\
 & 3 & 6 & 5 & 7
\end{array}
$$

Row 3 dominates Row 1 and 2

$$
\begin{array}{c|ccc}
 & 1 & 2 & 3 \\
\hline
3 & 6 & 5 & 7
\end{array}
$$

Coloumn 2 dominates Coloumn 1 & 3

$$
\begin{array}{c|c}
 & 2 \\
\hline
3 & 5
\end{array}
$$

$\therefore$ Game value $= v = 5$

Optimum strategy for player A $= \{0, 0, 1, 0\}$

Optimum strategy for player B $= \{0, 1, 0, 0, 0\}$

4. Reduce the game by dominance principles.

		B 1	2	3	4	5	6	max (min)
	1	4	2	0	2	1	1	0
	2	4	3	1	3	2	2	(1)
A	3	4	3	7	-5	1	2	-5
	4	4	3	4	-1	2	2	-1
	5	4	3	3	-1	2	2	-2
min(max)		4	3	7	3	(2)	2	

As min(max) $\neq$ max (min)

There is no saddle point.

Row 2 dominates Row 1

Row 4 dominates Row 5

	1	2	3	4	5	6
2	4	3	1	3	2	2
3	4	3	7	-5	1	2
4	4	3	4	-1	2	2

Cols 6, 5 & 4 dominates Col. 1 & 2

Column 5 dominates Col- 6.

	3	4	5
2	1	3	2
3	7	-5	1
4	4	-1	2

Average of Coloumn 3 & 4 dominates Col. 5.

$$\begin{bmatrix} \frac{1+3}{2} \\ \frac{7-5}{2} \\ \frac{4-1}{2} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3/2 \end{bmatrix}$$

	3	4
2	1	3
3	7	-5
4	4	-1

Avg. of Row 2 and 3 dominates Row 4

1	3
7	-5