Sequencing Model

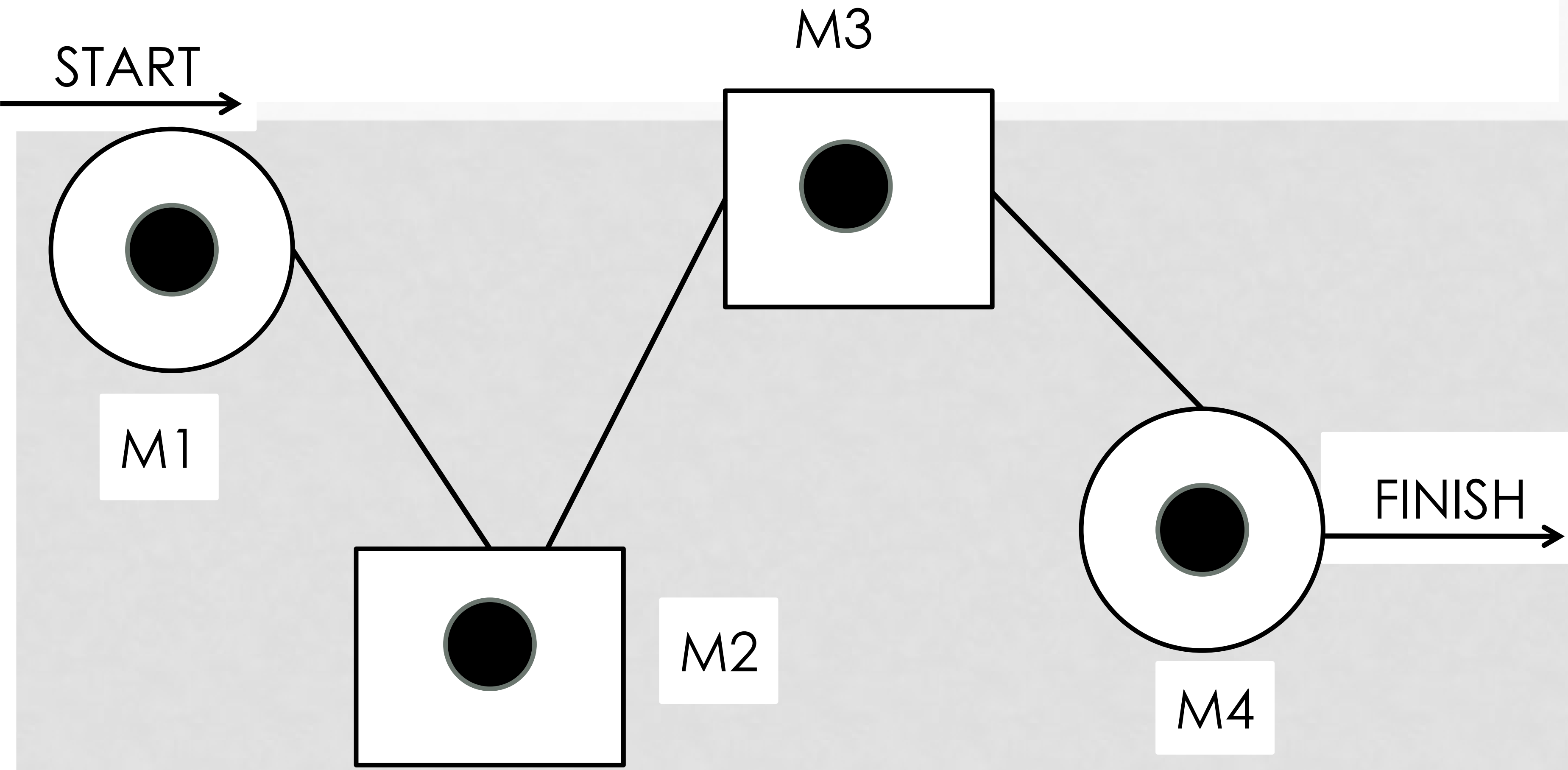
INTRODUCTION

- A sequence is the order in which the jobs are processed. Sequence problems arise when we are concerned with situations where there is a choice in which a number of tasks can be performed. A sequencing problem could involve:
- Jobs in a manufacturing plant.
- Aircraft waiting for landing and clearance.
- Maintenance scheduling in a factory.
- Programmes to be run on a computer.
- Customers in a bank & so-on.

TERMS USED

- ***Job :*** the jobs or items or customers or orders are the primary stimulus for sequencing. There should be a certain number of jobs say 'n' to be processed or sequenced.
- ***Number of Machines*** : A machine is characterized by a certain processing capability or facilities through which a job must pass before it is completed in the shop. It may not be necessarily a mechanical device. Even human being assigned jobs may be taken as machines. There must be certain number of machines say 'k' to be used for processing the jobs.

- ***Processing Time*** : Every operation requires certain time at each of machine. If the time is certain then the determination of schedule is easy. When the processing times are uncertain then the schedule is complex.
- ***Total Elapsed Time*** : It is the time between starting the first job and completing the last one.
- ***Idle time*** : it is the time the machine remains idle during the total elapsed time.
- ***Technological order*** : different jobs may have different technological order. It refers to the order in which various machines are required for completing the jobs e.g.



WHERE M1, M2, M3, M4 ARE MACHINES.

TYPES OF SEQUENCING PROBLEMS

There can be many types of sequencing problems which are as follows:

- Problem with ‘n’ jobs through one machine.
- Problem with ‘n’ jobs through two machines.
- Problem with ‘n’ jobs through three machines.
- Here the objective is to find out the optimum sequence of the jobs to be processed and starting and finishing time of various jobs through all the machines.

- No passing rule: it implies that passing is not allowed i.e. the same order of jobs is maintained over each machine. If each of the ‘n’ jobs is to be processed through ‘m’ machines in order of

M1 → M2 → M3 → M4 then this rule will mean that each job will go to machine M1 first then to M2 and lastly to M4 after M3.

- Static arrival pattern. If all the jobs arrive simultaneously.
- Dynamic arrival pattern. Where the jobs arrive continuously.

BASIC ASSUMPTIONS

Following are the basic assumptions underlying a sequencing problem:

- No machine can process more than one job at a time.
- The processing times on different machines are independent of the order in which they are processed.
- The time involved in moving a job from one machine to another is negligibly small.
- Each job once started on a machine is to be performed up to completion on that machine.
- All machines are of different types.
- All jobs are completely known and are ready for processing.
- A job is processed as soon as possible but only in the order specified.

These assumptions are considered to make the sequencing problem a simple one otherwise complicacy may arise.

PROCESSING 'N' JOBS THROUGH TWO MACHINES

- Let there be 'n' jobs each of which is to be processed through two machines say A & B, in the order AB. That is each job will go to machine A first and then to B in other words passing off is not allowed. All 'n' jobs are to be processed on A without any idle time. On the other hand the machine B is subject to its remaining idle at various stages.
- Let A1 A2………….An & B1 B2……..Bn be the expected processing time of n jobs on these two machines.

- ***Step 1.*** Select the smallest processing time occurring in list Ai or Bi, if there is a tie select either of the smallest processing time.
- ***Step 2.*** If the smallest time is on machine A, then place it at first place if it is for the B machine place the corresponding job at last. Cross off that job.
- ***Step 3.*** if there is a tie for minimum time on both the machines then select machine A first & machine B last and if there is tie for minimum on machine A (same machine) then select any one of these jobs first and if there is tie for minimum on machine B among and select any of these job in the last.
- ***Step 4.*** Repeat step 2 & 3 to the reduced set of processing times obtained by deleting the processing time for both the machines corresponding to the jobs already assigned.

- ***<u>Step 5</u>***. Continue the process placing the job next to the last and so on till all jobs have been placed and it is called optimum sequence.
- ***<u>Step 6.</u>*** after finding the optimum sequence we can find the followings

i. Total elapsed time = Total time between starting the first job of the optimum sequence on machine A and completing the last job on machine B.

ii. Idle time in machine A = Time when the last job in the optimum sequence is completed on Machine B – Time when the last job in the optimum sequence is completed on Machine A.

QUES.: IN A FACTORY, THERE ARE SIX JOBS TO PROCESS, EACH OF WHICH SHOULD GO TO MACHINES A & B IN THE ORDER AB. THE PROCESSING TIMINGS IN MINUTES ARE GIVEN, DETERMINE THE OPTIMAL SEQUENCING & TOTAL ELAPSED TIME.

JOBS	1	2	3	4	5	6
MACHINE A	7	4	2	5	9	8
MACHINE B	3	8	6	6	4	1

Solution: ***step 1*** ***:the least*** of all the times given in for job 6 in machine B. so perform job 6 in the end. It is last in the sequences. Now delete this job from the given data.

					6

- ***Step 2:*** Of the remaining timings now the minimum is for job 3 on machine A. so do the job 3 first. Now delete this job 3 also.

3					6

- ***Step 3:*** Now the smallest time is 3 minutes for job first on machine B. thus perform job 1 at the second last before job 6.

3				1	6

- ***Step 4***: after assigning job 1, we observe that the smallest value of 4 minutes is shared by job 2 on Machine A and job 5 on machine B. so perform job 2 first and job 5 in at the end end, i.e. job 2 after job 3 and job 5 before job 1.

3	2		5	1	6

- ***Step 5***: now the only job remaining is job 4 it shall be assigned the only space left in the sequence. Optimal resultant sequence of jobs is

3	2	4	5	1	6

THE TOTAL ELAPSED TIME T IS OBTAINED AS UNDER:

JOB SEQUENCE			MACHINE A		MACHINE B		IDLE TIME FOR MACHINE B
	MAC A	MAC B	IN	OUT	IN	OUT	
3	2	6	0	2	2	8	2
2	4	8	2	6	8	16	-
4	5	6	6	11	16	22	-
5	9	4	11	20	22	26	-
1	7	3	20	27	27	30	1
6	8	1	27	35	35	36	5

TOTAL IDLE TIME FOR MACHINE A IS 1 MINUTE (36-35)
AND FOR MACHINE B IS 2+1+5=8 MINUTES.

PROCESSING 'N' JOBS ON 3 MACHINES

- Let there be 'n' jobs each of which is to processed through three machines say A, B & C in the order ABC. No passing off jobs is permitted and actual processing time in all the three machine is known. The problem again is to find the optimum sequences which minimizes T (Processing Time)

JOB	MACHINE A	MACHINE B	MACHINE C
1	A1	B1	C1
2	A2	B2	C2
3	A3	B3	C3
i	Ai	Bi	Ci
n	An	Bn	Cn

- No general solution is available at present for such a case. However previous method given by Johnson can be applied if the following two conditions are satisfied.
- <u>***Condition 1***</u>: the minimum of the times for different jobs on machine A is at least equal to the maximum of the times of different jobs on machine B.
- <u>***Condition 2***</u>: The minimum of the times for different jobs on machine C is at least equal to the maximum of the times of different jobs in machine B.

i.e. Find (i) Minimum of Ai (ii) Minimum of Ci (iii) Maximum of Bi

Check (i) Ai ≥ Bi and/or (ii)Ci ≥ Bi

If the above two are not satisfied the method fails otherwise we can proceed.

Method: Replace the given problem with an equal problem involving 'n' jobs two machines here two machines fictitious say G & H and defines the corresponding processing time i.e. G_i & H_i as follows:

$G_i = A_i + B_i$, $H_i = B_i + C_i$

Thus for the job 1 $G_1 = A_1 + B_1$, $H_1 = B_1 + C_1$

For the job 2 $G_2 = A_2 + B_2$, $H_2 = B_2 + C_2$ so on

Now apply the same procedure and find out the optimal sequence which shall also be optimal for the original problem.

QUES.: A MACHINE OPERATOR HAS TO PERFORM 3 OPERATIONS, TURNING, THREADING & KNURLING ON THREE MACHINES A, B & C IN THE ORDER ABC. FIND THE OPTIMUM SEQUENCES WHEN WHEN THE TIME IN HOURS ARE GIVEN.

JOB	TURNING AI	THREADING BI	KNURLING CI
1	3	8	13
2	12	6	14
3	5	4	9
4	2	6	12
5	9	3	8
6	11	1	13

Solution: Min. of Ai = 2 >Max. of Bi= 8 not satisfied.

Min. of Ci = 8 > Max. of Bi = 8 satisfied.

Since one of the conditions are satisfied, the above problem can be converted into ‘n’ jobs, 2 machines.

Two fictitious operations G & H can be written as (in times)

Job	Gi = Ai + Bi	Hi = Bi + Ci
1	3 + 8 = 11	8 + 13 = 21
2	12 + 6 = 18	6 + 14 = 20
3	5 + 4 = 9	4 + 9 =13
4	2 + 6 = 8	6 + 12 = 18
5	9 + 3 = 12	3 + 8 = 11
6	11 + 1 = 12	1 + 13 = 14

- Examining the column Gi &Hi we find the smallest time is 8 of job 4 on G, thus we assign job 4 first as shown below now delete 4.

4					

- The next smallest time is 9 of job 3 on machine G. Hence we schedule job 3 as next to 4 also delete 3.

4	**3**				

- After assigning 4 & 3, we see that there are equal minimum values, processing time of 11 minutes under job 1 on G and job 5 on H. According to the rule, job 1 is scheduled next to 3 while job 5 is assigned last as shown below

4	**3**	**1**			**5**

- Now jobs 2 & 6 are left. The smallest value is 12 for job 6 on machine G so we assign job 6 after 1 and we get the optimum sequence as

4	3	1	6	2	5

- Total elapsed time for the original problem

JOB	TURNING		THREADING		KNURLING		IDLE TIME	
	IN	OUT	IN	OUT	IN	OUT	THREADING	KNURLING
4	0	2	2	8	8	20	2	8
3	2	7	8	12	20	29	-	-
1	7	10	12	20	29	42	-	-
6	10	21	21	22	42	55	1	-
2	21	33	33	39	55	69	11	-
5	33	42	42	45	69	77	3	-

- Thus the minimum elapsed time is 77 minutes
- Idle time for Turning = 35(77-42)
- Idle time for Threading = 2 + 1 + 11 + 3 + 32 =49
- Idle time for Knurling = 8 minutes.