CHAPTER I

HISTORICAL DEVELOPMENT OF OPERATIONS RESEARCH

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Historical Development of Operations Research

Development—Definition: Characteristics and Phases—Scientific Method—Types of Models—General methods for solving Operations Research Models.

1.1. INTRODUCTION

The subject **OPERATIONS RESEARCH** is a branch of mathematics—specially applied mathematics, used to provide a scientific base for management to take timely and effective decisions to their problems. It tries to avoid the dangers from taking decisions merely by guessing or by using thumb rules. Management is the multidimensional and dynamic concept. It is multidimensional, because management problems and their solutions have consequences in several dimensions, such as human, economic social and political fields. As the manager operates his system in an environment, which will never remain static, hence is dynamic in nature. Hence any manager, while making decisions, consider all aspects in addition to economic aspect, so that his solution should be useful in all aspects. The general approach is to ***analyse the problem in economic terms and then implement the solution if it does not aggressive or violent to other aspects like human, social and political constraints***.

Management may be considered as the process of integrating the efforts of a purposeful group, or organisation, whose members have at least one common goal. You have studied various schools of management in your management science. Most important among them which uses scientific basis for decision making are:

(*i*) The Decision theory or Decisional Management School, and

(*ii*) The Mathematical or Quantitative Measurement School.

The above-mentioned schools of management thought advocate the use of **mathematical methods** or **quantitative methods** for making decisions. Quantitative approach to management problems requires that decision problems be defined, analyzed, and solved in a conscious, rational, logical and systematic and scientific manner—based on data, facts, information and logic, and not on mere guess work or thumb rules. Here we use objectively measured decision criteria. Operations research is the body of knowledge, which uses mathematical techniques to solve management problems and make timely optimal decisions. Operations Research is concerned with helping managers and executives to make better decisions. Today's manager is working in a highly competitive and dynamic environment. In present environment, the manager has to deal with systems with complex interrelationship of various

factors among them as well as equally complicated dependence of the criterion of effective performance of the system on these factors, conventional methods of decision making is found very much inadequate. Though the common sense, experience, and commitment of the manager is essential in making decision, we cannot deny the role-played by scientific methods in making optimal decisions. Operation Research uses logical analysis and analytical techniques to study the behaviour of a system in relation to its overall working as resulting from its functionally interconnected constraints, whose parameters are recognized, quantified wherever possible relationships identified to the extent possible and alterative decisions are derived.

Conventional managers were very much worried about that an Operations Research analyst replace them as a decision maker, but immediately they could appreciated him due to his mathematical and logical knowledge, which he applies while making decisions. But operations research analyst list out alternative solutions and their consequences to ease manager's work of decision making. Operations research gives rationality to decision making with clear view of possible consequences.

The scope of quantitative methods is very broad. They are applied in defining the problems and getting solutions of various organisatons like, business, government organisations, profit making units and non-profit units and service units. They can be applied to verities of problems like deciding plant location, Inventory control, Replacement problems, Production scheduling, return on investment analysis (ROI), Portfolio selection, marketing research and so on. This book, deals with basic models of Operations Research and Quantitative Methods. The students have to go through advanced Operations Research books, to understand the scope of the subject.

Two important aspects of quantitative methods are:

(*a*) Availability of well-structured models and methods in solving the problems,

(*b*) The attitude of search, conducted on a scientific basis, for increased knowledge in the management of organisations.

Therefore, the attitude encompassed in the quantitative approaches perhaps more important than the specific methods or techniques. It is only by adopting this attitude that the boundaries and application of the quantitative approach can be advanced to include those areas where, at first glance, quantitative data and facts are hard to come by. Hence, quantitative approach has found place in traditional business and as well in social problems, public policy, national, international problems and interpersonal problems. In fact we can say that the application of quantitative techniques is not limited to any area and can be conveniently applied to all walks of life as far as decision making is concerned. **The quantitative approach does not preclude the qualitative or judgemental elements that almost always exert a substantial influence on managerial decision making. Quite the contrary. In actual practice, the quantitative approach must build upon, be modified by, and continually benefit from the experiences and creative insight of business managers.** In fact quantitative approach imposes a special responsibility on the manager. It makes modern manager to cultivate a managerial style that demand conscious, systematic and scientific analysis—and resolution—of decision problems.

In real world problems, we can notice that there exists a relationship among *intuition, judgement, science, quantitative attitudes, practices, methods and models,* as shown in Figure 1.1.

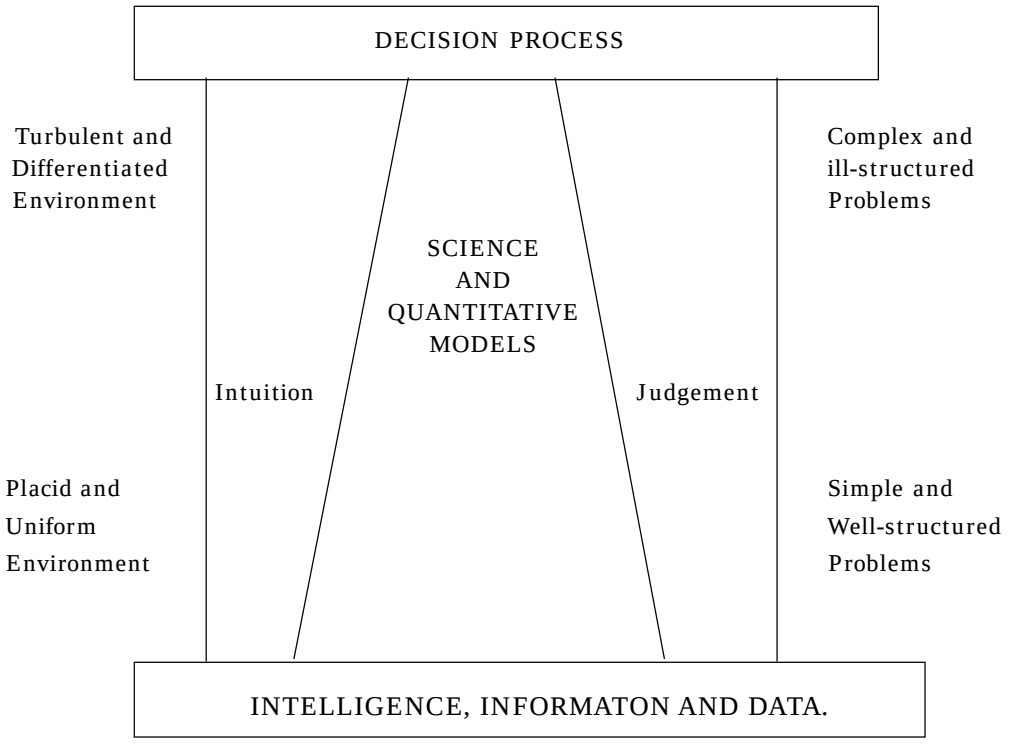

Figure 1.1. Qualitative Thinking and Quantitative models.

The figure depicts that higher the degree of complexity and the degree of turbulence in the environment, the greater is the importance of the qualitative approach to management. On the other hand, the lower the degree of complexity i.e. simple and well-structured problems, and lesser degree of turbulence in the environment, the greater is the potential of quantitative models. The advancement in quantitative approach to management problems is due to two facts. They are:

(*a*) Research efforts have been and are being directed to discover and develop more efficient tools and techniques to solve decision problems of all types.

(*b*) Through a continuous process of testing new frontiers, attempts have been made to expand the boundaries and application potential of the available techniques.

Quantitative approach is assuming an increasing degree of importance in the theory and practice of management because of the following reasons:

(*a*) Decision problems of modern management are so complex that only a conscious, systematic and scientifically based analysis can yield a realistic fruitful solution.

(*b*) Availability of list of more potential models in solving complex managerial problems.

(*c*) The most important one is that availability of high speed computers to solve large and complex real world problems in less time and at least cost and which help the managers to take timely decision.

One thing we have to remember here is that if managers are to fully utilize the potentials of management science models and computers, then problems will have to be stated in quantitative terms.

As far as the title of the subject is concerned, the terms **'quantitative approach', 'operations research', 'management science', 'systems analysis' and 'systems science'** are often used interchangeably. Whatever be the name of the subject, the syllabi and subject matter dealt with will be same. This analog to *'god is one but the names are different'.*

1.2. HISTORY OF OPERATIONS RESEARCH

Operations Research is a '**war baby'.** It is because, the first problem attempted to solve in a systematic way was concerned with *how to set the time fuse bomb to be dropped from an aircraft on to a submarine.* In fact the main origin of Operations Research was during the **Second World War.** At the time of Second World War, the military management in England invited a team of scientists to study the strategic and tactical problems related to air and land defense of the country. The problem attained importance because at that time the resources available with England was very limited and the objective was to win the war with available meager resources. The resources such as food, medicines, ammunition, manpower etc., were required to manage war and for the use of the population of the country. It was necessary to decide upon the most effective utilization of the available resources to achieve the objective. It was also necessary to utilize the military resources cautiously. Hence, the Generals of military, invited a team of experts in various walks of life such as scientists, doctors, mathematicians, business people, professors, engineers etc., and the problem of resource utilization is given to them to discuss and come out with a feasible solution. These specialists had a brain storming session and came out with a method of solving the problem, which they coined the name **"Linear Programming".** This method worked out well in solving the war problem. As the name indicates, the word **Operations** is used to refer to the problems of military and the word **Research** is use for inventing new method. As this method of solving the problem was invented during the war period, the subject is given the name **'OPERATIONS RESEARCH'** and abbreviated as **'O.R.'** After the World War there was a scarcity of industrial material and industrial productivity reached the lowest level. Industrial recession was there and to solve the industrial problem the method **linear programming** was used to get optimal solution. From then on words, lot of work done in the field and today the subject of O.R. have numerous methods to solve different types of problems. After seeing the success of British military, the United States military management started applying the techniques to various activities to solve military, civil and industrial problems. They have given various names to this discipline. Some of them are Operational Analysis, Operations Evaluation, Operations Research, System Analysis, System Evaluation, Systems Research, Quantitative methods, Optimisation Techniques and Management Science etc. But most widely used one is **OPERATIONS RESEARCH.** In industrial world, most important problem for which these techniques used is *how to optimise the profit or how to reduce the costs.* The introduction of Linear Programming and Simplex method of solution developed by American Mathematician George B. Dontzig in 1947 given an opening to go for new techniques and applications through the efforts and co-operation of interested individuals in academic field and industrial field. Today the scenario is totally different. A large number of Operations Research consultants are available to deal with different types of problems. In India also, we have O.R. Society of India (1959) to help in solving various problems. To day the Operations Research techniques are taught at High School levels. To quote some

Indian industries, which uses operations Research for problem solving are: M/s Delhi Cloth Mills, Indian Railways, Indian Airline, Hindustan Lever, Tata Iron and Steel Company, Fertilizers Corporation of India and Defense Organizations. In all the above organizations, Operations Research people act as staff to support line managers in taking decisions.

In one word we can say that Operations Research play a vital role in every organisation, especially in decision making process.

1.3. DECISION MAKING AND SOME ASPECTS OF DECISION

Many a time we speak of the word **decision,** as if we know much about decision. But what is decision? What it consists of? What are its characteristics? Let us have brief discussion about the word decision, as much of our time we deal with decision making process in Operations Research.

A **decision** is the conclusion of a process designed to weigh the relative uses or utilities of a set of alternatives on hand, so that decision maker selects the best alternative which is best to his problem or situation and implement it. **Decision Making** involves all activities and thinking that are necessary to identify the most optimal or preferred choice among the available alternatives. The basic requirements of decision making are: (*i*) A set of goals or objectives, (*ii*) Methods of evaluating alternatives in an objective manner, (*iii*) A system of choice criteria and a method of projecting the repercussions of alternative choices of courses of action. The evaluation of consequences of each course of action is important due to sequential nature of decisions.

The necessity of making decisions arises because of our existence in the world with various needs and ambitions and goals, whose resources are limited and some times scarce. Every one of us competes to use these resources to fulfill our goals. Our needs can be biological, physical, financial, social, ego or higher-level self-actualisation needs. One peculiar characteristics of decision making is the inherent conflict that desists among various goals relevant to any decision situation (for example, a student thinking of study and get first division and at the same time have youthhood enjoyment without attending classes, OR a man wants to have lot of leisure in his life at the same time earn more etc.). The process of decision-making consists of two phases. The first phase consists of formulation of goals and objectives, enumeration of environmental constraints, identification and evaluation of alternatives. The second stage deals with selection of optimal course of action for a given set of constraints. **In Operations Research, we are concerned with *how to choose optimal strategy under specified set of assumptions,* including all available strategies and their associated payoffs.**

Decisions may be classified in different ways, depending upon the criterion or the purpose of classification. Some of them are shown below:

I. Decisions (depending on the purpose)

PURPOSE:	Strategic	Administrative	Operational
	Related to external Environment	Resource utilisation of an organisation	Related to day-to-day Repetitive work

II. Decision (depending on the nature)

Programmed decisions	Non-Programmed decisions.
Meant for repetitive and well-structured problems	Meant for non-routine, novel, ill-structured problems
Inventory Problems, Product Mix Problems, etc.	Policy matters, Product market mix, Plant location etc.

III. Decision (depending on the persons involved)

Individual — Managerial

IV. Decisions (depending on the sphere of interest)

Economic — Social — Business — Political — Social

V. Decisions (depending on the time horizon)

Static	Dynamic
(One decision for entire planning period)	(Decisions are sequential)

Decisions may also be classified depending on the situations such as **degree of certainty.** For example, (*i*) Decision making under certainty, (*ii*) Decision making under uncertainty, and (*iii*) Decision making under risk. The first two are two extremes and the third one is falls between these two with certain probability distribution.

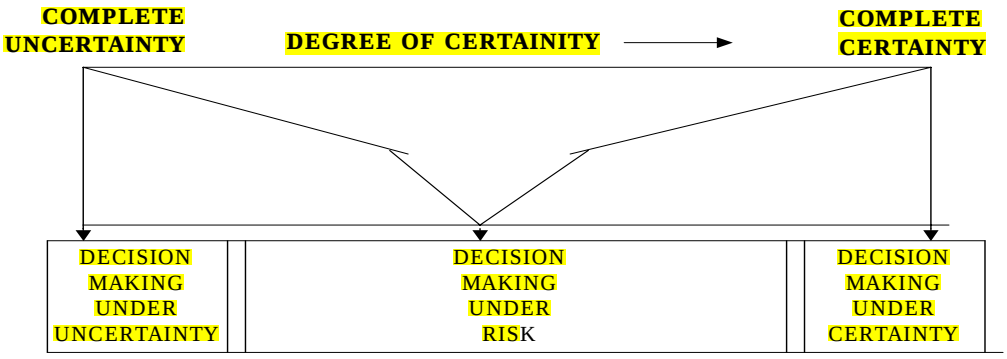

Figure 1.2. Decision basing on degree of certainty.

1.4. OBJECTIVE OF OPERATIONS RESEARCH

Today's situation in which a manager has to work is very complicated due to complexity in business organisations. To day's business unit have number of departments and each department work for fulfilling the objectives of the organisation. While doing so the individual objective of one of the department may be conflicting with the objective of the other department, though both working for achieving the common goal in the interest of the organisation. In such situations, it will become a very complicated issue for the general manager to get harmony among the departments and to allocate the available resources of all sorts to the departments to achieve the goal of the organisation. At the same time the environment in which the organization is operating is very dynamic in nature and the manager has to take decisions without delay to stand competitive in the market. At the same time a wrong decision or an untimely decision may be very costly. Hence the decision making process has become very complicated at the same time very important in the environment of conflicting interests and competitive strategies. Hence it is desirable for modern manager to use scientific methods with mathematical base while making decisions instead of depending on guesswork and thumb rule methods. Hence the knowledge of Operations Research is an essential tool for a manager who is involved in decision-making process. He must have support of knowledge of mathematics, statistics, economics etc., so that the decision he takes will be an optimal decision for his organisation. Operation Research provides him this knowledge and helps him to take quick, timely, decisions, which are optimal for the organisation. Hence the **objective** of operations research is:

"The objective of Operations Research is to provide a scientific basis to the decision maker for solving the problems involving the interaction of various components of an organisation by employing a team of scientists from various disciplines, all working together for finding a solution which is in the best interest of the organisation as a whole. The best solution thus obtained is known as optimal decision".

1.5 DEFINITION OF OPERATIONS RESEARCH

Any subject matter when defined to explain what exactly it is, we may find one definition. Always a definition explains what that particular subject matter is. Say for example, if a question is asked what is Boyel's law, we have a single definition to explain the same, irrespective of the language in which it is defined. But if you ask, what is Operations Research is? The answer depends on individual objective. Say for example a student may say that the Operations Research is technique use to obtain first class marks in the examination. If you ask a businessman the same question, he may say that it is the technique used for getting higher profit. Another businessman may say it is the technique used to capture higher market share and so on. Like this each individual may define in his own way depending on his objective. Each and every definition may explain one or another characteristic of Operations Research but none of them explain or give a complete picture of Operations Research. But in the academic interest some of the important definitions are discussed below.

(*a*) **Operations Research is the art of winning wars without actually fighting.**

— Aurther Clarke

This definition does not through any light on the subject matter, but it is oriented towards warfare. It means to say that the directions for fighting are planned and guidance is given from remote area, according to which the war is fought and won. Perhaps you might have read in Mahabharata or you might have seen some old pictures, where two kingdoms are fighting, for whom the guidance is given by the chief minister and the king with a chessboard in front of them. Accordingly war is fought in the warfront. Actually the chessboard is a model of war field.

(*b*) **Operations Research is the art of giving bad answers to problems where otherwise worse answers are given.** ***— T.L. Satty***

This definition covers one aspect of decision making, i.e. choosing the best alternative among the list of available alternatives. It says that if the decisions are made on guesswork, we may face the worse situation. But if the decisions are made on scientific basis, it will help us to make better decisions. Hence this definition deals with one aspect of decision making and not clearly tells what is operations research.

(*c*) **Operations Research is Research into Operations** ***— J. Steinhardt***

This definition does not give anything in clear about the subject of Operations Research and simply says that it is research in to operations. Operations may here be referred as military activities or simply the operations that an executive performs in his organisations while taking decisions. Research in the word means that finding a new approach. That is when an executive is involved in performing his operations for taking decisions he has to go for newer ways so that he can make a better decision for the benefit of his organisation.

(*d*) **Operations Research is defined as Scientific method for providing executive departments a quantitative basis for decisions regarding the operations under their control.** ***— P.M. Morse and G.E. Kimball***

This definition suggests that the Operations Research provides scientific methods for an executive to make optimal decisions. But does not give any information about various models or methods. But this suggests that executives can use scientific methods for decision making.

(*e*) **Operations Research is the study of administrative system pursued in the same scientific manner in which system in Physics, Chemistry and Biology are studied in natural sciences.**

This definition is more elaborate than the above given definitions. It compares the subject Operations Research with that of natural science subjects such as Physics, Chemistry and Biology, where while deciding any thing experiments are conducted and results are verified and then the course of action is decided. It clearly directs that Operations Research can also be considered as applied science and before the course of action is decided, the alternatives available are subjected to scientific analysis and optimal alternative is selected. But the difference between the experiments we conduct in natural sciences and Operations Research is: in natural sciences the research is rigorous and exact in nature, whereas in operations research, because of involvement of human element and uncertainty the approach will be totally different.

(*f*) **Operations Research is the application of scientific methods, techniques and tools to operation of a system with optimum solution to the problem.**

— Churchman, Ackoff and Arnoff.

This definition clearly states that the Operations Research applies scientific methods to find an optimum solution to the problem of a system. A system may be a production system or information system or any system, which involves men, machine and other resources. We can clearly identify that this definition tackles three important aspects of operations research i.e. application of scientific methods, study of a system and optimal solution. This definition too does not give any idea about the characteristics of Operations Research.

(*g*) **Operations Research is the application of the theories of Probability, Statistics, Queuing, Games, Linear Programming etc., to the problems of war, Government and Industry.**

This definition gives a list of various techniques used in Operations Research by various managers to solve the problems under their control. A manager has to study the problem, formulate the problem, identify the variables and formulate a model and select an appropriate technique to get optimal solution. We can say that Operations Research is a bunch of mathematical techniques to solve problems of a system.

(*h*) **Operations Research is the use of Scientific Methods to provide criteria or decisions regarding man-machine systems involving repetitive operations.**

This definition talks about man-machine system and use of scientific methods and decision-making. It is more general and comprehensive and exhaustive than other definitions. Wherever a study of system involving man and machine, the person in charge of the system and involved in decision making will use scientific methods to make optimal decisions.

(*i*) **Operations Research is applied decision theory. It uses any scientific, mathematical or logical mans to attempt to cope with problems that confront the executive, when he tries to achieve a thorough going rationally in dealing with his decision problem.** ***— D.W. Miller and M.K. Starr***

This definition also explains that Operations Research sues scientific methods or logical means for getting solutions to the executive problems. It too does not give the characteristics of Operations Research.

(*j*) **Operations Research is the application of the methods of science to complex problems arising in the direction and management of large systems of men, materials and money in industry, business, Government and defense. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance and risk, with which to predict and compare the outcome of alternative decisions strategies or controls. The purpose is to help management to determine its policy and actions scientifically.** ***— Operations Society of Great Britain***

A body, authoritative, hence it is more elaborate and says that operations research applies scientific methods to deal with the problems of a system where men, material and other resources are involved and the system under study may be industry, defense or business etc., gives this definition. It also say that the manager has to build a scientific model to study the system which

must be provided with facility to measure the outcomes of various alternatives under various degrees of risk, which helps the managers to take optimal decisions.

In addition to the above there are hundreds of definitions available to explain what Operations Research is? But many of them are not satisfactory because of the following reasons.

(*i*) Operations Research is not a well-defined science like Physics, Chemistry etc. All these sciences are having well defined theory about the subject matter, where as operations research do not claim to know or have theories about operations. Moreover, Operations Research is not a scientific research into the control of operations. It is only the application of mathematical models or logical analysis to the problem solving. Hence none of the definitions given above defines operations research precisely.

(*ii*) The objective of operations research says that the decisions are made by brain storming of people from various walks of life. This indicates that operations research approach is **inter-disciplinary** approach, which is an important character of operations research. This aspect is not included in any of the definitions hence they are not satisfactory.

(*iii*) The above discussed definitions are given by various people at different times and stages of development of operations research as such they have considered the field in which they are involved hence each definition is concentrating on one or two aspects. No definition is having universal approach.

But salient features of above said definitions are:

- Operations Research *uses Scientific Methods* for making decisions.
- It is interdisciplinary approach for solving problems and it uses the knowledge and experience of experts in various fields.
- While analyzing the problems all aspects are considered and examined and analyzed scientifically for finding the optimal solution for the problem on hand.
- As operations research has scientific approach, it improves the quality of answers to the problems.
- Operations research provides scientific base for decision making and provide scientific substitute for judgement and intuition.

1.6. CHARACTERISTICS OF OPERATIONS RESEARCH

After considering the objective and definitions of Operations Research, now let us try to understand what are the characteristics of Operations Research.

(*a*) Operations Research is an *interdisciplinary* team approach

The problems an operations research analyst face is heterogeneous in nature, involving the number of variables and constraints, which are beyond the analytical ability of one person. Hence people from various disciplines are required to understand the operations research problem, who applies their special knowledge acquired through experience to get a better view of cause and effects of the events in the problem and to get a better solution to the problem on hand. This type of team approach will reduce the risk of making wrong decisions.

(*b*) **Operations Research increases the creative ability of the decision maker**

Operations Research provides manager mathematical tools, techniques and various models to analyse the problem on hand and to evaluate the outcomes of various alternatives and make an optimal choice. This will definitely helps him in making better and quick decisions. A manager, without the knowledge of these techniques has to make decisions by thumb rules or by guess work, which may click some times and many a time put him in trouble. Hence, a manager who uses Operations Research techniques will have a better creative ability than a manager who does not use the techniques.

(*c*) **Operations Research is a systems approach**

A business or a government organisation or a defense organisation may be considered as a system having various sub-systems. The decision made by any sub-system will have its effect on other sub-systems. Say for example, a decision taken by marketing department will have its effect on production department. When dealing with Operations Research problems, one has to consider the entire system, and characteristics or sub-systems, the inter-relationship between sub-systems and then analyse the problem, search for a suitable model and get the solution for the problem. Hence we say Operations Research is a Systems Approach.

1.7. SCOPE OF OPERATIONS RESEARCH

The scope aspect of any subject indicates, the limit of application of the subject matter / techniques of the subject to the various fields to solve the verities of the problems. But we have studied in the objective, that the subject Operations Research will give scientific base for the executives to take decisions or to solve the problems of the systems under their control. The system may be business, industry, government or defense. Not only this, but the definitions discussed also gives different versions. This indicates that the techniques of Operations Research may be used to solve any type of problems. The problems may pertain to an individual, group of individuals, business, agriculture, government or defense. Hence, we can say that there is no limit for the application of Operations Research methods and techniques; they may be applied to any type of problems. Let us now discuss some of the fields where Operations Research techniques can be applied to understand how the techniques are useful to solve the problems. In general we can state that whenever there is a problem, simple or complicated, we can use operations research techniques to get best solution.

(*i*) **In Defense Operations**

In fact, the subject Operations Research is the baby of World War II. To solve war problems, they have applied team approach, and come out with various models such as resource allocation model, transportation model etc.

In any war field two or more parties are involved, each having different resources (manpower, ammunition, etc.), different courses of actions (strategies) for application. Every opponent has to guess the resources with the enemy, and his courses of action and accordingly he has to attack the enemy. For this he needs scientific, logical analysis of the problem to get fruitful results. Here one can apply the techniques like *Linear Programming, Game theory, and inventory models etc.* to win the game. In fact in war filed every situation is a competitive situation. More over each party may have different bases, such as Air force, Navy and Army.

The decision taken by one will have its effect on the other. Hence proper co-ordination of the three bases and smooth flow of information is necessary. Here operations research techniques will help the departmental heads to take appropriate decisions.

(*ii*) **In Industry**

After the World War II, the, industrial world faced a depression and to solve the various industrial problems, industrialist tried the models, which were successful in solving their problems. Industrialist learnt that the techniques of operations research can conveniently applied to solve industrial problems. Then onwards, various models have been developed to solve industrial problems. Today the managers have on their hand numerous techniques to solve different types of industrial problems. In fact *decision trees, inventory model, Linear Programming model, Transportation model, Sequencing model, Assignment model and replacement models* are helpful to the managers to solve various problems, they face in their day to day work. These models are used to minimize the cost of production, increase the productivity and use the available resources carefully and for healthy industrial growth. An industrial manager, with these various models in his hand and a computer for workout the solutions (to day various packages are available to solve different industrial problems) quickly and preciously.

(*iii*) **In Planning for Economic Growth**

In India we have five year planning for steady economic growth. Every state government has to prepare plans for balanced growth of the state. Various secretaries belonging to different departments has to co-ordinate and plan for steady economic growth. For this all departments can use Operations Research techniques for planning purpose. The question like how many engineers, doctors, software people etc. are required in future and what should be their quality to face the then problems etc. can be easily solved.

(*iv*) **In Agriculture**

The demand for food products is increasing day by day due to population explosion. But the land available for agriculture is limited. We must find newer ways of increasing agriculture yield. So the selection of land area for agriculture and the seed of food grains for sowing must be meticulously done so that the farmer will not get loss at the same time the users will get what they desire at the desired time and desired cost.

(*v*) **In Traffic control**

Due to population explosion, the increase in the number and verities of vehicles, road density is continuously increasing. Especially in peak hours, it will be a headache to control the traffic. Hence proper timing of traffic signaling is necessary. Depending on the flow of commuters, proper signaling time is to be worked out. This can be easily be done by the application of *queuing theory*.

(*vi*) **In Hospitals**

Many a time we see very lengthy queues of patient near hospitals and few of them get treatment and rest of them have to go without treatment because of time factor. Sometimes we have problems non-availability of essential drugs, shortages of ambulances, shortage of beds etc. These problems can be conveniently solved by the application of operations research techniques.

The above-discussed problems are few among many problems that can be solved by the application of operation research techniques. This shows that Operations Research has no limit on its scope of application.

1.8. PHASES IN SOLVING OPERATIONS RESEARCH PROBLEMS OR STEPS IN SOLVING OPERATIONS RESEARCH PROBLEMS

Any Operations Research analyst has to follow certain sequential steps to solve the problem on hand. The steps he has to follow are discussed below:

First he has to study the situation and collect all information and formulate the statement of the problem. Hence the first step is the *Formulation of the problem.* The Figure 1.3 shows the various steps to be followed.

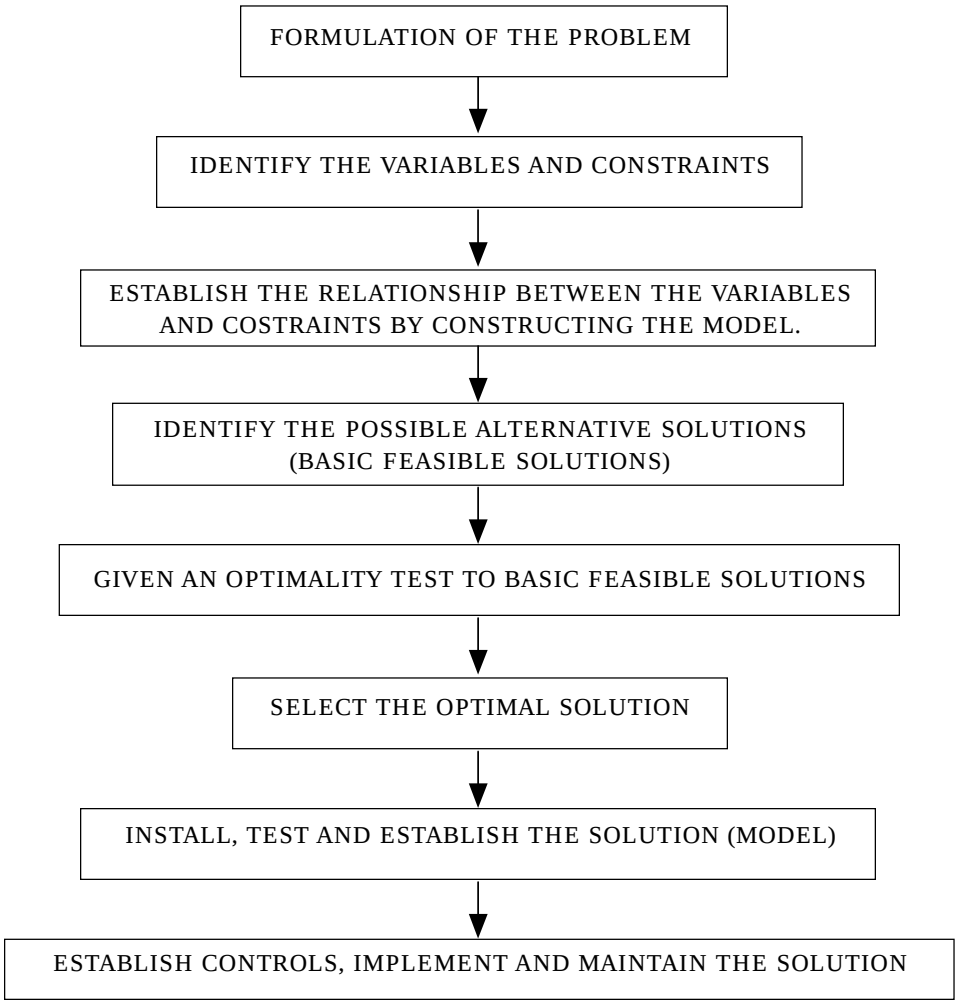

Figure 1.3. Phases of Solving Operations Research Problems.

1.8.1. Formulation of the Problem

The Operations Research analyst or team of experts first have to examine the situation and clearly define what exactly happening there and identify the variables and constraints. Similarly identify what is the objective and put them all in the form of statement. The statement must include: a) a precise description goals or objectives of the study, b) identification of controllable and uncontrollable variables, and c) restrictions of the problem. The team should consult the personals at the spot and collect information, if something is beyond their reach, they have to consult duty engineers available and understand the facts and formulate the problem. Let us consider the following statement:

Statement: A company manufactures **two products X and Y**, by using the **three machines A, B, and C**. Each unit of X takes **1 hour** on machine A, **3 hours** on machine B and **10 hours** on machine C. Similarly, product Y takes **one hour, 8 hours and 7 hours** on Machine A, B, and C respectively. In the coming planning period, **40 hours of machine A**, **240 hours of machine B** and **350 hours of machine C** is available for production. Each unit of **X brings a profit of Rs. 5** and **Y brings Rs. 7 per unit**. **How much of X and Y are to be manufactured** by the company **for maximizing the profit**?

The team of specialists prepares this statement after studying the system. As per requirement this must include the variables, constraints, and objective function.

1.8.2. Variables

The Company is manufacturing two products X and Y. These are the two variables in the problem. When they are in the problem statement they are written in *capital letters*. Once they are entered in the model small letters (lower case) letters are used (i.e. x and y). We have to find out how much of X and how much of Y are to be manufactured. Hence they are variables. In linear programming language, these are known as **competing candidates.** Because they compete to use or consume available resources.

1.8.3. Resources and Constraints

There are three machines A, B, and C on which the products are manufactured. These are known as resources. The capacity of machines in terms of machine hours available is the available resources. The competing candidates have to use these available resources, which are limited in nature. Now in the above statement, machine A has got available 40 hours and machine B has available a capacity of 240 hours and that of machine C is 350 hours. The products have to use these machine hours in required proportion. That is one unit of product X consumes one hour of machine A, 3 hours of machine B and 10 hours of machine C. Similarly, one unit of Y consumes one hour of machine B, 8 hours of machine B and 7 hours of machine C. These machine hours given are the available resources and they are limited in nature and hence they are **constraints** given in the statement.

1.8.4. Objective of the Problem

To maximise the profit how much of X and Y are to be manufactured? That is **maximization of the profit or maximization of the returns** is the objective of the problem. For this in the statement it is given that the profit contribution of X is Rs. 5 per unit and that of product Y is Rs. 7 per unit.

1.8.5. To Establish Relationship Between Variables and Constraints and Build up a Model

Let us say that company manufactures x units of X and y units of Y. Then as one unit of x consumes one hour on machine A and one unit of y consumes one hour on machine A, the total consumption by manufacturing x units of X and y units of Y is, 1x + 1y and this should not exceed available capacity of 40 hours. Hence the **mathematical relationship in the form of mathematical model is 1x + 1y ≤ 40**. This is for resource machine A. Similarly for machine B and machine C we can formulate the mathematical models. They appear as shown below:

3x + 8y ≤ 240 for machine B **and 10x + 7y ≤ 350** for machine C. Therefore, the mathematical model for these resources are:

$1x + 1y \leq 40$

$3x + 8y \leq 240$ and

$10x + 7y \leq 350$.

Similarly for objective function as the company manufacturing x units of X and y units of Y and the profit contribution of X and Y are Rs. 5 and Rs. 7 per unit of X and Y respectively, the total profit earned by the company by manufacturing x and y units is **5x + 7y.** This we have to maximise. Therefore **objective function is Maximise 5x + 7y.** At the same time, we have to remember one thing that the company can manufacture any number of units or it may not manufacture a particular product, for example say x = 0. But it cannot manufacture negative units of x and y. Hence one more constraint is to be introduced in the model i.e. **a non-negativity constraint.** Hence the mathematical representation of the contents of the statement is as given below:

Maximise Z = 5x + 7y Subject to a condition → **OBJECTIVE FUNCTION**
(written as s.t.)

1x + 1y ≤ 40
3x + 8y ≤ 240
10x + 7y ≤ 350 and → **STRUCTURAL CONSTRAINTS**

Both x and y are ≥ 0 → **NON-NEGATIVITY CONSTRAINT**

1.8.6. Identify the Possible Alternative Solutions (or known as Basic Feasible Solutions or Simply BFS)

There are various methods of getting solutions. These methods will be discussed later. For example we go on giving various values (positive numbers only), and find various values of objective function. All these are various Basic Feasible Solutions. For example x = 0,1,2,3, etc. and y = 0, 1, 2, 3 etc are all feasible values as far as the given condition is concerned. Once we have feasible solutions on hand go on asking is it maximum? Once we get maximum value those values of x and y is optimal values. And the value of objective function is **optimal value of the objective function.** These two steps we shall discuss in detail in the next chapter.

1.8.7. Install and Maintain the Solution

Once we get the optimal values of x and y and objective function instructions are given to the concerned personal to manufacture the products as per the optimal solution, and maintain the same until further instructions.

1.9 MEANING AND NECESSITY OF OPERATIONS RESEARCH MODELS

Management deals with reality that is at once complex, dynamic, and multifaceted. It is neither possible nor desirable, to consider each and every element of reality before deciding the courses of action. It is impossible because of time available to decide the courses of action and the resources, which are limited in nature. Moreover in many cases, it will be impossible for a manager to conduct experiment in real environment. For example, if an engineer wants to measure the inflow of water into a reservoir through a canal, he cannot sit on the banks of canal and conduct experiment to measure flow. He constructs a similar model in laboratory and studies the problem and decides the inflow of water. Hence for many practical problems, a model is necessary. We can **define an operations research model as some sort of mathematical or theoretical description of various variables of a system representing some aspects of a problem on some subject of interest or inquiry. The model enables to conduct a number of experiment involving theoretical subjective manipulations to find some optimum solution to the problem on hand.**

Let us take a very simple example. Say you have a small child in your house. You want to explain to it what is an elephant. You can say a story about the elephant saying that it has a trunk, large ears, small eyes etc. The child cannot understand or remember anything. But if you draw small drawing of elephant on a paper and show the trunk, ears, eyes and it will grasp very easily the details of elephant. When a circus company comes to your city and take elephants in procession, then the child if observe the procession, it will immediately recognize the elephant. This is the exact use of a model. In your classrooms your teacher will explain various aspects of the subject by drawing neat sketches on the black board. You will understand very easily and when you come across real world system, you can apply what all you learnt in your classroom. Hence through a model, we can explain the aspect of the subject/problem/system. The inequalities given in section 1.8.5 is a mathematical model, which explains clearly the manufacturing system, given in section 1.8.1. (***Here we can say a model is a relationship among specified variables and parameters of the system***).

1.9.1. Classification of Models

The models we use in operations research may broadly classified as:

(*i*) Mathematical and Descriptive modes, and

(*ii*) Static and Dynamic Models.

Mathematical and Descriptive Models

(*i*) **Descriptive Model**

A descriptive model explains or gives a description of the system giving various variables, constraints and objective of the system or problem. In article 1.8.1 gives the statement of the problem, which is exactly a descriptive model. The drawback of this model is as we go on reading and proceed; it is very difficult to remember about the variables and constraints, in case the problem or description of the system is lengthy one. It is practically impossible to keep on reading, as the manager has to decide the course of action to be taken timely. Hence these models, though necessary to understand the system, have limited use as far as operations research is concerned.

(*ii*) **Mathematical Model**

In article, 1.8.2 we have identified the variables and constraints and objective in the problem statement and given them mathematical symbols x and y and a model is built in the form of an inequality of $\leq$ type. Objective function is also given. This is exactly a mathematical model, which explains the entire system in mathematical language, and enables the operations research person to proceed towards solution.

1.9.2. Types of Models

Models are also categorized depending on the *structure, purpose, nature of environment, behaviour, by method of solution and by use of digital computers.*

(*a*) Classification by Structure

(*i*) *Iconic Models:* These models are scaled version of the actual object. For example a toy of a car is an iconic model of a real car. In your laboratory, you might have seen Internal Combustion Engine models, Boiler models etc. All these are iconic models of actual engine and boiler etc. They explain all the features of the actual object. In fact a globe is an iconic model of the earth. These models may be of enlarged version or reduced version. Big objects are scaled down (reduced version) and small objects, when we want to show the features, it is scaled up to a bigger version. In fact it is a descriptive model giving the description of various aspects of real object. As far as operations research is concerned, it of less use. The advantages of these models are: It is easy to work with an iconic model in some cases, these are easy to construct and these are useful in describing static or dynamic phenomenon at some definite time. The limitations are, we cannot study the changes in the operation of the system. For some type of systems, the model building is very costly. It will be sometimes very difficult to carry out experimental analysis on these models.

(*ii*) *Analogue Model:* In this model one set of properties are used to represent another set of properties. Say for example, blue colour generally represents water. Whenever we want show water source on a map it is represented by blue colour. Contour lines on the map is also analog model. Many a time we represent various aspects on graph by deferent colours or different lines all these are analog models. These are also not much used in operations research. The best examples are warehousing problem and layout problems.

(*iii*) *Symbolic Models or Mathematical Models:* In these models the variables of a problem is represented by mathematical symbols, letters etc. To show the relationships between variables and constraints we use mathematical symbols. Hence these are known as symbolic models or mathematical models. These models are used very much in operations research. Examples of such models are Resource allocation model, Newspaper boy problem, transportation model etc.

(*b*) Classification by utility

Depending on the use of the model or purpose of the model, the models are classified as *Descriptive, Predictive and Prescriptive models.*

(*i*) *Descriptive model:* The descriptive model simply explains certain aspects of the problem or situation or a system so that the user can make use for his analysis. It will not give full details and clear picture of the problem for the sake of scientific analysis.

(*ii*) *Predictive model:* These models basing on the data collected, can predict the approximate results of the situation under question. For example, basing on your performance in the

examination and the discussions you have with your friends after the examination and by verification of answers of numerical examples, you can predict your score or results. This is one type of predictive model.

(*iii*) *Prescriptive models:* We have seen that predictive models predict the approximate results. But if the predictions of these models are successful, then it can be used conveniently to prescribe the courses of action to be taken. In such case we call it as Prescriptive model. Prescriptive models prescribe the courses of action to be taken by the manager to achieve the desired goal.

(*c*) Classification by nature of environment

Depending on the environment in which the problem exists and the decisions are made, and depending on the conditions of variables, the models may be categorized as *Deterministic models and Probabilistic models.*

(*i*) *Deterministic models:* In this model the operations research analyst assumes complete certainty about the values of the variables and the available resources and expects that they do not change during the planning horizon. All these are deterministic models and do not contain the element of uncertainty or probability. The problems we see in Linear Programming, assumes certainty regarding the values of variables and constraints hence the Linear Programming model is a Deterministic model.

(*ii*) *Probabilistic or Stochastic models:* In these models, the values of variables, the pay offs of a certain course of action cannot be predicted accurately because of element of probability. It takes into consideration element of risk into consideration. The degree of certainty varies from situation to situation. A good example of this is the sale of insurance policies by Life Insurance Companies to its customers. Here the failure of life is highly probabilistic in nature. The models in which the pattern of events has been compiled in the form of probability distributions are known as Probabilistic or Stochastic models.

(*d*) Classification depending on the behaviour of the problem variables

Depending on the behaviour of the variables and constraints of the problem they may be classified as Static models or Dynamic models.

(*i*) *Static models:* These models assumes that no changes in the values of variables given in the problem for the given planning horizon due to any change in the environment or conditions of the system. All the values given are independent of the time. Mostly, in static models one decision is desirable for the given planning period.

(*ii*) *Dynamic models:* In these models the values of given variables goes on changing with time or change in environment or change in the conditions of the given system. Generally, the dynamic models a series of interdependent decisions during the planning period.

(*e*) Classification depending on the method of getting the solution

We may use different methods for getting the solution for a given model. Depending on these methods, the models are classified as Analytical Models and Simulation Models.

(*i*) *Analytical models:* The given model will have a well-defined mathematical structure and can be solved by the application of mathematical techniques. We see in our discussion that the Resource allocation model, Transportation model, Assignment model, Sequencing model

etc. have well defined mathematical structure and can be solved by different mathematical techniques. For example, Resource allocation model can be solved by Graphical method or by Simplex method depending on the number of variables involved in the problem. All models having mathematical structure and can be solved my mathematical methods are known as Analytical Models.

(*ii*) *Simulation models:* The meaning of simulation is *imitation*. These models have mathematical structure but cannot be solved by using mathematical techniques. It needs certain experimental analysis. To study the behaviour of the system, we use random numbers. More complex systems can be studied by simulation. Studying the behaviour of laboratory model, we can evaluate the required values in the system. Only disadvantage of this method is that it does not have general solution method.

1.9.3. Some of the Points to be Remembered while Building a Model

- When we can solve the situation with a simple model, do not try to build a complicated model.
- Build a model that can be easily fit in the techniques available. Do not try to search for a technique, which suit your model.
- In order to avoid complications while solving the problem, the fabrication stage of modeling must be conducted rigorously.
- Before implementing the model, it should be validated / tested properly.
- Use the model for which it is deduced. Do not use the model for the purpose for which it is not meant.
- Without having a clear idea for which the model is built do not use it. It is better before using the model; you consult an operations research analyst and take his guidance.
- Models cannot replace decision makers. It can guide them but it cannot make decisions. Do not be under the impression, that a model solves every type of problem.
- The model should be as accurate as possible.
- A model should be as simple as possible.
- Benefits of model are always associated with the by process by which it is developed.

1.9.4. Advantages of a Good Model

(*i*) A model provides logical and systematic approach to the problem.

(*ii*) It provides the analyst a base for understanding the problem and think of methods of solving.

(*iii*) The model will avoid the duplication work in solving the problem.

(*iv*) Models fix the limitation and scope of an activity.

(*v*) Models help the analyst to find newer ways of solving the problem.

(*vi*) Models saves resources like money, time etc.

(*vii*) Model helps analyst to make complexities of a real environment simple.

(*viii*) Risk of tampering the real object is reduced, when a model of the real system is subjected to experimental analysis.

(*ix*) Models provide distilled economic descriptions and explanations of the operation of the system they represent.

1.9.5. Limitations of a Model

(*i*) Models are constructed only to understand the problem and attempt to solve the problem; they are not to be considered as real problem or system.

(*ii*) The validity of any model can be verified by conducting the experimental analysis and with relevant data characteristics.

1.9.6. Characteristics of a Good Model

(*i*) The number of parameters considered in a model should be less to understand the problem easily.

(*ii*) A good model should be flexible to accommodate any necessary information during the stages of building the model.

(*iii*) A model must take less time to construct.

(*iv*) A model may be accompanied by lower and upper bounds of parametric values.

1.9.7. Steps in Constructing a Model

(*i*) *Problem environment analysis and formulation:* One has to study the system in all aspects, if necessary make relevant assumptions, have the decision for which he is constructing the model in mind and formulate the model.

(*ii*) *Model construction and assumptions:* Identify the main variables and constraints and relate them logically to arrive at a model.

(*iii*) *Testing the model:* After the formulation, before using check the model for its validity.

1.10. METHODS OF SOLVING OPERATIONS RESEARCH PROBLEMS

There are three methods of solving an operations research problem. They are:

(*i*) Analytical method, (*ii*) Iterative method, (*iii*) The Monte-Carlo Technique.

(*i*) *Analytical method:* When we use mathematical techniques such as differential calculus, probability theory etc. to find the solution of a given operations research model, the method of solving is known as analytical method and the solution is known as analytical solution. Examples are problems of inventory models. This method evaluates alternative policies efficiently.

(*ii*) *Iterative method (Numerical Methods):* This is trial and error method. When we have large number of variables, and we cannot use classical methods successfully, we use iterative process. First, we set a trial solution and then go on changing the solution under a given set of conditions, until no more modification is possible. The characteristics of this method is that the trial and error method used is laborious, tedious, time consuming and costly. The solution we get may not be accurate one and is approximate one. Many a time we find that after certain number of iterations, the solution cannot be improved and we have to accept it as the expected optimal solution.

(iii) Monte-Carlo method: This method is based on the random sampling of variable's values from a distribution of the variable. This uses sampling technique. A table of random numbers must be available to solve the problems. In fact it is a simulation process.

1.11. SOME IMPORTANT MODELS (PROBLEMS) WE COME ACROSS IN THE STUDY OF OPERATIONS RESEARCH

1.11.1. Linear Programming Model

This model is used for resourced allocation when the resources are limited and there are number of competing candidates for the use of resources. The model may be used to maximise the returns or minimise the costs. Consider the following two situations.

(*a*) A company which is manufacturing verities of products by using available resources, want to use resources optimally and manufacture different quantities of each type of product, which yield different returns, so as to maximise the returns.

(*b*) A company manufactures different types of alloys by purchasing the three basic materials and it want to maintain a definite percentage of basic materials in each alloy. The basic materials are to be purchased from the sellers and mix them to produce the desired ally. This is to be done at minimum cost.

Both of them are resource allocation models, the case (a) is maximisation problem and the case (b) is minimisation problem.

(*c*) Number of factories are manufacturing the same commodities in different capacities and the commodity is sent to various markets for meeting the demands of the consumers, when the cost of transportation is known, the linear programming helps us to formulate a programme to distribute the commodity from factories to markets at minimum cost. The model used is transportation model.

(*d*) When a company has number of orders on its schedule, which are to be processed on same machines and the processing time, is known, then we have to allocate the jobs or orders to the machines, so as to complete all the jobs in minimum time. This we can solve by using assignment model.

All the above-discussed models are Linear Programming Models. They can be solved by application of appropriate models, which are linear programming models.

1.11.2. Sequencing Model

When a manufacturing firm has some job orders, which can be processed on two or three machines and the processing times of each job on each machine is known, then the problem of processing in a sequence to minimise the cost or time is known as Sequencing model.

1.11.3. Waiting Line Model or Queuing Model

A model used for solving a problem where certain service facilities have to provide service to its customers, so as to avoid lengthy waiting line or queue, so that customers will get satisfaction from effective service and idle time of service facilities are minimised is waiting line model or queuing model.

1.11.4. Replacement Model

Any capital item, which is continuously used for providing service or for producing the product is subjected to wear and tear due to usage, and its efficiency goes on reducing. This reduction in efficiency can be predicted by the increasing number of breakdowns or reduced productivity. The worn out parts or components are to be replaced to bring the machine back to work. This action is known as maintenance. A time is reached when the maintenance cost becomes very high and the manger feels to replace the old machine by new one. This type of problems known as replacement problems and can be solved by replacement models.

1.11.5. Inventory Models

Any manufacturing firm has to maintain stock of materials for its use. This stock of materials, which are maintained in stores, is known as inventory. Inventory is one form of capital or money. The company has to maintain inventory at optimal cost. There are different types of inventory problems, depending the availability and demand pattern of the materials. These can be solved by the application of inventory models.

In fact depending on the number of variables, characteristics of variables, and the nature of constraints different models are available. These models, we study when we go chapterwise.

QUESTIONS

1. Trace the history of Operations Research.
2. Give a brief account of history of Operations Research.
3. Discuss the objective of Operations Research.
4. "Operations Research is a bunch of mathematical techniques to break industrial problems – Critically comment.
5. What is a Operations Research model? Discuss the advantages of limitation of good Operations Research model.
6. Discuss three Operations Research models.
7. What is a decision and what are its characteristics ?
8. Briefly explain the characteristics of Operations Research.
9. Discuss the various steps used in solving Operations Research problems.
10. Discuss the scope of Operations Research.