LPP-Graphical Method

Graphical solution is limited to linear programming models containing only two decision variables.

Graphical methods provide visualization of how a solution for a linear problem is obtained

Graphical methods can be classified under two categories

1. Iso-Profit line method
2. Extreme point evaluation method

While solving LPP by graphical method

1. **Formulate the problem, if required.**
2. **Convert inequalities into the equalities and find out the X and Y intercepts.**
3. **Plot the constraint equations using intercepts identified.**
4. **Identify the common feasible region using the nature of inequalities. The common feasible area is required to be bounded for feasible and optimal solution to the problem.**
5. **Find out the value of an objective function by substituting the corner points of the common feasible area. The coordinate which will results in the maximum value of an objective function will be the optimum solution for maximization problem and the coordinate which will results in the minimum value of an objective function will be the optimum solution for minimization problem.**

PROBLEM 1

A company manufactures two products, X and Y by using three machines A, B and C. Machine A has 4 hours of capacity available during the coming week. Similarly, the available capacity of machines B and C during the coming week is 24 and 35 hours respectively. One unit of product X requires one hour of machine A, 3 hours of machine B and 10 hours of machine C. Similarly one unit of product Y requires 1 hour, 8 hour and 7 hours of machine A, B and C respectively. When one unit of X is sold in the market, it yields a profit of Rs 5 per product and that of Y is Rs 7 per unit. Solve the problem by using graphical method to find the optimal product mix.

Solution

Machines	Products (Time required in hours).		Available capacity in hours.
	X	Y	
A	1	1	4
B	3	8	24
C	10	7	35
Profit per unit in Rs.	5	7	

Let the company manufactures x units of X and y units of Y, then the LP model is

$$\text{Maximize } z = 5x + 7y$$

Subjected to constraints

$$x + y \le 4$$

$$3x + 8y \le 24$$

$$10x + 7y \le 35$$

$$x, y \ge 0$$

Converting inequalities into equations and finding out X and Y intercepts we get

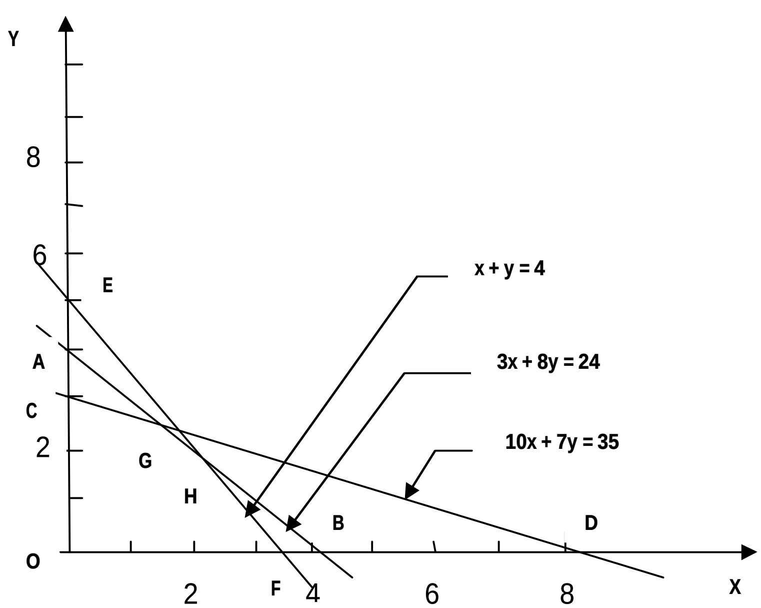
$x + y = 4$ A= (0,4) B= (4,0)

$3x + 8y = 24$ C= (0,3) D= (8,0)

$10x + 7y = 35$ E= (0,5) F= (3.5,0)

Point G is the intersection of $x + y = 4$ and $10x + 7y = 35$, hence G= (1.6, 2.4)

Point H is the intersection of $x + y = 4$ and $3x + 8y = 24$, hence H= (2.5, 1.5)



The area bounded by OCGHFO is the common feasible area.

Substituting the values of these points in the objective function we get

At point C= (0,3); z= 21, At point G= (1.6, 2.4); z= 24.8,

At point H= (2.5, 1.5); z= 23 At point F= (3.5,0); z= 17.5

As the value of the objective function is the maximum at point G, hence the optimal solution for the problem is the company should manufacture **1.6** units of X and **2.4** units of Y.

The optimum value of Max z **= Rs 24.8**

PROBLEM 2

The cost of materials A and B is Re 1 per unit respectively. We have to manufacture an alloy by mixing these materials. The process of preparing the alloy is carried out on three facilities X, Y and Z. Facilities X and Z are machines, whose capacities are limited. Y is a furnace, where heat treatment takes place and the material must use a minimum given time (even if it uses more than the required there is no harm). Material A requires 5 hours of machine X and 1 hour of machine Z. Both A and B are to be heat treated at last one hour in furnace Y. The available capacities of X, Y and Z are 50 hours, 1 hour and 4 hours respectively. Find how much of A and B are mixed so as to minimize the cost.

Solution

Let x and y are the number of units of material A and B to be mixed to minimize the cost.

$$\text{Minimize } z = x + y$$

Subjected to constraints

$$5x + 5y \leq 50$$

$$x + y \geq 1$$

$$y \leq 4$$

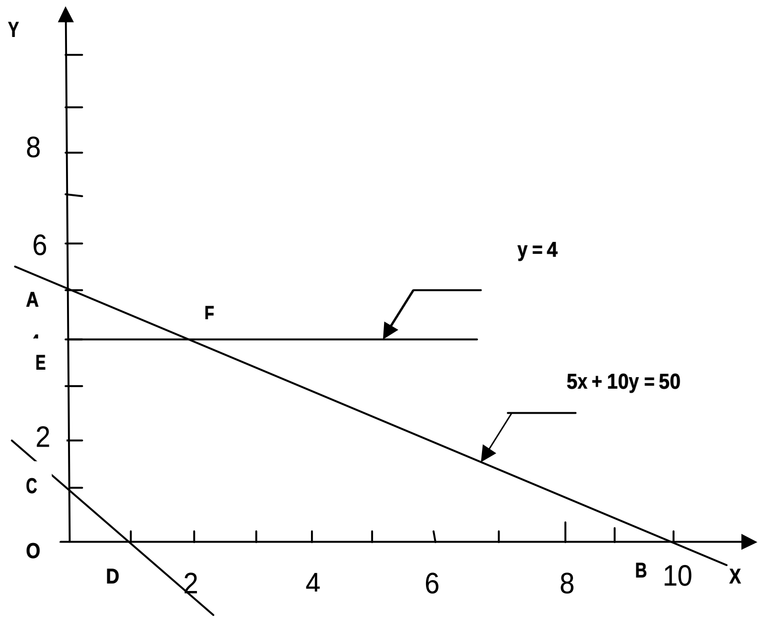
$$x, y \geq 0$$

Converting inequalities into equations and finding out X and Y intercepts we get

$5x + 10y = 50$ A= (0,5) B= (10,0)

$x + y = 1$ C= (0,1) D= (1,0)

$y = 4$ E= (0,4)



The area bounded by CEFBD the common feasible area.

Point F is the intersection of y = 4 and 5x + 10y = 50, hence F= (2,4)

Substituting the values of these points in the objective function we get

At point C= (0,1); z= 1, At point D= (1, O); z= 1,

At point B= (10, 0); z= 10 At point F= (2,4); z= 6

At point E= (0,4); z= 4

As the value of the objective function is the minimum at point C and D, hence the optimal solution for the problem is the company should manufacture **1** units of X and **1** units of Y. Moreover every value on the line CD is the optimal value; therefore the problem has **infinite optimum solutions**.

The optimum value of Min z **= Rs 1**

PROBLEM 3

$$\text{Max } z = 0.75x + y$$

Subjected to constraints

$$x + y \geq 0$$

$$-0.5x + y \leq 1$$

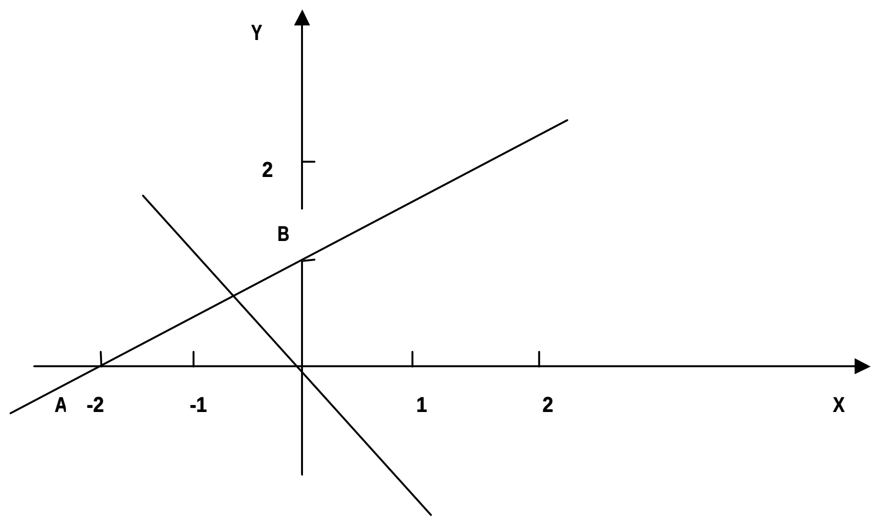
$$x, y \geq 0$$

Solution

Converting inequalities into equations and finding out X and Y intercepts we get

$x + y = 0$ i. e. a line passing through origin

$-0.5x + y = 1$ A= (-2,0) B= (0,1)



The polygon is not closed one. i. e. the feasible area is unbounded. Thus there is no finite maximum value of z, the problem has **unbounded solution.**

PROBLEM 4

Solve the following problem graphically.

$$\text{Minimise } Z = 200x + 500y \quad \dots (1)$$

subject to the constraints:

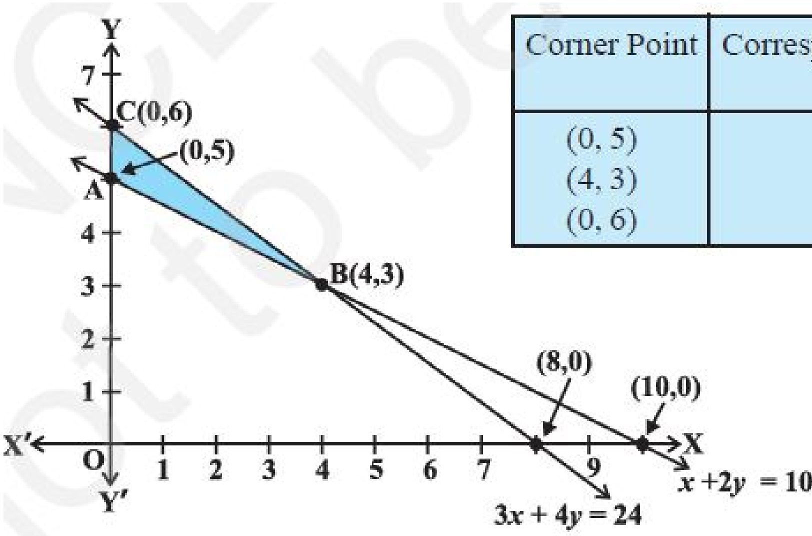
$$x + 2y \geq 10 \quad \dots (2)$$

$$3x + 4y \leq 24 \quad \dots (3)$$

$$x \geq 0, y \geq 0 \quad \dots (4)$$

Solution

The shaded region in the figure is the feasible region ABC determined by the system of constraints (2) to (4), which is bounded. The coordinates of corner points A, B and C are (0,5), (4,3) and (0,6) respectively.



Corner Point	Corresponding value of Z	
(0, 5)	2500	
(4, 3)	**2300** ⟵	Minimum
(0, 6)	3000	

Now we evaluate z = 200x + 500y

Hence, minimum value of z is 2300 attained at the point (4,3)

PROBLEM 5

Solve the following problem graphically.

Minimise and Maximise $Z = 3x + 9y$ ... (1)

subject to the constraints: $x + 3y \le 60$ ... (2)

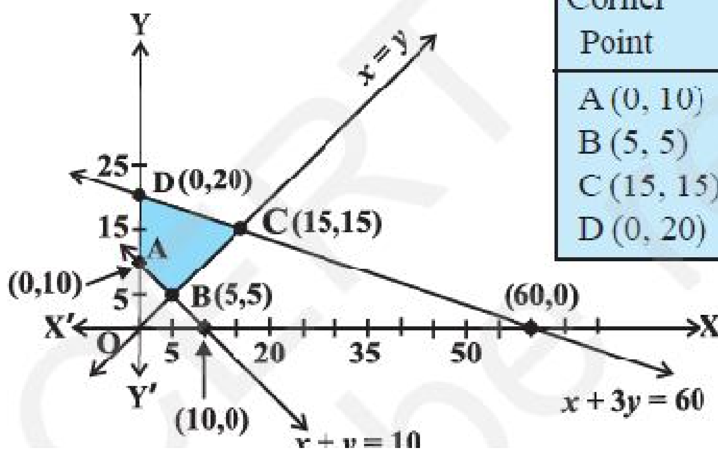
$x + y \ge 10$ ... (3)

$x \le y$ ... (4)

$x \ge 0, y \ge 0$ ... (5)

Solution

First of all, let us graph the feasible region of the system of linear inequalities (2) to (5). The feasible region ABCD is shown in figure. Note that the region is bounded. The coordinates of the corner points A, B, C and D are (0,10), (5,5), (15,15) and (0,20) respectively.



Corner Point	Corresponding value of $Z = 3x + 9y$	
A (0, 10)	90	
B (5, 5)	60	← Minimum
C (15, 15)	180	← Maximum (Multiple optimal solutions)
D (0, 20)	180	

We now find the minimum and maximum value of z. From the table, we find that the minimum value of z is 60 at the point B (5,5) of the feasible region.

The maximum value of z on the feasible region occurs at two corner points C (15,15) and D(0,20) and it is 180 in each case.

It is observed that in the problem has multiple optimal solutions at the corner points C and D, i.e. the both points produce same maximum value 180. In such cases, you can see that every point on the line segment CD joining the two corner points C and D also give the same maximum value. Same is also true in the case if the two points produce same minimum value.

PROBLEM 6

Solve the following problem graphically for minimum value of objective function.

$$Z = -50x + 20y \qquad \dots (1)$$

subject to the constraints:

$$2x - y \geq -5 \qquad \dots (2)$$

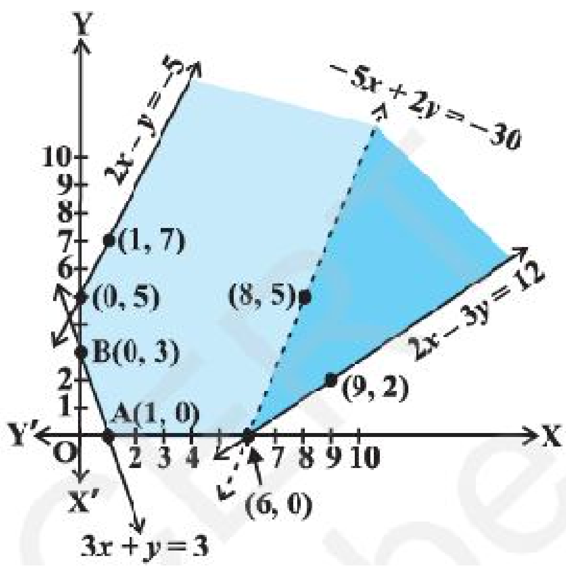
$$3x + y \geq 3 \qquad \dots (3)$$

$$2x - 3y \leq 12 \qquad \dots (4)$$

$$x \geq 0, y \geq 0 \qquad \dots (5)$$

Solution

The feasible region of the system of inequalities (2) to (5) is shown in the figure. The feasible region is **unbounded.**



Corner Point	Z = – 50x + 20y	
(0, 5)	100	
(0, 3)	60	
(1, 0)	–50	
(6, 0)	–300 ⟵	smallest

PROBLEM 7

Solve the following problem graphically for minimum value of objective function.

Minimize z = 3x + 2y

Subject to constraints

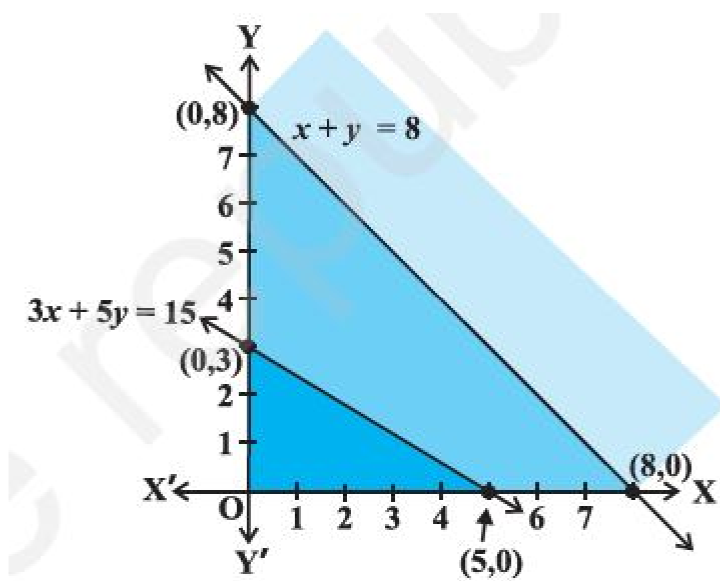
$$x + y \geq 8 \qquad \ldots (1)$$

$$3x + 5y \leq 15 \qquad \ldots (2)$$

$$x > 0,\ y > 0 \qquad \ldots (3)$$

Solution

Let us graph the inequalities for the constraints (1) to (3).



As there is no point satisfying all the constraints simultaneously. Thus, the problem is having no feasible region and hence **no feasible solution.**

PROBLEM 8

A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 liters and 10 liters per hectare. Further, no more than 800 liters of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society?

Solution

Let x hectare of land be allocate to crop X and y hectare to crop Y.

Profit per hectare on crop X = Rs 10,500

Profit per hectare on crop Y = Rs 9000

Therefore, total profit = Rs (10500x + 9000y)

The mathematical formulation of the problem is as follows:

Maximise $Z = 10500\,x + 9000\,y$

subject to the constraints:

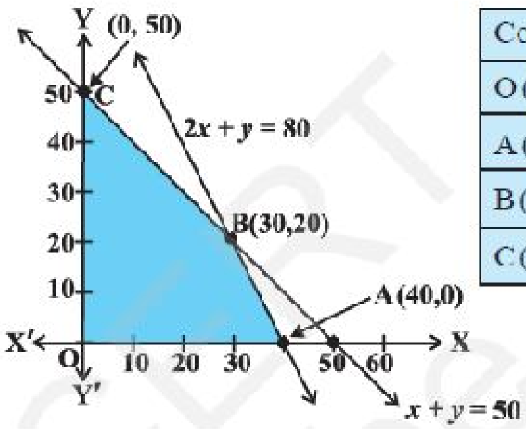
$x + y \leq 50$ (constraint related to land) ... (1)

$20x + 10y \leq 800$ (constraint related to use of herbicide)

i.e. $2x + y \leq 80$ ... (2)

$x \geq 0,\ y \geq 0$ (non negative constraint) ... (3)

Let us draw the graph of the system inequalities (1) to (3). The feasible region OABC is shown in figure, which is bounded.



Corner Point	$Z = 10500x + 9000y$	
O(0, 0)	0	
A(40, 0)	420000	
B(30, 20)	**495000** ←	Maximum
C(0,50)	450000	

The coordinates of the corner points O, A, B and C are (0,0), (40,0), (30,20) and (0,50). The maximum value of an objective function is at point B (30,20). Hence, the society will get the maximum profit of Rs 4,95,000 by allocating 30 hectares for cop X and 20 hectares for crop Y.

PROBLEM 9

A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 hours for fabricating and 1 hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realize a maximum profit? What is the maximum profit per week?

Solution

The mathematical formulation of the problem is

Maximise $Z = 8000x + 12000y$ ... (1)

subject to the constraints:

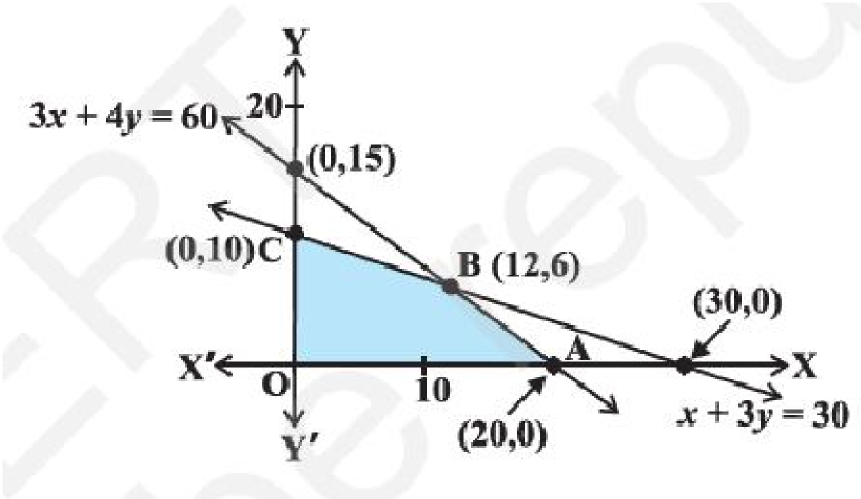
$9x + 12y \leq 180$ (Fabricating constraint)

i.e. $3x + 4y \leq 60$ ... (2)

$x + 3y \leq 30$ (Finishing constraint) ... (3)

$x \geq 0, y \geq 0$ (non-negative constraint) ... (4)

The feasible region OABC is determined by the linear inequalities (2) to (4) is shown as below.



We find that maximum value of z is Rs 1,68,000 at B (12,6). Hence the company should produce 12 units of Model A and 6 pieces of Model B to realize maximum profit and maximum profit is Rs 1,68,000.