Importance of Float (Slack) and Critical Path

1. Slack or Float shows how much allowance each activity has, i.e how long it can be delayed without affecting completion date of project
2. Critical path is a sequence of activities from start to finish with zero slack. Critical activities are activities on the critical path.
3. Critical path identifies the minimum time to complete project
4. If any activity on the critical path is shortened or extended, project time will be shortened or extended accordingly

Importance of Float (Slack) and Critical Path (cont)

5. So, a lot of effort should be put in trying to control activities along this path, so that project can meet due date. If any activity is lengthened, be aware that project will not meet deadline and some action needs to be taken.
6. If can spend resources to speed up some activity, do so only for critical activities.
7. Don’t waste resources on non-critical activity, it will not shorten the project time.
8. If resources can be saved by lengthening some activities, do so for non-critical activities, up to limit of float.
9. Total Float belongs to the path

Problem 2

Consider the network shown in Fig. 14.14 which consists of the following activities:

Activity	:	1-2	1-3	2-3	2-5	3-4	3-6	4-5	4-6	5-6	6-7
Duration (weeks)	:	15	15	3	5	8	12	1	14	3	14

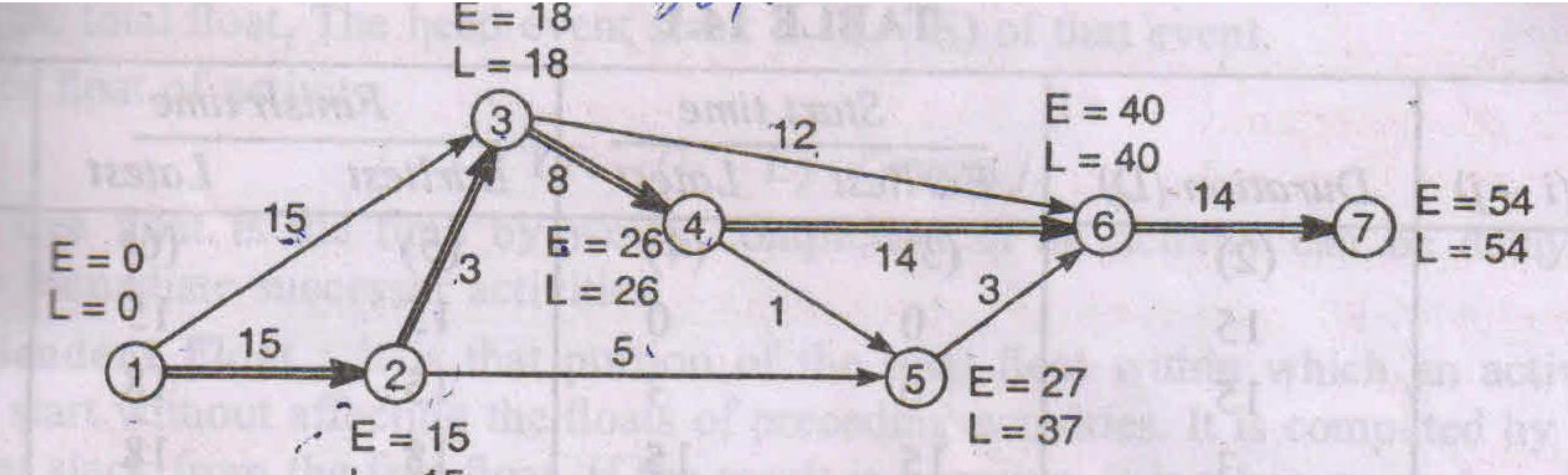


Fig. 14.14

E= EARLIEST START TIME
L= LATEST FINISH TIME

- **For forward pass observe no. of incoming arrows for the particular node**
- **For backward pass observe no. of outgoing arrows for the particular node.**

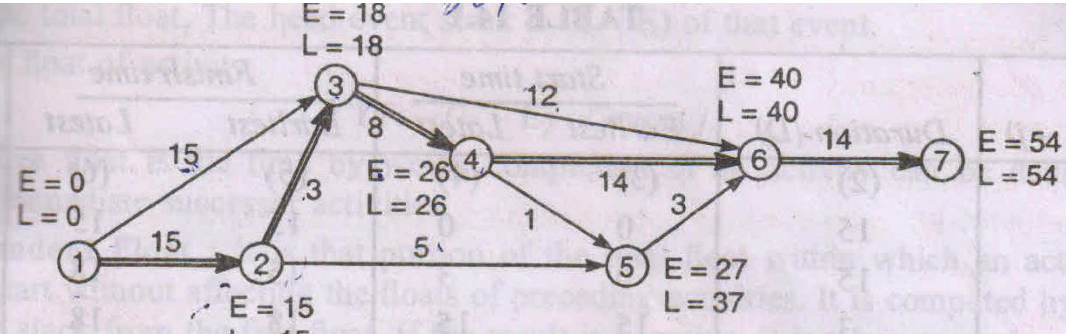


Fig. 14.14

TABLE 14.1

Activity (i – j)	Duration (D)	Start time Earliest	Start time Latest	Finish time Earliest	Finish time Latest	Total Float
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1-2	15	0	0	15	15	0
1-3	15	0	3	15	18	3
2-3	3	15	15	18	18	0
2-5	5	15	32	20	37	17
3-4	8	18	18	26	26	0
3-6	12	18	28	30	40	10
4-5	1	26	36	27	37	10
4-6	14	26	26	40	40	0
5-6	3	27	37	30	40	10
6-7	14	40	40	54	54	0

Problem 3

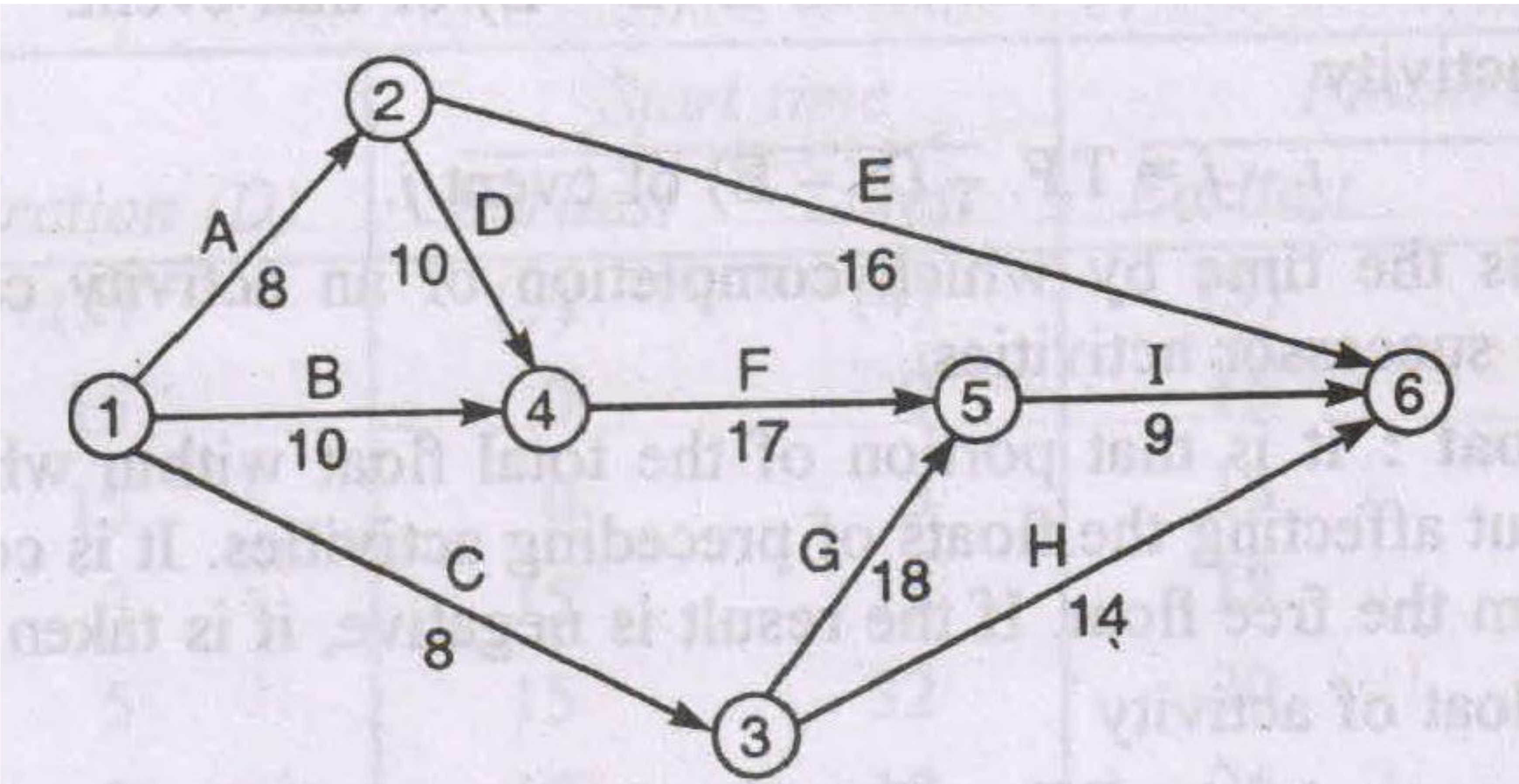
Tasks A, B, C,..., H, I constitute a project. The precedence relationships are

$$A < D;\ A < E;\ B < F;\ D < F;\ C < G;\ C < H;\ F < I;\ G < I.$$

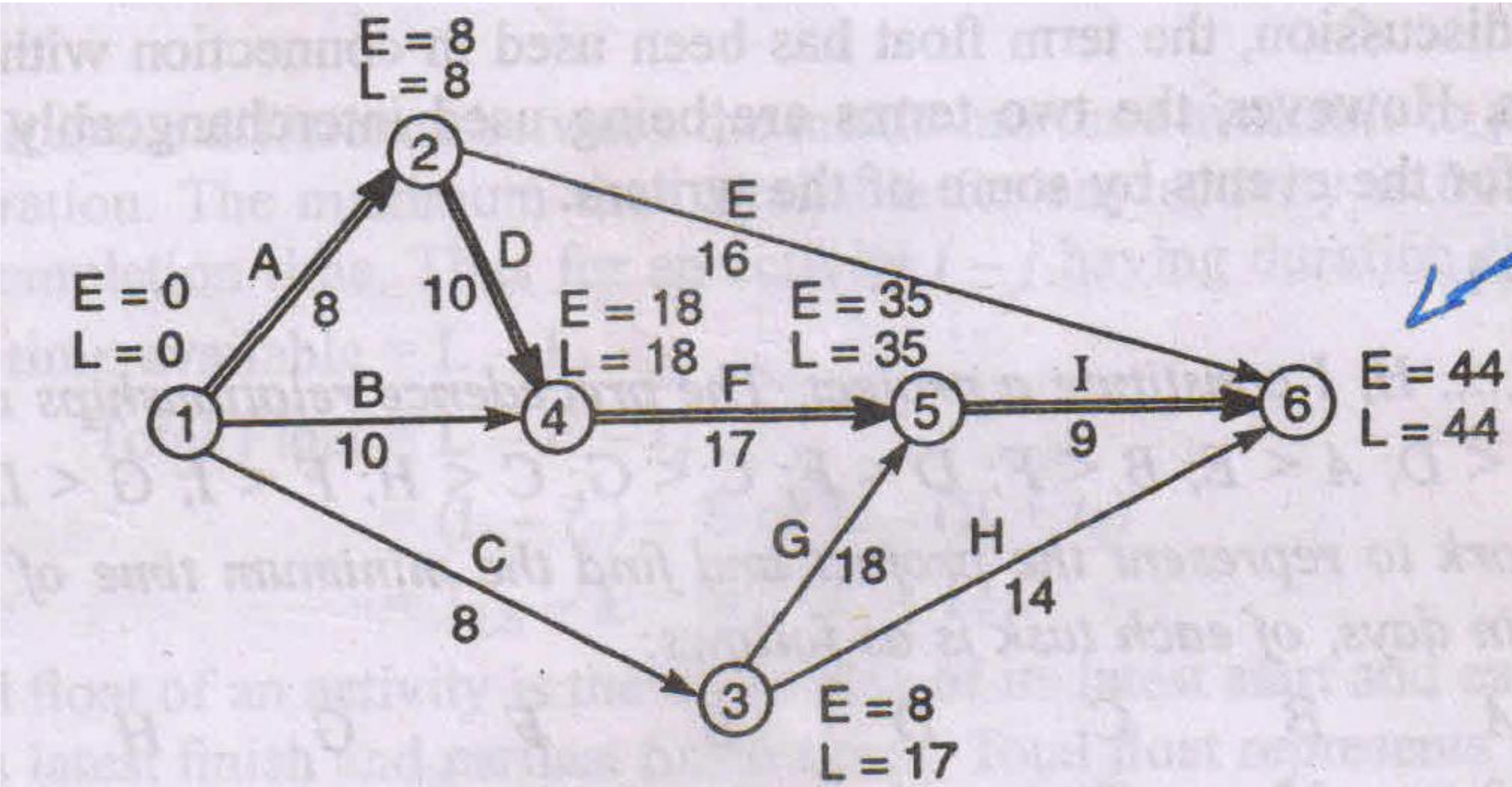
Draw a network to represent the project and find the minimum time of completion of the ect when time, in days, of each task is as follows:

Task	A	B	C	D	E	F	G	H	I
Time	8	10	8	10	16	17	18	14	9

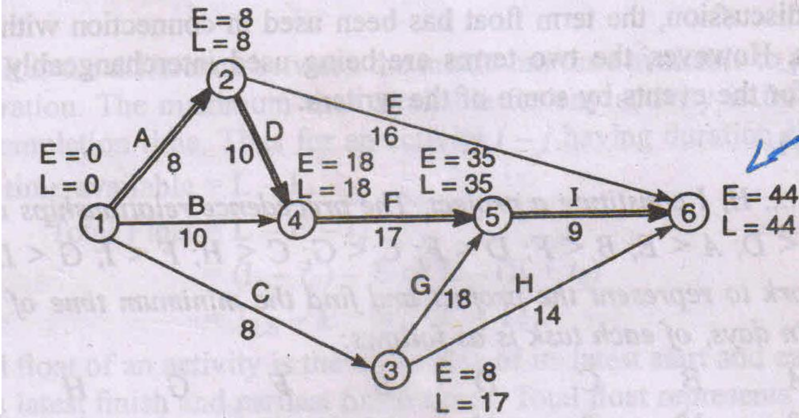
Also identify the critical path.



1
2
3
4
5
6
A
8
B
10
C
8
D
10
E
16
F
17
G
18
H
14
I
9



- For forward pass observe no. of incoming arrows for the particular node
- For backward pass observe no. of outgoing arrows for the particular node.



Method 1. The network analysis table is compiled as below.

TABLE 14.2

Activity	Duration	Start time Earliest	Start time Latest	Finish time Earliest	Finish time Latest	Total float
1-2	8	0	0	8	8	0
1-3	8	0	9	8	17	9
1-4	10	0	8	10	18	8
2-4	10	8	8	18	18	0
2-6	16	8	28	24	44	20
3-5	18	8	17	26	35	9
3-6	14	8	30	22	44	22
4-5	17	18	18	35	35	0
5-6	9	35	35	44	44	0

Activities 1-2, 2-4, 4-5 and 5-6 having zero float are the critical activities and 1-2-4-5-6 is the critical path.

Method 2. For identifying the critical path, the following conditions are checked. If an activity satisfies all the three conditions, it is critical.

(*i*) E = L for the tail event.

(*ii*) E = L for the head event.

(*iii*) $E_j - E_i = L_j - L_i = t_{ij}$.

Activities 1-2, 2-4, 4-5 and 5-6 satisfy these conditions. Other activities do not fulfil all the conditions. The critical path is, therefore, 1-2-4-5-6.

Method 3. The various paths and their durations are:

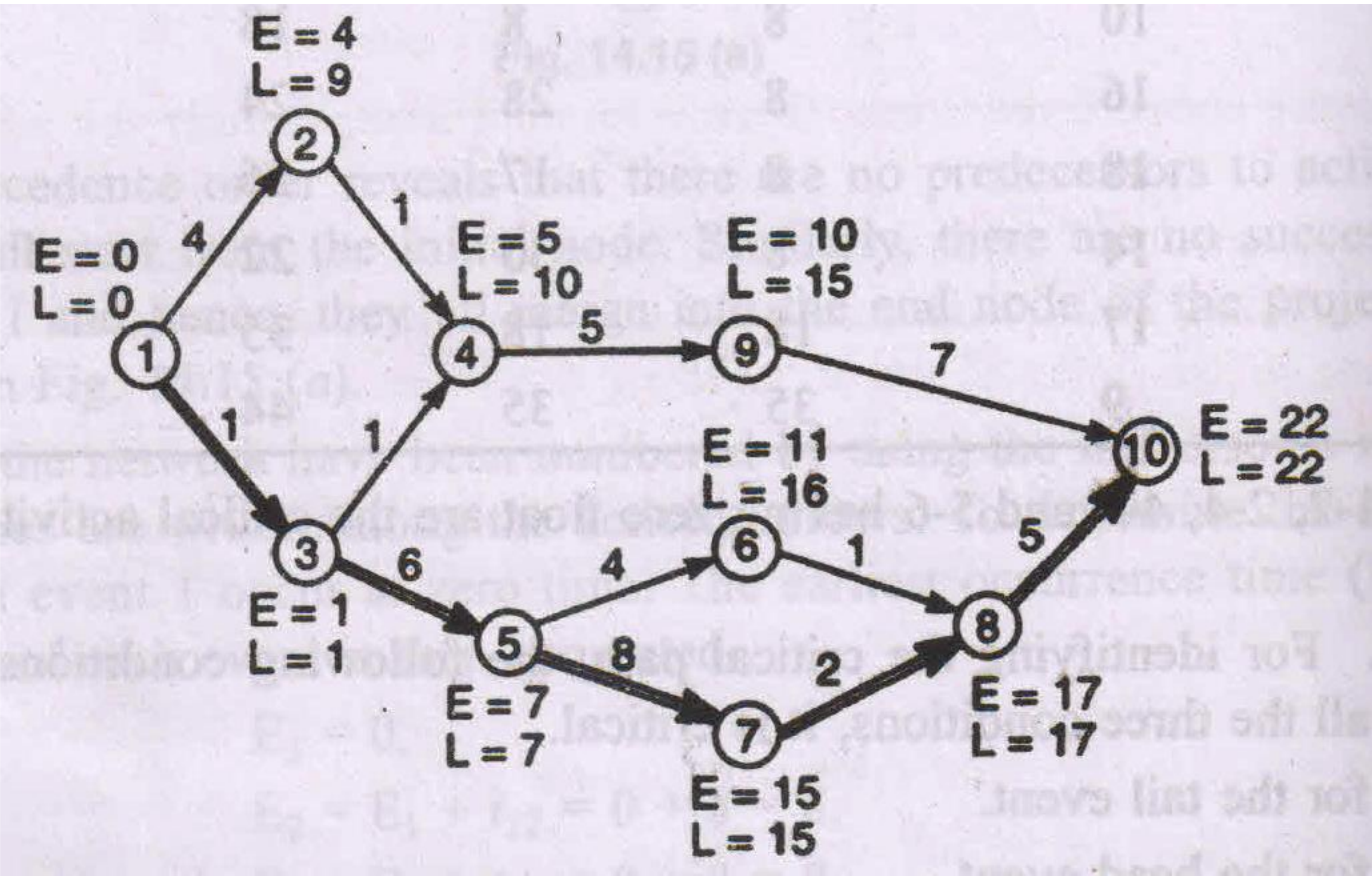
Path	*Duration (days)*
1–2–6	8 + 16 = 24
1–2–4–5–6	8 + 10 + 17 + 9 = 44
1–4–5–6	10 + 17 + 9 = 36
1–3–5–6	8 + 18 + 9 = 35
1–3–6	8 + 14 = 22

Path 1-2-4-5-6, the longest in time involving 44 days, is the critical path. It represented by lines in Fig. 14.15 (*b*).

Problem 4

A project schedule has the following characteristics:

Activity	*Time (weeks)*	*Activity*	*Times (weeks)*
1 - 2	4	5 - 6	4
1 - 3	1	5 - 7	8
2 - 4	1	6 - 8	1
3 - 4	1	7 - 8	2
3 - 5	6	8 - 10	5
4 - 9	5	9 - 10	7



E = 0
L = 0
1
4
E = 4
L = 9
2
1
E = 5
L = 10
4
5
E = 10
L = 15
9
7
E = 22
L = 22
10
1
E = 1
L = 1
3
1
6
E = 7
L = 7
5
4
E = 11
L = 16
6
1
8
E = 15
L = 15
7
2
E = 17
L = 17
8
5

TABLE 14.3

Activity	Duration (weeks)	Start time		Finish time		Total float
		Earliest	Latest	Earliest	Latest	
1 - 2	4	0	5	4	9	5
1 - 3	1	0	0	1	1	0
2 - 4	1	4	9	5	10	5
3 - 4	1	1	9	2	10	8
3 - 5	6	1	1	7	7	0
4 - 9	5	5	10	10	15	5
5 - 6	4	7	12	11	16	5
5 - 7	8	7	7	15	15	0
6 - 8	1	11	16	12	17	5
7 - 8	2	15	15	17	17	0
8 - 10	5	17	17	22	22	0
9 - 10	7	10	15	17	22	5

Path 1-3-5-7-8-10 with project duration of 22 weeks is the critical path.

Problem 5

The utility data for a network are given below. Determine the total, free, independent and fering floats and identify the critical path.

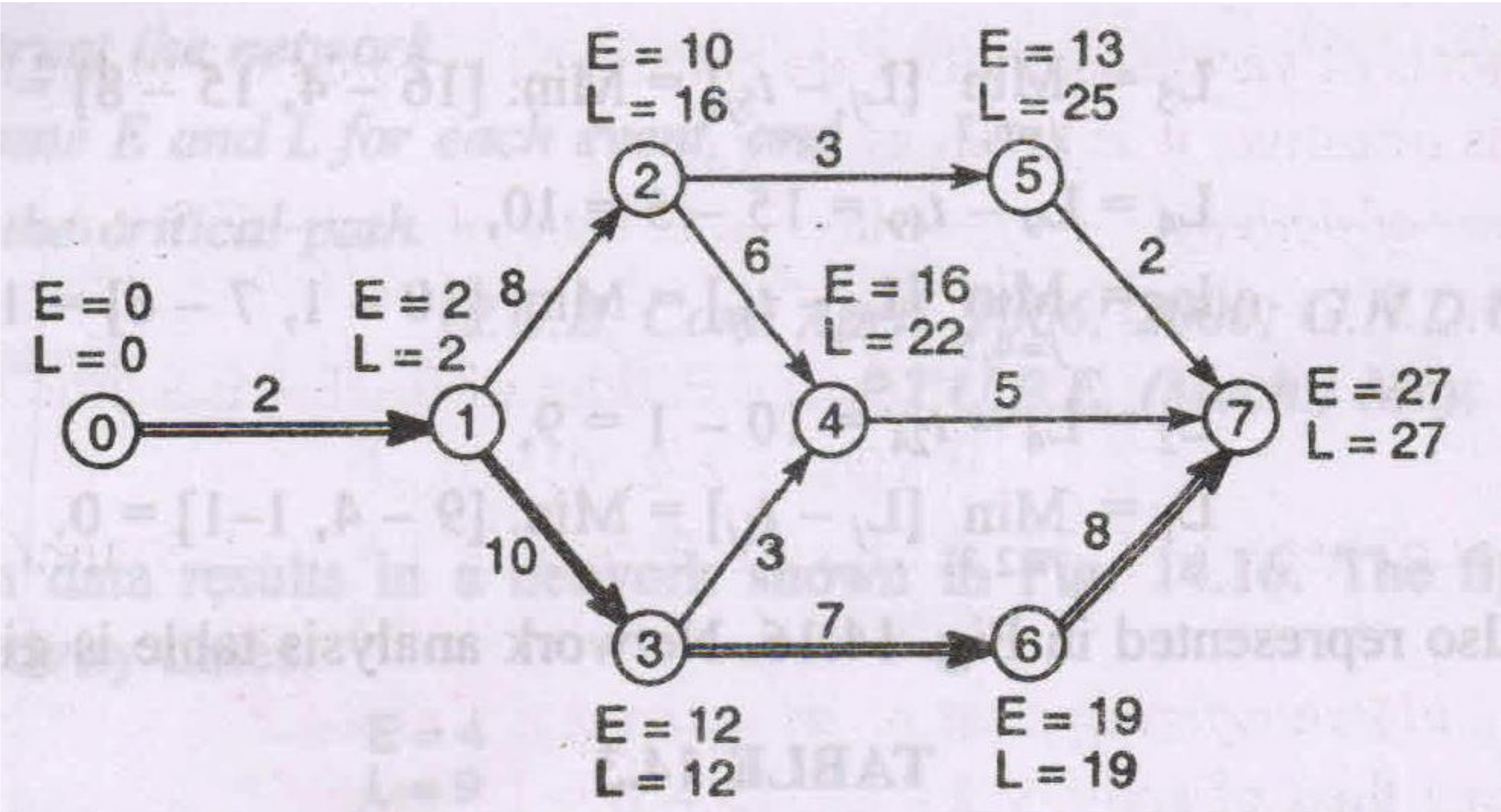
Activity :	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration :	2	8	10	6	3	3	7	5	2	8

Total Float= It is the difference between maximum time available to perform the activity and the activity duration.

Free Float: It is the time by which completion of an activity can be delayed without delaying its immediate successor activities.

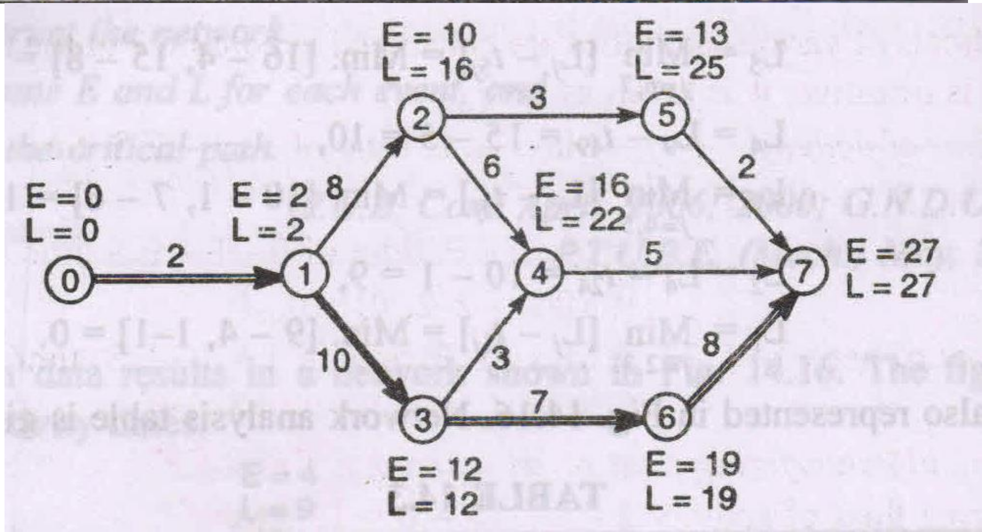
Independent Float= It is the time by which an activity can be delayed for start without affecting immediate preceding activities.

Interfering Float= it refers to that portion of an activity float which can not be consumed without adversely affecting the floats of the subsequent activities.


E = 0
L = 0
E = 2
L = 2
E = 10
L = 16
E = 13
L = 25
E = 16
L = 22
E = 27
L = 27
E = 12
L = 12
E = 19
L = 19
0
1
2
3
4
5
6
7
2
8
10
3
6
3
7
5
2
8

Activity	Duration	Start time		Finish time		Float			
		Earliest	Latest	Earliest	Latest	Total	Free	Independent	Interfering
1	2	3	4	5	6	7	8	9	10
0 - 1	2	0	0	2	2	0	0	0	0
1 - 2	8	2	8	10	16	6	0	0	6
1 - 3	10	2	2	12	12	0	0	0	0
2 - 4	6	10	16	16	22	6	0	$-6 \approx 0$	6
2 - 5	3	10	22	13	25	12	0	$-6 \approx 0$	12
3 - 4	3	12	19	15	22	7	1	1	6
3 - 6	7	12	12	19	19	0	0	0	0
4 - 7	5	16	22	21	27	6	6	0	0
5 - 7	2	13	25	15	27	12	12	0	0
6 - 7	8	19	19	27	27	0	0	0	0

Free float = total float - head event slack
Independent Float = free float – tail event slack
Interfering Float= Total Float – Free Float



Problem 6

For the network given in Fig. 14.18, determine the total, free, independent and interfering floats for each activity. Times for activities are in months.

[*H.P.U.B.Tech. (Mech.) Nov., 2010; P.U.B.E. (T.I.T.) Nov., 2006; B.Com. April, 2007*]

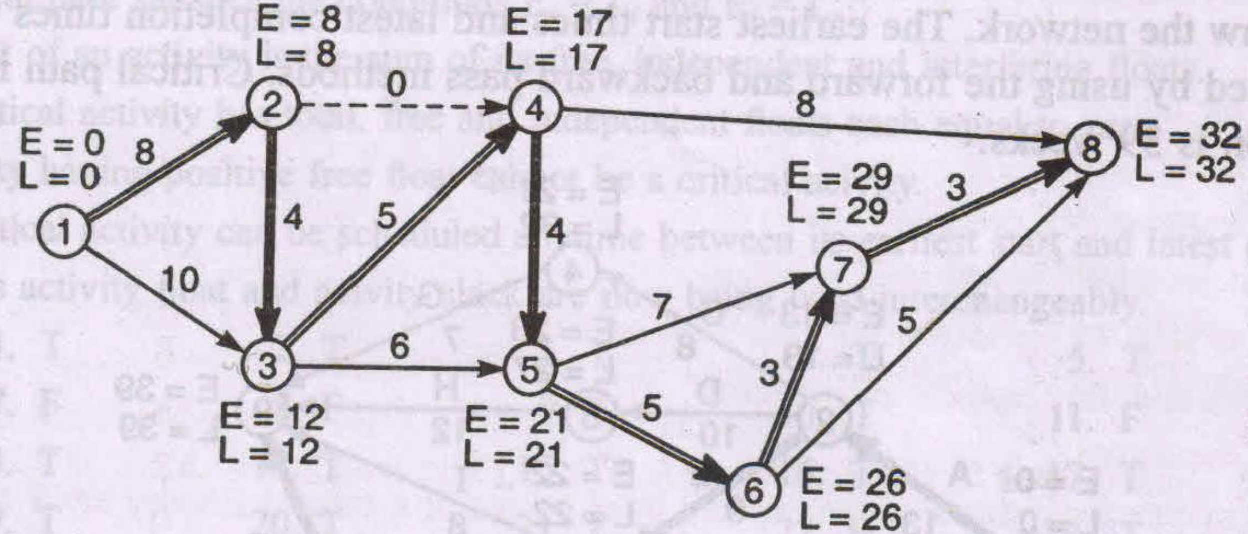


TABLE 14.5

Activity	Duration (months)	Start time		Finish time		Float			
		Earliest	Latest	Earliest	Latest	Total	Free	Independent	Interfering
1-2	8	0	0	8	8	0	0	0	0
1-3	10	0	2	10	12	2	2	2	0
2-3	4	8	8	12	12	0	0	0	0
2-4	0	8	17	8	17	9	9	9	0
3-4	5	12	12	17	17	0	0	0	0
3-5	6	12	15	18	21	3	3	3	0
4-5	4	17	17	21	21	0	0	0	0
4-8	8	17	24	25	32	7	7	7	0
5-6	5	21	21	26	26	0	0	0	0
5-7	7	21	22	28	29	1	1	1	0
6-7	3	26	26	29	29	0	0	0	0
6-8	5	26	27	31	32	1	1	1	0
7-8	3	29	29	32	32	0	0	0	0

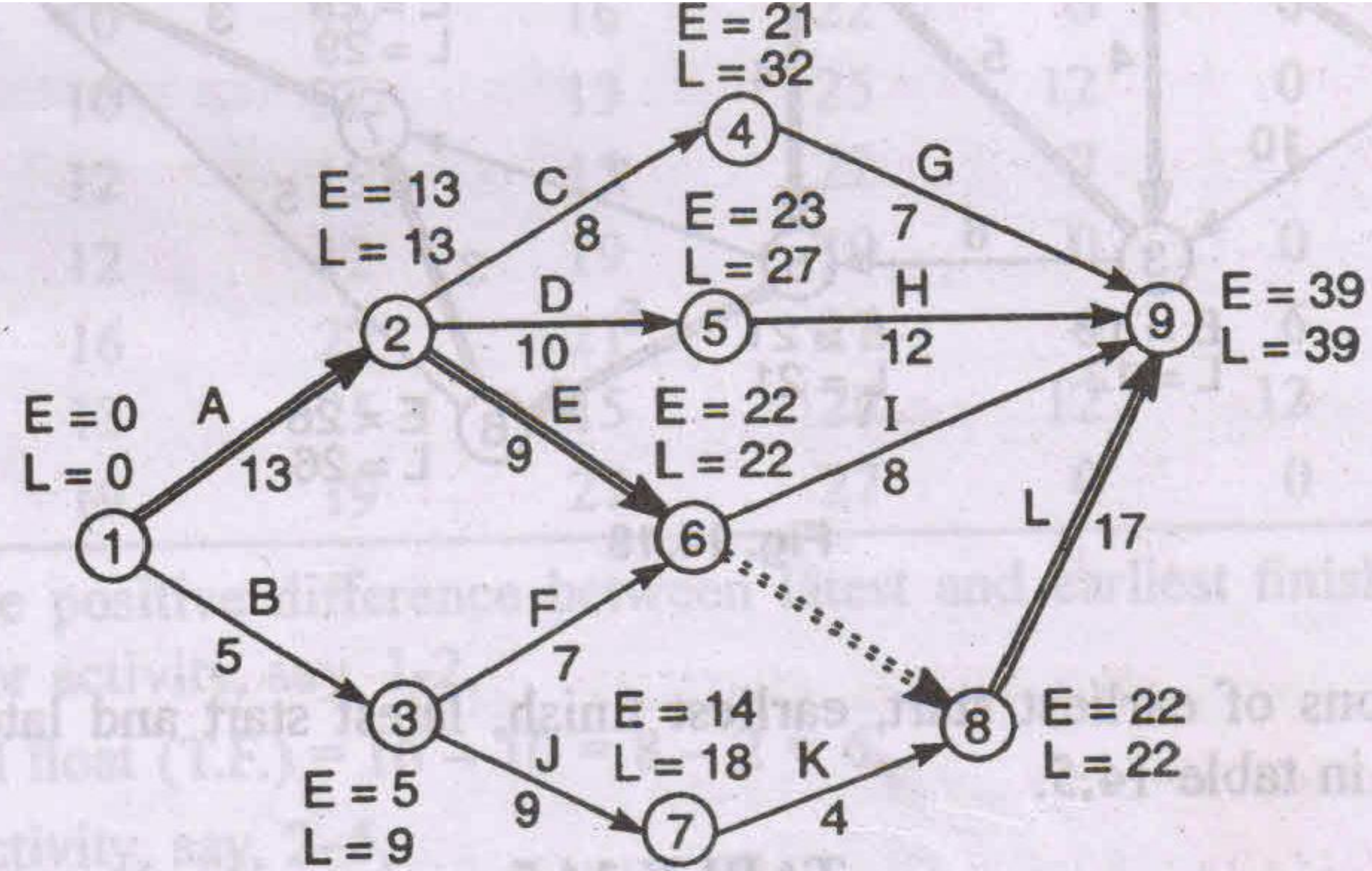
Problem 7

Estimated times for the jobs of a project are given below:

Job	:	A	B	C	D	E	F	G	H	I	J	K
Time (weeks)	:	13	5	8	10	9	7	7	12	8	9	4

The constraints governing the jobs are as follows:

A and B are start jobs; A controls C, D and E; B controls F and J; G depends upon C; H depends on D; E and F control I and L; K follows J; L is also controlled by K; G, H, I and L are the last jobs. Draw the network, determine project duration and the critical path.



E = 0
L = 0
A
13
B
5
E = 13
L = 13
C
8
D
10
E
9
E = 5
L = 9
F
7
J
9
E = 21
L = 32
E = 23
L = 27
E = 22
L = 22
E = 14
L = 18
K
4
G
7
H
12
I
8
L
17
E = 39
L = 39
E = 22
L = 22
1
2
3
4
5
6
7
8
9

PERT

PERT is designed for scheduling complex projects that involve many inter-related tasks. it improves planning process because:

1. **It forms planner to define the projects various components activities.**
2. **It provides a basis for normal time estimates & yet allows for some measure of optimism or pessimism in estimating the completion dates.**
3. **It shows the effects of changes to overall plans they contemplated.**
4. **It provides built in means for ongoing evaluation of the plan.**

ESTIMATING ACTIVITY TIMES

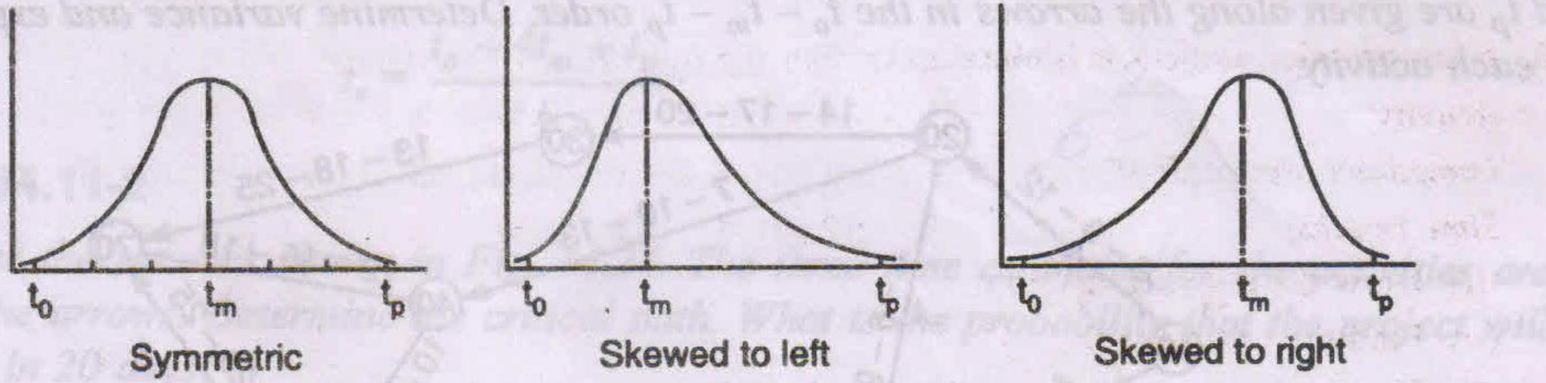
- **Optimistic time (t_0**) : is that time estimate of an activity when everything is assumed to go as per plan. In other words it is the estimate of minimum possible time which an activity takes in completion under ideal conditions.

- **Most likely time (t_m)** : the time which the activity will take most frequently if repeated number of times.

- **Pessimistic time (tp)** : the unlikely but possible performance time if whatever could go wrong , goes wrong in series. In other words it is the longest time the can take.

Frequency Distribution Curve for PERT

We have three time estimates for a PERT activity, the optimistic (t_0), pessimistic (t_p) and the most likely time (t_m). In the range from optimistic to pessimistic, there can be a number of time estimates for the activity. If a frequency distribution curve for the activity times is plotted, it will look like the one shown in figure 14.22. It is assumed to be a β-distribution curve with a unimodal point occurring at t_m and its end points occurring at t_0 and t_p. The most likely time need not be the midpoint of t_0 and t_p and hence the frequency distribution curve may be skewed to the left, skewed to the right or symmetric. The assumption of β-distribution, however, is not flawless and the calculations made on this assumption may, sometimes, be in error to the tune of even 30%.



EXPECTED TIME

- The times are combined statically to develop the expected time t_e .

$$t_e = \frac{to + 4tm + tp}{6}$$

Standard deviation of the time of the time required to complete the project

$$= \frac{tp - to}{6}$$

Though the curve is not fully described by the mean (μ) and the standard deviation (σ), yet in PERT the following relations are approximated for μ and σ:

Variance $$V = \sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$$

or $$\sigma = \frac{t_p - t_o}{6},$$

and $$\mu = \frac{t_0 + 4t_m + t_p}{6}.$$

Expected time or *average time* of an activity is taken equal to mean. This is the time that the activity is expected to consume while executed. Thus

$$t_e = \mu = \frac{t_0 + 4t_m + t_p}{6}.$$

The expected time is then used as the activity duration and the critical path is obtained by the analytical method explained earlier.

STEPS INVOLVED IN PERT

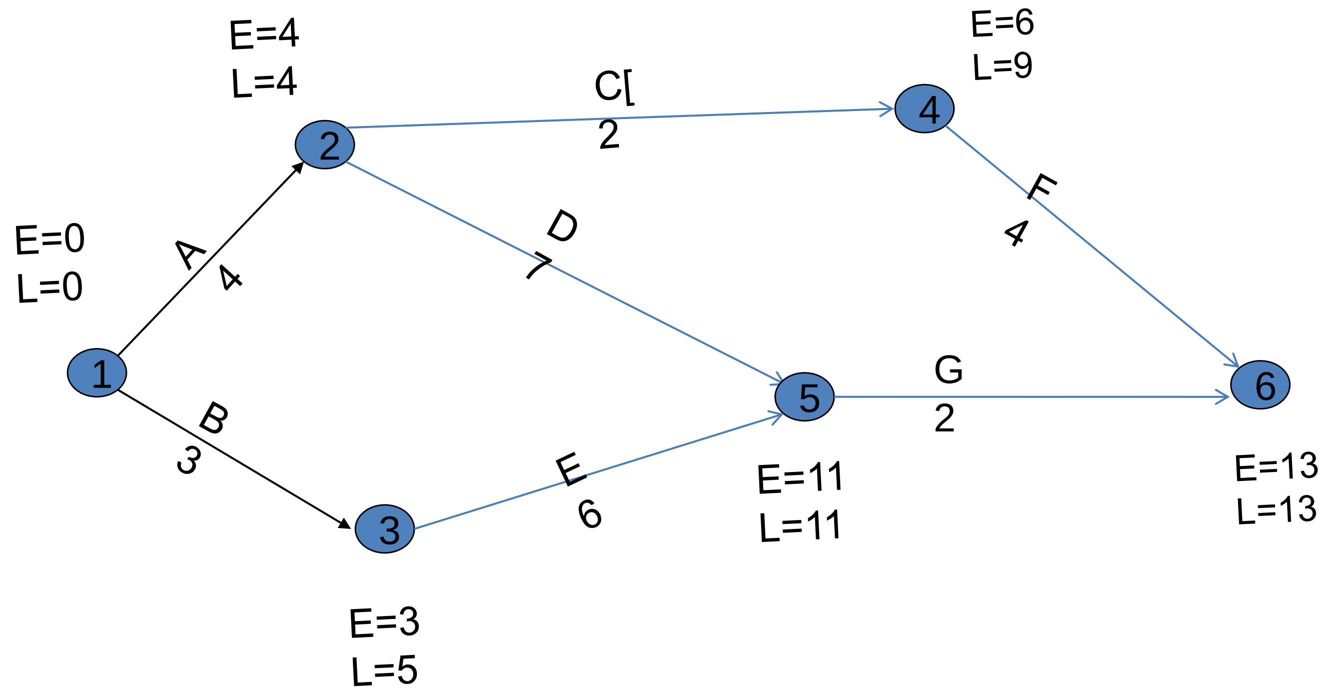
- Develop list of activities.
- A rough network for PERT is drawn.
- Events are numbered from left to right.
- Time estimates for each activity are obtained.
- Expected time for each activity is calculated : to+4tm+tp / 6
- Using these expected times calculate earliest & latest finish & start times of activities.
- Estimate the critical path.
- Using this estimate compute the probability of meeting a specified completion date by using the standard normal equation

$$Z = \frac{\text{Due date – expected date of completion}}{\text{standard deviation of critical path}}$$

Problem 1

Activity	Predecessor activity	Optimistic (to)	Most likely (tm)	Pessimistic (tp)
A	-	2	3	10
B	-	2	3	4
C	A	1	2	3
D	A	4	6	14
E	B	4	5	12
F	C	3	4	5
G	D,E	1	1	7

Network with LS & LF time



Activity	Expected time $T_e = \frac{t_o + 4t_m + t_p}{6}$
A(1-2)	4
B(1-3)	3
C(2-4)	2
D(2-5)	7
E(3-5)	6
F(4-6)	4
G(5-6)	2

CRITICAL PATH : 1-2-5-6 or A-D-G

activity	te	Es	Ef = Es +te	LS = Lf -te	Lf	Total float	Free float	Independent float
1-2	4	0	4	0	4	0	0	0
1-3	3	0	3	2	5	2	0	0
2-4	2	4	6	7	9	3	0	0
2-5	7	4	11	4	11	0	0	0
3-5	6	3	9	5	11	2	2	0
4-6	4	6	10	9	13	3	3	0
5-6	2	11	13	11	13	0	0	0

Total float = Ls – Es or Lf – Ef

Free float = total float - head event slack

Independent Float = free float – tail event slack

Consider the network shown Fig. 14.23. For each activity, the three time estimates t_o, t_m and t_p are given along the arrows in the $t_o - t_m - t_p$ order. Determine variance and expected time for each activity.

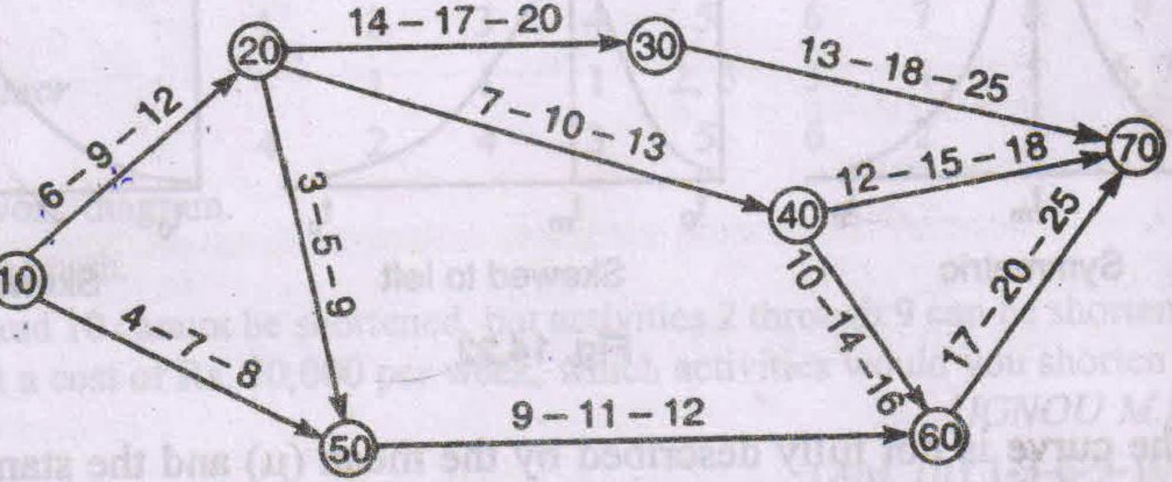
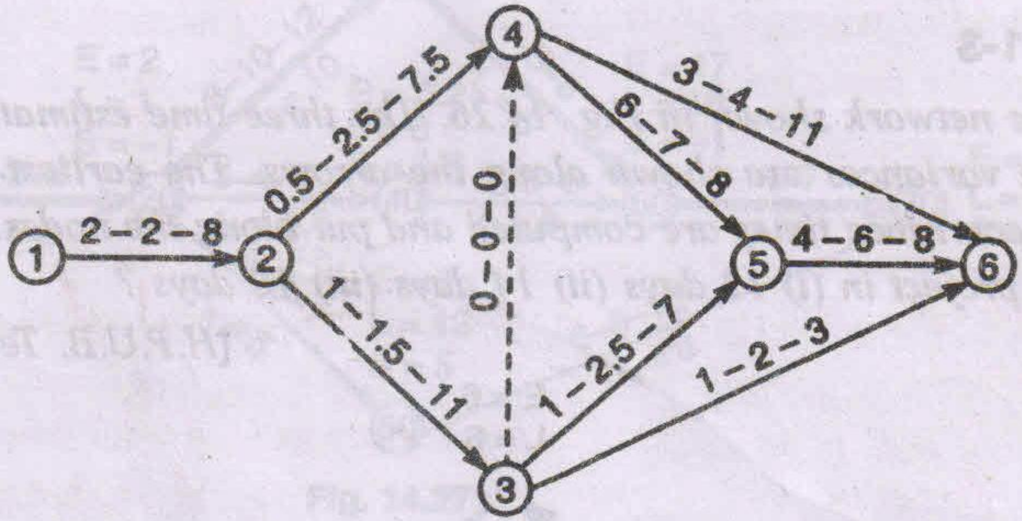


Fig. 14.23

Activity i-j		t_o	t_m	t_p	$\sigma^2 = V = \left(\frac{t_p - t_o}{6}\right)^2$	$t_e = \frac{t_o + 4t_m + t_p}{6}$
Predecessor event *i*	Successor event *j*					
10	20	6	9	12	1.00	9.0
10	50	4	7	8	0.44	6.7
20	30	14	17	20	1.00	17.0
20	40	7	10	13	1.00	10.0
20	50	3	5	9	1.00	5.33
30	70	13	18	25	4.00	18.33
40	60	10	14	16	1.00	13.67
40	70	12	15	18	1.00	15.00
50	60	9	11	12	0.25	10.83
60	70	17	20	25	1.78	20.33

Consider the network shown in Fig. 14.24. The three time estimates for the activities are given along the arrows. Determine the critical path. What is the probability that the project will be completed in 20 days?



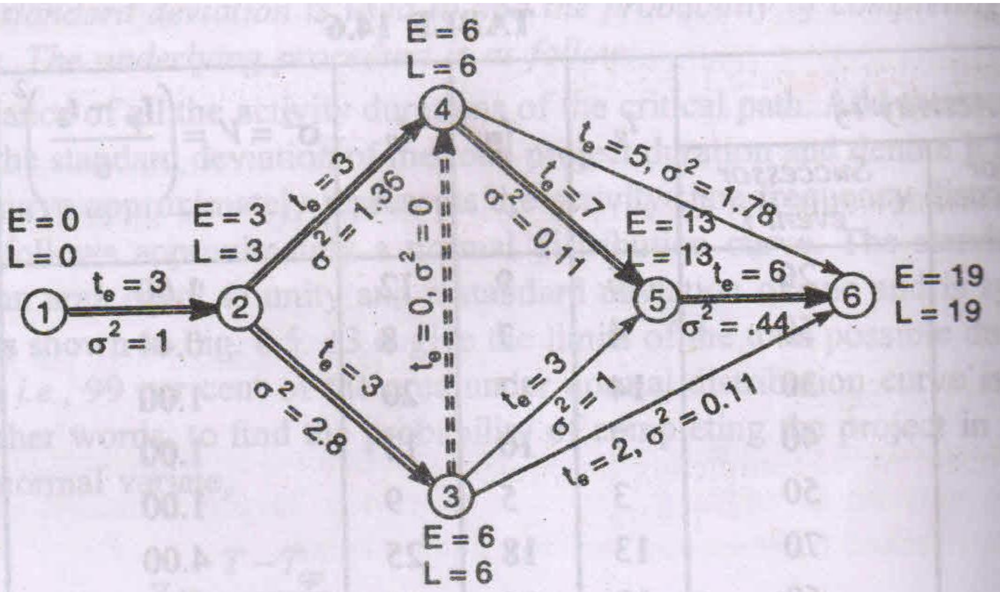


Fig. 14.25

Expected duration of the project, T_{cp} = 19 days.

Contractual obligation time, T = 20 days,

Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$ and $t_e = \frac{t_o + 4t_m + t_p}{6}$.

Standard deviation of the project,

$$\sigma = \sqrt{\Sigma \sigma_{ij}^2} \text{ for all } i\text{-}j \text{ on the critical path.}$$

$\therefore$ σ for path 1-2-4-5-6 $= \sqrt{1 + 1.36 + 0.11 + 0.44} = 1.70$,

σ for path 1-2-3-4-5-6 $= \sqrt{1 + 2.8 + 0 + 0.11 + 0.44} = 2.08$.

$\therefore$ $\sigma = 2.08$ is chosen as it is higher of the two values.

$\therefore$ Normal deviate, $Z = \frac{T - T_{cp}}{\sigma} = \frac{20 - 19}{2.08} = 0.48$.

From table C-2, probability = 68.44%.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Consider the network shown in Fig. 14.26. The three time estimates, the expected durations and the variances are shown along the arrows. The earliest expected times latest allowable occurrence times are computed and put along the nodes. What is the prob of completing the project in (i) 12 days (ii) 14 days (iii) 10 days ?

[H.P.U.B. Tech. (Mech.) Nov.