Optimal values of the above variables are

$$Q^* = \sqrt{\frac{2C_0R}{C_c} \cdot \frac{(C_s + C_c)}{C_s}}$$

$$Q_1^* = \sqrt{\frac{2C_0R}{C_c} \cdot \frac{C_s}{C_s + C_c}}$$

$$Q_2^* = Q^* - Q_1^*$$

$$t^* = \frac{Q^*}{R}$$

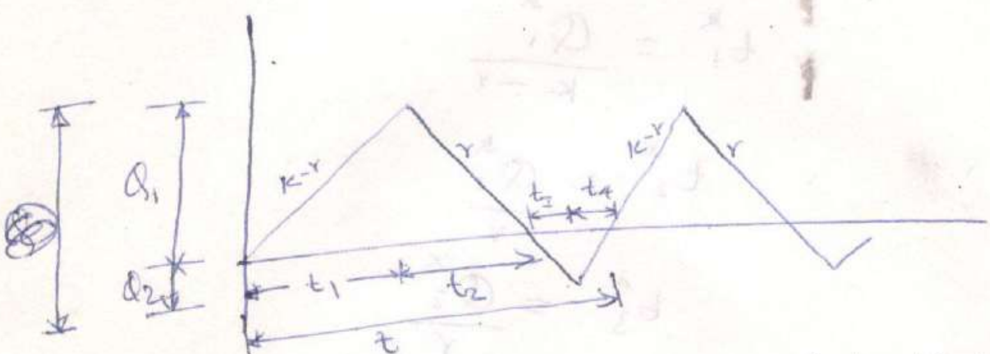
$$t_1^* = \frac{Q_1^*}{R}$$

$$t_2^* = \frac{Q_2^*}{R}$$

$$\text{No. of orders/period} = \frac{R}{Q^*}.$$

Model 4: Manufacturing Model with Shortages.

In this model, an item is produced and consumed simultaneously for a portion of the cycle time. During the remaining cycle time, only the consumption of the item takes place. The cost of production per unit is the same irrespective of the production lot size. Stock-out is permitted in this model, and it is assumed that the stock-out units will be satisfied from the units which will be produced at a later date, with a penalty. This is called backordering. The opn of this model is shown ~~to~~ in fig.



The variables which are used in this model are given below:

r - Demand of an item/period.
K - Production rate of the item (no. of units produced/period)
C_o - Cost/setup.
C_c - Carrying cost/unit/period.
C_s - Shortage cost/unit/period.
t - Total cycle time
~~p - Cost of production/unit.~~
t_1 - Period of production as well as consumption of the item satisfying period's requirement
t_2 - period of consumption only.
t_3 - Period of shortage.
t_4 - Period of production as well as consumption of the item satisfying backorder.

Optimal values of the above variables are:

Economic batch quantity $Q^* = \sqrt{\frac{2C_0}{C_c} \cdot \frac{kr}{k-r} \cdot \frac{C_c+C_s}{C_s}}$

Maximum inventory $Q_1^* = \sqrt{\frac{2C_0}{C_c} \cdot \frac{r(k-r)}{k} \cdot \frac{C_s}{C_c+C_s}}$

Maximum stockout $Q_2^* = \sqrt{\frac{2C_0 C_c}{C_s(C_c+C_s)} \cdot \frac{r(k-r)}{k}}$

$$Q_1^* = \frac{k-r}{k} Q^* - Q_2^*$$

$$t^* = \frac{Q^*}{r}$$

$$t_1^* = \frac{Q_1^*}{k-r}$$

$$t_2^* = \frac{Q_1^*}{r}$$

$$t_3^* = \frac{Q_2^*}{r}$$

$$t_4^* = \frac{Q_2^*}{k-r}$$

Quantity Discount

Probl: Find optimum order quantity for a product for which price breaks are given below:

Quantity	Unit Cost (Rs.)
$0 \leq Q_1 \leq 500$	15.00
$500 \leq Q_2$	14.50

Monthly demand for product is 250 units. Cost of carrying is 2% of unit cost and cost of ordering is Rs. 300.

Soln: Given $R = 250$, $C_c = 2\%$ of unit cost.

$C_o = Rs. 300$.

$P_2 = 14.50$

$$Q_2^* = \sqrt{\frac{2RC_o}{i \cdot P_2}} = \sqrt{\frac{2 \times 250 \times 300}{14.5 \times 0.02}}$$

$$Q_2^* = 719 \text{ units}$$

here, we find $Q_2^* > 500$

Hence $\underline{EOQ = 719 \text{ units}}$

2) Find optimum order quantity for a product for which price breaks are as follows:

Quantity	Price (Rs.)
Less than 500	10.00
500 or above	9.25

Monthly demand is 200 units. Cost of storage is 2% of unit cost. Cost of ordering is Rs. 100.

Soln: $R = 200$ units $\quad C_c = 2\%$ of unit cost

$C_o = Rs. 100$.

$$Q_{o2}^* = \sqrt{\frac{2RC_o}{C_c}} = \sqrt{\frac{2\times 200\times 100}{0.02\times 9.25}}$$

$$= 464.9 \simeq 465 \text{ unit}$$

we find $Q_2^* < 500$, Hence calculate Q_1^*

$$\underline{\underline{Q_1^*}} = \sqrt{\frac{2\times 200\times 100}{0.02\times 10}} = 447.2 \simeq \underline{\underline{447 \text{ unit}}}$$

Estimate ~~the~~ the total cost under the two condi''
viz. $Q_1^* = 447$ & $b_1 = 500$.

$$(T.C)_{\substack{1\\Q_1=447}} = r\times P + \frac{R}{Q}\times C_o + \frac{Q_1^*}{2}\times C_c$$

$$= \cancel{447}\ 200\times 10 + \frac{200}{447}\times 100 + \frac{447}{2}\times 0.02\times 10$$

$$= Rs\ 2089.44$$

$$(T.C)_{\substack{2\\b_1=500}} = 200\times 9.25 + \frac{200}{500}\times 100 + \frac{500}{2}\times 0.02\times 9.25$$

$$= Rs.\ 1936.25$$

As. $(T.C)_{\substack{2\\b_1=500}} < (T.C)_{\substack{1\\Q_1^*=447}}$

$$\underline{EOQ = 500}$$

Price Break Method when there are Two Price Breaks

Problem 1: Determine a decision rule using the basic purchasing E.O.Q. model for an annual demand of 20,000 units, ordering cost of Rs. 200 per order, carrying cost of 10% ~~per~~ of unit price. The basic price is Rs. 8.00 per unit. This price is, in effect, for all orders of less than 5000 units. Orders for 5000 units and above but less than 10,000 units may be purchased for Rs. 7.50 per unit. Orders for 10,000 or more may be purchased for Rs. 7.25 per unit.

Sol^n:

Given $R = 20{,}000$, $C_o = $ Rs 200 / order,

$C_c = 10\%$ of unit price.

Quantity	Price per unit
$q < 5000$	$P_1 = 8$
$5000 \le q < 10{,}000$	$P_2 = 7.50$
$10{,}000 \le q$	$P_3 = 7.25$

$$Q_3^* = \sqrt{\frac{2RC_o}{C_c}} = \sqrt{\frac{2 \times 20{,}000 \times 200}{0.1 \times 7.25}}$$

$$= 3322 \text{ (approx.)}$$

we find, $$Q_2^* = \sqrt{\frac{2RC_o}{C_c}} = \sqrt{\frac{2 \times 20{,}000 \times 200}{0.75}}$$

$(C_c = 0.1 \times 7.5)$

$$= 3265$$

As $Q_2^* < 5000$

$$Q_1^* = \sqrt{\frac{2RC_o}{C_c}} = \sqrt{\frac{2 \times 20{,}000 \times 200}{0.1 \times 8}}$$

$$Q_1^* = 3162$$

Hence EOQ can be either Q_{01}, b_1 or b_2

$$(T.C)(Q_1^*) = 20,000 \times 8 + \frac{20,000}{3162} \times 200 + \frac{3162}{2} \times 0.8$$

$$= 1,62,530$$

$$(T.C)(b_1 = 5000) = 20,000 \times 7.5 + \frac{20,000}{5000} \times 200 + \frac{5000}{2} \times 0.75$$

$$= 1,52,675$$

$$(T.C)(b_2 = 10,000) = 20,000 \times 7.25 + \frac{20,000}{10,000} \times 200 + \frac{10,000}{2} \times 0.725$$

$$= \text{Rs. } 149,025$$

As we find minimum total cost occurs when order quantity $= 10,000$

$$\therefore \underline{E.O.Q = 10,000}$$