Unit V: Game Theory (Graphical Method)

The graphical method is used to solve the games whose payoff matrix has

- Two rows and n columns (2 x n)
- m rows and two columns (m x 2)

Algorithm for solving 2 x n matrix games

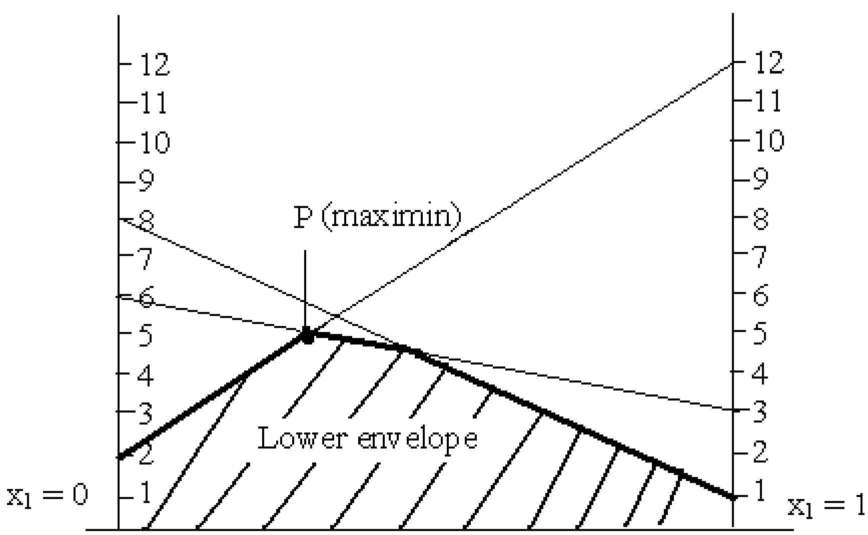
- Draw two vertical axes 1 unit apart. The two lines are $x_1 = 0$, $x_1 = 1$
- Take the points of the first row in the payoff matrix on the vertical line $x_1 = 1$ and the points of the second row in the payoff matrix on the vertical line $x_1 = 0$.
- The point a_{1j} on axis $x_1 = 1$ is then joined to the point a_{2j} on the axis $x_1 = 0$ to give a straight line. Draw 'n' straight lines for j=1, 2… n and determine the highest point of the lower envelope obtained. This will be the **maximin point**.
- The two or more lines passing through the maximin point determines the required 2 x 2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Example 1

Solve by graphical method

$$\begin{array}{c c} & \begin{array}{ccc} B1 & B2 & B3 \end{array} \\ \begin{array}{c} A1 \\ A2 \end{array} & \begin{bmatrix} 1 & 3 & 12 \\ 8 & 6 & 2 \end{bmatrix} \end{array}$$

Solution



$$\begin{array}{c|cc|c} & B2 & B3 & \\ \hline A1 & 3 & 12 & 4 \\ A2 & 6 & 2 & 9 \\ \hline & 10 & 3 & \end{array}$$

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{6 - 72}{5 - 18}$$

V = 66/13

S_A = (4/13, 9 /13)

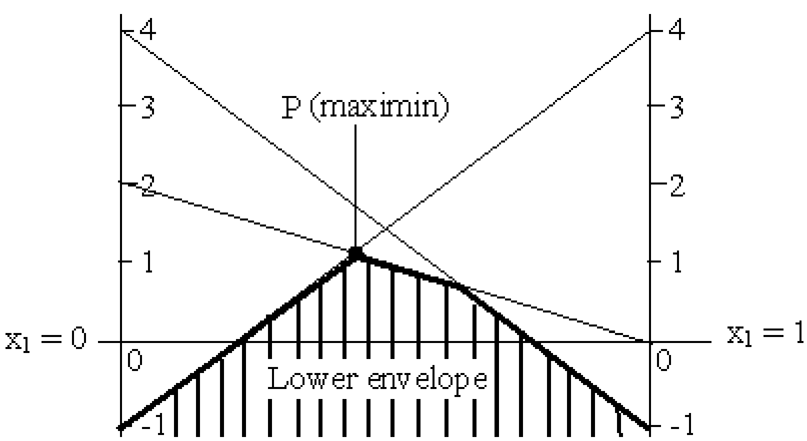
S_B = (0, 10/13, 3 /13)

Example 2

Solve by graphical method

$$\begin{array}{c} \\ A1 \\ A2 \end{array} \begin{array}{c} \begin{array}{ccc} B1 & B2 & B3 \end{array} \\ \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix} \end{array}$$

Solution



$$\begin{array}{c|cc|c} & B1 & B3 & \\ \hline A1 & 4 & 0 & 3 \\ A2 & -1 & 2 & 4 \\ \hline & 2 & 5 & \end{array}$$

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{8 - 0}{6 + 1}$$

V = 8/7

S_A = (3/7, 4 /7)

S_B = (2/7, 0, 5 /7)

Algorithm for solving m x 2 matrix games

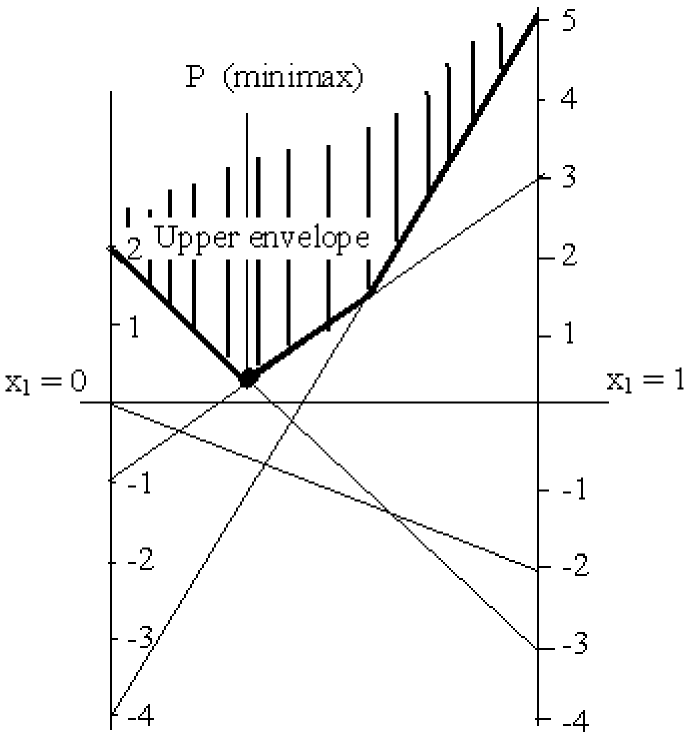
- Draw two vertical axes 1 unit apart. The two lines are x_1 =0, x_1 = 1
- Take the points of the first row in the payoff matrix on the vertical line x_1 = 1 and the points of the second row in the payoff matrix on the vertical line x_1 = 0.
- The point a_{1j} on axis x_1 = 1 is then joined to the point a_{2j} on the axis x_1 = 0 to give a straight line. Draw 'n' straight lines for j=1, 2… n and determine the lowest point of the upper envelope obtained. This will be the **minimax point**.
- The two or more lines passing through the minimax point determines the required 2 x 2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Example 1

Solve by graphical method

$$\begin{array}{cc} & \begin{array}{cc} B1 & B2 \end{array} \\ \begin{array}{c} A1 \\ A2 \\ A3 \\ A4 \end{array} & \begin{bmatrix} -2 & 0 \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix} \end{array}$$

Solution



$$\begin{array}{c|cc|c} & B1 & B2 & \\ \hline A2 & 3 & -1 & 5 \\ A3 & -3 & 2 & 4 \\ \hline & 3 & 6 & \end{array}$$

$$V = \frac{a_{11}\, a_{22} - a_{21}\, a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{6-3}{5+4}$$

V = 3/9 = 1/3

S_A = (0, 5 /9, 4/9, 0)

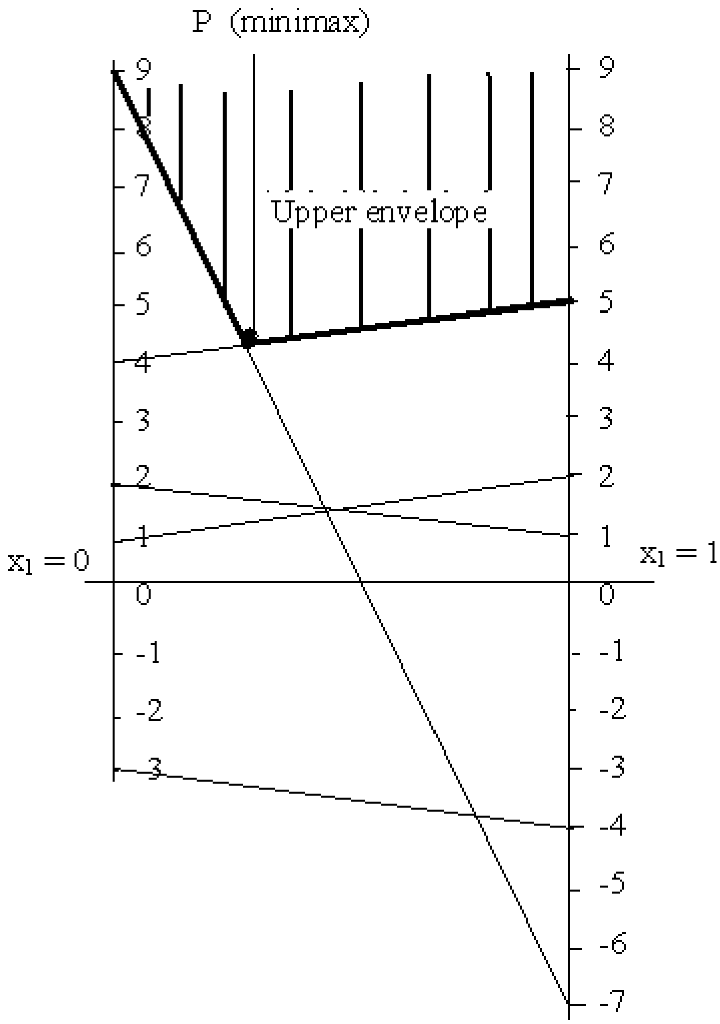
S_B = (3/9, 6 /9)

Example 2

Solve by graphical method

$$\begin{array}{c|cc} & B1 & B2 \\ \hline A1 & 1 & 2 \\ A2 & 5 & 4 \\ A3 & -7 & 9 \\ A4 & -4 & -3 \\ A5 & 2 & 1 \end{array}$$

Solution



$$\begin{array}{c|cc|c} & B1 & B2 & \\ \hline A2 & 5 & 4 & 16 \\ A3 & -7 & 9 & 1 \\ \hline & 5 & 12 & \end{array}$$

$$V - \frac{a_{11}\, a_{22} - a_{21}\, a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} - \frac{45 + 28}{14 + 3}$$

$V = 73/17$

$S_A = (0, 16/17, 1/17, 0, 0)$

$S_B = (5/17, 12/17)$