7

Inventory Control

7.1 INTRODUCTION

Inventory is essential to provide flexibility in operating a system or organization. An inventory can be classified into raw materials inventory, work-in-process inventory and finished goods inventory. The raw material inventory removes dependency between suppliers and plants. The work-in-process inventory removes dependency between various machines of a product line. The finished goods inventory removes dependency between plants and its customers or market. The main functions of an inventory are: smoothing out irregularities in supply, minimizing the production cost and allowing organizations to cope up with perishable materials.

Some important termenologies of inventory control are discussed now.

Inventory decisions. The following two basic *inventory decisions* are generally taken by managers.

1. When to replenish the inventory of an item?
2. How much of an item to order when the inventory of that item is to be replenished?

Costs of inventory systems. The following costs are associated with the inventory system.

1. Purchase price/unit
2. Ordering cost/order
3. Carrying cost/unit/period
4. Shortage cost/unit/period.

Costs trade off. If we place frequent orders, the cost of ordering will be more, but the inventory carrying cost will be less. On the other hand, if we place less frequent orders, the ordering cost will be less, but the carrying cost will be more. In Fig. 7.1, for an increase in Q (order size), the carrying cost increases and the ordering cost decreases. The total cost curve represents the sum of ordering cost and carrying cost for each order size. The order size at which the total cost is minimum is called *economic order quantity* (EOQ) or *optimal order size* (Q^*).

7.2 MODELS OF INVENTORY

There are different models of inventory. The inventory models can be classified into *deterministic* models and *probabilistic* models. The various deterministic models are:

(a) Purchase model with instantaneous replenishment and without shortages;

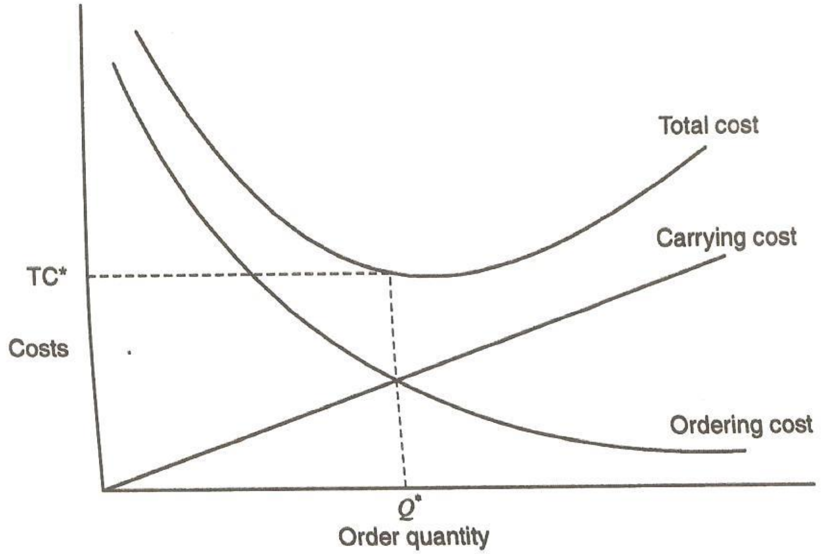

Fig. 7.1 Trade-off between costs.

(b) Manufacturing model without shortages;

(c) Purchase model with instantaneous replenishment and with shortages;

(d) Manufacturing model with shortages;

These models are explained in the following sections.

7.2.1 Purchase Model with Instantaneous Replenishment and without Shortages

In this inventory model, orders of equal size are placed at periodical intervals. The items against an order are replenished instantaneously and the items are consumed at a constant rate. The purchase price per unit is same irrespective of order size.

Let us suppose, D = Annual demand in units
C_o = Ordering cost/order
C_c = Carrying cost/unit/year
P = Purchase price per unit
Q = Order size

The corresponding purchase model can be represented as shown in Fig. 7.2. From the above assumptions, we have:

$$\text{The number of orders/year} = \frac{D}{Q}.$$

$$\text{Average inventory} = \frac{Q}{2}$$

$$\text{Cost of ordering/year} = \frac{D}{Q}C_o.$$

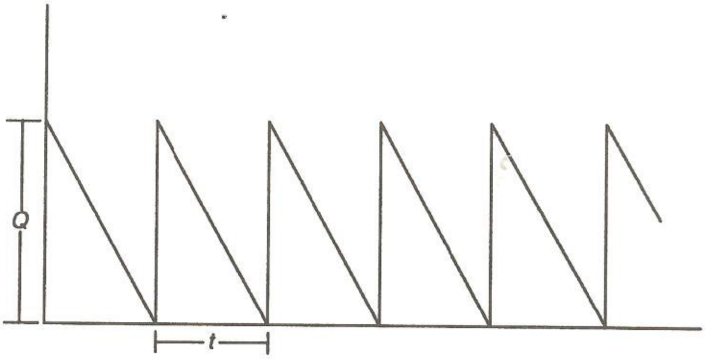

Fig. 7.2 Purchase model without shortage.

$$\text{Cost of carrying/year} = \frac{Q}{2}C_c$$

$$\text{Purchase cost/year} = DP$$

Therefore,

$$\text{Total inventory cost/year} = \frac{D}{Q}C_o + \frac{Q}{2}C_c + DP$$

Differentiating with respect to Q yields

$$\frac{d}{dQ}(\text{TC}) = \frac{-D}{Q^2}C_o + \frac{C_c}{2}$$

Differentiating it again with respect to Q yields

$$\frac{d^2}{dQ^2}(\text{TC}) = \frac{2D}{Q^3}C_o$$

Since the second derivative is positive, the optimal value for Q is obtained by equating the first derivative to zero. Therefore,

$$\frac{-D}{Q^2}C_o + \frac{C_c}{2} = 0 \quad \text{or} \quad Q^2 = \frac{2C_oD}{C_c}$$

Hence, the optimal order size is

$$Q^* = \sqrt{\frac{2C_oD}{C_c}}$$

and

$$\text{Total number of orders per year} = \frac{D}{Q^*}$$

where

$$\text{Time between orders} = \frac{Q^*}{D}$$

Example 7.1 Ram Industry needs 5,400 units/year of a bought-out component which will be used in its main product. The ordering cost is Rs. 250 per order and the carrying cost per unit per year is Rs. 30. Find: the economic order quantity (EOQ), the number of orders per year and the time between successive orders.

Solution

$$D = 5{,}400 \text{ units/year}$$
$$C_o = \text{Rs. } 250\text{/order}$$
$$C_c = \text{Rs. } 30\text{/unit/year}$$

Therefore, the economic order quantity

$$\text{EOQ } (Q^*) = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2 \times 250 \times 5{,}400}{30}} = 300 \text{ units}$$

where

$$\text{Number of orders/year} = \frac{D}{Q^*} = \frac{5{,}400}{300} = 18$$

and

$$\text{Time between successive orders} = \frac{Q^*}{D} = \frac{300}{5{,}400}$$
$$= 0.0556 \text{ year}$$
$$= 0.6672 \text{ month}$$
$$= 20 \text{ days (approx.)}$$

Example 7.2 Alpha Industry needs 15,000 units per year of a bought-out component which will be used in its main product. The ordering cost is Rs. 125 per order and the carrying cost per unit per year is 20% of the purchase price per unit. The purchase price per unit is Rs. 75. Find: economic order quantity, number of orders per year and time between successive orders.

Solution We have

$$D = 15{,}000 \text{ units/year}$$
$$C_o = \text{Rs. } 125\text{/order}$$
$$\text{Purchase price/unit} = \text{Rs. } 75$$
$$C_c = \text{Rs. } 75 \times 0.20$$
$$= \text{Rs. } 15\text{/unit/year}$$

Therefore, the economic order quantity is

$$\text{EOQ} = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2 \times 125 \times 15{,}000}{15}} = 500 \text{ units}$$

and

$$\text{Number of orders/year} = \frac{D}{Q^*} = \frac{15{,}000}{500} = 30$$

Time between successive orders is obtained as

$$\frac{Q^*}{D} = \frac{500}{15,000} = 0.033 \text{ year} = 0.4 \text{ month} = 12 \text{ days}$$

7.2.2 Manufacturing Model without Shortages

If a company manufactures an item which is required for its main product, then the corresponding model of inventory is called *manufacturing model*. In this model, shortages are not permitted. The rate of consumption of the item is assumed to be uniform throughout the year. The item is produced and consumed simultaneously for a portion of the cycle time. During the remaining cycle time, only the consumption of the item takes place and the cost of production per unit is same irrespective of production lot size.

Let us suppose,

r = Annual demand in units
k = Production rate of the item (total number of units produced/year)
C_o = Cost per set-up
C_c = Carrying cost/unit/year
p = Cost of production/unit
t_1 = Period of production as well as consumption of the item
t_2 = Period of consumption only
t = Cycle time (i.e. $t = t_1 + t_2$) .

The operation of the manufacturing model without shortages is shown as in Fig. 7.3.

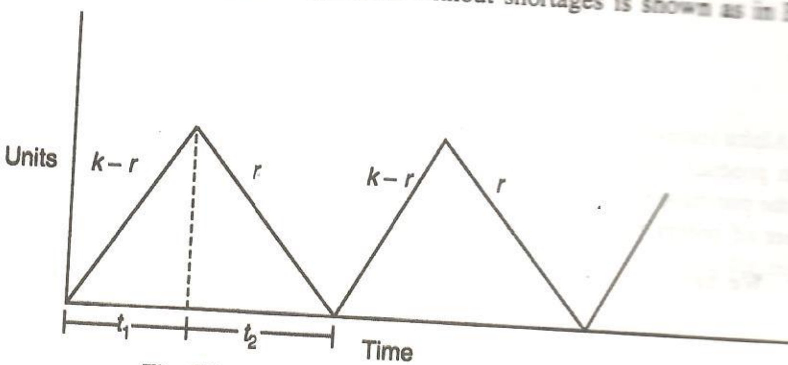

Fig. 7.3 Manufacturing model without shortages.

During the period t_1, the item is produced at the rate of k units per period and simultaneously it is consumed at the rate of r units per period. During this period, the inventory is built at the rate of $k - r$ units per period. During the period t_2, the production of the item is discontinued but the consumption of the same item is continued. Hence, the inventory is decreased at the rate of r units per period during this time t_2. The various formula to be applied for this kind of situation are given below.

$$\text{Economic batch quantity (EBQ or } Q^*) = \sqrt{\frac{2C_o r}{C_c[1 - (r/k)]}}$$

$$\text{Period of production as well as consumption, } t_1^* = \frac{Q^*}{k}$$

$$\text{Period of consumption only, } t_2^* = \frac{Q^*[1-(r/k)]}{r} = \frac{(k-r)t_1}{r}$$

$$\text{Cycle time } t = t_1^* + t_2^*$$

$$\text{Number of set-ups per year} = \frac{r}{Q^*}$$

Example 7.3 An automobile factory manufactures a particular type of gear within the factory. This gear is used in the final assembly. The particulars of this gear are: demand rate r = 14,000 units/year, production rate k = 35,000 units/year, set-up cost, C_o = Rs. 500 per set-up and carrying cost, C_c = Rs. 15/unit/year.

Find the economic batch quantity (EBQ) and cycle time.

Solution Applying the required formulae, we have the economic batch quantity

$$Q^* = \sqrt{\frac{2C_o r}{C_c[1-(r/k)]}}$$

$$= \sqrt{\frac{2 \times 500 \times 14{,}000}{15[1-(14{,}000/35{,}000)]}}$$

$$= 1{,}247.22 \text{ units}$$

$$= 1{,}248 \text{ (approx.)}$$

Now, the period of production as well as consumption

$$t_1^* = \frac{Q^*}{k}$$

$$= \frac{1{,}248}{35{,}000}$$

$$= 0.0357 \text{ year}$$

$$= 0.4284 \text{ month}$$

$$= 13 \text{ days (approx.)}$$

and the period of consumption

$$t_2^* = \frac{Q^*}{r}\left(1-\frac{r}{k}\right)$$

$$= \frac{1{,}248}{14{,}000}\left(1-\frac{14{,}000}{35{,}000}\right)$$

$$= 0.0535 \text{ year}$$

$$= 0.642 \text{ month}$$

$$= 20 \text{ days (approx.)}$$

Therefore, the cycle time is

$$t = t_1^* + t_2^* = 13 + 20 = 33 \text{ days}$$

Also

$$\text{The number of set-ups per year} = \frac{r}{Q^*} = \frac{14,000}{1,248} = 11.22$$

7.2.3 Purchase Model with Instantaneous Replenishment and with Shortages

In this model, an item on order will be received instantaneously and it is consumed at a constant rate. The purchase price per unit is same irrespective of order size. If there is no stock at the time of receiving a request for the item, it is assumed that it will be satisfied at a later date with a penalty. This is called *backordering*. The model is shown as in Fig. 7.4.

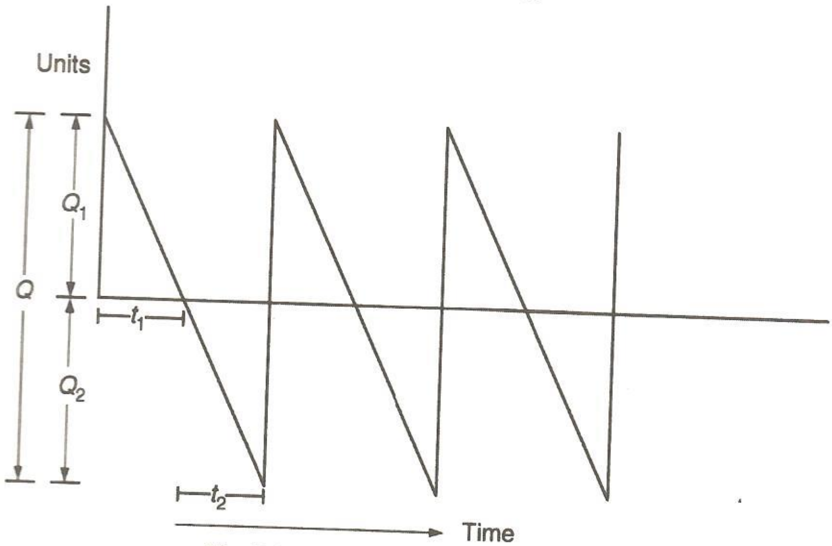

Fig. 7.4 Purchase model with shortages.

The variables which are to be used in this model are:

D = Demand/period

C_c = Carrying cost/unit/period

C_o = Ordering cost/order

C_s = Shortage cost/unit/period

Q = Order size

Q_1 = Maximum inventory

Q_2 = Maximum stock-out

t_1 = Period of positive stock

t_2 = Period of shortage

t = Cycle time $(t_1 + t_2)$

Optimal values of the above variables are:

$$Q^* = \sqrt{\frac{2C_o D}{C_c} \; \frac{C_s + C_c}{C_s}}$$

$$Q_1^* = \sqrt{\frac{2C_o D}{C_c} \; \frac{C_s}{C_s + C_c}}$$

$$Q_2^* = Q^* - Q_1^*$$

$$t^* = \frac{Q^*}{D}$$

$$t_1^* = \frac{Q_1^*}{D}$$

$$t_2^* = \frac{Q_2^*}{D}$$

where

$$\text{Number of orders/period} = \frac{D}{Q^*}$$

Example 7.4 The annual demand for a component is 7,200 units. The carrying cost is Rs. 500/unit/year, the ordering cost is Rs. 1,500 per order and the shortage cost is Rs. 2,000/unit/year. Find the optimal values of economic order quantity, maximum inventory, maximum shortage quantity, cycle time (t), inventory period (t_1) and, shortage period (t_2).

Solution We have

$$D = 7{,}200 \text{ units/year}$$

$$C_c = \text{Rs. } 500\text{/unit/year}$$

$$C_o = \text{Rs. } 1{,}500\text{/order}$$

$$C_s = \text{Rs. } 2{,}000\text{/unit/year}$$

Therefore,

$$\text{Economic order quantity } Q^* = \sqrt{\frac{2C_o D}{C_c} \; \frac{C_s + C_c}{C_s}}$$

$$= \sqrt{\frac{2 \times 1{,}500 \times 7{,}200}{500} \; \frac{2{,}000 + 500}{2{,}000}}$$

$$= 233 \text{ units (approx.)}$$

$$\text{Maximum inventory } Q_1^* = \sqrt{\frac{2C_0D}{C_c}\,\frac{C_s}{C_s + C_c}}$$

$$= \sqrt{\frac{2 \times 1{,}500 \times 7{,}200}{500}\,\frac{2{,}000}{2{,}000 + 500}}$$

$$= 186 \text{ units (approx.)}$$

$$\text{Maximum stock-out } Q_2^* = Q^* - Q_1^* = 233 - 186 = 47 \text{ units}$$

$$\text{Cycle time } t^* = \frac{Q^*}{D} = \frac{233}{7{,}200} \times 365 = 12 \text{ days (approx.)}$$

$$\text{Period of positive stock } t_1^* = \frac{Q_1^*}{D} = \frac{186}{7{,}200} \times 365 = 10 \text{ days (approx.)}$$

$$\text{Period of shortage } t_2^* = t^* - t_1^* = 12 - 10 = 2 \text{ days}$$

$$\text{Number of orders per year} = \frac{D}{Q^*} = \frac{7{,}200}{233} = 30.9$$

7.2.4 Manufacturing Model with Shortages

In this model, an item is produced and consumed simultaneously for a portion of the cycle time. During the remaining cycle time, only the consumption of the item takes place. The cost of production per unit is the same irrespective of the production lot size. Stock-out is permitted in this model, and it is assumed that the stock-out units will be satisfied from the units which will be produced at a later date, with a penalty. This is called *backordering*. The operation of this model is shown in Fig. 7.5.

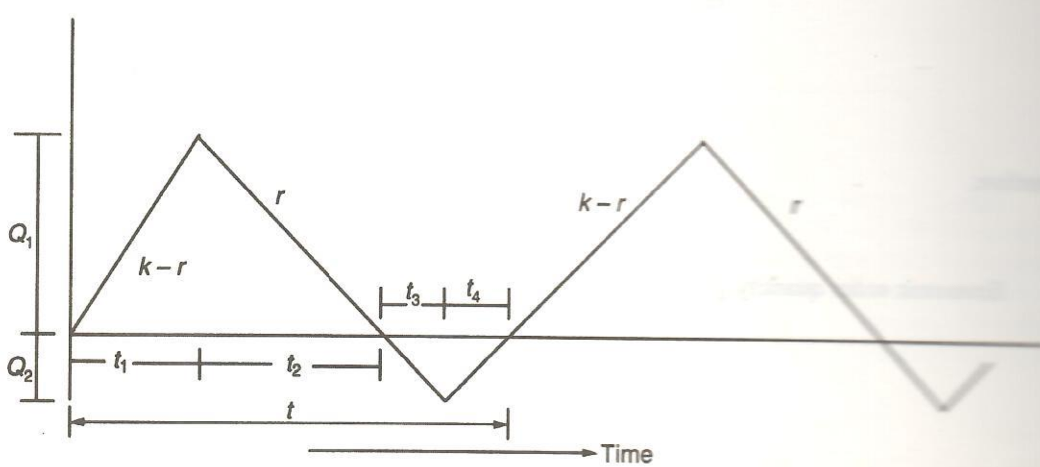

Fig. 7.5 Manufacturing model with shortages.

The variables which are used in this model are given below:

r = Demand of an item/period

k = Production rate of the item (number of units produced/period)

C_o = Cost/set-up

C_c = Carrying cost/unit/period

C_s = Shortage cost/unit/period

t = Total cycle time

p = Cost of production/unit

t_1 = Period of production as well as consumption of the item satisfying period's requirement

t_2 = Period of consumption only

t_3 = Period of shortage

t_4 = Period of production as well as consumption of the item satisfying back order

The formulae for the optimal values of the above variables are presented below:

$$\text{Economic batch quantity } Q^* = \sqrt{\frac{2C_o}{C_c}\,\frac{kr}{k-r}\,\frac{C_c + C_s}{C_s}}$$

$$\text{Maximum inventory } Q_1^* = \sqrt{\frac{2C_o}{C_c}\,\frac{r(k-r)}{k}\,\frac{C_s}{C_c + C_s}}$$

$$\text{Maximum stock out } Q_2^* = \sqrt{\frac{2C_oC_c}{C_s(C_c + C_s)}\,\frac{r(k-r)}{k}}$$

$$Q_1^* = \frac{k-r}{k}Q^* - Q_2^*$$

$$t^* = \frac{Q^*}{r}$$

$$t_1^* = \frac{Q_1^*}{k-r}$$

$$t_2^* = \frac{Q_1^*}{r}$$

$$t_3^* = \frac{Q_2^*}{r}$$

$$t_4^* = \frac{Q_2^*}{k-r}$$

Example 7.5 The demand for an item is 6,000 units per year. Its production rate is 1,000 units per month. The carrying cost is Rs. 50/unit/year and the set-up cost is Rs. 2,000 per set-up. The shortage cost is Rs. 1,000 per unit per year. Find various parameters of the inventory system.

Solution Here

$$r = 6{,}000 \text{ units/year}$$
$$k = 1{,}000 \times 12 = 12{,}000 \text{ units/year}$$
$$C_o = \text{Rs. } 2{,}000\text{/set-up}$$
$$C_c = \text{Rs. } 50\text{/unit/year}$$
$$C_s = \text{Rs. } 1{,}000\text{/unit/year}$$

Therefore,

$$Q^* \text{(EBQ)} = \sqrt{\frac{2C_o}{C_c}\,\frac{kr}{k-r}\,\frac{C_c + C_s}{C_s}}$$

$$= \sqrt{\frac{2 \times 2{,}000}{50}\,\frac{12{,}000 \times 6{,}000}{12{,}000 - 6{,}000}\,\frac{50 + 1{,}000}{1{,}000}}$$

$$= 1004 \text{ units (approx).}$$

$$Q_2^* = \sqrt{\frac{2C_oC_c}{C_s(C_c + C_s)}\,\frac{r(k-r)}{k}}$$

$$= \sqrt{\frac{2 \times 2{,}000 \times 50}{1{,}000(50 + 1{,}000)}\,\frac{6{,}000(12{,}000 - 6{,}000)}{12{,}000}}$$

$$= 24 \text{ units (approx.)}$$

$$Q_1^* = \frac{k-r}{k}Q^* - Q_2^*$$

$$= \frac{12{,}000 - 6{,}000}{12{,}000} \times 1{,}004 - 24 = 478 \text{ units}$$

$$t^* = \frac{Q^*}{r} \times 365 = \frac{1{,}004}{6{,}000} \times 365 = 61 \text{ days (approx.)}$$

$$t_1^* = \frac{Q_1^*}{k-r} \times 365 = \frac{478}{12{,}000 - 6{,}000} \times 365 = 29 \text{ days (approx.)}$$

$$t_2^* = \frac{Q_1^*}{r} \times 365 = \frac{478}{6{,}000} \times 365 = 29 \text{ days (approx.)}$$

$$t_3^* = \frac{Q_2^*}{r} \times 365 = \frac{24}{6{,}000} \times 365 = 1.5 \text{ days (approx.)}$$

$$t_4^* = \frac{Q_2^*}{k-r} \times 365 = \frac{24}{12{,}000 - 6{,}000} \times 365 = 1.5 \text{ days (approx.)}$$