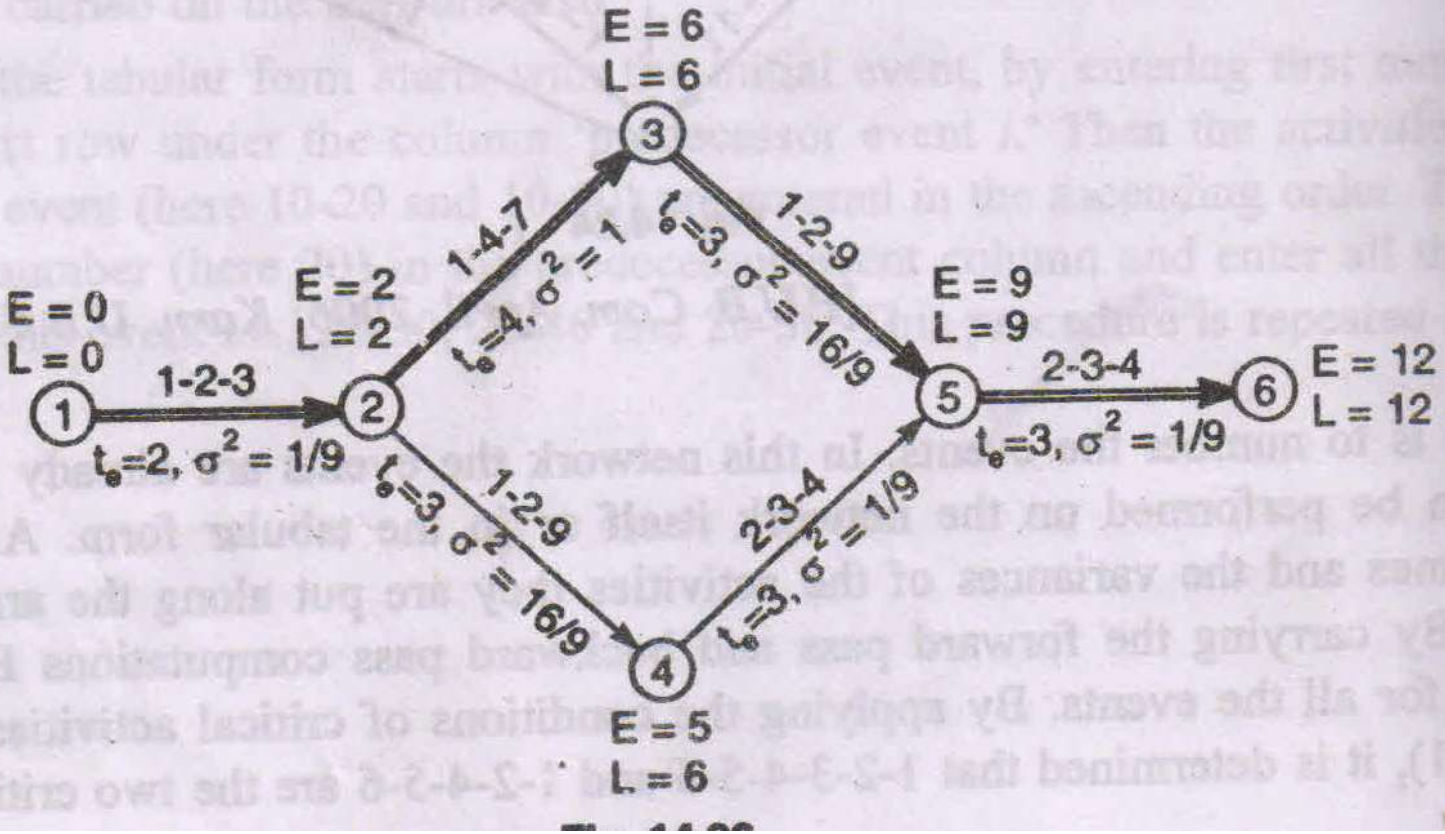
Consider the network shown in Fig. 14.26. The three time estimates, the expected activity durations and the variances are shown along the arrows. The earliest expected times and the latest allowable occurrence times are computed and put along the nodes. What is the probability of completing the project in (i) 12 days (ii) 14 days (iii) 10 days ?

[*H.P.U.B. Tech. (Mech.) Nov.*



We identify that the path 1-2-3-5-6 is the critical path and expected project length is 12 days.

(i) Here, $T_{cp} = 12$ days, $T = 12$ days.

Standard deviation for the project length, $\sigma = \sqrt{\Sigma\sigma_{ij}^2}$ for all ij on the critical path.

$\therefore \quad \sigma = \sqrt{1/9+1+16/9+1/9} = 1.73.$

$\therefore$ Normal deviate, $Z = \dfrac{T - T_{cp}}{\sigma} = \dfrac{12-12}{1.73} = 0.$

$\therefore$ Probability of completing the project (from table C-2) = 50%.

(ii) Here, $T = 14$ days.

$\therefore \quad Z = \dfrac{14-12}{1.73} = 1.16.$

$\therefore$ Corresponding probability = 87.7%.

(iii) Here, $T = 10$ days.

$\therefore \quad Z = \dfrac{10-12}{1.73} = -1.16.$

$\therefore$ Corresponding probability = 1 – 0.877 = 0.123 = 12.3%.

EXAMPLE 14.11-5

The time estimates (in weeks) for the activities of a PERT network are given below

Activity	t_o	t_m	t_p
1-2	*1*	*1*	*7*
1-3	*1*	*4*	*7*
1-4	*2*	*2*	*8*
2-5	*1*	*1*	*1*
3-5	*2*	*5*	*14*
4-6	*2*	*5*	*8*
5-6	*3*	*6*	*15*

(a) Draw the project network and identify all the paths through it.

(b) Determine the expected project length.

(c) Calculate the standard deviation and variance of the project length.

(d) What is the probability that the project will be completed

(i) at least 4 weeks earlier than expected time?

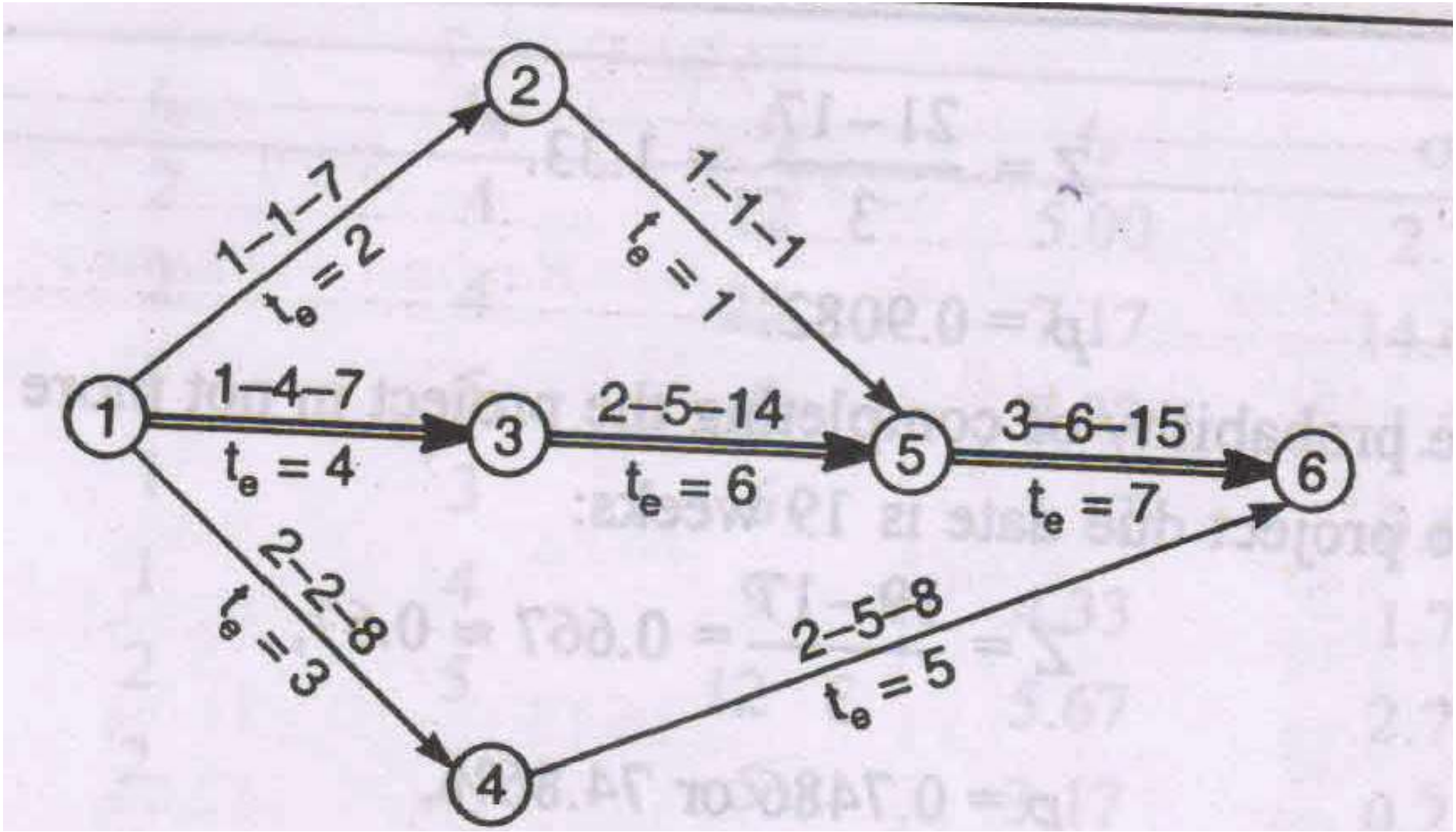
(ii) no more than 4 weeks later than expected time?

(e) If the project due date is 19 weeks, what is the probability of not meeting the due date

(f) Find the probability that the project will be completed on schedule if the scheduled completion time is 20 weeks.

(g) What should be the scheduled completion time for the probability of completion is 90%?

[P.U.B. Com. April, 2007, 2010; G.N.D.U. B. Com. April, 2008; Jammu U. B.E (Mech.) 2004; Pbi.U. B.Tech., 1999; Karn.U.B.E. (Mech.) 1998; P.U. B.E. (Elect.) 1997; (E&EC)



Solution

(*a*) The network for the given data is drawn in Fig. 14.28. The various paths through the network are

1-2-5-6,

1-3-5-6

and 1-4-6.

Fig. 14.28

(*b*) For determining the expected project length, the expected activity times need to be ...ated. The same, along with the variances, are computed below.

Activity	t_o	t_m	t_p	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	1	1	7	2	1
1-3	1	4	7	4	1
1-4	2	2	8	3	1
2-5	1	1	1	1	0
3-5	2	5	14	6	4
4-6	2	5	8	5	1
5-6	3	6	15	7	4

Length of path 1-2-5-6 = 2 + 1 + 7 = 10,

...ength of path 1-3-5-6 = 4 + 6 + 7 = 17, and

...ength of path 1-4-6 = 3 + 5 = 8.

Since 1-3-5-6 has the longest duration, it is the critical path of the network.

∴ The expected project length = 17 weeks

(c) Variance of the project length is the sum of the variances of the activities on the critical

$$V_{cp} = V_{1\text{-}3} + V_{3\text{-}5} + V_{5\text{-}6} = 1+4+4 = 9.$$

$\therefore$ $\sigma = 3$ weeks.

(d) (*i*) Probability that the project will be completed at least 4 weeks earlier than expected

Expected time = 17 weeks,

and scheduled time = 17–4 = 13 weeks.

$\therefore$ The standard normal deviate,

$$Z = \frac{13-17}{3} = -1.33.$$

For $Z = -1.33$, probability is $1 - 0.9082 = 0.0918$ or the probability of completing the at least 4 weeks earlier than expected time *i.e.*, within 13 weeks is ≤ 9.18 %.

(*ii*) Probability that the project will be completed no more than 4 weeks later than expected

Expected time = 17 weeks.

$\therefore$ Scheduled time = 17 + 4 = 21 weeks.

$$\therefore \quad Z = \frac{21-17}{3} = 1.33.$$

$$\therefore \quad p = 0.9082.$$

Therefore, the probability of completing the project in not more that 21 weeks is

(*e*) When the project due date is 19 weeks:

$$Z = \frac{19-17}{3} = 0.667 \approx 0.67,$$

for which, $p = 0.7486$ or 74.86%.

$\therefore$ The probability of meeting the due date is 74.86% and the probability of not meeting the due date is 25.14%.

(*f*) Scheduled time = 20 weeks.

$$\therefore \quad Z = \frac{20-17}{3} = 1, \text{ for which } p = 84.13\%.$$

(*g*) Value of Z for $p = 0.9$ is = 1.28.

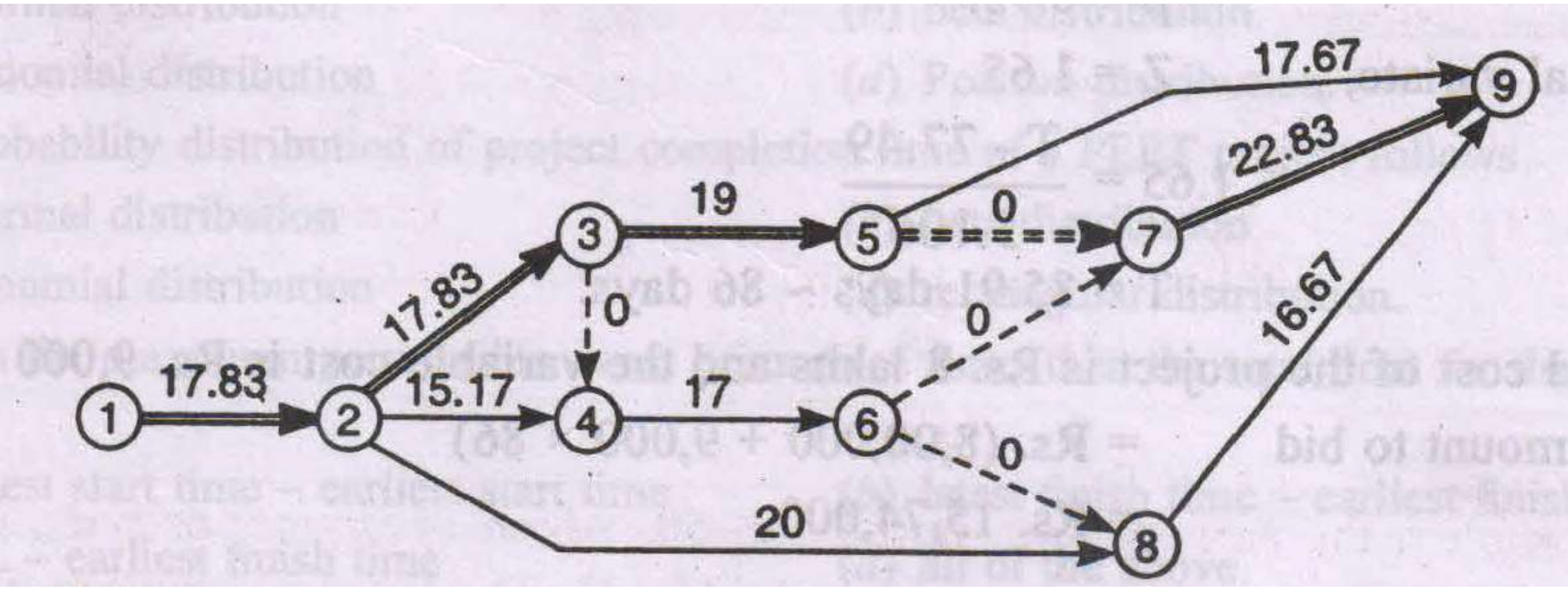
$$\therefore \quad 1.28 = \frac{T-17}{3} \text{ or } T = 17 + 3.84 = 20.84 \text{ weeks.}$$

A civil engineering firm has to bid for the construction of a dam. The activities and their estimates are given below:

Activity	Optimistic	Most likely	Pessimistic
1-2	14	17	25
2-3	14	18	21
2-4	13	15	18
2-8	16	19	28
3-4 (dummy)	0	0	0
3-5	15	18	27
4-6	13	17	21
5-7 (dummy)	0	0	0
5-9	14	18	20
6-7 (dummy)	0	0	0
6-8 (dummy)	0	0	0
7-9	16	20	41
8-9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide a 95% of probability of at best breaking-even. The fixed costs for the project are eight lacs and the variable costs are 9,000 every day spent working on the project. The duration is in days and the costs are in rupees.

What amount should the firm bid under this policy? (You may perform the calculations on duration etc., upto two decimal places). [H.P.U. B.Tech. (Mech.) June, 2010]



Various paths are

1-2-3-5-7-9	:	77.49 days,
1-2-3-5-9	:	72.33 days,
1-2-3-4-6-7-9	:	75.49 days,
1-2-3-4-6-8-9	:	69.33 days,
1-2-8-9	:	54.5 days,
1-2-4-6-8-9	:	66.67 days,
1-2-4-6-7-9	:	72.83 days.

Activity	Duration			$t_e = \frac{t_o + t_p + 4t_m}{6}$	$V = \left[\frac{\quad}{\quad}\right.$
	Optimistic (t_o)	Pessimistic (t_p)	Most likely (t_m)		
1-2	14	25	17	17.83	
2-3	14	21	18	17.83	
2-4	13	18	15	15.17	
2-8	16	28	19	20	
3-4	0	0	0	0	
3-5	15	27	18	19	
4-6	13	21	17	17	
5-7	0	0	0	0	
5-9	14	20	18	17.67	
6-7	0	0	0	0	
6-8	0	0	0	0	
7-9	16	41	20	22.83	
8-9	14	22	16	16.67	

Hence 1-2-3-5-7-9 is the critical path with project duration of 77.49 days. Variance critical activities have been calculated.

Variance of the critical path, $V = 26.08$.

$\therefore$ S.D. of the critical path, $\sigma = \sqrt{26.08} = 5.106$ days.

Now we are to determine the time within which the project should be completed provide 95% probability of break-even.

Now $p = 0.95$.

$\therefore$ Normal variate, $Z = 1.65$.

$\therefore$ $$1.65 = \frac{T - 77.49}{5.106}$$

or $T = 85.91$ days $\sim$ 86 days.

The fixed cost of the project is Rs. 8 lakhs and the variable cost is Rs. 9,000 per

$\therefore$ The amount to bid = Rs. (8,00,000 + 9,000 × 86)

= Rs. 15,74,000.

EXAMPLE 14.11-6

A PERT network is shown in Fig. 14.29. The activity times in days are given along the arrows. The scheduled times for some important events are given along the nodes. Determine the critical path and probabilities of meeting the scheduled dates for the specified events. Tabulate the results and determine slack for each event.

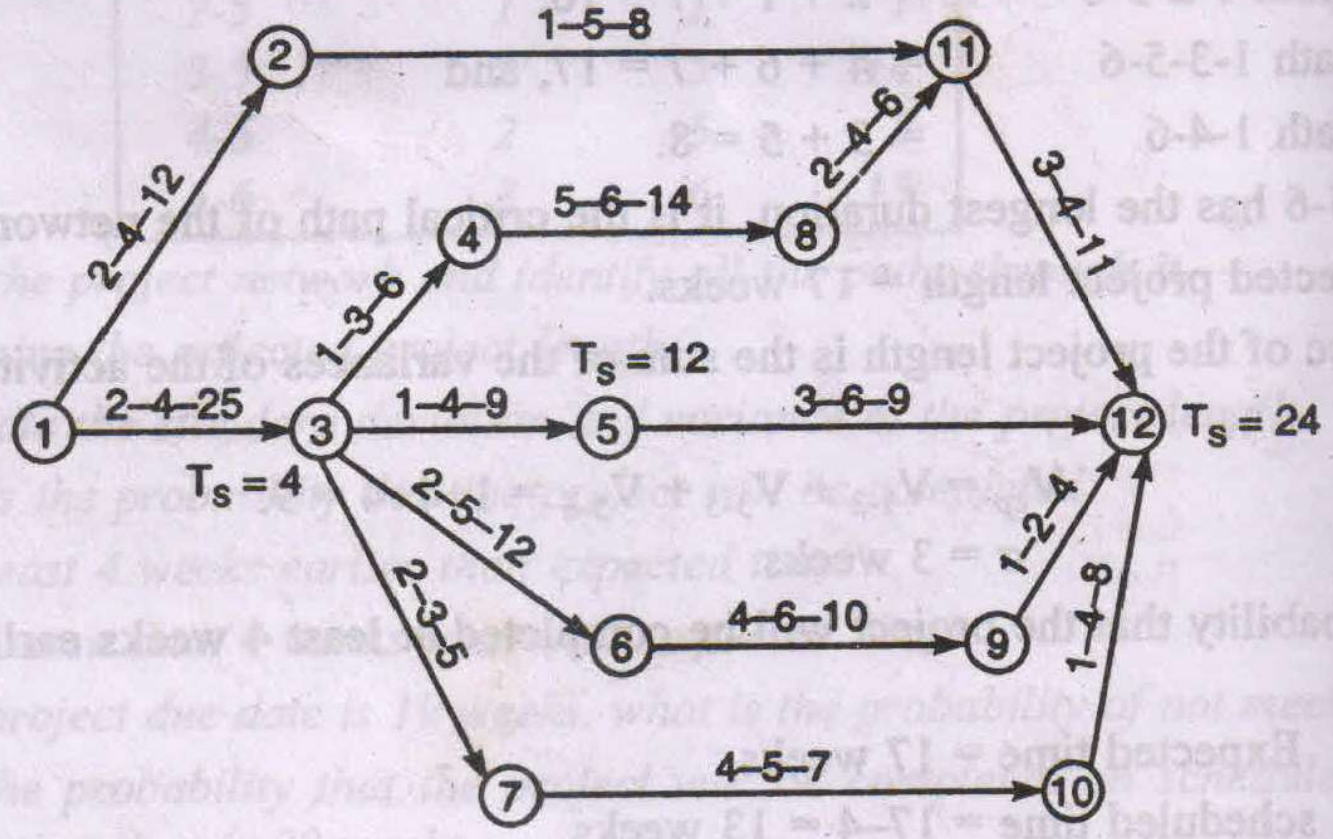


Fig. 14.29

Activity	t_o	t_m	t_p	t_e	σ^2
1 - 2	2	4	12	5.00	2.78
1 - 3	2	4	25	7.17	14.69
2 - 11	1	5	8	4.83	1.36
3 - 4	1	3	6	3.17	0.69
3 - 5	1	4	9	4.33	1.78
3 - 6	2	5	12	5.67	2.78
3 - 7	2	3	5	3.17	0.25
4 - 8	5	6	14	7.17	2.25
5 - 12	3	6	9	6.00	1.00
6 - 9	4	6	10	6.33	1.00
7 - 10	4	5	7	5.17	0.25
8 - 11	2	4	6	4.00	0.44
9 - 12	1	2	4	2.17	0.25
10 - 12	1	4	8	4.17	1.36
11 - 12	3	4	11	5.00	1.78

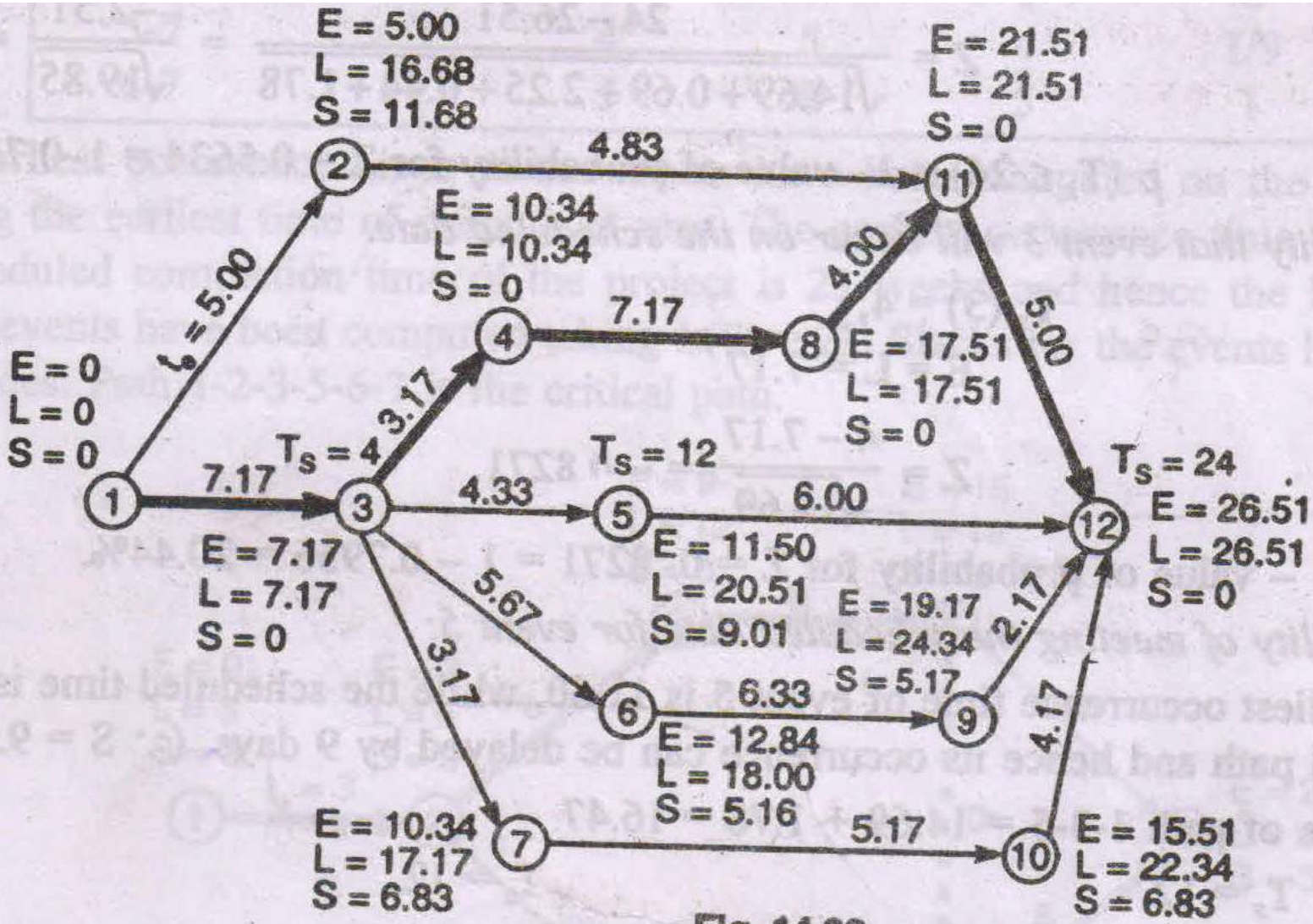


Fig. 14.30

Table 14.8 represents the network analysis. Floats for the activities in question are calculated last column. Critical path is 1-3-4-8-11-12 and the project completion time is 26.51 days.

Activity	Expected duration	Start time		Finish time		Total float
		Earliest	Latest	Earliest	Latest	
1 - 2	5.00	0	11.68	5.00	16.68	11.68
1 - 3	7.17	0	0	7.17	7.17	0
2 - 11	4.83	5.00	16.68	9.83	21.51	11.68
3 - 4	3.17	7.17	7.17	10.34	10.34	0
3 - 5	4.33	7.17	16.18	11.50	20.51	9.01
3 - 6	5.67	7.17	12.33	12.84	18.00	5.16
3 - 7	3.17	7.17	14.00	10.34	17.17	6.83
4 - 8	7.17	10.34	10.34	17.51	17.51	0
5 - 12	6.00	11.50	20.51	17.50	26.51	9.01
6 - 9	6.33	12.84	18.01	19.17	24.34	5.17
7 - 10	5.17	10.34	17.17	15.51	22.34	6.83
8 - 11	4.00	17.51	17.51	21.51	21.51	0
9 - 12	2.17	19.17	24.34	21.34	26.51	5.17
10 - 12	4.17	15.51	22.34	19.68	26.51	6.83
11 - 12	5.00	21.51	21.51	26.51	26.51	0

Probability of completing the project in the scheduled completion time of 24 days (T_s (12) = 24):

$$Z = \frac{24 - 26.51}{\sqrt{14.69 + 0.69 + 2.25 + 0.44 + 1.78}} = \frac{-2.51}{\sqrt{19.85}} = -0.5634$$

$\therefore$ $p\ (T_s \leq 24)$ = 1– value of probability for Z = 0.5634 = 1–0.7146 = 29.54%.

Probability that event 3 will occur on the scheduled date:

T_s (3) = 4,

E = L = 7.17.

$$\therefore \quad Z = \frac{4 - 7.17}{\sqrt{14.69}} = -0.8271.$$

$\therefore$ p = 1 – value of probability for Z = 0. 8271 = 1 – 0.7956 = 20.44%.

Probability of meeting the scheduled date for event 5:

The earliest occurrence time of event 5 is 11.50, while the scheduled time is 12. Event 5 is not on critical path and hence its occurrence can be delayed by 9 days. ($\because$ S = 9.01).

Variance of path 1-3-5 = 14.69 + 1.78 = 16.47.

T_s = 12,

E = 11.50.

$$\therefore \quad Z = \frac{12 - 11.50}{\sqrt{16.47}} = 0.123.$$

$\therefore$ Probability = 54.89%.

T_s = 12,

L = 20.51.

$$\therefore \quad Z = \frac{12 - 20.51}{\sqrt{16.47}} = -2.1.$$

$\therefore$ Probability = 1–0.982 = 0.018 = 1.8%.

Thus the probability of meeting the scheduled date in case of event 5 is less than or equal to 54.89% with minimum of 1.8% *i.e.*, it lies between 1.8 % and 54.89 %.

Problem 2

JOB	OPTIMISTIC	MOST LIKELY	PESSIMISTIC
1-2	1	4	7
1-3	5	10	15
2-4	3	3	3
2-6	1	4	7
3-4	10	15	26
3-5	2	4	6
4-5	5	5	5
5-6	2	5	8

1. Draw the network &find expected project completion time.
2. What is the probability that it would be completed in 41 days.

Variance of critical path

Activity	To	Tp	variance
1-3	5	15	2.77
3-4	10	26	7.11
4-5	5	5	0
5-6	2	8	1
			Total= 10.88

So, standard deviation = 3.30

Probability for completing the job in 41 days.

$$Z = \frac{\text{DUE DATE} - \text{EXPECTED DATE OF COMPLETION}}{\text{S.D. OF CRITICAL PATH}}$$

$$= \frac{41 - 36}{3.30}$$

$$= 1.51$$

The tabulated value of corresponding to calculated value i.e. 1.51 is .4345

So probability is .5 + .4345 = .9345 i.e. 93.45% that project will be completed on 41 day.

Problem 3

activity	Preceding activity	to	tm	tp
A	-	2	3	10
B	-	2	3	4
C	A	1	2	3
D	A	4	6	14
E	B	4	5	12
F	C	3	4	5
G	D,E	1	1	7

- **Find the expected duration and variance of each activity.**
- **What is the expected project length?**
- **Calculate the variance &standard deviation of the project length.**

activity	to	Tm	tp	te	variance
A	2	3	10	4	16/9
B	2	3	4	3	1/9
C	1	2	3	2	1/9
D	4	6	14	7	25/9
E	4	5	12	6	16/9
F	3	4	5	4	1/9
G	1	2	7	2	1

Various paths &expected project length

A-C-F = 4+2+4 = 10

A-D-G = 4+7+2 = 13 CRITICAL PATH

B-E-G = 3+6+2 = 11

Thus critical path is A-D-G with an expected length of 13 days. Hence the expected project length is 13 days.

Project variance = 16/9 + 25/9 + 1 = 50/9

PERT For Dealing With Uncertainty

- So far, times can be estimated with relative certainty, confidence

- For many situations this is not possible, e.g Research, development, new products and projects etc.

- Use 3 time estimates

  m= most likely time estimate, mode.

  a = optimistic time estimate,

  b = pessimistic time estimate, and

Expected Value (TE) = $(a + 4m + b) / 6$

Variance (V) = $((b - a) / 6)^2$

Std Deviation (δ) = SQRT (V)

Precedences And Project Activity Times

Activity	Immediate Predecessor	Optimistic Time	Most Likely Time	Pessimistic Time	EXP TE	Var V	S.Dev σ
a	-	10	22	22	20	4	2
b	-	20	20	20	20	0	0
c	-	4	10	16	10	4	2
d	a	2	14	32	15	25	5
e	b,c	8	8	20	10	4	2
f	b,c	8	14	20	14	4	2
g	b,c	4	4	4	4	0	0
h	c	2	12	16	11	5.4	2.32
I	g,h	6	16	38	18	28.4	5.33
j	d,e	2	8	14	8	4	2

The complete network

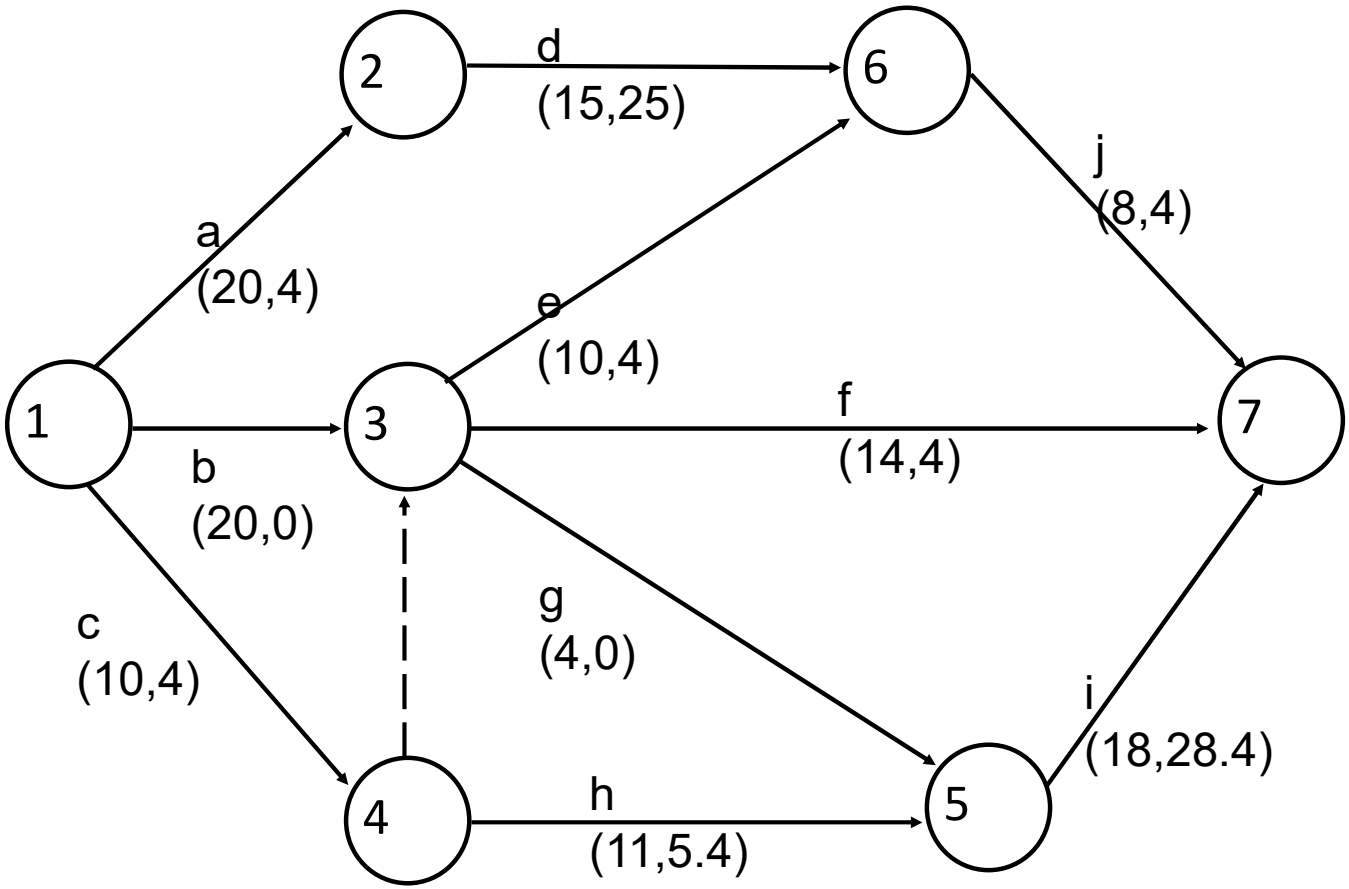
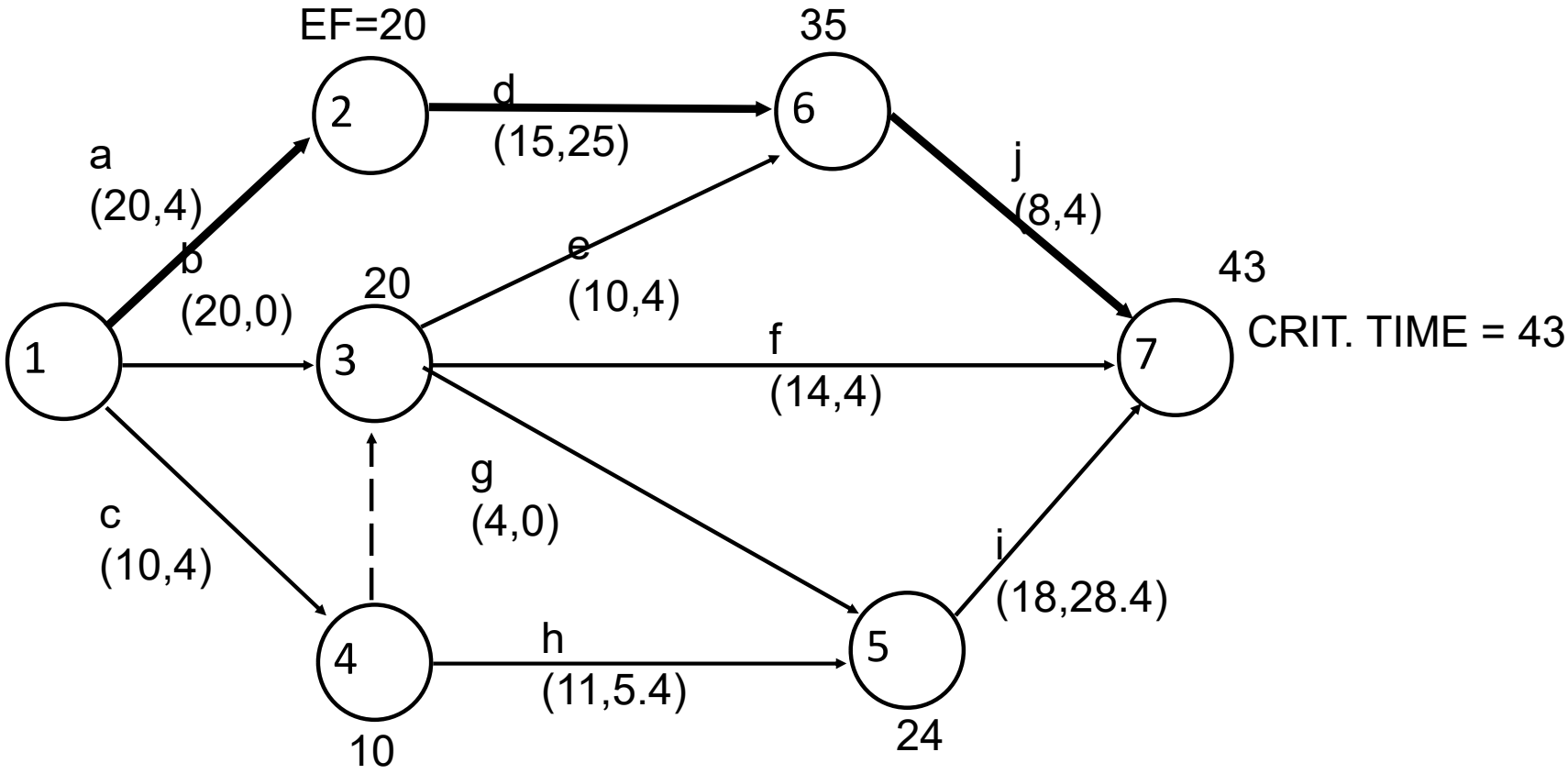


Figure 8-13 The complete Network



Critical Path Analysis (PERT)

Activity	LS	ES	Slacks	Critical ?
a	**0**	**0**	**0**	**Yes**
b	1	0	1	
c	4	0	4	
d	**20**	**20**	**0**	**Yes**
e	25	20	5	
f	29	20	9	
g	21	20	1	
h	14	10	4	
i	25	24	1	
j	**35**	**35**	**0**	**Yes**

Assume, PM promised to complete the project in the fifty days.
What are the chances of meeting that deadline?
Calculate Z, where

$$Z = (D-S) / \sqrt{V}$$

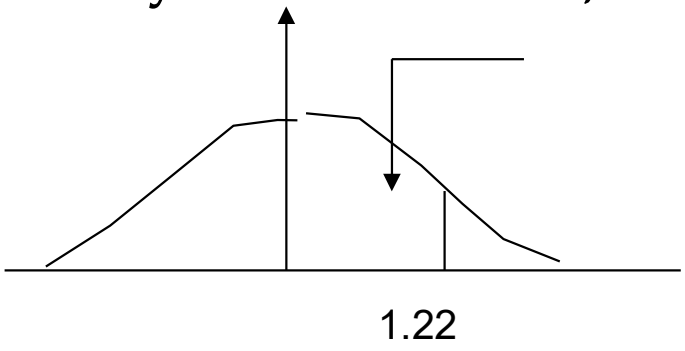
Example,

D = 50; S(Scheduled date) = 20+15+8 =43; V = (4+25+4) =33

Z = (50 – 43) / 5.745

= 1.22 standard deviations.

The probability value of Z = 1.22, is 0.888



What deadline are you 95% sure of meeting

Z value associated with 0.95 is 1.645

D = S + 5.745 (1.645)
= 43 + 9.45
= 52.45 days

Thus, there is a 95 percent chance of finishing the project by 52.45 days.

Comparison Between CPM and PERT

	CPM	PERT
1	Uses network, calculate float or slack, identify critical path and activities, guides to monitor and controlling project	Same as CPM
2	Uses one value of activity time	Requires 3 estimates of activity time Calculates mean and variance of time
3	Used where times can be estimated with confidence, familiar activities	Used where times cannot be estimated with confidence. Unfamiliar or new activities
4	Minimizing cost is more important	Meeting time target or estimating percent completion is more important
5	Example: construction projects, building one off machines, ships, etc	Example: Involving new activities or products, research and development etc

BENEFITS OFCPM / PERT NETWORK

Consistent framework for planning, scheduling, monitoring, and controlling project.

- Shows interdependence of all tasks, work packages, and work units.
- Helps proper communications between departments and functions.
- Determines expected project completion date.
- Identifies so-called critical activities, which can delay the project completion time.

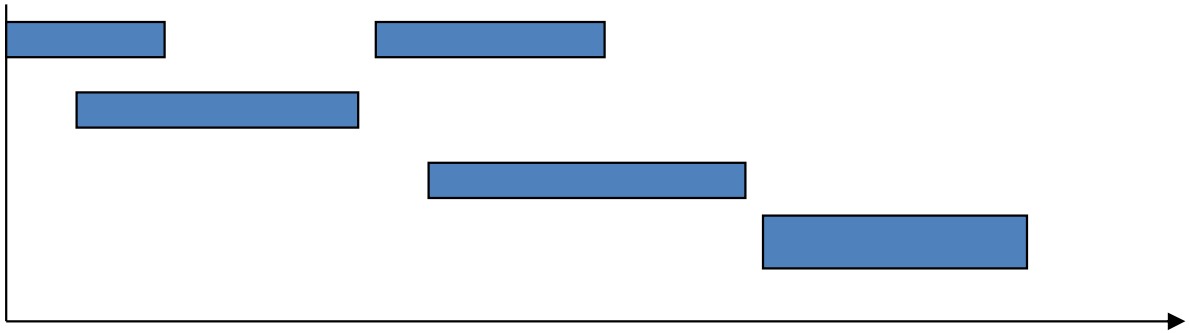
BENEFITS OFCPM / PERT NETWORK (cont.)

- Identified activities with slacks that can be delayed for specified periods without penalty, or from which resources may be temporarily borrowed
- Determines the dates on which tasks may be started or must be started if the project is to stay in schedule.
- Shows which tasks must be coordinated to avoid resource or timing conflicts.
- Shows which tasks may run in parallel to meet project completion date

Gantt Charts

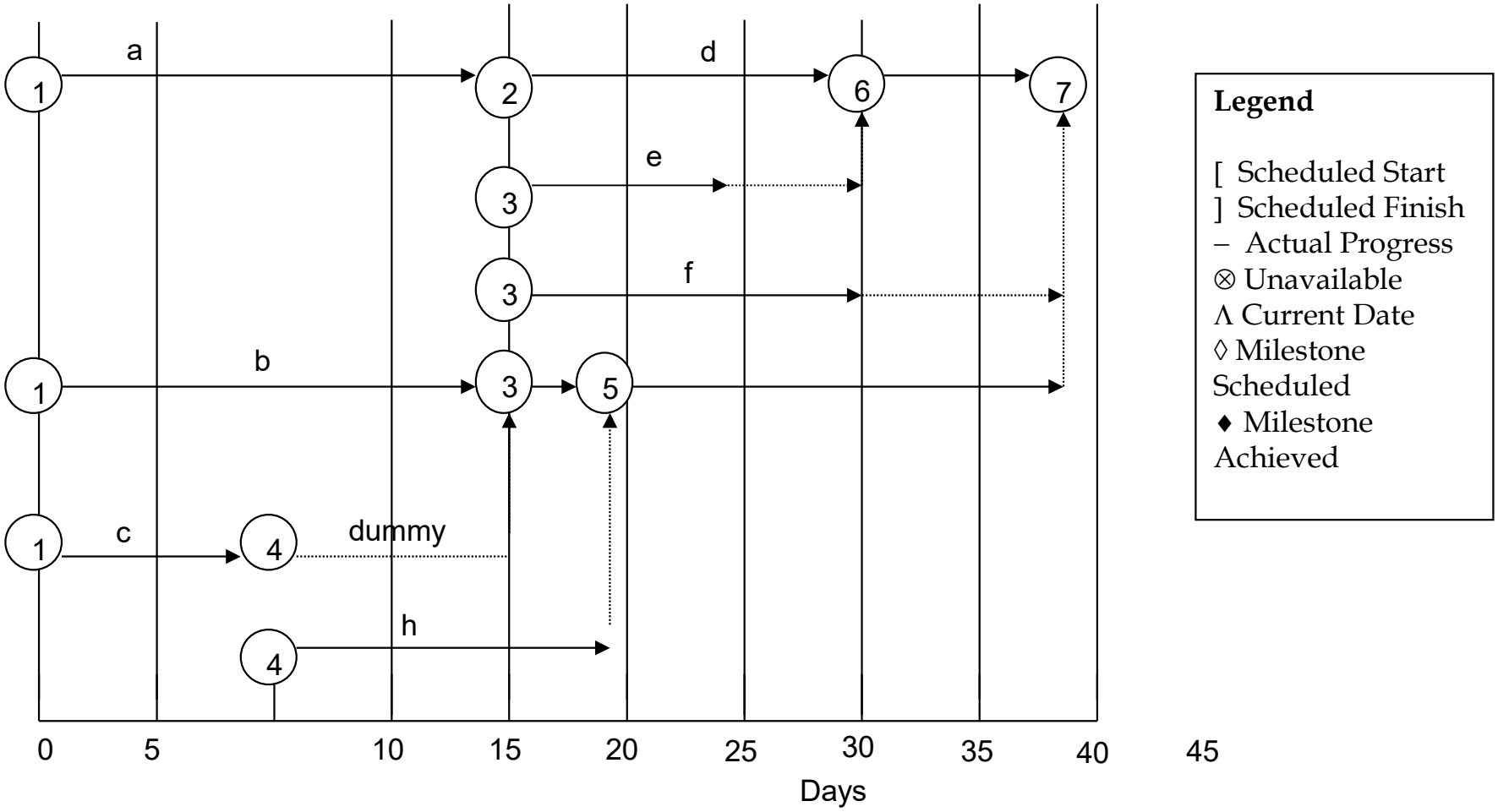
- Since 1917; Useful for showing work vs time in form of bar charts

e.g.

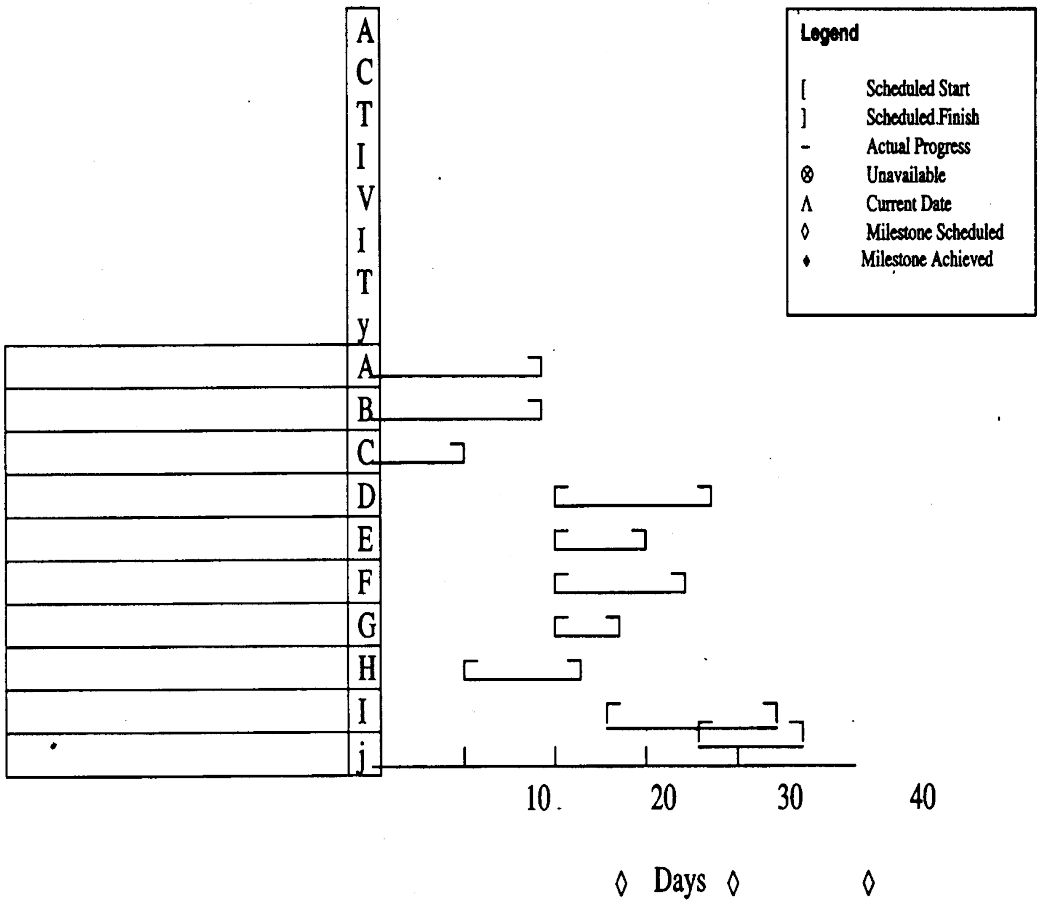


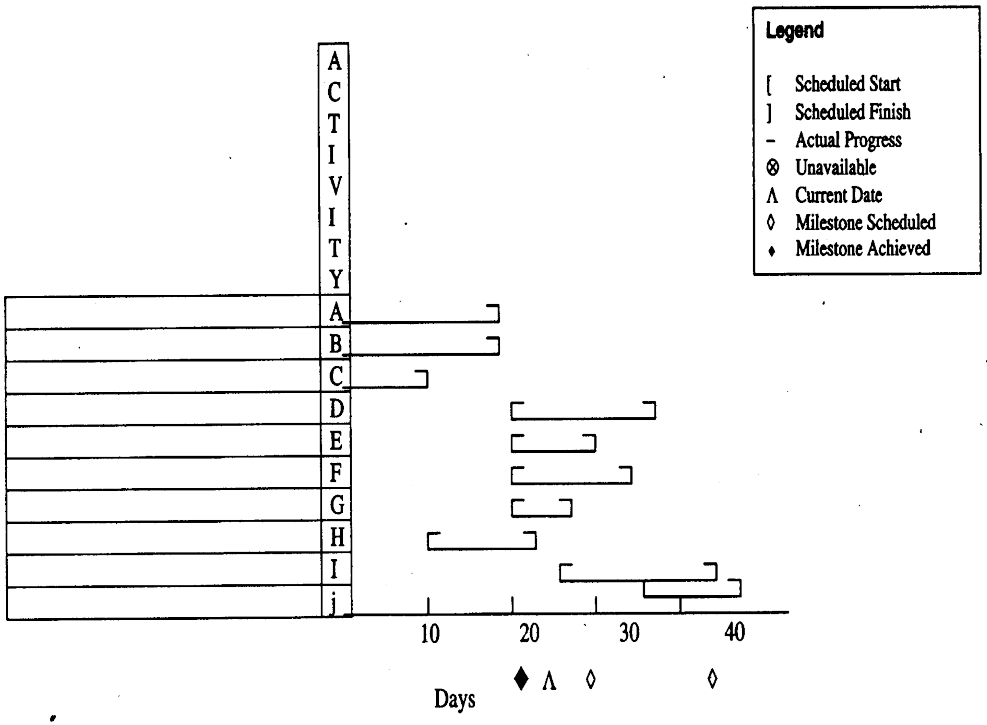
- Can draw directly or from CPM/PERT network

Modified PERT/CPM diagram from network

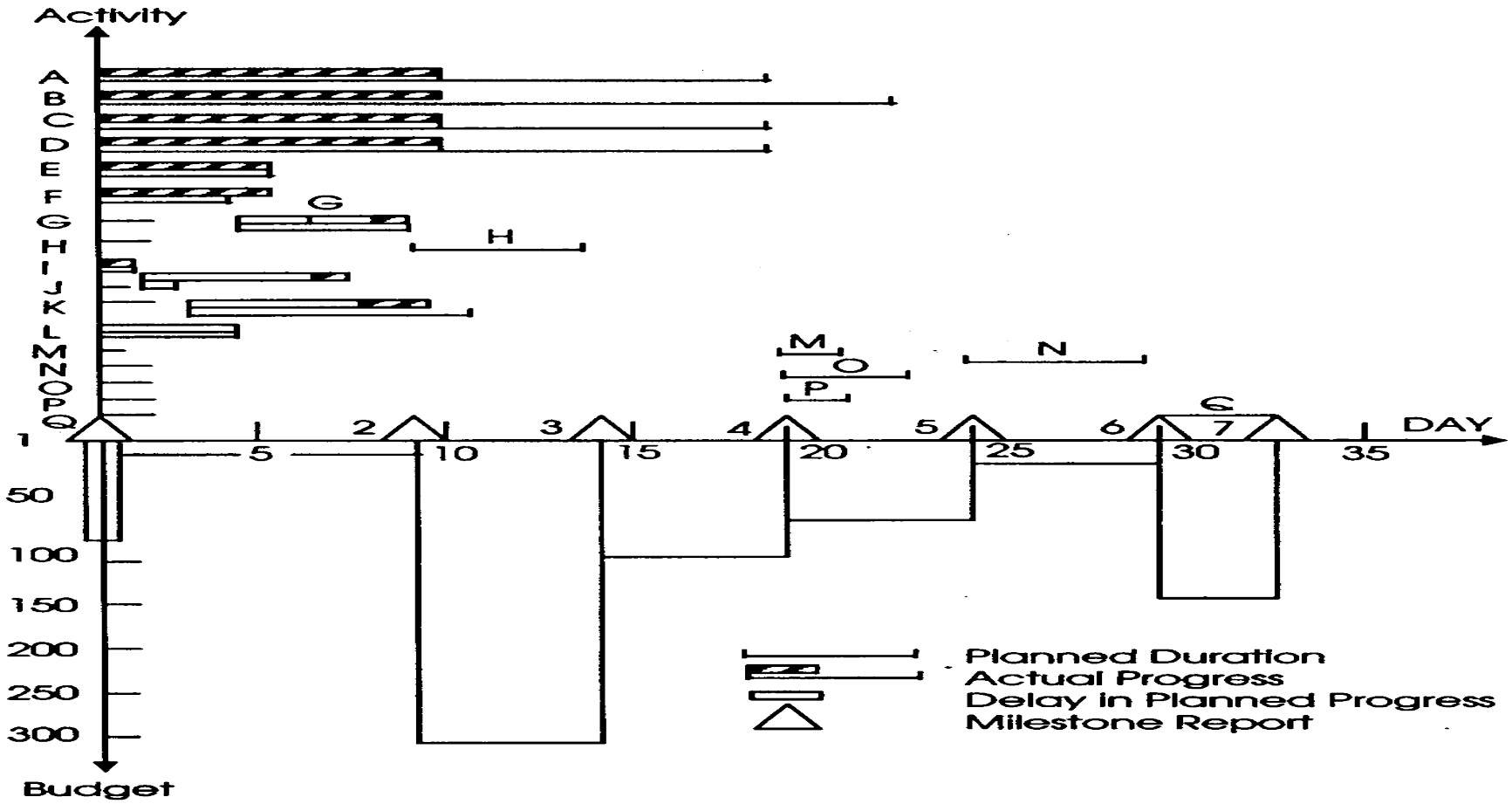


GANTT CHART


Legend
[Scheduled Start
] Scheduled Finish
– Actual Progress
⊗ Unavailable
Λ Current Date
◊ Milestone Scheduled
♦ Milestone Achieved
ACTIVITY
A
B
C
D
E
F
G
H
I
j
10
20
30
40
◊ Days ◊ ◊



Gantt chart showing progress of project on day 22.



Relating the budget to the Gantt chart schedule

Gantt Charts and CPM/PERT Networks

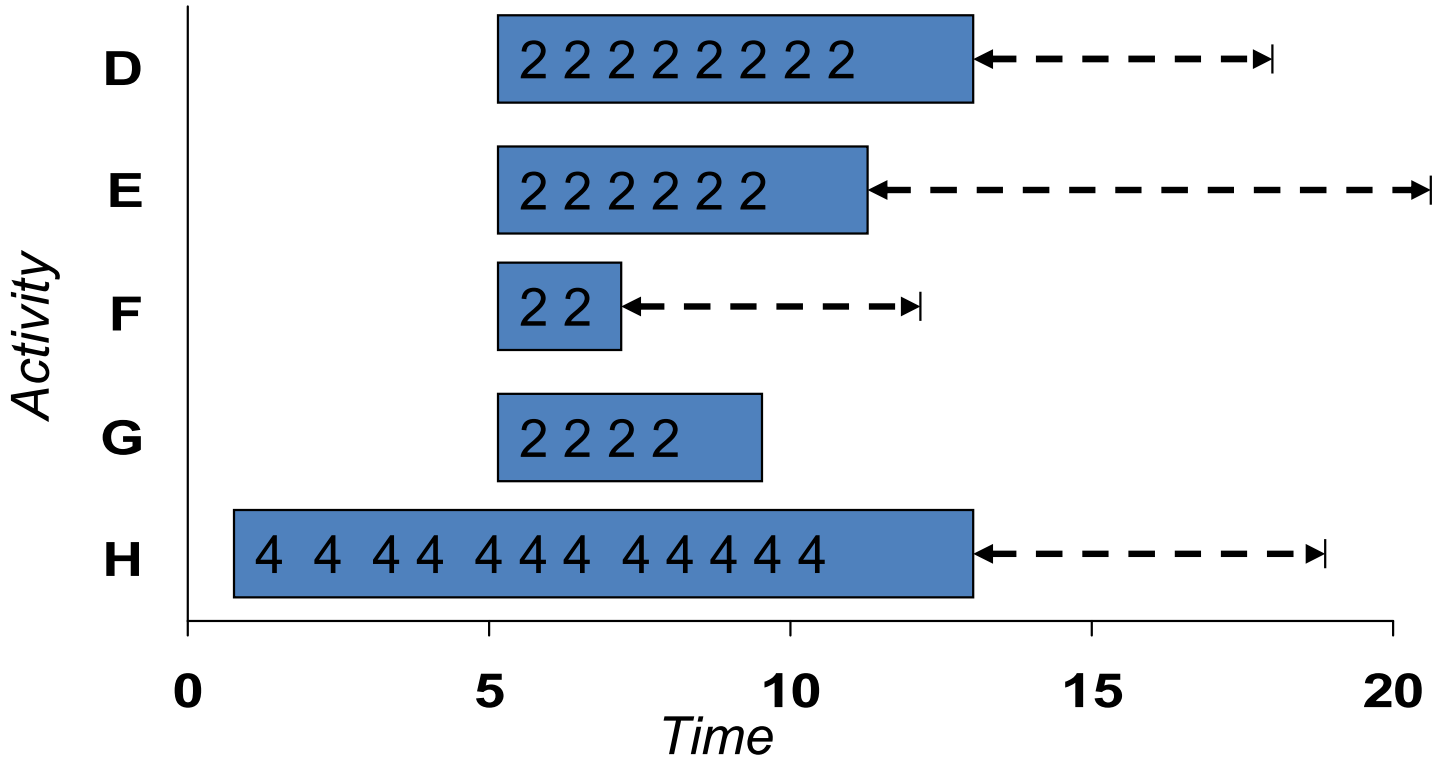
Gantt Charts:

- Even though a lot of info, easy to read and , understand to monitor and follow progress.
- Not very good for logical constraints
- Should be used to **COMPLEMENT** networks, not replace

RESOURCE ANALYSIS AND SCHEDULING

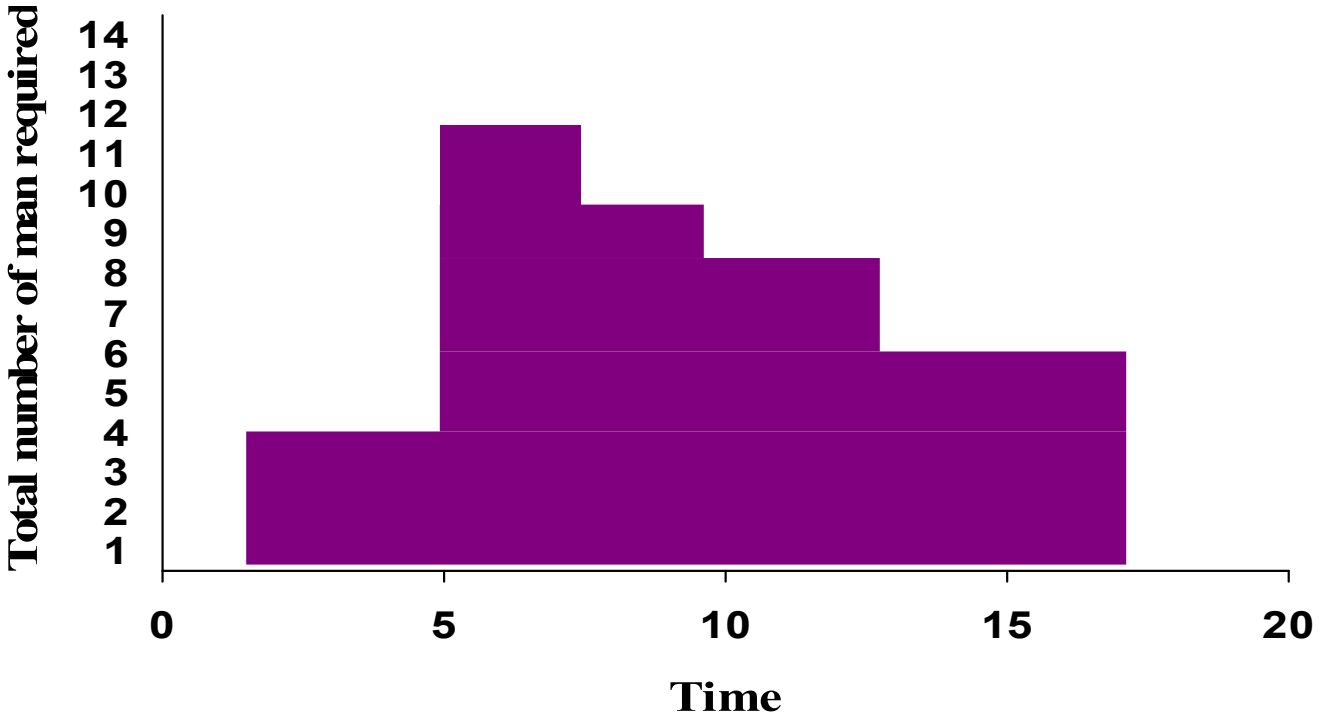
- Ability to carry out projects depend on the availability of resources
- Analyze resource implication
  -How requirements can be met and changes needed
- Use resources efficiently
- Use network to give information about time, resources and cost

Activities D, E, F, G and H require fitters.
Construct a bar chart with activities at their EST indicating person required and total float.



Add up across all activities to get the total number of men required.

Convert the bar chart to a histogram



Resource analysis before scheduling

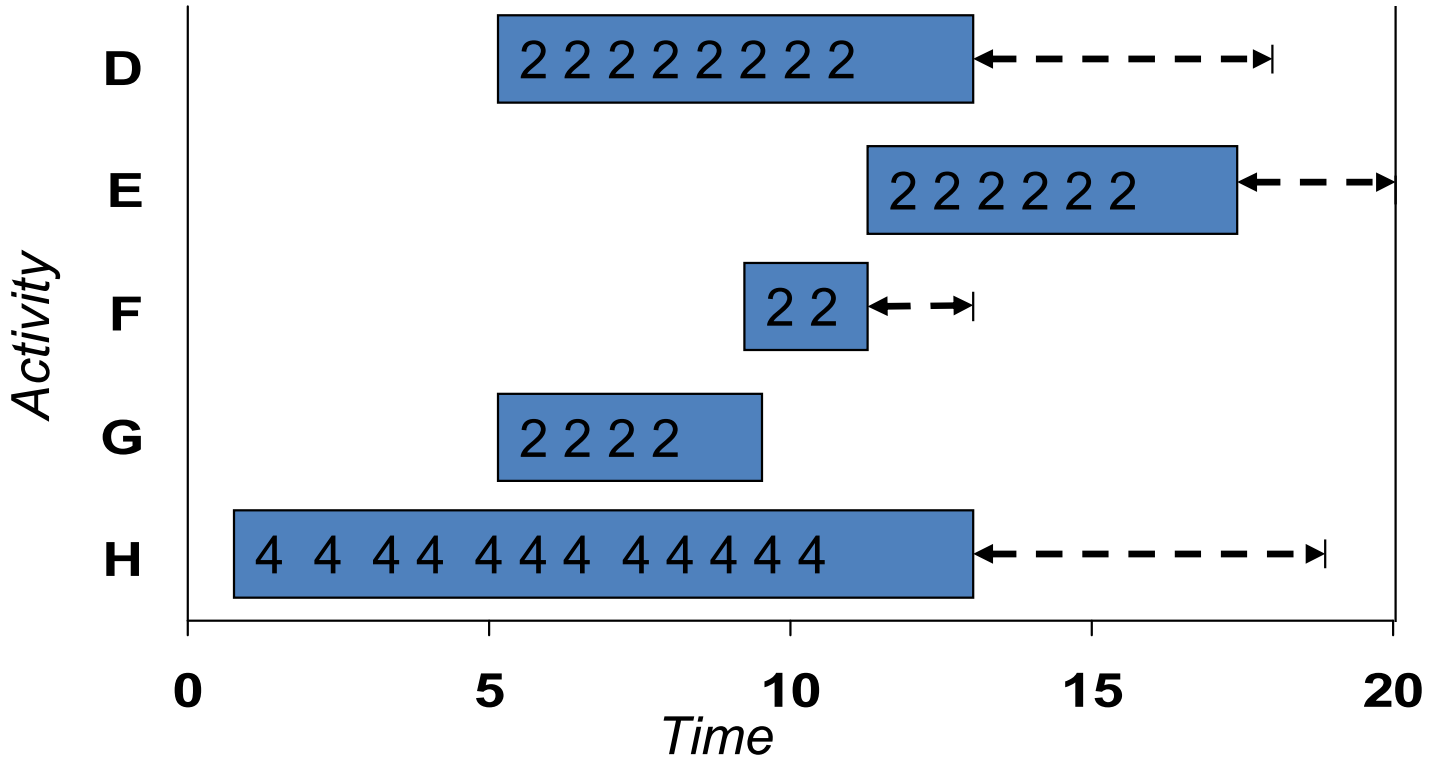
Shows: i) Variation from week to week (fitters)

ii) Maximum number of person required (12) during week 5-6

Examine resource implication.

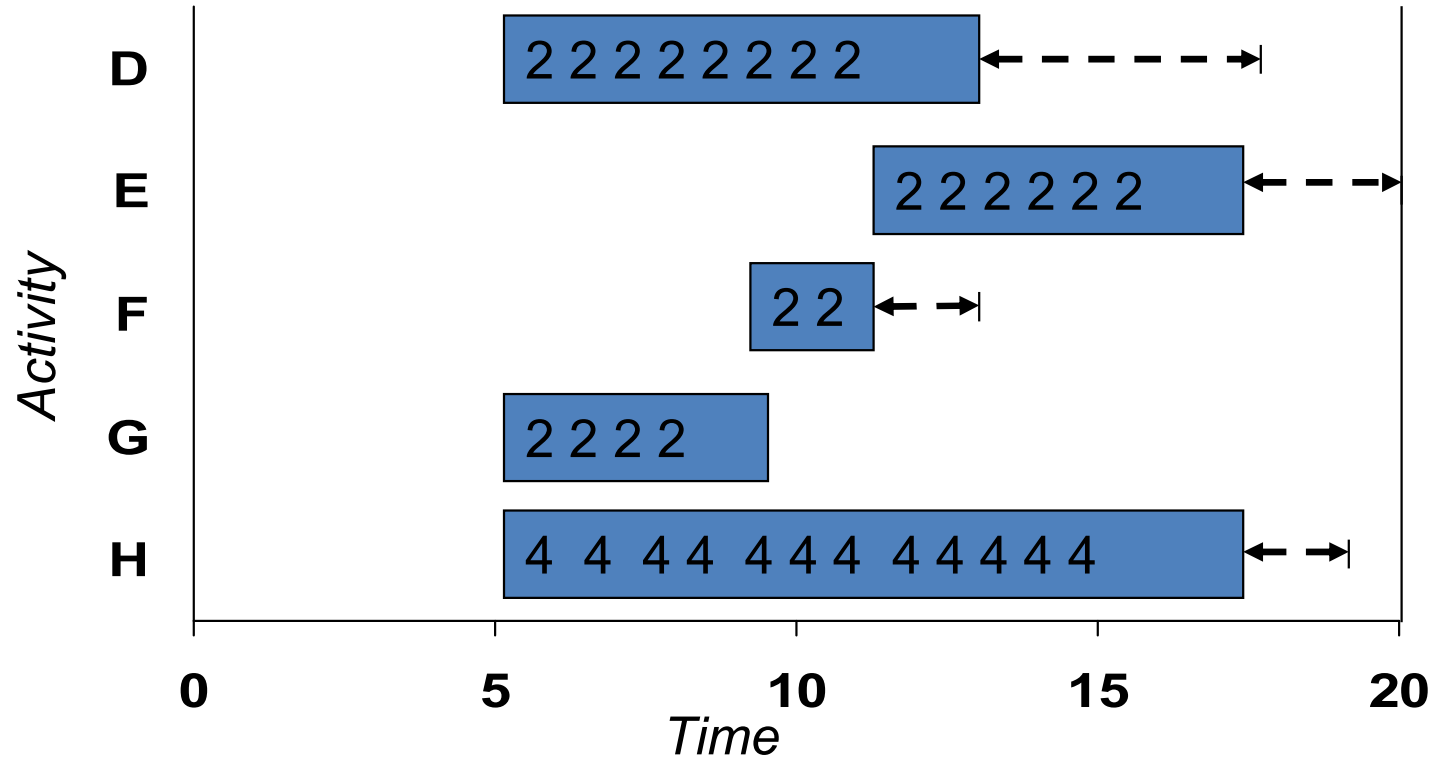
Example
If only 8 fitters are available at any period during the projects:

New bar chart:



Additional Restriction – no fitters available until the end of week 5.

Revised Schedule:



Resource constraints relates to:

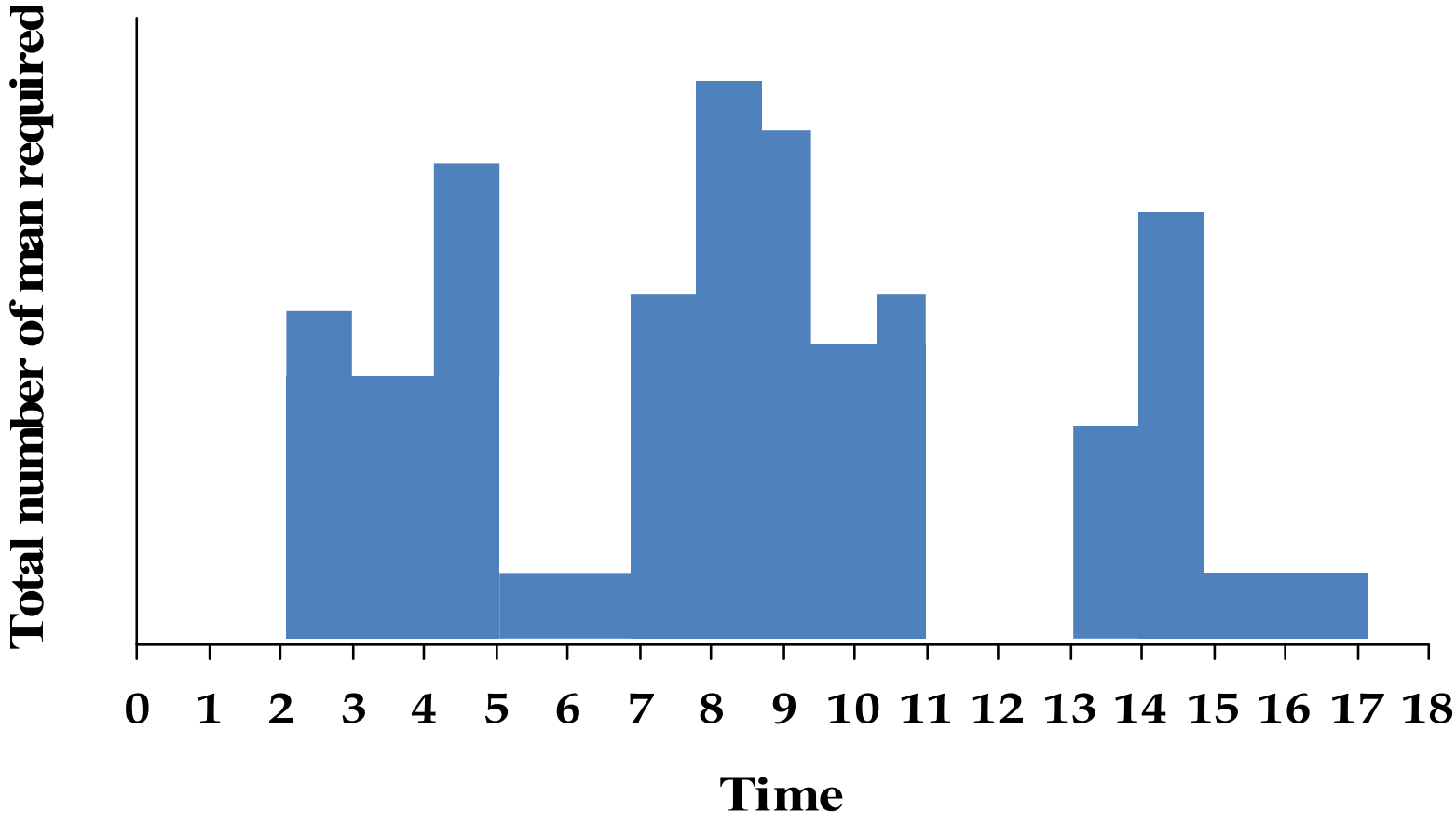
1. Variations in resource requirements
2. Resource availability

Smaller variations:

1. Easier control of the job
2. Better utilization of resources

Big variations:

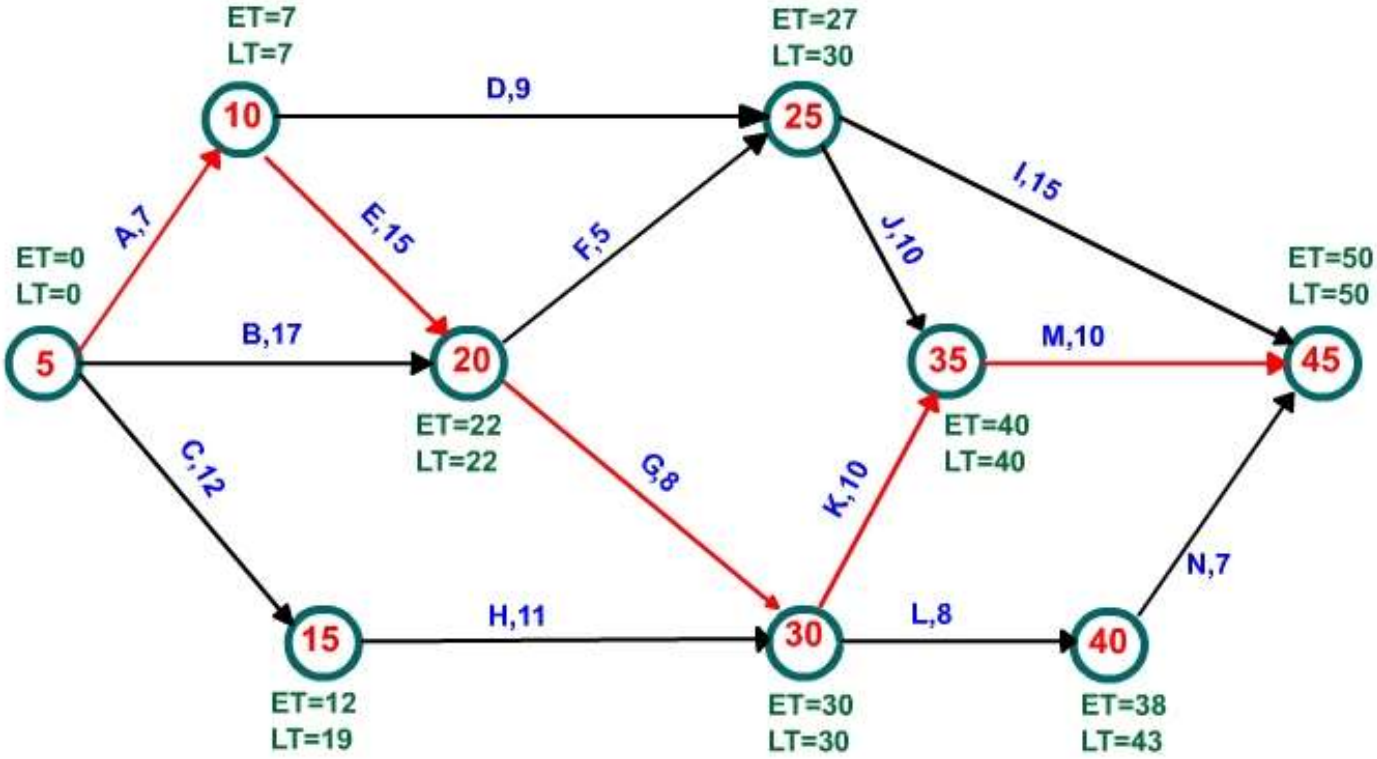
1. Frequent moving of manpower
2. Require close control
3. Affect efficiency



Histogram showing large resource variations

Calculation of Time Estimates in CPM

In the project network given in figure below, activities and their durations are specified at the activities. Find the critical path and the project duration.



- **Calculations in Network Analysis**
- The following calculations are required in network analysis in order to prepare a schedule of the project.
- Total completion time of the project
- Earliest time when each activity can start (i.e. earlist start time)
- Earliest time when each activity can finish (i.e. earlist finished time)
- Latest time when each activity can be started without delaying the project (i.e. latest start time)
- Latest time when each activity can be finished without delaying the project (i.e. latest finish time)
- Float on each activity (i.e. time by which the completion of an activity can be delayed without delaying the project)
- Critical activity and critical path

Predecessor Event i	Successor Event j	t_E^{i-j}	$(EST)_{ij}$	$(EFT)_{ij}$	$(LST)_{ij}$	$(LFT)_{ij}$	S(i) Slack
5	10	7	0	7	0	7	0
5	15	12	0	12	7	19	-
5	20	17	0	17	5	22	-
10	20	15	7	22	7	22	0
10	25	9	7	16	21	30	-
15	30	11	12	23	19	30	7
20	25	5	22	27	25	30	-
20	30	8	22	30	22	30	0
25	35	10	27	37	30	40	3
25	45	15	27	42	35	50	-
30	35	10	30	40	30	40	0
30	40	8	30	38	35	43	-
35	45	10	40	50	40	50	0
40	45	7	38	45	43	50	5