LPP-Simplex

LPP with any number of decision variables can be solved by simplex.

Prerequisites to solve a LPP by simplex

1. Objective function should be of maximization type. In case of minimization, convert it to maximization just by multiplying with (-1).
2. All the constraints should be less than or equal to (≤) type. In case of greater than or equal to (≥) type constraint, convert it to less than or equal to (≤) type by multiplying with (-1).
3. Right hand side of the constraint should be positive.

While solving by simplex

1. **Convert given LPP in standard form. The constraints less than or equal to (≤) type will be converted to equal to (=) type by adding slack variables. Accordingly modify objective function by assigning 0 cost coefficient to the slack variables used. Number of slack variables used will be equal to number of constraints.**
2. **Let's start to built up simplex table. The first column will be the basis or basic variable. At the beginning, it is assumed that Initial basic feasible solution starts with the slack variables added.**
3. **The next column will be C_B, it is for the cost coefficient of variables in the basis from objective function. For the initial solution it would be 0 for all the slack variables.**
4. **The next column will be X_B, it is the right hand side of the corresponding variable in the constraints.**
5. **Open the number of columns as per the given decision variables in the problem and in addition to it open number of columns as per the number of slack variables used.**
6. **Enter the coefficients in the table prepared by using the standard form developed. If some variable is missing in the standard form then the coefficient for that variable is required to be entered as 0.**
7. **On the top of the simplex table write the corresponding C_j values. These are the corresponding cost coefficients of the variables used in the objective function, which can be easily identified from the objective function of the standard form.**
8. **It is observed that, at the end (right hand side) of the table developed, there is an unit matrix of the order equal to number of slack variables added.**
9. **Let's calculate the value of z, by using $Z= \sum C_B X_B$**

10. **Let's calculate net evaluations for each column by using relation $\sum C_B X_k - C_j$, if all the net evaluations are found to be greater than or equal to zero, the solution is said to be optimum.**

Improving the solution

11. **Select the most negative net evaluation, and mark the corresponding variable as entering variable.**
12. **Take the ratio of X_B / X_k, and identify the minimum positive ratio which is greater than zero. It will be a leaving variable.**
13. **Extend the simplex table for the next iteration by incorporating changes due to leaving and entering variable.**
14. **Identify the key element in a previous iteration, with any transformation and observe 1 (one) at the key element in the next iteration.**
15. **By using the key element for the remaining places of entering column, observe 0 (zero).**
16. **Calculate net evaluations, and if required observe 10 to 15 as required.**

PROBLEM 1

A company makes two kinds of leather belts- Belt A and Belt B. Belt A is a high quality belt and Belt B is of lower quality. The respective profits are Rs 4 and Rs 3 per unit. The production of each of type A requires twice as much as a belt of type B, and if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle and only 400 of these are available per day. There are only 700 buckles a day available for belt B.

What should be the daily production of each type of belt? Formulate this problem as LPP and solve it using simplex method.

Let x_1 and x_2 be the number of units of belt A and belt B respectively.

Max $z = 4x_1 + 3x_2$

$2x_1 + x_2 \leq 1000$

$2x_1 + x_2 \leq 800$

$x_1 \leq 400$

$x_2 \leq 700$

$x_1, x_2 \geq 0$

Constraint inequalities are converted into equations by introducing slack variable S_1, S_2, S_3 and S_4

Max $z = 4x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

$2x_1 + x_2 + 0S_1 = 1000$

$x_1 + x_2 + 0S_2 = 800$

$x_1 + 0S_3 \leq 400$

$x_2 + 0S_4 \leq 700$

x_1, x_2, S_1, S_2, S_3 and $S_4 \geq 0$

		C_j	4	3	0	0	0	0	Min$\{x_B/x_k\}>0$
Basis	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	
S_1	0	1000	2	1	1	0	0	0	
S_2	0	800	1	1	0	1	0	0	
S_3	0	400	1	0	0	0	1	0	
S_4	0	700	0	1	0	0	0	1	
	Z= 0		-4	-3	0	0	0	0	

This starting simplex table

		C_j	4	3	0	0	0	0	Min$\{x_B/x_k\}>0$
Basis	C_B	X_B	x_1	X_2	S_1	S_2	S_3	S_4	
S_1	0	1000	2	1	1	0	0	0	1000/2=500
S_2	0	800	1	1	0	1	0	0	800/1=800
S_3	0	400	$\boxed{1}$	0	0	0	1	0	400/1=400 $\rightarrow$
S_4	0	700	0	1	0	0	0	1	---
	Z= 0		-4 $\uparrow$	-3	0	0	0	0	

SBFS

		C_j	4	3	0	0	0	0	Min{x_B/x_k}>0
Basis	C_B	X_B	x_1	X_2	S_1	S_2	S_3	S_4	
S_1	0	1000	2	1	1	0	0	0	1000/2=500
S_2	0	800	1	1	0	1	0	0	800/1=800
S_3	0	400	$\boxed{1}$	0	0	0	1	0	400/1=400
S_4	0	700	0	1	0	0	0	1	---
	Z= 0		-4	-3	0	0	0	0	
S_1	0	200	0	$\boxed{1}$	1	0	-2	0	200/1=200
S_2	0	400	0	1	0	1	-1	0	400/1=400
x_1	4	400	1	0	0	0	1	0	---
S_4	0	700	0	1	0	0	0	1	700/1=700
			0	-3	0	0	4	0	

TBFS

		C_j	4	3	0	0	0	0	Min{x_B/x_k}>0
Basis	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	
S_1	0	1000	2	1	1	0	0	0	1000/2=500
S_2	0	800	1	1	0	1	0	0	800/1=800
S_3	0	400	$\boxed{1}$	0	0	0	1	0	400/1=400
S_4	0	700	0	1	0	0	0	1	---
	Z= 0		-4	-3	0	0	0	0	
S_1	0	200	0	$\boxed{1}$	1	0	-2	0	200/1=200
S_2	0	400	0	1	0	1	-1	0	400/1=400
x_1	4	400	1	0	0	0	1	0	---
S_4	0	700	0	1	0	0	0	1	700/1=700
	Z=1600		0	-3	0	0	4	0	
x_2	3	200	0	1	1	0	-2	0	---
S_2	0	200	0	0	-1	1	$\boxed{1}$	0	200
x_1	4	400	1	0	0	0	1	0	400
S_4	0	500	0	0	-1	0	2	1	250
	Z=2200		0	0	3	0	-2	0	

Fourth Basic

		C_j	4	3	0	0	0	0	$\text{Min}\{x_B/x_k\}>0$
Basis	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	
S_1	0	1000	2	1	1	0	0	0	1000/2=500
S_2	0	800	1	1	0	1	0	0	800/1=800
S_3	0	400	$\boxed{1}$	0	0	0	1	0	400/1=400
S_4	0	700	0	1	0	0	0	1	---
	Z= 0		-4	-3	0	0	0	0	
S_1	0	200	0	$\boxed{1}$	1	0	-2	0	200/1=200
S_2	0	400	0	1	0	1	-1	0	400/1=400
x_1	4	400	1	0	0	0	1	0	---
S_4	0	700	0	1	0	0	0	1	700/1=700
	Z=1600		0	-3	0	0	4	0	
x_2	3	200	0	1	1	0	-2	0	---
S_2	0	200	0	0	-1	1	$\boxed{1}$	0	200
x_1	4	400	1	0	0	0	1	0	400
S_4	0	500	0	0	-1	0	2	1	250
	Z=2200		0	0	3	0	-2	0	
x_2	3	600	0	1	-1	2	0	0	
S_3	0	200	0	0	-1	1	1	0	
x_1	4	200	1	0	1	-1	0	0	
S_4	0	100	0	0	1	-2	0	1	
	Z=2600		0	0	1	2	0	0	

PROBLEM 2

Solve by using simplex.

Max $z = 3x_1 + 5x_2 + 4x_3$

$2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0$

Constraint inequalities are converted into equations by introducing slack variable S_1, S_2, and S_3

Max $z = 3x_1 + 5x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

$2x_1 + 3x_2 + S_1 \leq 8$

$2x_2 + 5x_3 + S_2 \leq 10$

$3x_1 + 2x_2 + 4x_3 + S_3 \leq 15$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Starting table

		C_j	3	5	4	0	0	0	$\text{Min}\{x_B/x_k\}>0$
Basis	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
S_1	0	8	2	3	0	1	0	0	
S_2	0	10	0	2	5	0	1	0	
S_3	0	15	3	2	4	0	0	1	
	Z= 0		-3	-5	-4	0	0	0	

Starting table

		C_j	3	5	4	0	0	0	$\text{Min}\{x_B/x_k\}>0$
Basis	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
S_1	0	8	2	$\boxed{3}$	0	1	0	0	8/3 →
S_2	0	10	0	2	5	0	1	0	10/2=5
S_3	0	15	3	2	4	0	0	1	15/2
	Z= 0		-3	↑-5	-4	0	0	0	

SBFS

		C_j	3	5	4	0	0	0	$\text{Min}\{x_B/x_k\}>0$
Basis	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
S_1	0	8	2	$\boxed{3}$	0	1	0	0	8/3 →
S_2	0	10	0	2	5	0	1	0	10/2=5
S_3	0	15	3	2	4	0	0	1	15/2
	Z= 0		-3	↑-5	-4	0	0	0	
x_2	5	8/3	2/3	1	0	1/3	0	0	---
S_2	0	14/3	-4/3	0	$\boxed{5}$	-2/3	1	0	(14/3)/5→
S_3	0	29/3	5/3	0	4	-2/3	0	1	
	Z= 40/3		1/3	0	↑-4	5/3	0	0	

TBFS

		C_j	3	5	4	0	0	0	Min$\{x_B/x_k\}>0$
Basis	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
S_1	0	8	2	$\boxed{3}$	0	1	0	0	8/3 →
S_2	0	10	0	2	5	0	1	0	10/2=5
S_3	0	15	3	2	4	0	0	1	15/2
	Z= 0		-3	↑-5	-4	0	0	0	
x_2	5	8/3	2/3	1	0	1/3	0	0	---
S_2	0	14/3	-4/3	0	$\boxed{5}$	-2/3	1	0	(14/3)/5 →
S_3	0	29/3	5/3	0	4	-2/3	0	1	(29/3)/4
	Z= 40/3		1/3	0	↑-4	5/3	0	0	
x_2	5	8/3	2/3	1	0	1/3	0	0	(8/3)/(2/3)=4
x_3	4	14/15	-4/15	0	1	-2/15	1/5	0	---
S_3	0	89/15	41/15	0	0	2/15	-4/5	1	(89/15)/(41/15)=89/41 →
	Z= 256/15		-11/15	0	0	17/15	4/5	0	

Fourth basic feasible solution

		C_j	3	5	4	0	0	0	Min$\{x_B/x_k\}>0$
Basis	C_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
S_1	0	8	2	$\boxed{3}$	0	1	0	0	8/3 →
S_2	0	10	0	2	5	0	1	0	10/2=5
S_3	0	15	3	2	4	0	0	1	15/2
	Z= 0		-3	↑-5	-4	0	0	0	
x_2	5	8/3	2/3	1	0	1/3	0	0	---
S_2	0	14/3	-4/3	0	$\boxed{5}$	-2/3	1	0	(14/3)/5 →
S_3	0	29/3	5/3	0	4	-2/3	0	1	(29/3)/4
	Z= 40/3		1/3	0	↑-4	5/3	0	0	
x_2	5	8/3	2/3	1	0	1/3	0	0	(8/3)/(2/3)=4
x_3	4	14/15	-4/15	0	1	-2/15	1/5	0	---
S_3	0	89/15	41/15	0	0	2/15	-4/5	1	(89/15)/(41/15)=89/41 →
	Z= 256/15		↑-11/15	0	0	17/15	4/5	0	
x_2	5	50/41	0	1	0	15/41	8/41	-10/41	
x_3	4	62/41	0	0	1	-6/41	5/41	4/41	
x_1	3	89/41	1	0	0	-2/41	-12/41	15/41	
	Z= 765/41		0	0	0	45/41	24/41	11/41	