(A) Transportation Problem with degenercy

Problem 1 – find the minimum/optimum transportation cost for the problem below.

	W_1	W_2	W_3	W_4	W_5	Available
F_1	7	6	4	5	9	40
F_2	8	5	6	7	8	30
F_3	6	8	9	6	5	20
F_4	5	7	7	8	6	10
Demand	30	30	15	20	5	100 / 100

As Σ Demand $= \Sigma$ Available

Given TP is a balanced T.P.

Initial Basic feasible Solution (VAM)

	7 (5)	6	4 (15)	5 (20)	9	~~40~~ ~~25~~ ~~5~~ [1] [1] [2]← [2]←
	8	5 (30)	6	7	8	~~30~~ [1] [2]←
	6 (15)	8	9	6	5 (5)	~~30~~ ~~15~~ [1] [1] [1] [1] [1]
	5 (10)	7	7	8	6	~~10~~ [1] [1] [1] [1] [1]
	~~30~~ ~~25~~ ~~15~~	~~30~~	~~15~~	~~20~~	~~5~~	
	[1]	[1]	[2]↑	[1]	[1]	
	[1]	[1]		[1]	[1]	
	[1]			[1]	[1]	
	[1]				[1]	
	[1]↑				[1]	

Total Transportation Cost (TTC)

$= Rs\left[(7\times5)+(4\times15)+(5\times20)+(5\times30)+(6\times15)+(5\times5)+(5\times10)\right]$

$= Rs\ (35+60+100+150+90+25+50) = Rs\ 510$

If $m+n-1 =$ No of allocations | Basic cells — TP without degenery

$m+n-1 \neq$ No of allocations | Basic cells — TP with degenery.

Check for optimality

	W_1	W_2	W_3	W_4	W_5	
f_1	7 $-\theta$ (5)	6 $\boxed{3}$	4 $+\theta$ (15)	5 (20)	9 $\boxed{3}$	0
f_2	8 $\boxed{-1}$ $+\theta$	5 (30)	6 $-\theta$ (ϵ)	7 $\boxed{0}$	8 $\boxed{0}$	2
f_3	6 (15)	8 $\boxed{6}$	9 $\boxed{6}$	6 $\boxed{2}$	5 (5)	-1
f_4	5 (10)	7 $\boxed{6}$	7 $\boxed{5}$	8 $\boxed{5}$	6	-2
	7	3	4	5	6	

i) $m+n-1 = 4+5-1 = 8 \neq$ No of Basic cells (7)

T.P. with degenery.

In case of degenery, select the minimum cost cell, such that it ~~wont~~ will not possible to have a close loop passing through the basic cells. Allocate ϵ ($\epsilon \to 0$ but $\epsilon \neq 0$) to the cell so that it will be a basic cell.

Allocating ϵ to (f_2, W_3) cell degenery can be removed.

(ii) All allocations are independent of each other i.e. it is not possible to have a closed loop passing through basic cells.

As all net evaluations are not positive (≥ 0), the obtained solution is not optimum.

Improving the solution

$$\text{Min} = \{(\epsilon - \theta), (5-\theta)\} = 0$$

$$\theta = \epsilon$$

7 (5)	6	4 (15)	5 (20)	8
8 (∈)	5 (30)	6	7	8
6 (15)	8	9	6	5 (5)
5 (10)	7	7	8	6

SBFS

$$TTC = Rs\left[(7\times5)+(4\times15)+(5\times20)+\underset{0}{\underline{(8\times\epsilon)}}+(5\times30)+(6\times15) + (5\times5)+(5\times10)\right]$$

$$= Rs\,(35+60+100+150+90+25+50) = Rs\,\underline{510}$$

Check for optimality

					u_i
7 (5)	6 $\boxed{2}$	4 $\boxed{0}$ (15)	5 (20)	8 $\boxed{2}$	7
8 (∈)	5 (30)	6 $\boxed{1}$	7 $\boxed{1}$	8 $\boxed{1}$	8
6 (15)	8 $\boxed{5}$	9 $\boxed{6}$	6 $\boxed{2}$	5 (5)	6
5 (10)	7 $\boxed{5}$	7 $\boxed{5}$	8 $\boxed{5}$	6	5
0	-3	-3	-2	-1	

As all net evaluations are positive (≥ 0), the obtained solution is optimum.

13 Unbalanced Transportation problem

Problem 2 Solve The T.P

		Stores 1	2	3	4	Supply
Factories	A	4	6	8	13	50
	B	13	11	10	8	70
	C	14	4	10	13	30
	D	9	11	13	8	50
	Demand	25	35	105	20	

Solution

Total supply = 50+70+30+50 = 200 units

Total Demand = 25+35+105+20 = 185 units

Given T.P. is balanced by introducing dummy coloumn with demand 15 units.

Obtaining IBFS by VAM we get

	1	2	3	4	Dummy
A	4 (25)	6 (5)	8 (20)	13	0
B	13	11	10 (70)	8	0
C	14	4 (30)	10	13	0
D	9	11	13 (15)	8 (20)	0 (15)

T.P. without degeneracy. and the solution is found to be optimum.

T.T.C = Rs 1465

(c) Profit Maximization Problem

(1) Solve the T.P. for maximization

	A	B	C	D	
1	90	90	100	110	200
2	50	70	130	85	100
	75	100	100	30	

Solution

Unbalanced & Maximization.

If IBFS is to obtained by VAM, then instead of obtaining the difference in between two least, Obtain the difference in between two highest and assign to the highest cell instead of lowest.

90 (70)	90 (100)	100	110 (30)	200
50	70	130 (100)	85	100
0 (5)	0	0	0	05
75	100	100	30	

IBFS

Above solution is obtained by VAM and found to be optimum.

Zmax = Rs 31, 600

Profit maximization problem

Solve the problem for maximization.

Sales agencies

		1	2	3	4	5	Supply
factories	1	15	17	12	11	11	140
	2	5	9	7	15	7	190
	3	14	15	16	20	10	115
Demand		74	94	69	39	119	

Let us convert maximization to minimization just by multiplying the matrix by (−1), we get –

−15	−17	−12	−11	−11
−5	−9	−7	−15	−7
−14	−15	−16	−20	−10

— Minimization

For more simplicity let us substract whole matrix from 20 so that all values would be positive

5	3	8	9	9	140
15	11	13	5	13	190
6	5	4	0	10	115
74	94	69	39	119	

Balance it and solve

While calculating final answer refer to original matrix.

Restricted Transportation Problem

Assign huge cost ∞ or M to the restricted ~~sale~~ cell and solve.