1) Purchase Model with Instantaneous Replenishment and without shortages.

In ~~this~~ inventory model, orders of equal size are placed at periodical interval. The items against an order are replenished instantaneously and the items are consumed at a constant rate. The purchase price per unit is same irrespective of order size.

Let us suppose.

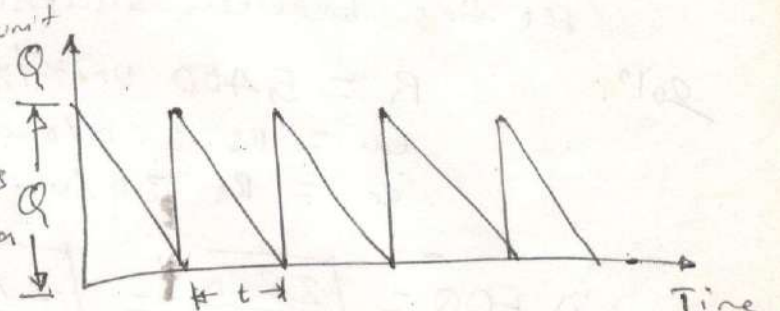
R - Annual Demand, in units.

C_o - Ordering cost / order, units

C_c - Carrying cost / unit / year

P - Purchase price per unit.

Q - Order size.



Purchase model without shortage.

The number of orders/year $= \frac{R}{Q}$

Average inventory $= \frac{Q}{2}$.

cost of ordering / year $= \frac{R}{Q} \times C_o$

Cost of carrying / year $= \frac{Q}{2} \times C_c$

Purchase cost / year $= RP$.

$\therefore$ Total Inventory cost / year $= \frac{R}{Q} \times C_o + \frac{Q}{2} C_c + RP$

Differentiating w.r.t Q

$$\frac{d}{dQ}(TC) = -\frac{R}{Q^2} C_o + \frac{C_c}{2}$$

Diff. w.r.t. Q

$$\frac{d^2(TC)}{dQ^2} = +\frac{2}{Q^3} R C_o$$

$\therefore$ second derivative is +ve

$\therefore$ the optimal value for Q is obtained by equating the first derivative is zero.

$$\therefore \frac{R}{Q^2} C_o = \frac{C_c}{2}$$

$$Q^2 = \frac{2 R C_o}{C_c}$$

Total number of orders per year $= \frac{R}{EOQ} = \frac{R}{Q^*}$

Time between orders $= \frac{EOQ}{R}$.

Ex. 1. Ram Industry needs 5,400 units/year of a bought-out component which will be used in its main product. The ordering cost is Rs. 250 per order and the carrying cost per unit per year is Rs. 30. Find : EOQ, the no. of orders per year and the time between successive orders.

Sol^n^:

$R = 5{,}400$ units/year.
$C_o =$ Rs. 250/order
$C_c =$ Rs. 30/unit/year.

EOQ, No. of orders/year & time, t.

i) $EOQ = \sqrt{\frac{2RC_o}{C_c}} = \sqrt{\frac{2 \times 5400 \times 250}{30}}$

$= 300$ unit

ii) No. of order/year $= \frac{R}{EOQ} = \frac{5400}{300} = 18$

& Time bet^n^ successive orders $= \frac{EOQ}{R} = \frac{300 \text{ units}}{5400 \text{ units/year}}$

$= 0.0556$ year

$= 0.6672$ month

$= 20$ days (app

2) Alpha industry needs 15,000 units per year of a bought-out component which will be used in its main product. The ordering cost is Rs. 125 per order and the carrying cost per unit per year is 20% of the purchase price per unit. The purchase price per unit is Rs. 75. Find: EOQ, no. of orders per year & time bet^n^ successive orders :

Sol^n^: $R = 15{,}000$ units/year
$C_o = 125$ Rs./order
Purchase price/unit = Rs. 75
$C_c = 0.2 \times$ Rs. 75
= Rs. 15/unit/year

$\therefore EOQ = \sqrt{\frac{2RC_o}{C_c}} = \sqrt{\frac{2 \times 15000 \times 12}{15}}$

$= 500$ units

No. of orders/year $= \frac{R}{EOQ} = \frac{15{,}000}{500} = 30$

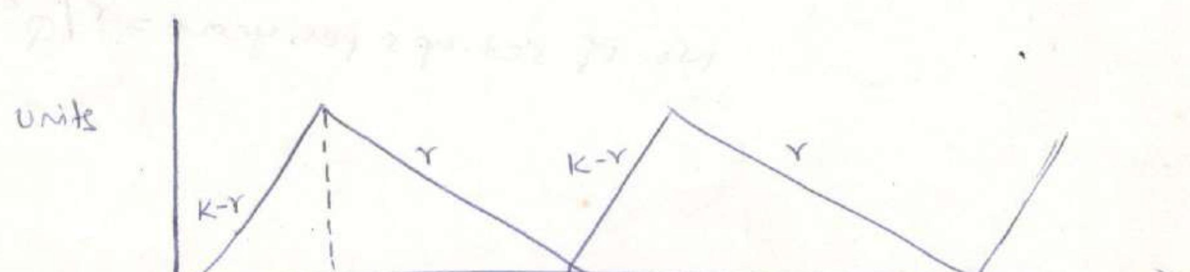
Manufacturing Model without shortages.

If a company manufactures an item which is required for its main product, then the corresponding model of inventory is called manufacturing model.

In this model, shortages are not permitt The rate of consumption of the item is assumed to be uniform throughout the year. The item is produced and consumed simultaneously for a part of the cycle time. During the remaining cycle time, only the consumption of the item takes place and the cost of production per unit is same irrespective of production lot size.

Let us suppose

r – Annual demand in units.
K – production rate of the item
C_o – cost per set-up.
C_c – carrying cost /unit /year
P – cost of production /unit
t_1 – period of production as well as consumption of the item
t_2 – period of consumption only
t – cycle time (i.e $t = t_1 + t_2$)



Manufacturing model without shortages.

During the period t, the item is produced at the rate of K units per period and simultaneously it is consumed at the rate of r units per period. During this period, the inventory is built at the rate of K-r units per period. During the period t_2, the production of the item is discontinued but the consumption of the same item is continued. Hence, the inventory is decrease at the rate of r units per period during this time. The various formula to be applied for this kind of situation are given below.

Economic batch quantity (EBQ or Q^*)

$$= \sqrt{\frac{2 C_0 r}{C_c [1-(r/K)]}}$$

Period of production as well as consumption.

$$t_1^* = \frac{Q^*}{K}$$

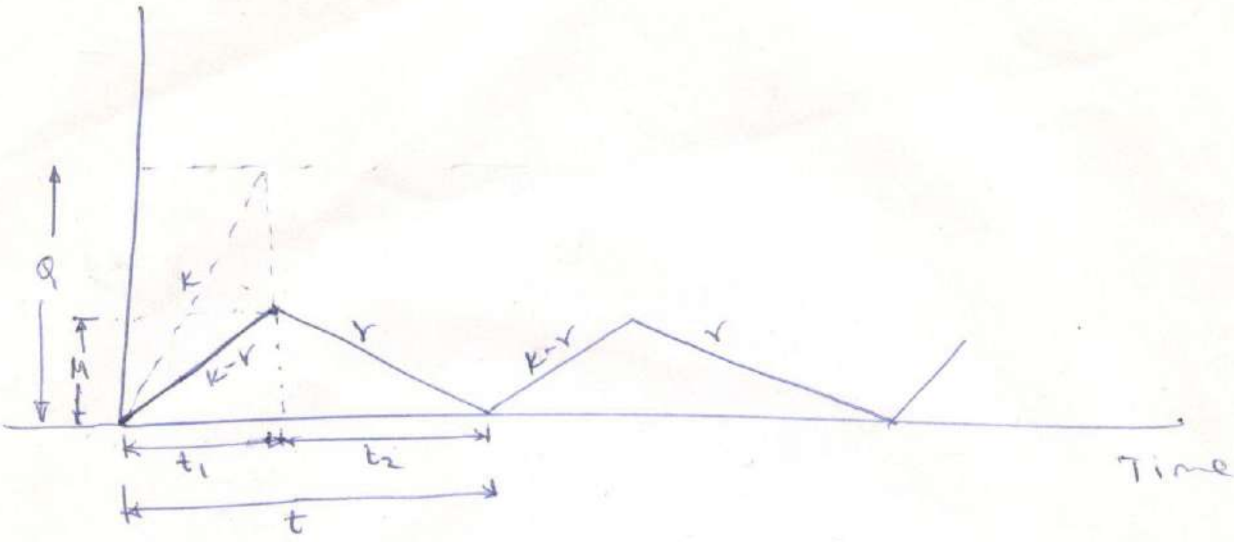
Period of consumption only, $t_2^* = \frac{Q^* (1 - r/K)}{r}$

$$= \frac{K-r}{r} t_1$$

cycle time $t = t_1^* + t_2^*$.

No. of set-ups per year $= r/Q^*$.

II) Manufacturing Model without shortages



$M = (k - r)t_1$

$Q = kt_1$ or $t_1 = \frac{Q}{k}$ or $t_1 = \frac{Q^*}{k}$.

$\therefore \boxed{M = \frac{(k-r)}{k} Q}$

Holding Cost / of — ordering cost / of — set up cost

$T\cdot C$ = Carryg. Cost + ~~Holding Cost~~ + Procurement Cost :

$$= \frac{Q}{2}\left(\frac{k-r}{k}\right) C_c + \frac{R}{Q} C_o + R\cdot P.$$

$$\frac{d(T\cdot C)}{dQ} = \frac{C_c}{2}\left(\frac{k-r}{k}\right) - \frac{R}{Q^2} C_o$$

$$\frac{d^2(T\cdot C)}{dQ^2} = + 2\frac{R}{Q^3} C_o.$$

So, $\frac{R}{Q^2} C_o = \frac{C_c}{2}\left(\frac{k-r}{k}\right)$

or $Q^2 = \frac{2RC_o}{C_c}\left(\frac{k}{k-r}\right)$

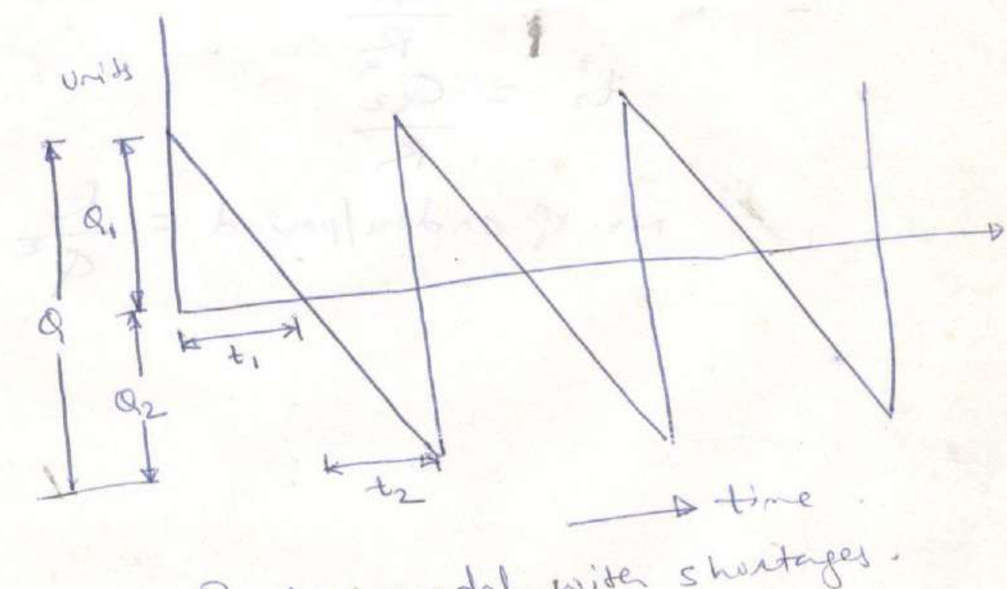
or $Q^* = \sqrt{\frac{2RC_o}{C_c}\left(\frac{k}{k-r}\right)}$ or $\sqrt{\frac{2RC_o}{C_c[1-(r/k)]}}$

$$T\cdot C = \sqrt{\frac{2RC_o}{C_c}\left(\frac{k}{k-r}\right)}\ \frac{C_c}{2}\left(\frac{k-r}{k}\right) + \frac{R}{\sqrt{\frac{2RC_o}{C_c}\left(\frac{k}{k-r}\right)}} \cdot C_o.$$

$$= \sqrt{\frac{RC_oC_c}{2}\cdot\left(\frac{k-r}{k}\right)} + \sqrt{\frac{RC_oC_c}{\ }\cdot\left(\frac{k-r}{\ }\right)}$$

Purchase Model with Instantaneous Replenishment and with shortages.

In this model, an item on order will be received instanteneously and it is consumed at a constant rate. The purchase price per unit is same irrespective of order size. If there is no stock at the time of receiving a request for the item, it is assumed that it will be satisfied at a later date with a penalty. This is called backordering. The model is shown as in Fig.



Purchase model with shortages.

The variables which are to be used.

- R - Demand / period
- C_c - Carrying cost / unit / period.
- C_o - Ordering cost / order
- C_s - shortage cost / unit / period
- Q - Order size
- Q_1 = Maximum inventory
- Q_2 - Maximum stock-out
- t_1 - period of positive stock
- period of shortage