LPP-Duality and Sensitivity

Duality in LPP

The given original problem is called the primal problem. The primal can be rewritten by transposing the rows and columns. Inverting the problem dual problem will result.

For a given problem (primal) we can obtain a dual for it. If the obtained dual is considered as the original problem (Primal) then the dual associated with it will the starting problem given.

For obtaining dual for the give problem

1. Express the given problem as standard primal problem. (objective function maximization and all constraints should of less than or equal to (≤) type)
2. Obtain the dual such that the coefficient matrix of the constraints of the dual is the transpose of the primal.
    a. The constant $C_1, C_2, C_3, \ldots C_n$ in the objective fuction of the primal appears in the right hand side of the constraints of the dual.
    b. The constants $b_1, b_2, \ldots b_m$ in the right hand side of the constraints of the primal appear in the objective function of the dual.
    c. The objective function has to be of minimization type
    d. The constraints are required to be greater than or equal to type only.
3. The variables in both problems are non-negative.

PROBLEM 1

Obtain dual for the following

Max $Z = 3x_1 + 5x_2$

Subject to

$2x_1 + 6x_2 \leq 50$

$3x_1 + 2x_2 \leq 35$

$5x_1 - 3x_2 \leq 10$

$x_1, x_2 \geq 0$

Solution

As the given problem has all the constraints less than or equal to type and objective function is maximization type, it is standard primal. Hence the dual of the problem can be

Min $Z = 50y_1 + 35y_2 + 10y_3$

Subject to

$2y_1 + 3y_2 + 5y_3 \geq 3$

$6y_1 + 2y_2 - 3y_3 \geq 5$

$y_1, y_2, y_3 \geq 0$

Let us consider the obtained dual is the given problem and let's obtain dual for it

Expressing the given problem as the standard primal problem; we get

Max $Z' = (-Z) = -50y_1 - 35y_2 - 10y_3$

Subject to

$-2y_1 - 3y_2 - 5y_3 \leq -3$

$-6y_1 - 2y_2 + 3y_3 \leq -5$

$y_1, y_2, y_3 \geq 0$

The dual for the above problem can be written as

Min $Z= -3x_1 - 5x_2$

Subject to

$-2x_1 - 6x_2 \geq -50$

$-3x_1 - 2x_2 \geq -35$

$-5x_1 + 3x_2 \geq -10$

$x_1, x_2 \geq 0$

It is found that the obtained dual is equivalent to the original problem from we have started.

PROBLEM 2

Construct the dual of the problem

Minimize $Z = x_2 + 3x_3$

Subject to

$2x_1 + x_2 \leq 3$

$x_1 + 2x_2 + 6x_3 \geq 5$

$-x_1 + x_2 + 2x_3 = 2$

$x_1, x_2, x_3 \geq 0$

Solution

Converting the given problem into standard primal.

Greater than equal to constraint can be converted into less than equal to by multiplying by (-1).

Equal to constraint will be written once as less than equal to and greater than equal to

Maximize $Z' = (-Z) = -0x_1 - x_2 - 3x_3$

Subject to

$2x_1 + x_2 + 0\,x_3 \leq 3$

$-x_1 - 2x_2 - 6x_3 \leq -5$

$-x_1 + x_2 + 2x_3 \leq 2$

$x_1 - x_2 - 2x_3 \leq -2$

$x_1, x_2, x_3 \geq 0$

The dual of the problem is

Min $Z_y = 3y_1 – 5y_2 + 2y_3' - 2y_3''$

Subject to

$2y_1 – y_2 + y_3' + y_3'' \geq 0$

$y_1 – 2y_2 + y_3' - y_3'' \geq -1$

$-6y_2 + 2y_3' - 2y_3'' \geq -3$

$y_1, y_2, y_3', y_3'' \geq 0$

Sensitivity or post optimality analysis

The changes in parameters of the problem may be discrete or continuous. The study of the effect of discrete changes in parameters on the optimal solution is called the sensitivity analysis or the post optimality analysis. Alternatively, the current optimal solution may be investigated.

The changes in the parameters of a linear programming problem include

1. Changes in the right-hand side of the constraints or availability of resources.
2. Changes in the cost/profit coefficients of decision variables (C_j).
3. Addition of new variables.
4. Changes in the coefficients of constraints (a_{mn})
5. Addition of new constraints
6. Deletion of variables
7. Deletion of constraints.

Generally, these parameter changes result in one of the following three caes:

1. The optimal solution remains unchanged.
2. The basic variables remain unchanged but their values changes.
3. The basic variables as well as their values are changed.

While dealing with these changes, one important objective is to find the maximum extent ot which a parameter or a set of parameters can be changed so that the current optimal solution remains optimal. In other words, the objective is to determine how sensitive the optimal solution to the changes in those parameters. Such analysis is called sensitivity.

Theory questions

Write short note on

1. Duality in LPP
2. Sensitivity analysis

Theory questions on Introduction to operations research

1. State the characteristics of operations research.
2. Explain scientific method of in operations research.
3. Explain necessity of operations research in industries.
4. Explain scope of operations research.

5. Explain the role of operations research in decision making.
6. Explain scope of operations research in management.
7. Explain scope of operations research in financial management.
8. Explain applications of various operations research techniques.
9. Describe the various phases or methodology or approach of operations research.
10. What do you understand by models in operations research? Explain classification of models in brief.
11. State the various characteristics of a good model.
12. State advantages and limitations of a model.
13. Write short note on “Role of computers in operations research”.
14. State the various limitations of operations research.