Chapter 1 - Basic Measure Theory

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Exercise Solutions

Exercise 1.1.3

Exercise 1.3.1

Exercise 1.3.3

Exercise 1.4.1

Exercise 1.4.2 Let f be as in **Ex 1.4.1** and let $(\mathbb{R}, \sigma(f), \lambda)$ be our measure space, we define the function $g : \mathbb{R} \to \mathbb{R}$ such that

$$g(x) = \begin{cases} 2 & \text{if } x = 1 \\ -2 & \text{if } x = -1 \\ |x| & \text{otherwise} \end{cases}$$

Now, f and g only differ in $\{1, -1\}$ which clearly has null measure. However, we have that $2 \in g^{-1}(1)$ and $-2 \notin g^{-1}(1)$ which implies that the set $g^{-1}(1)$ cannot be measurable (this assertion can be deduced from the solution of **Ex 1.4.1**) which conclues the proof.

Exercise 1.4.3 The differentiability of f implies it's continuity which in turn implies it's measurability. Now define the sequence of functions

$$f_n(c) = \frac{f(c + \frac{1}{n}) - f(c)}{(c + \frac{1}{n}) - c}$$

It is easy to verify that these are measurable functions and given the existance of the limit of the difference quotient of f we have $f'(c) = \limsup_{n\to\infty} f_n(c)$ for all $c \in \mathbb{R}$ which implies the measurability of f' and concludes the proof.

Exercise 1.4.5

Useful Properties