Implementation of S.O.T.A. Linear PieceWise modeling formulation

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Abstract

Small case study implementing the paper Huchette and Vielma (2017). Different functions were minimized and maximized using analytical and the proposed method in the paper. Finally, a small hydropower economic dispatch problem was solved.

1 Overview

The paper Huchette and Vielma (2017) proposed a new method for linearizing non-linear functions for the field of Operations Research (OR). The most direct approach for linearizing functions is using N binary variables for activating one of the N segments of the linearization. This takes a considerably large amount of binary variables. Huchette and Vielma (2017) proposed two methods for reducing the problem's complexity. In the first method, the number of binary variables is reduced while in the second one, the binary variables are replaced by integer variables. Alongside the new linearization methods Huchette and Vielma (2017) comes with an on-the-shelf solution for using the paper in form of a Julia Lang Library: PiecewiseLinearOpt.

The document is organized in the following way:

- **Model description:** PiecewiseLinearOpt and the model implementation are described.
- **Testing:** Different functions are maximized/minimized using the implementation of Huchette and Vielma (2017), its on-the-shelf library, and analytical methods.
- **Hydro Case Study:** A small Hydro coordination model is analyzed. The non-linear function that relates the potential energy of the water with electrical energy is linearized using the proposed methods.

2 Model description

2.1 PiecewiseLinearOpt

The proposed library is mainly composed of the function *piecewiselinear* which model a univariate and bivariate function using the breakpoints given by the user. The function allows the user to choose which linearization method to use, two of the methods are the ones proposed in Huchette and Vielma (2017)

- model: JuMP mathematical model where the linearized function should be attached to
- z,x,y: variables of the optimization problem.
- x_range, y_range: 1D vector of evenly distributed points used as the linearization breakpoints for $f(x, y)$.
- (u,v) -> f(u,v): mathematical function to linearized.

- method=:ZigZag: Linearization method selector

Listing 1: model of $z = f(x, y)$

```
z = piecewiselinear(model, x, y, x_range, y_range, (u,v) -> f(u,v),method=:ZigZag)
```

2.2 myPiecewiseLinearOpt

In order to implement the two proposed methods in Huchette and Vielma (2017) a function was developed following the PiecewiseLinearOpt style.

- f: mathematical function to linearized.
- model_type: It selects between the 2 proposed methods.
- id: auxiliary identification for those models where different functions should be linearized.
- The rest of the parameters are the same as the function piecewiselinear 1

Listing 2: model of $z = f(x, y)$

```
z = myPiecewiseLinearOpt(model, x, y, z, x_range, y_range, f, model_type, id)
```

3 Testing

Different non-linear functions are maximized/minimized all of them are analyzed using the same breakpoints. For the analysis, all functions had 8 segments. In table 1 the number of variables for each mathematical model is shown. As the number of segments is the same for all the experiments the results displayed on the tables 1 are the same. It can be observed that the function PiecewiseLinearOpt tends to have more variables than the implementation of the paper. This is because our implementation only allows power of two segments while PiecewiseLinearOpt allows all.

Model	Continuous	Integer
my ZZI	84	8 (2 binary)
my ZZB	84	8 (8 binary)
PiecewiseLinearOpt: default	84	12 (12 binary)
PiecewiseLinearOpt: ZZB	84	12 (12 binary)
PiecewiseLinearOpt: ZZI	85	12 (6 binary)

Table 1: Number of variables for bivariate formulations

Model	Continuous	Integer
PiecewiseLinearOpt: default	11	8 (8 binary)
PiecewiseLinearOpt: ZZB	11	3 (3 binary)
PiecewiseLinearOpt: ZZI	11	3 (0 binary)

Table 2: Number of variables for univariate formulations

3.1 HelloWorld Univariate

Minimization of the equation 1.

$$f(x) = sin(x) \tag{1}$$

	PiecewiseLinearOpt	Paper Implementation	Analytical
Solution	[1.570 ; 1]	NA	[1.570 ; 1]

Table 3: Comparison table

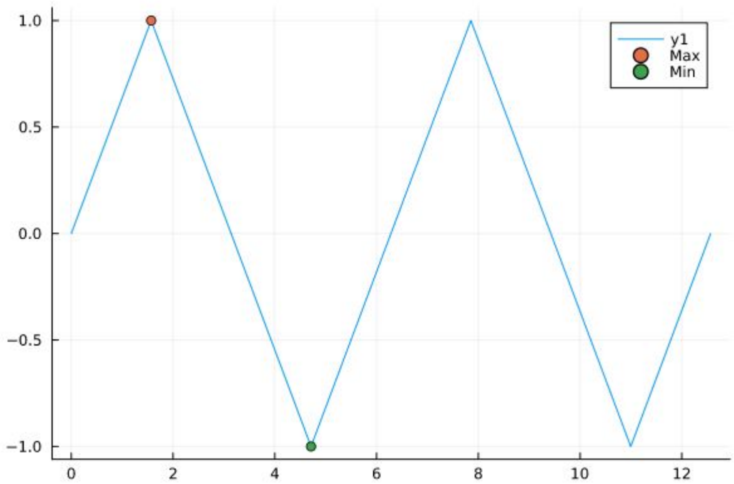

Figure 1: 1 Plot

3.2 HelloWorld Bivariate

$$f(x, y) = e^{(x+y)} \tag{2}$$

	PiecewiseLinearOpt(default)	ZZI	ZZB	Analytical
Min	[-4.0;-4.0;0.0]	[-4.0;-4.0;0.0]	[-4.0;-4.0;0.0]	[-4.0;-4.0;0.0]

Table 4: Comparison table

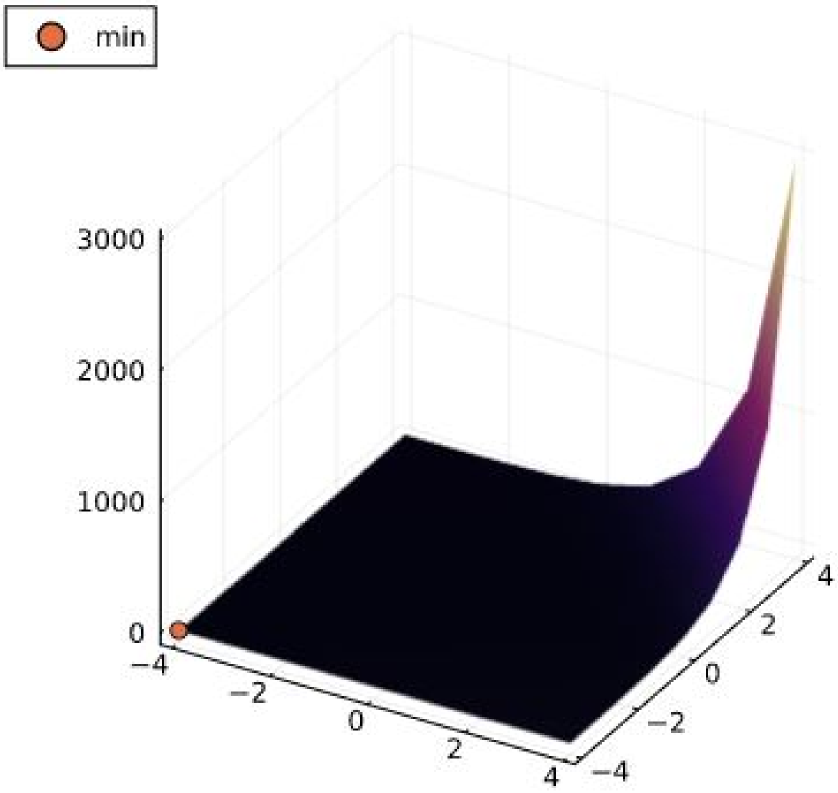

Figure 2: 2Plot

3.3 Highly non-convex a)

$$f(x,y) = 3(1-x)^2 e^{-(x^2)-(y+1)^2} - 10(\frac{x}{5} - x^3 - y^5)e^{-x^2-y^2} - \frac{1}{3}e^{-(x+1)^2-y^2} \tag{3}$$

	PiecewiseLinearOpt	Paper Implementation ZZI	Paper Implementation ZZB	Analytical
Min	[0.0;-2.0;-4.75]	[-2.0;0.0;-4.75]	[-2.0;0.0;-4.75]	[0.0;-2.0;-4.75]
Max	[0.0;2.0;5.85]	[2.0;0.0;5.85]	[2.0;0.0;5.85]	[0.0;2.0;5.85]

Table 5: Comparison table

3.4 Highly non-convex b)

Two optimization problems where a variable is minimized or maximized subject to two equations.

$$f(x,y) = 3(1-x)^2 e^{-(x^2)-(y+1)^2} - 10(\frac{x}{5} - x^3 - y^5)e^{-x^2-y^2} - \frac{1}{3}e^{-(x+1)^2-y^2} \tag{4}$$

$$\begin{aligned} \text{min} \quad & z \\ \text{Subject to} \quad & x + y + 3 \geq z \\ & f(x,y) = z \end{aligned}$$

	PiecewiseLinearOpt	Paper Implementation ZZI	Paper Implementation ZZB	Analytical
Min	[0.0;-2.0;-4.75]	[-2.0;0.0;-4.75]	[-2.0;0.0;-4.75]	NA
Max	[0.18;2.0; 5.18]	[2.0;0.18; 5.18]	[2.0;0.18; 5.18]	NA

Table 6: Comparison table

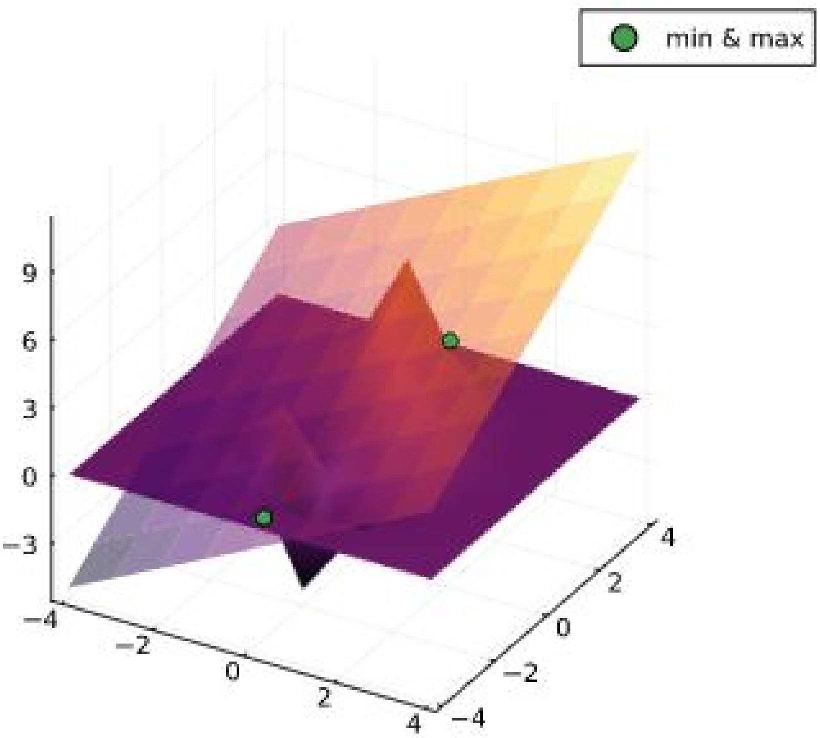

Figure 3: Plot of the mathematical space subject to minimization or maximization

4 Hydrothermal Coordination

Small Economic dispatch model with a thermal power plant and a hydropower plant.

4.1 PiecewiseLinearOpt

NOT WORKING

4.2 Paper Implementation

	Cost[€]	Time[s]			Number of Variables	
Model		Total	Presolve	Solve	Cont.	Int.
my ZZB	5.422877056363e+04	738	57	681	400k	384 (384 binary)
my ZZI	5.422877056363e+04	377	45	332	400k	384 (48 binary)

Table 7: Solution comparation

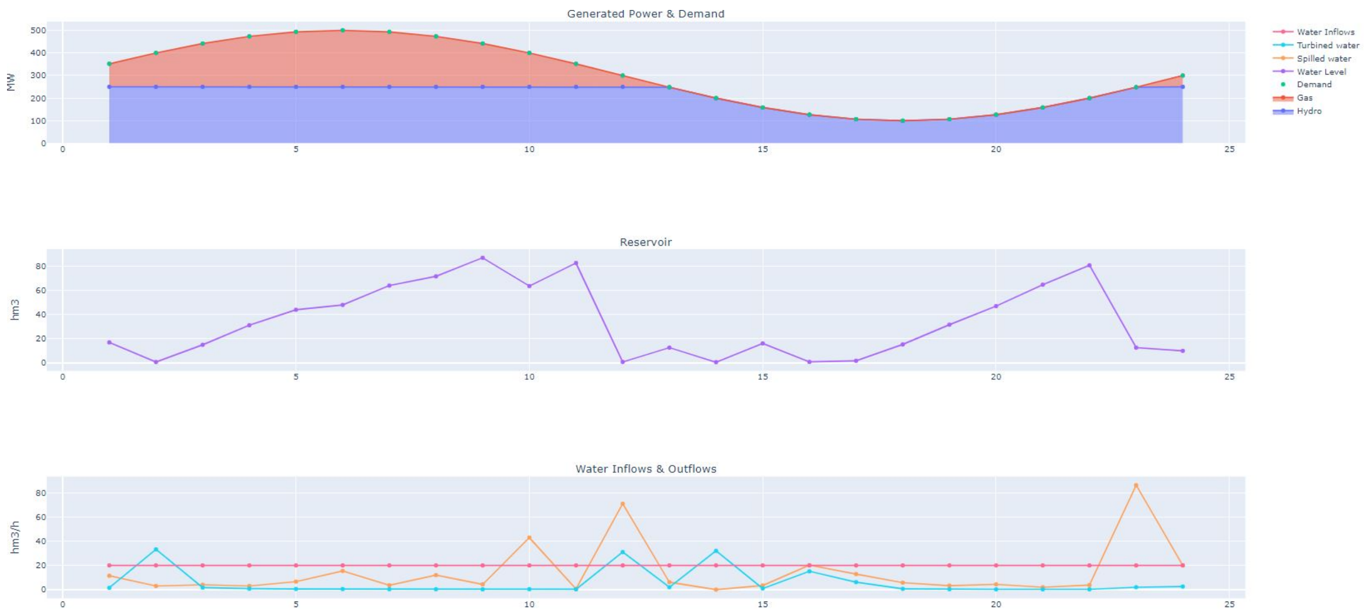

Figure 4: Model solution using ZZB approach

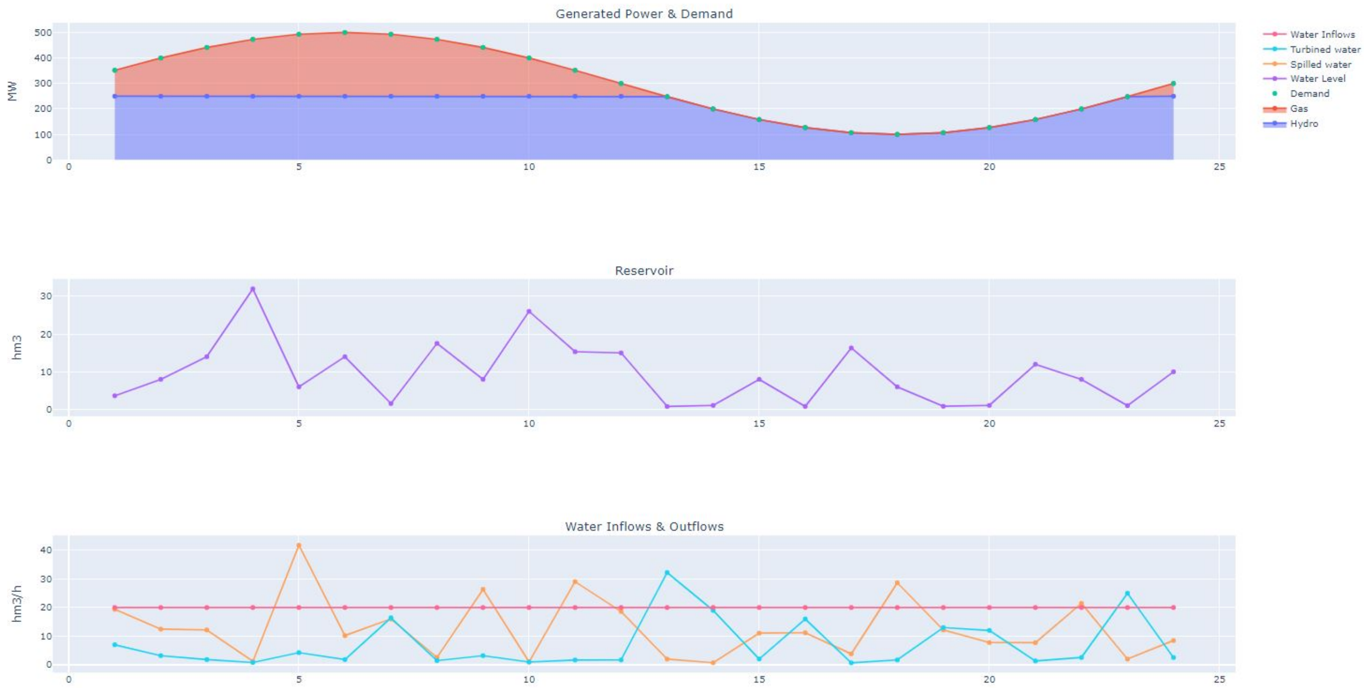

Figure 5: Plot of the mathematical space subject to minimization or maximization

4.3 Conclutions

Dual variables are not accessible due to the use of binary variables on Gurobi. The paper implementation x,y solution is swap.

References

Huchette, J., & Vielma, J. P. (2017). *Nonconvex piecewise linear functions: Advanced formulations*

and simple modeling tools. arXiv. Retrieved from https://arxiv.org/abs/1708.00050
DOI: 10.48550/ARXIV.1708.00050