



## Contextual merit-order dispatch under uncertain supply

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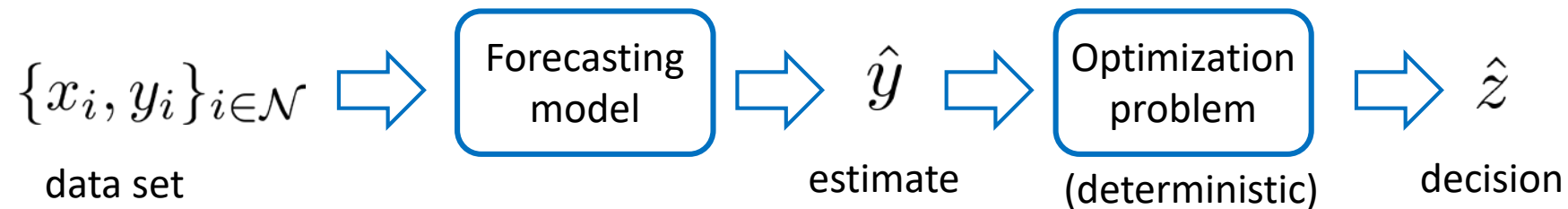


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# Decision making under uncertainty in Power Systems

- Decision making under **uncertainty** plays an important part in **Power Systems**
- Still today one of the most **common practices** is the “**Predict, then Optimize**” framework:

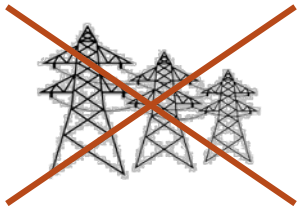


- This is an **approximation** of the **stochastic problem** but is **computationally less expensive**.
- Can be improved if the **forecasting model** takes into account the **optimization problem**.

# A two stage electricity market

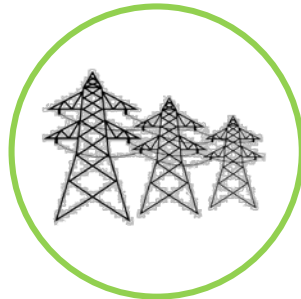
## Demonstrative two-stage electricity market:

**Forward  
market**



(Economic Dispatch)

**Real-time  
market**



(Pipeline network model)

- The uncertainty is produced by the **unknown Net Demand**  $L$
- This setup resembles some European markets

## Demonstrative framework characteristic:

- Reserve capacity is free but their use is not
- Max. reserve capacity fixed
- Stylized pipeline network model
- No wind spillage or load shedding
- No ramping or intertemporal constraints
- We consider Net Demand  $L$  (**uncertain**)



$$\text{Demand} - \text{Renewable} = \text{Net Demand } L$$

# Forward and Real-time market models

## Forward market model

(Economic Dispatch)

$$\begin{aligned} \min_{p_g, g \in G} \sum_{g \in G} C_g p_g \\ \text{s.t. } \sum_{g \in G} p_g = \hat{L} \\ 0 \leq p_g \leq \bar{P}_g, \quad \forall g \in G \end{aligned}$$

$\hat{L}$  is a point estimate

## Real-time market model

$$\begin{aligned} \min_{\Xi} \sum_{g=1}^G (C_g^u r_g^u - C_g^d r_g^d) \\ \text{s.t. } 0 \leq p_g^* + r_g^u - r_g^d \leq \bar{P}_g, \quad \forall g \in G \\ 0 \leq r_g^u \leq R_g^u, \quad \forall g \in G \\ 0 \leq r_g^d \leq R_g^d, \quad \forall g \in G \\ \sum_{g \in G(b)} (p_g^* + r_g^u - r_g^d) = \sum_{d \in D(b)} L_{di} + \sum_{l: o(l)=b} f_l - \sum_{l: e(l)=b} f_l, \quad \forall b \in B \\ |f_l| \leq \bar{F}_l, \quad \forall l \in \Lambda \end{aligned}$$

# Improving the framework

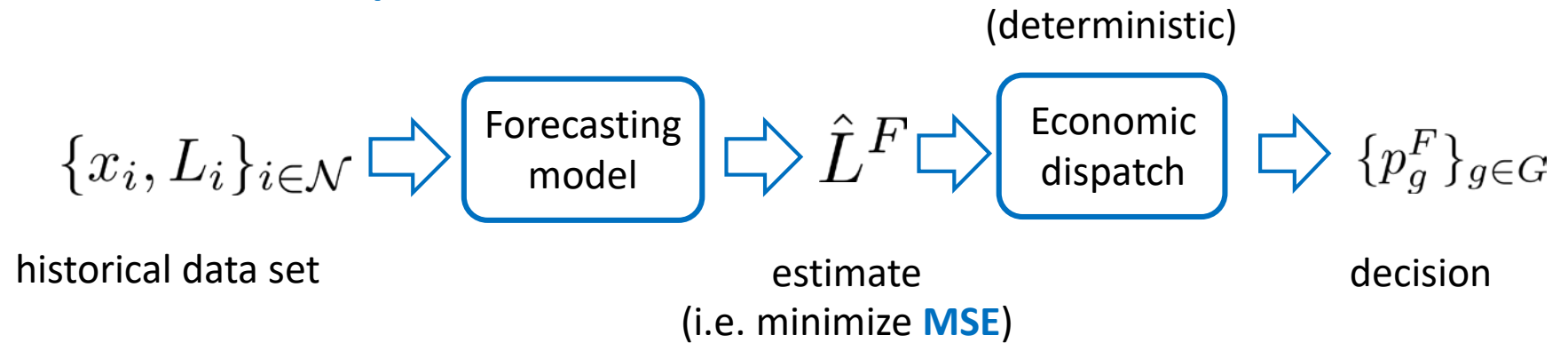
## Forward market model

(Economic Dispatch)

$$\begin{aligned} \min_{p_g, g \in G} \sum_{g \in G} C_g p_g \\ \text{s.t. } \sum_{g \in G} p_g = \hat{L} \\ 0 \leq p_g \leq \bar{P}_g, \quad \forall g \in G \end{aligned}$$

$\hat{L}$  is a point estimate  
 $\{p_g\}_{g \in G}$  dispatch

## Predict, then Optimize:



## Our target:

- Learn  $\hat{L}$  as a linear function  $g(x)$  of the features:  $\hat{L} = g(x) = \sum_j q^j x^j$
- When **learning** consider both **forward** and **real-time market** problems
- More specifically, we consider:  $\hat{L}^P = \underline{q_0} + \underline{q_1} \hat{L}^F$

We learn the  $q$

# Contextual merit-order dispatch model (P-MC)

$$\min_{\mathbf{q}, \Upsilon} \frac{1}{N} \sum_{i \in \mathcal{N}} \sum_{g \in G} (C_g p_{gi} + C_g^u r_{gi}^u - C_g^d r_{gi}^d)$$

Minimize the total cost over the training set of N samples

Forward  
market

$$\text{s.t. } \sum_{g \in G} p_{gi} = \hat{L}_i, \quad \forall i \in \mathcal{N}$$

Forecast

$$\hat{L}_i = q_0 + q_1 \hat{L}_i^F, \quad \forall i \in \mathcal{N}$$

Forward Energy balance

Real-time  
market

$$0 \leq p_{gi} + r_{gi}^u - r_{gi}^d \leq \bar{P}_{gi}, \quad \forall i \in \mathcal{N}, \quad \forall g \in G$$

$$0 \leq r_{gi}^u \leq R_g^u, \quad \forall i \in \mathcal{N}, \quad \forall g \in G$$

$$0 \leq r_{gi}^d \leq R_g^d, \quad \forall i \in \mathcal{N}, \quad \forall g \in G$$

True value

Reserve constraints

$$\sum_{g \in G(b)} (p_{gi} + r_{gi}^u - r_{gi}^d) = \sum_{d \in D(b)} L_{di} + \sum_{l: o(l)=b} f_{li} - \sum_{l: e(l)=b} f_{li}, \quad \forall i \in \mathcal{N}, \quad \forall b \in B$$

$$|f_{li}| \leq \bar{F}_l, \quad \forall i \in \mathcal{N}, \quad \forall l \in \Lambda$$

Network constraints  
(pipeline)

$$u_{gi} \bar{P}_g \leq p_{gi} \leq u_{(g-1)i} \bar{P}_g, \quad \forall i \in \mathcal{N}, \quad \forall g \in G : g > 1$$

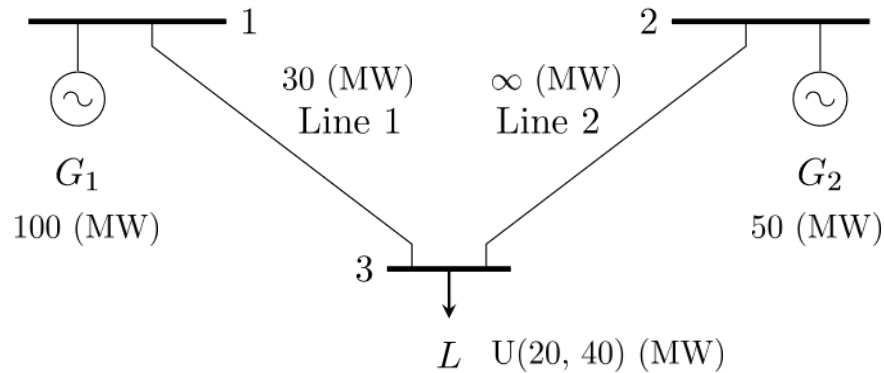
$$u_{gi} \bar{P}_g \leq p_{gi} \leq \bar{P}_g, \quad \forall i \in \mathcal{N}, \quad g = 1$$

$$u_{(g-1)i} \leq u_{gi}, \quad \forall i \in \mathcal{N}, \quad \forall g \in G : g > 1$$

$$\underline{u_{gi}} \in \{0, 1\}, \quad \forall i \in \mathcal{N}, \quad \forall g \in G$$

Generator's merit order  
(cheaper first)

# Toy example (I) introduction



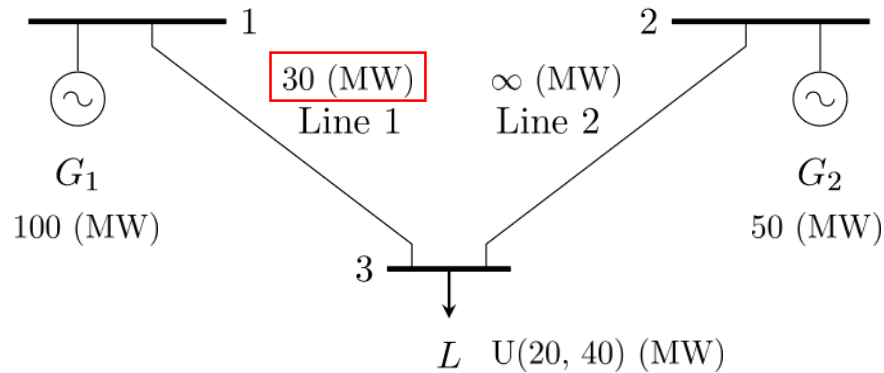
	$C$	$C^u$	$C^d$	
$G_1$	5	30	-20	(€/ MW)
$G_2$	15	20	10	

- Reserve up and down equals the max. capacity:  $\bar{P} = R^u = R^d$
- We have available a **forecast**  $\hat{L}^F$  that minimizes the **MSE**.
- We can use this **forecast as** the only **feature**:  $x = \hat{L}^F$

Joint distribution  $d(L^F, L)$

Probability (%)	$L^F$ (MW)	$L$ (MW)
50%	25	U(20, 30)
50%	35	U(30, 40)

# Toy example (II) results



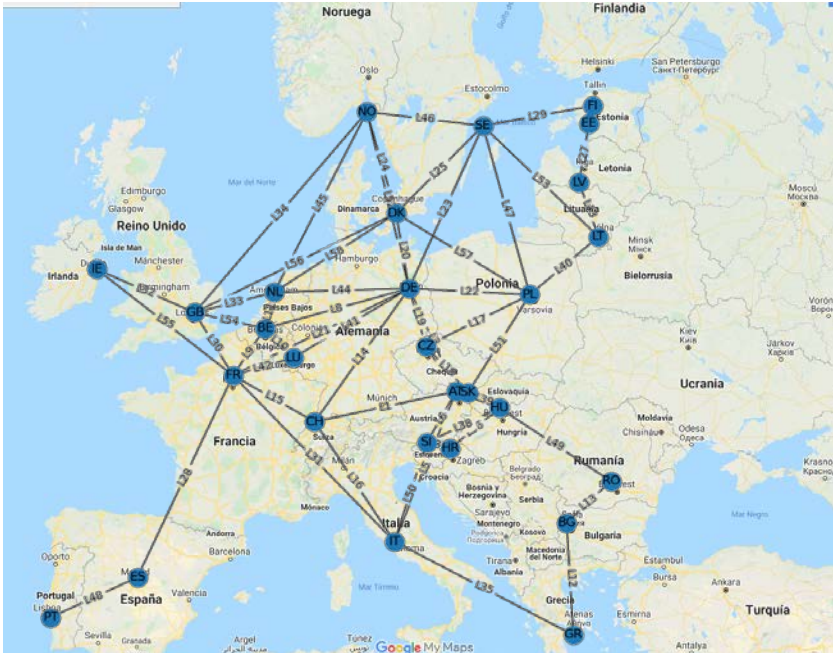
	$C$	$C^u$	$C^d$	
$G_1$	5	30	-20	(€/ MW)
$G_2$	15	20	10	

Probability (%)	$\hat{L}^F$ (MW)	$L$ (MW)	$\hat{L}^P$ (MW)	Improvement %
50%	25	U(20, 30)	22.8	5.77%
50%	35	U(30, 40)	30.0	35.54%

- The minimizer of the **Mean Square Error** is not the best estimator under the “**Predict, then Optimize**” framework
- **P-MC** model produces an estimate  $\hat{L}^P$  tailored to the two-stage market **optimization problem**.



# Case study: stylized European network model



- Stylized **European pipeline network model**
- 28 nodes, 1 node per country
- Line and generation capacities from the report (\*)
- 1 base and 1 peak dispatchable generator per node
  - **Base**: cheap non-flexible (Nuclear, Hard coal, Oil ...\*)
  - **Peak**: expensive but flexible (Natural Gas, Waste ...\*)
- Historical data of demand and renewables from **entsoe**
- We compute the aggregated net demand of the whole system
- Training / test set 100 / 50 samples, 10 different data sets

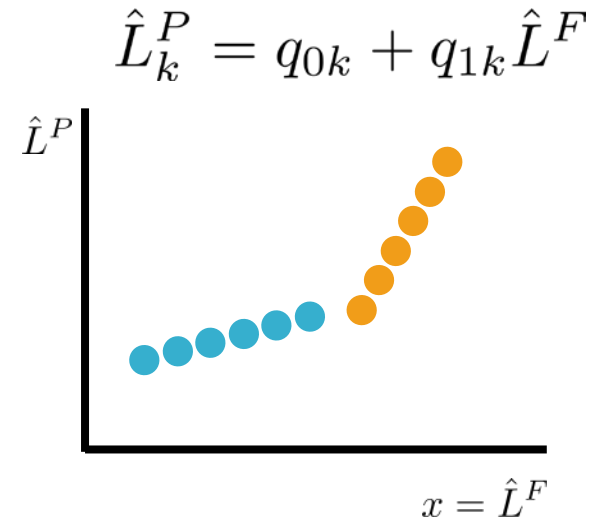
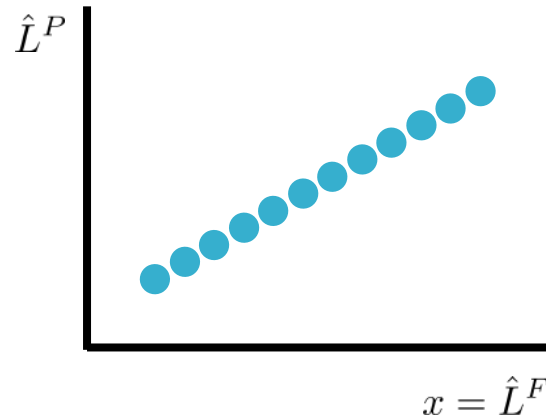
(\*) Nahmmacher, P., Schmid, E., & Knopf, B. (2014). *Documentation of LIMES-EU – A long-term electricity system model for Europe*. Potsdam Institute of Climate Impact Research (PIK)

# Clustering techniques to improve P-MC

## Technique 1:

### K-Means

Euclidian distance



Split the data set in several subsamples

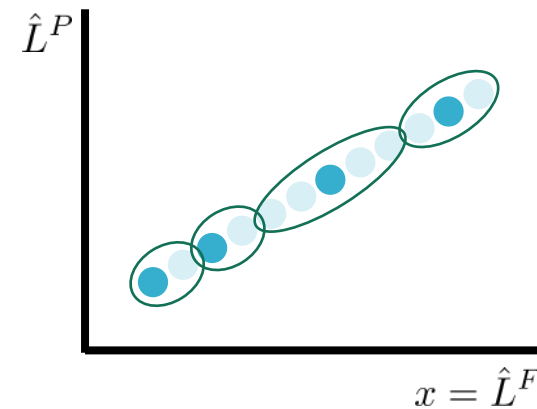
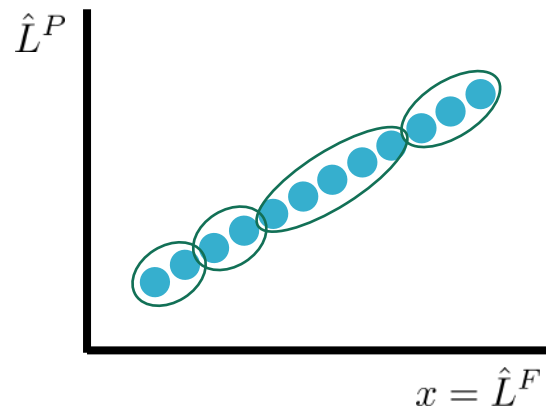


**Increase flexibility**

## Technique 2:

### K-Medoids

PAM



Find the most representative samples



**Reduce computational burden**

# Numerical results

We use as **benchmark** the aggregated forecast (**net demand**) issued by European TSOs (**F-MC**)

## Cost saving results (%) (Improvement of P-MC)

## Computational time P-MC (seconds)

k-Means									
$K$		1	2	5	7	1	2	5	7
$r$	k-Medoids 100%	2.83%	4.29%	4.74%	4.75%	2127.7	283.7	75.9	28.0
	50%	2.67%	4.23%	4.39%	4.06%	180.0	27.2	7.4	5.5
	20%	2.38%	4.12%	4.12%	3.97%	8.3	3.2	1.1	1.4

**K: number of clusters** the data set is divided in through k-Means

**r: sample reduction** through the k-Medoids technique

**We achieve a substantial cost reduction with respect to the F-MC benchmark**

# Conclusions

- Our model prescribe the net demand that a two-stage electricity market should clear in order to minimize the expected total cost of the system.
- Our simulations showcase the benefits of using several cluster techniques to face different operating points of the system.
- Numerical experiments conducted on a stylized model of the European electricity market reveal the cost savings implied by this approach are well above 2%.



# THANKS!

Checkout more at:



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Morales, Juan M., **Miguel A. Muñoz**, and Salvador Pineda. "*Prescribing net demand for electricity market clearing.*" arXiv preprint arXiv:2108.01003 (2021).

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