UNIVERSIDAD DE CASTILLA-LA MANCHA

DEPARTAMENTO DE INGENIERÍA ELÉCTRICA,
ELECTRÓNICA, AUTOMÁTICA Y COMUNICACIONES

MEDIUM-TERM ELECTRICITY TRADING FOR RISK-AVERSE POWER PRODUCERS VIA STOCHASTIC PROGRAMMING

TESIS DOCTORAL

AUTOR: SALVADOR PINEDA MORENTE

DIRECTOR: ANTONIO J. CONEJO NAVARRO

Ciudad Real, Mayo de 2011

UNIVERSIDAD DE CASTILLA-LA MANCHA

DEPARTMENT OF ELECTRICAL ENGINEERING

MEDIUM-TERM ELECTRICITY TRADING FOR RISK-AVERSE POWER PRODUCERS VIA STOCHASTIC PROGRAMMING

PhD THESIS

AUTHOR: SALVADOR PINEDA MORENTE

SUPERVISOR: ANTONIO J. CONEJO NAVARRO

Ciudad Real, May 2011

A mis padres, Salvador y Koky

Acknowledgments

This dissertation would not have been possible without the support and inestimable help of the following people and institutions.

First, I would like to express my deepest gratitude to Professor Antonio J. Conejo for his invaluable direction and supervision work, for having always time for discussion despite his busy schedule, for his patience and professionalism, and above all, for teaching me all the things that need to be done to become (someday) a *good* researcher.

I wish to thank the Ministerio de Educación y Ciencia for its financial support through the BES-2007-17102 grant of "Formación de Personal Investigador FPI" associated with project DPI2006-08001. This grant also enabled me to undertake part of my PhD studies at the University of Texas in Austin (USA) and at the Norwegian University of Science and Technology in Trondheim (Norway).

I wish to thank the Universidad de Castilla-La Mancha for allowing me the use of its facilities and for the financial support by means of the "Ayudas a la Investigación para la realización de Tesis Doctorales".

It was an honor for me to spend three months with Professor Ross Baldick and his research group at the University of Texas in Austin (USA) from February through April 2009. During this stay, he did not only make relevant suggestions about the work developed in this dissertation, but also included me in inspiring discussion about electricity markets.

I also very much appreciate the support and input I received from the research group headed by Professor Stein-Erik Fleten at the Norwegian University of Science and Technology in Trondheim (Norway) from February through April 2010. No doubt I benefited from the intense and productive discussions I had with various people there. And, of course, thanks for teaching me cross-country skiing.

I am also indebted to many of my colleagues at the Escuela Ténica Superior de Ingenieros Industriales of Ciudad Real for their support and friendship. To Natalia and Jose Manuel, for helping me to organize "juergas andaluzas" when needed. To Raquel, for proving me that Nefertiti was born in London. To Miguel and Rafa, for introducing me into the subtleties of the "manchega" lifestyle. To Juanmi, foor helpping my to impruve the inglishh of tis diserttation.

To Virginia, for having always time to go for an “illegal”. To Carlos, for surviving fours years sharing the office with me. To Cesar, for completing our Big Bang Theory. To Lina, for always knowing a good gossip. To Edu, for being the Bill Gates of the lab. And of course, I could never forget the great moments I spent during these years with Amparo, Luzmi, Mayte, Jalal, Luis, Sergio, Ricardo, Fran, Victor Hugo, Claudia, etc.

Me gustara también aprovechar esta oportunidad para decirle a mi familia que todo esto no habría sido posible sin su apoyo. A mis padres Salvador y Koky, por llevar casi tres décadas mostrándome incesantemente el camino a seguir. Por estar ahí siempre que os necesito y, por supuesto, por hacer el esfuerzo de entender de qué va todo esto del doctorado y la tesis. A mis hermanas María y Cristina, por, cada una a su manera, quererme incondicionalmente. A Amparo, por ser una persona tan especial en mi vida. A mis abuelos, por enseñarme que hay cosas que solo se aprenden con los años.

Por último, me gustaría agraceder a mis amigos por ayudarme a desconectar de la programación estocástica y el portfolio selection a base de risas y buenos momentos. A Juanmi, por compartir tantas cosas conmigo y por estar ahí siempre, “dude, you are the best!”. A Ampi, por aportarme tanto con tan poco. To Sarah, for filling my life with joy and love. A Pei, Alberto, Estela, Pepe, Estela, por hacer de la amistad algo que realmente merece la pena.

Contents

List of Figures

List of Tables

Notation

The main notation used in this thesis is listed below. Other symbols are defined as required in the text.

Indices and Numbers

b	Index of power blocks of the piecewise linear production cost function of the generating unit, running from 1 to N_B.
c	Index of forward contracts spanning the decision-making horizon, running from 1 to N_C.
c_1	Index of forward contracts spanning period 1, running from 1 to N_{C_1}.
c_2	Index of forward contracts spanning period 2, running from 1 to N_{C_2}.
i	Index of generating units owned by the power producer, running from 1 to N_I.
o	Index of call/put option contracts spanning the decision-making horizon, running from 1 to N_O.
t	Index of time steps spanning the decision-making horizon, running from 1 to N_T.
ω	Index of scenarios, running from 1 to N_Ω.

Real Variables

$C^{\mathrm{G}}_{(\omega)}$	Total production cost incurred by the power producer in scenario ω (€).

$C^{\mathrm{G}}_{(i,\omega,t)}$ Production cost incurred by generating unit i in time step t and scenario ω (€).

$D_{(\omega)}$ Payment received from the insurance contract in scenario ω (€).

$P_{(c)}$ Power sold through forward contract c (MW).

$P^{1}_{(c_1)}$ Power sold through forward contract c_1 at stage 1 (MW).

$P^{1}_{(c_2)}$ Power sold through forward contract c_2 at stage 1 (MW).

$P^{2}_{(c_2,\omega)}$ Power sold through forward contract c_2 at stage 2 in scenario ω (MW).

P^{D} Power traded through derivatives (MW).

$P^{\mathrm{G}}_{(i,\omega,t)}$ Power produced by generating unit i in time step t and scenario ω (MW).

$P^{\mathrm{G}}_{(i,\omega,t,b)}$ Power generated from the bth power block of the piecewise linear production cost function of generating unit i in time step t and scenario ω (MW).

$P_{(o)}$ Power traded through put/call option contract o (MW).

$P^{\mathrm{P}}_{(\omega,t)}$ Power sold in the pool by the power producer in time step t and scenario ω (MW).

ζ Auxiliary variable used to calculate the CVaR of the insurer profit probability distribution (€).

$\eta_{(\omega)}$ Auxiliary variable related to scenario ω and used to calculate the CVaR of the producer profit probability distribution (€).

ξ Auxiliary variable used to calculate the CVaR of the producer profit probability distribution (€).

$\rho_{(w)}$ Auxiliary variable related to scenario ω and used to calculate the CVaR of the insurer profit probability distribution (€).

$\Pi_{(\omega)}$ Total producer profit in scenario ω (€).

$\Pi^{\mathrm{P}}_{(\omega)}$ Revenue of the power producer due to the trading activity in the pool in scenario ω (€).

Π^{F} Revenue of the power producer due to forward contracting (€).

$\Pi^{\mathrm{I}}_{(\omega)}$ Revenue of the power producer due to insurance contracting in scenario ω (€).

$\Pi^{\mathrm{S}}_{(\omega)}$ Total insurer profit in scenario ω (€).

$\Pi^{\mathrm{O}}_{(\omega)}$ Revenue of the power producer due to option contracting in scenario ω (€).

$\Pi^{\mathrm{D}}_{(\omega)}$ Revenue of the power producer due to derivatives contracting in scenario ω (€).

Binary Variables

s^{P} Binary variable that is equal to 1 if the power producer signs an insurance against unexpected unit failures, and 0 otherwise.

s^{S} Binary variable that is equal to 1 if the insurer signs an insurance against unexpected unit failures, and 0 otherwise.

$u_{(i,\omega,t)}$ Binary variable that is equal to 1 if generating unit i is online during time step t and scenario ω, and 0 otherwise.

$y_{(o,\omega)}$ Binary variable that is equal to 1 if the producer exercises the option o in scenario ω, and 0 otherwise.

Random Parameters

$k_{(i,t)}$ Availability of generating unit i in time step t (1 if available and 0 otherwise).

$\lambda^{\mathrm{P}}_{(t)}$ Pool price in time step t (€/MWh).

$\lambda^{2}_{(c_2)}$ Energy price of forward contract c_2 at stage 2 (€/MWh).

Constants

$A_{(i)}$ Coefficient of the piecewise linear production cost function of generating unit i (€/h).

$l_{(o)}$ Parameter that is equal to +1 if option o is a put option, and to −1 if option o is a call option instead.

$L_{(c)}$ Duration of forward contract c (h).

$L_{(c_1)}$ Duration of forward contract c_1 (h).

$L_{(c_2)}$ Duration of forward contract c_2 (h).

$L_{(t)}$ Duration of time step t (h).

$L_{(o)}$ Duration of put/call option contract o (h).

M^{I} Premium of the insurance contract (€).

P^{I} Power level of the insurance contract (MW).

$P^{\mathrm{Min}}_{(i)}$ Minimum power output of generating unit i (MW).

$P^{\mathrm{Max}}_{(i)}$ Capacity of generating unit i (MW).

$P^{\mathrm{Max}}_{(i,b)}$ Size of the bth power block of the piecewise linear production cost function of generating unit i (MW).

α^{P} Risk aversion level of the power producer.

α^{S} Risk aversion level of the insurer.

$\lambda_{(c)}$ Energy price of forward contract c (€/MWh).

$\lambda^{1}_{(c_1)}$ Energy price of forward contract c_1 at stage 1 (€/MWh).

$\lambda^{1}_{(c_2)}$ Energy price of forward contract c_2 at stage 1 (€/MWh).

$\lambda_{(i,b)}$ Slope of the bth power block of the piecewise linear production cost function of generating unit i (MW).

λ^{I} Strike price of the insurance contract (€/MWh).

$\lambda^{\mathrm{O}}_{(o)}$ Option price of put/call option contract o (€/MWh).

$\lambda^{\mathrm{S}}_{(o)}$ Strike price of put/call option contract o (€/MWh).

$\pi_{(\omega)}$ Probability of occurrence of scenario ω.

Sets

$F_{(t)}$ Set of forward contracts c available during time step t.

$F^1_{(t)}$ Set of forward contracts c_1 available during time step t.

$F^2_{(t)}$ Set of forward contracts c_2 available during time step t.

$G_{(\omega)}$ Set of time steps in scenario ω in which both the pool price is higher than the strike price of the insurance contract and the generating unit is forced out.

$O_{(t)}$ Set of option contracts available during time step t.

$S_{(\omega)}$ Set of scenarios whose uncertain parameter values during period 1 are equal to those associated with scenario ω.

Acronyms

CVaR Conditional Value-at-Risk.

EP Expected Profit.

FOR Forced Outage Rate.

MTTF Mean Time To Failure.

MTTR Mean Time To Repair.

VaR Value-at-Risk.

Chapter 1

Introduction

1.1 Risk Everywhere

In our daily life, we make decisions facing uncertain information. For example, when leaving home we have to decide whether or not to take our umbrellas without being sure if it will rain, what time we will drive to our destination to avoid traffic jams, or even if we should fill up the tanks of our cars today or wait until tomorrow in the hope that the gas price will decrease. In all these situations, we face some kind of *risk* affecting our objective which can be described as not to get wet, to avoid traffic jams, or to save some money, respectively. Considering the above targets, we can adopt a high conservative position (to take our umbrellas with us, to get up very early to drive to our destinations, or to fill up our cars today to avoid the possibility of paying more as a result of an increase in the gas price), or a risky one (to leave the umbrellas at home, to stay in bed and driving to our destinations at the same time everybody else would probably do, or to wait until tomorrow to fill up our cars).

In a larger scale, most of the decisions in economic activities have to be made without full knowledge of the information involved. This lack of information translates into risk, which is defined as the deviation of the actual outcome from its desired value. Therefore, for proper decision making, the uncertainty affecting our objective has to be represented using probabilistic models. Once this is done, we have to select an appropriate measure to quantify the risk caused by the unknown information. The final step consists in designing a mathematical model to identify optimal decisions taking into account the risk involved as well as its impact on our target.

The wrong consideration of any of these steps may lead to serious consequences. For example, relevant economists around the world state that the current financial crisis has been a result of inappropriate risk management.

Within this framework, decision-making problems related to electricity markets are not exempt from risk, but quite the opposite. The particular physical characteristics of electricity together with the still ongoing development of electricity markets turn risk management into a very outstanding issue.

1.2 Electricity Markets

In this section, we present a brief description of the electricity market organization and the market agents, focusing on the main trading floors to buy and sell electricity. The purpose here is to explain how competition entails different types of risk in electricity markets, as well as to describe the available instruments to hedge against these risks.

1.2.1 Market Agents

We succinctly describe below the main agents participating in electricity markets as well as the Independent System Operator (ISO) and the Market Operator (MO):

- *Power producers*, which are the entities owning the generating units, thus being in charge of the electricity production. A producer can sell electricity either to the electricity market (pool and derivatives market) or directly to consumers or retailers through bilateral contracts. The objective of the producers is to attain the maximum profit from the sale of electricity.

- *Retailers*, also known as marketers, which are entities that purchase electricity to sell it to those consumers that do not participate directly in the electricity market. The objective of the retailers is to maximize the profit obtained from selling electricity to their costumers.

- *Consumers*, which are entities that purchase electricity either directly from power producers or retailers through bilateral contracts or in the pool in order to supply their respective demands.

- *Independent System Operator* (ISO), which is an entity in charge of the technical management of the power system. The Independent System Operator must provide equal access to the grid to all consumers, retailers, and producers.

- *Market Operator* (MO), which is responsible for the financial management of the system. Roughly, the market operator collects electric energy production offers and purchase bids, settles prices, and assigns energy production/consumption levels to each generating company, consumer, and retailer.

1.2.2 Pool

The pool generally comprises a day-ahead market and several shorter-term markets known as adjustment markets. The energy traded in the pool is mostly negotiated in the day-ahead market, while adjustment markets are used to make final adjustments to the amount of energy cleared in the day-ahead market. In the pool, producers submit offers involving a series of energy blocks and their corresponding minimum selling prices for every hour of the market horizon and every generating unit. At the same time, retailers and consumers submit bids involving a set of energy blocks and their corresponding maximum buying prices for every hour of the market horizon.

The Market Operator collects purchase bids and sale offers and clears the market using a market-clearing procedure. This procedure results in market-clearing prices, as well as production and consumption schedules. In this dissertation, we consider a single hourly market-clearing price, which is referred to as pool price.

The pool price exhibits high volatility in most day-ahead markets around the world due to the non-storability character of electricity, the high variation of the demand level with the hour of the day, the day of the week, and the week of the year; the inelasticity of the electricity demand, the stepwise price offers submitted by generating units, and the required real-time electricity balance between production and consumption. The high volatility of the electricity pool price constitutes one of the major risk factors affecting agents participating in electricity markets, and is referred to as *price risk*.

1.2.3 Derivatives Market

A derivative can be defined as a generic term to describe a wide variety of financial instruments whose values *derive* from the price of one or more underlying assets [125]. Derivatives products include standardized, exchange-listed products, and over-the-counter instruments, and can be used to either increase or decrease risk. Basic derivatives instruments fall into two categories: forward-based, such as forward contracts and futures; and option-based, including put and call options, exotic options, etc.

Derivatives can be mainly traded in two different ways:

- In an *exchange* market, where market agents can trade pre-defined and standardized derivatives contracts. The key advantage of exchange markets is that each agent trades with a central authority, without knowing the party it is trading with in fact. Therefore, trading in an exchange market is free of the counterparty risk, which is the risk of not being paid by the contract counterparty. In order to avoid the counterparty risk, market agents taking part in an exchange market are required to pay a margin fee that ensures their corresponding payments. The downside of trading in these markets lies on the higher transaction cost to be paid by the participants, which is due to the margin fee imposed by the exchange. Among the most developed exchange markets for electricity derivatives in Europe, we highlight NordPool in Scandinavia [1], EEX in Germany [2], and OMIP in the Iberian Peninsula [3].

- *Over-the-counter* (OTC), which refers to contracts privately traded between two parties without going through an exchange market or any other intermediary. The advantages of OTC derivatives are the flexibility provided to market participants to design products that suit their specific needs, and the smaller transaction cost. On the other hand, trading derivatives over-the-counter involves a significant counterparty risk. However, this risk can be logically reduced by trading with participants with enough financial stability.

Most derivatives can be traded either in an exchange or over-the-counter in electricity markets. Usually, the number of contracts traded in the exchange is higher than those traded over-the-counter. In contrast, the electricity volume traded in OTC exceeds that negotiated in the exchange market. For instance, in the European Energy Exchange (EEX), the electricity volume traded during the year 2009 in the exchange market and in OTC were equal to 285 TWh and 740 TWh, respectively.

Next, we define the most common derivatives traded in electricity markets:

- *Electricity forwards*, which are the simplest derivatives and consist in agreements between two parties to buy or sell a given power quantity during a pre-specified future delivery period and at a fixed price. Since the price of a given forward contract is stable as it is pre-specified in advance, a forward contract is an appropriate financial instrument to hedge against the risk related to the inherent volatility of pool prices.

- *Electricity futures*, which, like electricity forwards, are agreements between two parties to buy or sell electricity during a certain future time period at a fixed price. The main difference between forwards and futures is that whereas forward contracts are usually traded over-the-counter (OTC) and are non-standardized products, futures contracts are traded on an exchange where the standard characteristics of the different available products are specified. Moreover, these two types of derivatives differ in the quantity of power to be delivered. Particulary, the quantity associated with electricity futures contracts is often significantly smaller than that in forward contracts.

- *Electricity options*. While forward and futures contracts necessarily imply the sale or purchase of the agreed electricity during their respective delivery periods, electricity options give the right, but not the obligation, to sell or buy a particular amount of electricity during a certain future time period, at a fixed price called *strike price*. On the other hand, the flexibility provided by an electricity option entails an *extra cost*, referred to as *option price*, which has to be paid whether or not the option is exercised.

Most of the derivatives in electricity markets can be settled either physically or financially. For example, a physical forward contract traded over-the-counter necessarily involves the physical delivery of the agreed power level during each hour of the contract period. In this case, the buyer of the forward contract pays to the seller the pre-specified price times the power level times the duration of the contract. On the other hand, a financial forward contract traded in an exchange market is usually cash settled. This way, the buyer of a forward contract pays to or receives from the exchange the difference between the contract price minus the average pool price during the delivery period times the agreed power level times the contract duration. Likewise, the seller of a contract pays to the exchange the corresponding amount if the average pool price is higher than the contract price, and receives such amount otherwise. It is relevant to note that signing a financial forward contract boils down to making a "bet" on the average pool price during a given future period.

Signing either a physical or a financial forward contract is equivalent to certain extent. For example, let us suppose two power producers: producer A and producer B. Producer A acquires a physical forward contract to sell 100 MW of electricity at 20 €/MWh during January. Therefore, it has the obligation to deliver 100 MW during each hour of January, thus obtaining a revenue equal to $20€/\text{MWh} \cdot 100\text{MW} \cdot 31\text{days} \cdot 24\text{h/day} = €1,488,000$. On the other hand, producer B signs a financial forward contract to sell 100 MW at 20 €/MWh during the same period. Then, the contract revenue received by producer B is determined as $(20 - \overline{\lambda}^{\text{P}}) \cdot 100 \cdot 744$ €, where $\overline{\lambda}^{\text{P}}$ is the average pool price during January. In addition, if this producer physically sells 100 MW in the pool during each hour of January, it gets a revenue equal to $\overline{\lambda}^{\text{P}} \cdot 100 \cdot 744$ €. Adding up the revenues obtained by the producer B through the financial forward contract and the pool, we come to the conclusion that both power producers make the same amount of money (€1,488,000) and deliver 100 MW of electricity during every hour of January. Note that this equivalence between physical and financial derivatives is valid provided that there exists a sufficiently liquid pool market. If the pool is not liquid enough, some market agents may acquire physical contracts in order to ensure either the delivery of their respective produced electricity or the supply of their respective electricity needs.

Assuming a pool liquid enough, market agents may decide between physical and financial derivatives based on company policies, tax involved, etc. In this dissertation, we present an energy-oriented study and therefore, all the derivatives considered hereinafter imply the physical delivery of the electricity.

It is worth mentioning that *insurances*, albeit not being derivatives, play an important role for the risk management in electricity markets. Traditionally, insurances have been used in other fields like agriculture, transportation, etc., to hedge against the risk caused by the possibility of suffering a rare event with catastrophic consequences for a company. Basically, an insurance is defined as a transfer of risk from one agent to another in exchange for a given payment known as *premium*. In current electricity markets, and taking into account the wide range of available derivatives, insurance companies have focused their efforts on providing products to protect against very specific risks that are not covered by other derivatives or financial contracts. For instance, some insurance companies offer products for power producers to hedge against the risk caused by unexpected failures of generating units [95].

1.3 Power Producer Risks

Within an electricity market environment, producers make the following decisions:

- *Long-term decisions*: These are the most important decisions made by power producers and comprise a horizon from 2 to 15 years. Basically, in the long run, power producers need to assess the profitability of making investment in new generating facilities within a given power system. The uncertainties affecting these decisions are related to the estimation of future electricity demand, fuel prices, possible network expansion, regulatory changes, and the development of new generating technologies such as renewable sources.

- *Short-term decisions*: Power producers have also to make decisions concerning the immediate future. Short-term decision making varies from the next day to the next 2-3 weeks and primarily involves determining the production planning for each generating unit owned by the power producer, as well as its offering strategy in the pool. Operating characteristics such as physical restrictions of generating units (generating limits, ramping constraints, minimum up- and down-time constraints, etc.) [12], and start-up and shut-down costs have to be accounted for to make short-term decisions.

- *Medium-term decisions*: Once investment decisions have been made, power producers have to determine the optimal operating plans for their generating units. In this sense, they have to decide their participation in electricity markets, such as the derivatives market, as well as the acquisition of new physical or financial instruments to sell their production. Since selling periods of electricity derivatives vary from one week to several years, they can be consistently included in medium-term decision-making models. In these models, long-term decisions of power producers such as the generating portfolio of the power producer (number and type of generating units) as well as the characteristics of the generating units (cost and capacity) are assumed to be fixed. Moreover, and for the sake of simplicity, short-term physical constraints such as ramping limits and minimum up- and down-time requirements are usually disregarded in medium-term horizons. Short-term electricity trading directly impacts the contracting involvement of power producers in the derivatives market and therefore, it has to be accounted for in medium-term decision-making models. In most cases, the short-term trading of power producers is simplified by just considering a day-ahead market.

In this dissertation, we focus on medium-term decision-making models of power producers. Consequently, we examine below the most important risks faced by a power producer in the mid-term prospect [138]:

- *Market risk*, which is defined as the variation of a portfolio value due to changes in the market parameters. This includes the risk related to the interest-rate and foreign exchange fluctuation, as well as the so-called price risk, which refers to the risk caused by the volatility of the price of a commodity, i.e., the electricity in the case we are dealing with.

- *Operational risk*, which refers to potential losses involving the people, systems, and processes through which a company operates. For instance, from a power producer perspective, the unexpected failure of a generating unit is a form of operational risk known as availability risk.

- *Volume risk*, which is defined as the risk originated from the unpredictable nature of the future electricity production. As an example, this risk is especially relevant for hydro-power producers.

- *Credit risk*, which is the risk of loss arising from the non-payment of the contract counterparty, and it is also called counterparty risk.

- *Liquidity risk*, which comprises the risk that a market agent may not be able to sell or buy electric energy due to inadequate market depth, i.e., the risk associated with insufficient transaction volume.

- *Regulatory risk*, which covers all kind of risks caused by unexpected changes of regulation by the administration that may affect the company financial results.

Note that the risks above presented are common for most of the commodity markets. In this dissertation, however, we only consider those types that are of special interest in electricity markets, and more particularly to power producers. First, we deal with the risk related to the electricity pool price which, given the special physical characteristics of electricity as a commodity, exhibits the highest volatility among all commodities traded in energy markets. Henceforth, we refer to this risk as *price risk*. Second, we come with the risk associated to unexpected failures of generating units, also called *availability risk*, which is of especial interest for power producers due to the non-storability of electricity. We consider these two risks for two reasons: their major impact on the power producer profit and the fact that they can be efficiently managed using the wide variety of available derivatives and financial contracts in the electricity sector.

1.4 Motivation

Current electricity markets are evolving towards organizations in which the competition among different market players is present in almost all their facets with the ultimate aim of reducing the electricity cost for end-consumers. However, this competitive framework has led the market players to reconsider the way they understand, measure, and manage the risk affecting their respective profit distributions. This change is especially relevant for power producers since they have to decide their most effective strategies for long, medium, and short-term decision making.

While making decisions in competitive electricity markets, one of the major concerns involving electricity producers is to identify the different risks that they have to face. Once these risks are identified, the decision makers must design mathematical models able to characterize the uncertain parameters pertaining to each risk involved.

The next step is one of the most important and complex, and consists in determining how those risks impact the producer objectives. At this stage, two different, although related tasks, have to be carried out. First, an appropriate risk measure that quantifies the effect of the risks on the profit distribution of the power producer has to be selected. Second, and depending on the selected risk measure, the power producer must define such an abstract index as the risk aversion of the company. In general terms, the risk aversion can be defined as the reluctance of a company to make decisions that, with a certain probability, may lead to relatively unfavorable outcomes.

Once the different sources of risk are identified and characterized, and an appropriate risk measure is selected to quantify them, the power producer has to evaluate the alternative strategies to hedge against those risks. In this respect, the correct understanding and modeling of derivatives and insurances available in electricity markets becomes an important task. Finally, the power producer has to build a mathematical model to decide the most appropriate strategy to optimize the profit objectives of the company in accordance with its risk aversion level.

1.5 Literature review

This section provides a review of the most relevant technical literature to this thesis. First, references regarding the trading of derivatives in electricity markets are included. Subsequently, relevant references on stochastic programming, scenario generation and reduction, and risk management are presented. Finally, references on decision-making models for power producers are provided.

1.5.1 Electricity Markets and Derivatives

Since the restructuring process of the electricity sector began in the early 1990s, several authors have analyzed its causes and consequences [109,163]. One of the major changes is the inclusion of competition in the electricity business, giving rise to new types of risks to be faced by market participants. In reference [7], the main risks involved in competitive markets are presented. More specifically, references [47, 116] present an assessment of the risks pertaining to electricity markets.

Within a competitive environment, the special features of electricity as a commodity lead to a significantly high volatility of the resulting market price. The modeling of electricity prices in a competitive environment is analyzed in [34]. In this vein, the literature is rich on papers addressing the modeling and forecasting of electricity pool prices. Reference [43] describes three techniques to forecast electricity pool prices that comprise ARIMA, dynamic regression and transfer function models. A transfer function model to predict electricity prices based on both past electricity prices and demands is provided in [129]. References [46, 130] propose forecasting tools based on time series analysis [33, 139]. Finally, references [137, 170] propose artificial neural network approaches to forecast electricity pool prices.

The key economic and financial concepts regarding derivatives markets can be found in [93, 94, 183]. More specifically, references [52, 109, 125] provide a characterization of the derivatives traded in electricity markets.

Parallel to the emergence and trading of electricity derivatives around the world, several works study and analyze their performance in electricity markets. Reference [174] studies the application of the theory of financial markets to electricity markets. Reference [106] provides a negotiation framework for bilateral contracting in electricity markets. The role and efficiency of forward contracts in the electricity market of Australia is analyzed in [9, 121, 186]; the efficiency of the Iberian electricity futures market is evaluated in [36]; using the Nordic Power Exchange as the test market, references [31, 122] study the valuation of the electricity derivatives, the authors in [148] analyze the characteristics of the long-term electricity forward price series using a vector autoregressive model, and [66] proposes a method for constructing approximated forward price curves; in reference [164] the authors investigate the statistical properties of the wholesale electricity spot and futures prices traded on the New York Mercantile Exchange; the pricing of electricity futures at the European Energy Exchange (EEX) is analyzed in [35,184]; the impact of the futures market in England and Wales is discussed in [75]. Reference [39] assesses the effects of firm physical bilateral agreements on the wholesale market prices of the Italian system. Finally, it is worth mentioning reference [117], which presents a survey on physical delivery vs. cash settlement in futures contracts.

Regarding electricity derivatives, experts are mostly concerned about the relationship between the futures and spot prices, which is usually referred to as risk premium. In [20, 55, 144], the authors investigate the presence of risk premium in the principal European energy markets. Reference [22] proposes an equilibrium model to explain the risk premium in the PJM electricity market, and in [119] the authors conduct an empirical analysis of forward prices in this market. The information embedded in the difference between the forward and spot prices in the Spanish electricity market is analyzed in [71].

1.5.2 Decision Making under Uncertainty

The competitive market framework together with the wide variety of derivatives available in electricity markets have changed the modus operandi of electricity market agents, especially that of power producers. In this line, references [16, 63, 118, 133, 179, 180] propose the use of sophisticated mathematical models for market agents to make decisions under uncertainty in electricity markets.

1.5.2.1 Stochastic Programming

One of the main mathematical tools used to model decision-making under uncertainty is stochastic programming. The basic concepts and procedures of stochastic programming can be found in [28, 48, 89, 99–101, 149, 161]. In order to solve a stochastic mathematical programming problem, a deterministic equivalent model is usually needed. References [177] and [181] provide the deterministic equivalent for a stochastic programming problem with probabilistic criteria or with fixed recourse, respectively. In [120], the authors analyze the structure of stochastic integer programming. The application of decomposition techniques to stochastic programming is described in [90]. Reference [156] elaborates on the stability of stochastic programming problems. Relevant features of the multi-stage stochastic programming are discussed in [72, 87, 112, 176]. In reference [59], the authors compare the multi-stage stochastic programming with stochastic dynamic programs with discrete time. An appraisal of approximation and solution methods for multi-stage linear stochastic programming problems is presented in [111]. Some important indexes to quantify the quality of the optimal solution of a stochastic programming problem are defined and discussed in [27, 62, 64]. Some general uses of stochastic programming are reviewed in [178], while reference [189] describes some of the most relevant applications of stochastic programming in financial problems. More related to the scope of this thesis, references [19, 21, 182] apply the stochastic programming theory to model specific aspects of electricity markets.

1.5.2.2 Scenario Generation and Reduction

The generation of scenario trees to achieve an adequate representation of the uncertain parameters in stochastic optimization problems has been the subject of recent studies. In [92], Høyland and Wallace propose a method to generate a scenario tree satisfying specific statistical properties. A scenario tree generation method based on the clustering of scenarios is proposed in [57]. The work developed in [58, 77] constitute relevant references on scenario reduction within a stochastic programming framework. Moreover, Heitsch and Römisch have significantly contributed to scenario generation and reduction techniques through their publications on this topic [83–86]. The influence of the probability distance definition on the performance of scenario reduction procedure is analyzed in [88]. Kaut and Wallace evaluate in [102] the different methods to reduce scenario sets in stochastic programming. In [91], the authors propose a heuristic procedure to generate a scenario tree characterizing multidimensional distributions. Reference [131] presents algorithms to generate scenarios particularly useful in electricity market-related problems. Some applications of scenario generation to financial optimization are illustrated in [79, 141].

1.5.2.3 Risk Management

The way in which risk is quantified is equally relevant when making decisions under uncertainty. The main concepts and definitions regarding risk measures can be found in [142,150,151]. The first quantitative treatment of the tradeoff between profit and risk, called mean-variance portfolio analysis, was introduced by Nobel laureate Henry M. Markowitz in [124] and revisited in [169]. This approach measures risk as the profit variance. In [13], Artzner et al. introduce a set of four desirable properties for risk measures. Thus, any risk index satisfying these four properties is said to be coherent. Subsequently, Föllmer and Schied presents in [68] the concept of a convex measure of risk as an extension of the concept of a coherent risk measure. The analysis of the Conditional Value-at-Risk as a coherent measure of risk is carried out by Rockafellar in [153–155]. References [140,160] carry out a comparison between the Value-at-Risk and the Conditional Value-at-Risk as alternative risk measures. References [123, 159] provide an analysis of how optimization problems including convex risk measures can be solved. In references [60, 61], Eirnoch and Römish propose a multiperiod extension of the Conditional Value-at-Risk. Finally, reference [69] summarizes of the main risk measures used in finance.

1.5.3 Power Producer Models

Within an electricity market framework, reference [108] presents detailed analysis and characterizations of the decision-making tools that consumers, retailers, and producers need to participate in a competitive electricity market. More specific decision models for large consumers can be found in [38, 42, 143]. Likewise, references [37, 188] propose mathematical models to obtain optimal decisions for retailers.

Power producers are probably the market players that have changed more significantly their decision-making models in the long, medium, and short term within competitive electricity markets. As to the long-term planning for power producers, references [11, 32, 98, 105, 114, 166, 172] describe different investment models. In relation to trading in electricity markets, references [15, 45, 115, 147, 158, 167] present different approaches to derive optimal offers for a producer operating in an electricity spot market. A short-term model for hydro-power producers is described in [65]. Highly topical works are those that deal with the short-term trading of wind power producers in electricity markets, [17,127,146].

1.5.3.1 Forward Contracting

In the medium term, power producers have to decide their involvement in the derivatives market. For example, they have to determine whether its production is sold in the pool at uncertain prices or through forward contracts at fixed prices. Kaye et al. illustrate in their pioneering work [103] the use of forward contracts to hedge against the risk of profit volatility. Reference [29] proposes solutions inspired in the field of financial risk management for electric energy contract evaluation. In [73], the author explores the effect of the strategic behavior of some market agents on forward contracting. Reference [171] provides statistical evidence of the reduction in risk obtained through forward contracts. Reference [138] presents a methodology for optimal hedging of a power portfolio with generation assets and energy contracts. In [78], Guan et al. examine a mean-variance risk management model for power portfolio selection in futures and spot markets. The model proposed in [162] integrates the unit commitment model with financial decision making by including the forwards and spot market activity within the scheduling decision model. The influence of forward contracts on the optimal offering by power producers is discussed in [40, 152]. Reference [14] addresses the optimal involvement in derivatives markets by power producers to hedge against the pool price volatility via a heuristic optimization technique. In [44], Conejo et al. propose a two-stage stochastic programming model to determine the optimal involvement of a power producer in the futures market. The usage of derivatives by hydro-power producers to hedge against both the electricity price risk and the uncertain reservoir level is examined in [30,67,126,165]. Reference [8] concludes that forward contracts reduce the market power of producers in electricity markets.

1.5.3.2 Impact of Unit Failures

Some references, [49, 50, 81], study the impact of forced outages on the offering strategies of power producers in day-ahead markets. Further, in [97], the authors propose a solution approach for the optimal generation investment problem considering the uncertain availability of additional generating units. However, no reference has been found that considers the effect of such forced outages on the optimal involvement of a power producer in electricity derivatives markets.

The characterization of the availability of generating units as a stochastic process requires a basic knowledge of reliability analysis in power systems. In this respect, it is worth mentioning the work by Billinton, [24–26]. References [54, 173] aim to provide the fundamental concepts in reliability engineering. The authors in [18] evaluate the investment in back-up generators to hedge against the adverse effects of power outages.

1.5.3.3 Insurance Contracting

Apart from forward contracts, insurance products constitute an efficient way to evade the risks involved in electricity markets while bringing a new opportunity to insurance companies to expand their business. In [56, 107] insurance contracts are assessed as financial instruments. In these papers, game theory and equilibrium models are used to calculate the parameters defining the insurance contract between the insurer and the insured. References [70, 134] advocate the use of a certain type of insurance for enhancing the reliability of supply to consumers, whose aim is to allocate the risk of forced outages to the distribution provider rather than to the consumer. The introduction of insurance mechanisms to evade market risk and reduce economic losses for generators is analyzed in [82]. In [95, 96], Jiang et al. present an insurance product against financial losses caused by forced outages. The insurance bargaining process between the insurer and the insured is discussed in [175].

1.5.3.4 Option Contracting

Options are also significant derivatives to be considered in mid-term decision making by power producers. In [74] the authors investigate the development of an options market for power trading. Reference [190] shows that options reduce the price risk and allow market participants to increase their potential profits. Since electricity cannot be stored, the well-known Black-Scholes equation [187] is not generally an appropriate method for pricing electricity derivatives. In this context, reference [113] proposes a heuristic algorithm to appraise electricity options. References [10, 152] study the impact of options and forward contracts on the offering strategies of electricity market agents. References [135, 136] discuss the possibility of mitigating the risks faced by retailers using electricity options. The use of an option to buy electricity by large consumers to hedge against price increases is explored in [23]. Reference [41] deals with the design of forward contracts bundled with financial options for electricity market risk management. In [132], Oren et al. propose a model to use electricity options for demand-side management. An analytical framework for the valuation of options contracts for physical delivery that enable risk-sharing among market participants is developed in [168]. The valuation of a rich family of electricity swing options is carried out in [80, 104]. Some relevant references that study real options in electricity markets are [51, 53].

1.6 Problem Description and Solution Methodology

The main purpose of this dissertation is to develop mathematical programming models for the decision making of a power producer within a medium-term horizon. Using these models, a power producer can determine its optimal involvement in the available derivatives market and in the pool.

The power producer is modeled as a price-taker in both the pool and the derivatives market. The generating units owned by the power producer are thermal units, each of them characterized by a quadratic cost function and a maximum and minimum power outputs. Within a medium-term horizon planning, other technical constraints such as ramping limits and minimum up and down times are disregarded for the sake of tractability.

It is relevant to note that power producers have to make decisions regarding the derivatives market without perfect information about some parameters that affect the payoff of those derivatives. Consequently, the appropriate modeling of these uncertain parameters is a key issue in the design of medium-term decision-making tools. Stochastic programming provides an appropriate modeling framework in which problems of decision making under uncertainty are adequately formulated.

Stochastic programming relies on the knowledge of the distribution functions of the uncertain parameters. These distributions can be used to generate a finite set of possible outcomes, referred to as scenarios, which represent plausible realizations of the uncertain parameters with associated weights or probabilities of occurrence. Once the uncertain parameters are characterized by scenario sets, it is generally possible to formulate a mathematical programming problem that identifies optimal decisions according to a given objective function.

In the stochastic optimization problems considered in this dissertation, two sources of uncertainty are taken into account. The first one is the pool price of electricity which, given the special features of the commodity electricity, exhibits a significantly high volatility. Although usually ignored, this dissertation also considers the uncertainty related to the availability of the generating units owned by power producers.

The number of scenarios needed to properly represent an uncertain parameter is usually very large, which may make the resulting stochastic optimization problem computationally intractable. For this reason, scenario reduction techniques are used to trim down the number of scenarios while keeping the statistical properties of the initial scenario set as much as possible. Part of this dissertation is devoted to the development of a scenario reduction technique that efficiently reduces the scenario set representing the pool price and the availability of generating units for decision-making models of power producers.

Market agents may make their decisions in order to obtain the maximum expected profit. However, it might be more appropriate to make decisions that avoid the possibility of obtaining very unfavorable outcomes due to the realization of an adverse scenario. Within a stochastic programming framework, different risk measures to model the variability of the profit distribution can be considered. In the proposed models, we use the Conditional Value-at-Risk for this purpose due to its good mathematical properties.

Using the stochastic programming theory as the mathematical framework, this dissertation is aimed at presenting models to identify optimal decisions of power producers concerning to the derivatives market and the pool. Specifically, the problems tackled in this thesis are:

- To determine the optimal quantity of electricity to be sold through the available forward contracts considering the uncertainty related to the pool price and the availability of the generating units. The risk aversion level of the producer is accounted for in the model.

- To assess the acquisition of an insurance contract against unexpected failures of generating units. To this end, the power producer takes into account the uncertainty of both the pool prices and the availability of generating units, and assumes a specific risk aversion level.

- To appraise the usefulness of the available options to sell or buy electricity to hedge against the risk related to the pool price volatility and the occurrence of unexpected unit failures. Optimal decisions of a power producer regarding electricity options depend on its risk aversion level.

1.7 Thesis Objectives

The general objective of this thesis is to develop a mathematical framework to characterize and quantify the main risks affecting power producers as well as to identify the appropriate strategies to manage them.

The objectives are stated below:

- General objectives:

    1. To formulate a two-stage stochastic programming problem with recourse to model the decision-making process of a power producer selling energy in an electricity market, which incorporates risk aversion using the Conditional Value-at-Risk. In addition, we derive the mixed-integer linear programming equivalent of the proposed model.

2. To formulate a three-stage stochastic programming problem to model the decision-making process of a power producer that sells energy in different points in time throughout the study horizon. This formulation includes the modeling of risk aversion via the Conditional Value-at-Risk.

- Objectives regarding scenario generation and reduction:

  1. To present two probabilistic models to properly characterize the electricity pool price and the unexpected failures suffered by generating units, respectively.
  2. To propose a scenario reduction technique that, taking into account the optimization problem to be solved, selects a reduced scenario set that accurately approximates the statistical characteristics of the original set while making the resulting stochastic optimization problem computationally tractable. This technique is used to reduce scenario sets representing both the pool price and the availability of generating units.
  3. To design a scenario reduction technique specially tailored to account for the risk aversion level of the decision maker into the reduction process.

- Objectives pertaining to forward contracting:

  1. To mathematically model the forward contracts traded in electricity markets to buy or sell energy at a fixed price throughout a future time horizon.
  2. Taking into account the uncertainty corresponding to the pool price and the availability of generating units, to formulate a two-stage stochastic optimization model to determine the optimal mix of pool and forward involvement of a power producer so that the Conditional Value-at-Risk of its profit distribution is maximized for a given risk aversion level.

- Objectives pertaining to insurance contracting:

  1. To mathematically characterize insurances against unexpected unit failures using a stochastic optimization framework.

2. To formulate a two-stage stochastic optimization model that allows analyzing the impact of the acquisition of an insurance against unexpected unit failures of generating units on the profit distribution of a power producer. Moreover, the proposed model can be used to determine the maximum premium that a power producer is willing to pay for a given insurance contract and risk aversion level taking into account both the price and availability uncertainties.

- Objectives pertaining to option contracting:

  1. To propose a mathematical model to characterize electricity options giving the holder the right to buy or sell electricity within a three-stage stochastic optimization framework.
  2. To illustrate how electricity options can be used by a power producer to manage the risk associated with both the volatility of pool prices and the occurrence of unexpected unit failures.
  3. To formulate a three-stage stochastic optimization model to determine the optimal mix of pool, forward contracts, and options for a power producer that seeks to maximize the Conditional Value-at-Risk of its profit distribution for a given risk aversion level.

1.8 Chapter Outline

The chapters of the thesis are outlined as follows:

Chapter 1 provides an overview of electricity markets, emphasizing the difference between the pool and the derivatives market. Then, the main type of risk faced by power producers in electricity markets are described. Next, the motivation of this thesis is stated. A detailed literature review of the topics pertaining to this dissertation follows. The chapter continues by describing the problems tackled in this thesis and summarizing the methods selected to solve them. Subsequently, the main objectives of the thesis are listed. Finally, the outline of the chapters is presented.

Chapter 2 has two main purposes. On the one hand, to provide the mathematical basis of stochastic optimization for two-stage and multi-stage problems. On the other, to formulate a general medium-term decision-making model of a power producer within a stochastic optimization framework. The risk measure selected for this model is also discussed in the chapter.

Chapter 3 describes the main procedures used to generate the scenario sets characterizing the uncertain parameters faced by power producers, i.e., the pool price and the availability of generating units. Besides, in this chapter we propose two scenario reduction techniques and compare them with existing ones.

Chapter 4 presents a stochastic optimization model to determine the optimal involvement of a risk-averse power producer in the futures market of electricity. Particular emphasis is placed on the impact of the forced outage rate of generating units on the forward contracting decisions of the power producer. An illustrative example and a realistic case study are provided to test the proposed formulation.

Chapter 5 proposes a stochastic optimization model that assesses the impact of the acquisition of an insurance against forced outages of generating units on both forward contract decisions and the profit distribution of a risk-averse power producer. A procedure to determine the maximum premium that a power producer is willing to pay for a given insurance considering both the price and availability risks is developed. The performance of the proposed model is equally tested through an illustrative example and a realistic case study.

Chapter 6 examines the trading of options in an electricity market, focusing on the impact of these contracts on the power producer profit. Using a multi-stage stochastic formulation, we determine the optimal decisions of a risk-averse power producer to sell and buy electricity through options. An illustrative example and a realistic case study are presented to analyze the proposed formulation.

Chapter 7 concludes this thesis providing a summary, several noteworthy conclusions, and the main contributions of the work. Some suggestions for further research are also provided.

Appendix A illustrates a procedure to efficiently aggregate the 24 hourly values representing the pool price evolution throughout one day into a smaller number of multi-hour price values.

Appendix B provides an efficient implementation of the forward selection procedure for scenario reduction.

Chapter 2

Decision Making under Uncertainty for Power Producers

2.1 Introduction

As described in the previous chapter, the electricity market restructuring process has lead to a significant increase in competition among market participants, thus augmenting their uncertainty and risk exposure. Moreover, the especial features of electricity as a commodity give rise to prices even more volatile and with a higher probability of spikes than those in other energy markets such as gas or oil. As an example, the standard deviations of the daily return between January 1999 and September 2002 for the electricity price in the PJM interconnection, the natural gas price in the Henry Hub, and the crude oil price in West Texas Intermediate (WTI) were 32.2%, 4.3%, and 2.6%, respectively. These two facts make the treatment of uncertainty in decision-making models for electricity market participants even more critical.

A first attempt may be to obtain decisions based on the average values of the random parameters, i.e., to solve the deterministic expected-value problem. However, since the 1970s several works in the technical literature show that this is not the best approach to deal with uncertainty, and propose *stochastic programming* as an effective alternative [28, 89, 101].

Throughout this chapter we present the basic concepts and notions required to understand the stochastic models proposed in this dissertation to make informed decisions by electricity power producers.

2.2 Stochastic Programming

Stochastic programming is a mathematical framework to formulate and solve optimization problems under uncertainty, that is, mathematical programming problems in which some of the parameters involved are not fully known at the time of the decision making. In this sense, *stochastic* has to be understood as the opposite of *deterministic*, while the term *programming* refers to modeling the problem as a linear or non-linear programming problem. The goal of stochastic programming is to chose the "best" decision from the set of available alternatives considering that some input data are uncertain. All these features make stochastic programming an adequate tool to solve decision-making models for electricity market agents [178, 179].

Within a stochastic programming framework, two different, although related, mathematical models need to be formulated. First, the model intended to answer the question *"which would be the best decision?"* has to be designed. We refer to this model as *decision model* and its performance is tested through *Out-of-Sample* studies [178]. Second, an adequate model has to be developed to describe the behavior of the uncertain parameters that are outside of the decision-maker control but still affect its decisions. We refer to this model as *descriptive model* since its objective is to provide a description as good as possible of the uncertain parameters involved in the decision model. Descriptive models are usually assessed by comparing the predictions fabricated out of them with actual observations.

Only a few stochastic programming models can be solved if the uncertain parameters are described using continuous probability distributions. Therefore, these parameters need to be first approximated by discrete distributions with a limited number of outcomes that are referred to as *scenarios*, and represent possible *states of the world* in a future time. Each scenario has an associated probability of occurrence and the sum of the probabilities of all scenarios must be equal to 1. In this regard, the choice of a *descriptive model* that properly represents the uncertain parameters becomes a crucial issue to solve stochastic programming models. There exists a wide diversity of techniques to generate scenarios representing stochastic processes [102]. The methods to generate scenarios characterizing the stochastic variables that affect the medium-term decision making of power producers are described in Chapter 3.

Another important question here is *"how many scenarios should be generated to adequately represent a given uncertain parameter?"*. In principle, it can be thought that the higher the number of scenarios, the better the characterization of the uncertain parameters. However, a scenario set too large may entail tractability complications. For this reason, Section 3.6 of Chapter 3 focuses on the analysis of different *scenario reduction techniques* to solve decision-making stochastic models of power producers in a reasonable time.

Stochastic programming models are classified into recourse and chance-constrained models. The main characteristic of the latter is that constraints must be satisfied with a given probability [149], whereas in recourse models constraints need to hold for all possible realizations of the stochastic processes. In order to solve both recourse and chance-constrained stochastic models, the *deterministic equivalent program* (DEP) has to be formulated first. This task is more complicated in chance-constrained models because it involves the manipulation of probabilistic distributions [177]. There is no a clear answer to the question of which model is better, as it depends on the characteristics of the problem to be solved. Throughout this dissertation, stochastic linear programming models with recourse are used to derive optimal decisions for power producers due to both their adequacy and their simplicity [89, 101].

Recourse problems are defined as stochastic models in which the decisions to be made can be divided into two groups. The first group comprises those decisions that must be made before the value of the stochastic parameters is known, i.e., facing the uncertainty involved in the problem. The decisions that can be delayed until partial or total information about uncertain parameters is available form the second group. The term "recourse" refers to the fact that decisions made after some uncertain parameters are known provide the decision maker with the opportunity to adapt to each specific combination of the realized values of these parameters together with the decisions made beforehand.

Recourse problems are classified by its number of *stages*. A stage is understood as a point in time when decisions are made or information is revealed, and the number of stages depends on how the sequence of decisions is made in relation to how the information is gradually revealed over time. In this sense, decisions that are made based on the same information on uncertain parameters correspond to the same stage. The time between two decision points or stages is referred to as *period* [72].

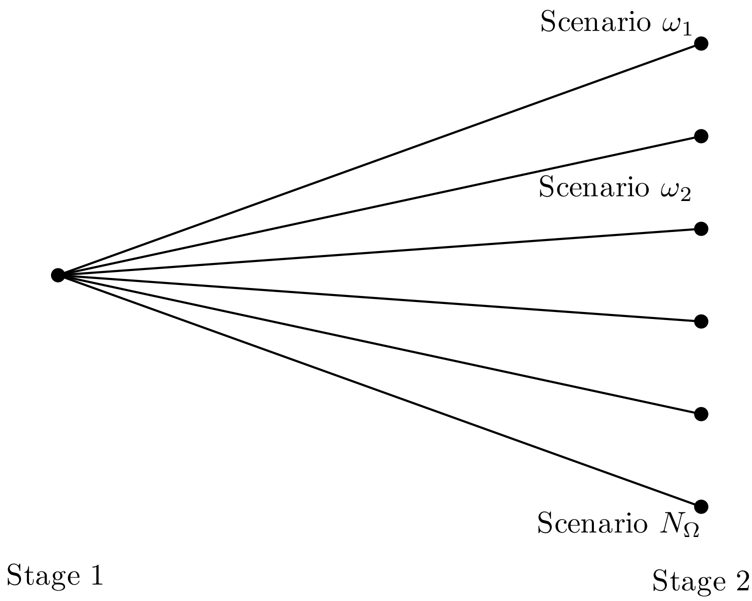


Figure 2.1: Two-stage scenario tree

2.2.1 Two-stage Stochastic Programming with Recourse

The simplest recourse problem is the two-stage stochastic programming problem. In this model, some decisions are made before knowing the actual values of the stochastic parameters, while other decisions depend on each possible realization of these parameters through the whole study horizon. The sequence of decisions and events in a two-stage stochastic problem can be summarized as

$$\text{decisions} \rightarrow \text{realization of stochastic processes} \rightarrow \text{decisions.}$$

Fig. 2.1 illustrates a generic two-stage scenario tree . The root node represents those decisions that have to be made facing the uncertainty involved in the optimization problem, also known as *first-stage* or *here-and-now* decisions . On the other hand, each final node corresponds to those decisions that depend on the realization of the stochastic processes throughout the study horizon, which are referred to as *second-stage* or *wait-and-see* decisions. The branches represent the plausible realizations (scenarios) of the stochastic processes.

As previously discussed, the stochastic processes involved in stochastic programming models are usually characterized via a set of possible scenarios. Therefore, a stochastic process ξ is mathematically denoted by a set of pairs $(\xi_{(\omega)}, \pi_{(\omega)})$, where ω represents the scenario index and varies from 1 to the total number of scenarios considered N_Ω, and $\pi_{(\omega)}$ corresponds to the probability of occurrence of each possible realization $\xi_{(\omega)}$. First-stage and second-stage decisions are represented by vectors x and $y_{(\omega)}$, respectively. Therefore, the sequence of decisions and events in a two-stage stochastic programming problem can be formally expressed as

$$x \to \xi_{(\omega)} \to y_{(\omega)}.$$

In mathematical programming terms, a two-stage stochastic programming model has the following general form [28]:

$$\begin{aligned}
\underset{x}{\text{Maximize}} \quad & z = c^T x + E[h(x,\omega)] \\
\text{subject to} \quad & Ax \geq b \qquad (2.1)\\
& x \geq 0 \\
\text{where} \quad & h(x,\omega) = \underset{y_{(\omega)}}{\text{Maximize}} \quad g_{(\omega)}^T y_{(\omega)} \\
& \text{subject to} \quad W_{(\omega)} y_{(\omega)} \geq r_{(\omega)} - T_{(\omega)} x, \quad \forall \omega \qquad (2.2)\\
& \qquad\qquad\qquad y_{(\omega)} \geq 0, \quad \forall \omega,
\end{aligned}$$

where $c, A, b, g_{(\omega)}, W_{(\omega)}, r_{(\omega)}$, and $T_{(\omega)}$ are vectors and matrices of correct size.

It is worth mentioning that in this general two-stage stochastic model, uncertainty affects all vectors and matrices appearing in optimization problem (2.2), known as second-stage problem, subproblem, or recourse subproblem. Therefore, each possible scenario contains the values of the elements of all these vectors and matrices.

In essence, the objective of the above stochastic programming problem consists in determining the first-stage decisions (x) that are *best positioned* against all possible realizations of the stochastic processes involved. In other words, optimal first-stage decisions have to be selected so that second-stage decisions ($y_{(\omega)}$) can be adapted to the realization of the stochastic processes so as to avoid unfavorable outcomes.

If uncertain parameters are characterized via a set of discrete scenarios, two-stage stochastic programming problems with recourse can be formulated using the corresponding deterministic equivalent program (DEP) , which is deeply discussed in [181] and has the following general form,

$$\underset{x,y_{(\omega)}}{\text{Maximize}} \quad z = c^T x + \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} g_{(\omega)}^T y_{(\omega)} \qquad (2.3a)$$

$$\text{subject to} \quad Ax \geq b \tag{2.3b}$$

$$W_{(\omega)} y_{(\omega)} \geq r_{(\omega)} - T_{(\omega)} x, \quad \forall \omega \tag{2.3c}$$

$$x \geq 0 \tag{2.3d}$$

$$y_{(\omega)} \geq 0, \quad \forall \omega. \tag{2.3e}$$

Note that this alternative formulation enables the use of a wide variety of commercial software to solve two-stage stochastic models.

2.2.2 Multi-stage Stochastic Programming with Recourse

In two-stage stochastic problems, some decisions (here-and-now decisions) are made before the uncertain parameters are disclosed, whereas other decisions (wait-and-see decisions) are deferred in time until the outcomes of the stochastic processes are revealed. Multi-stage recourse problems deal with models in which this "decide-observe-decide" pattern is repeated more than once [28,67]. For example, the sequence of a three-stage stochastic programming is as follows,

decisions → realization of stochastic processes → decisions → realization of stochastic processes → decisions.

Fig. 2.2 depicts an illustrative scenario tree for a three-stage stochastic problem. Each branch represents a particular realization of the stochastic processes between two consecutive stages. For example, each branch between stages 1 and 2 corresponds to a possible outcome of stochastic processes during period 1. Likewise, each node corresponds to the decisions to be made in each stage. This way, decisions in stage 1 are made facing uncertainty affecting periods 1 and 2; decisions made in stage 2 depend on the realization of the stochastic processes during period 1, but they are independent of the possible scenario realizations during period 2; and third-stage decisions are selected depending on each realization of the stochastic processes throughout the study horizon.

The decision sequence of a three-stage stochastic programming can be formally expressed as

$$x^1 \rightarrow \xi^1_{(\omega)} \rightarrow x^2(x^1, \xi^1_{(\omega)}) \rightarrow \xi^2_{(\omega)} \rightarrow x^3(x^1, \xi^1_{(\omega)}, x^2, \xi^2_{(\omega)}),$$

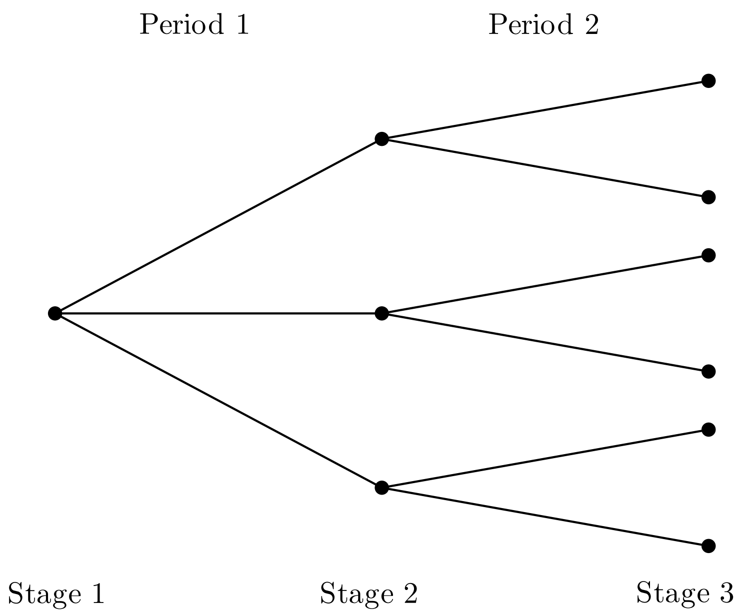


Figure 2.2: Three-stage scenario tree

where:

- x^1 represents the first-stage variables whose values have to be decided facing the uncertainty throughout the whole study horizon.

- $\xi^1_{(\omega)}$ stands for the realization of the stochastic processes ξ^1 during period 1.

- $x^2(x^1, \xi^1_{(\omega)})$ are the second-stage decisions which depend on both the firs-stage decisions and the realization of the stochastic processes during period 1, but whose values have to be decided facing the uncertainty affecting period 2.

- $\xi^2_{(\omega)}$ corresponds to the realization of the stochastic processes ξ^2 during period 2.

- $x^3(x^1, \xi^1_{(\omega)}, x^2, \xi^2_{(\omega)})$ denotes the third-stage decisions, which are made depending on first-stage and second-stage decisions as well as on the realization of the stochastic processes during the whole study horizon.

According to a scenario based formulation, the deterministic equivalent program of a three-stage stochastic programming problem is mathematically expressed as follows:

$$
\begin{aligned}
\underset{x^1, x^2_{(\omega)}, x^3_{(\omega)}}{\text{Maximize}} \quad & z = c^{1,T} x^1 + \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} c^{2,T}_{(\omega)} x^2_{(\omega)} + \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} c^{3,T}_{(\omega)} x^3_{(\omega)} && \text{(2.4a)}\\
\text{subject to} \quad & Ax \geq b && \text{(2.4b)}\\
& T^{1,1}_{(\omega)} x^1 + T^{1,2}_{(\omega)} x^2_{(\omega)} \geq r^1_{(\omega)}, \quad \forall \omega && \text{(2.4c)}\\
& T^{2,1}_{(\omega)} x^1 + T^{2,2}_{(\omega)} x^2_{(\omega)} + T^{2,3}_{(\omega)} x^3_{(\omega)} \geq r^2_{(\omega)}, \quad \forall \omega && \text{(2.4d)}\\
& x^2_{(\omega)} = x^2_{(\omega')}, \quad \forall (\omega, \omega') : \xi^1_{(\omega)} = \xi^1_{(\omega')} && \text{(2.4e)}\\
& x^1 \geq 0 && \text{(2.4f)}\\
& x^2_{(\omega)} \geq 0, \quad \forall \omega && \text{(2.4g)}\\
& x^3_{(\omega)} \geq 0, \quad \forall \omega. && \text{(2.4h)}
\end{aligned}
$$

In multi-stage stochastic programming problems it is important to establish the non-anticipatory nature of decisions [28]. That is, if the realizations of the stochastic processes are identical up to stage k, then the values of decision variables must be identical up to stage k. In the three-stage stochastic programming problem (2.4), the so called *nonanticipativity conditions* are imposed via constraints (2.4e), which force that second-stage decisions are made depending on the realization of the stochastic processes during period 1 ($\xi^1_{(\omega)}$), but still facing the uncertainty affecting period 2 ($\xi^2_{(\omega)}$).

2.2.3 Risk Management

The stochastic optimization models described above are called *risk-neutral* models since optimal decisions are determined in order to maximize an expected value without considering any measure of *deviation* or *risk* of the probability distribution of the objective function (e.g., profit) caused by the uncertainty of the parameters involved.

However, most decisions are made based not only on the expected outcome, but also on the possibility of obtaining undesirable results because of the realization of an adverse scenario. Thus, it is advisable to make decisions seeking to control also the "shape" of the targeted probability distribution in order to decrease the probability of unfavorable outcomes. The challenge here is then to condense the information contained in a probability distribution into a single quantity that properly represents the risk inherent in such a distribution. There are several approaches to quantify the risk through a functional $\Re(\cdot)$

that associates a real number with a given probability distribution, i.e.,

$$\Re : \xi \to (-\infty, \infty], \tag{2.5}$$

where ξ represents a stochastic process with finite mean and standard deviation.

As stated in [13, 155], it is desirable that a *risk measure* satisfies the following properties:

1. $\Re(\kappa) = \kappa$, for all constants κ.
2. $\Re((1-\kappa)\xi + \kappa\xi') \leq (1-\kappa)\Re(\xi) + \kappa\Re(\xi')$ for $\kappa \in (0,1)$ (convexity).
3. $\Re(\xi) \leq \Re(\xi')$ when $\xi \leq \xi'$ (monotonicity).
4. $\Re(\xi) \leq 0$ when $\|\xi_k - \xi\|_2$ with $\Re(\xi_k) \leq 0$ (closedness).
5. $\Re(\kappa\xi) = \kappa\Re(\xi)$ for $\kappa > 0$ (positive homogeneity).

The simplest way of measuring the risk of a probability distribution is using its standard deviation, i.e.,

$$\Re(\xi) = \sigma(\xi), \tag{2.6}$$

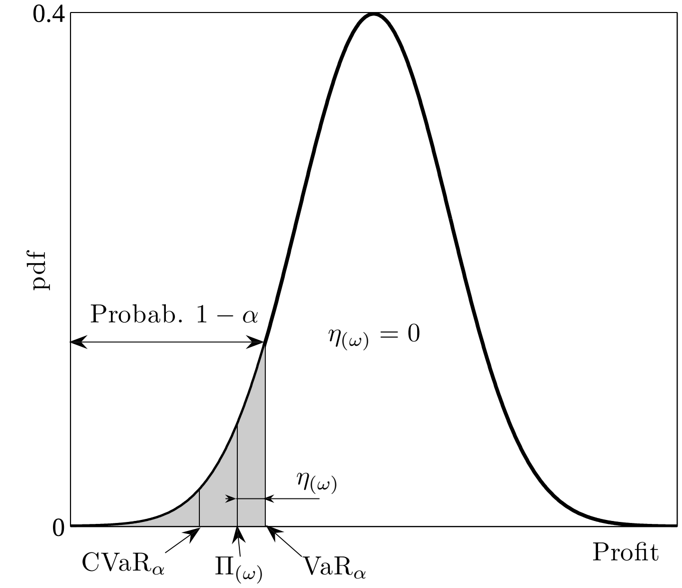
where $\sigma(\cdot)$ represents the standard deviation of a random variable. However, this risk functional so obtained is not a coherent measure of risk because the monotonicity property is not satisfied.

A different approach consists in defining a risk functional based on the α-quantile of a probability distribution ($q_\alpha(\xi)$), i.e.,

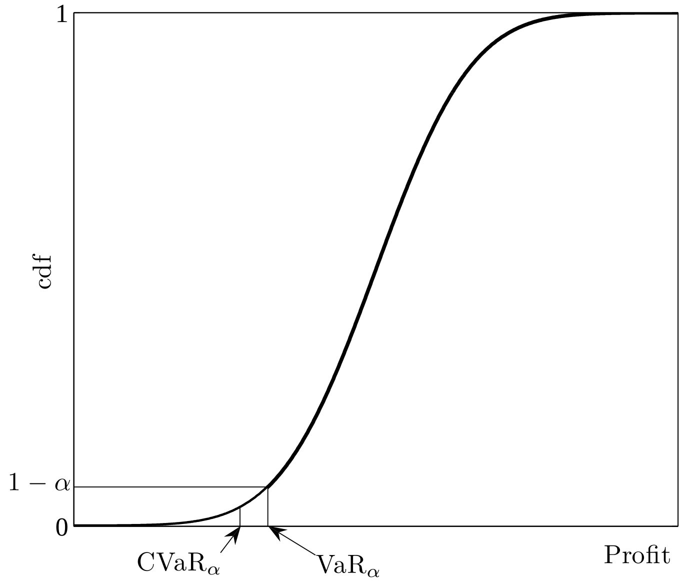
$$\Re(\xi) = q_\alpha(\xi) = \min\{\eta : F_\xi(\eta) \geq 1-\alpha\}, \alpha \in (0,1), \tag{2.7}$$

where $F_\xi(\cdot)$ is the cumulative distribution function of random variable ξ. This functional, also referred to as *Value-at-Risk* with a confidence level α (VaR_α), does not hold the convexity axiom and therefore, it is not a coherent risk measure.

The *Conditional Value-at-Risk* for a confidence level α (CVaR_α) for a profit distribution is defined as the conditional expectation of the values of the probability distribution ξ lower than the α-quantile. Figure 2.3 represents the probability density function (pdf) and the cumulative distribution function (cdf) of a profit distribution. The values of the VaR and the CVaR for a confidence level α are also shown. The CVaR_α of a probability distribution



(a) pdf



(b) cdf

Figure 2.3: Illustration of the VaR and the CVaR

fulfills the five properties mentioned above and as such, is a coherent risk measure [140,160]. For this reason, the CVaR is the risk measure used throughout this dissertation, i.e.,

$$\Re(\xi) = \mathrm{CVaR}_\alpha(\xi). \tag{2.8}$$

Within a stochastic programming framework in which uncertainty is characterized via a set of scenarios, the CVaR_α of a profit distribution can be calculated solving the following optimization problem [153]:

$$\underset{\zeta,\eta_{(\omega)}}{\text{Maximize}} \quad \mathrm{CVaR}_\alpha(\Pi) = \zeta - \frac{1}{1-\alpha}\sum_{\omega=1}^{N_\Omega} \pi_{(\omega)}\eta_{(\omega)} \tag{2.9a}$$

$$\text{subject to} \tag{2.9b}$$

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \quad \forall\omega \tag{2.9c}$$

$$0 \leq \eta_{(\omega)}, \quad \forall\omega, \tag{2.9d}$$

where $\Pi_{(\omega)}$ is the profit in scenario ω with an associated probability equal to $\pi_{(\omega)}$ and ζ is an auxiliary variable whose optimal value corresponds to the value of the VaR_α. Once optimization problem (2.9) is solved, for those scenarios with a profit lower than the VaR_α (shadow area in Figure 2.3(a)), the value of the auxiliary variable $\eta_{(\omega)}$ represents the difference between the VaR_α and the value of the profit in scenario ω; for the rest of scenarios, $\eta_{(\omega)}$ is equal to zero. Note that the optimization model (2.9) to be solved to determine the CVaR is a linear programming problem.

2.3 Power Producer Decision Model

Power producers participating in electricity markets have to make their decisions facing the high level of uncertainty affecting both the economic and physical aspect of power systems. Specifically, a power producer has to decide the optimal strategy to sell the electricity produced by its generating units considering the uncertainty related to the pool price as well as the probability of suffering unexpected failures of its generating units.

Within a stochastic framework, this section provides a general two-stage decision model for a risk-averse power producer to make informed decisions pertaining to both the pool and the derivatives market.

The stochastic optimization model of a power producer that has to decide how much of its production should be sold in the pool or through the derivatives market to maximize the CVaR of its profit distribution for a confidence level α basically boils down to

$$\underset{x,y_{(\omega)}}{\text{Maximize}} \quad z = \mathrm{CVaR}_\alpha(\Pi) \tag{2.10a}$$

$$\text{subject to} \quad \Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{D}}_{(\omega)} - C^{\mathrm{G}}_{(\omega)} \tag{2.10b}$$

$$(x, y_{(\omega)}) \in \Delta^{\mathrm{P}} \tag{2.10c}$$

$$(x, y_{(\omega)}) \in \Delta^{\mathrm{D}} \tag{2.10d}$$

$$(x, y_{(\omega)}) \in \Delta^{\mathrm{G}}, \tag{2.10e}$$

where the total profit of the power producer per scenario ($\Pi_{(\omega)}$) is the sum of the profit obtained from selling electricity in the pool ($\Pi^{\mathrm{P}}_{(\omega)}$), plus the income achieved in the derivatives market ($\Pi^{\mathrm{D}}_{(\omega)}$), minus the generating cost ($C^{\mathrm{G}}_{(\omega)}$). Δ^{P}, Δ^{D}, and Δ^{G} represent, respectively, the set of constraints imposed by the pool, the derivatives market, and the technical constraints of the generating units on first-stage and second-stage variables, i.e., x and $y_{(\omega)}$.

To extend the proposed formulation to a multi-stage stochastic programming problem, nonanticipativity constraints need to be included. For example, a three-stage stochastic decision-making model of a power producer can be formulated as follows

$$\underset{x^1, x^2_{(\omega)}, x^3_{(\omega)}}{\text{Maximize}} \quad z = \mathrm{CVaR}_{\alpha}(\Pi) \tag{2.11a}$$

$$\text{subject to} \quad \Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{D}}_{(\omega)} - C^{\mathrm{G}}_{(\omega)} \tag{2.11b}$$

$$(x^1, x^2_{(\omega)}, x^3_{(\omega)}) \in \Delta^{\mathrm{P}} \tag{2.11c}$$

$$(x^1, x^2_{(\omega)}, x^3_{(\omega)}) \in \Delta^{\mathrm{D}} \tag{2.11d}$$

$$(x^1, x^2_{(\omega)}, x^3_{(\omega)}) \in \Delta^{\mathrm{G}} \tag{2.11e}$$

$$x^2_{(\omega)} = x^2_{(\omega')}, \quad \forall (\omega, \omega') : \xi^1_{(\omega)} = \xi^1_{(\omega')}, \tag{2.11f}$$

where x^1, $x^2_{(\omega)}$, and $x^3_{(\omega)}$ represent the first-stage, second-stage, and third-stage decision variables, respectively. $\xi^1_{(\omega)}$ denotes the realization of the stochastic variables over the time period between the first and second stages and therefore, constraint (2.11f) enforces that two-stage variables associated with two scenarios with the same "history" up to the second stage have the same value.

In the remaining of this chapter, we elaborate on the formulation of the decision-making model of a power producer using a two-stage stochastic programming model. However, this formulation can be extended to a three-stage stochastic programming problem by including new variables (second-stage decisions) and the appropriate nonanticipativity constraints.

2.3.1 Decision Variables

The electricity derivatives typically traded by power producers (forward contracts, options, or insurances) have long delivery periods, e.g., the next month, the next quarter, or even the next year. Therefore, decisions related to the derivatives market have to be made facing a high level of uncertainty and thus, they are first-stage variables (x) in the models proposed in this dissertation.

On the other hand, decisions pertaining to the pool and the operation of the generating units owned by the producer have to be decided only one day in advance, thereby facing a much lower level of uncertainty if compared to derivatives market decisions. For this reason, pool and operation decisions are assumed to be made with perfect information, i.e., once the uncertain parameters realize. Consequently, these decisions are modeled as second-stage variables, $y_{(\omega)}$.

2.3.2 Objective Function

In decision-making problems, risk can be included either in the objective function, as the mean-variance approach in [124, 169], or in the constraints by bounding the value of the risk function. The simplest way to incorporate a risk measure into the objective function is using a weighting factor that represents the trade-off between the expected outcome and the risk measure as follows,

$$z = E[\xi] + \beta\Re(\xi), \tag{2.12}$$

where z is the objective function to be maximized, and β represents the weighting factor. This way, a value of β equal to zero leads to a risk-neutral attitude in which only the expected outcome is maximized, whereas a high value of β corresponds to a risk-averse position, in which the decisions are made to maximize both the expected value and the risk measure.

If the CVaR is employed as the risk measure, the use of the objective function formulation (2.12) has the two following disadvantages:

- Objective function (2.12) involves the confidence level of the CVaR (α) and the weighting factor of the objective function (β). In most cases, α is fixed to a value between 0.9 and 0.95, and the risk aversion of the decision maker is simulated by changing the value of the weighting parameter β from 0 up to a "high enough level". Both the selection of the parameter α and the "high enough" value of the parameter β are arbitrary selected.

- The weighting parameter β implements a trade-off between the maximization of the expected profit and the risk measure, and it is clear that an increase in the value of β increases the weight of the risk measure in the objective function of the optimization model, thus resulting in a higher risk-averse attitude. However, β does not have a clear mathematical or physical meaning, which is a drawback when comparing the optimal decisions of two different power producers with the same level of risk aversion. In such a case, fixing the parameter β to the same value does not imply the same risk aversion for both decision makers.

For these two reasons, the objective function proposed in this thesis comes down to the CVaR_α of the profit probability distribution, i.e.,

$$z = \text{CVaR}_\alpha(\Pi). \tag{2.13}$$

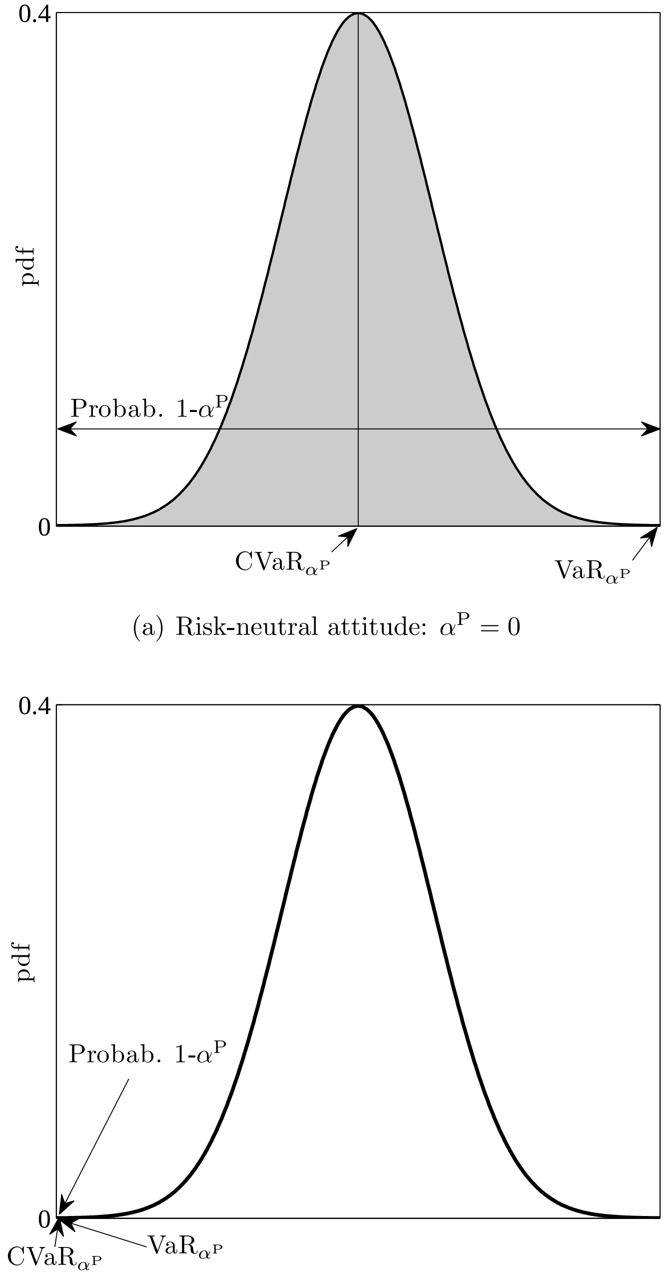
Being so, the risk aversion of the decision maker is solely controlled by the confidence level α, which is denoted by α^{P} hereafter. Considering (2.7), a value of α^{P} equal to zero makes the VaR equal to the highest value of the distribution and consequently, the CVaR is equivalent to the expected value of the probability distribution (risk-neutral attitude, Figure 2.4(a)). On the other hand, as depicted in Figure 2.4(b), a value of α^{P} close to 1 makes both the VaR and the CVaR equal to the lowest possible outcome (risk-averse attitude). Therefore, the risk aversion level of the decision maker is implemented through a single parameter α^{P}, varying from 0 to 1.

Furthermore, using objective function (2.13) allows comparing the optimal decisions of two different power producers with the same value of α^{P}, because in that case, both decision makers seek to maximize the expected value of the $100(1-\alpha^{\text{P}})\%$ worst possible outcomes.

Next, we explain the different terms making up the profit of a power producer, which is expressed as

$$\Pi_{(\omega)} = \Pi^{\text{P}}_{(\omega)} + \Pi^{\text{D}}_{(\omega)} - C^{\text{G}}_{(\omega)}, \quad \forall \omega. \tag{2.14}$$

As previously stated, $\Pi^{\text{P}}_{(\omega)}$ represents the profit that the power producer obtains from participating in the pool. Pool decisions are made with perfect information and therefore, the profit associated with trading in this market is



(a) Risk-neutral attitude: $\alpha^{\mathrm{P}} = 0$

(b) Risk-averse attitude: $\alpha^{\mathrm{P}} \to 1$

Figure 2.4: Risk aversion modeling through the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$

formulated as

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} P^{\mathrm{P}}_{(\omega,t)} L_{(t)}, \quad \forall \omega, \tag{2.15}$$

where $\lambda^{\mathrm{P}}_{(\omega,t)}$ represents the pool price during time step t and in scenario ω, and $P^{\mathrm{P}}_{(\omega,t)}$ is a continuous variable corresponding to the quantity in MW that the power producer sells in the pool during time step t and in scenario ω. N_T is the total number of time steps within the study horizon and $L_{(t)}$ is the duration of time step t.

The mathematical expression of the profit obtained in the derivatives market ($\Pi^{\mathrm{D}}_{(\omega)}$) is a function of each particular derivative or contract. For consistency, its description is postponed to the following chapters in which different derivatives and contracts are considered.

The cost term $C^{\mathrm{G}}_{(\omega)}$ is equal to the sum of the production cost of the different generating units owned by the power producer, which are typically modeled by the following quadratic function,

$$C^{\mathrm{G}}_{(i,\omega,t)} = \Big(a_{(i)} + b_{(i)} P^{\mathrm{G}}_{(i,\omega,t)} + c_{(i)} (P^{\mathrm{G}}_{(i,\omega,t)})^2\Big) L_{(t)}, \quad \forall i, \forall \omega, \forall t, \tag{2.16}$$

where variable $P^{\mathrm{G}}_{(i,\omega,t)}$ is the power generated by unit i, during time step t, and in scenario ω; and $a_{(i)}$, $b_{(i)}$, and $c_{(i)}$ are coefficients that depend on each particular generating unit.

Expression (2.16) can be approximated by a piecewise linear function in order to maintain the model linearity [12]. In doing so, the production cost (2.16) is reformulated as follows

$$C^{\mathrm{G}}_{(i,\omega,t)} = \Big(A_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} \lambda_{(i,b)} P^{\mathrm{G}}_{(i,\omega,t,b)}\Big) L_{(t)}, \quad \forall i, \forall \omega, \forall t, \tag{2.17}$$

where b is the index of the blocks comprising the piecewise linear function, and runs from 1 up to the total number of blocks (N_B). $A_{(i)}$ and $\lambda_{(i,b)}$ are, respectively, the no-load cost and the cost slopes per block b of each generating unit i, and variable $P^{\mathrm{G}}_{(i,\omega,t,b)}$ stands for the power generated within block b of each unit i during time step t and in scenario ω. Figure 2.5 shows the approximation of the quadratic production cost of a thermal unit by a piecewise linear function.

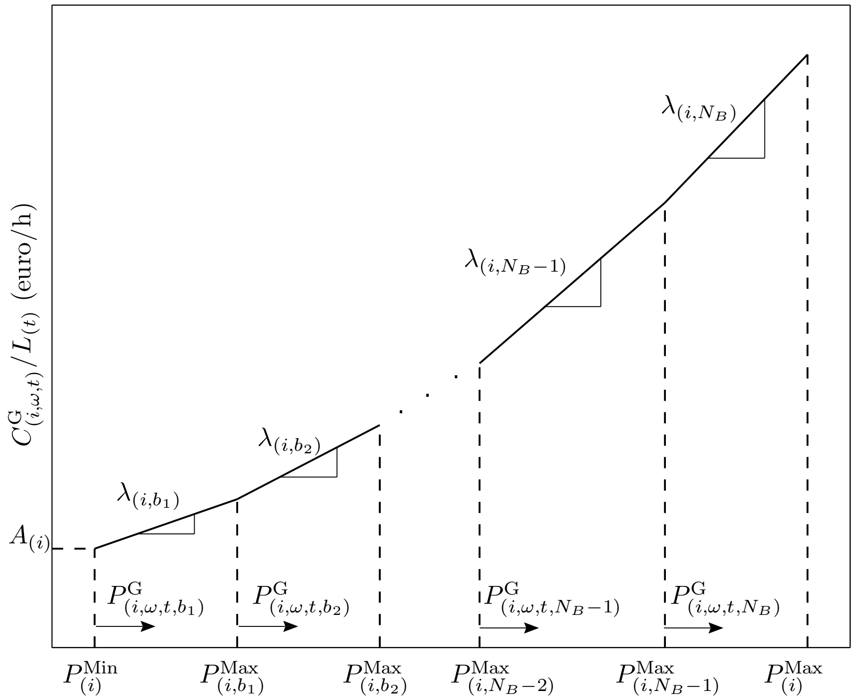


Figure 2.5: Piecewise linear production cost of a thermal generating unit

2.3.3 Constraints

Some technical limitations of thermal generating units, such as ramping constraints and minimum up and down times, can be disregarded in medium-term decision-making models. In contrast, the technical constraint that bounds the electricity produced by a thermal unit between a minimum and a maximum value has to be considered. Within a stochastic framework in which the possibility of unexpected failures is accounted for, this constraint is formulated as

$$u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\mathrm{Min}}_{(i)} \leq P^{\mathrm{G}}_{(i,\omega,t)} \leq u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\mathrm{Max}}_{(i)}, \quad \forall i, \forall \omega, \forall t, \tag{2.18}$$

where variable $P^{\mathrm{G}}_{(i,\omega,t)}$ represents the power generated by unit i, in scenario ω, and during time step t. $P^{\mathrm{Min}}_{(i)}$ and $P^{\mathrm{Max}}_{(i)}$ are the minimum and maximum power outputs of generating unit i, respectively, and $u_{(i,\omega,t)}$ is a binary variable that indicates the start-up and shut-down status of generating unit i, during time step t, and in scenario ω. Finally, $k_{(i,\omega,t)}$ is a parameter used to model unexpected failures, being 0 if the generating unit i suffers from an unexpected failure during time step t and in scenario ω, and 1 otherwise.

If the quadratic production cost is approximated by a piecewise linear function, the total generated power by unit i is computed as

$$P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{Min}}_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} P^{\mathrm{G}}_{(i,\omega,t,b)} \qquad \forall i, \forall \omega, \forall t \tag{2.19}$$

$$0 \leq P^{\mathrm{G}}_{(i,\omega,t,1)} \leq P^{\mathrm{Max}}_{(i,b_1)} - P^{\mathrm{Min}}_{(i)} \qquad \forall i, \forall \omega, \forall t \tag{2.20}$$

$$0 \leq P^{\mathrm{G}}_{(i,\omega,t,b)} \leq P^{\mathrm{Max}}_{(i,b)} - P^{\mathrm{Max}}_{(i,b-1)} \qquad \forall i, \forall \omega, \forall t, \forall b = 2, ..., N_B, \tag{2.21}$$

where $P^{\mathrm{Max}}_{(i,b)}$ is the upper limit of the b-th power block of the piecewise linear production cost function.

Despite the fact that there exist purely financial derivatives, in this dissertation we only consider those derivatives that force the physical delivery of the committed electricity and therefore, the amount of energy sold in the pool and derivatives market is limited to the total capacity of the available generating units. This condition is enforced by the following constraint,

$$\sum_{i=1}^{N_I} P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{P}}_{(\omega,t)} + P^{\mathrm{D}}, \quad \forall \omega, \forall t, \tag{2.22}$$

where P^{D} represents the power committed to be sold through derivatives and which may or may not depend on the scenario realization and N_I is the number of generating units. Like the profit term $\Pi^{\mathrm{D}}_{(\omega)}$, the formal description of the power sold through derivatives is comprehensively explained in the upcoming chapters.

Arbitrage is defined as the practice of making profit by the simultaneous purchase and sale of the same commodity and is caused by potential price discrepancies of this commodity in different markets [163]. In this dissertation, the prices of the different mechanisms to buy or sell electricity are assumed to be set according to a non-arbitrage approach and consequently, it is reasonable to assume that no arbitrage opportunities exist. For a power producer, buying energy in the pool to sell it in some derivatives market or vice versa is considered as arbitrage. In order to avoid arbitrage actions in the models proposed in this dissertation, the power producer can buy energy in the pool only in those scenarios and time steps in which one or more generating units are forced out, i.e.,

$$\sum_{i=1}^{N_I} (k_{(i,\omega,t)} - 1) P^{\mathrm{Max}}_{(i)} \leq P^{\mathrm{P}}_{(\omega,t)}, \quad \forall \omega, \forall t. \tag{2.23}$$

Therefore, if all generating units are online, the power producer can only sell electricity in the pool. On the other hand, if some generating units suffer from unexpected failures, the power producer can buy in the pool the amount of energy needed to replace the forced out units. Additional constraints to avoid arbitrage in the derivatives market are described throughout this dissertation.

2.3.4 General Model

Arranging all equations described above, a general two-stage stochastic model of a risk-averse power producer is presented below,

$$\text{Maximize}_{P^{\mathrm{G}}_{(i,\omega,t,b)},P^{\mathrm{P}}_{(\omega,t)},P^{D},u_{(i,\omega,t)},\zeta,\eta_\omega}$$

$$\zeta - \frac{1}{1-\alpha^{\mathrm{P}}}\sum_{\omega=1}^{N_\Omega}\pi_{(\omega)}\eta_{(\omega)} \tag{2.24a}$$

subject to

$$\Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{D}}_{(\omega)} - C^{\mathrm{G}}_{(\omega)}, \qquad \forall\omega \tag{2.24b}$$

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T}\lambda^{\mathrm{P}}_{(\omega,t)}P^{\mathrm{P}}_{(\omega,t)}L_{(t)}, \qquad \forall\omega \tag{2.24c}$$

$$C^{\mathrm{G}}_{(\omega)} = \sum_{i=1}^{N_I}\sum_{t=1}^{N_T}C^{\mathrm{G}}_{(i,\omega,t)}, \qquad \forall\omega \tag{2.24d}$$

$$C^{\mathrm{G}}_{(i,\omega,t)} = \Big(A_{(i)}u_{(i,\omega,t)} + \sum_{b=1}^{N_B}\lambda_{(i,b)}P^{\mathrm{G}}_{(i,\omega,t,b)}\Big)L_{(t)}, \qquad \forall i,\forall\omega,\forall t \tag{2.24e}$$

$$P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{Min}}_{(i)}u_{(i,\omega,t)} + \sum_{b=1}^{N_B}P^{\mathrm{G}}_{(i,\omega,t,b)}, \qquad \forall i,\forall\omega,\forall t \tag{2.24f}$$

$$0 \le P^{\mathrm{G}}_{(i,\omega,t,1)} \le P^{\mathrm{Max}}_{(i,b_1)} - P^{\mathrm{Min}}_{(i)}, \qquad \forall i,\forall\omega,\forall t \tag{2.24g}$$

$$0 \le P^{\mathrm{G}}_{(i,\omega,t,b)} \le P^{\mathrm{Max}}_{(i,b)} - P^{\mathrm{Max}}_{(i,b-1)}, \qquad \forall i,\forall\omega,\forall t,\forall b=2,...,N_B \tag{2.24h}$$

$$u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\mathrm{Min}}_{(i)} \le P^{\mathrm{G}}_{(i,\omega,t)} \le u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\mathrm{Max}}_{(i)}, \qquad \forall i,\forall\omega,\forall t \tag{2.24i}$$

$$\sum_{i=1}^{N_I}P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{P}}_{(\omega,t)} + P^{\mathrm{D}}, \qquad \forall\omega,\forall t \tag{2.24j}$$

$$\sum_{i=1}^{N_I}(k_{(i,\omega,t)}-1)P^{\mathrm{Max}}_{(i)} \le P^{\mathrm{P}}_{(\omega,t)}, \qquad \forall\omega,\forall t \tag{2.24k}$$

$$P^{\mathrm{D}} \in \Delta^{\mathrm{D}} \tag{2.24l}$$

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \qquad \forall\omega \qquad (2.24\text{m})$$

$$0 \leq \eta_{(\omega)}, \qquad \forall\omega. \qquad (2.24\text{n})$$

Objective function (2.24a) to be maximized is the CVaR of the profit distribution for a risk aversion level α^{P}. Constraints (2.24b)–(2.24e) define the profit of the producer for each scenario ω. The production level of generating unit i is modeled through constraints (2.24f)–(2.24i). The energy balance is imposed by constraint (2.24j). The arbitrage between the pool and the derivatives market is avoided through constraints (2.24k)–(2.24l). To compute the CVaR of the profit distribution, constraints (2.24m)–(2.24n) are needed.

2.4 Summary

Stochastic programming represents an adequate mathematical framework to formulate optimization models affected by uncertain parameters. In this chapter, we introduce the basic concepts and definitions of stochastic programming. In addition, we present the general formulations of a two-stage and a three-stage stochastic programming problem. The choice of the CVaR as the risk measure used in the models proposed in this dissertation is also discussed in this chapter.

All the models presented in this thesis are intended to identify optimal decisions for power producers. The purpose of the last part of this chapter is to highlight the common features of these models as well as to formulate a general decision-making model for a risk-averse power producer within a stochastic framework.

Chapter 3

Scenario Generation and Reduction

3.1 Introduction

Stochastic programming encompasses those optimization problems in which some of the parameters involved are uncertain. Continuous probability distributions, e.g., normal distributions, may be used to approximately represent the stochastic behavior of these parameters. However, even under such an assumption, stochastic programming including continuous probability distributions can only be solved for some simple cases. Therefore, in order to tackle most practical problems, the distributions of the stochastic processes need to be approximated by discrete distributions with a limited number of possible outcomes, which are referred to as *scenarios*.

Representing the uncertain parameters involved in a stochastic programming problem using a scenario set entails some advantages, e.g., the flexibility to jointly model different stochastic processes like the electricity price, the availability of a thermal unit, and the uncertain power production of a wind farm, among many others. Moreover, the decision-making pattern *decide-observe-decide* can be easily incorporated into a scenario-based stochastic programming formulation. However, the main disadvantage of using a scenario tree to represent the stochastic parameters lies in determining how good this approximation is in terms of the solution of the stochastic model. In other words, the use of an inappropriate scenario tree may lead to a solution of the stochastic programming problem far from optimal.

In this chapter, we design two methodologies to generate a scenario tree representing the uncertainty related to the pool price of electricity as well as the probability of suffering from unexpected failures of a generating unit. Although each of these procedures is explained to generate a scenario tree corresponding to both a two-stage and a three-stage stochastic programming problem, they can be equally used to build scenario trees that properly characterize the uncertainty involved in stochastic programming models with more than three stages.

The number of scenarios needed to accurately represent the plausible realizations of the stochastic processes pertaining to a decision-making problem is generally large, which may render the associated stochastic programming model intractable. Therefore, scenario reduction techniques are needed to trim down significantly the number of scenarios while keeping as intact as possible the stochastic information embedded in such scenarios. In the second part of this chapter, we propose two novel scenario reduction procedures that advantageously compare with the existing ones for electricity-market problems.

3.2 Stochastic Processes

Within a modeling framework based on stochastic programming, input data affected by uncertainty are conceptually described as stochastic processes. A stochastic process $\xi = \{\xi_{(t)}, t \in T\}$ is a collection of interrelated random variables. That is, for each t in the index set T, $\xi_{(t)}$ is a random variable. We often interpret t as time and call $\xi_{(t)}$ the state of the process at time t.

In this dissertation, two stochastic processes are considered, namely the pool price of electricity and the availability of a generating unit i. If we denote the pool price of electricity in time t by $\lambda^{\mathrm{P}}_{(t)}$, then the set of random variables $\lambda^{\mathrm{P}} = \{\lambda^{\mathrm{P}}_{(t)}, t = 1, 2, \ldots, N_T\}$ represents a continuous stochastic process. Likewise, if $k_{(i,t)}$ corresponds to the availability of generating unit i in time t, the collection of random variables $k_{(i)} = \{k_{(i,t)}, t = 1, 2, \ldots, N_T\}$ constitutes a discrete stochastic process.

Both continuous and discrete stochastic processes can be characterized by a finite set of real vectors referred to as *scenarios*. For instance, the continuous stochastic process λ^{P} can be mathematically modeled as $\lambda^{\mathrm{P}} = \{\lambda^{\mathrm{P}}_{(\omega)}, \omega = 1, 2, \ldots, N_\Omega\}$, where ω is the scenario index varying from 1 to the total number of scenarios N_Ω. Associated with each realization $\lambda^{\mathrm{P}}_{(\omega)}$ of the stochastic process λ^{P}, there exists a probability of occurrence denoted by $\pi_{(\omega)}$ such that $\pi_{(\omega)} =$

$P(\lambda^{\mathrm{P}} = \lambda^{\mathrm{P}}_{(\omega)})$ with $\sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} = 1$. Likewise, the discrete stochastic process $k_{(i)}$ can be characterized as $k_{(i)} = \{k_{(i,\omega)}, \omega = 1, 2, \ldots, N_\Omega\}$.

Note that, unlike continuous stochastic processes, a discrete stochastic process can be perfectly represented by a finite set of scenarios resulting from the combinations of all the discrete values that its component random variables can adopt. However, given the high number of time steps and generating units that are usually considered in medium-term decision-making models of power producers, the discrete stochastic process representing the availability of a generating unit needs also to be approximated by a reduced scenario set. In the following sections, we describe the procedures to generate and reduce scenarios representing the continuous and the discrete stochastic processes considered, i.e., the pool prices of electricity and availability of generating units, respectively.

3.3 Scenario Generation

Prior to solving a stochastic optimization problem, we have to carefully evaluate the different procedures available in the technical literature to generate a scenario set that properly characterizes the stochastic processes involved in such a problem. Next, we present a brief description of the most relevant scenario generation procedures [102]:

- *Sampling*, which consists in a continuous sampling process from the original distribution functions of the stochastic process involved [110].

- *Moment matching*, which generates discrete distributions that preserve a prefixed set of statistical properties (e.g., moments, correlation matrix, percentiles, etc.) of the original stochastic process [91].

- *Path-based methods*, which produce complete paths (scenarios) using econometric and time series models. The set of scenarios so obtained is named *fan*. Once the fan is generated, the scenarios are clustered to form the scenario tree [57].

- *Scenario reduction techniques*, which, although not considered as "pure" scenario generation methods, seek to find a scenario subset that keeps as much as possible the statistical properties of the initial distribution in terms of some *probability metric* [83, 84, 86].

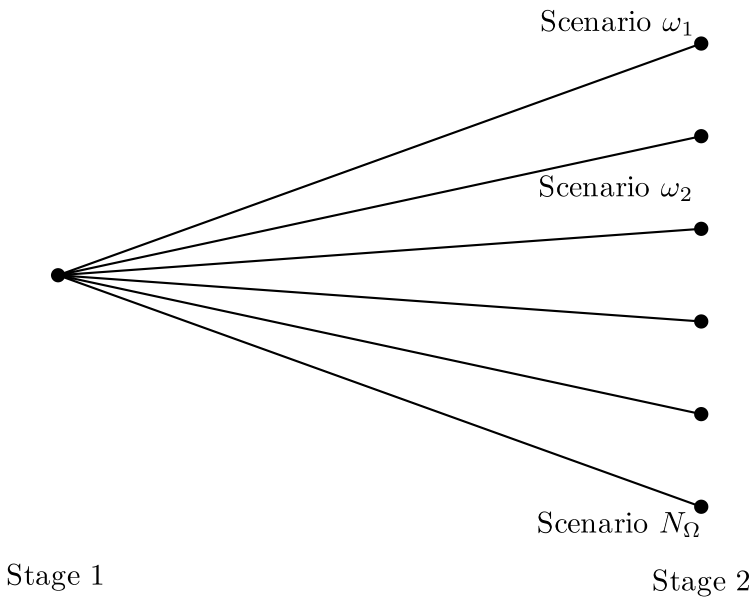


Figure 3.1: Scenario generation for two-stage stochastic programming problems

In this dissertation, we combine path-based methods with scenario reduction techniques to generate two sets of scenarios representing, respectively, the pool price of electricity and the availability of the generating units owned by the power producer. Specifically, pool price scenarios are generated using ARIMA models [43,46] . On the other hand, availability scenarios are obtained considering that both the time between two consecutive failures and the time to repair a failure follow exponential distributions [24, 54].

No matter which of the above techniques is used to characterize the stochastic processes, the decision-making pattern throughout the study horizon needs to be accounted for to build the scenario tree. For instance, in a two-stage stochastic optimization problem (Fig. 3.1) we have only two groups of decisions: those made before the realization of the stochastic processes (first-stage decisions) and those made once the values of random variables are known (second-stage decisions). Due to this decision-making structure, we generate scenarios that span the whole study horizon in just one go.

On the other hand, if we deal with a three-stage stochastic problem (Fig.3.2), some decisions are made at some point in the middle of the study horizon (second-stage decisions). For this reason, the scenario generation algorithm needs to be implemented as follows: first, a scenario set representing the uncertainty during period 1 is generated; then, and for each one of the scenarios

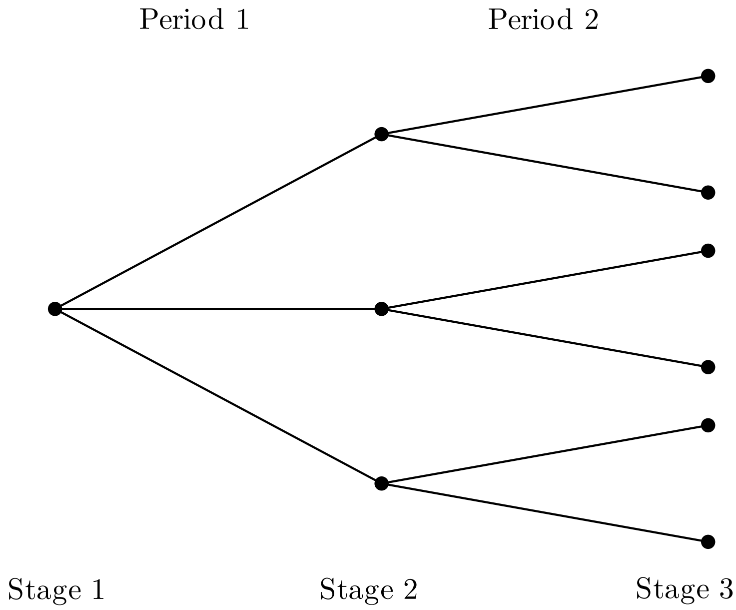


Figure 3.2: Scenario generation for three-stage stochastic programming problems

spanning period 1, a second scenario set characterizing the uncertainty during period 2 is produced using the same procedure but considering the values contained in each scenario corresponding to period 1 as certain. The way in which the information disclosed in period 1 is taken into account to generate a scenario tree for period 2 depends on the scenario generation technique itself. Finally, note that this procedure can be generalized to create scenario trees for stochastic optimization models with more than three stages.

In the following sections, we present procedures to generate scenario trees that represent both the pool price of electricity and the availability of generating units for two-stage and three-stage stochastic models.

3.4 Pool Price Scenario Generation

In time series analysis, *autoregressive integrated moving average* processes (ARIMA) are a class of stochastic processes used to fit time series data either to better understand the data or to forecast future points in the series [33, 139]. These processes are of interest in econometrics and in engineering applications. Several works in the technical literature illustrate the good performance of ARIMA models to explain and forecast next-day electricity prices [43, 46, 130].

An ARIMA(p,d,q) model of the electricity pool price λ^{P} is mathematically expressed as

$$\Big(1-\sum_{i=1}^{p}\phi_i B^i\Big)(1-B)^d\lambda^{\mathrm{P}}_{(t)}=\Big(1+\sum_{j=1}^{q}\theta_j B^j\Big)\varepsilon_{(t)}, \tag{3.1}$$

with p autoregressive parameters $\phi_1,\phi_2,\ldots,\phi_p$ and q moving average parameters $\theta_1,\theta_2,\ldots,\theta_q$; d corresponds to the number of non-seasonal differences, and B is the lag operator, i.e., $B^u(\lambda^{\mathrm{P}}_{(t)})=\lambda^{\mathrm{P}}_{(t-u)}$.

The estimation and adjustment of ARIMA models basically comprises the following phases:

- Phase 1: Formulation of the model assuming normality and stationarity for the underlying stochastic process.
- Phase 2: Identification of the model terms.
- Phase 3: Estimation of the model parameters.
- Phase 4: Model validation, which essentially boils down to checking the normality and stationarity assumptions.

Further details on ARIMA models, e.g., the inclusion of terms to deal with seasonal time series, can be found in [33, 139].

Once an appropriate ARIMA model has been fitted to the stochastic process under consideration λ^{P}, the value of the forecast of the stochastic process for time t, denoted by $\widehat{\lambda^{\mathrm{P}}}_{(t)}$, is determined as

$$\widehat{\lambda^{\mathrm{P}}}_{(t)}=\Phi\Big(\lambda^{\mathrm{P}}_{(t-1)},\lambda^{\mathrm{P}}_{(t-2)},\ldots,\lambda^{\mathrm{P}}_{(t-p)}\Big)+\Theta\Big(\varepsilon_{(t-1)},\varepsilon_{(t-2)},\ldots,\varepsilon_{(t-q)}\Big)+\varepsilon_{(t)}, \tag{3.2}$$

that is, as a function of the values of the pool price λ^{P} in time steps previous to t $\Big(\lambda^{\mathrm{P}}_{(t-1)},\lambda^{\mathrm{P}}_{(t-2)},\ldots,\lambda^{\mathrm{P}}_{(t-p)}\Big)$, the error terms in time steps prior to t $\Big(\varepsilon_{(t-1)},\varepsilon_{(t-2)},\ldots,\varepsilon_{(t-q)}\Big)$, and the error term corresponding to the time step t, $\varepsilon_{(t)}$. Note that $\Phi(\cdot)$ and $\Theta(\cdot)$ are polynomials resulting from isolating variable $\lambda^{\mathrm{P}}_{(t)}$ from expression (3.1). In an ARIMA model, the error term, also referred to as *innovation*, is assumed to be a white noise, i.e., $\varepsilon_{(t)}\sim N(0,\sigma_\varepsilon)$. Therefore, the forecast value $\widehat{\lambda^{\mathrm{P}}}_{(t)}$ is obtained by setting the error term $\varepsilon_{(t)}$ to 0. To compute the forecast value $\widehat{\lambda^{\mathrm{P}}}_{(t+1)}$, we consider that $\lambda^{\mathrm{P}}_{(t)}$ is known and equal to its forecast value $\widehat{\lambda^{\mathrm{P}}}_{(t)}$, set the error term $\varepsilon_{(t+1)}$ to 0, and evaluate expression (3.2) for time step t+1. Repeating this procedure, a vector containing the forecast pool prices for a certain future time interval can be obtained.

Additionally to this forecasting procedure, ARIMA models can also be used to generate scenario trees for stochastic optimization problems. For instance, we can generate a two-stage tree of N_Ω scenarios representing the electricity price uncertainty during a future horizon spanning N_T time steps following the algorithm below:

- Step 1: Set to zero the scenario counter: $\omega \leftarrow 0$.

- Step 2: Update the scenario counter and set to zero the time step counter: $\omega \leftarrow \omega + 1, t \leftarrow 0$.

- Step 3: Update the time step counter: $t \leftarrow t + 1$.

- Step 4: Randomly generate the error term: $\varepsilon_{(t)} \leftarrow N(0, \sigma_\varepsilon)$.

- Step 5: Obtain $\lambda^{\mathrm{P}}_{(\omega,t)}$ by evaluating expression (3.2).

- Step 6: If $t < N_T$ go to Step 3, else go to Step 7.

- Step 7: If $\omega < N_\Omega$ go to Step 2, else the scenario generation process concludes.

The output of this algorithm is a $N_\Omega \times N_T$ matrix in which each element $\lambda^{\mathrm{P}}_{(\omega,t)}$ represents the value of the stochastic process λ^{P} in time step t and scenario ω.

To extend the above algorithm to generate a scenario tree characterizing the pool price within a multi-stage stochastic programming framework, we first need to know the number of stages, the duration of each period, and the number of scenarios to be generated for each period. For instance, let us consider a three-stage stochastic programming problem with durations in time steps for periods 1 and 2 being N_{T1} and N_{T2}, respectively. Additionally, suppose that $N_{\Omega 1}$ denotes the number of scenarios representing the pool price during period 1, and $N_{\Omega 2}$ corresponds to the number of price scenarios generated for period 2 for each one of the scenarios generated for period 1. As a result, the total number of scenarios is equal to $N_{\Omega 1} \times N_{\Omega 2}$. We detail below the algorithm to generate a three-stage scenario tree representing the pool price of electricity:

- Step 1: Set to zero the scenario counter of period 1: $\omega_1 \leftarrow 0$.

- Step 2: Update the scenario counter of period 1 and set to zero the time step counter: $\omega_1 \leftarrow \omega_1 + 1, t \leftarrow 0$.

- Step 3: Update the time step counter: $t \leftarrow t + 1$.
- Step 4: Randomly generate the error term: $\varepsilon_{(t)} \leftarrow N(0, \sigma_\varepsilon)$.
- Step 5: Obtain $\lambda^{\mathrm{P},1}_{(\omega_1,t)}$ by evaluating expression (3.2).
- Step 6: If $t < N_{T1}$ go to Step 3, else go to Step 7.
- Step 7: Consider scenario ω_1, i.e., $\lambda^{\mathrm{P},1}_{(\omega_1,t)}, t = 1, \ldots, N_{T1}$ as certain and its associated series of errors, namely, $\varepsilon_{(\omega_1,t)}, t = 1, \ldots, N_{T1}$.
- Step 8: Set to zero the scenario counter of period 2: $\omega_2 \leftarrow 0$.
- Step 9: Update the scenario counter of period 2 and set to N_{T1} the time step counter: $\omega_2 \leftarrow \omega_2 + 1, t \leftarrow N_{T1}$.
- Step 10: Update the time step counter: $t \leftarrow t + 1$.
- Step 11: Randomly generate the error term: $\varepsilon_{(t)} \leftarrow N(0, \sigma_\varepsilon)$.
- Step 12: Obtain $\lambda^{\mathrm{P},2}_{(\omega_1,\omega_2,t)}$ by evaluating expression (3.2).
- Step 13: If $t < N_{T1} + N_{T2}$ go to Step 10, else go to Step 14).
- Step 14: If $\omega_2 < N_{\Omega 2}$ go to Step 9, else go to Step 15.
- Step 15: If $\omega_1 < N_{\Omega 1}$ go to Step 2, else the scenario generation process concludes.

The output of this algorithm is a $N_{\Omega 1} \times N_{T1}$ matrix in which each element $\lambda^{\mathrm{P},1}_{(\omega_1,t)}$ represents the value of the stochastic process λ^{P} during period 1 (time step t and scenario ω_1), and a $N_{\Omega 1} \times N_{\Omega 2} \times N_{T1}$ matrix in which each element $\lambda^{\mathrm{P},2}_{(\omega_1,\omega_2,t)}$ corresponds to the realization of the electricity pool price during period 2 (time step t and scenario (ω_1, ω_2)).

3.5 Availability Scenario Generation

In this section, we present a Markov model [54] to characterize the availability of a single generating unit based on the following assumptions:

- The failures of any number of generating units are statistically independent.

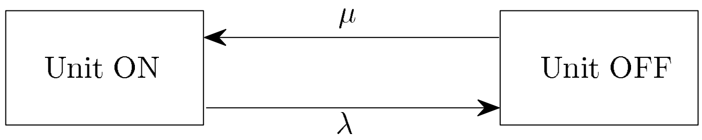


Figure 3.3: Two-state model of the generating unit

- The generating unit failure and repair rates are constant.

- A repaired generating unit is as good as new.

Fig. 3.3 illustrates the Markov model characterizing the availability of a generating unit, where λ and μ are the failure rate and the repair rate, respectively.

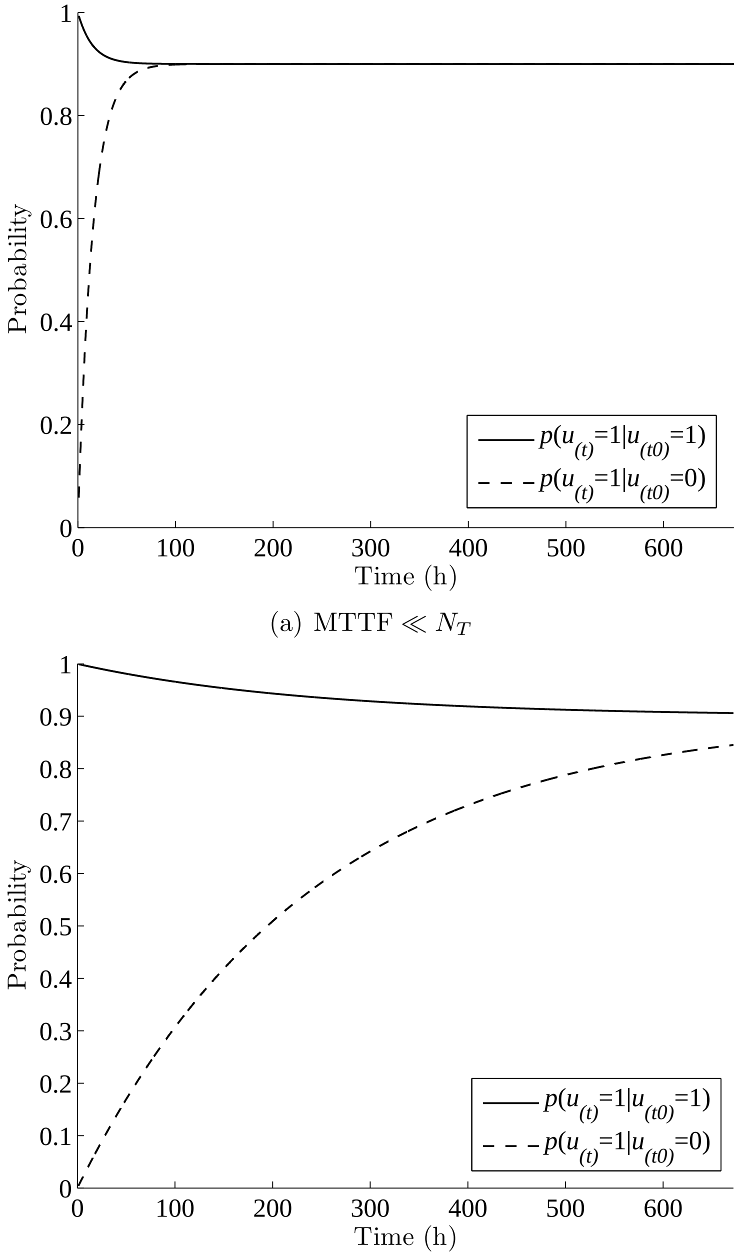
Using the Laplace transform to solve the equations resulting from the Markov method [54], we obtain the following equation to compute the probability that the unit is available in time step t:

$$p(k_{(t)} = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu \cdot (k_{(t_0)} - 1) + \lambda}{\lambda + \mu} e^{-(\lambda+\mu)(t-t_0)}, \tag{3.3}$$

where $k_{(t_0)}$ is equal to 1 if the unit is available at t_0 and 0 otherwise, λ is the generating unit failure rate, and μ is the generating unit repair rate. The probability that the generating unit is unavailable at t is equal to $1 - p(k_{(t)} = 1)$.

The failure rate (λ) and repair rate (μ) are equal to the inverse of the mean time to failure (MTTF) and the mean time to repair (MTTR), in that order, i.e., $\lambda = \frac{1}{\mathrm{MTTF}}$ and $\mu = \frac{1}{\mathrm{MTTR}}$. The MTTF and MTTR of a generating unit are defined as the average time between two consecutive unexpected failures and the average time to repair an unexpected failure, respectively, and both of them are obtained from historical data. The forced outage rate (FOR) is the percentage of time that the generating unit is unavailable, i.e., $\mathrm{FOR} = \frac{\mathrm{MTTR}}{\mathrm{MTTF} + \mathrm{MTTR}}$, [24].

Considering equation (3.3), note that for t high enough, the probability that the unit is available is independent of its initial status $k_{(t_0)}$ and equal to $\frac{\mu}{\lambda + \mu}$. On the other hand, for t small enough, this probability strongly depends on the initial status of the unit. In order to visualize this relevant effect, Fig. 3.4 depicts the probability that a generating unit with $\mathrm{FOR} = 10\%$ is available during one month depending on its MTTF and its initial status. In Fig. 3.4(a), the MTTF of the generating unit is much smaller than the time horizon N_T and therefore, the probability that the unit is available in



(b) MTTF $\gg N_T$

Figure 3.4: Availability probability functions

the future depends on its initial status only for the first few time steps of the time horizon. For time steps higher than 100, the probability that the unit is available is practically the same regardless of the initial status of the unit. On the other hand, in Fig. 3.4(b), the MTTF of the generating unit is much larger than N_T and therefore, the probability that the unit is available in the future significantly depends on its initial status during the whole time horizon.

Consequently, for a given time horizon and FOR, the higher the MTTF, the more influence has the current status of the unit on the prediction of its future availability. This fact becomes essential to make decisions in multi-stage stochastic programming as discussed in Chapter 6.

Next, we describe the procedure to generate a set of N_Ω scenarios representing the availability of a generating unit with a given FOR, MTTF, and MTTR for a two-stage stochastic programming problem involving N_T time steps:

- Step 1: Set to zero the scenario counter: $\omega \leftarrow 0$.

- Step 2: Update the scenario counter and set to zero the time step counter: $\omega \leftarrow \omega + 1, t \leftarrow 0$.

- Step 3: Update the time step counter: $t \leftarrow t + 1$.

- Step 4: Determine the probability that the generating unit is online in time t according to (3.3).

- Step 5: Randomly simulate a Bernoulli distribution with a probability of success equal to $p(k_{(t)} = 1)$ to obtain the value of the unit status at time t, $k_{(\omega,t)}$.

- Step 6: If $t < N_T$ go to Step 3, else go to Step 7.

- Step 7: If $\omega < N_\Omega$ go to Step 2, else the scenario generation procedure concludes.

The extension of the above procedure to a three-stage stochastic programming framework is provided below. To this end, we assume again that N_{T1} and N_{T2} are the duration of period 1 and 2, respectively. Likewise, $N_{\Omega 1}$ and $N_{\Omega 2}$ correspond, respectively, to the number of scenarios representing the unit availability during period 1 and the number of availability scenarios for period 2 for each one of the scenarios generated for period 1. Consequently, the total number of availability scenarios is equal to $N_{\Omega 1} \times N_{\Omega 2}$.

- Step 1: Set to zero the scenario counter of period 1: $\omega_1 \leftarrow 0$.

- Step 2: Update the scenario counter of period 1 and set to zero the time step counter: $\omega_1 \leftarrow \omega_1 + 1, t \leftarrow 0$.

- Step 3: Update the time step counter: $t \leftarrow t + 1$.

- Step 4: Determine the probability that the generating unit is online in time t using (3.3).

- Step 5: Randomly simulate a Bernoulli distribution with a probability of success equal to $p(k^1_{(t)} = 1)$ to obtain the value of the unit status at time t within period 1, $k^1_{(\omega_1,t)}$.

- Step 6: If $t < N_{T1}$ go to Step 3, else go to Step 7.

- Step 7: Consider the status of the generating unit at the end of period 1, $k^1_{(\omega_1,N_{T1})}$, as the initial status to generate the availability scenarios for period 2.

- Step 8: Set to zero the scenario counter for period 2: $\omega_2 \leftarrow 0$.

- Step 9: Update the scenario counter of period 2 and set to N_{T1} the time step counter: $\omega_2 \leftarrow \omega_2 + 1, t \leftarrow N_{T1}$.

- Step 10: Update the time step counter: $t \leftarrow t + 1$.

- Step 11: Determine the probability that the generating unit is online in time t using (3.3).

- Step 12: Randomly simulate a Bernoulli distribution with a probability of success equal to $p(k^2_{(t)} = 1)$ to obtain the value of the unit status at time t within period 2, $k^2_{(\omega_1,\omega_2,t)}$.

- Step 13: If $t < N_{T1} + N_{T2}$ go to Step 10, else go to Step 14.

- Step 14: If $\omega_2 < N_{\Omega 2}$ go to Step 9, else go to Step 15.

- Step 15: If $\omega_1 < N_{\Omega 1}$ go to Step 2, else the scenario generation procedure concludes.

Note that given the discrete nature of the status of a generating unit, this stochastic process could be exactly characterize by a finite set of scenarios. However, for the sake of tractability, a smaller number of scenarios is usually generated.

3.6 Scenario Reduction

As previously explained, plausible realizations of the stochastic processes describing uncertain parameters throughout a decision-making horizon are represented via a finite set of scenarios. The two scenario generation techniques presented in this chapter are based on the repeated generation of random numbers following the probability distributions that best represent the stochastic processes. As a result, the higher the number of scenarios, the more accurate the approximation of the finite scenario set to the stochastic processes involved and therefore, the closer to the optimal value the solution to the optimization problem is. However, a large number of scenarios may result in high computational burden or even intractability. For this reason, a *scenario reduction technique* is needed to identify a reduced scenario set that keeps as much as possible the stochastic properties of the original one.

Several works in the technical literature [58, 77, 83, 84, 86, 88] deal with the scenario reduction issue. In this chapter, we propose an alternative scenario reduction technique based on the values of the objective functions of single-scenario problems. The proposed technique is applied to reduce both the continuous and discrete stochastic processes involved in the decision making of power producers. Moreover, given that most decision makers are risk averse, we propose a modified version of the novel scenario reduction technique to account for the risk aversion of the decision maker when reducing the original scenario set.

In two-stage stochastic linear programming problems, it is possible to reduce a large scenario set to a smaller one that is close to the original one if measured by a so-called *probability distance*. Under mild conditions on the problem data, it can be shown that the optimal value of the smaller problem (considering the reduced scenario set) is closer to the value of the solution to the original problem (considering the original scenario set) if the scenario sets are sufficiently close in terms of this probability distance [57, 150].

If Q is a probability distribution defined over the scenario set Ω, the aim of a scenario reduction technique consists in determining a scenario subset $\Omega_S \subset \Omega$ with cardinality denoted by N_{Ω_S} and reassigning the probabilities of the preserved scenarios so that the corresponding reduced probability distribution Q' defined over the subset Ω_S is the closest to the original distribution Q in terms of a probability distance.

The most common probability distance used in stochastic optimization is the *Kantorovich distance*, $D_K(\cdot)$, defined between two probability distributions Q and Q' by the problem

$$D_K(Q, Q') = \inf_{\eta} \left\{ \int_{\Omega \times \Omega} c(\omega, \omega')\eta(d\omega, d\omega') : \right.$$
$$\left. \int_{\Omega} \eta(\cdot, d\omega') = Q, \quad \int_{\Omega} \eta(d\omega, \cdot) = Q' \right\}. \tag{3.4}$$

Problem (3.4) is known as *Monge-Kantorovich mass transportation problem* [150]. In (3.4), $c(\omega, \omega')$ is a nonnegative, continuous, symmetric function, often referred to as *cost function*, and the infimum is taken over all joint probability distributions defined on $\Omega \times \Omega$. Note that $D_K(\cdot)$ can only be properly called Kantorovich distance if function $c(\omega, \omega')$ is given by a norm.

In the case where Q and Q' are finite distributions corresponding to the sets of scenarios Ω and Ω_S, respectively, the Kantorovich distance is obtained by solving

$$D_K(Q, Q') = \min \left\{ \sum_{\substack{\omega \in \Omega \\ \omega' \in \Omega_S}} c(\omega, \omega')\eta(\omega, \omega') : \eta(\omega, \omega') \geq 0, \ \forall \omega \in \Omega, \forall \omega' \in \Omega_S, \right.$$
$$\left. \sum_{\omega' \in \Omega_S} \eta(\omega, \omega') = \pi_{(\omega)}, \ \forall \omega \in \Omega, \sum_{\omega \in \Omega} \eta(\omega, \omega') = \tau_{(\omega')}, \ \forall \omega' \in \Omega_S \right\}, \tag{3.5}$$

where $\pi_{(\omega)}$ and $\tau_{(\omega')}$ represent the probabilities of scenarios ω and ω' in sets Ω and Ω_S, respectively.

For a two-stage problem with stochasticity affecting the objective function and right-hand sides, the Kantarovich distance can be equivalently computed as [150]

$$D_K(Q, Q') = \sum_{\omega \in \Omega \setminus \Omega_S} \pi_\omega \min_{\omega' \in \Omega_S} c(\omega, \omega'). \tag{3.6}$$

As outlined in [83], expression (3.6) can be used to derive several heuristics for generating reduced scenario sets that are close to an original set in terms of the Kantarovich distance. Specifically, two different heuristic algorithms can be developed: the *backward reduction* or the *forward selection* [83]. In the backward reduction, the reduced set Ω_S is obtained by removing scenarios one at a time from the initial set Ω. On the other hand, the *forward selection* starts with an empty set ($\Omega_S = \emptyset$) and, in each step, includes in Ω_S the scenario that minimizes the Kantorovich distance between the reduced and original sets.

Given the high computational burden of the stochastic optimization models proposed in this dissertation, the forward selection is the heuristic algorithm used to reduce the scenario sets representing the stochastic processes involved. A detailed description of the forward selection algorithm is provided in Appendix B.

Note that the above scenario reduction technique is a heuristic, with no known performance guarantee. The reduced scenario set produced by the forward selection algorithm is not guaranteed to be the closest in the Kantorovich distance to the original set (over all reduced sets of the same cardinality). Moreover, we have no guarantee that the reduced set gives a good approximation to the optimal value of the original problem. Nevertheless, the empirical results reported in the literature (e.g., in [83]) indicate that the reduced sets yielded by the forward selection algorithm perform well in practice.

Next we explain the three scenario reduction techniques sketched in this section. The first one is a well-established algorithm presented in [83]; the second one is based on the objective functions of single-scenario problems [128]; and in the third one, we take account of the risk aversion of the decision maker in the scenario reduction process [145]. The three techniques are based on the forward selection algorithm whose mathematical formulation is stated in Appendix B.

3.6.1 Technique A

Let us consider the vector of random variables $\xi = \{\xi_1, \ldots, \xi_{N_T}\}$. For instance, this random vector may correspond to a stochastic process representing pool prices, demands or unit availability over a year divided into N_T time steps. We assume that this random vector is characterized by an initial set of scenarios $\{\xi_{(\omega)}, \forall \omega \in \Omega\}$.

According to [83], we can define the distance between two scenarios as the norm of the difference between the two vectors containing the values of the stochastic process for these two scenarios, i.e.,

$$c(\omega, \omega') = \|\xi_{(\omega)} - \xi_{(\omega')}\|. \tag{3.7}$$

The working of the scenario reduction technique based on the difference between pairs of random vectors is as follows:

Table 3.1: Example of Technique A: initial pool price scenario set

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$ (€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$ (€/MWh)	$\pi_{(\omega)}$
ω_1	20	20	0.1667
ω_2	20	9	0.1667
ω_3	15	15	0.1667
ω_4	9	20	0.1667
ω_5	10	9	0.1667
ω_6	9	9	0.1667

- Step 1: Compute the distance matrix $c(\omega, \omega')$ according to expression (3.7).

- Step 2: Following the fast forward selection algorithm, select the scenario that if included in the reduced set of scenarios minimizes the distance (3.6) between the resulting new reduced set (Ω_S) and the original one (Ω) using the function $c(\omega, \omega')$.

- Step 3: Repeat Step 2 until a sufficient number of scenarios is selected.

- Step 4: Update the probability associated with the selected scenarios.

Example 1

In this example, we illustrate how Technique A works to identify a reduced scenario set representing the pool prices within a stochastic decision model for a power producer. The optimization model used in this example is a simplified version of the general model (2.24), in which the available derivatives are just forward contracts.

We consider a producer owning one generating unit with a power capacity of 150 MW, zero minimum power output, and a linear cost of 8 €/MWh. This producer has to decide whether to sell its production in the pool at uncertain prices or through two forward contracts at fixed prices. The delivery period of these forward contracts spans a study horizon of 2 hours.

Table 3.1 lists the six equiprobable scenarios that characterize the pool price uncertainty throughout the two-hour horizon, and Fig. 3.5 plots them. The beginning and the end of each line represent the pool price realization during the first and the second hour of the study horizon, respectively.

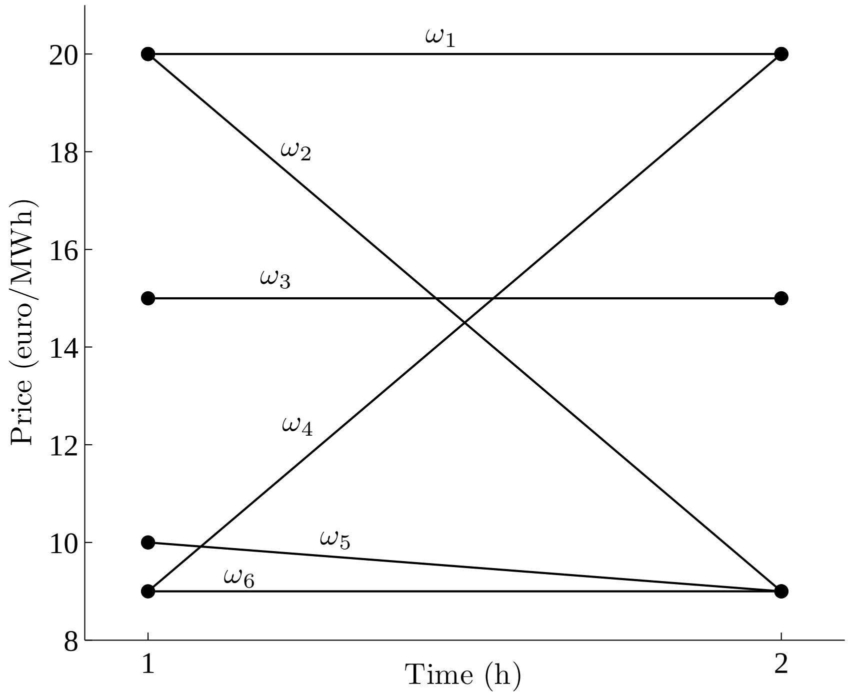


Figure 3.5: Example of Technique A: initial pool price scenario set

Table 3.2: Example of Technique A: forward contract data

# forward contract	$\lambda_{(c)}$ (€/MWh)	$P_{(c)}^{\mathrm{Max}}$ (MW)
c_1	13.5	50
c_2	9.5	50

Note that there is a scenario of high pool prices (ω_1), a scenario of medium pool prices (ω_3), two scenarios of low pool prices (ω_5 and ω_6), a scenario of decreasing pool prices (ω_2), and a scenario of increasing pool prices (ω_4).

On the other hand, the price ($\lambda_{(c)}$) and the maximum power to be sold ($P_{(c)}^{\mathrm{Max}}$) through the two available forward contracts are provided in Table 3.2.

First we solve optimization problem (2.24) for two risk aversion levels ($\alpha^{\mathrm{P}} = 0$ and $\alpha^{\mathrm{P}} = 0.66$) considering the six initial pool price scenarios. Table 3.3 shows the first-stage variables and the objective function of the power producer for these two cases.

Note that since the expected pool price during the two hours is equal to 13.75 €/MWh, which is a value higher than the fixed prices of the two forward contracts, a risk-neutral producer would sell all its energy production in the pool to maximize its expected profit. On the other hand, a risk-averse producer

Table 3.3: Example of Technique A: results considering the initial scenario tree

α^{P}	$P_{(c_1)}$(MW)	$P_{(c_2)}$(MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (€)
0	0	0	1725.00
0.66	50	50	835.29

would rather sell its production through forward contracts to hedge against the possibility of having low profits due to low pool price realizations.

Next, we illustrate the performance of Technique A to reduce the pool price scenarios provided in Table 3.1 to a smaller set of 3 scenarios according to the fast forward selection algorithm presented in Appendix B.

- *Step 0*: According to Technique A, the distance between two scenarios is determined as the norm (e.g., the Euclidean norm) of the difference between the corresponding two vectors representing the realization of the stochastic process throughout the study horizon. For instance, the distance between scenario ω_2 and ω_5, i.e., $c(\omega_2, \omega_5)$, is computed as

$$c(\omega_2, \omega_5) = \|\lambda^{\mathrm{P}}_{(\omega_2)} - \lambda^{\mathrm{P}}_{(\omega_5)}\| = \sqrt{\left(\lambda^{\mathrm{P}}_{(\omega_2,t_1)} - \lambda^{\mathrm{P}}_{(\omega_5,t_1)}\right)^2 + \left(\lambda^{\mathrm{P}}_{(\omega_2,t_2)} - \lambda^{\mathrm{P}}_{(\omega_5,t_2)}\right)^2} = $$
$$= \sqrt{(20-10)^2 + (9-9)^2} = 10 \text{ €/MWh}.$$

Proceeding this way, we determine the distance matrix $c(\omega, \omega')$, which results in

$$c(\omega, \omega') = \begin{pmatrix} 0 & 11.0 & 7.1 & 11.0 & 14.9 & 15.6 \\ 11.0 & 0 & 7.8 & 15.6 & 10.0 & 11.0 \\ 7.1 & 7.8 & 0 & 7.8 & 7.8 & 8.5 \\ 11.0 & 15.6 & 7.8 & 0 & 11.1 & 11.0 \\ 14.9 & 10.0 & 7.8 & 11.1 & 0 & 1.0 \\ 15.6 & 11.0 & 8.5 & 11.0 & 1.0 & 0 \end{pmatrix} \text{ €/MWh}.$$

- *Step 1*: For each candidate scenario ω to be selected from Ω, we compute the Kantorovich distance between the reduced set (whose only element in this step is the candidate scenario ω) and the original one, i.e.,

$$d_{(\omega_1)} = \pi_{(\omega_2)} c(\omega_1, \omega_2) + \pi_{(\omega_3)} c(\omega_1, \omega_3) + \pi_{(\omega_4)} c(\omega_1, \omega_4) + \pi_{(\omega_5)} c(\omega_1, \omega_5) +$$
$$+ \pi_{(\omega_6)} c(\omega_1, \omega_6) = 9.9 \text{ €/MWh}$$

$$\begin{aligned}
d_{(\omega_2)} &= \pi_{(\omega_1)}c(\omega_2,\omega_1)+\pi_{(\omega_3)}c(\omega_2,\omega_3)+\pi_{(\omega_4)}c(\omega_2,\omega_4)+\pi_{(\omega_5)}c(\omega_2,\omega_5)+\\
&\quad +\pi_{(\omega_6)}c(\omega_2,\omega_6)=9.2 \text{ €/MWh}\\
d_{(\omega_3)} &= \pi_{(\omega_1)}c(\omega_3,\omega_1)+\pi_{(\omega_2)}c(\omega_3,\omega_2)+\pi_{(\omega_4)}c(\omega_3,\omega_4)+\pi_{(\omega_5)}c(\omega_3,\omega_5)+\\
&\quad +\pi_{(\omega_6)}c(\omega_3,\omega_6)=6.5 \text{ €/MWh}\\
d_{(\omega_4)} &= \pi_{(\omega_1)}c(\omega_4,\omega_1)+\pi_{(\omega_2)}c(\omega_4,\omega_2)+\pi_{(\omega_3)}c(\omega_4,\omega_3)+\pi_{(\omega_5)}c(\omega_4,\omega_5)+\\
&\quad +\pi_{(\omega_6)}c(\omega_4,\omega_6)=9.4 \text{ €/MWh}\\
d_{(\omega_5)} &= \pi_{(\omega_1)}c(\omega_5,\omega_1)+\pi_{(\omega_2)}c(\omega_5,\omega_2)+\pi_{(\omega_3)}c(\omega_5,\omega_3)+\pi_{(\omega_4)}c(\omega_5,\omega_4)+\\
&\quad +\pi_{(\omega_6)}c(\omega_5,\omega_6)=7.5 \text{ €/MWh}\\
d_{(\omega_6)} &= \pi_{(\omega_1)}c(\omega_6,\omega_1)+\pi_{(\omega_2)}c(\omega_6,\omega_2)+\pi_{(\omega_3)}c(\omega_6,\omega_3)+\pi_{(\omega_4)}c(\omega_6,\omega_4)+\\
&\quad +\pi_{(\omega_5)}c(\omega_6,\omega_5)=7.8 \text{ €/MWh.}
\end{aligned}$$

The first scenario selected is then the one that minimizes the above calculated Kantorovich distance, i.e.,

$$\begin{aligned}
\Omega_S^{[1]} &= \{\omega_3\}\\
\Omega_J^{[1]} &= \{\omega_1,\omega_2,\omega_4,\omega_5,\omega_6\}.
\end{aligned}$$

- *Step 2*: Next, we update the distance matrix as follows

$$\begin{aligned}
c^{[2]}(\omega_1,\omega_2) &= \min\{c(\omega_1,\omega_2),c(\omega_1,\omega_3)\}=\min\{11.0,7.1\}=7.1\\
c^{[2]}(\omega_1,\omega_4) &= \min\{c(\omega_1,\omega_4),c(\omega_1,\omega_3)\}=\min\{11.0,7.1\}=7.1\\
c^{[2]}(\omega_1,\omega_5) &= \min\{c(\omega_1,\omega_5),c(\omega_1,\omega_3)\}=\min\{14.9,7.1\}=7.1\\
c^{[2]}(\omega_1,\omega_6) &= \min\{c(\omega_1,\omega_6),c(\omega_1,\omega_3)\}=\min\{15.6,7.1\}=7.1\\
c^{[2]}(\omega_2,\omega_1) &= \min\{c(\omega_2,\omega_1),c(\omega_2,\omega_3)\}=\min\{11.0,7.8\}=7.8\\
c^{[2]}(\omega_2,\omega_4) &= \min\{c(\omega_2,\omega_4),c(\omega_2,\omega_3)\}=\min\{15.6,7.8\}=7.8\\
c^{[2]}(\omega_2,\omega_5) &= \min\{c(\omega_2,\omega_5),c(\omega_2,\omega_3)\}=\min\{10.0,7.8\}=7.8\\
c^{[2]}(\omega_2,\omega_6) &= \min\{c(\omega_2,\omega_6),c(\omega_2,\omega_3)\}=\min\{10.0,7.8\}=7.8\\
c^{[2]}(\omega_4,\omega_1) &= \min\{c(\omega_4,\omega_1),c(\omega_4,\omega_3)\}=\min\{11.0,7.8\}=7.8\\
c^{[2]}(\omega_4,\omega_2) &= \min\{c(\omega_4,\omega_2),c(\omega_4,\omega_3)\}=\min\{15.6,7.8\}=7.8\\
c^{[2]}(\omega_4,\omega_5) &= \min\{c(\omega_4,\omega_5),c(\omega_4,\omega_3)\}=\min\{11.1,7.8\}=7.8\\
c^{[2]}(\omega_4,\omega_6) &= \min\{c(\omega_4,\omega_6),c(\omega_4,\omega_3)\}=\min\{11.0,7.8\}=7.8\\
c^{[2]}(\omega_5,\omega_1) &= \min\{c(\omega_5,\omega_1),c(\omega_5,\omega_3)\}=\min\{14.9,7.8\}=7.8
\end{aligned}$$

$$\begin{aligned}
c^{[2]}(\omega_5,\omega_2) &= \min\{c(\omega_5,\omega_2), c(\omega_5,\omega_3)\} = \min\{10.0, 7.8\} = 7.8\\
c^{[2]}(\omega_5,\omega_4) &= \min\{c(\omega_5,\omega_4), c(\omega_5,\omega_3)\} = \min\{11.1, 7.8\} = 7.8\\
c^{[2]}(\omega_5,\omega_6) &= \min\{c(\omega_5,\omega_6), c(\omega_5,\omega_3)\} = \min\{1.0, 7.8\} = 1.0\\
c^{[2]}(\omega_6,\omega_1) &= \min\{c(\omega_6,\omega_1), c(\omega_6,\omega_3)\} = \min\{15.6, 8.5\} = 8.5\\
c^{[2]}(\omega_6,\omega_2) &= \min\{c(\omega_6,\omega_2), c(\omega_6,\omega_3)\} = \min\{11.0, 8.5\} = 8.5\\
c^{[2]}(\omega_6,\omega_4) &= \min\{c(\omega_6,\omega_4), c(\omega_6,\omega_3)\} = \min\{11.0, 8.5\} = 8.5\\
c^{[2]}(\omega_6,\omega_5) &= \min\{c(\omega_6,\omega_5), c(\omega_6,\omega_3)\} = \min\{1.0, 8.5\} = 1.0.
\end{aligned}$$

Consequently,

$$c^{[2]}(\omega,\omega') = \begin{pmatrix} 0 & 7.1 & 7.1 & 7.1 & 7.1 & 7.1\\ 7.8 & 0 & 7.8 & 7.8 & 7.8 & 7.8\\ 7.1 & 7.8 & 0 & 7.8 & 7.8 & 8.5\\ 7.8 & 7.8 & 7.8 & 0 & 7.8 & 7.8\\ 7.8 & 7.8 & 7.8 & 7.8 & 0 & 1.0\\ 8.5 & 8.5 & 8.5 & 8.5 & 1.0 & 0 \end{pmatrix} \text{€/MWh}.$$

Considering the new distance matrix $c^{[2]}(\omega,\omega')$, we compute for each candidate scenario ω belonging to $\Omega_J^{[1]}$, the Kantorovich distance between the reduced set (whose elements are ω_3 and the candidate scenario ω) and the original one, i.e.,

$$\begin{aligned}
d^{[2]}_{(\omega_1)} &= \pi_{(\omega_2)}c^{[2]}(\omega_2,\omega_1) + \pi_{(\omega_4)}c^{[2]}(\omega_4,\omega_1) + \pi_{(\omega_5)}c^{[2]}(\omega_5,\omega_1) +\\
&\quad +\pi_{(\omega_6)}c^{[2]}(\omega_6,\omega_1) = 5.3 \text{ €/MWh}\\
d^{[2]}_{(\omega_2)} &= \pi_{(\omega_1)}c^{[2]}(\omega_1,\omega_2) + \pi_{(\omega_4)}c^{[2]}(\omega_4,\omega_2) + \pi_{(\omega_5)}c^{[2]}(\omega_5,\omega_2) +\\
&\quad +\pi_{(\omega_6)}c^{[2]}(\omega_6,\omega_2) = 5.2 \text{ €/MWh}\\
d^{[2]}_{(\omega_4)} &= \pi_{(\omega_1)}c^{[2]}(\omega_1,\omega_4) + \pi_{(\omega_2)}c^{[2]}(\omega_2,\omega_4) + \pi_{(\omega_5)}c^{[2]}(\omega_5,\omega_4) +\\
&\quad +\pi_{(\omega_6)}c^{[2]}(\omega_6,\omega_4) = 5.2 \text{ €/MWh}\\
d^{[2]}_{(\omega_5)} &= \pi_{(\omega_1)}c^{[2]}(\omega_1,\omega_5) + \pi_{(\omega_2)}c^{[2]}(\omega_2,\omega_5) + \pi_{(\omega_4)}c^{[2]}(\omega_4,\omega_5) +\\
&\quad +\pi_{(\omega_6)}c^{[2]}(\omega_6,\omega_5) = 4.0 \text{ €/MWh}\\
d^{[2]}_{(\omega_6)} &= \pi_{(\omega_1)}c^{[2]}(\omega_1,\omega_6) + \pi_{(\omega_2)}c^{[2]}(\omega_2,\omega_6) + \pi_{(\omega_4)}c^{[2]}(\omega_4,\omega_6) +\\
&\quad +\pi_{(\omega_5)}c^{[2]}(\omega_5,\omega_6) = 4.0 \text{ €/MWh}.
\end{aligned}$$

Both ω_5 and ω_6 minimizes the Kantarovich distance, being the first of them the new selected scenario. Thus, it follows that

$$\begin{aligned}\Omega_S^{[2]} &= \{\omega_3, \omega_5\}\\ \Omega_J^{[2]} &= \{\omega_1, \omega_2, \omega_4, \omega_6\}.\end{aligned}$$

- *Step 3*: following the same procedure as in Step 2, the new distance matrix is

$$c^{[3]}(\omega, \omega') = \begin{pmatrix} 0 & 7.1 & 7.1 & 7.1 & 7.1 & 7.1 \\ 7.8 & 0 & 7.8 & 7.8 & 7.8 & 7.8 \\ 7.1 & 7.8 & 0 & 7.8 & 7.8 & 8.5 \\ 7.8 & 7.8 & 7.8 & 0 & 7.8 & 7.8 \\ 7.8 & 7.8 & 7.8 & 7.8 & 0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0 \end{pmatrix} \text{€/MWh}.$$

Likewise, the new Kantorovich distances are equal to

$$\begin{aligned}
d_{(\omega_1)}^{[3]} &= \pi_{(\omega_2)} c^{[2]}(\omega_2, \omega_1) + \pi_{(\omega_4)} c^{[2]}(\omega_4, \omega_1) + \pi_{(\omega_6)} c^{[2]}(\omega_6, \omega_1) = 2.8 \text{ €/MWh}\\
d_{(\omega_2)}^{[3]} &= \pi_{(\omega_1)} c^{[2]}(\omega_1, \omega_2) + \pi_{(\omega_4)} c^{[2]}(\omega_4, \omega_2) + \pi_{(\omega_6)} c^{[2]}(\omega_6, \omega_2) = 2.7 \text{ €/MWh}\\
d_{(\omega_4)}^{[3]} &= \pi_{(\omega_1)} c^{[2]}(\omega_1, \omega_4) + \pi_{(\omega_2)} c^{[2]}(\omega_2, \omega_4) + \pi_{(\omega_6)} c^{[2]}(\omega_6, \omega_4) = 2.7 \text{ €/MWh}\\
d_{(\omega_6)}^{[3]} &= \pi_{(\omega_1)} c^{[2]}(\omega_1, \omega_6) + \pi_{(\omega_2)} c^{[2]}(\omega_2, \omega_6) + \pi_{(\omega_4)} c^{[2]}(\omega_4, \omega_6) = 3.8 \text{ €/MWh}.
\end{aligned}$$

The last selected scenario is ω_2 and therefore,

$$\begin{aligned}\Omega_S^{[3]} &= \Omega_S^* = \{\omega_3, \omega_5, \omega_2\}\\ \Omega_J^{[3]} &= \Omega_J^* = \{\omega_1, \omega_4, \omega_6\}.\end{aligned}$$

- *Step 4*: The last step of the algorithm consists in assigning the probabilities of the non-selected scenarios belonging to Ω_J^* to the closet scenario in Ω_S^*, i.e.,

Table 3.4: Example of Technique A: reduced pool price scenario set

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$ (€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$\pi_{(\omega)}$
ω_2	20	9	0.1667
ω_3	15	15	0.5
ω_5	10	9	0.3333

$$
\begin{aligned}
j(\omega_1) &\in \arg\,\min\{c(\omega_3,\omega_1), c(\omega_5,\omega_1), c(\omega_2,\omega_1)\}; j(\omega_1) = \omega_3 \\
j(\omega_4) &\in \arg\,\min\{c(\omega_3,\omega_4), c(\omega_5,\omega_4), c(\omega_2,\omega_4)\}; j(\omega_4) = \omega_3 \\
j(\omega_6) &\in \arg\,\min\{c(\omega_3,\omega_6), c(\omega_5,\omega_6), c(\omega_2,\omega_6)\}; j(\omega_6) = \omega_5.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
J(\omega_3) &= \{\omega_1,\omega_4\} \\
J(\omega_5) &= \{\omega_6\} \\
J(\omega_2) &= \emptyset.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\pi^*_{(\omega_3)} &= \pi_{(\omega_3)} + \sum_{\omega' \in J(\omega_3)} \pi_{(\omega')} = \pi_{(\omega_3)} + \pi_{(\omega_1)} + \pi_{(\omega_4)} = 0.5 \\
\pi^*_{(\omega_5)} &= \pi_{(\omega_5)} + \sum_{\omega' \in J(\omega_5)} \pi_{(\omega')} = \pi_{(\omega_5)} + \pi_{(\omega_6)} = 0.3333 \\
\pi^*_{(\omega_2)} &= \pi_{(\omega_2)} + \sum_{\omega' \in J(\omega_2)} \pi_{(\omega')} = \pi_{(\omega_2)} = 0.1667.
\end{aligned}
$$

Table 3.4 contains the reduced set of three scenarios and their probabilities determined by Technique A.

Fig. 3.6 illustrates the reduced scenario set obtained by Technique A. In this figure, the scenarios belonging to the reduced set are plotted in continuous lines, while the dotted lines represent the non-selected scenarios. Moreover, the color of a dotted line relates each non-selected scenario to its corresponding

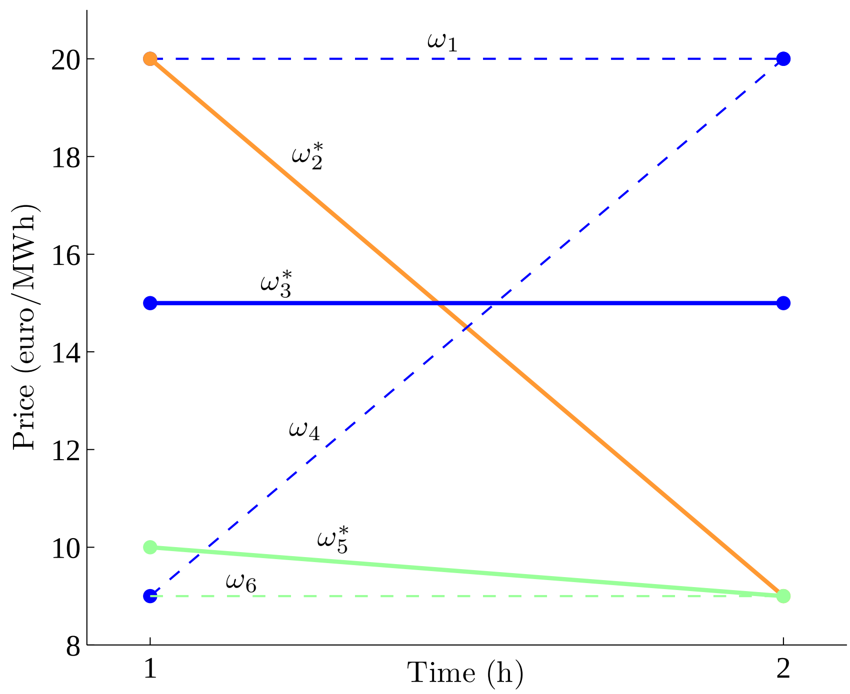


Figure 3.6: Example of Technique A: reduced pool price scenario set

selected one. For instance, ω_3 is the closet scenario according to $c(\omega, \omega')$ to the non-selected scenario ω_1 and ω_4. In other words, the selected scenario ω_3 represents the information embodied in ω_1, ω_3, and ω_4.

Note that Technique A selects the decreasing scenario ω_2; the medium scenario ω_3 on behalf of the high pool price scenario ω_1, the increasing one ω_4, and itself; and one of the low pool price scenarios ω_5 that also represents the other low pool price scenario ω_6.

3.6.2 Technique B

This technique includes information of the optimization problem to be solved into the scenario reduction procedure from the solution to single-scenario optimization problems.

Let us consider a two-stage stochastic programming problem described by its general form (2.3) and denoted by SP, which includes the random vector ξ previously defined. We also define DP_E as the deterministic problem associated with SP and obtained by replacing in SP the stochastic process ξ by its expected value $E\{\xi\}$. Let us denote the optimal value of the first-stage variables in problem DP_E as $\hat{x}$. Further, $DP_{(\omega)}$ corresponds to the single-scenario problem that results if the stochastic process ξ in problem SP is replaced by its single realization in scenario ω, $\xi_{(\omega)}$, and the first-stage variables are fixed to that obtained from DP_E, i.e., $\hat{x}$. The optimal objective function of each single-scenario optimization problem $DP_{(\omega)}$ is denoted by $z_{(\omega)}$.

The scenario reduction technique proposed in this section defines the distance between two scenarios as the absolute value of the difference between the objective functions of the corresponding single-scenario optimization problems if the first-stage decisions are fixed to those obtained from the associated deterministic problem, i.e.,

$$c(\omega, \omega') = |z_{(\omega)} - z_{(\omega')}|. \tag{3.8}$$

From expression (3.8), we can conclude that, unlike Technique A, the structure of the stochastic optimization problem as well as its objective function and constraints are accounted for by Technique B to compute the distance between two scenarios, thus yielding a *better* reduced scenario set. On the other hand, note that although the computational burden to determine the single-scenario objective functions $z_{(\omega)}$ is very low, it is definitely higher than that needed to obtain the norm-based scenario distances of Technique A.

The working of this scenario reduction technique is as follows:

- Step 1: Compute the average scenario of the stochastic process ξ, $E\{\xi\}$.

- Step 2: Solve the deterministic problem, DP_E, associated with the original stochastic problem, SP, to obtain the optimal values of the first-stage variables, $\hat{x}$.

- Step 3: Solve the single-scenario problem $DP_{(\omega)}$ corresponding to each scenario $\omega \in \Omega$. The optimal objective function value of this problem, $z_{(\omega)}$, characterizes the scenario ω.

- Step 4: Compute the distance matrix $c(\omega, \omega')$ according to expression (3.8).

- Step 5: Select the scenario that if included in the reduced set of scenarios minimizes the distance (3.6) between this new reduced set (Ω_S) and the original one (Ω) using the function $c(\omega, \omega')$ defined in (3.8).

- Step 6: Repeat Step 5 until a sufficient number of scenarios is selected.

- Step 7: Update the probability associated with the selected scenarios.

Table 3.5: Example of Technique B: objective function of single-scenario problems

# scenario	$z_{(\omega)}$ (€)	$\pi_{(\omega)}$
ω_1	3600	0.1667
ω_2	1950	0.1667
ω_3	2100	0.1667
ω_4	1950	0.1667
ω_5	450	0.1667
ω_6	300	0.1667

Example 2

Using the same stochastic optimization problem and data presented in Example 1, this new example is aimed at illustrating the functioning of Technique B to reduce the pool price scenario set provided in Table 3.1.

The first step is to determine the average pool price scenario, i.e.,

$$E\{\lambda^{\mathrm{P}}\} = (13.83, 13.67) \text{ €/MWh}.$$

Next, we solve the deterministic problem, DP_E, to obtain the first-stage decisions, which result in

$$\begin{aligned} P_{(c_1)} &= 0 \text{ MW} \\ P_{(c_2)} &= 0 \text{ MW}. \end{aligned}$$

Subsequently, we solve separately six single-scenario optimization problems to determine the producer profit corresponding to each scenario if first-stage decisions are fixed to those yielded by the associated deterministic problem. The scenario profit and its probability are provided in Table 3.5.

Based on these objective function values we can determine the distance matrix $c(\omega, \omega')$ according to (3.8). For example, the distance between scenario ω_1 and scenario ω_4 is calculated as

$$c(\omega_1, \omega_4) = |z_{(\omega_1)} - z_{(\omega_4)}| = |3600 - 1950| = \text{€}1650.$$

Table 3.6: Example of Technique B: reduced pool price scenario set

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$ (€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$\pi_{(\omega)}$
ω_2	20	9	0.5
ω_5	10	9	0.3333
ω_1	20	20	0.1667

Proceeding analogously, the distance matrix $c(\omega, \omega')$ is computed,

$$c(\omega,\omega') = \begin{pmatrix} 0 & 1650 & 1500 & 1650 & 3150 & 3300 \\ 1650 & 0 & 150 & 0 & 1500 & 1650 \\ 1500 & 150 & 0 & 150 & 1650 & 1800 \\ 1650 & 0 & 150 & 0 & 1500 & 1650 \\ 3150 & 1500 & 1650 & 1500 & 0 & 150 \\ 3300 & 1650 & 1800 & 1650 & 150 & 0 \end{pmatrix} €.$$

If we use this distance matrix in the fast forward selection algorithm presented in Appendix B, the reduced scenario set obtained as well as the scenario probabilities are those shown in Table 3.6.

Likewise, Fig. 3.7 plots both the selected and non-selected scenarios by Technique B. Note that this technique selects a scenario associated with a high profit, ω_1, a scenario corresponding to a medium profit, ω_2, which embodies other two scenarios, ω_3 and ω_4, with very similar objective functions (see Table 3.5), and a low-profit scenario, ω_5, representing the other low-profit scenario, ω_6.

It is worth mentioning that due to the large distance between scenarios ω_2 and ω_3 according to Technique A, which quantifies the distance based on the pool prices values in each hour, both of them are selected to belong to the reduced scenario set in Example 1. In contrast, note that these two scenarios have very similar single-scenario objective functions and therefore, Technique B only includes one of them in the reduced scenario set.

3.6.3 Technique C

As stated in Section 2.2.3, the Conditional Value-at-Risk is the risk measure used in the stochastic optimization models proposed in this dissertation. In line with the definition of the CVaR, only those scenarios with a profit lower than the VaR are needed to calculate the value of the CVaR. Therefore, this scenario

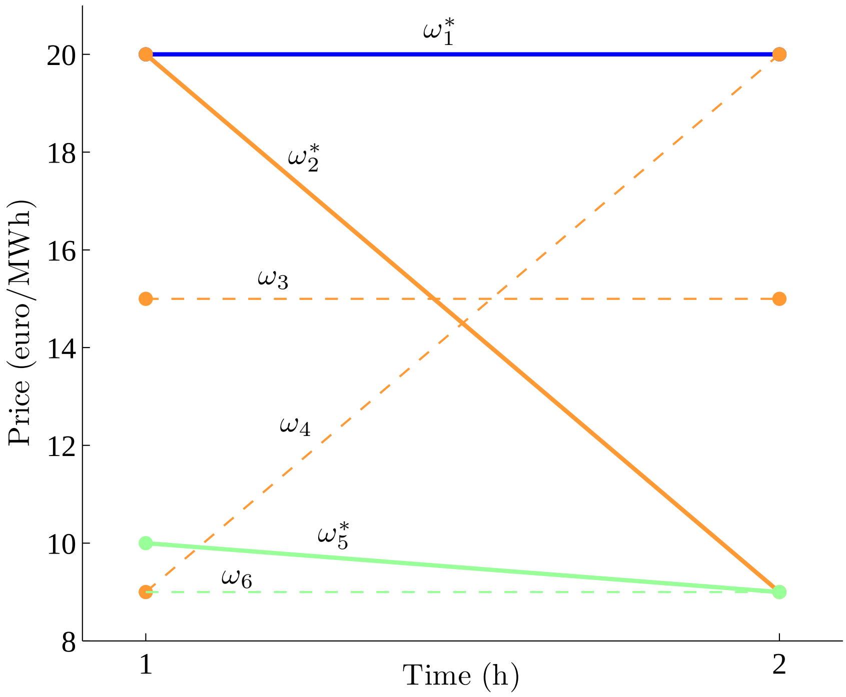


Figure 3.7: Example of Technique B: reduced pool price scenario set

reduction technique consists in reducing the original scenario set considering the values of variable $\eta_{(\omega)}$. This way, all scenarios with a profit higher than the VaR ($\eta_{(\omega)} = 0$) are merged into one single scenario with a probability equal to α, whereas the scenarios with a profit lower than VaR are reduced based on the values of $\eta_{(\omega)}$ (Fig. 3.8).

Accordingly, Technique C defines the distance between two scenarios as

$$c(\omega, \omega') = |\eta_{(\omega)} - \eta_{(\omega')}|. \tag{3.9}$$

Note that for the risk-neutral situation ($\alpha = 0$), Technique B and C are equivalent.

The working of this scenario reduction technique is as follows:

- Step 1: Compute the average scenario of the stochastic process ξ, $E\{\xi\}$.

- Step 2: Solve the deterministic problem, DP_E, associated with the original stochastic problem, SP, to obtain the optimal values of the first-stage variables, $\hat{x}$.

- Step 3: Solve the single-scenario problem $DP_{(\omega)}$ corresponding to each scenario $\omega \in \Omega$. The optimal objective function value of this problem,

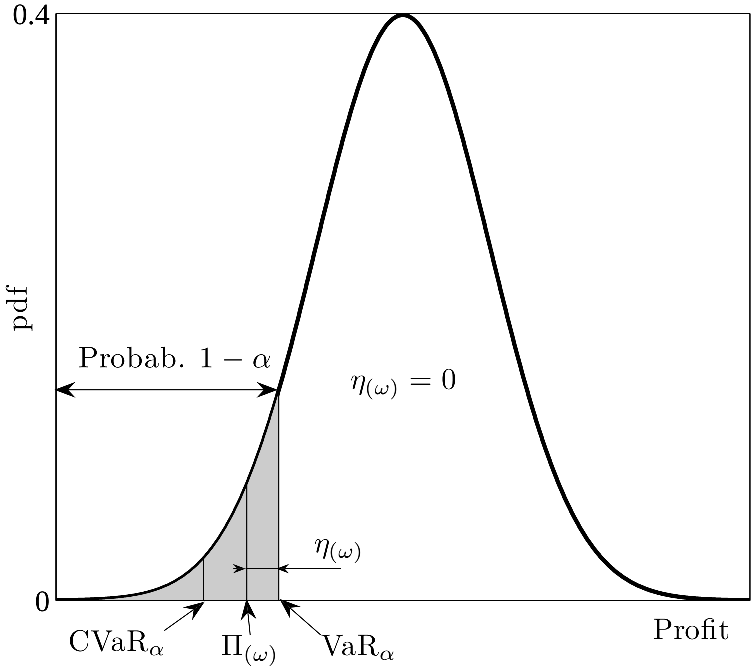


Figure 3.8: Example of Technique C: illustration of the variable η_ω

$z_{(\omega)}$, characterizes the scenario ω. The set of pairs $(z_{(\omega)}, \pi_{(\omega)})$ determines the probability distribution of the profit obtained through the set of deterministic problems.

- Step 4: Calculate the value of the CVaR_α of the profit probability distribution obtained in Step 3, and compute the values of the auxiliary variables $\eta_{(\omega)}$ associated with all scenarios of the original set.

- Step 5: Compute the distance matrix, $c(\omega, \omega')$ according to expression (3.9).

- Step 6: Select the scenario that if included in the reduced set of scenarios minimizes the distance (3.6) between this new reduced set (Ω_S) and the original one (Ω) using the function $c(\omega, \omega')$ defined in (3.9).

- Step 7: Repeat Step 6 until a sufficient number of scenarios is selected.

- Step 8: Update the probability associated with the selected scenarios.

It is important to note that this technique is based on solving single-scenario problems whose first-stage decisions are fixed to those obtained from the corresponding deterministic expected-value problem DP_E (Step 2 of the proposed algorithm). These decisions may not be equal to the optimal first-stage decisions that would be obtained if we solve the target stochastic optimization problem SP. Consequently, the pdf of the profit corresponding to the single-scenario problems obtained in step 3 may also differ from the pdf of the profit obtained is the stochastic optimization with the original scenario set is solved. In this sense, the higher the level of similarity between these two probability distributions (e.g., in terms of their moments), the higher the accuracy of the proposed algorithm to obtain a reduced scenario set considering the risk aversion level of the decision maker.

Note also that the proposed technique ignores all scenarios with a profit higher than the VaR. Consequently, non-exact estimates for the first-stage variables yield by the corresponding deterministic expected-value problem may cause some relevant scenarios for the calculation of the CVaR (those with $\eta_{(\omega)} \neq 0$) to be neglected. For this reason, in the calculation of variables $\eta_{(\omega)}$ (Step 4 of the algorithm), the value of the parameter α is reduced a quantity $\Delta\alpha$ to decrease the probability of neglecting potentially relevant scenarios (see Fig. 3.8). The higher the value of $\Delta\alpha$, the lower the probability of ignoring scenarios with $\eta_{(\omega)} \neq 0$. However, the higher the value of $\Delta\alpha$, the closer to 0 the value of α is and therefore, the less the risk aversion of the decision maker is accounted for in the scenario reduction process. Note that the modification of the parameter α affects only the scenario reduction stage, whereas its original value is maintained to solve the stochastic problem with the reduced scenario set. The values of $\Delta\alpha$ used in this dissertation are:

$$\Delta\alpha = \begin{cases} 0 & \text{if} \quad 0 < \alpha \leq 0.1 \\ 0.1 & \text{if} \quad 0.1 < \alpha \leq 0.5 \\ 0.2 & \text{if} \quad 0.5 < \alpha \leq 0.7 \\ 0.3 & \text{if} \quad 0.7 < \alpha \leq 1. \end{cases} \tag{3.10}$$

Example 3

Using the same example as in Section 3.6.1, we show below how Technique B can be modified to take into account the risk aversion of the decision maker in the reduction scenario procedure. In this example, the risk aversion level of the producer α^{P} is equal to 0.66.

The first steps of this procedure are identical to the corresponding ones in Technique B, i.e., the algorithm starts by obtaining the first-stage decisions cor-

Table 3.7: Example of Technique C: optimal values of $\eta_{(\omega)}$

# scenario	$\eta_{(\omega)}$ (€)
ω_1	0
ω_2	0
ω_3	0
ω_4	0
ω_5	1500
ω_6	1650

responding to the deterministic problem DP_E and by solving the optimization problem $DP_{(\omega)}$ for each scenario to determine its optimal objective function value, $z_{(\omega)}$.

Next, we solve optimization problem (2.24) to calculate the CVaR_α for a value of α equal to the risk aversion parameter of the producer (α^{P}) minus the value of $\Delta\alpha$ given by (3.10), i.e.,

$$\alpha = \alpha^{\text{P}} - \Delta\alpha = 0.66 - 0.2 = 0.46.$$

The value of the $\text{CVaR}_{\alpha=0.46}$ and $\text{VaR}_{\alpha=0.46}$ for the profit probability distribution provided in Table 3.5 are €977.78 and €1950, respectively. Likewise, the optimal values of the auxiliary variables $\eta_{(\omega)}$ are presented in Table 3.7.

Note that the optimal values of $\eta_{(\omega)}$ corresponding to scenarios whose objective function $z_{(\omega)}$ is higher than the value of $\text{VaR}_{\alpha=0.46}$ are equal to 0. On the other hand, for those scenarios with an objective function $z_{(\omega)}$ lower than the $\text{VaR}_{\alpha=0.46}$, the optimal value of $\eta_{(\omega)}$ can be determined as the difference between the objective function value and the $\text{VaR}_{\alpha=0.46}$.

The distance matrix obtained by using expression (3.9) is

$$c(\omega,\omega') = \begin{pmatrix} 0 & 0 & 0 & 0 & 1500 & 1650 \\ 0 & 0 & 0 & 0 & 1500 & 1650 \\ 0 & 0 & 0 & 0 & 1500 & 1650 \\ 0 & 0 & 0 & 0 & 1500 & 1650 \\ 0 & 0 & 0 & 0 & 1500 & 1650 \\ 1500 & 1500 & 1500 & 1500 & 0 & 150 \\ 1650 & 1650 & 1650 & 1650 & 150 & 0 \end{pmatrix} \text{€}.$$

Table 3.8 provides the scenarios and the probabilities of the reduced sce-

Table 3.8: Example of Technique C: reduced pool price scenario set

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$ (€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$\pi_{(\omega)}$
ω_1	20	20	0.6666
ω_5	10	9	0.1667
ω_6	9	9	0.1667

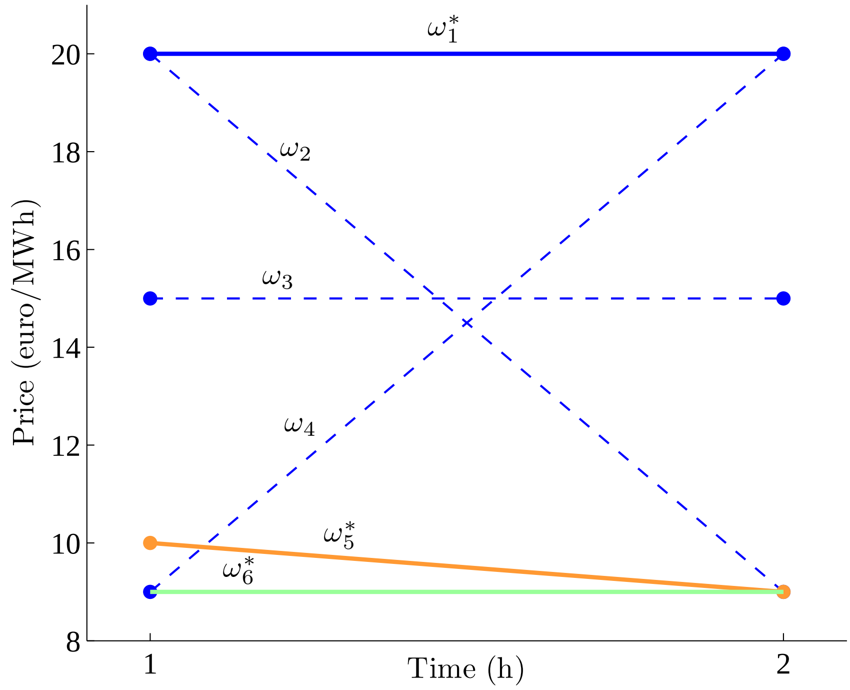


Figure 3.9: Example of Technique C: reduced pool price scenario set

nario set obtained if the fast forward selection algorithm is applied considering the distance matrix presented above.

Fig. 3.9 depicts the reduced scenario set obtained by Technique C. Note that a risk aversion α^{P} equal to 0.66 means that the producer is making its decisions to maximize the expected profit of the *worst* scenarios of the distribution. Therefore, the proposed Technique C selects one scenario, ω_1, to represent all the *good* scenarios, ω_2, ω_3, and ω_4, and the two scenarios of low pool prices, ω_5 and ω_6. This way, Technique C incorporates all the unfavorable scenarios into the reduced scenario set since risk-averse decisions are made based on these scenarios.

3.6.4 Comparison

The main purpose of the three techniques reported in Sections 3.6.1, 3.6.2, and 3.6.3 is to build a reduced scenario set that results in an objective function value (the CVaR in this case) as close as possible to that obtained with the original set. For this reason, these three techniques are compared based on the evolution of the CVaR error with the cardinality of the reduced scenario set. The lower the CVaR error value, the more efficient the scenario reduction technique is. The evolution of the CVaR error with the number of scenarios in the reduced set is determined as follows:

- Step 1: Select the number of scenarios in the reduced scenario set (N_{Ω_S}). This number is progressively increased from one to the total number of scenarios in the original set to obtain the evolution of the CVaR error with the cardinality of the reduced scenario set.

- Step 2: Use one of the considered scenario reduction techniques to select N_{Ω_S} scenarios according to its corresponding distance definition and the fast forward selection algorithm.

- Step 3: Solve the considered stochastic optimization problem taking into account only the N_{Ω_S} scenarios selected in the previous step, and obtain the value of first-stage decisions.

- Step 4: Fix the values of the first-stage decisions to those calculated in Step 3 and solve one single-scenario optimization problem for each scenario of the original set.

- Step 5: Compute the value of the CVaR_α of the profit distribution determined in the preceding step.

- Step 6: Calculate the relative error between the CVaR obtained in Step 5 and the CVaR corresponding to the stochastic model solved with the total number of scenarios. Note that the latter value of the CVaR is independent of the scenario reduction technique and thus, is used as benchmark.

It is relevant to note that in the simulations carried out in the examples and case studies reported below, the number of scenarios in the original set is

Table 3.9: Comparison example: $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ values and errors (in brackets)

α^{P}	Original tree	Technique A	Technique B	Technique C
0	1725.00	1700.00 (1.45%)	1725.00 (0%)	1725.00 (0%)
0.66	835.29	820.59 (1.76%)	820.59 (1.76%)	835.29 (0%)

small enough for the stochastic optimization problem to be solvable considering all scenarios and therefore, the CVaR error can be calculated to compare the different scenario reduction techniques. However, in most practical applications, the stochastic optimization problem with the original scenario set is intractable. In this case, we may use initially a small reduced scenario set that makes the problem tractable. Then, the cardinality of this initial set should be increased until the CVaR stabilizes.

Example 4

In this example, we make use of the procedure presented above to compare scenario reduction Techniques A, B, and C according to the objective function obtained if the pool price uncertainty is characterized by the reduced scenario set given by each technique and indicated in Tables 3.4, 3.6, and 3.8, respectively. Table 3.9 provides the value of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ calculated in Step 5 of the procedure in this subsection for two risk aversion levels ($\alpha^{\mathrm{P}} = 0$ and $\alpha^{\mathrm{P}} = 0.66$) and for the reduced scenario set yielded by each scenario reduction technique. The second column presents the objective function obtained if the original scenario set is considered, and the relative error calculated in Step 6 for each technique is shown in parentheses.

Firstly, note that the objective functions provided by Technique A for both risk aversion levels, $\alpha^{\mathrm{P}} = 0$ and $\alpha^{\mathrm{P}} = 0.66$, differs from those obtained with the original scenario tree. Secondly, since Technique B incorporates the characteristics of the optimization problem to be solved into the scenario reduction procedure, the objective function obtained with the reduced scenario set provided by this technique for $\alpha^{\mathrm{P}} = 0$ is equal to the CVaR obtained with the original scenario set. However, observe that this technique has an error different from 0% for $\alpha^{\mathrm{P}} = 0.66$. Finally, the use of Technique C to select the reduced scenario tree allows accounting for the risk aversion level of the decision maker in the scenario reduction procedure and therefore, the error achieved with this technique for the risk-averse situation ($\alpha^{\mathrm{P}} = 0.66$) is equal to 0%.

3.7 Case Study 1: Reduction of Pool Price Scenarios

This section is aimed at illustrating the performance of the two proposed scenario reduction techniques (Techniques B and C) vs. the existing one (Technique A) [83] to select a smaller scenario set characterizing the pool price of electricity within a power producer decision-making model. We use the general problem (2.24) presented in Section 2.3.4 of Chapter 2, but considering that the only available derivatives are forward contracts to sell electricity during a given future time period. Moreover, the forced outage rate of the generating unit is neglected in this case study, i.e., the only source of uncertainty involved in the stochastic optimization model is the pool price. Further details about the optimization problem to select the optimal forward contracts of a power producer can be found in Chapter 4.

The case study reported below considers a planning horizon of one month. Historical data from the Spain zone of the electricity market of the Iberian Peninsula [4] in year 2000 are used to adjust the parameters of both the autoregressive and moving average parts of an ARIMA model. Once these parameters are estimated, 200 equally probable pool price scenarios are generated according to the procedure explained in Section 3.4 by randomly simulating the non-explicable part of the pool price series, which consists in a Gaussian random variable of zero mean and a standard deviation equal to 0.19. For the sake of tractability, the 24 pool prices of each day are aggregated in six values per day as explained in Appendix A.

The producer is assumed to own only one generating unit with capacity and minimum power output of 450 and 50 MW, respectively. The quadratic production cost of this unit is approximated by the set of four linear blocks characterized in Table 3.10. Moreover, the risk aversion parameter of the producer (α^{P}) varies from 0 (risk-neutral case) to 0.9 (highly risk-averse case).

Five forward contracts are available during the considered month: one spans the whole month and the other four cover each one of the weeks of the month. The maximum quantity that can be sold through each forward contract is 200 MW for the monthly contract and 150 MW for each of the weekly ones. The forward contract data are given in Table 3.11.

Table 3.10: Scenario reduction case study 1: piecewise linear production cost

# block	$P_{(b)}^{\mathrm{Max}}$ (MW)	$\lambda_{(b)}$ (€/MWh)
b_1	150	12.0
b_2	250	12.5
b_3	350	13.0
b_4	450	13.5

Table 3.11: Scenario reduction case study 1: forward contract quantity and duration

# contract	Power (MW)	Duration
c_1	200	Month
c_2	150	Week 1
c_3	150	Week 2
c_4	150	Week 3
c_5	150	Week 4

Note that in the stochastic optimization model used in this case study, the first-stage variables are the amounts of power sold through the available forward contracts. Each one of these first-stage variables depends mainly on the value of each forward contract price relative to the average pool price throughout its delivery period. For this reason, a set of different prices for each forward contract are considered to analyze the performance of the proposed scenario reduction techniques for high, medium and low forward contract prices. For this purpose, a *critical price* is calculated per forward contract as the price for which the optimal objective function value is the same independently of whether or not the power producer signs the considered forward contract. Therefore, if the price of the contract is much higher than the critical price, the producer sells the maximum power possible through such a forward contract. On the other hand, if the forward contract price is much smaller than the critical price the producer reduces the power sold through the contract below the maximum level, eventually becoming 0 if the contract price is low enough. However, if the forward contract price is close to its critical price, the pool price scenarios belonging to the reduced set can significantly alter first-stage decisions and therefore, the accuracy of the scenario reduction technique becomes a very outstanding issue.

To calculate the value of the critical price of a given forward contract two optimization problems are solved considering that the other forward contracts are not available. In the first one, the power sold through the contract is fixed to 0 MW ($P_{(c)} = 0$). The value of the objective function of this problem is denoted by $\mathrm{CVaR}_{P_{(c)}=0}$. In the second one, the forward contract price is set to 0 €/MWh ($\lambda_{(c)} = 0$) and the power sold through the contract is fixed to its maximum value ($P_{(c)} = P_{(c)}^{\mathrm{Max}}$). The value of the objective function of this problem is referred to as $\mathrm{CVaR}_{\lambda_{(c)}=0}$. Taking into account that the CVaR is a coherent risk measure [140], and therefore positively homogeneous, if the price of the contract is increased a given value and the power sold through the contract is maintained to its maximum value, the CVaR increases a quantity equal to the new price times the maximum power sold times the contract duration, i.e.,

$$\mathrm{CVaR}(\lambda_{(c)}) = \mathrm{CVaR}_{\lambda_{(c)}=0} + \lambda_{(c)} \cdot P_{(c)}^{\mathrm{Max}} \cdot T_{(c)}, \tag{3.11}$$

where $P_{(c)}^{\mathrm{Max}}$ and $T_{(c)}$ are the maximum power that can be sold and the duration of the forward contract c, respectively. Therefore, the critical price $\lambda_{(c)}^{*}$ is mathematically determined by making expression (3.11) equal to $\mathrm{CVaR}_{P_{(c)}=0}$, i.e.,

$$\begin{aligned}
&\mathrm{CVaR}_{P_{(c)}=0} = \mathrm{CVaR}(\lambda_{(c)}^{*}) \\
&\mathrm{CVaR}_{P_{(c)}=0} = \mathrm{CVaR}_{\lambda_{(c)}=0} + \lambda_{(c)}^{*} \cdot P_{(c)}^{\mathrm{Max}} \cdot T_{(c)} \\
&\lambda_{(c)}^{*} = \frac{\mathrm{CVaR}_{P_{(c)}=0} - \mathrm{CVaR}_{\lambda_{(c)}=0}}{P_{(c)}^{\mathrm{Max}} \cdot T_{(c)}}.
\end{aligned} \tag{3.12}$$

Note that the critical price for each forward contract is computed considering that the rest of forward contracts are not available and therefore, its calculation does not include the possible interactions between different contracts. Thus, we can use the value obtained by (3.12) as an approximation to the actual critical forward price.

Table 3.12 provides the critical price of the five available contracts for the risk-neutral case ($\alpha^{\mathrm{P}} = 0$) and for a highly risk-averse case ($\alpha^{\mathrm{P}} = 0.9$). Note that the higher the risk aversion of the producer, the lower the critical price of any forward contract. The reason is that a very risk-averse producer is willing to hedge its profits against the pool price volatility through a forward contract even if the price of such a contract is low.

For each forward contract, ten different uniformly distributed prices between the critical prices for $\alpha^{\mathrm{P}} = 0.9$ and $\alpha^{\mathrm{P}} = 0$ are considered. These ten

Table 3.12: Scenario reduction case study 1: critical price of forward contracts (€/MWh)

# contract	$\alpha^{\mathrm{P}} = 0$	$\alpha^{\mathrm{P}} = 0.9$
c_1	26.88	20.23
c_2	24.52	20.77
c_3	26.98	20.58
c_4	28.04	19.67
c_5	27.90	20.40

Table 3.13: Scenario reduction case study 1: sets of prices for forward contracts (€/MWh)

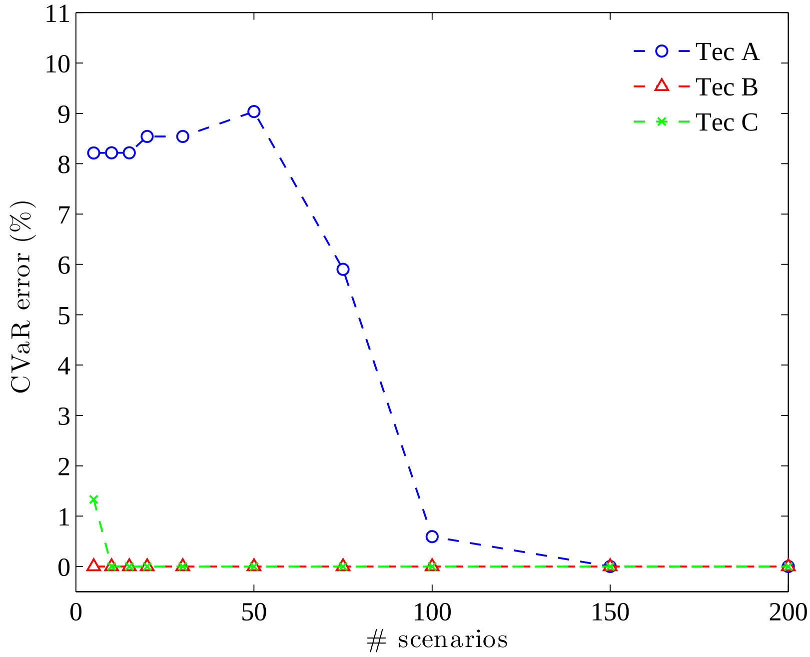
	# contract				
Set	c_1	c_2	c_3	c_4	c_5
1	19.73	20.27	20.08	19.17	19.90
2	20.58	20.80	20.90	20.21	20.84
3	21.43	21.33	21.72	21.25	21.79
4	22.28	21.85	22.55	22.29	22.73
5	23.13	22.38	23.37	23.33	23.68
6	23.98	22.91	24.19	24.38	24.62
7	24.83	23.44	25.01	25.42	25.57
8	25.68	23.96	25.84	26.46	26.51
9	26.53	24.49	26.66	27.50	27.46
10	27.38	25.02	27.48	28.54	28.40

sets of contract prices are listed in Table 3.13.

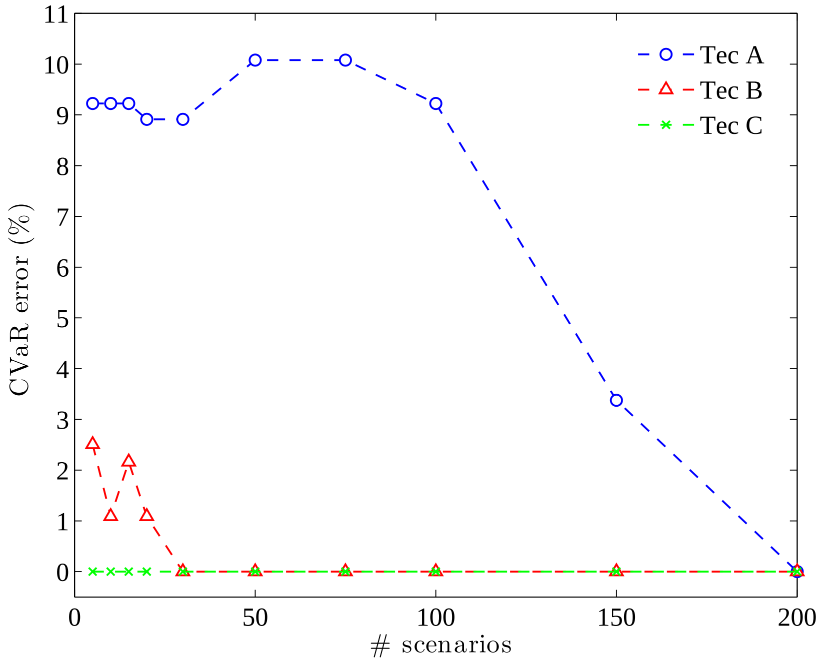
The aim of solving ten different optimization problems (one for each set of contract prices) for each value of the risk aversion α^{P} is to consider a variety of relevant cases. Although the critical prices have been only calculated for $\alpha^{\mathrm{P}} = 0$ and $\alpha^{\mathrm{P}} = 0.9$ (extreme cases), there exists in fact a critical price for each value of α^{P} and each contract. However, these values are between the two critical prices in Table 3.12 and thus, they are adequately represented using the ten price sets of Table 3.13.

Six values for the risk aversion parameter ($\alpha^{\mathrm{P}} = 0, 0.2, 0.4, 0.6, 0.8, 0.9$) and ten values for contract prices (Table 3.13) are then considered. Therefore, the three scenario reduction techniques described in Sections 3.6.1, 3.6.2, and 3.6.3 are tested on sixty different optimization problems.

Fig. 3.10 compares the evolution of the CVaR error (computed as explained in Section 3.6.4) with respect to the number of scenarios in the reduced set for



(a) Contract set 6. $\alpha^{\mathrm{P}} = 0.2$



(b) Contract set 2. $\alpha^{\mathrm{P}} = 0.8$

Figure 3.10: Scenario reduction case study 1: CVaR error evolution with the number of scenarios in the reduced set for selected α^{P} and contract price values

each scenario reduction technique, two values of the risk aversion parameter (α^{P}), and two sets of contract prices. Note that techniques B and C outperform Technique A for both low and high values of the risk aversion parameter ($\alpha^{\mathrm{P}} = 0.2$ and $\alpha^{\mathrm{P}} = 0.8$), thereby requiring fewer scenarios to reach a low value of the CVaR error. For instance, if $\alpha^{\mathrm{P}} = 0.2$, the number of scenarios needed by techniques A, B and C to reduce the CVaR error below 1% are 98, 5, and 7, respectively. If the risk aversion parameter is higher ($\alpha^{\mathrm{P}} = 0.8$), the difference is even higher, being required 185, 21, and 5 scenarios, respectively. Note that for a low value of α^{P}, Techniques B and C perform similarly, whereas for the highly risk-averse case, Technique C reduces the error of the CVaR faster than Technique B. The reason for this is that Technique C takes into account the risk aversion parameter α^{P} in the algorithm to select the reduced scenario set.

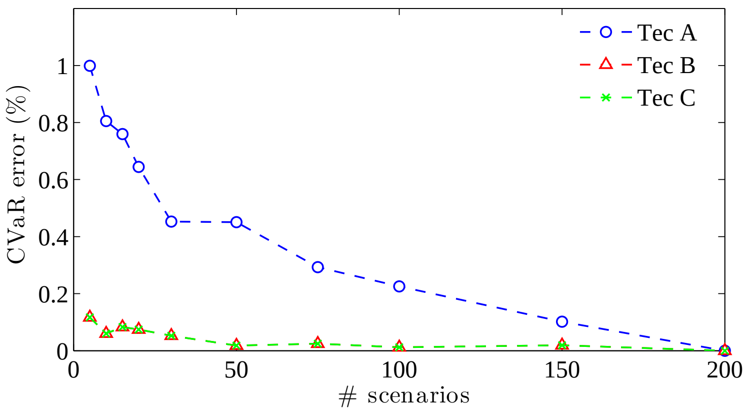
To quantify the performance of the techniques using the 60 considered optimization problems, a mean value of the CVaR error over the contract price sets is computed for each value of α^{P}. Fig. 3.11 depicts this average error for three values of the risk aversion parameter ($\alpha^{\mathrm{P}} = 0, 0.6, 0.9$). From this figure we can infer that the proposed techniques, namely Techniques B and C, produce reduced scenario sets that generally yield CVaR values closer to that obtained with the original scenario set (200 scenarios). Moreover, the higher the risk aversion parameter α^{P}, the more effective Technique C is in comparison with Technique B.

Lastly, Fig. 3.12 represents the mean value of the CVaR error over all values of the risk aversion parameter and forward prices for the three techniques. It can be concluded that, at least on average, Techniques B and C result in significantly smaller CVaR errors than those obtained using Technique A.

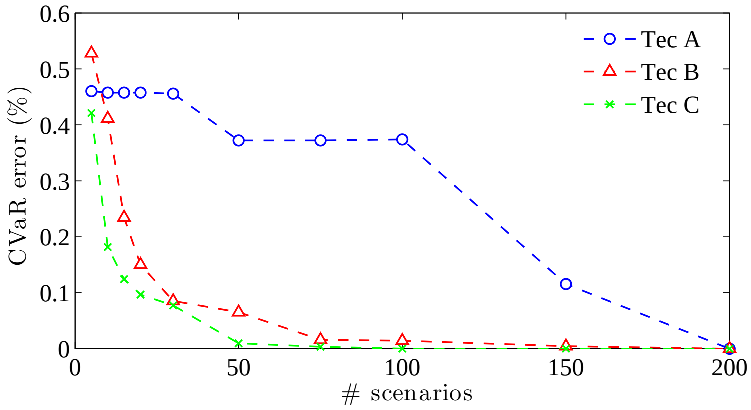
3.8 Case Study 2: Reduction of Availability Scenarios

In this section, the proposed scenario reduction techniques are used to determine small scenario sets representing the uncertainty associated with the unexpected forced outages of the generating units owned by the power producer. As in the previous case study, we use the general model (2.24) to evaluate the performance of the different scenario reduction techniques presented in this chapter.

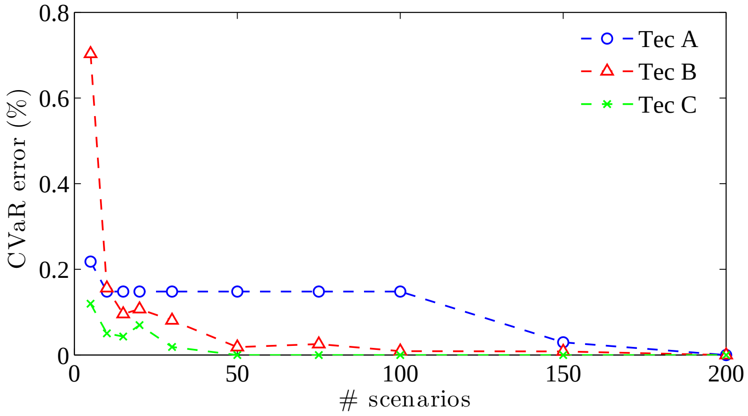
The power producer is assumed to own one generating unit and the consid-



(a) $\alpha^{\mathrm{P}} = 0$



(b) $\alpha^{\mathrm{P}} = 0.6$



(c) $\alpha^{\mathrm{P}} = 0.9$

Figure 3.11: Scenario reduction case study 1: average CVaR error evolution with the number of scenarios in the reduced set and selected α^{P} values

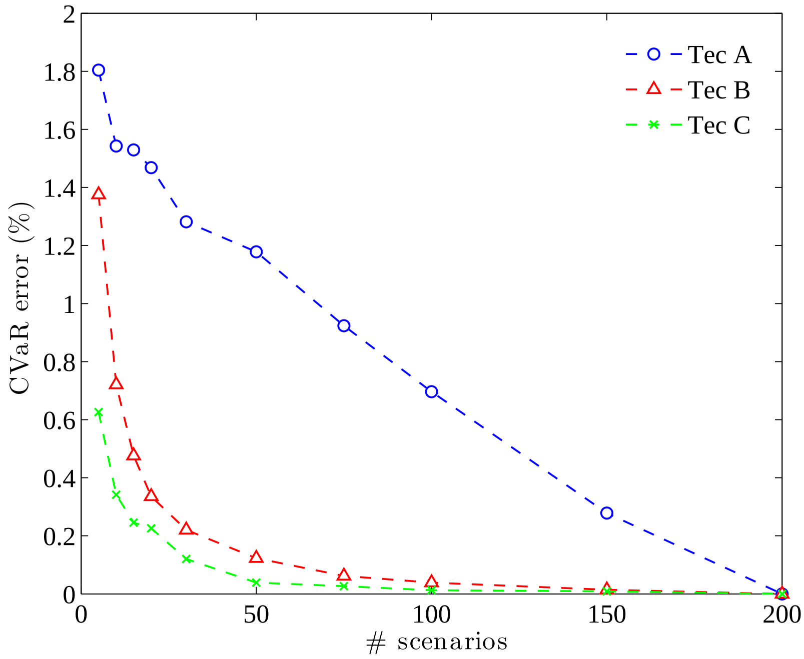


Figure 3.12: Scenario reduction case study 1: average CVaR error evolution with the number of scenarios in the reduced set

Table 3.14: Scenario reduction case study 2: unit availability data

FOR (%)	MTTF (h)	MTTR (h)
5	500	26
10	400	45
20	300	75

ered planning horizon expands one month. Since the purpose of this section is to assess the performance of the three described techniques to reduce availability scenario sets, the uncertainty related to pool prices is characterized by the same set of 20 scenarios in all cases. This set is generated according to the procedure detailed in Section 3.4. As in the previous case study, the 24 hourly pool prices are aggregated in six time steps per day according to the procedure proposed in Appendix A.

Table 3.14 provides the data of the three different values of the Forced Outage Rate (FOR), Mean Time to Failure (MTTF), and Mean Time to Repair (MTTR) considered in this case study. Using these values and following the method provided in Section 3.5, three sets of 50 availability scenarios spanning the whole month are generated.

Once both pool price and availability scenarios are generated, the final

Table 3.15: Scenario reduction case study 2: sets of prices for forward contracts (€/MWh)

	# contract				
Set	c_1	c_2	c_3	c_4	c_5
1	17.0	15.0	16.8	18.5	19.0
2	18.0	16.0	17.8	19.5	20.0
3	19.0	17.0	18.8	20.5	21.0
4	20.0	18.0	19.8	21.5	22.0
5	21.0	19.0	20.8	22.5	23.0
6	22.0	20.0	21.8	23.5	24.0
7	23.0	21.0	22.8	24.5	25.0
8	24.0	22.0	23.8	25.5	26.0
9	25.0	23.0	24.8	26.5	27.0
10	26.0	24.0	25.8	27.5	28.0

scenario set is built by combining all possible availability scenarios with all possible price scenarios. For consistency, since the 24 hourly pool prices of each day are aggregated in six time steps per day, the availability of the production unit in each time step and scenario is calculated as the average value of the corresponding hourly availabilities. Note that if different from 0 and 1, this average value has to be logically rounded to one of these two values. Besides the value of FOR, the rest of characteristics of the generating unit are identical to those presented in Section 3.7. Further, the same six values for the risk aversion parameter ($\alpha^{\mathrm{P}} = 0, 0.2, 0.4, 0.6, 0.8, 0.9$) are considered.

We also use the same five forward contracts as in case study 1. However, their critical prices change due to the forced outage rate of the generating unit. Ten uniformly distributed contract prices are determined for each contract between its highest critical price ($\alpha = 0$ and FOR $= 5\%$) and its lowest one ($\alpha = 0.9$ and FOR $= 20\%$). Table 3.15 contains the resulting 10 forward price sets.

The three scenario reduction techniques presented in this chapter are thus used to reduce the set of scenarios representing the availability of the unit and to compute the CVaR error for 180 optimization problems (3 values of FOR, 6 risk aversion parameters, and 10 sets of forward prices). Note that to calculate the first-stage decisions associated with any of the reduced availability scenario set, the original 20 pool price scenario set is considered.

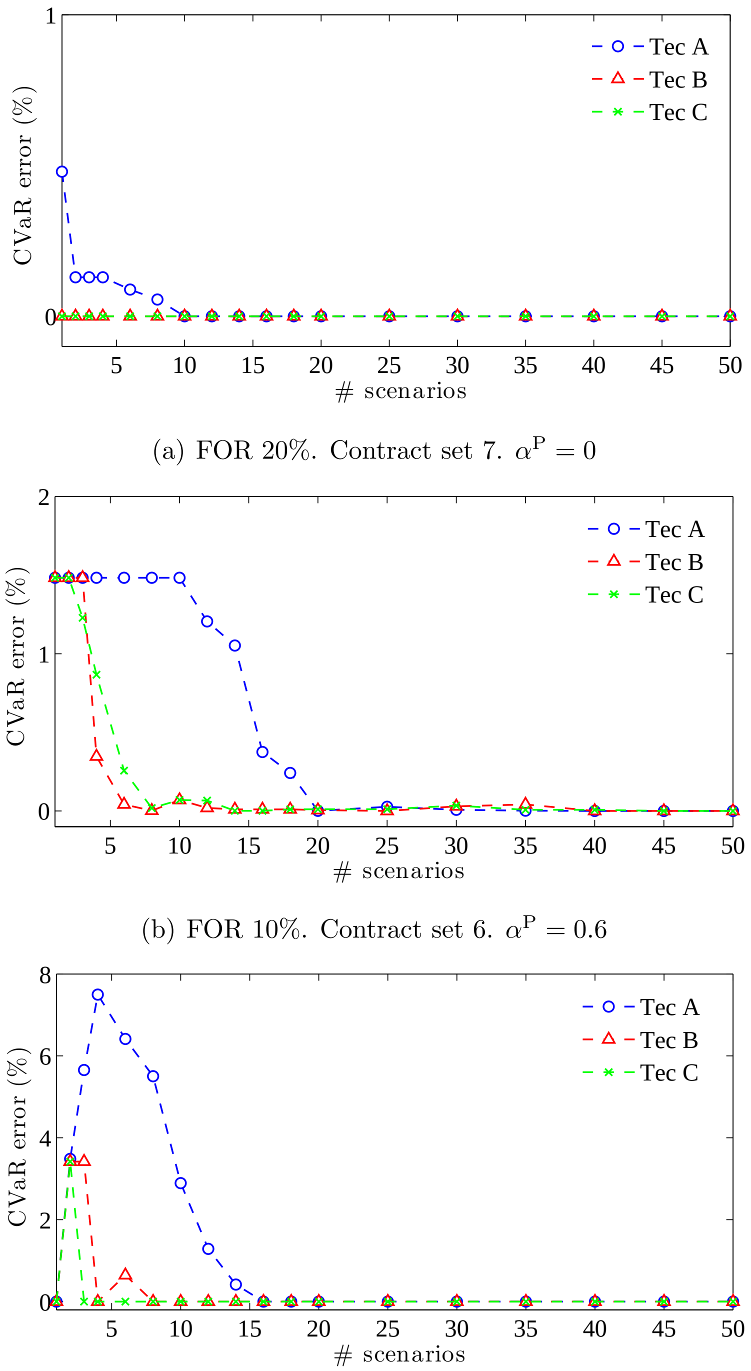
An important issue regarding Techniques B and C when reducing discrete availability scenarios is how to define the average scenario used to compute the first-stage decisions of the associated deterministic average-value problem DP_E. Selecting an adequate average scenario is important for the good performance of the proposed techniques since the single-scenario problems are solved by fixing the first-stage decisions to those obtained from the deterministic problem. An appropriate manner to generate the average availability scenario is to consider that the time between two consecutive failures and the time to repair a failure are equal to their mean values, i.e., MTTF and MTTR, respectively. Therefore, the average availability scenario used to solve the deterministic problem is built as

$$E\{k_{(\omega)}\} = [\,\underbrace{1\,1\,1\,\cdots\,1\,1}_{\text{MTTF ones}}\;\underbrace{0\,0\,0\,\cdots\,0\,0}_{\text{MTTR zeros}}\;\ldots\;\underbrace{1\,1\,1\,\cdots\,1\,1}_{\text{MTTF ones}}\,].$$

The evolution of the CVaR error obtained with the three scenario reduction techniques and for three values of the forced outage rate, risk aversion parameters, and selected forward price sets are depicted in Fig. 3.13. First, note that Techniques B and C generally outperform Technique A. Moreover, as the risk aversion parameter α^{P} becomes higher, Technique C performs better than Technique B.

Fig. 3.14 represents, for each value of FOR, the evolution of the average CVaR error over all values of forward prices and risk aversion parameters. Observe that for a forced outage rate equal to 5%, the probability of an unexpected failure is so small that the CVaR error is low even for a small set of availability scenarios (note that the scale of the vertical axis of the three plots are different). Therefore, no definite conclusions can be drawn as for the scenario reduction techniques. On the other hand, for higher FOR values (10% and 20%) the CVaR error associated with the reduced availability scenario sets is high enough to compare the three scenario reduction techniques. In this regard, Techniques B and C yield notably lower values of the CVaR error than Technique A.

Fig. 3.15 depicts, for selected values of the risk aversion parameter α^{P}, the average CVaR error over all forward prices and forced outage rates. Note that Techniques B and C are equivalent in the risk-neutral case ($\alpha^{\mathrm{P}} = 0$). For $\alpha^{\mathrm{P}} = 0.6$, Techniques B and C result in similar values for the CVaR error, and exhibit a better performance than Technique A (again, note that the scale of the vertical axis of the three plots are different). However, Technique C significantly outperforms the other two techniques for high values of the risk

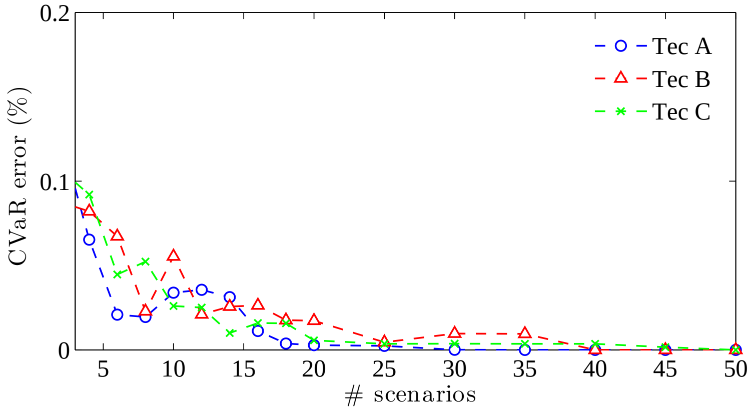


(a) FOR 20%. Contract set 7. $\alpha^{\mathrm{P}} = 0$

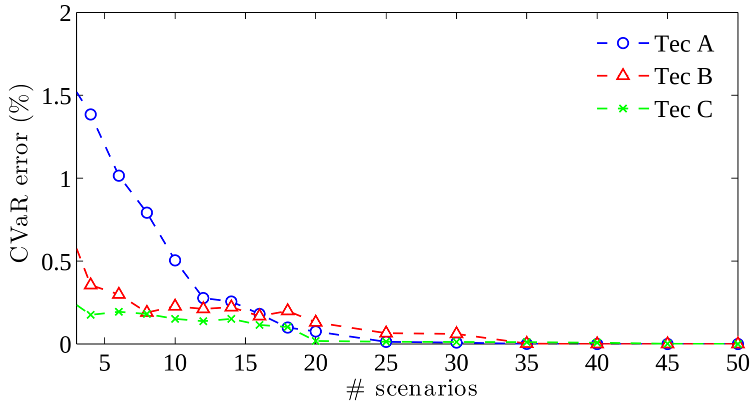
(b) FOR 10%. Contract set 6. $\alpha^{\mathrm{P}} = 0.6$

(c) FOR 20%. Contract set 10. $\alpha^{\mathrm{P}} = 0.9$

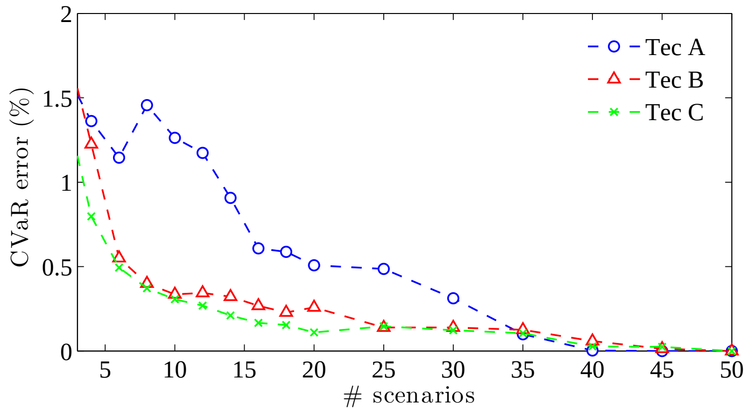
Figure 3.13: Scenario reduction case study 2: CVaR error evolution with the number of scenarios in the reduced set for selected FOR, α^{P}, and contract price values



(a) FOR 5%

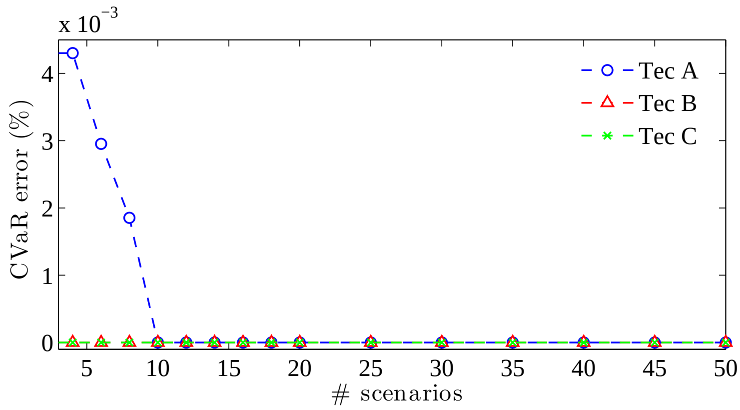


(b) FOR 10%

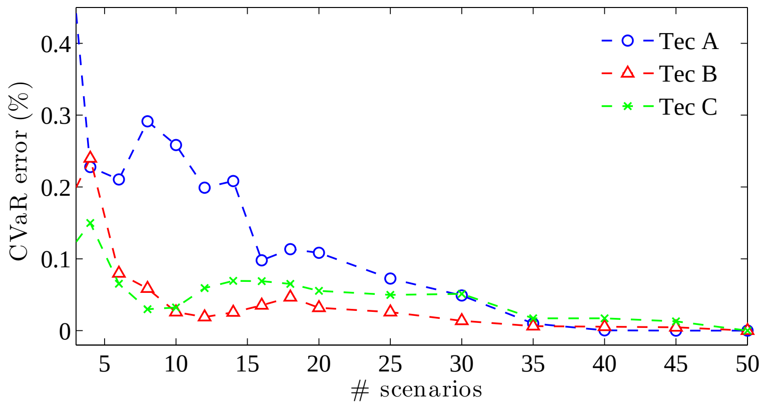


(c) FOR 20%

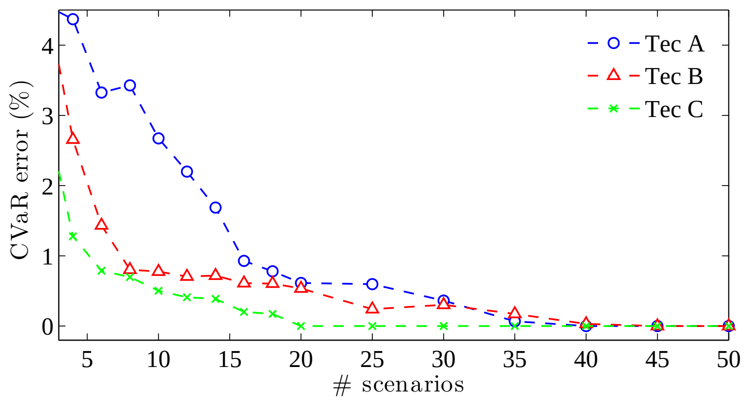
Figure 3.14: Scenario reduction case study 2: average CVaR error evolution with the number of scenarios in the reduced set for selected FOR values



(a) $\alpha^{\mathrm{P}} = 0$



(b) $\alpha^{\mathrm{P}} = 0.6$



(c) $\alpha^{\mathrm{P}} = 0.9$

Figure 3.15: Scenario reduction case study 2: average CVaR error evolution with the number of scenarios in the reduced set for selected α^{P} values

aversion parameter, e.g., $\alpha^{\mathrm{P}} = 0.9$.

An average value of the CVaR error is calculated using the 180 optimization problems considered. Fig. 3.16 depicts the evolution of this average CVaR

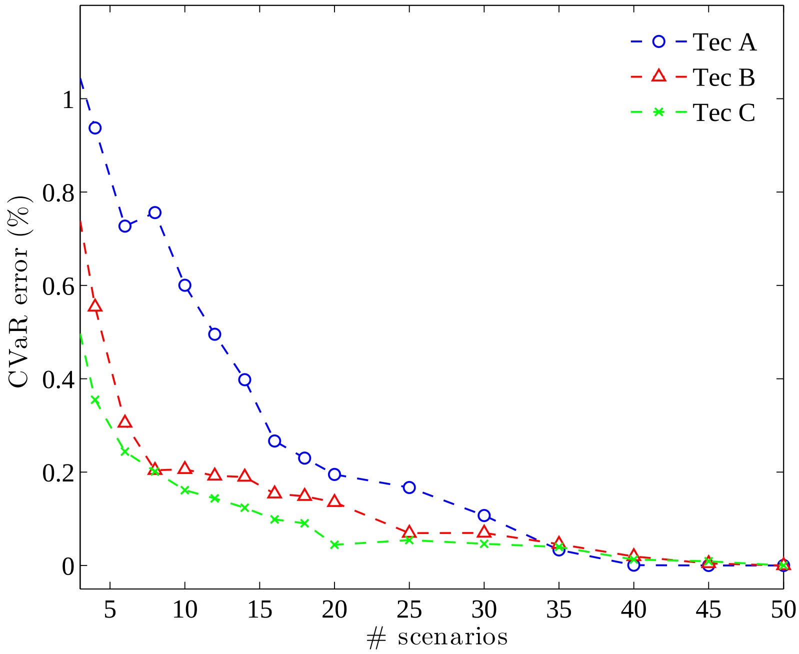


Figure 3.16: Scenario reduction case study 2: average CVaR error evolution with the number of scenarios in the reduced set

error with respect to the cardinality of the reduced availability scenario set. Note that the two scenario reduction techniques proposed in this dissertation, i.e., Techniques B and C, reduce the original availability scenario set more effectively than the technique described in [83], namely Technique A. In fact, the number of availability scenarios needed by Techniques A, B, and C to reduce the CVaR error below 0.1% are 31, 23, and 16, respectively.

3.9 On Computational Burden and Tractability

Note that the two scenario reduction techniques proposed in this chapter (Techniques B and C) entail a higher computational burden than that proposed in [83] (Technique A), as they require the solution to all single-scenario optimization problems to determine their corresponding distance matrices. Accordingly, the computational times needed by Techniques B and C are significantly higher than that required by Technique A.

More especifically, using CPLEX 11.0.1 [6] under GAMS [5] on a Linux-based server with one processor clocking at 2.6 GHz and 32 GB of RAM, the computational time required by Techniques B and C to solve 200 single-scenario optimization problems is lower than 2 minutes, while the corresponding time needed by Technique A is negligible.

However, two observations need to be made in favor of the proposed techniques. Firstly, it should be noted that the solutions to the single-scenario problems are independent and therefore, the computational burden of Techniques B and C can be reduced as much as needed by using parallel computation strategies. Secondly, it is worth mentioning that the use of a scenario reduction technique must not be necessarily seen as a way to gain in computational efficiency, but rather as a mean to obtain an approximate solution to a problem that becomes computationally intractable if the original scenario tree is used.

The computational time required by the fast forward selection algorithm presented in Appendix B increases significantly with the cardinality of the reduced scenario set. For instance, this algorithm requires 30 seconds to select 5 scenarios out of an original set of 50 availability scenarios, and 2 minutes to build a reduced set of 40 scenarios.

3.10 Summary

Numerous problems in engineering and science are exposed to uncertainty due to the randomness of the phenomena involved. Stochastic programming is a framework for modeling and solving optimization problems that include uncertain parameters. In order to solve efficiently a stochastic optimization model, the uncertain parameters have to be characterized by a set of scenarios, which represent different plausible realizations of the random parameters in the future.

In this chapter, we present two techniques to generate scenario sets characterizing the uncertain parameters involved in decision-making models for power producers. The first technique makes use of the ARIMA model theory to explain and forecast the behavior of the electricity pool price. On the other hand, the characterization of a generating unit operational status via a Markov model allows us to generate a scenario set representing its availability. Furthermore, we explain in this chapter how these two techniques can be used to generate scenarios for both two-stage and multi-stage stochastic optimization problems.

Logically, a larger number of scenarios characterizes more accurately the stochastic phenomena involved at the cost of increasing the computational burden of the stochastic optimization problem. For this reason, the selection of an adequate scenario reduction technique that makes the corresponding model tractable while keeping as much as possible the information contained in the original scenario set becomes a fundamental issue when solving stochastic optimization problems.

This chapter proposes two scenario reduction techniques to trim down the size of a scenario tree representing a stochastic process within a two-stage stochastic optimization framework. Further research needs to be done to use the proposed techniques to reduce scenario sets pertaining to stochastic programming problems with more than two stages.

The first proposed technique accounts for information on the structure of the optimization problem to be solved to select the scenarios belonging to the reduced set. On the other hand, the second technique explicitly consider the risk aversion of the decision maker when reducing the number of scenarios.

The algorithm of the two proposed scenario reduction techniques as well as the technique traditionally used in the technical literature are illustrated in detail with three electricity-market examples. Likewise, the good performance of the proposed techniques is highlighted and discussed via two real-world trading problems pertaining to electricity markets. In the first one, we compare three techniques to reduce the scenario set characterizing a continuous stochastic process, namely, the pool price of electricity; while in the second one, the scenario set representing a discrete stochastic process, i.e., the availability of a generating unit, is reduced by the three techniques described in this chapter.

Chapter 4

Forward Contracting Considering Unit Failures

4.1 Introduction

Among all energy commodities, electricity is the one with the most volatile pool price, which turns decision making by electricity market participants into a complex issue. For instance, a power producer that sells its production in the pool faces the risk associated with such a high price volatility, thus obtaining either high or low profits depending on the specific electricity price realization. Alternatively, there exists a wide variety of medium-term agreements, such as forward contracts [52, 93], which allow market participants to sell or buy electricity at fixed prices during a certain delivery period in the future (a month, a quarter, etc.). This way, a power producer that sells some of its electricity production through forward contracts reduces the risk associated with pool prices.

Although usually disregarded in medium-term decision models, unexpected unit failures may have a significant impact on power producer profits. If the producer sells its production in the pool, an unexpected unit failure entails no additional selling in this market, thereby being the profit during the time steps in which the generating unit is down equal to zero. On the other hand, if the producer has signed a forward contract, which necessarily involves the obligation to sell the agreed electricity during each hour of the delivery period, the consequences of an unexpected unit failure may be even more important. During the forced outage of the generating unit, the power producer must buy enough electricity in the pool to meet its contracting selling obligations. Note that if the pool price during these time steps happens to be high, the profit of the power producer cannot only be low, but also negative.

Therefore, increasing the power to be sold through forward contracts has opposite effects when it comes to managing the two risks faced by power producers, namely the price and availability risks. In other words, whereas selling the production through forward contracts reduces the risk associated with pool price volatility, it also increases the risk of suffering low profits as a consequence of unexpected generating unit failures.

In this chapter, we introduce a two-stage stochastic programming model to determine the optimal amount of power that a risk-averse producer should sell through forward contracts taking into account the uncertainty of pool prices and unit availability.

4.2 Pool Trading

In most electricity markets, the pool is the main trading floor to sell and buy electricity and usually comprises a day-ahead market, in which energy transactions for the next day are traded, and several shorter-term markets known as adjustment markets, which enable market agents to modify the energy sold or purchased in the day-ahead market.

In the day-ahead market, producers' sale offers and consumers' and retailers' purchase bids are cleared so as to obtain the market-clearing price as well as the production and consumption schedules for the 24 hours of the following day. Due to the particular features of the electric energy as a commodity, such as non-storability, inelastic demand, and network constraints, electricity pool prices turn out to be highly volatile.

In medium-term decision-making models, the level of uncertainty involved in day-ahead decisions is much smaller than that affecting other types of decisions such as the energy to be sold through electricity derivatives or the trading of financial products. For this reason, decisions on pool trading are assumed to be made with perfect information. In other words, in the medium-term decision-making model described in this thesis, power producers decide the quantity they sell in the pool knowing the resulting price in this market, while in fact, they would have to send their offers consisting of quantity-price pairs without knowing a priori whether or not their offers will be accepted.

The assumption that a power producer trades in the day-ahead market knowing the resulting pool price does not mean that the pool price realization during the study horizon is known. In fact, the pool price represents a source of uncertainty that power producers have to face when making medium-term decisions. Thus, within a stochastic framework, the pool price is modeled through the stochastic process $\lambda^{\mathrm{P}}_{(\omega,t)}$, which represents the clearing price of energy in the pool during time step t and in scenario ω. The set of scenarios that characterizes pool price realizations in this market is built according to the methodology presented in Section 3.4 of Chapter 3.

The profit that the power producer obtains from participating in the pool ($\Pi^{\mathrm{P}}_{(\omega)}$) is formulated as

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} P^{\mathrm{P}}_{(\omega,t)} L_{(t)}, \quad \forall \omega, \tag{4.1}$$

where $P^{\mathrm{P}}_{(\omega,t)}$ is a continuous variable corresponding to the quantity in MW that the power producer sells in the pool during time step t and in scenario ω, N_T is the total number of time steps within the study horizon, and $L_{(t)}$ is the duration of time step t.

4.3 Forward Contracts

4.3.1 Definition and Characteristics

A *forward contract* is one of the simplest derivatives, being an agreement to buy or sell electricity during a certain time horizon in the future at a fixed price [93]. These types of contracts constitute an alternative to the pool [44, 103, 171], in which electricity is usually sold and bought on a daily basis at varying prices. A forward contract requires that one party assumes a so called *long position*, thus agreeing to buy the electricity during a certain specified future time period for a fixed price, as well as another party that agrees to sell electricity during the same time period and for the same price, thereby taking a so called *short position*.

Like a forward contract, a *futures contract* is an agreement between two parties to buy or sell electricity during a certain future time period for a fixed price. The main difference between both derivatives is that whereas forward contracts are usually traded over-the-counter (OTC) and are non-standardized products, futures contract are traded on an *exchange* that specifies the standard characteristics of the different available products. In this dissertation, given the minor differences between forward and futures contracts, we refer to both of them as forward contracts.

In some electricity markets, forward contract settlement is carried out based on an underlying reference price, thus being understood as purely financial contracts. However, in this thesis, for the sake of simplicity we only consider electricity forward contracts that necessarily involve the physical delivery of the agreed amount of electricity during each hour of the delivery period.

4.3.2 Trading of Forward Contracts

There exists a wide variety of tradable forward contracts involving different commodities, such as agricultural ones, precious metals, currencies or energy. Electricity forward contracts were first introduced by the New York Mercantile Exchange (NYMEX) on March 29, 1996, and since then, they have been gradually included in electricity markets worldwide. In most markets, electricity forward contracts are traded continuously, i.e., each agent can see other agents' orders that are currently active in the exchange at any time to decide its own orders, which are executed in accordance with a price-time criteria. In other words, offers to sell electricity at lower prices are executed first and, if there is more than one offer with the same price, the first submitted offer is executed first. Forward contracts at NordPool are traded continuously [1]. In other electricity markets such as OMIP in the Iberian Peninsula [3] or EEX in Germany [2], the trading of forward contracts starts with an auction process in which offers to buy and sell electricity through each forward contract are matched to obtain the corresponding settlement price. This auction process has usually a duration of less than one hour and is followed by a continuous trading for the rest of the trading day.

In the decision-making model proposed in this thesis we assume that the power producer can check the prices of all available forward contracts and that its decisions to sell energy do not affect these prices. Therefore, a forward contract is described by:

- A quantity of energy to be sold or bought.
- The period of delivery (next week, next month, etc.).
- The price at which the energy is sold or bought during each hour of the delivery period.

4.3.3 Forward Contract Revenue

Mathematically, the revenue obtained from selling though forward contracts in the derivatives market (Π^{F}) is formulated as

$$\Pi^{\mathrm{F}} = \sum_{c=1}^{N_C} \lambda_{(c)} P_{(c)} L_{(c)}, \tag{4.2}$$

where $\lambda_{(c)}$ and $L_{(c)}$ are the price and the duration of forward contract c, N_C is the total number of available forward contracts, and $P_{(c)}$ is a continuous variable that represents the amount of electricity that the power producer sells through forward contract c during each time step of its delivery period.

The first remarkable observation from equation (4.2) is the fact that the revenue obtained from selling electricity through forward contracts does not depend on scenario realizations. That is, knowing the price and the duration of a certain forward contract, the power producer can obtain an assured revenue by selling some of its production through this forward contract. Although the *actual revenue* associated with a forward contract is independent of the pool price, there also exists a *potential revenue* of a forward contract linked to the question *"could the power producer have done better?"*. It should be noted in this sense that, if the average pool price at the time of delivery of the contract is higher than the forward price, the forward contract represents an economic loss for the power producer since it could have achieved a higher profit by selling its production in the pool. On the other hand, a realized average pool price lower than the forward price tells us that the power producer made a good decision by selling its production through the forward contract. Note that potential losses associated with forward contracts make a power producer less competitive because it means that it has sold electricity at a lower price than some of its competitors.

4.3.4 Price vs. Availability Risk

A risk-neutral power producer that aims to obtain the highest expected profit possible over all scenario realizations should sell energy through a forward contract only if its known price is higher than the expected pool price throughout the delivery period of the forward contract. On the contrary, a risk-averse power producer is willing to sell electricity at a price lower than the expected pool price in order to reduce the possibility of getting low profits or financial

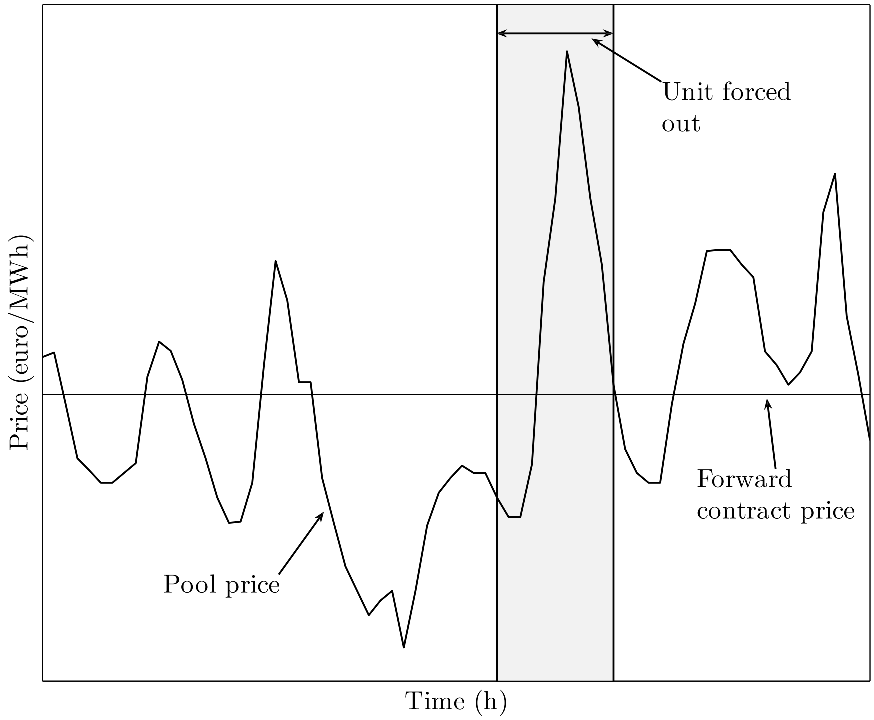


Figure 4.1: Availability risk of forward contracts

losses caused by unfavorable pool price realizations. In doing so, the expected profit is reduced to cut down the risk associated with the pool price.

The above statements are made under the assumption that the generating units owned by the power producer do not fail. However, one important question arises here: *"do unit outages have an impact on the forward contracting decisions of a power producer?"*. A preliminary answer to this question is found in Fig. 4.1. In this figure, the pool price evolution during one week as well as the price of a forward contract spanning the whole week are depicted. The shadow area represents the unexpected forced outage of the generating unit owned by the power producer. Suppose that the power producer decides to sell its production through the weekly forward contract, which involves an obligation to deliver the agreed energy during each hour of the week. In that case, during the time steps in which the unit is forced out, the only alternative of the producer to meet its forward contract commitments consists in buying the corresponding energy in the pool. Since the pool price during those time steps happens to be particularly high, the power producer incurs significant financial losses.

Therefore, although forward contracts can be seen as derivatives to reduce the risk associated with the pool price volatility, selling electricity through forward contracts increases the risk of suffering severe financial losses due to unexpected forced outages of the generating units. Being so, a power producer should determine the appropriate amount of electricity to be sold through forward contracts aiming to reduce the price risk without significantly increasing its availability risk.

4.4 Arbitrage

In order to avoid arbitrage in the decision-making models presented in this chapter, the power producer can buy energy in the pool only in those scenarios and time steps in which one or more of its generating units are forced out, i.e.,

$$\sum_{i=1}^{N_I}(k_{(i,\omega,t)} - 1)P^{\mathrm{Max}}_{(i)} \leq P^{\mathrm{P}}_{(\omega,t)}, \quad \forall\omega, \forall t. \tag{4.3}$$

Therefore, if all generating units are available, the power producer can only sells electricity in the pool. On the other hand, if some generating unit suffers from an unexpected failure, the power producer can buy in the pool the amount of energy needed to replace the production of the forced out unit.

Similarly, power producers can only sign forward contracts to sell its production, i.e.,

$$0 \leq P_{(c)}, \quad \forall c. \tag{4.4}$$

4.5 Energy Balance

Taking into account that the forward contracts considered in this model entail the physical delivery of electricity, the generated power by the units owned by the power producer in time step t and in scenario ω has to be equal to the power sold in the pool plus the power sold through forward contracts that are active during time step t, i.e.,

$$\sum_{i=1}^{N_I} P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{P}}_{(\omega,t)} + \sum_{c\in F_{(t)}} P_{(c)}, \quad \forall\omega, \forall t, \tag{4.5}$$

where $F_{(t)}$ constitutes the set of forward contracts available during time step t.

4.6 Power Producer Model for Forward Selection

4.6.1 Assumptions

The following assumptions are made to formulate the decision-making model of a power producer. Although most of them have already been discussed throughout the dissertation, they are summarized below for the reader's convenience.

1. The power producer only owns thermal units, which are dispatchable and involve a generation cost different from zero.

2. Thermal units operate within a minimum and a maximum power output and are characterized by a quadratic generation cost function, which is approximated by a piecewise linear function. Other operating constraints such as production ramp limits or minimum up and down times can be disregarded in medium-term decision-making models.

3. The power producer can sell its production in two markets: the pool, which consists in a day-ahead market with very volatile prices; and in a derivatives market, in which forward contracts to sell electricity at fixed prices are available.

4. The considered power producer behaves as a price taker in both the pool and the futures market, i.e., power producer decisions pertaining to these two markets do not have any influence on their corresponding clearing prices.

5. Since in medium-term models the level of uncertainty involving day-ahead pool decisions is much smaller than that of making decisions in the futures market, we consider that day-ahead pool decisions are made with perfect information.

6. Two sources of uncertainty affecting medium-term decisions are considered: the pool price and the availability of generating units. Moreover, these stochastic processes are assumed to be uncorrelated, i.e., the occurrence of unit failures does not affect pool prices and vice versa.

7. Arbitrage is avoided in the proposed model and therefore, the power producer can only sell electricity in both the pool and the futures market. As an exceptional case, the power producer can buy electricity in the pool to meet its contracting selling obligations when one or more of its generating units are forced out.

4.6.2 Uncertainty Characterization

In this section, uncertainties associated with both pool price and generating unit availability are characterized in the form of scenarios.

4.6.2.1 Pool Prices

The resulting price in the pool in time step t is modeled as a stochastic process denoted by $\lambda^{\mathrm{P}}_{(t)}$. The collection λ^{P} of dependent random variables $\lambda^{\mathrm{P}}_{(t)}$ throughout the decision horizon represents a stochastic process, i.e., $\lambda^{\mathrm{P}} = \{\lambda^{\mathrm{P}}_{(t)}, t = 1, ..., N_T\}$, where N_T is the total number of time steps.

The pool price evolution during the decision horizon is unknown and is thus characterized via a set of scenarios. Firstly, according to the procedure presented in Section 3.4 of Chapter 3, historical pool price data are used to estimate the parameters of an ARIMA model, and after that, hourly pool prices scenarios are generated. In this sense, each pool price scenario ϖ is a vector of N_T pool prices that represents a possible realization of the random variable $\lambda^{\mathrm{P}}_{(t)}$ over the study horizon. Each scenario ϖ has a probability of occurrence $\pi_{(\varpi)}$ such that the summation of the probabilities over all pool price scenarios is equal to 1, i.e., $\sum_{\varpi=1}^{N_\varpi} \pi_{(\varpi)} = 1$.

Seeking to reduce the computational burden to solve the stochastic optimization model considered in this chapter, while trying to keep as much information as possible of the pool price contained in the scenario set, the 24 values representing the pool price during each day of the study horizon are aggregated into a lower number of time steps. In Appendix A, a procedure to aggregate pool prices into a given number of time steps per day is proposed. Roughly speaking, this procedure determines the number of hours of each time step in order to minimize the average error between the hourly prices and the approximated ones over all possible scenarios.

Initially, a large enough number of scenarios to represent the pool price variability is generated. However, solving the optimization problem with a large number of scenarios may result in high solution times or even intractability. For this reason, a scenario reduction technique [128, 145] is needed to identify a reduced scenario set that keeps, as much as possible, the stochastic properties of the original scenario set. The scenario reduction technique applied to pool price scenarios in this thesis is described and analyzed in Section 3.6.2 of Chapter 3.

4.6.2.2 Generating Unit Availability

The status of a generating unit i during time step t and in scenario f is denoted by $k_{(i,f,t)}$, being equal to 0 if the unit is forced out, and 1 otherwise. The initial scenario set representing the availability of a generating unit is generated based on its reliability parameters, i.e., FOR (Forced Outage Rate), MTTF (Mean Time To Failure) and MTTR (Mean Time To Repair). Each availability scenario f is a $N_I \times N_T$ matrix of 0/1 values that represents the availability of the N_I generating units throughout the study horizon. Each scenario f has a probability of occurrence $\pi_{(f)}$ such that the summation of the probabilities over all availability scenarios is equal to 1, i.e., $\sum_{f=1}^{N_f} \pi_{(f)} = 1$. Details about the procedure to generate availability scenarios can be found in Section 3.5 of Chapter 3. It should be pointed out that the higher the number of generating units owned by the power producer (N_I), the higher the number of availability scenario (N_f) that is required to encompass all possible combinations of unexpected failures affecting different units.

Since the frequency of changes in the status of generating units is much lower than that related to pool price variability, it makes sense to assume that the stochastic process $k_{(i,f,t)}$ representing the availability of generating unit i can be aggregated according to the time step duration calculated to aggregate pool price scenarios. If the average value of the availability for a given time step is different from 0 or 1, it is rounded to be equal to one of these two values. For example, if the vector (1 1 1 0 0 0 0) represents the hourly unit status throughout a 7-hour time step, the aggregated unit status corresponding to that time step is equal to 0.

In the same way as pool price scenarios, the number of scenarios representing the availability of each generating unit can be trimmed down by using the scenario reduction technique proposed in Section 3.6.2 of Chapter 3.

4.6.2.3 Scenario Tree

Each scenario has to contain information about both the pool price evolution and the availability of the generating units throughout the study horizon. Therefore, the scenario tree is constructed by combining all possible pool price scenarios with all possible availability scenarios, i.e., each scenario ω in the tree is defined as a pair (ϖ,f) representing the occurrence of the pool price scenario ϖ and the availability scenario f. The probability $\pi_{(\omega)}$ of each scenario ω is equal to

$$\pi_{(\omega)} = \pi(\varpi, f) = \pi_{(\varpi)} \cdot \pi_{(f)}, \tag{4.6}$$

where, of necessity, it holds that $\sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} = 1$. Let N_ϖ and N_f be the number of price and availability scenarios, respectively, the total number of scenarios in the final tree is therefore equal to $N_\Omega = N_\varpi \times N_f$.

Each scenario is thus made up of one vector and one matrix:

1. Pool price vector, which contains the pool price for each time step t throughout the study horizon, i.e., $(\lambda^{\mathrm{P}}_{(\omega,t_1)}, \lambda^{\mathrm{P}}_{(\omega,t_2)}, ..., \lambda^{\mathrm{P}}_{(\omega,N_T)})$.

2. Availability matrix, whose values are equal to 0 or 1 depending on whether or not the generating unit i is forced out in time step t, i.e.,

$$\begin{pmatrix} k_{(i_1,\omega,t_1)} & k_{(i_1,\omega,t_2)} & \cdots & k_{(i_1,\omega,N_T)} \\ k_{(i_2,\omega,t_1)} & k_{(i_2,\omega,t_2)} & \cdots & k_{(i_2,\omega,N_T)} \\ \vdots & \vdots & \ddots & \vdots \\ k_{(N_I,\omega,t_1)} & k_{(N_I,\omega,t_2)} & \cdots & k_{(N_I,\omega,N_T)} \end{pmatrix}.$$

The set of pool price and availability scenarios, together with the decision timing of the power producer, can be arranged in a two-stage scenario tree as illustrated in Fig. 4.2.

4.6.3 Variables

Depending on the futures market trading mechanism, forward contract agreements can be signed from several years up to some days before its delivery period. Therefore, in the decision-making model proposed in this dissertation, the amount of electricity sold through a forward contract ($P_{(c)}$) is a first-stage variable because its value has to be determined without knowing the realization of the stochastic processes involved.

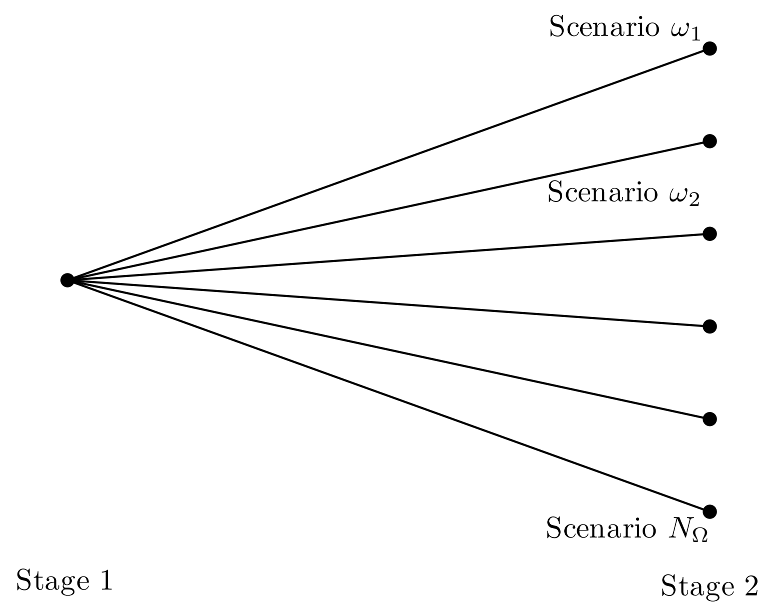


Figure 4.2: Two-stage scenario tree to model forward contracting

On the other hand, the following variables can adapt their values to each particular scenario realization of the random parameters and therefore, constitute the set of second-stage variables of the proposed stochastic optimization model:

1. Start-up and shut-down plan of each generating unit throughout the study horizon ($u_{(i,\omega,t)}$).

2. Scheduled power output for each generating unit throughout the study horizon ($P^{\mathrm{G}}_{(i,\omega,t)}$).

3. Amount of generated power to be sold in the pool for each time step of the study horizon ($P^{\mathrm{P}}_{(\omega,t)}$).

4.6.4 Objective Function

As discussed in Section 2.3.2 of Chapter 2, the objective function to be maximized by a power producer to determine its medium-term decisions is the Conditional Value-at-Risk (CVaR) of its profit probability distribution (Π) for a confidence level (α^{P}) [153], i.e.,

$$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}(\Pi) = \zeta - \frac{1}{1-\alpha^{\mathrm{P}}} \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \eta_{(\omega)}, \tag{4.7}$$

where $\pi_{(\omega)}$ represents the probability of scenario ω, and ζ and $\eta_{(\omega)}$ are two auxiliary variables to calculate the CVaR of the probability distribution as explained in Section 2.2.3 of Chapter 2. The producer profit per scenario ω is denoted as $\Pi_{(\omega)}$ and mathematically formulated as

$$\Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{F}} - C^{\mathrm{G}}_{(\omega)}. \tag{4.8}$$

$\Pi^{\mathrm{P}}_{(\omega)}$ represents the profit that the power producer obtains from participating in the pool and has the following form

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} P^{\mathrm{P}}_{(\omega,t)} L_{(t)}, \tag{4.9}$$

where $\lambda^{\mathrm{P}}_{(\omega,t)}$ represents the uncertain pool price during time step t and in scenario ω, and $P^{\mathrm{P}}_{(\omega,t)}$ is a continuous variable corresponding to the power that the producer sells in the pool during time step t and in scenario ω. N_T is the total number of time steps.

The revenue obtained from the futures market Π^{F} is independent of the scenario realization and is computed as

$$\Pi^{\mathrm{F}} = \sum_{c=1}^{N_C} \lambda_{(c)} P_{(c)} L_{(c)}, \tag{4.10}$$

where $\lambda_{(c)}$, $L_{(c)}$, and $P_{(c)}$ are the price, the duration, and the amount of electricity to be sold through forward contract c, respectively.

The cost term $C^{\mathrm{G}}_{(\omega)}$ is equal to the sum of the production cost of the different generating units owned by the power producer, i.e.,

$$C^{\mathrm{G}}_{(\omega)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} C^{\mathrm{G}}_{(i,\omega,t)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} \Big(A_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} \lambda_{(i,b)} P^{\mathrm{G}}_{(i,\omega,t,b)} \Big) L_{(t)}, \tag{4.11}$$

where $P^{\mathrm{G}}_{(i,\omega,t,b)}$ represents the power generated by unit i from block b during time step t and in scenario ω. $A_{(i)}$ and $\lambda_{(i,b)}$ are the no-load cost and block-slopes to approximate the quadratic production cost function of unit i.

4.6.5 Constraints

The maximization of the CVaR of the producer profit is subject to the constraints presented below.

The generated power by a unit is bounded by its maximum and minimum power output,

$$u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\text{Min}}_{(i)} \leq P^{\text{G}}_{(i,\omega,t)} \leq u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\text{Max}}_{(i)}, \quad \forall i, \forall \omega, \forall t. \tag{4.12}$$

If the quadratic function cost of each unit is approximated by a piecewise linear function, the following constraints are needed,

$$P^{\text{G}}_{(i,\omega,t)} = P^{\text{Min}}_{(i)}u_{(i,\omega,t)} + \sum_{b=1}^{N_B} P^{\text{G}}_{(i,\omega,t,b)}, \qquad \forall i, \forall \omega, \forall t \tag{4.13}$$

$$0 \leq P^{\text{G}}_{(i,\omega,t,b_1)} \leq P^{\text{Max}}_{(i,b_1)} - P^{\text{Min}}_{(i)}, \qquad \forall i, \forall \omega, \forall t \tag{4.14}$$

$$0 \leq P^{\text{G}}_{(i,\omega,t,b)} \leq P^{\text{Max}}_{(i,b)} - P^{\text{Max}}_{(i,b-1)}, \qquad \forall i, \forall \omega, \forall t, \forall b = b_2, ..., N_B. \tag{4.15}$$

Arbitrage between the pool and the futures market is explicitly avoided by the following constraints,

$$\sum_{i=1}^{N_I}(k_{(i,\omega,t)} - 1)P^{\text{Max}}_{(i)} \leq P^{\text{P}}_{(\omega,t)}, \qquad \forall \omega, \forall t \tag{4.16}$$

$$0 \leq P_{(c)}, \qquad \forall c. \tag{4.17}$$

The energy balance for the producer is formulated as

$$\sum_{i=1}^{N_I} P^{\text{G}}_{(i,\omega,t)} = P^{\text{P}}_{(\omega,t)} + \sum_{c \in F_{(t)}} P_{(c)}, \quad \forall \omega, \forall t. \tag{4.18}$$

Finally, the use of the CVaR adds the following constraints to the optimization model,

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \qquad \forall \omega \tag{4.19}$$

$$0 \leq \eta_{(\omega)}, \qquad \forall \omega, \tag{4.20}$$

where $\eta_{(\omega)}$ and ζ are two auxiliary variables needed to calculate the optimal value of the CVaR [153].

4.6.6 Formulation

The proposed formulation of the risk-constrained optimal trading problem for a power producer involving both the pool and the derivatives market is

$$\text{Maximize}_{P^{\mathrm{G}}_{(i,\omega,t,b)},P^{\mathrm{P}}_{(\omega,t)},P_{(c)},u_{(i,\omega,t)},\zeta,\eta_{(\omega)}}$$

$$\zeta - \frac{1}{1-\alpha^{\mathrm{P}}}\sum_{\omega=1}^{N_\Omega}\pi_{(\omega)}\eta_{(\omega)} \tag{4.21a}$$

subject to

$$\Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{F}} - C^{\mathrm{G}}_{(\omega)}, \qquad \forall\omega \tag{4.21b}$$

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T}\lambda^{\mathrm{P}}_{(\omega,t)}P^{\mathrm{P}}_{(\omega,t)}L_{(t)}, \qquad \forall\omega \tag{4.21c}$$

$$\Pi^{\mathrm{F}} = \sum_{c=1}^{N_C}\lambda_{(c)}P_{(c)}L_{(c)} \tag{4.21d}$$

$$C^{\mathrm{G}}_{(\omega)} = \sum_{i=1}^{N_I}\sum_{t=1}^{N_T}C^{\mathrm{G}}_{(i,\omega,t)}, \qquad \forall\omega \tag{4.21e}$$

$$C^{\mathrm{G}}_{(i,\omega,t)} = \Big(A_{(i)}u_{(i,\omega,t)} + \sum_{b=1}^{N_B}\lambda_{(i,b)}P^{\mathrm{G}}_{(i,\omega,t,b)}\Big)L_{(t)}, \qquad \forall i,\forall\omega,\forall t \tag{4.21f}$$

$$u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\mathrm{Min}}_{(i)} \le P^{\mathrm{G}}_{(i,\omega,t)} \le u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\mathrm{Max}}_{(i)}, \qquad \forall i,\forall\omega,\forall t \tag{4.21g}$$

$$P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{Min}}_{(i)}u_{(i,\omega,t)} + \sum_{b=1}^{N_B}P^{\mathrm{G}}_{(i,\omega,t,b)}, \qquad \forall i,\forall\omega,\forall t \tag{4.21h}$$

$$0 \le P^{\mathrm{G}}_{(i,\omega,t,b_1)} \le P^{\mathrm{Max}}_{(i,b_1)} - P^{\mathrm{Min}}_{(i)}, \qquad \forall i,\forall\omega,\forall t \tag{4.21i}$$

$$0 \le P^{\mathrm{G}}_{(i,\omega,t,b)} \le P^{\mathrm{Max}}_{(i,b)} - P^{\mathrm{Max}}_{(i,b-1)}, \qquad \forall i,\forall\omega,\forall t,\forall b = b_2,...,N_B \tag{4.21j}$$

$$\sum_{i=1}^{N_I}(k_{(i,\omega,t)}-1)P^{\mathrm{Max}}_{(i)} \le P^{\mathrm{P}}_{(\omega,t)}, \qquad \forall\omega,\forall t \tag{4.21k}$$

$$0 \le P_{(c)}, \qquad \forall c \tag{4.21l}$$

$$\sum_{i=1}^{N_I}P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{P}}_{(\omega,t)} + \sum_{c\in F_{(t)}}P_{(c)}, \qquad \forall\omega,\forall t \tag{4.21m}$$

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \le 0, \qquad \forall\omega \tag{4.21n}$$

$$0 \le \eta_{(\omega)}, \qquad \forall\omega, \tag{4.21o}$$

$$u_{(i,\omega,t)} \in \{0,1\}, \qquad \forall i,\forall\omega,\forall t. \tag{4.21p}$$

Objective function (4.21a) maximizes the CVaR of the probability distribution of the producer profit for a confidence level α^{P}. The total profit, as well as the profit obtained in the pool, the revenue of forward contracts, and the production cost are determined according to equations (4.21b)–(4.21f). Constraints (4.21g) bound the total power produced by each generating unit. Further, the power produced from each block b is determined according to equations (4.21h)–(4.21j). The arbitrage between the pool and the futures market is avoided with constraints (4.21k) and (4.21l). Constraints (4.21m) enforce the energy balance in each scenario and time step. In order to maximize the CVaR of the profit distribution, equations (4.21n) and (4.21o) are required. Lastly, constraints (4.21p) are binary variable declarations.

4.7 Forward Contracting Example 1

This section is aimed to present some preliminary theoretical implications of the proposed optimization model for risk-neutral producers that facilitates the correct understanding of some results obtained in the upcoming example and case study.

Consider a power producer that owns a single generating unit with a maximum capacity equal to P^{Max} and a linear cost equal to C. For simplicity, the minimum output of the unit is set to 0. Considering a study horizon of 1 hour, this producer can sell its production either in the pool or through a forward contract. The pool price in the next hour is characterized by a set of scenarios ($\lambda^{\mathrm{P}}_{(\omega)}$) whose average value is equal to $\overline{\lambda^{\mathrm{P}}}$. Moreover, we determine the average value of the pool price that exceeds the linear cost of the generating unit C as

$$\overline{\lambda^{\mathrm{P}}}^{\,*} = \frac{\sum_{\omega \in H} \pi_{(\omega)} \lambda^{\mathrm{P}}_{(\omega)}}{\sum_{\omega \in H} \pi_{(\omega)}}, \tag{4.22}$$

where

$$H = \{\omega : \lambda^{\mathrm{P}}_{(\omega)} \geq C\}.$$

The price of the forward contract is equal to $\lambda_{(c)}$. Finally, the probability that the generating unit is forced out during the next hour is denoted as π_f.

Therefore, the expected profit of the producer if its total production is sold in the pool is determined as

$$\varepsilon_\omega\{\Pi_{(\omega)}\} = \pi_f \cdot 0 + (1 - \pi_f) \cdot \left(\overline{\lambda^{\mathrm{P}}}^{\,*} - C\right) \cdot P^{\mathrm{Max}}. \tag{4.23}$$

Note that if the generating unit fails, the corresponding profit is equal to 0. On the other hand, if the unit is online the revenue obtained is proportional to $\overline{\lambda^{\mathrm{P}}}^{\,*}$ since the producer sells its production in the pool provided that the pool price is higher than its linear cost C.

Next, suppose that the producer sells its production through the forward contract. In this case, its profit is computed as

$$\varepsilon_\omega\{\Pi_{(\omega)}\} = \pi_f \cdot \left(\lambda_{(c)} - \overline{\lambda^{\mathrm{P}}}\right) \cdot P^{\mathrm{Max}} + (1 - \pi_f) \cdot (\lambda_{(c)} - C) \cdot P^{\mathrm{Max}}. \tag{4.24}$$

If the unit fails the producer has to buy the committed energy in the pool. On the other hand, if the unit is online, the producer sells its production at $\lambda_{(c)}$ with a cost of C.

We can determine the price of the forward contract that makes the expected profit achieved if the production is sold in the pool equal to the expected profit obtained by selling electricity through the forward contract. To do so, we make equation (4.23) equal to (4.24), thus obtaining the following expression for this *critical* price of the forward contract

$$\lambda^*_{(c)} = \overline{\lambda^{\mathrm{P}}}^{\,*} + \pi_f \cdot \left(\overline{\lambda^{\mathrm{P}}} - \overline{\lambda^{\mathrm{P}}}^{\,*}\right). \tag{4.25}$$

Therefore, the optimal decision of a risk-neutral producer is to sign the forward contract to sell electricity if its price $\lambda_{(c)}$ is higher than $\lambda^*_{(c)}$.

Note that either if the generating unit does not fail ($\pi_f = 0$) or the pool price is always higher than C ($\overline{\lambda^{\mathrm{P}}} = \overline{\lambda^{\mathrm{P}}}^{\,*}$), this critical forward price is equal to $\overline{\lambda^{\mathrm{P}}}^{\,*}$. Moreover, even for $\pi_f \neq 0$, due to the usual low forced outage rate of generating units (below 10%) and the low probability of having pool price lower than the production cost C, the critical forward price value is usually very close to the value obtained if the unit does not fail, i.e., $\overline{\lambda^{\mathrm{P}}}^{\,*}$. In other words, under such circumstances, the term $\pi_f \cdot \left(\overline{\lambda^{\mathrm{P}}} - \overline{\lambda^{\mathrm{P}}}^{\,*}\right)$ is generally negligible.

Therefore, this simplified theoretical example allows us to state that, considering low values of FOR and a low probability of having pool prices lower than the cost of the generating unit, optimal forward contracting decisions of risk-neutral producers for FOR = 0% are, if not the same, very similar to

Table 4.1: Forward contracting example 2: pool prices scenarios and their probabilities

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$(€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$\pi_{(\omega)}$
ω_1	20	30	0.2
ω_2	18	22	0.6
ω_3	16	14	0.2

those obtained by including the probability of suffering from unexpected unit failures (FOR $\neq$ 0%).

4.8 Forward Contracting Example 2

In this section, we present an example of reduced size to illustrate the forward contract trading issue of a power producer. In this example, we consider a power producer owning a single generating unit that has to decide between selling its total production in the pool or through a forward contract that spans the whole study horizon of two hours.

The pool price scenarios and their probabilities are provided in Table 4.1. Note that each scenario represents the realization of high, medium, and low pool prices.

Instead of selling its electricity production in the pool at uncertain prices, the power producer can sell it through a forward contract that spans the two hours and whose fixed price ($\lambda_{(c_1)}$) is set to the average pool price over the study horizon, i.e., 20 €/MWh.

The generating unit is characterized by a capacity (P^{Max}) of 100 MW and a linear production cost (C) of 12 €/MWh. For the sake of simplicity, the minimum output of the unit is considered equal to 0 MW.

Next, we compare the profit distributions obtained if the generated power is sold either in the pool or through the available forward contract for two different cases. In the first one, the generating unit is assumed to be online during the two hours, whereas in the second one unexpected failures can occur with a certain probability.

Table 4.2: Forward contracting example 2: power producer profit distribution if its production is sold in the pool and unit failures are ignored

# scenario	$\Pi_{(\omega)}$(€)	$\pi_{(\omega)}$
ω_3	600	0.2
ω_2	1600	0.6
ω_1	2600	0.2

4.8.1 Non-failing Generating Unit Case

Next, we calculate the profit of the power producer if the generating unit failures are neglected.

Considering that the producer sells all its production in the pool, we calculate the profit associated with each possible realization of the pool price as the revenue obtained in the pool minus the production cost, i.e.,

$$\Pi_{(\omega)} = \sum_{t=t_1}^{t_2} (\lambda^{\text{P}}_{(\omega,t)} P^{\text{Max}} - C P^{\text{Max}}). \tag{4.26}$$

Note that the pool price is always higher than the production cost and therefore, the optimal decision is to produce at full capacity. For example, the profit obtained if the first pool price scenario realizes is equal to

$$\begin{aligned}\Pi_{(\omega_1)} &= (\lambda^{\text{P}}_{(\omega_1,t_1)} P^{\text{Max}} - C P^{\text{Max}}) + (\lambda^{\text{P}}_{(\omega_1,t_2)} P^{\text{Max}} - C P^{\text{Max}}) = \\ &= (20 \cdot 100 - 12 \cdot 100) + (30 \cdot 100 - 12 \cdot 100) = €2600.\end{aligned}$$

Table 4.2 contains the profits, sorted in ascending order, associated with each pool price realization as well as its probability.

On the other hand, if the power producer sells its production through the forward contract, its profit is not subject to any uncertainty and is computed as

$$\begin{aligned}\Pi_{(\omega)} = \sum_{t=t_1}^{t_2} (\lambda_{(c_1)} P^{\text{Max}} - C P^{\text{Max}}) = (20 \cdot 100 - 12 \cdot 100) + \\ + (20 \cdot 100 - 12 \cdot 100) = €1600, \forall \omega.\end{aligned}$$

Considering the results in Table 4.2, the expected profit if the power producer sells its production in the pool is equal to €1600, which is exactly the revenue obtained if the forward contract is acquired. Therefore, a risk-neutral

Table 4.3: Forward contracting example 2: optimal forward decisions, CVaR and expected profit if unit failures are ignored

α^{P}	$P_{(c_1)}$ (MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (€)	EP (€)
0	0	1600	1600
0.5	100	1600	1600
0.9	100	1600	1600

producer, which makes its decisions seeking to maximize the expected profit, does not have a preference between these two alternatives. However, a risk-averse producer, which is concerned about the possibility of experiencing low profits, would decide to sell its production through the forward contract in order to avoid the possibility of obtaining a profit of €600 if the third pool price scenario realizes. Note that in this case, the producer gives up the possibility of having a higher profit, i.e., €2600, if the first pool price scenario realizes.

Table 4.3 shows the results obtained by solving optimization problem (4.21) for three risk aversion levels and without considering unexpected unit failures. This table provides the optimal power sold through the available forward contract, as well as the CVaR and the expected value of the producer profit distribution.

Note that an increase in the risk aversion level leads the producer to sell its production through the available forward contract to reduce the probability of having low profits because of the realization of low pool prices. Therefore, forward contracts constitute an instrument to reduce the risk associated with the volatility of pool prices.

4.8.2 Failing Generating Unit Case

In this second case, the generating unit owned by the producer suffers from unexpected failures. Given the short study horizon, the values of the MTTF and MTTR of the generating unit are set to 2 h and 1 h, respectively, which gives rise to an unrealistic value of FOR equal to 33%. Table 4.4 lists the four possible availability scenarios of the generating unit, whose probabilities are calculated according to equation (3.3) of Chapter 3.

The final scenario tree is built by combining all possible pool price scenarios provided in Table 4.1 with all availability scenarios in Table 4.4, and is characterized in Table 4.5.

Table 4.4: Forward contracting example 2: availability scenarios and their probabilities

# scenario	$k_{(f,t_1)}$	$k_{(f,t_2)}$	$\pi_{(f)}$
f_1	1	1	0.5476
f_2	1	0	0.1924
f_3	0	1	0.1534
f_4	0	0	0.1066

Table 4.5: Forward contracting example 2: pool price and availability scenarios with their probabilities

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$(€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$k_{(\omega,t_1)}$	$k_{(\omega,t_2)}$	$\pi_{(\omega)}$
ω_1	20	30	1	1	0.10952
ω_2	20	30	1	0	0.03848
ω_3	20	30	0	1	0.03068
ω_4	20	30	0	0	0.02132
ω_5	18	22	1	1	0.32856
ω_6	18	22	1	0	0.11544
ω_7	18	22	0	1	0.09204
ω_8	18	22	0	0	0.06396
ω_9	16	14	1	1	0.10952
ω_{10}	16	14	1	0	0.03848
ω_{11}	16	14	0	1	0.03068
ω_{12}	16	14	0	0	0.02132

Table 4.6: Forward contracting example 2: power producer profit distribution if its production is sold in the pool and unit failures are considered

# scenario	$\Pi_{(\omega)}$(€)	$\pi_{(\omega)}$
ω_4	0	0.02132
ω_8	0	0.06396
ω_{12}	0	0.02132
ω_{11}	200	0.03068
ω_{10}	400	0.03848
ω_6	600	0.11544
ω_9	600	0.10952
ω_2	800	0.03848
ω_7	1000	0.09204
ω_5	1600	0.32856
ω_3	1800	0.03068
ω_1	2600	0.10952

Next, we calculate the profit distribution if the power producer sells its whole production in the pool. The profit associated with a scenario in which the unit does not fail is calculated according to equation (4.26). For example,

$$\Pi_{(\omega_5)} = (\lambda^{\mathrm{P}}_{(\omega_5,t_1)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) k_{(\omega_5,t_1)} + (\lambda^{\mathrm{P}}_{(\omega_5,t_2)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) k_{(\omega_5,t_2)} =$$
$$= (18 \cdot 100 - 12 \cdot 100) \cdot 1 + (22 \cdot 100 - 12 \cdot 100) \cdot 1 = €1600.$$

However, if the unit fails, the revenue of those time steps during which the unit is forced out is equal to 0. For instance,

$$\Pi_{(\omega_{10})} = (\lambda^{\mathrm{P}}_{(\omega_{10},t_1)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) k_{(\omega_{10},t_1)} + (\lambda^{\mathrm{P}}_{(\omega_{10},t_2)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) k_{(\omega_{10},t_2)} =$$
$$(16 \cdot 100 - 12 \cdot 100) \cdot 1 + (14 \cdot 100 - 12 \cdot 100) \cdot 0 = €400.$$

Table 4.6 provides the profit distribution of the power producer if its entire production is sold in the pool. The expected profit in this case is equal to €1145.

If the producer sells its production through the forward contract, unlike the case in which unit failures were disregarded, the resulting profit is not unique, but depends on whether or not the unit fails. In other words, if the unit suffers from an unexpected failure during some time steps, the producer must buy in the pool the amount of electricity needed to comply with its forward contracting obligation. For example, the profit associated with scenario ω_7 is

Table 4.7: Forward contracting example 2: power producer profit distribution if its production is sold through the forward contract and unit failures are considered

# scenario	$\Pi_{(\omega)}$(€)	$\pi_{(\omega)}$
ω_4	-1000	0.02132
ω_2	-200	0.03848
ω_8	0	0.06396
ω_6	600	0.11544
ω_3	800	0.03068
ω_7	1000	0.09204
ω_{12}	1000	0.02132
ω_{11}	1200	0.03068
ω_{10}	1400	0.03848
ω_1	1600	0.10952
ω_5	1600	0.32856
ω_9	1600	0.10952

determined as

$$\begin{aligned}\Pi_{(\omega_7)} &= (\lambda_{(c_1)}P^{\text{Max}} - \lambda^{\text{P}}_{(\omega_7,t_1)}P^{\text{Max}}) + (\lambda_{(c_1)}P^{\text{Max}} - CP^{\text{Max}}) = \\ &= (20 \cdot 100 - 18 \cdot 100) + (20 \cdot 100 - 12 \cdot 100) = €1000.\end{aligned}$$

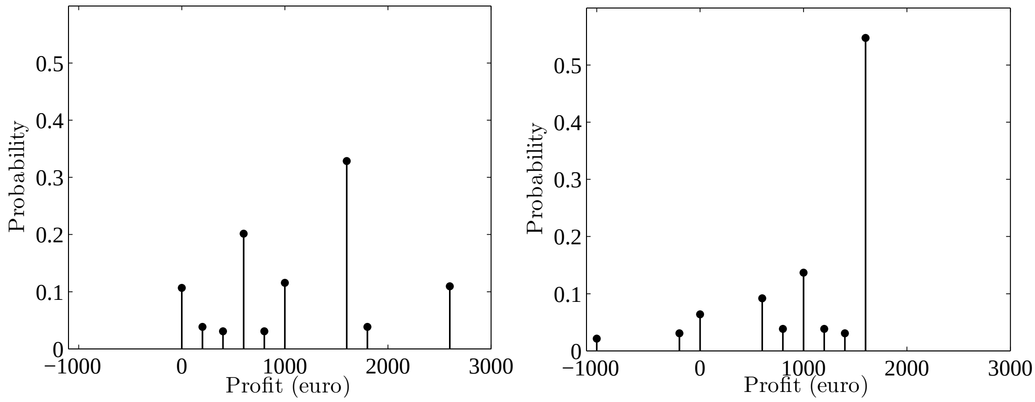
In this case, and given that the pool price during the forced outage of the unit is lower than the forward price, the profit drop is not very significant. On the other hand, if scenario ω_4 realizes, the profit of the producer falls to

$$\begin{aligned}\Pi_{(\omega_4)} &= (\lambda_{(c_1)}P^{\text{Max}} - \lambda^{\text{P}}_{(\omega_4,t_1)}P^{\text{Max}}) + (\lambda_{(c_1)}P^{\text{Max}} - \lambda^{\text{P}}_{(\omega_4,t_2)}P^{\text{Max}}) = \\ &= (20 \cdot 100 - 20 \cdot 100) + (20 \cdot 100 - 30 \cdot 100) = -€1000.\end{aligned}$$

The profit distribution of the producer if it sells its full production through the forward contract is provided in Table 4.7.

Although the average profit from selling through the forward contract is the same to that calculated if the producer sells its production in the pool, i.e., €1145, there are scenarios whose associated profits are lower than the expected profit, and even scenarios involving losses (scenario ω_4, for example). As expected, the scenarios with the highest losses are those in which high pool prices occur when the unit is forced out.

Fig. 4.3 shows the probability mass function of the profit depending on whether the producer sells its production in the pool (Fig. 4.3(a)) or through



(a) Electricity sold in the pool

(b) Electricity sold through a forward contract

Figure 4.3: Forward contracting example 2: probability mass function of the profit of the producer

the forward contract (Fig. 4.3(b)). Note that both figures are plotted using the same scale.

From these figures, it can be concluded that a risk-averse producer that aims to avoid the possibility of suffering low profits should sell its production in the pool rather than through a forward contract. In doing so, the lowest profit it obtains is equal to €0. Note that this decision is the opposite to that made if unit failures are neglected, since in that case, a risk-averse power producer should sell its production through a forward contract in order to hedge against pool price volatility.

The above analysis illustrates how the availability risk affects the producer profit distribution if all its production is sold in the pool or through the available forward contract.

Next, by solving optimization problem (4.21), we determine the optimal power quantity that the producer has to sell through the forward contract to maximize the CVaR of its profit distribution. Table 4.8 provides the power sold through the available forward contract (column 2), the CVaR (column 3), and the expected profit (column 4) for three different values of the producer risk aversion level α^{P}, namely, 0, 0.5 and 0.9.

Comparing Tables 4.3 and 4.8, we see that, for a value of the risk aversion parameter α^{P} equal to 0.9, the possibility of suffering from unexpected unit failures reduces the power sold through the forward contract from 100 MW to 0 MW. In doing so, the producer hedges against the availability risk.

This small example allows us to conclude that a power producer should

Table 4.8: Forward contracting example 2: forward decisions, CVaR and expected profit if unit failures are considered

α^{P}	$P_{(c_1)}$ (MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (€)	EP (€)
0	0	1145	1145
0.5	100	690	1145
0.9	0	0	1145

determine, according to its risk aversion level, the appropriate mix of pool and futures-market involvement to reduce the risk associated with the pool price volatility, but without significantly increasing the risk corresponding to unexpected unit failures.

4.9 Forward Contracting Case Study

In this section, the proposed optimization model to decide forward contracting decisions of a risk-averse power producer is tested on a realistic 8-week case study based on the electricity market of the Iberian Peninsula, which includes Spain and Portugal. The case study consists of three parts: in the first part, the impact of unit failures on the forward contracting decisions and objective function of a power producer is analyzed; the second part is aimed at assessing the performance of the proposed decision-making model via an Out-of-Sample analysis; and the last part presents the influence of the number of generating units on the producer decisions.

4.9.1 Impact of Unit Failures on Forward Contracting

We consider a producer owning a single thermal generating unit with maximum and minimum power output of 350 MW and 50 MW, respectively, and a piecewise linear cost function whose parameters are provided in Table 4.9.

Three different forced outage rates are considered in this case study to highlight the influence of unit failures on forward contracting decisions by power producers. In the first case, the generating unit does not fail, i.e., FOR is equal to 0%. Additionally, we consider a medium and a high value of FOR, whose outage parameters are provided in Table 4.10 [76].

If the forced outage rate of the generating unit is equal to 0%, only one scenario is needed to characterize its availability status. On the other hand, if

Table 4.9: Forward contracting case study: piecewise linear production cost function of the generating unit

# block	$P_{(b)}^{\text{Max}}$ (MW)	$\lambda_{(b)}$ (€/MWh)
1	140.0	10.08
2	227.5	10.66
3	280.0	11.09
4	350.0	11.72

Table 4.10: Forward contracting case study: forced outage parameters

FOR (%)	MTTF (h)	MTTF (h)
5	950	50
10	450	50

a generating unit with FOR value equal to 5% or 10% is considered, a set of 5000 availability scenarios is generated according to the procedure described in Section 3.5 of Chapter 3.

Historical pool price data of the Spanish part of the electricity market of the Iberian Peninsula during the year 2000 are used to adjust the parameters of an ARIMA model following the procedure explained in Section 3.4 of Chapter 3. This way, the uncertainty of the pool price throughout the 8-week horizon is characterized via a set of 200 scenarios generated by randomly simulating the innovation term of the ARIMA model.

For the sake of tractability, the 24 hours of each day of the study horizon are aggregated in 3 time steps according to the method proposed in Appendix A. The number of hours and the pool price of each time step are calculated in order to minimize the average error between the actual prices contained in the 200 scenarios and the aggregated prices. Note that the duration in hours of each time step is the same for all pool price scenarios.

Once the duration of each time step is decided to aggregate hourly time steps into 3 time steps per day, the availability of the generating unit associated with each time step and scenario is computed as the average value of the hourly availabilities associated with each time step. Note that if this average value is different from 0 and 1, it is rounded to be equal to one of these two values.

Since pool prices are assumed to be non-correlated with generating unit failures, the final scenario tree is formed by combining the 200 pool price scenarios with the 5000 availability scenarios, thus giving rise to a tree of one

Table 4.11: Forward contracting case study: forward contract prices

Case	$\lambda_{(c_1)}$ (€/MWh)	$\lambda_{(c_2)}$ (€/MWh)
A	21.75	22.75
B	22.85	23.85
C	22.15	23.15

million scenarios that renders the associated stochastic programming problem intractable. For this reason, both price and availability scenario sets are reduced according to the procedure proposed in Section 3.6.2 of Chapter 3 in two phases. Firstly, the 200 pool price scenarios are reduced to 30 considering that the generating unit never fails. In the same vein, considering only the average pool price scenario, a reduced availability set of 50 scenarios is obtained. Therefore, the final scenario tree includes 1500 scenarios created by combining the 30 pool price scenarios and the 50 availability scenarios.

Two forward contracts spanning the first four weeks and the last four weeks, respectively, are considered. We solve optimization problem (4.21) to determine the producer decisions for three different cases characterized by low (case A), high (case B), and medium (case C) forward prices with respect to the average pool price during their corresponding delivery periods. Forward contract prices are provided in Table 4.11.

Finally, the risk aversion level of the power producer is simulated by varying the value of α^{P} between 0 and 1.

By solving optimization problem (4.21), we first determine the results in the case that forward prices are lower than the expected pool prices (case A). Fig. 4.4 plots the summation of the power sold through the forward contract spanning the first four weeks plus the power sold through the forward contract spanning the last four weeks as a function of the risk aversion parameter of the producer (α^{P}) and for the three forced outages rates considered in this case study.

If the generating unit never fails (FOR = 0%), given that the forward prices of both contracts are lower than the expected pool price during their corresponding delivery periods, a risk-neutral producer ($\alpha^{\mathrm{P}} = 0$) sells all its production in the pool in order to maximize the expected profit over all possible scenarios. However, an increase in the risk aversion of the producer involves the maximization of the expected profit of the *worst* scenarios, which are characterized by the realization of low pool prices. For this reason, the higher

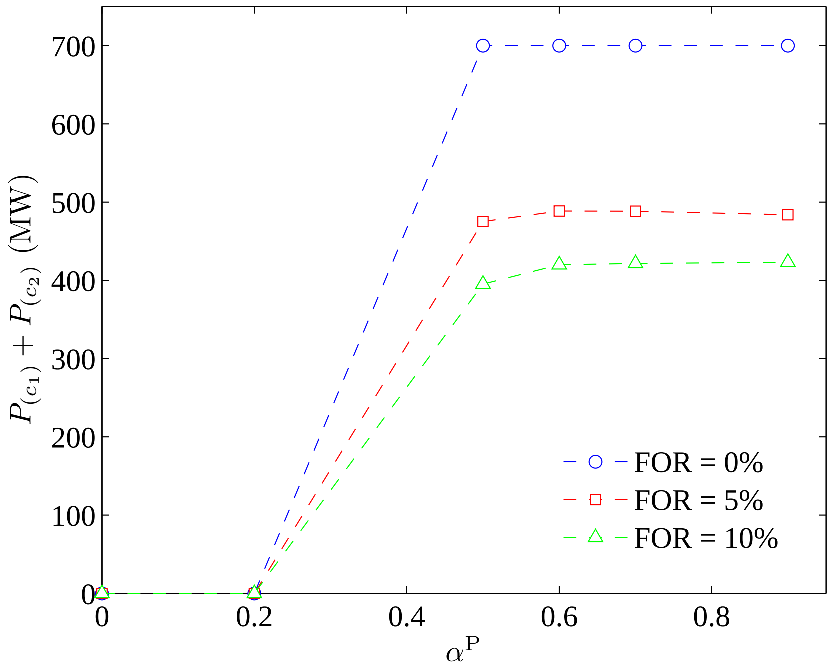


Figure 4.4: Forward contracting case study: impact of unit failures on the power sold through forward contracts (Case A)

the value of α^{P}, the more power is sold through forward contracts in order to reduce the possibility of obtaining low revenues caused by low pool prices.

As explained in Section 4.3.4, if generating unit failures are considered (FOR $= 5\%$ and 10%), the producer also faces the risk of the unit availability, a risk which, as opposed to that of price, increases if the production is sold through forward contracts. For this reason, as shown in Fig. 4.4, an increase in the forced outage rate of the generating unit for the same risk aversion level reduces the optimal production sold through forward contracts.

Note that in Fig. 4.4 the power sold through forward contracts is an increasing function with respect to the risk aversion level for all FOR values. In other words, an increase in the risk aversion level of the power producer always leads to an increase in the power sold through forward contracts, which means that in this particular case, the risk associated with pool price volatility is *dominant* over the risk associated with failures of the generating unit.

Figs. 4.5(a) and 4.5(b) plot the evolution of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ and the expected profit, respectively, as a function of the risk aversion parameter α^{P} and for all FOR values. Note that the first figure represents a different $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ for each value of α^{P}. That is, for $\alpha^{\mathrm{P}} = 0$ Fig. 4.5(a) illustrates the expected value of the profit distribution (which coincides with the value depicted in Fig. 4.5(b)), while for $\alpha^{\mathrm{P}} = 0.9$, it shows the optimal value of $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$. By definition, the $\mathrm{CVaR}_{\alpha_1^{\mathrm{P}}}$ of a profit distribution is always higher than the $\mathrm{CVaR}_{\alpha_2^{\mathrm{P}}}$ provided that $\alpha_1^{\mathrm{P}} \leq \alpha_2^{\mathrm{P}}$, and that is the reason why the CVaR decreases with α^{P}. Moreover, a higher value of FOR originates a lower value of the CVaR for the same risk aversion level.

Fig. 4.5(b) shows how an increase in the risk aversion entails a decrease in the expected value of the profit distribution. In this case, in which the forward contract prices are lower than the expected pool prices, it is logical that the higher the power sold through forward contracts (Fig. 4.4), the lower the expected profit. Note also that the expected profit decreases with the probability of suffering from generating unit failures.

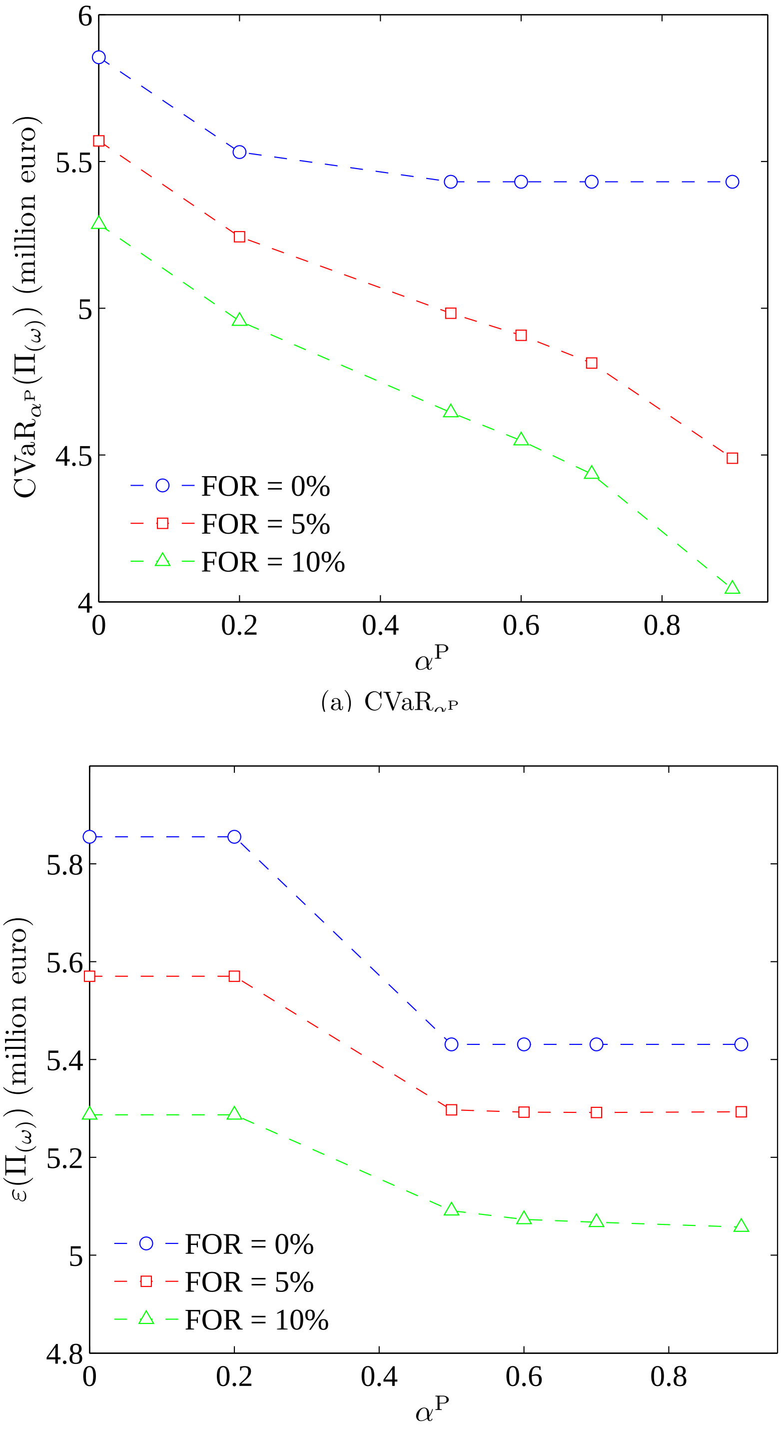
For completeness, Fig.4.6 shows two different representations of the efficient frontier of the producer for the three FOR values. In the first one (Fig. 4.6(a)), the expected profit is represented as a function of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ of the profit distribution for increasing values of the risk aversion parameter α^{P}. Note that whereas in Fig. 4.5(a) we show the optimal value of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ for different values of α^{P}, in Fig. 4.6(a) we represent the value of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ (which is the confidence level normally used in the technical literature) of the profit distribution obtained if forward contract decisions are made to maximize the value of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$.

In Fig. 4.6(b) we provide the expected profit as a function of the standard deviation of the profit distribution. As in the first efficient frontier, both the expected profit and the standard deviation of the profit are computed once forward contracting decisions are made to maximize the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$.

From these two efficient frontiers we conclude that while increasing the risk aversion level reduces the variability of the profit (characterized via the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ in Fig. 4.6(a) or the standard deviation of the profit in Fig. 4.6(b)), it also reduces the value of the expected profit for the power producer. Note also that an increase in the forced outage rate leads to a shift of the efficient frontier to areas with lower expected profit and higher profit variability.

Next, we analyze case B in which forward prices are higher than expected pool prices. Like Fig. 4.4, Fig. 4.7 plots the total power sold through forward contracts, which is obtained by solving optimization problem (4.21), as a function of the risk aversion level and for each forced outage rate. Note that in this case the vertical axis starts at 500 MW.

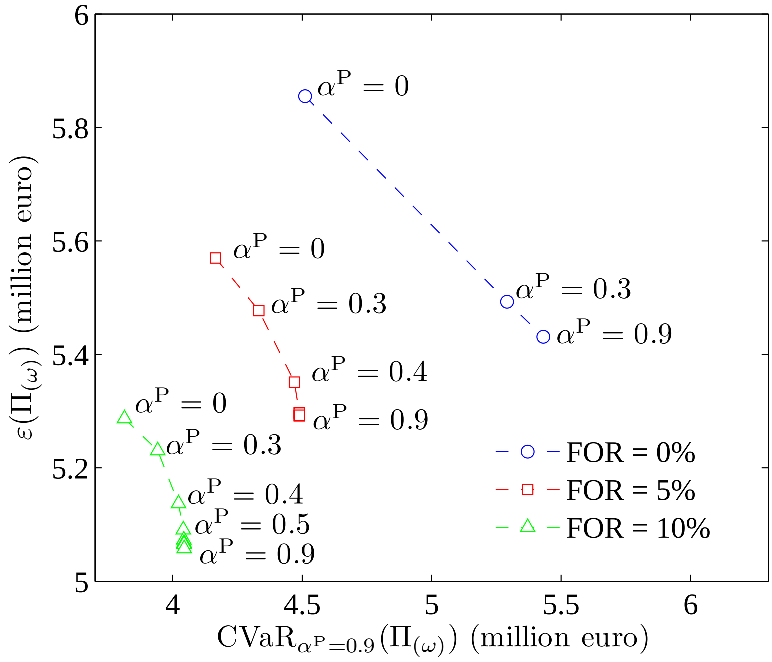
Given that the forward prices are higher than the expected pool prices,



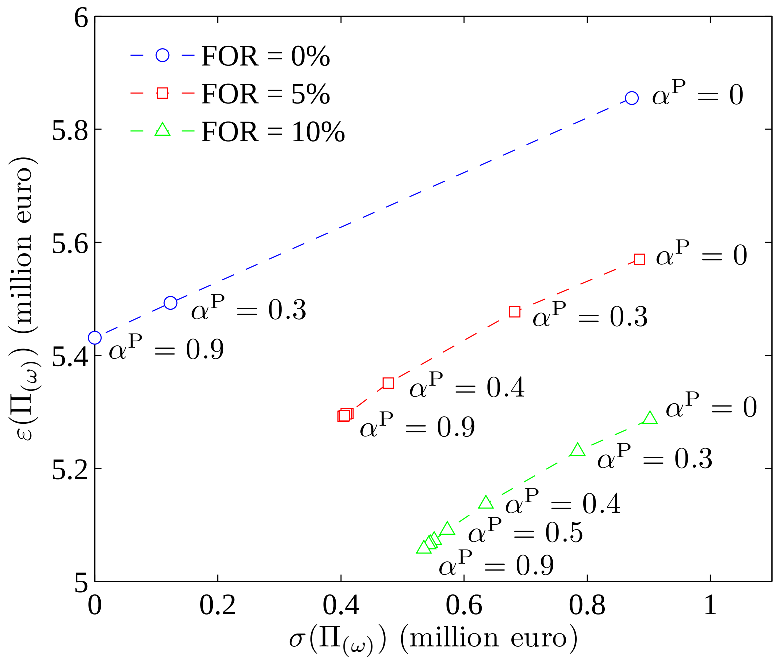
(a) $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$

(b) Expected profit

Figure 4.5: Forward contracting case study: impact of unit failures on the CVaR and the expected profit (Case A)



(a) Expected profit vs. $\mathrm{CVaR}_{\alpha^P=0.9}$



(b) Expected profit vs. standard deviation

Figure 4.6: Forward contracting case study: impact of unit failures on the efficient frontier (Case A)

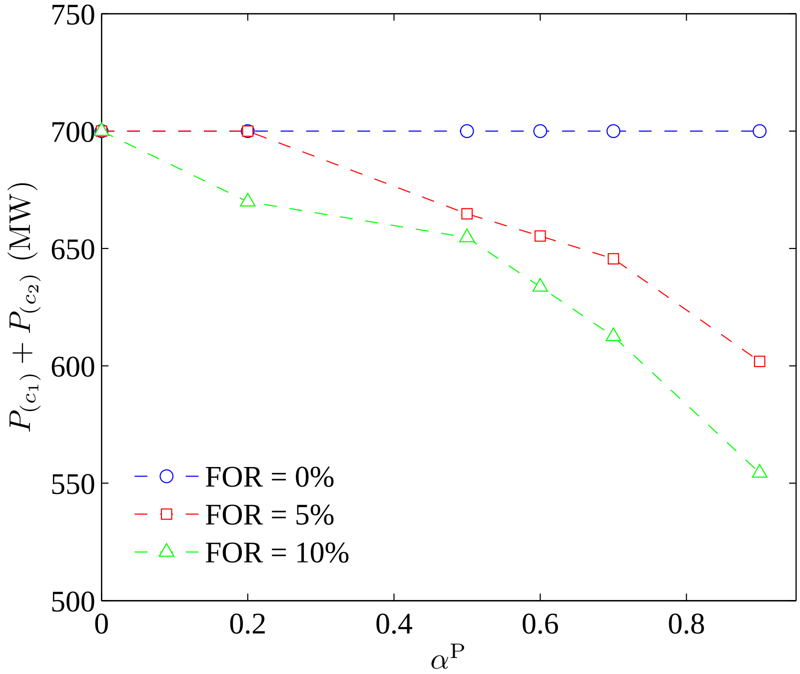
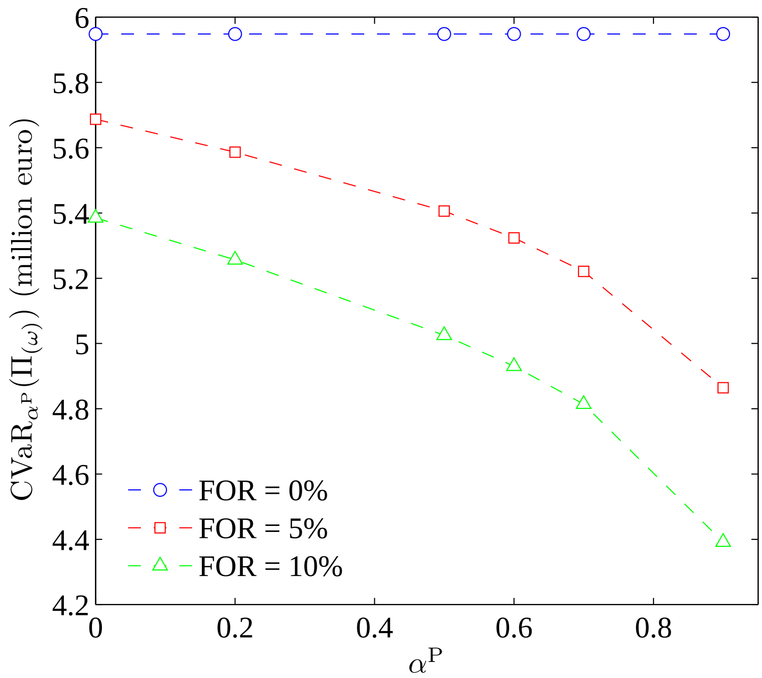


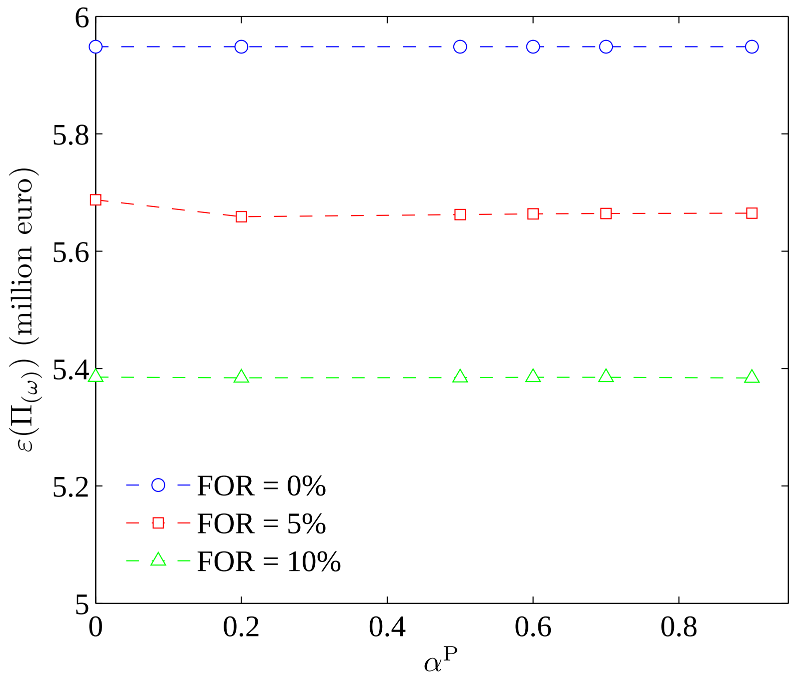
Figure 4.7: Forward contracting case study: impact of unit failures on the power sold through forward contracts (Case B)

a risk-neutral producer operating a generating unit that never fails (FOR = 0%) sells as much energy as possible through the forward contracts in order to maximize its expected profit. As discussed in Section 4.7, given the low values of real-world forced outage rates, the optimal decisions for FOR equal to 5% and 10% coincide with those made for FOR = 0%, i.e., the producer sells all its production through forward contracts. From this point on, and for FOR = 0%, an increase in the risk aversion of the producer does not modify its optimal decisions because the price risk, which is the only one involved in this case, is reduced by selling the production through forward contracts instead of in the pool.

On the other hand, for FOR equal to 5% and 10%, the possibility of suffering from unexpected unit failures, which may entail financial losses if the producer has signed forward contracts, causes a decrease in the power sold through forward contracts for high levels of risk aversion. Unsurprisingly, this drop is more significant for FOR = 10% than for FOR = 5%. Therefore, whereas in case A the predominance of price risk vs. availability risk is the reason why the power sold through forward contracts increases with the risk aversion, in case B the opposite effect holds, i.e., the availability risk is predominant over the price risk causing a decreasing evolution of the power sold



(a) CVaR$_{\alpha^P}$



(b) Expected profit

Figure 4.8: Forward contracting case study: impact of unit failures on the CVaR and the expected profit (Case B)

through forward contracts with the risk aversion level.

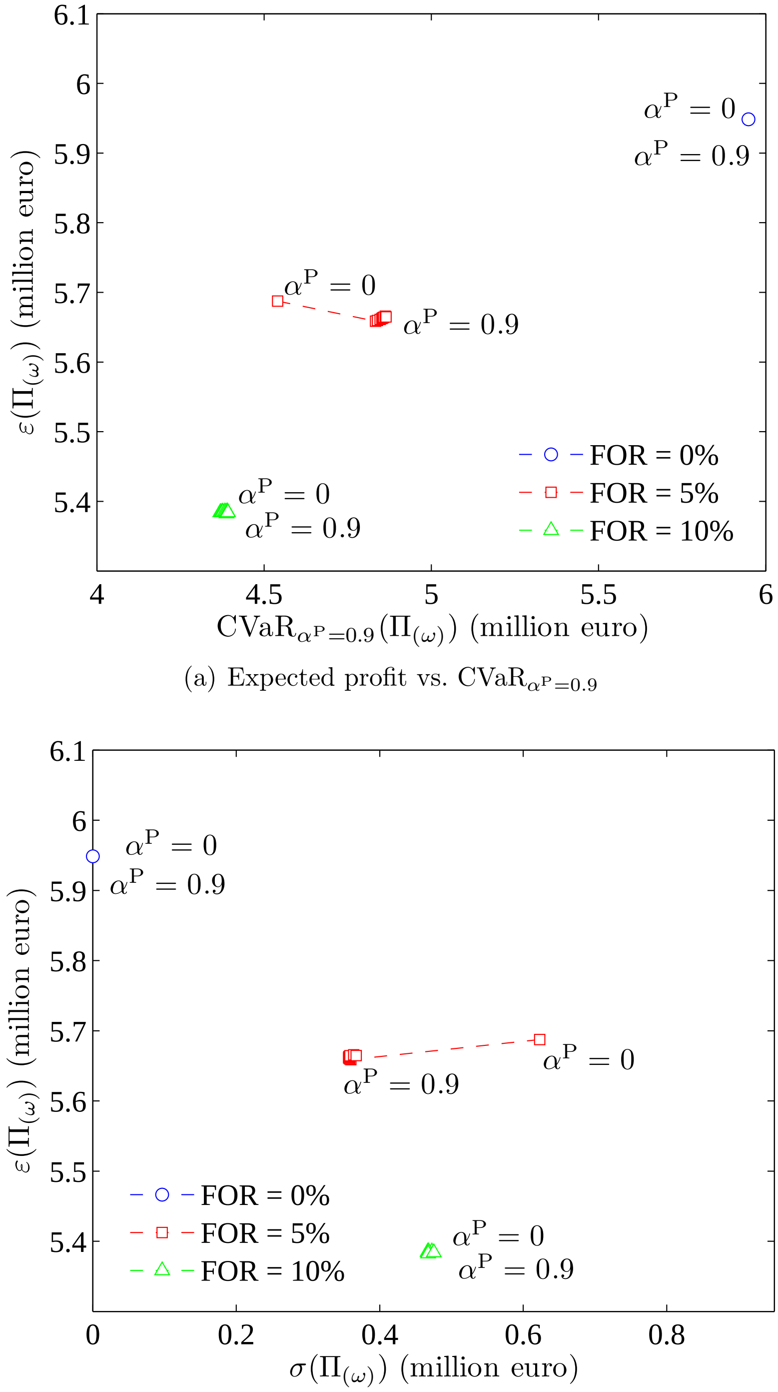
Figs. 4.8(a) and 4.8(b) illustrate the evolution of the CVaR$_{\alpha^P}$ and the expected profit, respectively, with the risk aversion level for case B. Both values show a decreasing trend if the risk aversion parameter α^{P} increases. Note that in this case, the forward prices are higher than the expected pool price during their corresponding delivery periods and therefore, the reduction of the power sold through forward contracts is the cause of the slight expected profit drop.

Efficient frontiers for case B are shown in Fig. 4.9. Given the forward prices of case B, optimal decisions hardly change and therefore, the variation of the expected profit and profit variability is not very significant either. Actually, for FOR = 0%, both efficient frontiers reduce to a single point.

Finally, in case C, we solve optimization problem (4.21) if the forward prices are set to values very close to the expected pool price during their delivery periods. The power sold through forward contracts for each value of risk aversion and FOR are provided in Fig. 4.10. In this case, the power sold through forward contracts increases with the risk aversion level for values of α^{P} between 0 and 0.5, and from this point on it decreases. The reason for this is that, for low levels of risk aversion, the price risk is predominant over the availability risk, thus leading to an increase in the power sold through forward contracts with α^{P}. However, for high levels of risk aversion, the risk associated with unit failures is prevalent over the price risk and therefore, the power sold through forward contracts decreases to reduce the financial losses caused by unexpected failures. In any case, for each value of risk aversion, the higher the probability of unit failures, the lower the power sold through forward contracts.

The evolution of the CVaR and the expected profit with the risk aversion level for case C is represented in Figs.4.11(a) and 4.11(b), respectively. Efficient frontiers for case C are provided in Fig. 4.12.

For completeness, Figs. 4.13(a) and 4.13(b) provide, for FOR = 5% and FOR = 10%, respectively, the power sold through forward contracts as a function of the risk aversion level for the considered forward prices in cases A, B, and C. Note how, for both FOR values, the power sold through forward contracts has an increasing, decreasing, and increasing-decreasing trend in cases A, B, and C, respectively. Moreover, Fig. 4.13 shows that higher forward prices lead to higher levels of power sold through forward contracts.



(a) Expected profit vs. $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$

(b) Expected profit vs. standard deviation

Figure 4.9: Forward contracting case study: impact of unit failures on the efficient frontier (Case B)

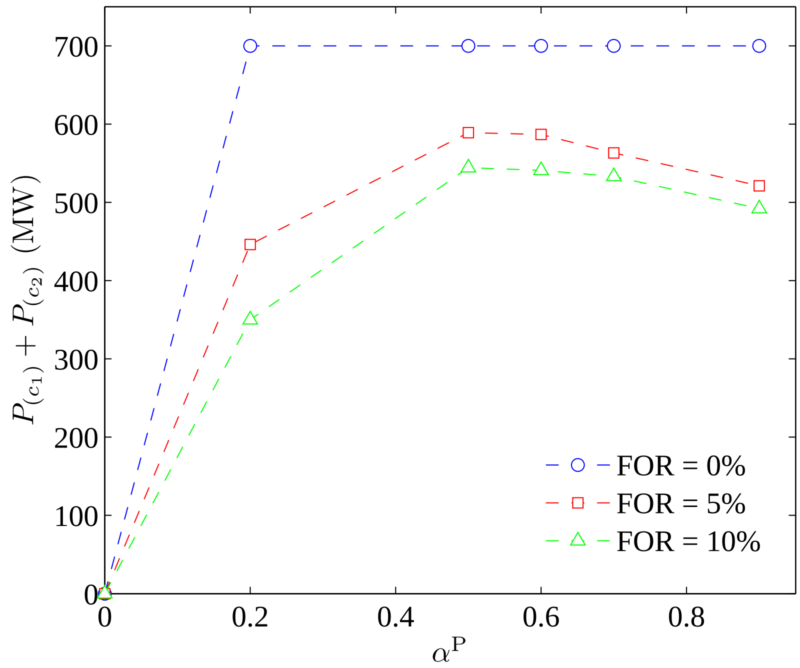
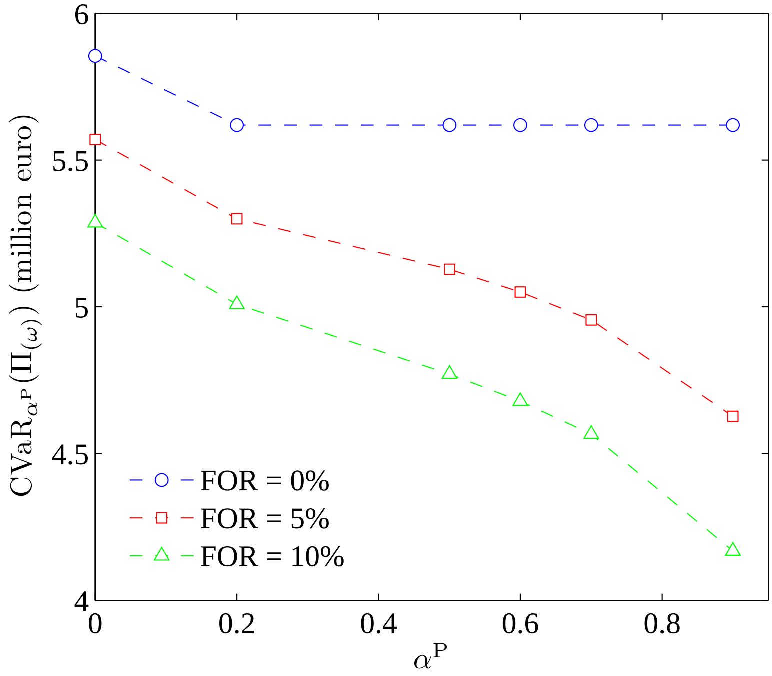


Figure 4.10: Forward contracting case study: impact of unit failures on the power sold through forward contracts (Case C)

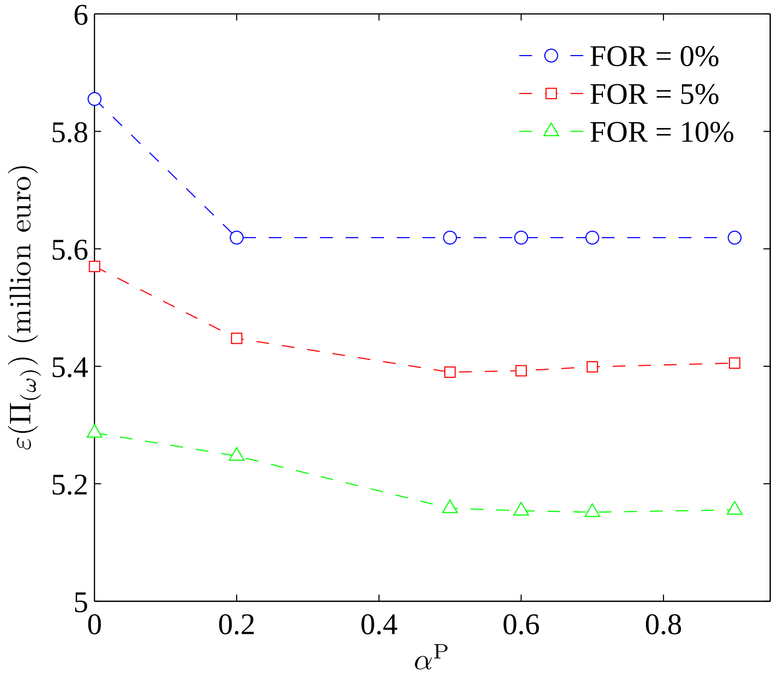
4.9.2 Out-of-Sample Analysis

The analysis previously reported is called *In-Sample* analysis, since the scenario tree used to determine the optimal forward contracting decisions is the same than the one employed to calculate the objective function of the producer and the efficient frontiers. In other words, we assume that the selected scenario tree represents precisely the stochastic processes involved in the decision-making model, i.e., the pool price and the unit availability. Complementarity, an *Out-of-Sample* analysis is important to further evaluate the performance of the proposed stochastic optimization model to make optimal forward contracting decisions.

Basically, an Out-of-Sample analysis consists in evaluating the quality of the optimal decisions obtained with a given scenario tree for either a larger scenario tree (which better represents the uncertain parameters involved) or the actual realization of such parameters throughout the study horizon [178]. In the Out-of-Sample study below, we analyze how optimal decisions that are determined by considering the possibility of unit failures perform better if compared to those obtained by ignoring unit failures (FOR = 0%), thus motivating the inclusion of unit availability uncertainty in mid-term decision-making models for power producers.

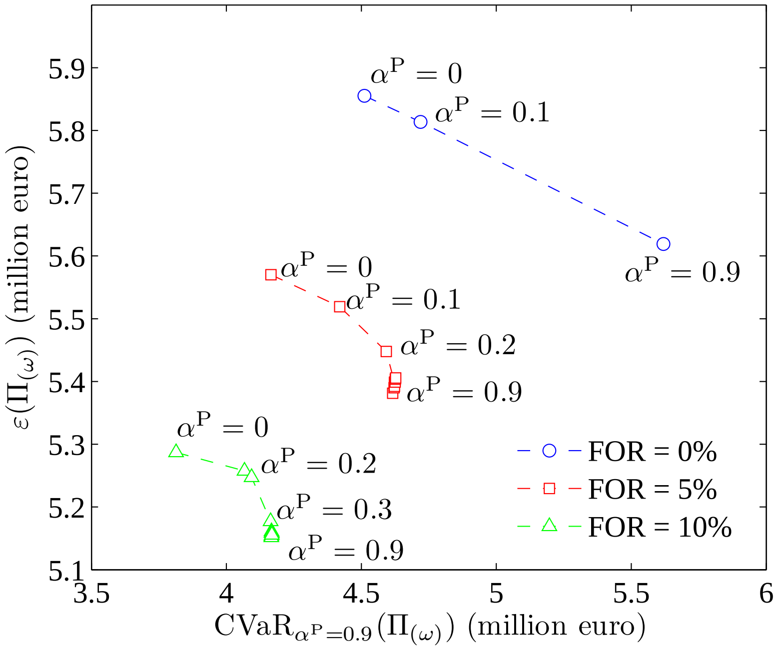


(a) $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$

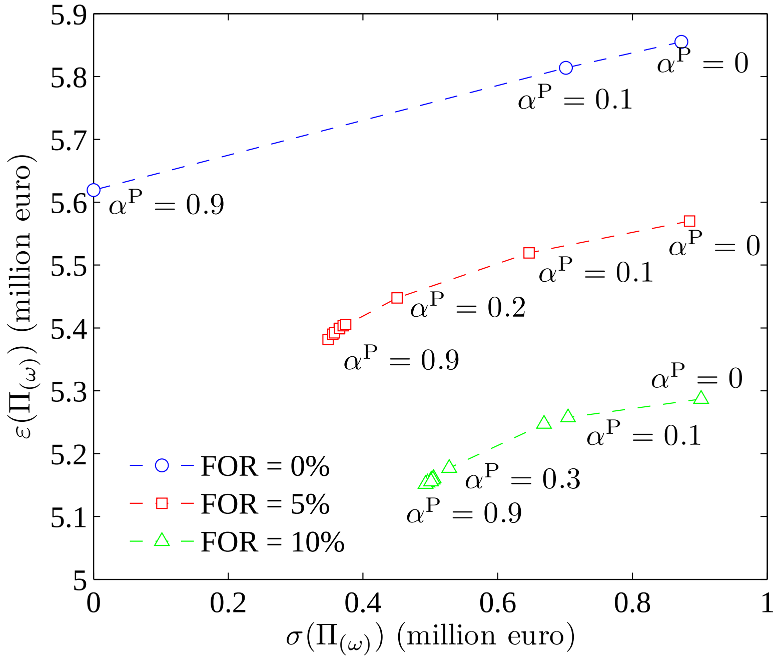


(b) Expected profit

Figure 4.11: Forward contracting case study: impact of unit failures on the CVaR and the expected profit (Case C)

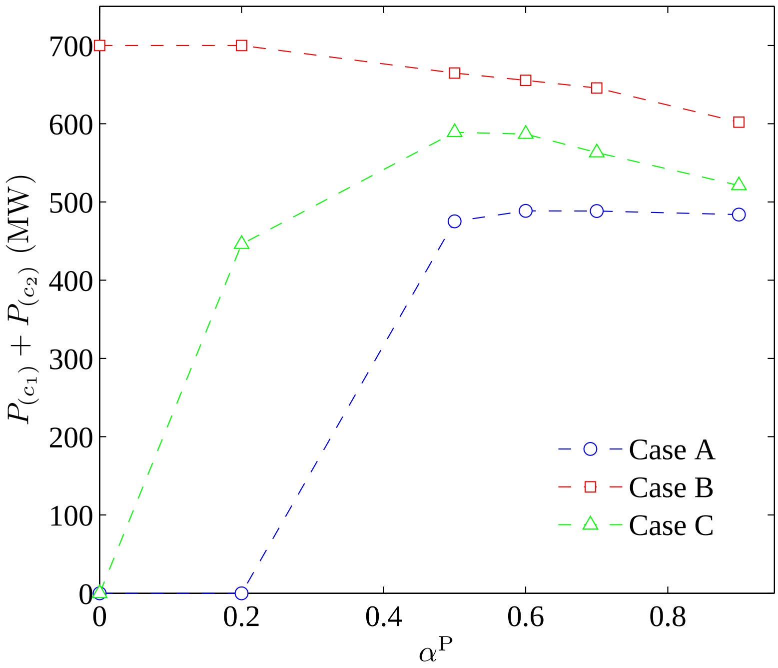


(a) Expected profit vs. $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$

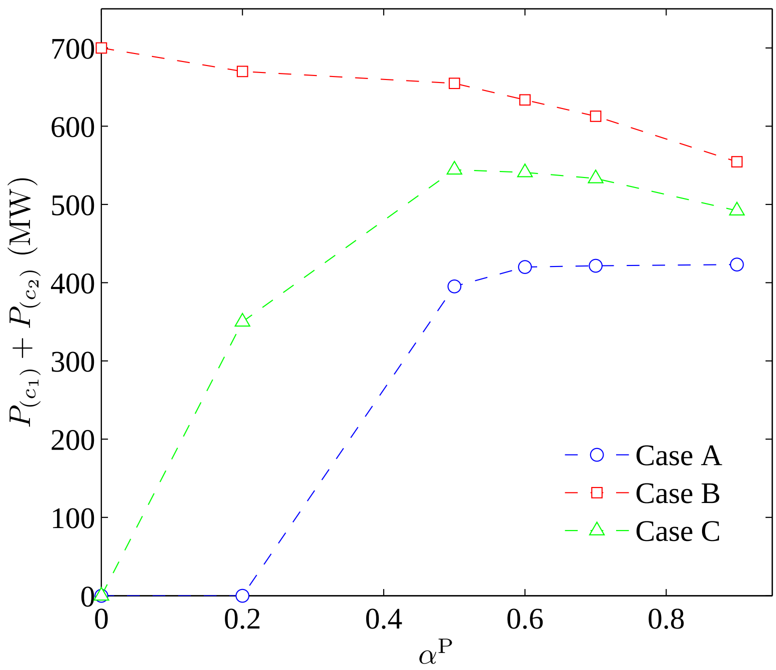


(b) Expected profit vs. standard deviation

Figure 4.12: Forward contracting case study: impact of unit failures on the efficient frontier (Case C)



(a) FOR = 5%



(b) FOR = 10%

Figure 4.13: Forward contracting case study: impact of forward prices on the power sold through forward contracts

The steps to test the quality of the optimal decisions are the following:

- Step 1: Solve optimization problem (4.21) to determine the optimal decisions considering the 30 pool price scenarios and the 50 unit availability scenarios obtained as previously described.

- Step 2: Solve optimization problem (4.21) to determine the optimal decisions of the producer considering the same 30 pool price scenarios but neglecting the possibility of suffering from unit failures.

- Step 3: Generate the scenario tree to compare Out-of-Sample the optimal decisions determined in Steps 1 and 2. In this respect, we consider the historical realization of the pool price during the study horizon to calculate the CVaR of the producer. As for unit failures, given that there is no available data of actual forced outages of generating units, we use the initial set of 5000 availability scenarios to better represent this source of uncertainty. Note that given the discrete nature of this stochastic process, a tree of 2^{N_T} scenarios perfectly represents this uncertain parameter from a theoretical point of view.

- Step 4: Calculate the value of the CVaR considering the actual realization of the pool price and the set of 5000 availability scenarios, and fixing the first-stage decisions variables to those obtained in Steps 1 and 2, respectively.

- Step 5: Compare the two values of the CVaR obtained in the previous step to evaluate the economic gain obtained by implementing in practice the optimal decisions obtained in Step 1.

This Out-of-Sample study is carried out considering twelve 8-week time horizons covering year 2001. The duration of each 8-week horizon is provided in Table 4.12.

In order to take into account different forward prices, as compared with the expected pool price, three sets of forward contract prices are considered. First, optimal decisions are determined if the forward contract prices are equal to the expected pool price during the delivery period of each contract. In the other two cases, the forward contract prices are set to the expected pool price ± 2 €/MWh, respectively, representing a relative variation between 5% and 12%.

Table 4.12: Forward contracting case study: time horizon of Out-of-Sample studies

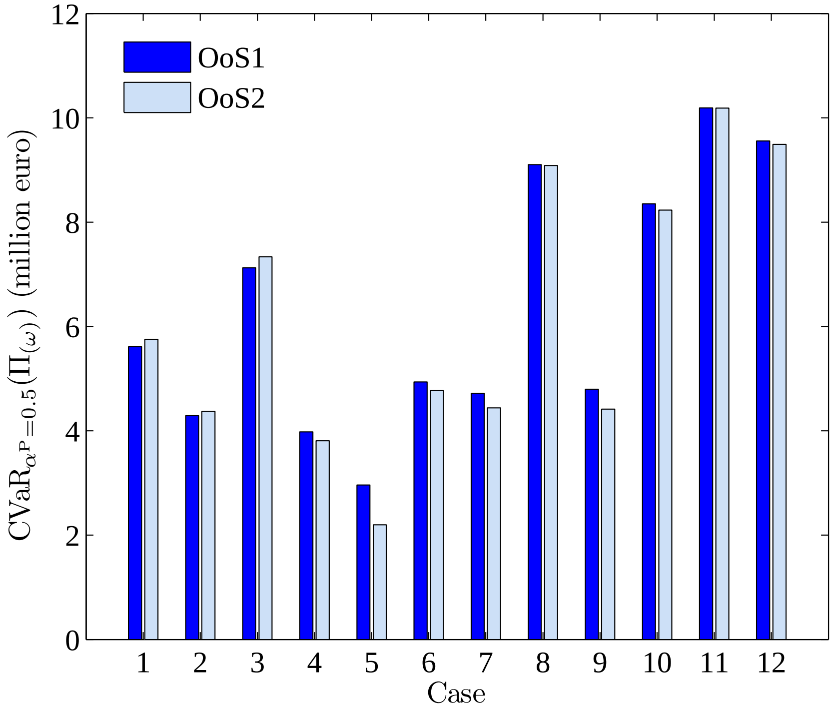
Case	Time horizon
1	week01–2001 to week08–2001
2	week05–2001 to week12–2001
3	week09–2001 to week16–2001
4	week13–2001 to week20–2001
5	week17–2001 to week24–2001
6	week21–2001 to week28–2001
7	week25–2001 to week32–2001
8	week29–2001 to week36–2001
9	week33–2001 to week40–2001
10	week37–2001 to week44–2001
11	week41–2001 to week48–2001
12	week45–2001 to week52–2001

Fig. 4.14 compares, for the twelve considered cases, the value of the Out-of-Sample CVaR if first-stage decisions are fixed to those determined in Step 1 (OoS1) with the Out-of-Sample CVaR associated with first-stage decisions of Step 2 (OoS2). In Fig. 4.14(a), the forced outage rate and the risk aversion level of the producer are equal to 5% and 0.5, respectively, and the forward prices are settled to the corresponding expected pool price. On the other hand, in Fig. 4.14(b), we consider a higher value of both the FOR (10%) and α^{P} (0.9), and the forward prices are 2 €/MWh lower than the expected pool price during the delivery periods of the forward contracts.

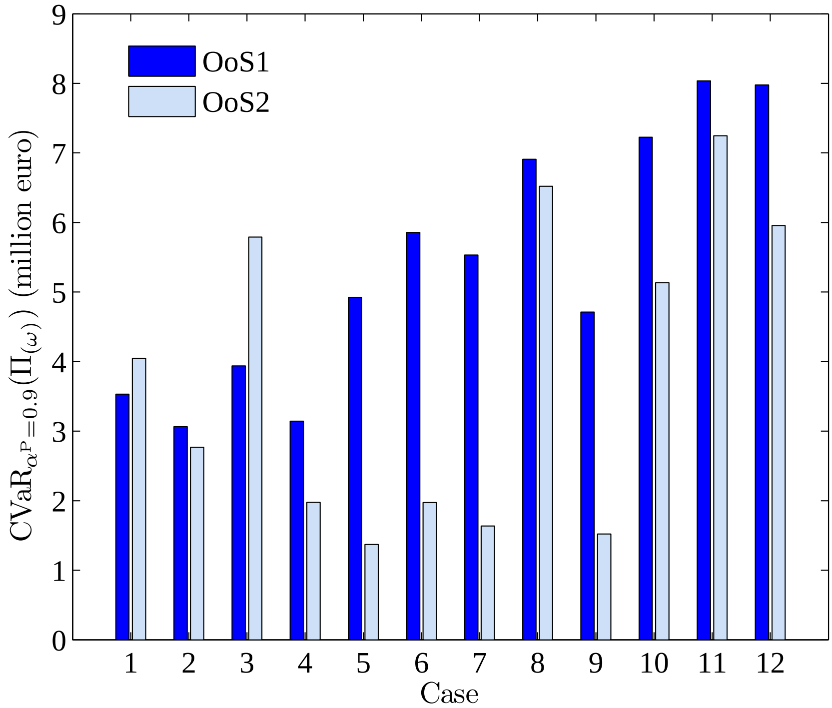
Observe that, for most of the cases of both figures, the CVaR obtained if first-stage decisions are made considering the mathematical modeling of the availability uncertainty are significantly higher. As expected, this increase is more considerable for higher forced outages rates. Moreover, a higher risk aversion of the producer, which means that decisions should be made to face up to the occurrence of unit failures, makes the modeling of the availability uncertainty essential to achieve optimal results.

From completeness, Fig. 4.15 shows the comparison in the expected profits for the same cases presented in Fig.4.14. Note that although the producer is not maximizing its expected profit (since α^{P} is not equal to 0), the improvement of the expected profit that is achieved accounting for the availability risk is just apparent in most of the cases.

Table 4.13 provides the average relative difference between the CVaR values

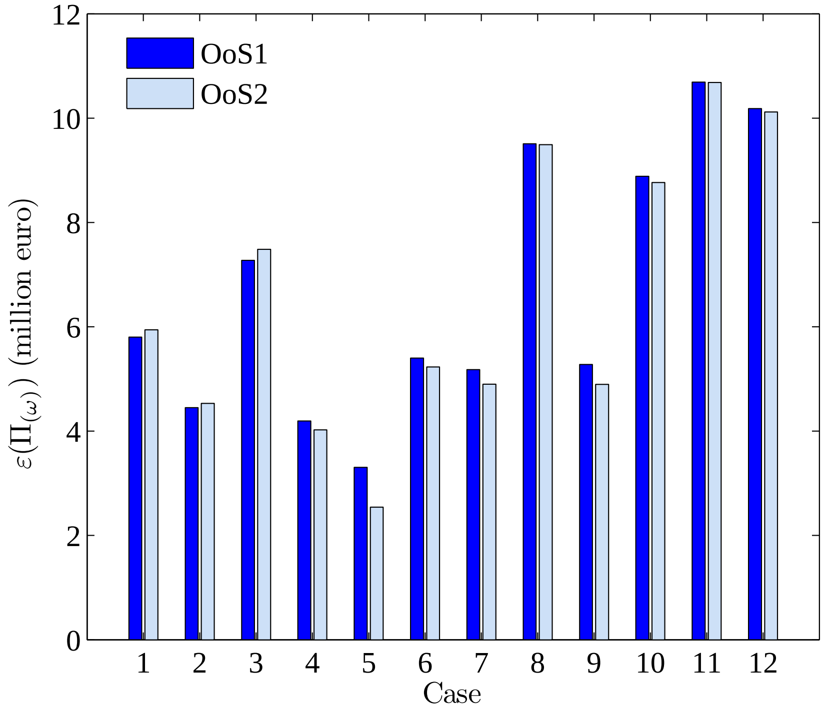


(a) FOR = 5%, α^{P} = 0.5, and forward prices equal to the expected pool price

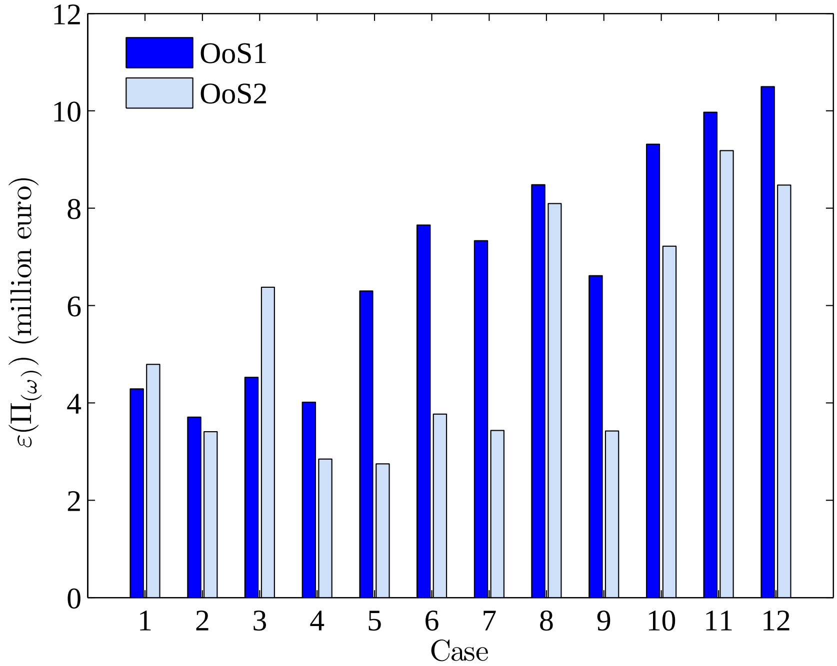


(b) FOR = 10%, α^{P} = 0.9, and forward prices equal to the expected pool price minus 2 €/MWh

Figure 4.14: Forward contracting case study: Out-of-Sample results. Variation of the CVaR



(a) FOR = 5%, $\alpha^{\mathrm{P}} = 0.5$, and forward prices equal to the expected pool price



(b) FOR = 10%, $\alpha^{\mathrm{P}} = 0.9$, and forward prices equal to the expected pool price minus 2 €/MWh

Figure 4.15: Forward contracting case study: Out-of-Sample results. Variation of the expected profit

Table 4.13: Forward contracting case study: Out-of-Sample results. CVaR increase

α^{P}	FOR = 5%	FOR = 10%
0	0	0.16
0.5	3.0	4.0
0.9	9.1	13.2

Table 4.14: Forward contracting case study: Out-of-Sample results. Expected profit increase

α^{P}	FOR = 5%	FOR = 10%
0	0	0.16
0.5	2.8	3.6
0.9	7.0	9.4

obtained in Step 4 for three different values of the risk aversion parameter ($\alpha^{\mathrm{P}} = \{0, 0.5, 0.9\}$) and two forced outage rates (5% and 10%). The average value is calculated considering 12 time horizons and 3 set of forward contract prices, i.e., a total number of 36 different cases.

If the producer is risk-neutral ($\alpha^{\mathrm{P}} = 0$), the Out-of-Sample results show that the objective functions obtained if decisions are made with or without considering unit availability uncertainty are nearly the same, as explained in Section 4.7. On the other hand, a higher risk aversion yields a difference in the CVaR value. For example, for $\alpha^{\mathrm{P}} = 0.5$ and FOR = 5%, the average increase in the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.5}$ of the profit distribution if decisions are made considering unit failures is equal to 3.0%. It is relevant to note that considering uncertainty on unit availability to make forward contract decisions becomes more important as the forced outage rate increases.

Likewise, Table 4.14 shows the average increase in the expected profit if decisions are made considering unexpected unit failures in the decision-making model. Although lower than the increase observed in the CVaR values, the improvement of the expected profit is significant.

Therefore, this Out-of-Sample study highlights the need for considering the uncertainty related to generating unit availability to make informed decisions pertaining to forward contracting by power producers.

4.9.3 Impact of the Number of Units on Producer Forward Contracting

This section is aimed at analyzing the influence of the number of generating units on the optimal forward contracting of a power producer. To this end, we compare the amount of electricity sold through forward contracts by two power producers owning a different number of the same type of generating unit. The first power producer has one generating unit with maximum and minimum power outputs of 350 MW and 50 MW, respectively, and whose piecewise linear cost function is provided in Table 4.9. We consider three forced outage rates for this generating unit: 0, 5, and 10%. On the other hand, the second power producer owns two generating units with the same technical and economic characteristics as those of the unit owned by the first producer.

In order to carry out the above comparison, special attention needs to be paid to the characterization of the availability uncertainty of the generating units. To characterize the availability risk faced by the first producer, a set of 5000 availability scenarios are initially generated and subsequently reduced to a final set of 50 scenarios. Each scenario contains the availability evolution throughout the study horizon of each generating unit. As stated in Section 4.6.2.2, a higher number of generating units owned by a power producer requires the generation of a higher number of scenarios representing the availability uncertainty. However, in this case, we also generate a set of 5000 scenarios to characterize the availability of the two generating units of the second power producer in order to keep the computational burden of the scenario reduction technique within reasonable limits. Moreover, the final scenario tree also comprises 50 availability scenarios to trim down the number of binary variables representing the start-up and shut-down of the units, which increases proportionally with the number of scenarios and generating units. Computationally speaking, the growth of the number of these binary variables constitutes the main reason for which optimization problem (4.21) may become unacceptably intensive in terms of computational time.

Both the scenario set representing the pool price uncertainty and the prices of forward contracts are the same as those considered in Section 4.9.1. Thus, optimization problem (4.21) is solved for each one of the three forward price sets of Table 4.11. For simplicity, only three values of the risk aversion parameter are used to solve the corresponding optimization problems, namely

$\alpha^{\mathrm{P}} = \{0, 0.5, 0.9\}$.

Tables 4.15, 4.16, and 4.17 contain, for each set of forward contract prices, the percentage of production sold through the two available forward contracts with respect to the maximum capacity of each power producer as a function of the FOR, risk aversion level, and number of generating units owned by each power producer.

Table 4.15: Forward contracting case study: percentage of production sold through forward contracts as a function of the number of units owned by the power producer (Case A)

α^{P}	FOR = 0%		FOR = 5%		FOR = 10%	
	$N_I = 1$	$N_I = 2$	$N_I = 1$	$N_I = 2$	$N_I = 1$	$N_I = 2$
0	0	0	0	0	0	0
0.5	100	100	68	73	56	65
0.9	100	100	69	78	60	70

Table 4.16: Forward contracting case study: percentage of production sold through forward contracts as a function of the number of units owned by the power producer (Case B)

α^{P}	FOR = 0%		FOR = 5%		FOR = 10%	
	$N_I = 1$	$N_I = 2$	$N_I = 1$	$N_I = 2$	$N_I = 1$	$N_I = 2$
0	100	100	100	100	100	100
0.5	100	100	95	96	94	95
0.9	100	100	86	87	79	82

Table 4.17: Forward contracting case study: percentage of production sold through forward contracts as a function of the number of units owned by the power producer (Case C)

α^{P}	FOR = 0%		FOR = 5%		FOR = 10%	
	$N_I = 1$	$N_I = 2$	$N_I = 1$	$N_I = 2$	$N_I = 1$	$N_I = 2$
0	0	0	0	0	0	0
0.5	100	100	83	84	75	81
0.9	100	100	77	81	71	76

From the results of these tables, we can infer that a power producer owning more than one generating unit with the same FOR can increase the power sold through forward contracts for the same risk aversion level. This is so because if a power producer owns more than one generating unit with identical FORs, say 5%, the probability that the producer loses its full capacity is lower than 5% since the occurrence of unexpected failures affecting all its generating units during the same time steps is unlikely. Consequently, the decrease in the availability risk that results from the increase in the number of generating units reduces the financial losses associated with forward contracting if unit failures occur and therefore, it allows the producer to increase the power sold through forward contracts.

As previously stated, the difference among the three considered cases lies in the dependence of the power sold through forward contracts with the risk aversion level. Whereas in case A the power increases if the risk aversion level does, it decreases for case B. Likewise, in case C we observe an increasing-decreasing behavior of the power sold through forward contracts. Note how, for these three cases, owning two generating units instead of only one has the same effect on forward contracting decisions, namely, an increase in the power sold through forward contracts.

4.9.4 Computational burden

To conclude this case study, we point out that the simulations results presented in this chapter have been obtained using CPLEX 12.1.0 [6] under GAMS [5] on a Sun Fire X4600M2 with 8 Quad-Core processors running at 2.9 GHz and 256 GB of RAM. The computational time required to solve the two-stage stochastic model (4.21) depends on whether or not generating unit failures are considered. If the generating unit never fails and the pool price uncertainty is characterized by a set of 30 scenarios, the computational time needed to identify the optimal forward contracting decisions is lower than five seconds. On the other hand, if the pool prices and the availability of the generating unit are represented, respectively, by a set of 30 and 50 scenarios, the computational time required to solve optimization problem (4.21) with a total number of 1500 scenarios is hovering around one hour.

4.10 Summary

Electricity markets involve risks that are faced by market participants. Among them, power producers have to deal with the risk associated with pool price volatility and the possibility of suffering from unexpected failures of its generating units. While forward contracts reduce the price risk by allowing the producers to sell its production at fixed prices, they also increase the financial losses obtained if unit failures occur. Therefore, a risk-averse power producer should decide its forward contracting decisions taking into account the uncertainty related to both pool prices and unit availability.

This chapter focuses on the design of a two-stage stochastic programming model to determine the optimal quantity that a power producer should sell through forward contracts considering the variability of pool prices as well as the possibility of suffering from unexpected unit failures. The optimal forward contracting decisions for the power producer obtained from the proposed model are aimed at maximizing the Conditional Value-at-Risk of the profit distribution for a given risk aversion level α^{P}. Uncertainties related to pool prices and unit availability are characterized via a set of scenarios, whose cardinality is conveniently reduced to attain computational tractability. The proposed stochastic programming model results in a large-scale mixed-integer linear programming problem that is efficiently solved using commercially available software.

The adequate performance of the proposed decision-making model to determine optimal forward contract decisions is demonstrated through both an illustrative example and a realistic case study based on the electricity market of the Iberian Peninsula. Additionally, an Out-of-Sample analysis is carried out to highlight the good performance of the decisions obtained from the proposed model using real-world data. Finally, the impact of the number of generating units on the forward contracting decisions of the producer is also discussed.

Chapter 5

Insuring Unit Failures in Electricity Markets

5.1 Introduction

Most electricity markets include a derivatives market, which is used by participants to reduce the risk associated with the high volatility of the pool price of electricity by selling or buying this commodity through forward contracts at fixed prices. For example, a power producer can sell part of its production through a forward contract to make part of its revenue stable regardless of the resulting pool price during the delivery period of such a contract. However, forward contracts imply the obligation to sell the agreed energy throughout the delivery period and therefore, an unexpected failure affecting the generating units of the producer may entail significant financial losses, especially if pool prices are high during the forced outage time periods. In other words, while selling electricity through forward contracts reduces the pool price risk faced by power producers, it also increases the risk associated with unit failures, known as availability risk.

Some insurers provide insurance contracts against unit failures. By means of these contracts, the insurer agrees to reduce the financial losses caused by unexpected failures of generating units in exchange for a premium. That is, the insurer has to pay the producer during those time steps in which the unit is forced out and the pool price is above a pre-specified level, thus reducing the availability risk faced by the power producer.

In this chapter, we present a two-stage stochastic programming model to analyze the impact of an insurance contract against unit failures on both the forward contracting decisions and the objective function of a risk-averse power producer. The ultimate purpose is to determine the convenience of acquiring such insurance by the power producer. Additionally, we use the proposed optimization model to compute the maximum premium that a power producer would be willing to pay for a given insurance contract.

5.2 Insurance Against Unit Failures

5.2.1 Definition and Characteristics

An insurance contract against unit failures is a financial instrument whereby a power producer receives a refund from the insurer if financial losses associated with unexpected production forced outages occur. In exchange, the producer has to pay a certain premium to the insurer at the beginning of the time period covered by the insurance.

Although there exists a wide variety of insurance contracts against unit failures, usually two conditions must be met for the insurer to have the obligation to pay the producer: the production unit is forced out and the pool price is higher than a strike price, which is agreed in advance. Therefore, the insurance contract against unit failures considered in this dissertation is characterized by:

- The premium M^{I} in €, which is the initial amount of money to be paid by the power producer to acquire the insurance contract.

- The strike price λ^{I} in €/MWh, which is the limit above which prices are considered *high prices*. That is, the strike price identifies the scenario in which a payment from the insurer to the power producer occurs.

- The insured power P^{I} in MW, which is agreed in advance and stands for the power quantity covered by the insurance contract.

- The time horizon during which the insurance contract is in effect, e.g., next month or next quarter.

Apart from this basic type of insurance contract against unit failures, there exist more complex insurance arrangements that include other conditions like a maximum payment by the insurer or a deductible capacity, according to which, the insurer does not pay if the total outage capacity is smaller than the

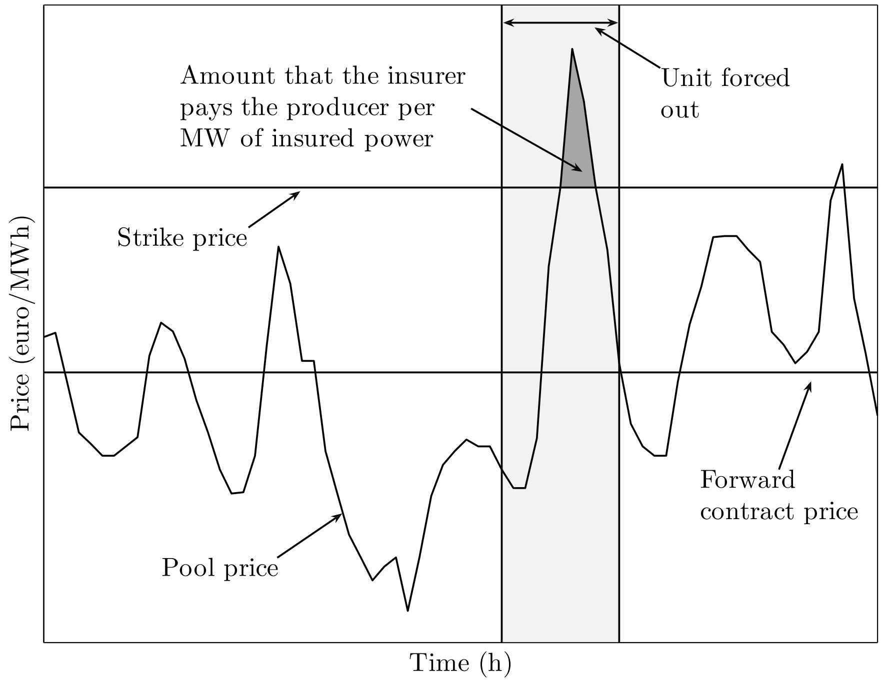


Figure 5.1: Insurance contract example

deductible capacity [95, 96]. However, these arrangements are not considered in this dissertation.

Fig. 5.1 represents an illustrative example of the insurance contract against unit failures considered in this dissertation. In this figure, pool prices during one week are depicted. During this time horizon, the unit is forced out from hour 40 to hour 50 (light gray area). If the producer has signed a weekly forward contract to sell electricity and its single production unit fails, it has to buy the energy in the pool to meet its contracting selling obligations, which may cause significant financial losses. However, if the producer has also acquired an insurance contract against unit failures, the insurer has to pay the producer the difference between the pool price and the strike price times the insured power in all the time steps in which the following two conditions are met: the pool price is higher than the strike price and the generating unit is forced out. In Fig. 5.1 the darker gray area represents the amount per MW of insured power that the insurer has to pay the producer.

The amount that the insurer has to pay the power producer if scenario ω realizes ($D_{(\omega)}$) is mathematically expressed as

$$D_{(\omega)} = \sum_{t \in G_{(\omega)}} P^{\mathrm{I}} (\lambda^{\mathrm{P}}_{(\omega,t)} - \lambda^{\mathrm{I}}) L_{(t)}, \tag{5.1}$$

where $G_{(\omega)}$ is the set of time steps in scenario ω in which the pool price is higher than the strike price and the production unit is forced out. Likewise, $\lambda^{\mathrm{P}}_{(\omega,t)}$ and $L_{(t)}$ are the pool price in time step t and scenario ω, and the duration in hours of time step t, respectively. Therefore, given a premium equal to M^{I}, the producer revenue associated with the insurance contract, which is denoted by $\Pi^{\mathrm{I}}_{(\omega)}$ is

$$\Pi^{\mathrm{I}}_{(\omega)} = s^{\mathrm{P}}(D_{(\omega)} - M^{\mathrm{I}}), \tag{5.2}$$

where s^{P} is a binary variable that is equal to 1 if the power producer signs the insurance contract, and 0 otherwise.

In relation to equations (5.1) and (5.2), note that the acquisition of the insurance contract necessarily entails the payment of the premium cost M^{I} for any scenario realization. However, the producer only receives a refund from the insurer if a scenario characterized by high pool prices and a unit failure realizes. Consequently, the question that arises here is: *How much is a power producer willing to reduce its profit under any scenario (as a result of the insurance premium) in order to receive an extra payment from the insurer if significant financial losses caused by unexpected unit failures occur?*

5.2.2 Hedging against Availability Risk

In the course of Chapter 4, two of the main risks faced by power producers were presented, namely, price and availability risks. Further, we concluded that while forward contracts are adequate derivatives to reduce price risk, selling energy through these contracts increases the risk related to unexpected failures of generating units, i.e., the availability risk.

In this chapter, an insurance contract against unit failures is considered as a financial instrument whereby a power producer can hedge specifically against the availability risk by reducing the financial losses if forced outages occur when the pool price is notably high. On the other hand, the purchase of an insurance contract also allows the power producer to reduce the price risk by increasing the power sold through forward contracts; a quantity that, without the insurance contract, would be bounded so as to reduce the availability risk. Therefore, although the main purpose of an insurance contract against unit failures is to hedge against the availability risk, it also reduces the price risk by allowing the power producer to increase the power sold through forward contracts.

5.3 Producer Model for Insurance Selection

5.3.1 Assumptions

The following assumptions are considered to formulate the decision-making model of a power producer if an insurance contract against unit failures is available. Although most of them have already been discussed throughout the dissertation, they are summarized below for the reader's convenience.

1. The power producer under study owns only thermal generating units, which are dispatchable sources of electricity with a quadratic production cost function that, in this thesis, is approximated by a piecewise linear function. The power generated by each unit is limited by its minimum and maximum power outputs, while ramp limits and minimum up and down times are short-term operating constraints disregarded in the proposed model.

2. The power producer can sell its production either at volatile prices in the pool, or at fixed prices through forward contracts. Moreover, in the model proposed in this dissertation, the arbitrage between these two markets is avoided and their corresponding prices are not affected by power producer decisions.

3. Two uncorrelated sources of uncertainty are involved in the proposed model: the pool price and the availability of the generating units. While forward and insurance contracting decisions are made facing the uncertainty of such parameters throughout the whole study horizon, pool decisions are assumed to be made with perfect information because of the comparatively much smaller uncertainty involved in the pool.

4. Although the proposed model in this dissertation is valid if several insurance contracts against unit failures are available, only one insurance spanning the whole study horizon is considered. This insurance is characterized by an initial premium, a strike price, and an insured power, parameters which are known at the beginning of the study horizon. The power producer has to decide whether or not the insurance is acquired according to the uncertainty related to the pool price and the availability of its generating units as well as its risk aversion level.

5. All the generating unit failures are assumed to be unexpected and unavoidable, thus disregarding the possibility of receiving any payment from the insurer as a result of either planned or *fake* outages of the generating units.

5.3.2 Uncertainty Characterization

As in Chapter 4, the two sources of uncertainty involved in the two-stage stochastic programming problem proposed in this chapter are the electricity pool price and the availability of generating units. Therefore, the uncertainty characterization of this model coincides with that provided in Section 4.6.2 of Chapter 4.

5.3.3 Variables

The proposed model, which is designed to obtain optimal decisions on insurance contracting, is formulated as a two-stage stochastic programming and as such, decisions variables are classified into two groups: first-stage decisions, which are made facing the uncertainty of the stochastic parameters; and second-stage decisions, whose values adapt to each particular realization of the uncertain processes. Within this particular model, these decisions are:

- First-stage variables:
  1. Power sold through each available forward contract ($P_{(c)}$).
  2. Binary variable to decide whether or not the available insurance contract is signed (s^{P}).
- Second-stage variables:
  1. Start-up and shut-down plan of each generating unit throughout the study horizon ($u_{(i,\omega,t)}$).
  2. Scheduled power output for each generating unit throughout the study horizon ($P^{\mathrm{G}}_{(i,\omega,t)}$).
  3. Amount of generated power to be sold in the pool for each time step of the study horizon ($P^{\mathrm{P}}_{(\omega,t)}$).

5.3.4 Objective Function

The objective function of the proposed optimization model for the power producer consists in maximizing the Conditional Value-at-Risk (CVaR) of its profit probability distribution Π for a given confidence level α^{P}, i.e.,

$$\text{Maximize} \quad \mathrm{CVaR}_{\alpha^{\mathrm{P}}}(\Pi) = \zeta - \frac{1}{1-\alpha^{\mathrm{P}}} \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \eta_{(\omega)}, \tag{5.3}$$

where $\pi_{(\omega)}$ represents the probability of scenario ω, and ζ and $\eta_{(\omega)}$ are two auxiliary variables that are needed to calculate the CVaR of the profit probability distribution as explained in Section 2.2.3 of Chapter 2.

The producer profit for each scenario, $\Pi_{(\omega)}$, comprises four components: the revenue obtained in the pool, $\Pi^{\mathrm{P}}_{(\omega)}$, the revenue obtained from the signed forward contracts, Π^{F}, the revenue associated with the insurance contract, $\Pi^{\mathrm{I}}_{(\omega)}$, and the production cost, $C^{\mathrm{G}}_{(\omega)}$, i.e.,

$$\Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{F}} + \Pi^{\mathrm{I}}_{(\omega)} - C^{\mathrm{G}}_{(\omega)}, \tag{5.4}$$

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} P^{\mathrm{P}}_{(\omega,t)} L_{(t)}, \tag{5.5}$$

$$\Pi^{\mathrm{F}} = \sum_{c=1}^{N_C} \lambda_{(c)} P_{(c)} L_{(c)}, \tag{5.6}$$

$$\Pi^{\mathrm{I}}_{(\omega)} = s^{\mathrm{P}}(D_{(\omega)} - M^{\mathrm{I}}) = s^{\mathrm{P}}\Big(\sum_{t \in G_{(\omega)}} P^{\mathrm{I}}(\lambda^{\mathrm{P}}_{(\omega,t)} - \lambda^{\mathrm{I}}) L_{(t)} - M^{\mathrm{I}} \Big), \tag{5.7}$$

$$C^{\mathrm{G}}_{(\omega)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} C^{\mathrm{G}}_{(i,\omega,t)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} (A_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} \lambda_{(i,b)} P^{\mathrm{G}}_{(i,\omega,t,b)}) L_{(t)}, \tag{5.8}$$

where $\lambda^{\mathrm{P}}_{(\omega,t)}$ represents the uncertain pool price during time step t and in scenario ω, and $P^{\mathrm{P}}_{(\omega,t)}$ is the power sold in the pool during time step t and in scenario ω; $L_{(t)}$ represents the duration in hours of time step t; $\lambda_{(c)}$, $L_{(c)}$, and $P_{(c)}$ are the price, the duration, and the amount of electricity to be sold through forward contract c, respectively; M^{I}, P^{I}, and λ^{I} are the premium, the insured power, and the strike price corresponding to the available insurance contract, while s^{P} is a binary variable to decide whether or not the insurance contract is signed; $P^{\mathrm{G}}_{(i,\omega,t,b)}$ represents the power generated by unit i from each block b during time step t and in scenario ω, and finally, $A_{(i)}$ and $\lambda_{(i,b)}$ are the no-load cost and block-slopes to approximate the quadratic production cost function of unit i.

5.3.5 Constraints

In this section, we formulate the constraints of the proposed stochastic optimization model to determine the optimal power sold by the power producer in both the pool and through forward contracts, as well as its decision on the acquisition of an insurance against unit failures. Note that most of the constraints have already been discussed in Chapter 4.

If unexpected failures are considered, the generated power by a unit is bounded by its maximum and minimum power output as follows,

$$u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\text{Min}}_{(i)} \leq P^{\text{G}}_{(i,\omega,t)} \leq u_{(i,\omega,t)}k_{(i,\omega,t)}P^{\text{Max}}_{(i)}, \quad \forall i, \forall \omega, \forall t. \tag{5.9}$$

In order to approximate the quadratic cost function by a piecewise linear function, the following constraints are needed:

$$P^{\text{G}}_{(i,\omega,t)} = P^{\text{Min}}_{(i)}u_{(i,\omega,t)} + \sum_{b=1}^{N_B} P^{\text{G}}_{(i,\omega,t,b)}, \qquad \forall i, \forall \omega, \forall t \tag{5.10}$$

$$0 \leq P^{\text{G}}_{(i,\omega,t,b_1)} \leq P^{\text{Max}}_{(i,b_1)} - P^{\text{Min}}_{(i)}, \qquad \forall i, \forall \omega, \forall t \tag{5.11}$$

$$0 \leq P^{\text{G}}_{(i,\omega,t,b)} \leq P^{\text{Max}}_{(i,b)} - P^{\text{Max}}_{(i,b-1)}, \qquad \forall i, \forall \omega, \forall t, \forall b = b_2, ..., N_B. \tag{5.12}$$

Arbitrage between the pool and the derivatives market is avoided by the following constraints:

$$\sum_{i=1}^{N_I}(k_{(i,\omega,t)} - 1)P^{\text{Max}}_{(i)} \leq P^{\text{P}}_{(\omega,t)}, \qquad \forall \omega, \forall t \tag{5.13}$$

$$0 \leq P_{(c)}, \qquad \forall c. \tag{5.14}$$

The following constraints enforce the electric energy balance for each time step and scenario:

$$\sum_{i=1}^{N_I} P^{\text{G}}_{(i,\omega,t)} = P^{\text{P}}_{(\omega,t)} + \sum_{c \in F_{(t)}} P_{(c)}, \quad \forall \omega, \forall t. \tag{5.15}$$

Lastly, the optimization of the CVaR requires the following constraints:

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \qquad \forall \omega \tag{5.16}$$

$$0 \leq \eta_{(\omega)}, \qquad \forall \omega, \qquad (5.17)$$

where $\eta_{(\omega)}$ and ζ are two auxiliary variables needed to calculate the optimal value of the CVaR for a confidence level α^{P} [153].

5.3.6 Formulation

The proposed formulation of the risk-constrained profit maximization problem for a power producer is

$$\text{Maximize}_{P^{\mathrm{G}}_{(i,\omega,t,b)}, P^{\mathrm{P}}_{(\omega,t)}, P_{(c)}, u_{(i,\omega,t)}, s^{\mathrm{P}}, \zeta, \eta_{(\omega)}}$$

$$\zeta - \frac{1}{1-\alpha^{\mathrm{P}}} \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \eta_{(\omega)} \qquad (5.18\text{a})$$

subject to

$$\Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{F}} + \Pi^{\mathrm{I}}_{(\omega)} - C^{\mathrm{G}}_{(\omega)}, \qquad \forall \omega \qquad (5.18\text{b})$$

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} P^{\mathrm{P}}_{(\omega,t)} L_{(t)}, \qquad \forall \omega \qquad (5.18\text{c})$$

$$\Pi^{\mathrm{F}} = \sum_{c=1}^{N_C} \lambda_{(c)} P_{(c)} L_{(c)}, \qquad (5.18\text{d})$$

$$\Pi^{\mathrm{I}}_{(\omega)} = s^{\mathrm{P}}(D_{(\omega)} - M^{\mathrm{I}}), \qquad \forall \omega \qquad (5.18\text{e})$$

$$D_{(\omega)} = \sum_{t \in G_{(\omega)}} P^{\mathrm{I}}(\lambda^{\mathrm{P}}_{(\omega,t)} - \lambda^{\mathrm{I}}) L_{(t)}, \qquad \forall \omega \qquad (5.18\text{f})$$

$$C^{\mathrm{G}}_{(\omega)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} C^{\mathrm{G}}_{(i,\omega,t)}, \qquad \forall \omega \qquad (5.18\text{g})$$

$$C^{\mathrm{G}}_{(i,\omega,t)} = \Big(A_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} \lambda_{(i,b)} P^{\mathrm{G}}_{(i,\omega,t,b)} \Big) L_{(t)}, \qquad \forall i, \forall \omega, \forall t \qquad (5.18\text{h})$$

$$u_{(i,\omega,t)} k_{(i,\omega,t)} P^{\mathrm{Min}}_{(i)} \leq P^{\mathrm{G}}_{(i,\omega,t)} \leq u_{(i,\omega,t)} k_{(i,\omega,t)} P^{\mathrm{Max}}_{(i)}, \qquad \forall i, \forall \omega, \forall t \qquad (5.18\text{i})$$

$$P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{Min}}_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} P^{\mathrm{G}}_{(i,\omega,t,b)}, \qquad \forall i, \forall \omega, \forall t \qquad (5.18\text{j})$$

$$0 \leq P^{\mathrm{G}}_{(i,\omega,t,b_1)} \leq P^{\mathrm{Max}}_{(i,b_1)} - P^{\mathrm{Min}}_{(i)}, \qquad \forall i, \forall \omega, \forall t \qquad (5.18\text{k})$$

$$0 \leq P^{\mathrm{G}}_{(i,\omega,t,b)} \leq P^{\mathrm{Max}}_{(i,b)} - P^{\mathrm{Max}}_{(i,b-1)}, \qquad \forall i, \forall \omega, \forall t, \forall b = b_2, ..., N_B \qquad (5.18\text{l})$$

$$\sum_{i=1}^{N_I} (k_{(i,\omega,t)} - 1) P^{\mathrm{Max}}_{(i)} \leq P^{\mathrm{P}}_{(\omega,t)}, \qquad \forall \omega, \forall t \qquad (5.18\text{m})$$

$$0 \leq P_{(c)}, \qquad \forall c \qquad (5.18n)$$

$$\sum_{i=1}^{N_I} P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{P}}_{(\omega,t)} + \sum_{c \in F_{(t)}} P_{(c)}, \qquad \forall \omega, \forall t \qquad (5.18o)$$

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \qquad \forall \omega \qquad (5.18p)$$

$$0 \leq \eta_{(\omega)}, \qquad \forall \omega. \qquad (5.18q)$$

$$u_{(i,\omega,t)} \in \{0,1\}, \qquad \forall i, \forall \omega, \forall t \qquad (5.18r)$$

$$s^{\mathrm{P}} \in \{0,1\}. \qquad (5.18s)$$

Objective function (5.18a) to be maximized is the CVaR of the producer profit probability distribution for a confidence level α^{P}. Equations (5.18b)–(5.18h) determine the total profit achieved by the producer as well as its components, namely, the profit obtained in the pool, the revenue from selling through forward contracts, the insurance profit, and the production cost. The total power produced by each generating unit, which is bounded by constraints (5.18i), is determined according to equations (5.18j)–(5.18l) as a result of the piecewise linear modeling of the production cost. The arbitrage between the pool and the derivatives market is avoided via constraints (5.18m) and (5.18n). The energy balance in each scenario and time step is enforced by constraints (5.18o). In order to maximize the CVaR of the profit distribution, equations (5.18p) and (5.18q) are needed. Finally, constraints (5.18r) and (5.18s) are binary variable declarations.

5.3.7 Maximum Premium of the Producer

The optimization model (5.18) is formulated so as to determine whether or not the power producer should acquire an insurance contract given its premium, strike price and insured power level. However, a power producer may also be interested in knowing the value of the maximum premium to pay in exchange for an insurance contract with a given strike price and insured power level. We refer to this amount, denoted by $M^{\mathrm{I,P}*}$, as *maximum premium* of the producer. Note that the maximum premium measures how much the producer is willing to reduce its expected profit in exchange for cutting down the losses caused by unexpected unit failures.

If the producer is risk neutral, the determination of the maximum premium is straightforward, being equal to the expected compensation associated with the insurance contract, i.e., $\sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} D_{(\omega)}$, which can be computed just knowing the parameters of the insurance together with the scenario set representing the pool price and the unit availability. Since the power producer seeks to maximize its expected profit, a premium higher than the expected compensation of the insurance would lead to a negative expected revenue and hence, to the rejection of such insurance. Conversely, a premium lower than the expected compensation would make attractive the insurance contract.

If the producer is risk averse, i.e., if its objective function consists in maximizing the CVaR of the profit distribution for a confidence level α^{P} different from zero, the determination of the maximum premium needs further elaboration. In this case, the maximum premium of the producer is calculated by comparing the values of the CVaR depending on whether or not the insurance contract is signed for different premium values. The procedure is detailed below.

In order to obtain the maximum premium of a risk-averse producer, first we determine the evolution of the CVaR as a function of the premium considering that the insurance contract is always signed. For this purpose, we solve optimization problem (5.18) in which the binary variable s^{P} and the premium M^{I} are fixed to 1 and 0, respectively, thus obtaining the value of the CVaR if a *free* insurance contract is considered, which is denoted by $\mathrm{CVaR}^{\mathrm{P}}_{M^{\mathrm{I}}=0}$. From this point on, assuming that the insurance is acquired for any premium M^{I}, the evolution of the CVaR with the premium can be easily determined by applying the positively homogeneous property stated in Section 2.2.3 of Chapter 2, i.e.,

$$\mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=1}(M^{\mathrm{I}}) = \mathrm{CVaR}^{\mathrm{P}}_{M^{\mathrm{I}}=0} - M^{\mathrm{I}}, \tag{5.19}$$

where $\mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=1}(M^{\mathrm{I}})$ denotes the CVaR value of the producer profit distribution provided that an insurance contract of premium M^{I} is signed.

Additionally, we determine the value of the CVaR if the insurance contract is not available by solving optimization problem (5.18). We refer to the corresponding objective function optimal value as $\mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=0}$.

Fig. 5.2 illustrates the lineal relationship between the CVaR and the premium if the insurance contract is signed ($\mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=1}(M^{\mathrm{I}})$) together with an horizontal line that represents the value of the objective function if the insurance contract is not available ($\mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=0}$).

We can state that the insurance contract is signed if, for a specific value of its premium, the CVaR obtained signing the insurance contract is higher than the CVaR of the profit distribution with no insurance contract, and it is not

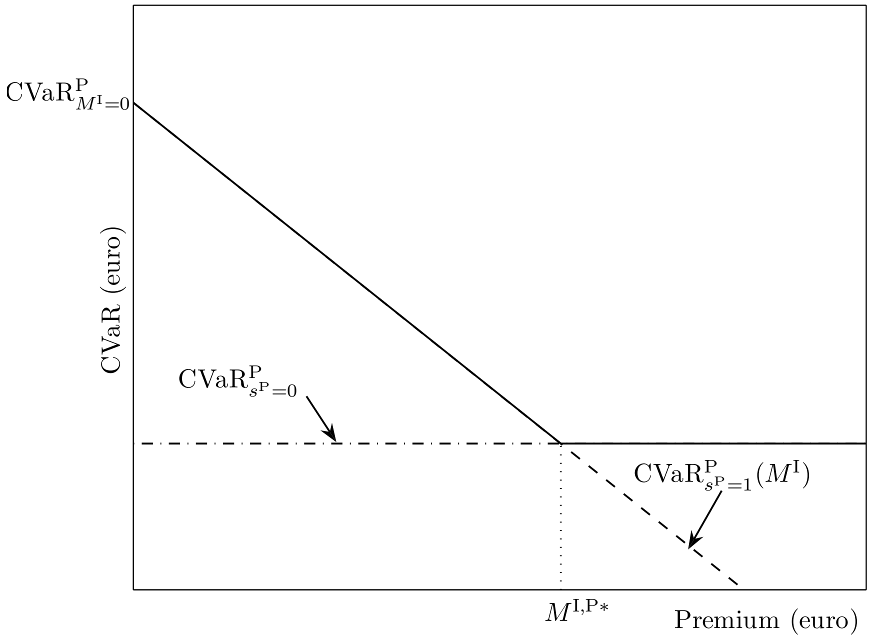


Figure 5.2: Determination of the maximum premium of the producer

signed otherwise. Therefore, the intersection of both lines in Fig. 5.2 represents the value of the premium for which the objective function obtained with and without insurance contract are the same, i.e., the value of the maximum premium that the power producer is willing to pay for the insurance contract ($M^{\mathrm{I,P}*}$). Note that beyond this premium, the CVaR is higher if the insurance is not signed.

Mathematically, the maximum premium of the producer coincides with the premium that makes expression (5.19) equal to the CVaR obtained if no insurance is available, i.e.,

$$\begin{aligned}
&\mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=1}(M^{\mathrm{I,P}*}) = \mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=0} \\
&\mathrm{CVaR}^{\mathrm{P}}_{M^{\mathrm{I}}=0} - M^{\mathrm{I,P}*} = \mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=0} \\
&M^{\mathrm{I,P}*} = \mathrm{CVaR}^{\mathrm{P}}_{M^{\mathrm{I}}=0} - \mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=0}. \qquad (5.20)
\end{aligned}$$

Note that $\mathrm{CVaR}^{\mathrm{P}}_{M^{\mathrm{I}}=0}$ is always higher than or equal to $\mathrm{CVaR}^{\mathrm{P}}_{s^{\mathrm{P}}=0}$. Although the value of the insurance premium is provided by the insurer, the calculation of the maximum premium for different cases provides relevant information related to the tradeoff between expected profit and risk faced by the power producer.

5.4 Insurer model

As a complementary point of view, we propose in this chapter a mathematical formulation to model the utility function of an insurer. For simplicity, we assume that the insurer only offers one insurance contract against unit failures to one power producer. In this case, the profit of the insurer can be expressed as the premium of the insurance minus the compensation the insurer must give to the producer, i.e.,

$$\Pi^{\mathrm{S}}_{(\omega)} = s^{\mathrm{S}}(M^{\mathrm{I}} - D_{(\omega)}), \tag{5.21}$$

where $\Pi^{\mathrm{S}}_{(\omega)}$ and s^{S} represent, respectively, the revenue of the insurer in scenario ω and the binary variable deciding whether or not the insurance contract is signed. M^{I} stands for the premium of the insurance contract and $D_{(\omega)}$ symbolizes the compensation from the insurer to the producer in scenario ω, which is computed as

$$D_{(\omega)} = \sum_{t \in G_{(\omega)}} P^{\mathrm{I}}(\lambda^{\mathrm{P}}_{(\omega,t)} - \lambda^{\mathrm{I}})L_{(t)}, \tag{5.22}$$

where $G_{(\omega)}$ is the set of time steps in scenario ω in which the pool price is higher than the strike price and the production unit of the insured power producer is forced out.

The utility of the insurer is modeled as the expectation of the lowest profits, determined by the CVaR of its profit probability distribution for a given confidence level α^{S}, which represents the risk aversion level of the insurer. The following optimization problem aims at maximizing the utility function of the insurer given the insurance contract characteristics, as well as the pool price distribution and the availability distribution of the generating units of the power producer:

$$\underset{s^{\mathrm{S}},\xi,\rho_{(w)}}{\text{Maximize}} \quad \mathrm{CVaR}_{\alpha^{\mathrm{S}}}(\Pi^{\mathrm{S}}) = \xi - \frac{1}{1-\alpha^{\mathrm{S}}} \sum_{\omega=1}^{N_{\Omega}} \pi_{(\omega)}\rho_{(\omega)} \tag{5.23a}$$

subject to

$$\Pi^{\mathrm{S}}_{(\omega)} = s^{\mathrm{S}}(M^{\mathrm{I}} - D_{(\omega)}), \qquad \forall\omega \tag{5.23b}$$

$$D_{(\omega)} = \sum_{t \in G_{(\omega)}} P^{\mathrm{I}}(\lambda^{\mathrm{P}}_{(\omega,t)} - \lambda^{\mathrm{I}})L_{(t)}, \qquad \forall\omega \tag{5.23c}$$

$$-\Pi^{\mathrm{S}}_{(\omega)} + \xi - \rho_{(\omega)} \leq 0, \qquad \forall\omega \tag{5.23d}$$

$$0 \leq \rho_{(\omega)}, \qquad \forall \omega \qquad (5.23e)$$

$$s^{\mathrm{S}} \in \{0, 1\}, \qquad (5.23f)$$

where ξ and $\rho_{(\omega)}$ are two auxiliary variables used for the calculation of the CVaR of the profit distribution of the insurer.

Objective function (5.23a) to be maximized is the CVaR of the profit distribution of the insurer for a confidence level α^{S}. Constraints (5.23b) and (5.23c) provide, respectively, the mathematical expressions of the utility of the insurer and the amount that the insurer must pay back to the producer in each scenario ω. As stated in Section 2.2.3 of Chapter 2, the optimization of the CVaR of the profit distribution of the insurer needs equations (5.23d) and (5.23e). Constraint (5.23f) declares s^{S} as a binary variable. Therefore, problem (5.23) allows determining the CVaR of the profit distribution of the insurer and whether or not the insurance contract is signed on its part.

5.4.1 Minimum Premium of the Insurer

Conversely to the maximum premium of the producer, the insurer has a *minimum premium* defined as the lowest quantity that the insurer is willing to receive to sign an insurance contract. In order to calculate this minimum premium, first we determine the profit distribution of the insurer and its CVaR if the insurance contract is signed by the insurer even with a premium M^{I} equal to 0. We denote the value of the objective function so obtained as $\mathrm{CVaR}^{\mathrm{S}}_{M^{\mathrm{I}}=0}$. Fixing the binary variable s^{S} to 1 and making use of the properties of the CVaR, the evolution of the objective function of the insurer as a function of the insurance premium is expressed as

$$\mathrm{CVaR}^{\mathrm{S}}_{s^{\mathrm{S}}=1}(M^{\mathrm{I}}) = \mathrm{CVaR}^{\mathrm{S}}_{M^{\mathrm{I}}=0} + M^{\mathrm{I}}, \qquad (5.24)$$

where $\mathrm{CVaR}^{\mathrm{S}}_{s^{\mathrm{S}}=1}(M^{\mathrm{I}})$ denotes the utility of the insurer considering that it signs an insurance contract with a premium equal to M^{I}. Note that the CVaR of the insurer increases with the premium.

Fig 5.3 shows the CVaR of the insurer as a function of the premium if the insurance is signed ($s^{\mathrm{S}} = 1$) and a horizontal line at 0 representing the value of the objective function of the insurer if the insurance contract is not signed. As in the case of the producer, the insurer signs the insurance contract for those premia higher than the value obtained at the intersection of both lines, which

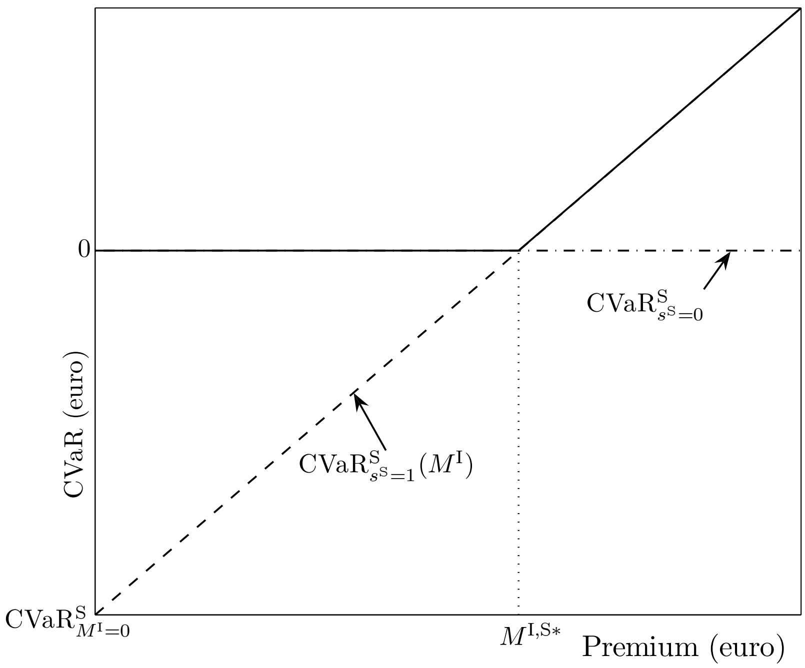


Figure 5.3: Determination of the minimum premium of the insurer

guarantees a nonnegative CVaR. For a lower premium, the insurer does not sign the insurance contract, which renders a CVaR equal to 0. Therefore, the intersection between the two lines in Fig. 5.3 defines the value of the minimum premium of the insurer ($M^{\mathrm{I,S}*}$).

Mathematically, the minimum premium of the insurer is obtained by making expression (5.24) equal to 0, which is the CVaR achieved if the insurance is not signed, that is:

$$\begin{aligned}
&\mathrm{CVaR}^{\mathrm{S}}_{s^{\mathrm{S}}=1}(M^{\mathrm{I,S}*}) = 0\\
&\mathrm{CVaR}^{\mathrm{S}}_{M^{\mathrm{I}}=0} + M^{\mathrm{I,S}*} = 0\\
&M^{\mathrm{I,S}*} = -\mathrm{CVaR}^{\mathrm{S}}_{M^{\mathrm{I}}=0}. \qquad (5.25)
\end{aligned}$$

Note that the value of $\mathrm{CVaR}^{\mathrm{S}}_{M^{\mathrm{I}}=0}$ is always lower than or equal to 0, thereby rendering a positive value for the minimum premium of the insurance company.

5.5 On Insurance Bargaining

Several works in the technical literature show that, either using cooperative [107] or non-cooperative [157, 175] games, the expected profit of the insurer is higher if it bargains with highly risk-averse clients. Although it is out of the scope of this chapter to present a game or an equilibrium model to characterize the complex bargaining process between an insurer and its client (a power producer in this case), we propose a simple but intuitive and practical approach to this bargaining process via the comparison of the maximum premium of the producer and the minimum premium of the insurer.

In Section 5.3.7, for given market conditions (pool prices), forced outage rates of generating units, and risk aversion level of the producer, we propose a procedure to determine the maximum premium that a power producer is willing to pay for a given insurance contract with a certain strike price and insured power level in order to maximize the CVaR of its profit probability distribution. Likewise, Section 5.4.1 is aimed at calculating the minimum premium that an insurer is willing to receive for a given insurance contract against unit failures. Insurance companies insure multiple and independent risks in order to diversify the global risk, then behaving as risk neutral. However, since the case considered in this dissertation focuses on a single power producer, the minimum premium is computed for different risk aversion levels of the insurer.

The insurance bargaining process proposed is based on comparing the maximum premium of the producer with the minimum premium of the insurer. In plain words, if the maximum premium of the producer is higher than the minimum premium of the insurer for a given insurance contract, we conclude that both parties can reach an agreement, and they do not otherwise. Moreover, the higher the difference between those values, the higher the willingness of both market agents to sign the insurance contract. This is clarified in the illustrative example reported in Section 5.6.

5.6 Insurance Contracting Example

In this section, we extend the example provided in Section 4.8 of Chapter 4 to illustrate the main features of insurance contracts against unit failures, as well as to highlight how these insurance contracts affect forward contracting decisions by the power producer. The example is divided into four parts: in the first part, the impact of an insurance contract against unit failures on the producer profit distribution is analyzed; in the second one, we illustrate how forward contracting decisions are modified if the producer signs an insurance; in the third part, we calculate the maximum premium of the producer; and in the last part, the maximum premium of the producer is compared with the

Table 5.1: Insurance contracting example: pool prices scenarios and their probabilities

# scenario	$\lambda^{\mathrm{P}}_{(\varpi,t_1)}$(€/MWh)	$\lambda^{\mathrm{P}}_{(\varpi,t_2)}$(€/MWh)	$\pi_{(\varpi)}$
ϖ_1	20	30	0.2
ϖ_2	18	22	0.6
ϖ_3	16	14	0.2

Table 5.2: Insurance contracting example: availability scenarios and their probabilities

# scenario	$k_{(f,t_1)}$	$k_{(f,t_2)}$	$\pi_{(f)}$
f_1	1	1	0.5476
f_2	1	0	0.1924
f_3	0	1	0.1534
f_4	0	0	0.1066

minimum premium of an insurance company so as to determine in which cases an agreement between both parties can be reached.

5.6.1 Impact on the Producer Profit Distribution

We consider a power producer owning one 100-MW generating unit with a linear cost of 12 €/MWh and with no minimum power output. Table 5.1 includes the pool price throughout the two-hour horizon considered in this example. The MTTF and the MMTR of the generating unit are set to 2 h and 1 h, respectively. The resulting availability scenarios are provided in Table 5.2. Likewise, the final scenario tree obtained by combining price and availability scenarios is provided in Table 5.3.

Table 5.4 provides the profit of the power producer for each scenario, sorted in ascending order, provided that it sells all its generation in the pool and there is no available insurance contract against unit failures. The third and fourth columns contain the probability $\pi_{(\omega)}$ and the cumulative probability $F_{(\omega)}$ of each scenario profit, respectively.

Note that the producer obtains a nil profit if the unit is forced out during the two hours of the study horizon (scenarios ω_4, ω_8, and ω_{12}). If the unit only fails during one hour, the profit depends on the pool price realization during the hour the unit is available. Finally, the highest profit is obtained when the generating unit does not fail and high pool prices realize during the study

Table 5.3: Insurance contracting example: pool price and availability scenarios with their probabilities

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$(€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$k_{(\omega,t_1)}$	$k_{(\omega,t_2)}$	$\pi_{(\omega)}$
ω_1	20	30	1	1	0.10952
ω_2	20	30	1	0	0.03848
ω_3	20	30	0	1	0.03068
ω_4	20	30	0	0	0.02132
ω_5	18	22	1	1	0.32856
ω_6	18	22	1	0	0.11544
ω_7	18	22	0	1	0.09204
ω_8	18	22	0	0	0.06396
ω_9	16	14	1	1	0.10952
ω_{10}	16	14	1	0	0.03848
ω_{11}	16	14	0	1	0.03068
ω_{12}	16	14	0	0	0.02132

Table 5.4: Insurance contracting example: power producer profit distribution if its entire production is sold in the pool with no insurance

# scenario	$\Pi_{(\omega)}$ (€)	$\pi_{(\omega)}$	$\mathrm{F}_{(\omega)}$
ω_4	0	0.02132	0.02132
ω_8	0	0.06396	0.08528
ω_{12}	0	0.02132	0.10660
ω_{11}	200	0.03068	0.13728
ω_{10}	400	0.03848	0.17576
ω_6	600	0.11544	0.29120
ω_9	600	0.10952	0.40072
ω_2	800	0.03848	0.43920
ω_7	1000	0.09204	0.53124
ω_5	1600	0.32856	0.85980
ω_3	1800	0.03068	0.89048
ω_1	2600	0.10952	1.00000

Table 5.5: Insurance contracting example: power producer profit distribution if its entire production is sold through the forward contract with no insurance

# scenario	$\Pi_{(\omega)}$ (€)	$\pi_{(\omega)}$	$\mathrm{F}_{(\omega)}$
ω_4	-1000	0.02132	0.02132
ω_2	-200	0.03848	0.05980
ω_8	0	0.06396	0.12376
ω_6	600	0.11544	0.23920
ω_3	800	0.03068	0.26988
ω_7	1000	0.09204	0.36192
ω_{12}	1000	0.02132	0.38324
ω_{11}	1200	0.03068	0.41392
ω_{10}	1400	0.03848	0.45240
ω_1	1600	0.10952	0.56192
ω_5	1600	0.32856	0.89048
ω_9	1600	0.10952	1.00000

horizon, i.e., in scenario ω_1.

Table 5.4 allows us to directly determine the VaR of the producer profit distribution. For instance, for a risk aversion of the producer α^{P} equal to 0.5, the VaR is equal to €1000 since 50% of the distribution is below that value. The CVaR of the profit distribution, defined as the weighted average of the $(1-\alpha^{\mathrm{P}}) \times 100\%$ lowest profits, is computed as [160]

$$\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.5} = \frac{0 \cdot 0.02132 + 0 \cdot 0.06396 + \ldots + 1000 \cdot 0.06080}{1-0.5} = €496.2.$$

Note that each scenario profit is multiplied by its associated probability except for the scenario that coincides with the VaR, whose probability is adjusted so that the sum of the probabilities of the considered scenarios is equal to $1-\alpha^{\mathrm{P}}$, i.e., 0.5. Proceeding in the same way we obtain the value of the VaR and CVaR for $\alpha^{\mathrm{P}} = 0.9$ are both equal to €0.

Table 5.5 provides the profit distribution of the producer if all its production is sold through a forward contract with a fixed price of 20 €/MWh and no insurance contract is signed.

In this case, for $\alpha^{\mathrm{P}} = 0.5$, the VaR is equal to €1600 and the CVaR is calculated as

$$\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.5} = \frac{-1000 \cdot 0.02132 + \ldots + 1600 \cdot 0.0476}{1-0.5} = €690.0.$$

Table 5.6: Insurance contracting example: CVaR comparison with no insurance

α^{P}	Pool (€)	Forward (€)
0	1145.0	1145.0
0.5	496.2	690.0
0.9	0.0	-290.2

For $\alpha^{\mathrm{P}} = 0.9$, the VaR and the CVaR of the profit distribution are equal to €0.0 and −€290.2, respectively.

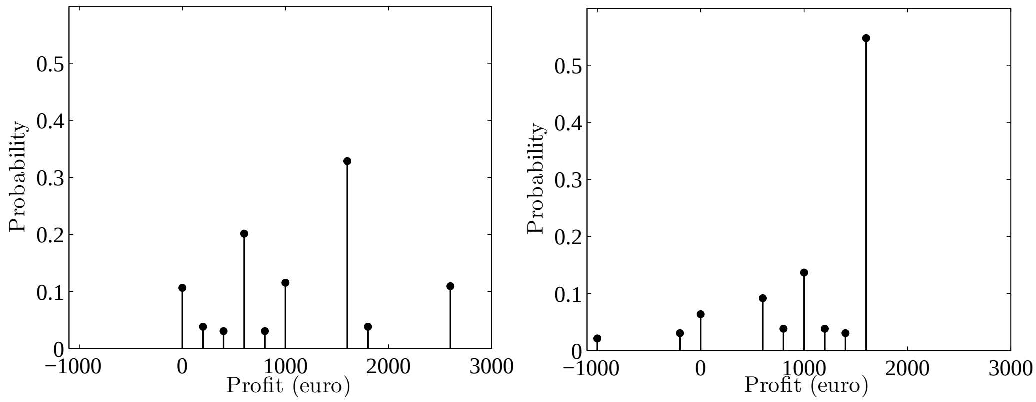
Table 5.6 shows the CVaR of the producer profit distribution for different risk aversion levels and depending on whether the production is sold in the pool or through the forward contract.

Although the profit distributions resulting from selling either in the pool or through the forward contract have the same expected profit (€1145.0), we can observe in Tables 5.4 and 5.5 that the second one includes negative profits as a consequence of unexpected unit failures that take place during the delivery period of the signed forward contract to sell electricity. This effect is also visible in the behavior of the CVaR in both cases as illustrated in Table 5.6. The $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.5}$, which represents the average profit of the 50% worst profit scenarios, is much higher if the producer sells its production through a forward contract. On the contrary, the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ is €0.0 if the electricity is sold in the pool, while it turns out to be negative if the forward contract is signed.

For illustration purposes, Figs. 5.4(a) and 5.4(b) show the probability mass function of the producer profit according to the information contained in Tables 5.4 and 5.5, respectively.

Next, we consider that the producer has the possibility of signing an insurance against unit failures with the following characteristics: a strike price of 20 €/MWh, an insured power of 100 MW, and a premium of €75. Whereas the premium of the insurance constitutes a fixed cost for the power producer, the economic compensation received from the insurer depends on each scenario realization according to equation (5.1). For instance, for scenario ω_8, in which the unit is forced out during the two hours of the study horizon, the compensation of the producer is computed as

$$D_{(\omega_8)} = 100 \cdot (22 - 20) = €200. \tag{5.26}$$



(a) Electricity sold in the pool (b) Electricity sold through a forward contract

Figure 5.4: Insurance contracting example: probability mass function of the producer profit with no insurance

Even though scenario ω_8 comprises an unexpected failure spanning the two hours of the study horizon, the insurer only pays the producer during the second hour since the pool price in the first hour happens to be lower than the strike price. Likewise, irrespective of the pool price realization, scenarios in which the generating unit is available during the whole study horizon, say ω_5, do not entail entail any compensation by the insurer. Thus,

$$D_{(\omega_5)} = €0. \tag{5.27}$$

The second column of Table 5.7 provides the compensation received by the producer for each scenario. The net revenue associated with the insurance contract, which is computed as the compensation received minus the premium, is shown in the third column. Note that the highest compensations received by the producer from the insurer correspond to scenarios ω_2 and ω_4, which are the scenario that lead to the lowest producer profits provided that its production is sold through the forward contract and the insurance contract is not available (Table 5.5).

Tables 5.8 and 5.9 provide the profit distributions of the producer if its entire production is sold either in the pool or through the forward contract, respectively, but considering that an insurance contract against unit failures for the two-hour study horizon is signed. From these tables, we observe that the insurance acquisition significantly improves the profit distribution if the forward contract is signed by reducing the probability of experiencing negative outcomes. It has, however, the opposite effect if the electricity is sold solely

Table 5.7: Insurance contracting example: payment and revenue associated with the insurance

# scenario	$D_{(\omega)}$(€)	$\Pi^{\mathrm{I}}_{(\omega)}$(€)
ω_1	0	−75
ω_2	1000	925
ω_3	0	−75
ω_4	1000	925
ω_5	0	−75
ω_6	200	125
ω_7	0	−75
ω_8	200	125
ω_9	0	−75
ω_{10}	0	−75
ω_{11}	0	−75
ω_{12}	0	−75

Table 5.8: Insurance contracting example: power producer profit distribution if its entire production is sold in the pool and the insurance is signed

# scenario	$\Pi_{(\omega)}$ (€)	$\pi_{(\omega)}$	$\mathrm{F}_{(\omega)}$
ω_{12}	−75	0.02132	0.02132
ω_8	125	0.06396	0.08528
ω_{11}	125	0.03068	0.11596
ω_{10}	325	0.03848	0.15444
ω_9	525	0.10952	0.26396
ω_6	725	0.11544	0.37940
ω_4	925	0.02132	0.40072
ω_7	925	0.09204	0.49276
ω_5	1525	0.32856	0.82132
ω_2	1725	0.03848	0.85980
ω_3	1725	0.03068	0.89048
ω_1	2525	0.10952	1.00000

in the pool since the acquisition of the insurance contract causes a negative profit in one of the scenarios, i.e., scenario ω_{12}.

This effect is even more clear in Table 5.10, where the CVaR of the profit distribution is provided for three different risk aversion levels and considering that the producer signs the insurance contract. Note that in this case, as opposed to what can be observed in Table 5.6, the CVaR values are higher if the producer sells its production through the forward contract instead of in the pool. This is so because the combination of selling electricity through a

Table 5.9: Insurance contracting example: power producer profit distribution if its entire production is sold through the forward contract and the insurance is signed

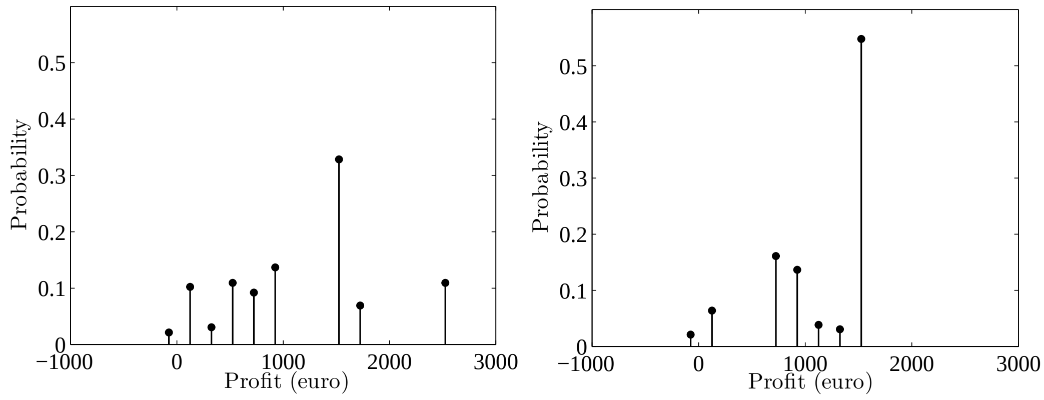
# scenario	$\Pi_{(\omega)}$ (€)	$\pi_{(\omega)}$	$\mathrm{F}_{(\omega)}$
ω_4	-75	0.02132	0.02132
ω_8	125	0.06396	0.08528
ω_2	725	0.03848	0.12376
ω_3	725	0.03068	0.15444
ω_6	725	0.11544	0.26988
ω_7	925	0.09204	0.36192
ω_{12}	925	0.02132	0.38324
ω_{11}	1125	0.03068	0.41392
ω_{10}	1325	0.03848	0.45240
ω_1	1525	0.10952	0.56192
ω_5	1525	0.32856	0.89048
ω_9	1525	0.10952	1.00000

Table 5.10: Insurance contracting example: CVaR comparison with insurance

α^{P}	Pool (€)	Forward (€)
0	1165.7	1165.7
0.5	559.7	806.4
0.9	82.4	170.7

forward contract plus acquiring an insurance against unit failures considerably reduces, respectively, the risk related to both the pool price volatility and the unexpected unit failures. In contrast, signing an insurance while selling electricity in the pool is a decision that forces the producer to face the possibility of experiencing low outcomes if low pool prices realize and the generating unit does not fail.

Fig. 5.5 depicts the probability mass function of the producer profit if it signs the insurance against unit failures and sells its production in the pool (Fig. 5.5(a)) or through the forward contract (Fig. 5.5(b)). By comparing Figs. 5.4 and 5.5 we realize that the effect of the insurance is comparatively more positive if the production is sold through the forward contract, because it drastically increases the profits associated with adverse scenario realizations without significantly reducing the highest profit of the distribution. On the other hand, if the producer sells its electricity in the pool, the purchase of the insurance barely improves the profit distribution while creating the possibility



(a) Electricity sold in the pool

(b) Electricity sold through a forward contract

Figure 5.5: Insurance contracting example: probability mass function of the producer profit with insurance

of experiencing negative profits as a consequence of the insurance premium.

5.6.2 Impact on Forward Contracting

Until now, we have illustrated the impact of an insurance contract against unit failures on the producer profit distribution depending on whether its full capacity is sold in the pool or through a forward contract. However, the producer can decide the optimal mix of pool and forward contract involvement according to its risk aversion level and of course, this optimal mix may be affected by the acquisition of an insurance contract.

In order to analyze the impact of an insurance contract on power producer decisions, we solve optimization problem (5.18) for the proposed example and two different cases. In the first case, there is no available insurance contract, while in the second one an insurance contract with a strike price equal to 20 €/MWh, a €75 premium, and insured power level of 100 MW is considered. The pool price and unit availability uncertainty is characterized by the scenario tree provided in Table 5.3. There is a forward contract spanning the two-hour horizon with a price equal to 20 €/MWh. Finally, the generating unit has a maximum capacity equal to 100 MW and a linear cost of 12 €/MWh.

Table 5.11 provides the power sold through the forward contract, the CVaR, and the expected profit of the producer for three different risk aversion levels on the assumption that there is no available insurance contract. Likewise, Table 5.12 shows the same results if an insurance contract is available. An extra column containing the binary variable s^{P} is included.

Table 5.11: Insurance contracting example: forward contracting decisions, CVaR and expected profit with no insurance

α^{P}	$P_{(c_1)}$ (MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (€)	EP (€)
0	0	1145	1145
0.5	100	690	1145
0.9	0	0	1145

Table 5.12: Insurance contracting example: forward contracting decisions, CVaR and expected profit with insurance (M^{I} = €75)

α^{P}	$P_{(c_1)}$ (MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (€)	EP (€)	s^{P}
0	0	1165.7	1165.7	1
0.5	100	806.4	1165.7	1
0.9	50	233.1	1165.7	1

Observe that although for both $\alpha^{\mathrm{P}} = 0$ and $\alpha^{\mathrm{P}} = 0.5$ the insurance contract improves the CVaR of the producer profit distribution, its acquisition does not have an impact on the production sold through the forward contract. In contrast, for $\alpha^{\mathrm{P}} = 0.9$, the insurance contract against unit failures allows the producer to increase the power sold through the forward contract to achieve a higher CVaR. Thus, the producer avoids having low profits since it hedges against both price and availability risks by selling part of its production through a forward contract and acquiring an insurance contract against unit failures, respectively.

5.6.3 Maximum Premium of the Producer

As explained in Section 5.3.7, to determine the maximum premium that a power producer is willing to pay for an insurance contract characterized by a given strike price and insured power level, two optimization problems need to be solved. In the first one, the CVaR of the producer profit distribution is determined if no insurance contract is available. In the second problem, we solve optimization problem (5.18) considering that the mentioned insurance contract is available and that its premium is equal to €0.

The CVaR values of the producer profit distribution for the two aforementioned problems for a strike price of 20 €/MWh, an insured power level of 100 MW, and three risk aversion levels are provided in Table 5.13. Moreover, the

Table 5.13: Insurance contracting example: CVaR and maximum premium of the producer

α^{P}	$\mathrm{CVaR}_{s^{\mathrm{P}}=0}$ (€)	$\mathrm{CVaR}_{M^{\mathrm{I}}=0}$ (€)	$M^{\mathrm{P,I}*}$ (€)
0	1145.0	1240.7	95.7
0.5	690.0	881.4	191.4
0.9	0.0	308.1	308.1

Table 5.14: Insurance contracting example: CVaR and minimum premium of the insurer

α^{S}	$\mathrm{CVaR}_{\alpha^{\mathrm{S}}}$ (€)	$M^{\mathrm{P,S}*}$ (€)
0	−95.7	95.7
0.5	−191.4	191.4
0.9	−678.4	678.4

third column of this table contains the maximum premium of the producer for each risk aversion level computed according to equation (5.20).

Note that the higher the risk aversion level of the power producer, the higher its maximum premium.

5.6.4 On Insurance Bargaining

Continuing with the same example, in this section we determine the minimum premium that an insurance company is willing to receive for the insurance contract against unit failures previously considered. Comparing this minimum premium of the insurer with the maximum premium of the producer we can conclude whether or not both parties can reach an agrement and sign the insurance contract.

As explained in Section 5.4.1, to determine the minimum premium of the insurer first we have to solve optimization problem (5.23) considering an insurance premium equal to €0. To this end, we consider the scenario tree described in Table 5.3, and an insurance contract with a strike price equal to 20 €/MWh and an insured power level of 100 MW. The optimization problem is solved for different values of the risk aversion level of the insurer, obtaining the CVaR values listed in Table 5.14. This table also includes the minimum premium of the insurer ($M^{\mathrm{P,S}*}$) determined according to equation (5.25).

Section 5.5 states that an agreement between the power producer and the insurer can be reached if the maximum premium that the producer is willing to pay for a given insurance contract is higher that the minimum premium that the insurer is willing to accept. Therefore, note that if the risk aversion level of both parties is the same, the insurance contract under consideration is not signed. However, if the risk aversion level of the insurer is lower than that of the producer, e.g., $\alpha^{\mathrm{P}} = 0.5$ and $\alpha^{\mathrm{S}} = 0$, the maximum premium of the producer (€191.4) is higher than the minimum premium of the insurer (€95.7) and as a result, an agreement is reached. This way, the insurance contract against unit failures is proved to be a mechanism to transfer risk from the producer to the insurer in exchange for a certain premium.

5.7 Insurance Contracting Case Study

In this section, we apply the proposed stochastic decision-making model to a realistic 8-week case study based on the electricity market of the Iberian Peninsula. The case study is divided into three parts: in the first part, the impact of an insurance contract against unit failures on the power producer decisions and objective function is analyzed; in the second part, we compute the maximum premium that the power producer is willing to pay for a given insurance contract; and the last part illustrates the bargaining process between the power producer and the insurer.

5.7.1 Impact on Forward Contracting and Objective Function of the Producer

For comparison purposes, we consider a power producer owing one single generating unit with the same technical and economic characteristics than those presented in Section 4.9 of Chapter 4, i.e., a maximum and minimum power output of 350 MW and 50 MW, respectively, and a piecewise linear cost function described in Table 5.15.

The proposed model is evaluated for the three forced outage rates previously used, i.e., 0%, 5%, and 10%, whose parameters are provided in Table 5.16.

In order to characterize the uncertainty related to the availability of the generating unit, a set of 5000 scenarios is generated for each forced outage rate according to the procedure described in Section 3.5 of Chapter 3. Likewise, the procedure explained in Section 3.4 of Chapter 3 is used to generate a set of 200

Table 5.15: Insurance contracting case study: piecewise linear modeling of the production cost function of the generating unit

# block	$P_{(b)}^{\text{Max}}$ (MW)	$\lambda_{(b)}$ (€/MWh)
1	140.0	10.08
2	227.5	10.66
3	280.0	11.09
4	350.0	11.72

Table 5.16: Insurance contracting case study: forced outage parameters

FOR (%)	MTTF (h)	MTTF (h)
5	950	50
10	450	50

pool price scenarios that represents the stochastic behavior of the pool price throughout the 8-week study horizon. To reduce the computational burden of the optimization model, the 24 hours of each day of the study horizon are aggregated in 3 time steps following the algorithm described in Appendix A.

In the same vein, the two original scenario sets characterizing the pool prices and the unit availability, respectively, are the same as those considered in the case study of Chapter 4. However, due to the fact that the scenario reduction technique used in this dissertation takes into account the structure of the optimization problem, the reduced sets of 30 and 50 scenarios representing, respectively, the pool price and the unit availability uncertainty, do not coincide with those obtained in the case study of Chapter 4.

As in that chapter, in this case study we consider two forward contracts spanning the first four weeks and the last four weeks of the study horizon, whose fixed prices are set to 21.75 €/MWh and 22.75 €/MWh, respectively.

Moreover, the producer has the possibility of signing an insurance contract against unexpected unit failures characterized by the following parameters: a premium of €50,000, a strike price equal to 25 €/MWh, and an insured power of 200 MW.

Finally, the risk aversion level of the producer is modeled by the parameter α^{P}, varying between 0 and 1.

The first part of this case study is aimed to assess the impact of an insurance contract against unit failures on both the objective function of the producer and its forward contracting decisions. With this aim in mind, we compare the

Table 5.17: Insurance contracting case study: forward contracting decisions, CVaR and expected profit with no insurance

FOR (%)	α^{P}	$P_{(c_1)}$ (MW)	$P_{(c_2)}$ (MW)	$P_{(c_1)} + P_{(c_2)}$ (MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (€ million)	EP (€ million)
	0	0	0	0	5.8551	5.8551
0	0.5	350	350	700	5.4310	5.4310
	0.9	350	350	700	5.4310	5.4310
	0	0	0	0	5.5700	5.5700
5	0.5	224	252	476	4.9829	5.2972
	0.9	223	261	484	4.4891	5.2930
	0	0	0	0	5.2869	5.2869
10	0.5	276	119	395	4.6451	5.0910
	0.9	247	176	423	4.0442	5.0577

results obtained by solving optimization problem (5.18) for two different cases depending on whether or not the producer has the possibility of signing the aforesaid insurance contract.

If the insurance contract is not available, Table 5.17 provides the power sold through the two available forward contracts (column 3 and 4), the total power sold through these forward contracts (column 5), the value of the CVaR of the profit distribution (column 6), and the expected profit (column 7) for different values of the forced outage rate (FOR) and the risk aversion parameter (α^{P}).

By simple inspection of this table, we come to the conclusion that an increase in the forced outage rate causes a reduction in the optimal power that the producer sells through forward contract so as to reduce the financial losses associated with unexpected unit failures.

In Table 5.18, similar results as those in Table 5.17 are provided but solving optimization problem (5.18) if an insurance contract with a premium of €50,000, a strike price equal to 25 €/MWh, and an insured power level of 200 MW is available. Note that an extra column has been added to indicate the value of the binary variable s^{P}. This variable is equal to 1 if the producer signs the insurance contract, and 0 otherwise. It should be noted that for all the cases in which the value of s^{P} is equal to 0, the results are identical to those provided in Table 5.17.

In light of the results shown in Tables 5.17 and 5.18, the following observations are in order. First, if the probability of unit failure is null, the producer does not sign the insurance contract for any level of risk aversion. However, if

Table 5.18: Insurance contracting case study: forward contracting decisions, CVaR and expected profit with insurance

FOR (%)	α^{P}	$P_{(c_1)}$ (MW)	$P_{(c_2)}$ (MW)	$P_{(c_1)} + P_{(c_2)}$ (MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (€ million)	EP (€ million)	s^{P}
	0	0	0	0	5.8551	5.8551	0
0	0.5	350	350	700	5.4310	5.4310	0
	0.9	350	350	700	5.4310	5.4310	0
	0	0	0	0	5.5700	5.5700	0
5	0.5	240	270	510	4.9867	5.2600	1
	0.9	237	280	517	4.5517	5.2571	1
	0	0	0	0	5.3017	5.3017	1
10	0.5	280	157	437	4.6830	5.0744	1
	0.9	271	200	471	4.1521	5.0438	1

Table 5.19: Insurance contracting case study: impact of an insurance contract on the CVaR and the expected profit of the producer

FOR (%)	α^{P}	$\triangle\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (%)	$\triangle$EP(%)
	0	0	0
0	0.5	0	0
	0.9	0	0
	0	0	0
5	0.5	0.2	−0.8
	0.9	1.3	−0.6
	0	0.2	0.2
10	0.5	0.6	−0.4
	0.9	2.7	−0.4

the unit FOR is equal to 5%, the producer only signs the insurance contract for high risk aversion levels. If the FOR increases up to 10%, it is optimal for the producer to sign the contract for all risk aversion levels. Therefore, for the cases in which the insurance contract is signed, the value of the objective function (CVaR) is higher than that obtained without signing the insurance contract. For clarity, Table 5.19 illustrates the variation, in percentage, of the CVaR and the expected profit if the insurance is acquired. Note that the CVaR change is greater if either the forced outage rate or the risk aversion level of the producer increases. In addition, for risk aversion levels different from 0, the insurance acquisition leads to a reduction in the expected profit of the producer due to the insurance premium cost.

Second, observe that the total power sold through the two forward contracts

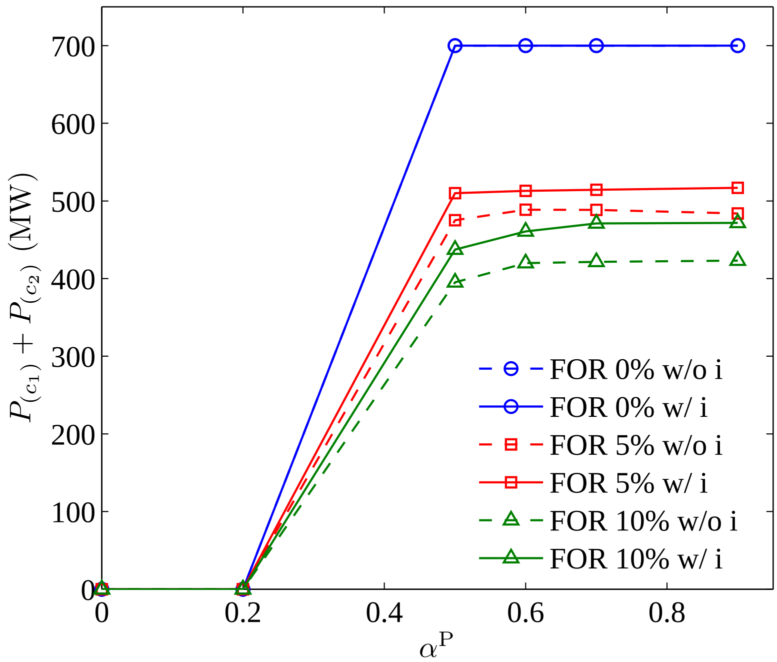


Figure 5.6: Insurance contracting case study: impact of an insurance contract on forward contracting decisions

is higher if the insurance contract is signed. For example, for FOR $= 10\%$ and $\alpha^{\mathrm{P}} = 0.9$, the total power sold through forward contracts increases from 423 MW to 471 MW if the insurance contract is signed. The reason for this is that the insurance contract reduces the financial losses of the producer if the generating unit fails during forward contract delivery periods and therefore, allows the producer to increase the energy sold through forward contracts to hedge against pool price volatility.

Third, note that in some cases the insurance contract is signed even though no energy is sold through forward contracts. This happens because the insured power is not directly linked to the power sold through forward contracts and as a consequence, the insurance may be profitable for high values of FOR even if all the energy production is sold in the pool.

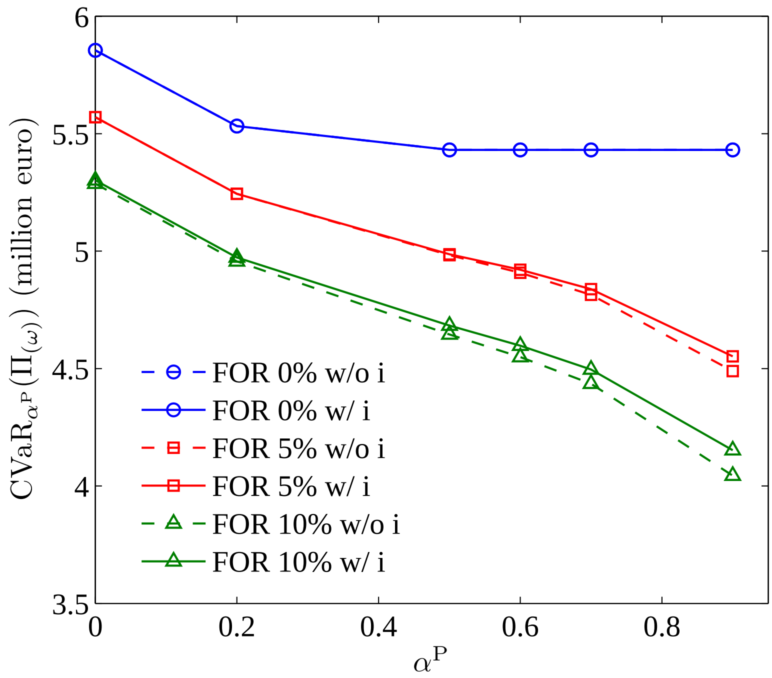
Fig. 5.6 illustrates the total power sold through forward contracts as a function of the risk aversion level (α^{P}) for the three forced outages rates considered and depending on whether there is an available insurance (w/ i) or not (w/o i).

Independently of the risk aversion level and the forced outage rate, the total power sold through forward contracts increases if the producer signs an insurance contract against unit failures. Note that for a forced outage rate of 10%, the acquisition of the insurance produces an increase in the power sold through forward contracts that almost reaches the level obtained if the insurance is not available and the FOR is equal to 5%. In short, signing an insurance contract against unit failures has a similar effect to that caused by a reduction of the forced outage rate of the generating unit. Thus, an insurance contract is a financial instrument to reduce the availability risk faced by power producers.

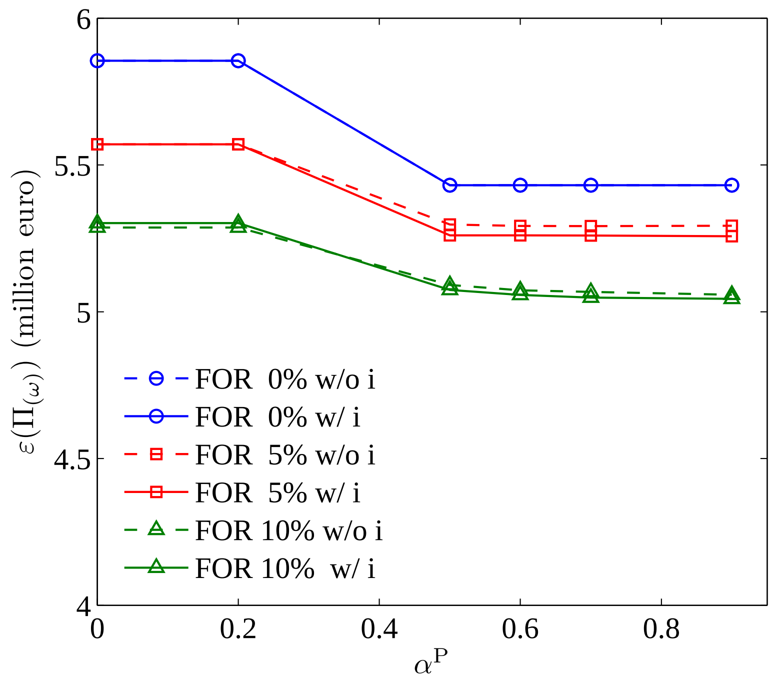
Figs. 5.7(a) and 5.7(b) show the variation of the CVaR and the expected profit as a function of the risk aversion level (α^{P}) if the producer has the possibility of signing an insurance contract.

For completeness, the change undergone by the probability mass function of the producer profit for $\alpha^{\mathrm{P}} = 0.5$ and FOR $= 5\%$ if the insurance is signed is plotted in Fig. 5.8. While the acquisition of the insurance reduces the probability of low profits due to the compensation received if adverse scenarios realize, it also reduces the probability of obtaining high profits as a consequence of the insurance premium cost. Besides, the probability of those scenarios close to the average value increases. Therefore, the profit distribution of the producer if the insurance is signed is a comparatively *less risky* distribution because of the reduction of its variability.

Finally, Fig. 5.9 depicts the impact of an insurance contract on the efficient frontier of the power producer. In the first figure, the expected profit is plotted as a function of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ for increasing values of α^{P}. For a value of FOR equal to 5%, the efficient frontier does not depend on whether or not the insurance contract is signed until $\alpha^{\mathrm{P}} = 0.4$. From this risk aversion value, however, the insurance becomes appealing to the power producer and hence the split into two of the red line in Fig. 5.9(a). Specifically, from this point, the acquisition of the insurance allows the producer to achieve a higher value of the CVaR in exchange for reducing its expected profit. If the forced outage rate increases (FOR $= 10\%$), the difference between the two efficient frontiers obtained depending on whether or not the insurance is available is even more apparent. Note that for $\alpha^{\mathrm{P}} = 0$, the acquisition of the insurance translates into a higher expected profit. In contrast, for higher values of α^{P}, signing the insurance against unit failures leads to higher values of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ and lower values of the expected profit. In Fig. 5.9(b) a second set of efficient frontiers representing the expected profit as a function of the standard deviation is presented. Observe that the acquisition of the insurance produces a displacement of the efficient frontiers to the left, that is, in the direction of decreasing



(a) CVaR_{α^P}



(b) Expected profit

Figure 5.7: Insurance contracting case study: impact of an insurance contract on the CVaR and the expected profit

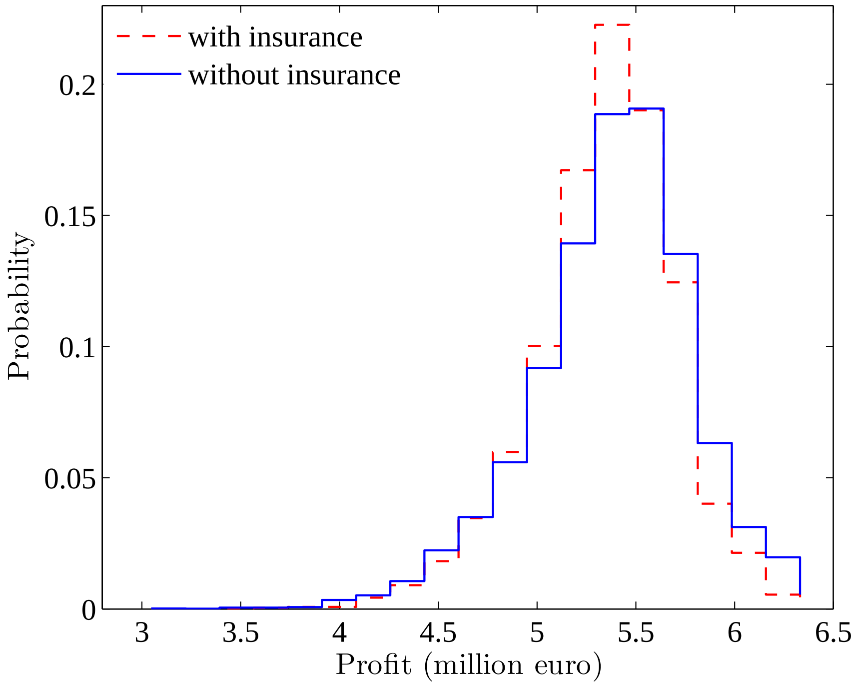


Figure 5.8: Insurance contracting case study: impact of an insurance contract on the profit distribution of the producer for $\alpha^{\mathrm{P}} = 0.5$ and $\mathrm{FOR} = 5\%$

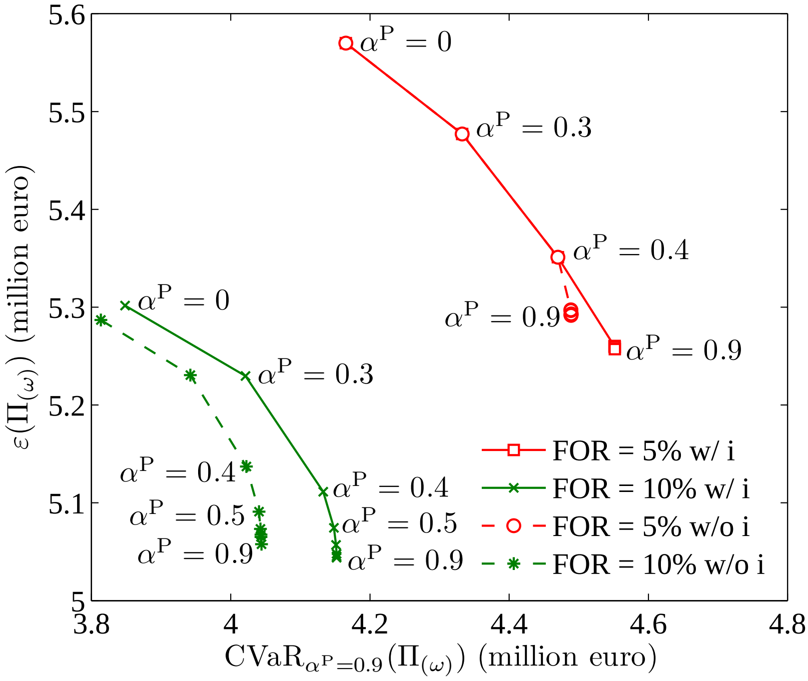
standard deviations.

5.7.2 Maximum Premium of the Producer

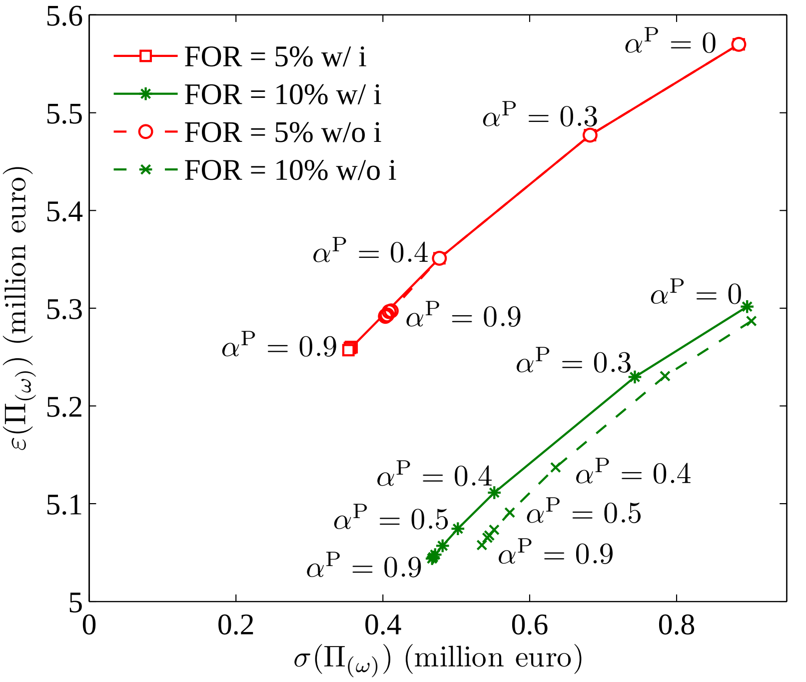
The impact of an insurance contract on the producer forward contracting decisions, objective function, and efficient frontiers strongly depends on the premium of the insurance. That is the reason why the calculation of the maximum premium that the producer is willing to pay for a given insurance becomes a key piece of information.

As stated in Section 5.3.7, to determine the maximum premium that the producer is willing to pay, we first solve optimization problem (5.18) to compute its objective function considering that the insurance premium is equal to €0. Table 5.20 provides the CVaR and the expected profit for a value of FOR equal to 5% and a risk aversion level of 0.5. The insured power is equal to 200 MW and two different strike prices are considered, namely, 20 €/MWh and 30 €/MWh.

Second, we compute the objective function of the producer if there is no available insurance contract by solving optimization problem (5.18) if the variable s^{P} is fixed to 0. For $\alpha^{\mathrm{P}} = 0.5$ and FOR $= 5\%$, the CVaR and the



(a) Expected profit vs. $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$



(b) Expected profit vs. standard deviation

Figure 5.9: Insurance contracting case study: impact of an insurance contract on the efficient frontier

Table 5.20: Insurance contracting case study: CVaR and expected profit of the producer if the insurance premium is equal to €0. FOR = 5%, $\alpha^{\mathrm{P}} = 0.5$, $P^{\mathrm{I}} = 200$ MW

	$\lambda^{\mathrm{I}} = 20$ €/MWh	$\lambda^{\mathrm{I}} = 30$ €/MWh
$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (million €)	5.09	5.01
EP (million €)	5.32	5.30

expected profit of the producer are equal to €4.98 million and €5.30 million, respectively.

Therefore, from equation (5.20) the maximum premium the producer is willing to pay for the insurance with a strike price of 20 €/MWh and an insured power of 200 MW is determined as

$$M^{\mathrm{I,P}*} = 5.09 - 4.98 = €0.11\text{million}. \tag{5.28}$$

Likewise, for a strike price equal to 30 €/MWh, the maximum premium is

$$M^{\mathrm{I,P}*} = 5.01 - 4.98 = €0.03\text{million}. \tag{5.29}$$

Logically, a higher strike price results in a lower probability that the producer is paid by the insurer and therefore, the maximum premium decreases.

In a similar way, the maximum premium of the producer can be calculated for different values of risk aversion level, forced outage rate, strike price and insured power level. Fig. 5.10 depicts the maximum premium of the producer as a function of the strike price for different values of risk aversion level and FOR. The value of the insured power level is fixed to 200 MW. Observe that the maximum premium of the producer ($M^{\mathrm{I,P}*}$) decreases with the strike price for the reason explained in the previous paragrah. Moreover, if the probability of an unexpected failure increases (higher FOR), the producer is willing to pay a higher premium for a given level of risk aversion and strike price in order to reduce its availability risk. Lastly, a higher risk aversion involves a higher maximum premium in order to reduce the probability of having low profit as a consequence of forced outages.

Fig. 5.11 shows the maximum premium of the producer as a function of the insured power level. In this case, the strike price is fixed to 25 €/MWh. The straightforward conclusion drawn from this figure is that the maximum premium of the producer increases with the insured power level.

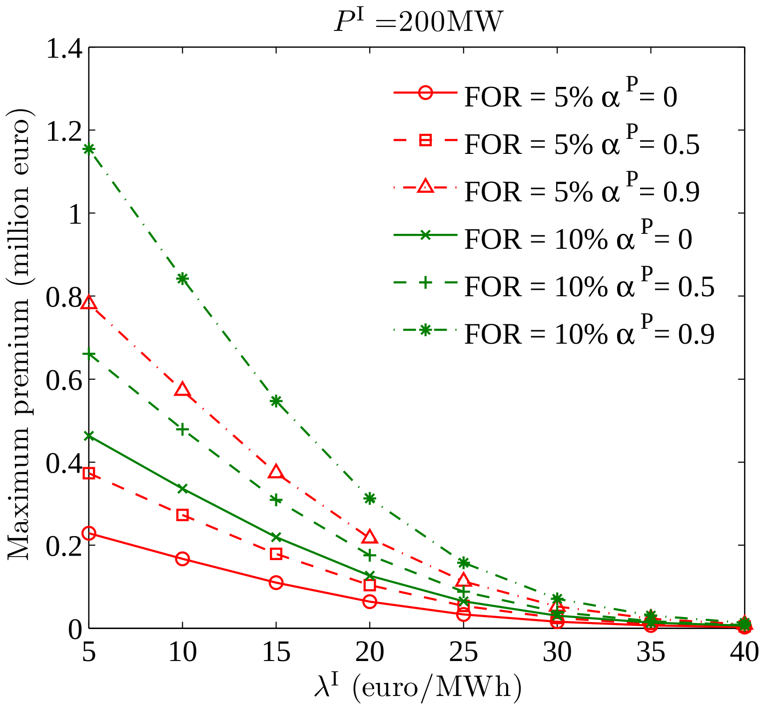


Figure 5.10: Insurance contracting case study: maximum premium of the producer vs. the strike price for different values of FOR and risk aversion. Insured power equal to 200 MW

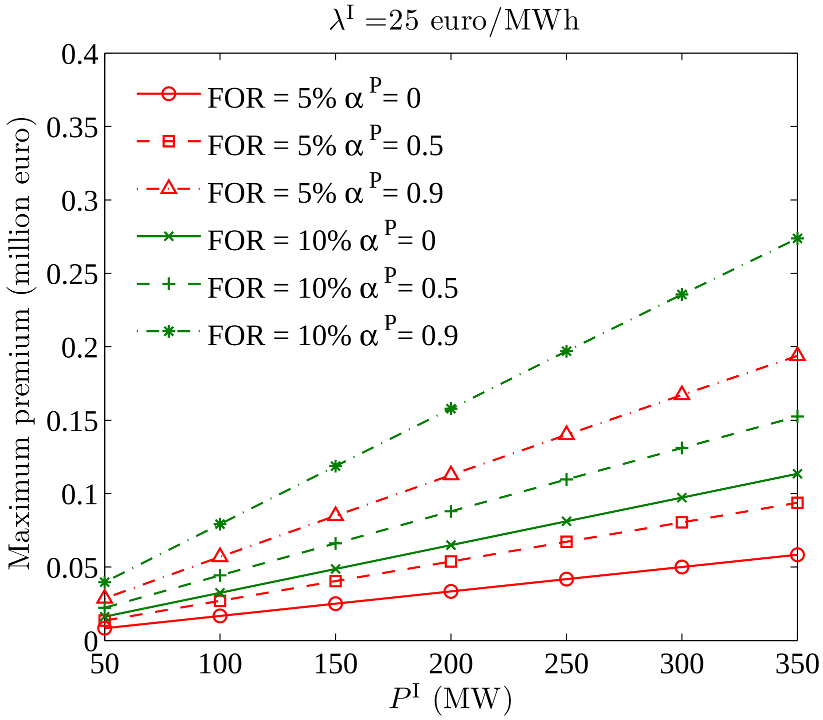


Figure 5.11: Insurance contracting case study: maximum premium of the producer vs. the insured power level for different values of FOR and risk aversion. Strike price equal to 25 €/MWh

5.7.3 On Insurance Bargaining

In order to understand the bargaining process between the power producer and the insurer, in this section we determine the minimum premium that an insurer is willing to receive for a given insurance contract. As an example, we calculate the minimum premium that an insurer with a risk aversion level equal to 0.5 is willing to receive for an insurance contract with a strike price equal to 20 €/MWh and an insured power level of 200 MW. The minimum premium is computed considering a forced outage rate of the generating unit owned by the power producer equal to 5%.

In keeping with the procedure stated in Section 5.4.1, first we calculate the value of the $\mathrm{CVaR}_{\alpha^{\mathrm{S}}=0.5}$ assuming that the insurance premium is equal to €0 by solving problem (5.23), i.e.,

$$\mathrm{CVaR}^{\mathrm{S}}_{M^{\mathrm{I}}=0} = -€0.09\text{million}. \tag{5.30}$$

Therefore, from equation (5.25), the minimum premium of the insurer is

$$M^{\mathrm{I,S}*} = -\mathrm{CVaR}^{\mathrm{S}}_{M^{\mathrm{I}}=0} = €0.09\text{million}. \tag{5.31}$$

Likewise, for a strike price equal to 30 €/MWh and keeping the insured power level equal to 200 MW, the minimum premium of the insurer is equal to

$$M^{\mathrm{I,S}*} = €0.02\text{million}. \tag{5.32}$$

Observe that a higher strike price, which involves a lower compensation from the insurer to the power producer during failure periods, causes a decrease in the minimum premium that the insurer is willing to receive.

Fig. 5.12 depicts, for different forced outage rates and risk aversion levels, the minimum premium of the insurer as a function of the strike price and for an insured power level equal to 200 MW.

Note that, qualitatively speaking, the impact of the strike price, the FOR, and the risk aversion level on the minimum premium of the insurer are very similar to those obtained for the maximum premium of the producer, i.e., an increase in the strike price causes a decrease in the minimum premium. What is more, the minimum premium of the insurer increases if the probability that the generating unit suffers from unexpected failures also grows. Finally, in order to improve its revenue if adverse scenarios realizes, the higher the risk

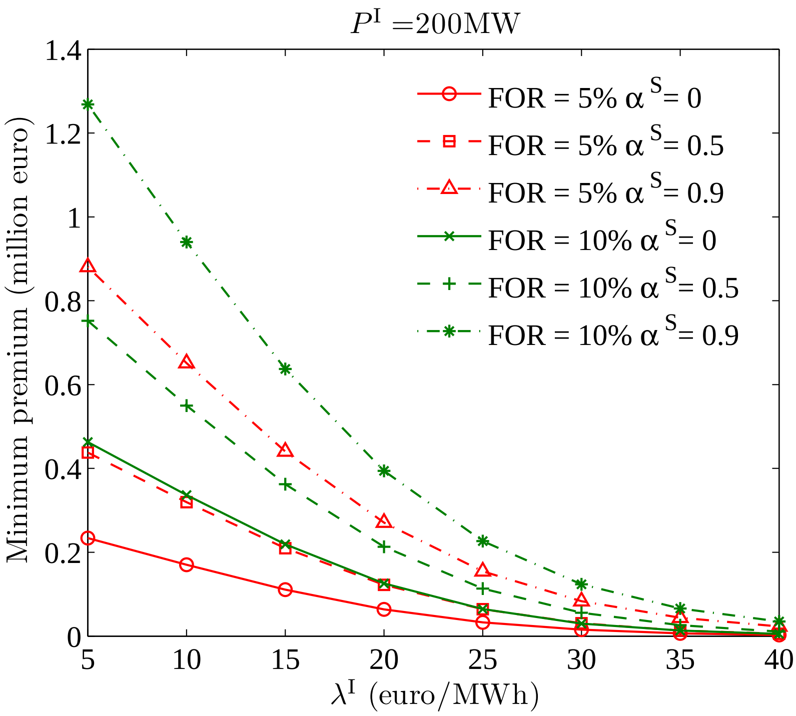


Figure 5.12: Insurance contracting case study: minimum premium of the insurer vs. the strike price for different values of FOR and risk aversion. Insured power equal to 200 MW

aversion level of the insurer, the higher its minimum premium.

For a strike price equal to 25 €/MWh, Fig 5.13 shows the dependence of the minimum premium of the insurer on the insured power level. Note that an increase in the insured power involves a proportional increase in the minimum premium of the insurer.

Two critical premia are calculated then: the maximum premium that the producer is willing to pay for a given insurance contract and the minimum premium that the insurer is willing to receive. In Table 5.21, the maximum premium of the producer is provided for a strike price equal to 25 €/MWh, an insured power of 200 MW, several risk aversion levels and two FOR values, namely, 5% and 10%. Similarly, Table 5.22 provides the corresponding minimum premium of the insurer.

As stated in Section 5.5, a given insurance contract is signed if the minimum premium of the insurer is lower than the maximum premium of the producer. It is impossible that both agents agree on signing the insurance contract if the maximum amount that the producer is willing to pay is lower than the minimum amount required by the insurer. Observe that if the value of the risk aversion of the producer (α^{P}) and the risk aversion of the insurer (α^{S})

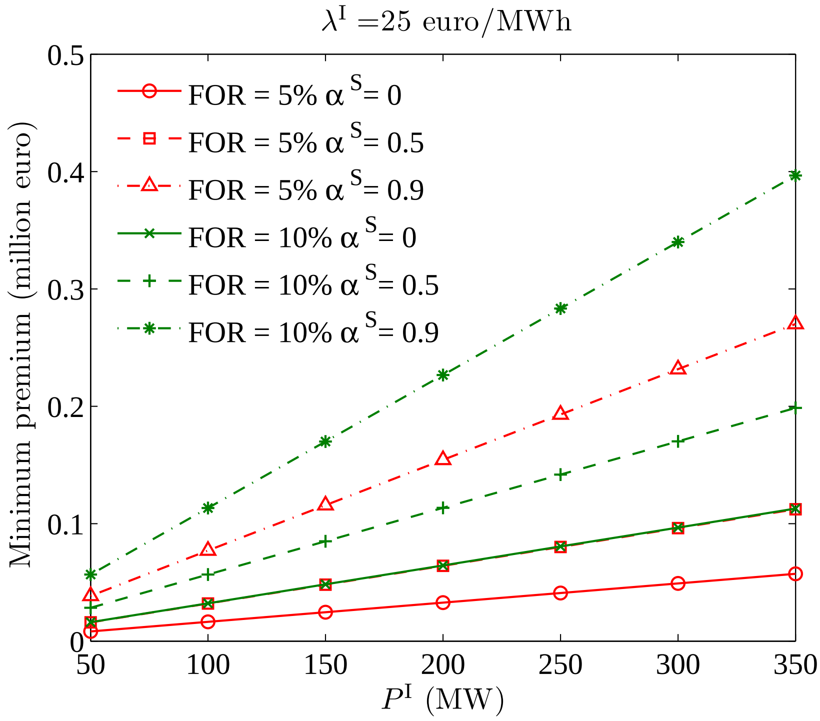


Figure 5.13: Insurance contracting case study: minimum premium of the insurer vs. the insured power level for different values of FOR and risk aversion. Strike price equal to 25 €/MWh

Table 5.21: Insurance contracting case study: producer maximum premium (€million) for $\lambda_{\mathrm{I}} = 25$ €/MWh and $P_{\mathrm{I}} = 200$ MW

α^{P}	FOR = 5%	FOR = 10%
0	0.033	0.065
0.1	0.034	0.065
0.2	0.034	0.066
0.3	0.038	0.069
0.4	0.046	0.078
0.5	0.054	0.088
0.6	0.063	0.098
0.7	0.074	0.111
0.8	0.089	0.126
0.9	0.113	0.158

Table 5.22: Insurance contracting case study: insurer minimum premium (€million) for $\lambda_{\text{I}} = 25$ €/MWh and $P_{\text{I}} = 200$ MW

α^{S}	FOR = 5%	FOR = 10%
0	0.033	0.065
0.1	0.036	0.071
0.2	0.041	0.080
0.3	0.047	0.090
0.4	0.055	0.101
0.5	0.064	0.114
0.6	0.076	0.129
0.7	0.091	0.149
0.8	0.115	0.178
0.9	0.154	0.227

are equal, the insurance contract is not signed for any of the FOR values. On the other hand if, for example, FOR = 5%, $\alpha^{\text{P}} = 0.9$, and $\alpha^{\text{S}} = 0.5$, the maximum premium of the producer (€0.113 million) is higher than the minimum premium of the insurer (€0.064 million), and an agreement can be reached in consequence.

In Tables 5.23 and 5.24, the relative difference between the maximum premium of the producer and the minimum premium of the insurer $(100\frac{M^{\text{I,P}*}-M^{\text{I,S}*}}{M^{\text{I,S}*}})$ is provided for FOR = 5% and FOR = 10%, respectively, and for all considered risk aversion levels of both market participants. If this difference is positive, both players are interested in signing the insurance contract. Likewise, the higher the difference between both premia, the more likely the agreement is. On the other hand, if the minimum premium of the insurer ($M^{\text{I,S}*}$) is higher than the maximum premium of the producer ($M^{\text{I,P}*}$), the corresponding entry in the table is 0, indicating that the insurance contract is not signed.

Observe that the insurance contract is signed for risk aversion levels of the insurer lower than those of the producer. This corroborates the fact that an insurance contract is a mechanism to transfer risk from one party (the producer) to another (the insurer) in exchange for a certain amount of money (the premium). For a given risk aversion of the producer (a specific column in Tables 5.23 and 5.24), observe that the lower the risk aversion of the insurer, the higher the relative premium difference, and therefore, the higher the possibility that a deal is reached between both parties.

Finally, note that the relative premium difference for FOR = 10% (Table

Table 5.23: Insurance contracting case study: premium difference $(100\frac{M^{\mathrm{I,P}*}-M^{\mathrm{I,S}*}}{M^{\mathrm{I,S}*}})$ in % for $\lambda_{\mathrm{I}} = 25$ €/MWh, $P_{\mathrm{I}} = 200$ MW, and FOR = 5%

		Producer risk aversion (α^{P})									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Insurer risk aversion (α^{S})	0	0	3	5	16	40	64	93	127	172	244
	0.1	0	0	0	5	26	48	74	104	144	209
	0.2	0	0	0	0	12	31	54	82	117	175
	0.3	0	0	0	0	0	15	35	59	90	141
	0.4	0	0	0	0	0	0	16	36	63	106
	0.5	0	0	0	0	0	0	0	16	39	76
	0.6	0	0	0	0	0	0	0	0	17	48
	0.7	0	0	0	0	0	0	0	0	0	23
	0.8	0	0	0	0	0	0	0	0	0	0
	0.9	0	0	0	0	0	0	0	0	0	0

Table 5.24: Insurance contracting case study: premium difference $(100\frac{M^{\mathrm{I,P}*}-M^{\mathrm{I,S}*}}{M^{\mathrm{I,S}*}})$ in % for $\lambda_{\mathrm{I}} = 25$ €/MWh, $P_{\mathrm{I}} = 200$ MW, and FOR = 10%

		Producer risk aversion (α^{P})									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Insurer risk aversion (α^{S})	0	0	1	3	7	21	36	52	72	96	145
	0.1	0	0	0	0	9	23	37	55	76	120
	0.2	0	0	0	0	0	10	22	38	58	97
	0.3	0	0	0	0	0	0	9	24	41	76
	0.4	0	0	0	0	0	0	0	10	26	57
	0.5	0	0	0	0	0	0	0	0	11	39
	0.6	0	0	0	0	0	0	0	0	0	22
	0.7	0	0	0	0	0	0	0	0	0	6
	0.8	0	0	0	0	0	0	0	0	0	0
	0.9	0	0	0	0	0	0	0	0	0	0

5.24) are lower than those for FOR = 5% (Table 5.23), meaning that the possibility that both parties achieve an agreement decreases for higher forced outage rates. This effect occurs as a consequence of the way the availability risk is handled by the power producer and the insurer. Apart from signing an insurance contract, a power producer can reduce its availability risk by other mechanisms such as decreasing the power sold through forward contracts, thus moderating the financial losses if unit failures occur. Nevertheless, an insurer that signs an insurance contract with a power producer has to completely face this availability risk and pay the producer any time the unit fails and the price is higher than the strike price. In other words, if the FOR increases, the only *defense mechanism* of the insurer consists in increasing the premium of the insurance. As it can be observed in Tables 5.21 and 5.22, an increase in the FOR value causes an increase in the minimum premium of the insurer higher than the increase in the maximum premium of the producer for the same risk aversion level.

Needless to say, in practice, insurance companies make agreements with more than one client, and this multi-contracting mechanism is how they diversify risk and reduce insurance premia.

5.7.4 Computational burden

The simulations results presented in this case study have been obtained using CPLEX 12.1.0 [6] under GAMS [5] on a Sun Fire X4600M2 with 8 Quad-Core processors running at 2.9 GHz and 256 GB of RAM. The computational time required to solve the two-stage stochastic optimization problem (5.18) if a set of 1500 scenarios is used to characterize the uncertainty associated with the pool price and the unit availability is lower than 10 minutes.

5.8 Summary

Power producers selling energy in electricity markets have to face both price and availability risks. Producers can manage price risk by selling more energy through forward contracts at fixed prices instead of selling it in the pool at uncertain pool prices. However, in doing that, the availability risk increases as a result of the producer obligation to deliver the agreed amount of electricity during the delivery period of the forward contracts, a requirement that involves significant financial losses if the unit fails and the pool price turns out to be comparatively high.

In this chapter, insurance contracts against unit failures are introduced as a financial instrument to reduce the availability risk faced by power producers. Taking into account the uncertainty related to the pool price as well as its risk aversion level and the forced outage rates of its generating units, a power producer should decide whether or not to sign a given insurance contract.

This chapter proposes a two-stage stochastic programming model to analyze the impact of an insurance contract against unit failures on both the forward contracting decisions and the objective function of a risk-averse power producer. Moreover, the proposed stochastic optimization model is used to compute the maximum premium that, according to its risk aversion level and forced outage rate, a power producer is willing to pay for a given insurance contract. For a certain risk aversion level, insurance decisions are made to maximize the Conditional Value-at-Risk of the profit distribution of the producer. Uncertainties affecting producer decisions, i.e., pool prices and generating unit availability, are modeled via a set of scenarios, whose cardinality is conveniently reduced to make the resulting optimization problem tractable and solvable in a reasonable time. The proposed stochastic programming model is formulated as a large-scale mixed-integer linear programming problem that is efficiently solved using commercially available software.

Additionally, the chapter includes an optimization model that, considering the scenario set characterizing the pool prices and the availability of the generating units, determines the CVaR of the profit distribution of an insurer that provides insurance contracts against unit failures to power producers. Based on this model, the minimum premium that the insurer is willing to receive for a given insurance contract is computed and compared to the maximum premium of the producer in order to give an insight into the insurance bargaining process.

The main aspects of the proposed optimization model are highlighted and discussed via an illustrative example and a real-world case study pertaining to the electricity market of the Iberian Peninsula.

Chapter 6

Managing Producer Risks Using Options

6.1 Introduction

Forward contracts are agreements to buy/sell a fixed amount of electricity at a given price throughout a certain time interval in the future. Selling electricity through a forward contract at a fixed price allows power producers to hedge against the risk due to pool price volatility. On the other hand, the main disadvantage of a forward contract is that its delivery is mandatory, i.e., if the power producer is unable to deliver the agreed amount of energy, it must buy the corresponding energy in the pool to comply with its contract obligations. If the pool price is high during these time steps, financial losses may occur.

Alternatively to sell electricity through forward contracts, a producer can sell its production through *options*. An option is a contract that gives the holder of the option the right (not the obligation) to buy/sell a specified energy amount during a certain future time and at a price called *strike price*. Therefore, an option provides more flexibility than a forward contract since the holder can decide whether or not the option is exercised depending on the availability of its generating units and/or the pool price behavior. Nonetheless, whereas signing a forward contract entails no cost, there is a non-refundable cost to acquire an option, which is called *option price*.

In this chapter, we analyze electricity options as instruments to manage the two main risks faced by price-taker power producers: price and production-availability risks. For this purpose, we design a multi-stage stochastic programming model that enables a risk-averse power producer to decide its optimal portfolio of forward contracts and options taking into account the pool price volatility, the unexpected generating unit failures, and the uncertain forward prices.

6.2 Options

While forward contracts necessarily require the sale or purchase of the agreed electricity during their delivery period at the price established beforehand, the word that better defines the nature of options is *choice*. The holder of an option does indeed observe how some uncertain parameters that affect its profit or utility function turn out during the time interval right after the acquisition of the option and based on that, decides whether or not to exercise the option to buy or sell electricity at the strike price. This way, the acquisition of an option reduces the risk faced by its holder. However, the flexibility provided by options is not free, but there is a cost associated with the purchase of an option.

6.2.1 Definition and Characteristics

Formally, an option is an agreement which gives the buyer the right, but not the obligation, to sell/buy a certain amount of electricity during a specified future time period, referred to as *delivery period*, and at a fixed price called *strike price*. Needless to say, buying an option has an additional cost (unlike a forward contract) called *option price*, which has to be paid even if the option is not exercised.

Depending on whether an option can be exercised on any business day up to and including the expiration date or only on the expiration date itself, options are classified into American or European options, respectively [93]. Due to the more flexibility offered by American options, their option prices are usually higher than those of European options.

There are two main types of options: *calls* and *puts*. A call option gives its holder the right to buy a given amount of electricity at the strike price. Conversely, a put option gives its holder the right to sell a given amount electricity at the strike price. In both cases, the seller of the option receives the nonrefundable option price from the holder of the option and assures the purchase/sale of the electricity at the strike price if the holder decides to exercise the signed option.

There are two sides to every option contract. On one side is the agent that takes the *long position*, i.e., it purchases either a put or a call option and therefore, the right to sell or buy the underlying commodity at the strike price, respectively. On the other side is the agent that assumes a *short position* by selling either a put or call option and undertaking the obligation to buy or sell, respectively, the underlying commodity at the strike price as long as the holder of the option exercises it. By combining both types of options (calls and puts) with both positions (long and short), the following four strategies are obtained:

- *Long put position.* A market agent that buys a put option adopts a long put position, thereby having the right, not the obligation, to sell electricity during the delivery period at the strike price.

- *Long call position.* A market agent that purchases a call option adopts a long call position and therefore, it has the right, not the obligation, to buy electricity during the delivery period at the strike price.

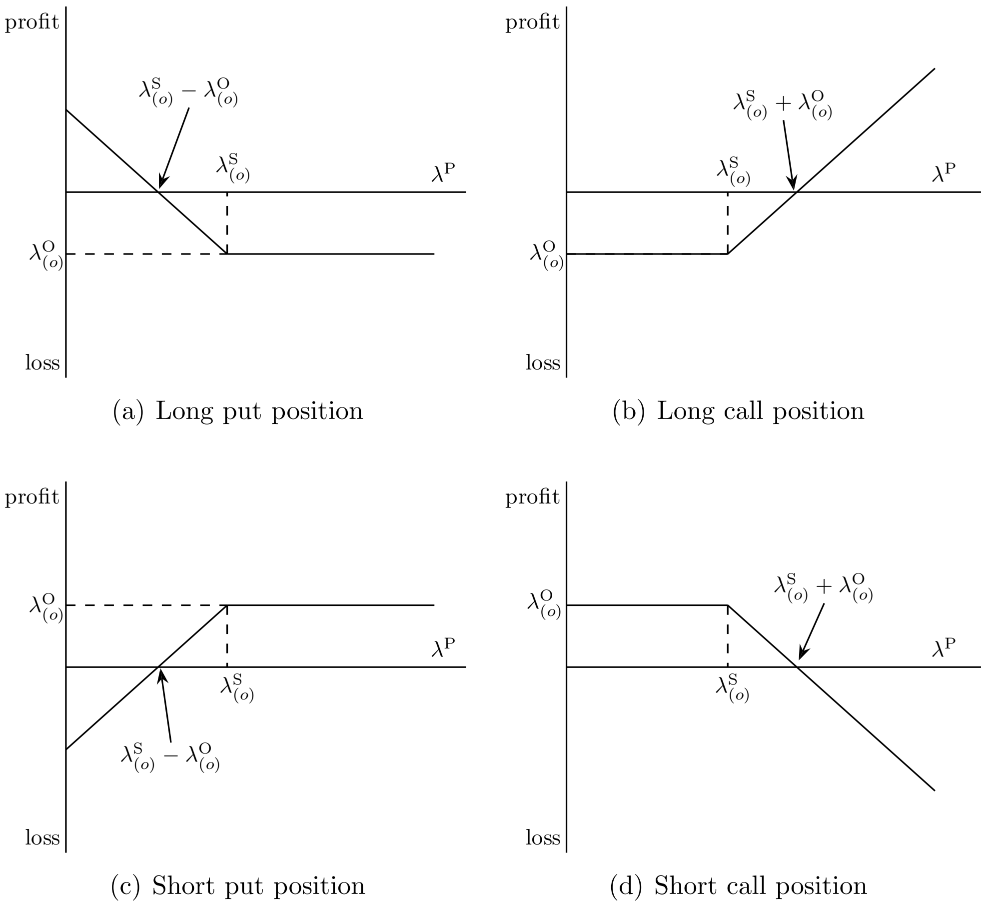
- *Short put position.* A market agent that sells a put option takes a short put position, thus having the obligation to buy the agreed electricity at the strike price if the holder of the option exercises it.

- *Short call position.* A market agent that sells a call option takes a short call position and therefore, it has the obligation to deliver the agreed electricity at the strike price if the holder of the option exercises it.

Fig. 6.1 illustrates the profit profile that results from each of these four positions. In these figures, $\lambda^{\mathrm{S}}_{(o)}$ and $\lambda^{\mathrm{O}}_{(o)}$ represent the strike price and the option price, respectively. Likewise, λ^{P} stands for the pool price of electricity.

Observe that both long positions limit the possible financial losses to the option price, thus representing the typical behavior of a risk-averse agent. On the other hand, the short positions correspond to risk-taker agents since, in exchange for a given *premium*, they are willing to assume the risk of the agent that buys the option.

In this dissertation, we consider the purchase of put and call European options by a risk-averse power producer. Such options give the buyer the right to decide, on the expiration date, whether or not to sell or buy electricity at the strike price, respectively.



(a) Long put position

(b) Long call position

(c) Short put position

(d) Short call position

Figure 6.1: Profit from positions in European options

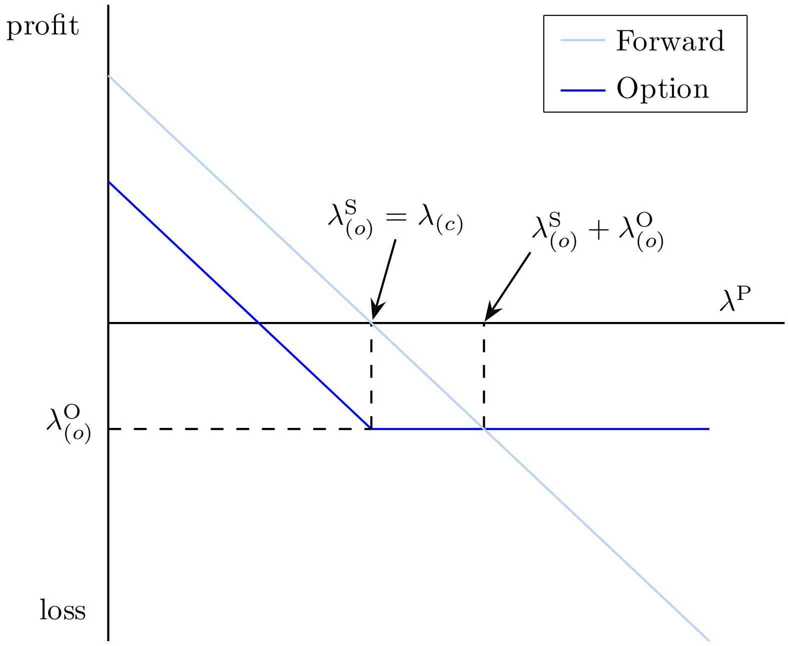


Figure 6.2: Forward contract vs. put option

6.2.2 Options vs. Forward Contracts

This section illustrates the main differences between a forward contract and an option. To do that, Fig. 6.2 compares the profit/loss of a producer that sells its production through a forward contract vs. the situation in which the producer purchases a put option to have the right to sell electricity depending on the pool price, which is denoted by λ^{P}. Note that the price of the forward contract ($\lambda_{(c)}$) is set to the strike price of the option ($\lambda^{\mathrm{S}}_{(o)}$). The option price is equal to $\lambda^{\mathrm{O}}_{(o)}$.

The first observation that can be made from this figure is that both the profit and the lost revenue related to the forward contract are “unlimited” and depend directly on the realization of the pool price. In contrast, if the producer purchases a put option, the lost revenue associated with high pool prices are bounded to the option price by not exercising the option. However, note that in exchange, the producer profit is reduced by the option price if low pool prices realize. Therefore, while the profit obtained from an option is “unlimited”, albeit lower than that obtained with a forward contract, its lost revenue is bounded by the option price.

6.2.3 Options to Hedge Against Risk

The most relevant feature of options is the delay existing between the time in which the option is signed, i.e., when the strike price and the volume of the option contract are established, and the time in which the holder of the option has to decide whether or not to exercise the option. The realization of the uncertain parameters between these two decision points allows the option holder to better characterize the uncertain parameters during the option delivery period to informatively decide whether or not to exercise the option. Below, we provide some insight into how options to sell or buy electricity can be used by power producers to reduce their price and availability risks.

First, to evaluate how a put option is used to hedge against the pool price risk, we consider that the generating unit owned by the power producer does not fail. To do so, we suppose that the power producer has acquired a put option that can be exercised at a certain subsequent decision time in the future. Observe that the realization of high pool prices prior to the exercising time of the option will probably lead to high pool prices during its delivery period and therefore, the producer decides not to exercise the option so as to sell its production in the pool at higher prices. On the other hand, falling pool prices between the purchase and the exercising times of the option encourage the power producer to exercise the put option to sell electricity at the prespecified strike price, which will be probably higher than the average pool price realization during the delivery period of the option. This way, the acquisition of the put option allows the producer to adapt its decisions according to the realization of the pool price to avoid low profit if pool prices are low (option is exercised), and to take advantage of the high pool price realizations (option is not exercised).

Next, a generating unit subject to failures is considered to show how put options can be additionally used to hedge the availability risk faced by power producers. The availability risk of a power producer is related to the realization of high pool prices during the time steps in which the producer suffers from an unexpected forced outage of its generating unit and has the obligation to deliver electricity at a prespecified price because of a signed forward contract. In order to hedge against this risk, a producer can sign a put option to sell electricity instead of a forward contract. Thus, if the generating unit suffers from an unexpected unit failure just before the delivery period of the option (which probably implies that the unit will be offline afterward), the producer does not exercise the option to avoid the possibility of buying in the pool at potentially high prices the energy agreed to be delivered through the put option contract. Conversely, if the unit is available just before the delivery period of the option, the producer may exercise it to sell electricity at a fixed price and hedge its profit against the realization of low pool prices. Therefore, we conclude that a put option allows a power producer to reduce the probability of having low profits as a consequence of unexpected forced outages of its generating units.

Finally, we analyze how a call option to buy electricity reduces the availability risk of power producers. Let us consider that a power producer has signed a forward contract to sell electricity and a call option to have the right to buy electricity during the same delivery period. In that case, if the generating unit owned by the producer is forced out just before the delivery period of both contracts and the pool price is expected to be high, the producer can exercise the call option to buy electricity. This way, the producer can comply with its contracting selling obligation by buying the electricity through the call option at the strike price, which will be probably lower than the average pool price during the delivery period. On the other hand, if either the generating unit does not fail or the pool prices are expected to decrease below the strike price, the call option is not exercised.

These are three representative situations of how options can be used to reduce both the price and the availability risks faced by power producers. Note, however, that the flexibility provided by options involves the payment of the option price, which has to be paid by the producer regardless of whether or not the option is exercised. Therefore, a power producer has to decide, according to the pool price variability, its availability parameters, and its risk aversion level, the acquisition of a put/call option given its strike and option prices.

6.2.4 Option Trading

The most common way to trade options is via standardized options contracts, which are listed by various futures and options exchanges. For instance, in EEX [2], market participants can trade options on all the forward contracts available in this exchange. As an example, for the monthly forward contract spanning December 2010, EEX offers several call and put options on this forward contract with different strike prices. Depending on the interest rate, the volatility of the pool price, the strike price, and the underlying price of the corresponding forward contract, EEX settles the option price for each put and call option.

Fig. 6.3 illustrates the option prices of different put and call options on the forward contract spanning December 2010 in EEX as recorded on November 7, 2010. The second row depicts the trading activity of the forward contract spanning December 2010, while the following rows correspond to the option trading activity for that month, where CXX00 and PYY00 stand for a call option of strike price equal to XX €/MWh and to a put option of strike price equal to YY €/MWh, respectively. For the forward contract, the second, third, and fourth column are the best bid, the best ask, and the number of contracts traded so far. The price of the last contract, the absolute change of this price with respect to the previous signed contract, and the time when the last contract was signed are provided in columns fifth, sixth, and seventh, respectively. The eighth and ninth column contains the traded volume in GWh and the settlement price in €/MWh.

For the rows corresponding to put and call options, the last column represents the value of the option price in €/MWh. For example, the option price of a call option with a strike price of 50 €/MWh (C5000) was equal to 0.317 €/MWh. Likewise, the put option with a strike price of 53 €/MWh (P5300) had an option price of 6.763 €/MWh. Note that increasing strike prices of call options correspond to decreasing option prices, since market agents are interested in having the right to buy electricity at low prices. Conversely, the higher the strike price of a put option, the higher the option price.

6.2.5 Option Revenue

Under a stochastic framework, the revenue obtained through N_O option contracts ($\Pi^{\mathrm{O}}_{(\omega)}$) is mathematically expressed as

$$\Pi^{\mathrm{O}}_{(\omega)} = \sum_{o=1}^{N_O} \Big(- \lambda^{\mathrm{O}}_{(o)} P_{(o)} + l_{(o)} y_{(o,\omega)} \lambda^{\mathrm{S}}_{(o)} P_{(o)} \Big) L_{(o)}, \tag{6.1}$$

where $\lambda^{\mathrm{S}}_{(o)}$, $\lambda^{\mathrm{O}}_{(o)}$, $P_{(o)}$, and $L_{(o)}$ are the strike price, the option price, the option power and the delivery period of option o. Variable $y_{(o,\omega)}$ identifies whether or not the producer exercises the option o if scenario ω realizes. The parameter $l_{(o)}$ is equal to $+1$ if option o is a put option and to -1 if it is a call option.

Name	Bid	Ask	Contr.	Price	Change	Time	Vol.	Price
Nov-10	-	-	-	-	-	-	-	48.61
Dec-10	46.45	46.67	30	46.55	0.27	12:36	7,440	46.28
C 4300	-	-	-	-	-	-	-	3.600
C 4400	-	-	-	-	-	-	-	2.825
C 4500	-	-	-	-	-	-	-	2.146
C 4600	-	-	-	-	-	-	-	1.574
C 4700	-	-	-	-	-	-	-	1.114
C 4800	-	-	-	-	-	-	-	0.760
C 4900	-	-	-	-	-	-	-	0.499
C 5000	-	-	-	-	-	-	-	0.317
C 5100	-	-	-	-	-	-	-	0.194
C 5200	-	-	-	-	-	-	-	0.115
C 5300	-	-	-	-	-	-	-	0.067
C 5400	-	-	-	-	-	-	-	0.038
C 5500	-	-	-	-	-	-	-	0.021
C 5600	-	-	-	-	-	-	-	0.011
C 5700	-	-	-	-	-	-	-	0.006
C 5800	-	-	-	-	-	-	-	0.003
C 5900	-	-	-	-	-	-	-	0.002
C 6000	-	-	-	-	-	-	-	0.001
C 6100	-	-	-	-	-	-	-	0.001
C 6200	-	-	-	-	-	-	-	0.001
C 6300	-	-	-	-	-	-	-	0.001
P 4300	-	-	-	-	-	-	-	0.321
P 4400	-	-	-	-	-	-	-	0.546
P 4500	-	-	-	-	-	-	-	0.866
P 4600	-	-	-	-	-	-	-	1.294
P 4700	-	-	-	-	-	-	-	1.834
P 4800	-	-	-	-	-	-	-	2.479
P 4900	-	-	-	-	-	-	-	3.218
P 5000	-	-	-	-	-	-	-	4.035
P 5100	-	-	-	-	-	-	-	4.912
P 5200	-	-	-	-	-	-	-	5.832
P 5300	-	-	-	-	-	-	-	6.783
P 5400	-	-	-	-	-	-	-	7.754
P 5500	-	-	-	-	-	-	-	8.736
P 5600	-	-	-	-	-	-	-	9.727
P 5700	-	-	-	-	-	-	-	10.721
P 5800	-	-	-	-	-	-	-	11.718
P 5900	-	-	-	-	-	-	-	12.716
P 6000	-	-	-	-	-	-	-	13.714
P 6100	-	-	-	-	-	-	-	14.713
P 6200	-	-	-	-	-	-	-	15.713
P 6300	-	-	-	-	-	-	-	16.712

Figure 6.3: Option trading in EEX

This way, if a put option is exercised, the producer profits come from selling electricity at the strike price. On the other hand, if a call option is exercised, the producer has to pay to acquire the power traded through the option at the strike price. N_O is the number of available option contracts.

6.3 Producer Model for Option Selection

6.3.1 Assumptions

The following assumptions are considered to formulate the multi-stage stochastic decision-making model of a power producer which has the opportunity to purchase put or call options. Although most of them have already been discussed throughout the dissertation, they are summarized below for the reader's convenience.

1. The generating units owned by the power producer are thermal units, being each one a dispachable source of electricity whose cost is modeled by a piecewise linear function and whose power output is limited by minimum and maximum bounds. Short-term operating constraints such as ramp limits and minimum up and down times are disregarded.

2. The power producer can sell its production in the pool at volatile prices, or at fixed prices through forward contracts or options in the derivatives market. For the sake of clarity, the arbitrage between these markets is avoided in the model proposed in this dissertation.

3. The prices of forward contracts and options are not affected by the decisions made by the power producer, which is assumed to behave as a price-taker.

4. Three uncorrelated sources of uncertainty are taken into account, namely, the pool price, the availability of generating units, and the forward prices. Forward and option contracting decisions are made facing the uncertainty of such parameters throughout the whole study horizon. However, the exercise of the acquired options is decided knowing the realization of the random parameters throughout part of the study horizon, but still facing the uncertainty affecting the rest of that horizon. Finally, pool decisions are assumed to be made with perfect information due to the comparatively smaller uncertainty involved in this market.

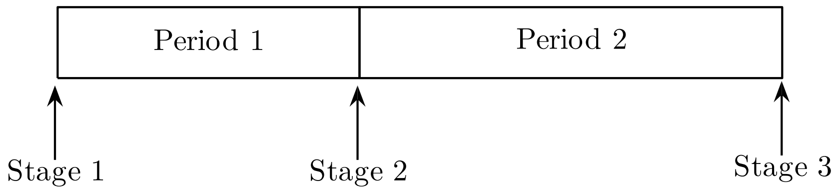


Figure 6.4: Time horizon and stages for option contracting

5. Although both physical and financial options are available in electricity markets, due to the energy-oriented approach of this dissertation, all the options considered in the proposed model imply the physical delivery of the option power.

6. Two types of options are considered: put and call options. Likewise, since the power producer is assumed to be risk-averse, the producer can purchase either put or call options to have the right to sell or buy electricity, respectively, but it cannot sell neither of these options to other market participants. In other words, we only consider power producers taking a long position on either put or call options.

7. Despite the fact that options can be traded any time up to their expiration date, we have simplified its trading by allowing the producer to sign options only at the beginning of the decision horizon for a certain option price. Likewise, if the option is exercised at the beginning of its delivery period, the holder of the option sells or buys electricity at the strike price, a price which is agreed at the time of acquiring the option.

6.3.2 Decision Framework

Decisions related to electricity options are made in different stages. First, the producer has to decide whether or not to sell/buy electricity through a given option, and subsequently, if the option has been purchased, the producer has to decide whether or not it is exercised. Since we consider European options, this second decision has to be made on the expiration date. Therefore, the study horizon is divided in two periods and three stages (see Fig. 6.4).

As an example, let us consider that the producer can trade its production at fixed prices through two forward contracts and through either a put or a call option at their corresponding strike and option prices. The first forward contract spans period 1 ($t = 1, \ldots, N_{T_1}$), which means that the producer must deliver the agreed energy during each hour of period 1. Additionally, a second forward contract spanning period 2 ($t = N_{T_1} + 1, \ldots, N_T$) is considered. Besides, the producer has the possibility of selling/buying electricity through a put/call option spanning period 2 ($t = N_{T_1} + 1, \ldots, N_T$). This option can be bought at the beginning of period 1 and exercised at the beginning of period 2. Therefore, the sequence of decisions is the following:

- Stage 1. Decision on the energy to be sold through the available forward contracts (whose delivery periods comprise either period 1 or 2), and the energy to be sold/bought through the available put/call option for period 2. These decisions are made accounting for all plausible realizations of the stochastic processes involved, i.e., knowing neither the pool price, nor the unit availability throughout the study horizon, nor forward contract prices for the second period.

- Stage 2. Once the pool price, the unit availability, and the forward prices throughout period 1 are known, the producer decides the amount of energy to sell in the pool and its production levels during this period, i.e., from $t = 1$ to $t = N_{T_1}$ (pool market decisions are considered to be made with perfect information). Additionally, the producer can modify the energy sold through the forward contract spanning period 2 depending on the updated forward and pool prices. Finally, the producer decides whether or not to exercise the purchased put/call option.

- Stage 3. Once pool prices and the unit availability during period 2 ($t = N_{T_1} + 1, \ldots, N_T$) become certain, the producer decides the energy quantity to be sold in the pool during period 2 as well as the energy produced by the unit from $t = N_{T_1} + 1$ to $t = N_T$.

In two-stage stochastic problems, some decisions (here-and-now decisions) are made before the uncertain parameters are known, while other decisions (wait-and-see decisions) are delayed until the stochastic processes are disclosed. Multi-stage recourse problems deal with models in which this "decide-observe-decide" pattern is repeated more than once and therefore, they prove to be an effective mathematical tool to determine option purchases in electricity markets.

The choice of a multi-stage stochastic programming framework to model options does not only modify the way in which variables and constraints are defined, but also changes the method to generate scenarios representing the uncertain parameters involved in the model. In this respect, Fig. 6.5 depicts an illustrative scenario tree for a three-stage stochastic problem. Each branch represents the realization of the stochastic processes between two consecutive stages. For example, every branch between stages 1 and 2 corresponds to a possible realization of the pool price, the unit availability, and the forward prices during period 1. Likewise, each node corresponds to the decisions to be made in each stage. Thus, decisions in stage 1, namely, the power sold through forward contracts spanning period 1, $P^1_{(c_1)}$, the power sold through forward contracts spanning period 2, $P^1_{(c_2)}$, and the power sold/bought through the put/call options, $P_{(o)}$, are made facing uncertainty affecting periods 1 and 2. Similarly, decisions made in stage 2 like the exercising of the options, $y_{(o,\omega)}$, the power re-traded through forward contracts spanning period 2, $P^2_{(c_2,\omega)}$, the generated power and the power sold in the pool during period 1, $P^{\mathrm{G}}_{(\omega,t)}$ and $P^{\mathrm{P}}_{(\omega,t)}, t = 1, \ldots, N_{T_1}$, depend on the realization of the stochastic processes during period 1, but they are independent of the possible scenario realizations during period 2. Finally, third-stage decisions, i.e., the generated power and the power sold in the pool during period 2, $P^{\mathrm{G}}_{(\omega,t)}$ and $P^{\mathrm{P}}_{(\omega,t)}, t = N_{T_1} + 1, \ldots, N_T$, are determined depending on each realization of the stochastic processes throughout the whole study horizon. In Section 6.3.7, we introduce and formulate the constraints that enforce the nonanticipativity conditions inherent to the above decide-observe-decide pattern.

6.3.3 Uncertainty Characterization

In order to appropriately characterize the uncertainty of the parameters involved in this model, the sequence of decisions of the multi-stage stochastic programming problem has to be taken into account to generate the scenario set. That is, the knowledge of the realization of a stochastic process during period 1 has to be properly accounted for to produce the scenario set representing the uncertainty of the same stochastic process during period 2.

Availability scenarios are built as follows. First, a scenario set representing the availability of the generating units during period 1 is generated according to the procedure explained in Section 3.5 of Chapter 3. Then, for each scenario generated for period 1, a new availability scenario set for period 2 is built considering the status of the unit in the last hour of period 1 ($k_{N_{T1}}$) as

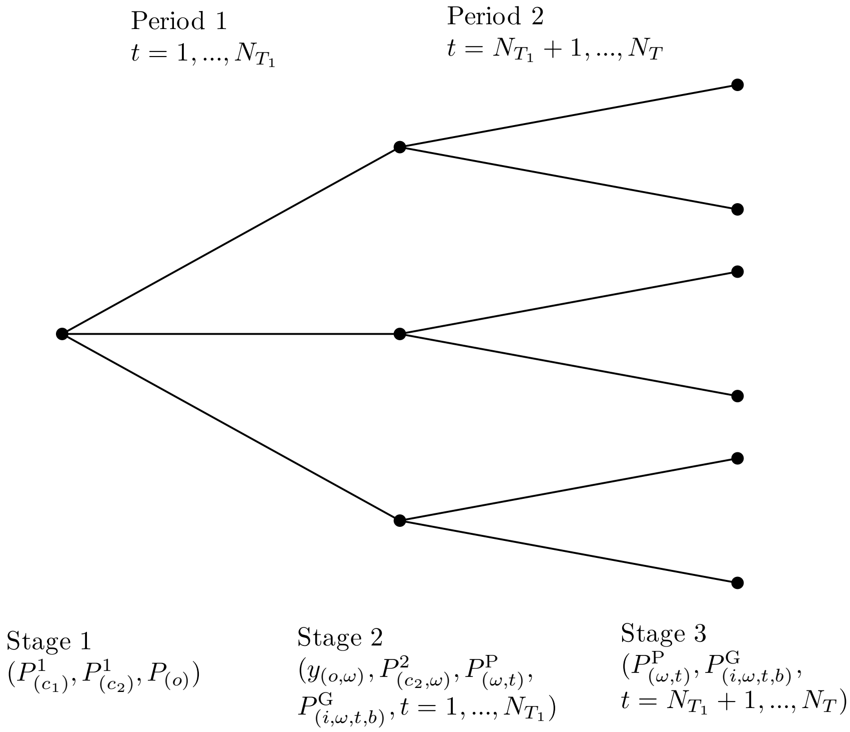


Figure 6.5: Three-stage scenario tree for option contracting

its initial status at the beginning of period 2. A detailed procedure to generate availability scenarios for multi-stage stochastic programming problems is presented in Section 3.5 of Chapter 3.

Pool price scenarios are built as follows. A scenario set representing the pool price during period 1 is generated by simulating the innovation term of the ARIMA model (Section 3.4 of Chapter 3). Then, the values of each pool price scenario for period 1 are taken as certain and introduced into the ARIMA model to generate each scenario set for period 2. This way, it is more likely that a scenario of high/low pool prices during period 1 gives rise to a scenario set of high/low prices during period 2. Section 3.4 of Chapter 3 provides a complete description of the procedure to generate pool price scenarios within a multi-stage stochastic approach.

Forward contracts delivered during period 2 can be traded in stages 1 and 2 at known and uncertain prices, respectively. Thus, a scenario set representing forward prices at the end of period 1 needs to be generated.

6.3.4 Variables

In the proposed multi-stage stochastic optimization problem, decisions are made in three different stages during the study horizon. First-stage decisions are made facing the uncertainty of the stochastic parameters during the whole study horizon. Second-stage variables are decided knowing the realization of the stochastic processes during period 1, but still facing the uncertainty of such parameters during period 2. Finally, once the information of the stochastic processes during the whole study horizon is revealed, third-stage decisions are made. For this particular model, the decision variables are:

- First-stage variables:
  1. Power sold through each available forward contract spanning either period 1 ($P^1_{(c_1)}$) or period 2 ($P^1_{(c_2)}$).
  2. Power level sold or bought through the available put or call options, respectively ($P_{(o)}$).
- Second-stage variables:
  1. Binary variable representing whether or not each option is exercised ($y_{(o,\omega)}$).
  2. Power re-traded through each forward contract spanning period 2 ($P^2_{(c_2,\omega)}$).
  3. Binary variable representing the start-up and shut-down plan of each generating unit during period 1 ($u_{(i,\omega,t)}, t = 1, \ldots, N_{T_1}$).
  4. Power output scheduled for each generating unit during period 1 ($P^{\mathrm{G}}_{(i,\omega,t)}, t = 1, \ldots, N_{T_1}$).
  5. Amount of generated power to be sold in the pool during period 1 ($P^{\mathrm{P}}_{(\omega,t)}, t = 1, \ldots, N_{T_1}$).
- Third-stage variables:
  1. Binary variable representing the start-up and shut-down plan of each generating unit during period 2 ($u_{(i,\omega,t)}, t = N_{T_1} + 1, \ldots, N_T$).
  2. Power output scheduled for each generating unit during period 2 ($P^{\mathrm{G}}_{(i,\omega,t)}, t = N_{T_1} + 1, \ldots, N_T$).
  3. Amount of generated power to be sold in the pool during period 2 ($P^{\mathrm{P}}_{(\omega,t)}, t = N_{T_1} + 1, \ldots, N_T$).

6.3.5 Objective Function

The objective function to be maximized is the Conditional Value-at-Risk (CVaR) of the profit probability distribution (Π) of the power producer for a given confidence level (α^{P}), i.e.,

$$\text{Maximize} \quad \mathrm{CVaR}_{\alpha^{\mathrm{P}}}(\Pi) = \zeta - \frac{1}{1-\alpha^{\mathrm{P}}} \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \eta_{(\omega)}, \tag{6.2}$$

where $\pi_{(\omega)}$ represents the probability of scenario ω, and ζ and $\eta_{(\omega)}$ are two auxiliary variables required to calculate the CVaR of the profit probability distribution as explained in Section 2.2.3 of Chapter 2.

The producer profit for each scenario, $\Pi_{(\omega)}$, includes four components: the revenue obtained in the pool, $\Pi^{\mathrm{P}}_{(\omega)}$, the revenue obtained from the signed forward contracts, Π^{F}, the revenue due to the signed options, $\Pi^{\mathrm{O}}_{(\omega)}$, and the production cost, $C^{\mathrm{G}}_{(\omega)}$, i.e.,

$$\Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{F}} + \Pi^{\mathrm{O}}_{(\omega)} - C^{\mathrm{G}}_{(\omega)}, \tag{6.3}$$

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} P^{\mathrm{P}}_{(\omega,t)} L_{(t)}, \tag{6.4}$$

$$\Pi^{\mathrm{F}} = \sum_{c_1=1}^{N_{C_1}} \lambda^1_{(c_1)} P^1_{(c_1)} L_{(c_1)} + \sum_{c_2=1}^{N_{C_2}} \Big(\lambda^1_{(c_2)} P^1_{(c_2)} + \lambda^2_{(c_2,\omega)} P^2_{(c_2,\omega)} \Big) L_{(c_2)} \tag{6.5}$$

$$\Pi^{\mathrm{O}}_{(\omega)} = \sum_{o=1}^{N_O} \Big(-\lambda^{\mathrm{O}}_{(o)} P_{(o)} + l_{(o)} y_{(o,\omega)} \lambda^{\mathrm{S}}_{(o)} P_{(o)} \Big) L_{(o)}, \tag{6.6}$$

$$C^{\mathrm{G}}_{(\omega)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} C^{\mathrm{G}}_{(i,\omega,t)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} (A_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} \lambda_{(i,b)} P^{\mathrm{G}}_{(i,\omega,t,b)}) L_{(t)}, \tag{6.7}$$

where $\lambda^{\mathrm{P}}_{(\omega,t)}$ represents the pool price during time step t and in scenario ω, $P^{\mathrm{P}}_{(\omega,t)}$ is the power sold in the pool during time step t and in scenario ω, $L_{(t)}$ is the duration in hours of time step t, and $\lambda^1_{(c_1)}$, $L_{(c_1)}$, and $P^1_{(c_1)}$ are, respectively, the price, the duration, and the amount of electricity to be sold through forward contract c_1 spanning period 1. Likewise, $\lambda^1_{(c_2)}$ and $P^1_{(c_2)}$ are the price and the power sold through forward contract c_2 at stage 1, $\lambda^2_{(c_2,\omega)}$ and $P^2_{(c_2,\omega)}$ represent the price and the power re-traded through forward contract c_2 at stage 2, i.e., once the stochastic parameters during period 1 realize, and $L_{(c_2)}$ stands for the duration of the forward contract c_2. Regarding options, $\lambda^{\mathrm{S}}_{(o)}$, $\lambda^{\mathrm{O}}_{(o)}$, $P_{(o)}$, and

$L_{(o)}$ are the strike price, the option price, the option power, and the duration of option o. Finally, $P^{\mathrm{G}}_{(i,\omega,t,b)}$ represents the power generated by unit i from production block b during time step t and in scenario ω, and $A_{(i)}$ and $\lambda_{(i,b)}$ are, in that order, the no-load cost and block-slopes to approximate the quadratic production cost function of unit i.

6.3.6 Constraints

We formulate below the constraints of the multi-stage stochastic optimization model design to determine the optimal power traded by the power producer in the pool, through forward contracts, and through the available put and call options.

If unexpected failures are taken into account, the power generated by a unit is bounded by its maximum and minimum power outputs as follows,

$$u_{(i,\omega,t)} k_{(i,\omega,t)} P^{\mathrm{Min}}_{(i)} \leq P^{\mathrm{G}}_{(i,\omega,t)} \leq u_{(i,\omega,t)} k_{(i,\omega,t)} P^{\mathrm{Max}}_{(i)}, \quad \forall i, \forall \omega, \forall t. \qquad (6.8)$$

To approximate the quadratic cost function by a piecewise linear function, the following constraints are needed:

$$P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{Min}}_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} P^{\mathrm{G}}_{(i,\omega,t,b)}, \qquad \forall i, \forall \omega, \forall t \qquad (6.9)$$

$$0 \leq P^{\mathrm{G}}_{(i,\omega,t,b_1)} \leq P^{\mathrm{Max}}_{(i,b_1)} - P^{\mathrm{Min}}_{(i)}, \qquad \forall i, \forall \omega, \forall t \qquad (6.10)$$

$$0 \leq P^{\mathrm{G}}_{(i,\omega,t,b)} \leq P^{\mathrm{Max}}_{(i,b)} - P^{\mathrm{Max}}_{(i,b-1)}, \qquad \forall i, \forall \omega, \forall t, \forall b = b_2, ..., N_B. \qquad (6.11)$$

The following constraints enforce the electric energy balance in each time step and scenario:

$$\sum_{i=1}^{N_I} P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{P}}_{(\omega,t)} + \sum_{c_1 \in F^1_{(t)}} P^1_{(c_1)} + \\ + \sum_{c_2 \in F^2_{(t)}} \left(P^1_{(c_2)} + P^2_{(c_2,\omega)} \right) + \sum_{o \in O_{(t)}} l_{(o)} y_{(o,\omega)} P_{(o)}, \qquad \forall \omega, \forall t. \qquad (6.12)$$

For the sake of clarity, we prevent arbitrage between the pool and the derivatives market. This condition is enforced through the following constraints:

$$\sum_{i=1}^{N_I}(k_{(i,\omega,t)} - 1)P^{\text{Max}}_{(i)} \leq P^{\text{P}}_{(\omega,t)}, \qquad \forall\omega, \forall t \tag{6.13}$$

$$0 \leq P^1_{(c_1)}, \qquad \forall c_1 \tag{6.14}$$

$$0 \leq P^1_{(c_2)}, \qquad \forall c_2 \tag{6.15}$$

$$0 \leq P^1_{(c_2)} + P^2_{(c_2,\omega)}, \qquad \forall c_2, \forall\omega \tag{6.16}$$

$$\sum_{i=1}^{N_I} P^{\text{G}}_{(i,\omega,t)} + \sum_{o\in O_{(t)}:l_{(o)}=-1} y_{(o,\omega)}P_{(o)} \leq \sum_{i=1}^{N_I} P^{\text{Max}}_{(i)} \qquad \forall\omega, \forall t. \tag{6.17}$$

Note that these constraints guarantee that the producer can only buy energy in the pool during those time steps in which the generating unit is forced out. Moreover, the producer may sell electricity using the available forward contracts spanning both period 1 and period 2. The set of constraints (6.17) enforces that if a call option is signed at the beginning of the study horizon and the producer exercises it, the sum of the power bought through the call option plus the generated electricity cannot be higher than the total capacity of the generating units. In doing so, we impose, for clarity purposes, that the producer exercises the call option if the probability of having an unit failure is significant enough.

Since put and call options are distinguished by means of the binary parameter $l_{(o)}$, the option power level is always greater than or equal to zero, i.e.,

$$0 \leq P_{(o)}, \qquad \forall o. \tag{6.18}$$

The formulation of the CVaR requires the following constraints:

$$-\Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \qquad \forall\omega \tag{6.19}$$

$$0 \leq \eta_{(\omega)}, \qquad \forall\omega, \tag{6.20}$$

where $\eta_{(\omega)}$ and ζ are two auxiliary variables needed to calculate the optimal value of the CVaR for a confidence level α^{P} [153].

A multi-stage stochastic programming formulation needs a set of constraints enforcing that second-stage decisions are made knowing the realization of the stochastic parameters during period 1, but still facing the uncertainty of these parameters afterward. These constraints are called nonanticipativity constraints and, for this particular model, are expressed as

$$P^{\mathrm{G}}_{(i,\omega,t,b)} = P^{\mathrm{G}}_{(i,\omega',t,b)}, \qquad \forall i, \forall \omega, \omega' : \omega' \in S_{(\omega)}, \forall t = 1, \ldots, N_{T_1}, \forall b \tag{6.21}$$

$$P^{\mathrm{P}}_{(\omega,t)} = P^{\mathrm{P}}_{(\omega',t)}, \qquad \forall \omega, \omega' : \omega' \in S_{(\omega)}, \forall t = 1, \ldots, N_{T_1} \tag{6.22}$$

$$P^{2}_{(c_2,\omega)} = P^{2}_{(c_2,\omega')}, \qquad \forall c_2, \forall \omega, \omega' : \omega' \in S_{(\omega)} \tag{6.23}$$

$$y_{(o,\omega)} = y_{(o,\omega')}, \qquad \forall o, \forall \omega, \omega' : \omega' \in S_{(\omega)}, \tag{6.24}$$

where $S_{(\omega)}$ is the set of scenarios whose uncertain parameter values during period 1 are equal to those associated with scenario ω, i.e.,

$$S_{(\omega)} = \{\omega' : (\lambda^{\mathrm{P}}_{(\omega',t)} = \lambda^{\mathrm{P}}_{(\omega,t)}, k_{(i,\omega',t)} = k_{(i,\omega,t)}, \forall i, \forall t = 1, ..., N_{T_1}) \text{ and } (\lambda^{2}_{(c_2,\omega')} = \lambda^{2}_{(c_2,\omega)}, \forall c_2)\}.$$

As explained in Section 3.5 of Chapter 3, a set of scenarios representing the availability of a generating unit during period 2 is built taking into account its availability during period 1 in the scenario generation procedure. According to equation (3.3), only the status of the generating unit in the last time step of period 1 is needed to generate availability scenarios for period 2. Therefore, all the availability scenario sets for period 2 sharing the same final unit status ($k_{(i,\omega,N_{T_1})}$) in period 1 are generated using the same probability distribution.

Note that the availability of a generating unit is a discrete stochastic process that can be characterized by a finite scenario set and therefore, the higher the number of scenarios, the closer the generated set to the "real" one is. Consequently, if a sufficiently large number of scenarios is generated, all the availability scenario sets for period 2 sharing identical final unit status in period 1 will be theoretically composed of the same availability scenarios. In that case, for a given realization of the pool price during period 1, two-stage decision variables associated with all the availability scenario sets for period 2 sharing the same final unit status in period 1 will be naturally the same.

However, since a relatively small number of scenarios must be considered to make the stochastic optimization problem computationally tractable, these availability scenarios sets for period 2 that are generated with the same final unit status in period 1 may be comprised of very different availability scenarios, thus probably leading to different two-stage decision variables. In order to *artificially* eliminate the possibility of obtaining results that contradict the theoretical statement in the previous paragraph, the following constraints are included in the stochastic optimization model:

$$P^2_{(c_2,\omega)} = P^2_{(c_2,\omega')} \qquad \forall c_2, \forall \omega, \omega' : (k_{(i,\omega,N_{T_1})} = k_{(i,\omega',N_{T_1})}, \forall i \text{ and } \lambda^{\mathrm{P}}_{(\omega,t)} = \lambda^{\mathrm{P}}_{(\omega',t)}, \forall t = 1, ..., N_{T_1}) \tag{6.25}$$

$$y_{(o,\omega)} = y_{(o,\omega')} \qquad \forall o, \forall \omega, \omega' : (k_{(i,\omega,N_{T_1})} = k_{(i,\omega',N_{T_1})}, \forall i \text{ and } \lambda^{\mathrm{P}}_{(\omega,t)} = \lambda^{\mathrm{P}}_{(\omega',t)}, \forall t = 1, ..., N_{T_1}). \tag{6.26}$$

These constraints enforce then that decisions on both the forward contracts traded in stage 2 and the exercise of the option are identical for the same status of the generating unit at the end of period 1 and the same pool prices during that period. Note that these constraints would not be needed either if the same reduced availability scenario set for period 2 is linked to every availability scenario in period 1 with identical unit status at the end of this period. Nevertheless, in doing so, the characterization of the uncertainty related to the unit availability during period 2 would be much more inaccurate for the same level of computational burden.

Finally, it is worth mentioning that, although formulated very similarly to nonanticipativity constraints (6.21)–(6.24), the purpose of constraints (6.25)–(6.26) is not to impose the way in which information is revealed over time, but to take advantage of the particular features of the availability scenario generation procedure to characterize more accurately the unit status in period 2 without increasing the computational burden of the optimization model.

6.3.7 Formulation

The proposed multi-stage stochastic formulation of the risk-constrained profit maximization problem for a power producer is

$$\text{Maximize}_{P^{\mathrm{G}}_{(i,\omega,t,b)}, P^{\mathrm{P}}_{(\omega,t)}, P^1_{(c_1)}, P^1_{(c_2)}, P^2_{(c_2,\omega)}, P_{(o)}, y_{(o,\omega)}, u_{(i,\omega,t)}, \zeta, \eta_{(\omega)}}$$

$$\zeta - \frac{1}{1-\alpha^{\mathrm{P}}} \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \eta_{(\omega)} \tag{6.27a}$$

subject to

$$\Pi_{(\omega)} = \Pi^{\mathrm{P}}_{(\omega)} + \Pi^{\mathrm{F}} + \Pi^{\mathrm{O}}_{(\omega)} - C^{\mathrm{G}}_{(\omega)}, \qquad \forall \omega \tag{6.27b}$$

$$\Pi^{\mathrm{P}}_{(\omega)} = \sum_{t=1}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} P^{\mathrm{P}}_{(\omega,t)} L_{(t)}, \qquad \forall \omega \tag{6.27c}$$

$$\Pi^{\mathrm{F}} = \sum_{c_1=1}^{N_{C_1}} \lambda^1_{(c_1)} P^1_{(c_1)} L_{(c_1)} + \sum_{c_2=1}^{N_{C_2}} \Big(\lambda^1_{(c_2)} P^1_{(c_2)} + \lambda^2_{(c_2,\omega)} P^2_{(c_2,\omega)}\Big) L_{(c_2)} \tag{6.27d}$$

$$\Pi^{\mathrm{O}}_{(\omega)} = \sum_{o=1}^{N_O} \Big(- \lambda^{\mathrm{O}}_{(o)} P_{(o)} + l_{(o)} y_{(o,\omega)} \lambda^{\mathrm{S}}_{(o)} P_{(o)}\Big) L_{(o)}, \qquad \forall \omega \tag{6.27e}$$

$$C^{\mathrm{G}}_{(\omega)} = \sum_{i=1}^{N_I} \sum_{t=1}^{N_T} C^{\mathrm{G}}_{(i,\omega,t)}, \qquad \forall \omega \tag{6.27f}$$

$$C^{\mathrm{G}}_{(i,\omega,t)} = \Big(A_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} \lambda_{(i,b)} P^{\mathrm{G}}_{(i,\omega,t,b)} \Big) L_{(t)}, \qquad \forall i, \forall \omega, \forall t \tag{6.27g}$$

$$u_{(i,\omega,t)} k_{(i,\omega,t)} P^{\mathrm{Min}}_{(i)} \leq P^{\mathrm{G}}_{(i,\omega,t)} \leq u_{(i,\omega,t)} k_{(i,\omega,t)} P^{\mathrm{Max}}_{(i)}, \qquad \forall i, \forall \omega, \forall t \tag{6.27h}$$

$$P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{Min}}_{(i)} u_{(i,\omega,t)} + \sum_{b=1}^{N_B} P^{\mathrm{G}}_{(i,\omega,t,b)}, \qquad \forall i, \forall \omega, \forall t \tag{6.27i}$$

$$0 \leq P^{\mathrm{G}}_{(i,\omega,t,b_1)} \leq P^{\mathrm{Max}}_{(i,b_1)} - P^{\mathrm{Min}}_{(i)}, \qquad \forall i, \forall \omega, \forall t \tag{6.27j}$$

$$0 \leq P^{\mathrm{G}}_{(i,\omega,t,b)} \leq P^{\mathrm{Max}}_{(i,b)} - P^{\mathrm{Max}}_{(i,b-1)}, \qquad \forall i, \forall \omega, \forall t, \forall b = b_2, ..., N_B \tag{6.27k}$$

$$\begin{aligned} \sum_{i=1}^{N_I} P^{\mathrm{G}}_{(i,\omega,t)} = P^{\mathrm{P}}_{(\omega,t)} + \sum_{c_1 \in F^1_{(t)}} P^1_{(c_1)} + \\ + \sum_{c_2 \in F^2_{(t)}} \Big(P^1_{(c_2)} + P^2_{(c_2,\omega)} \Big) + \sum_{o \in O_{(t)}} l_{(o)} y_{(o,\omega)} P_{(o)}, \quad \forall \omega, \forall t \end{aligned} \tag{6.27l}$$

$$\sum_{i=1}^{N_I} (k_{(i,\omega,t)} - 1) P^{\mathrm{Max}}_{(i)} \leq P^{\mathrm{P}}_{(\omega,t)}, \qquad \forall \omega, \forall t \tag{6.27m}$$

$$0 \leq P^1_{(c_1)}, P^1_{(c_2)}, P_{(o)} \qquad \forall c_1, \forall c_2, \forall o \tag{6.27n}$$

$$0 \leq P^1_{(c_2)} + P^2_{(c_2,\omega)}, \qquad \forall c_2, \forall \omega \tag{6.27o}$$

$$\sum_{i=1}^{N_I} P^{\mathrm{G}}_{(i,\omega,t)} + \sum_{o \in O_{(t)} : l_{(o)} = -1} y_{(o,\omega)} P_{(o)} \leq \sum_{i=1}^{N_I} P^{\mathrm{Max}}_{(i)} \qquad \forall \omega, \forall t \tag{6.27p}$$

$$- \Pi_{(\omega)} + \zeta - \eta_{(\omega)} \leq 0, \qquad \forall \omega \tag{6.27q}$$

$$0 \leq \eta_{(\omega)}, \qquad \forall \omega \tag{6.27r}$$

$$u_{(i,\omega,t)} \in \{0, 1\}, \qquad \forall i, \forall \omega, \forall t \tag{6.27s}$$

$$y_{(o,\omega)} \in \{0, 1\}, \qquad \forall o, \forall \omega \tag{6.27t}$$

$$P^{\mathrm{G}}_{(i,\omega,t,b)} = P^{\mathrm{G}}_{(i,\omega',t,b)}, \qquad \forall i, \forall \omega, \omega' : \omega' \in S_{(\omega)}, \forall t = 1, \ldots, N_{T_1}, \forall b \tag{6.27u}$$

$$P^{\mathrm{P}}_{(\omega,t)} = P^{\mathrm{P}}_{(\omega',t)}, \qquad \forall \omega, \omega' : \omega' \in S_{(\omega)}, \forall t = 1, \ldots, N_{T_1} \tag{6.27v}$$

$$P^2_{(c_2,\omega)} = P^2_{(c_2,\omega')}, \qquad \forall c_2, \forall \omega, \omega' : \omega' \in S_{(\omega)} \tag{6.27w}$$

$$y_{(o,\omega)} = y_{(o,\omega')}, \qquad \forall o, \forall \omega, \omega' : \omega' \in S_{(\omega)} \tag{6.27x}$$

$$P^2_{(c_2,\omega)} = P^2_{(c_2,\omega')} \qquad \forall c_2, \forall \omega, \omega' : (k_{(i,\omega,N_{T_1})} = k_{(i,\omega',N_{T_1})}, \forall i \text{ and } \lambda^{\mathrm{P}}_{(\omega,t)} = \lambda^{\mathrm{P}}_{(\omega',t)}, \forall t = 1, ..., N_{T_1}) \tag{6.27y}$$

$$y_{(o,\omega)} = y_{(o,\omega')} \qquad \forall o, \forall \omega, \omega' : (k_{(i,\omega,N_{T_1})} = k_{(i,\omega',N_{T_1})}, \forall i \text{ and } \lambda^{\mathrm{P}}_{(\omega,t)} = \lambda^{\mathrm{P}}_{(\omega',t)}, \forall t = 1, ..., N_{T_1}). \tag{6.27z}$$

Objective function (6.27a) is the CVaR of the profit probability distribution of the producer for a confidence level α^{P}. Equations (6.27b)–(6.27g) determine the total profit achieved by the producer as well as its components, i.e., the profit obtained in the pool, the revenue from forward contracts, the option revenue, and the production cost. The total power produced by each generating unit, which is bounded by constraints (6.27h), is obtained according to equations (6.27i)–(6.27k). The energy balance in each scenario and time step is enforced through constraints (6.27l). The arbitrage among the pool and the derivatives market (forward contracts and options) is avoided using constraints (6.27m)–(6.27p). To maximize the CVaR of the profit distribution, equations (6.27q) and (6.27r) are needed. Constraints (6.27s) and (6.27t) are binary variable declarations. Constraints (6.27u)–(6.27x) are nonanticipativity conditions. Finally, two-stage decisions corresponding to scenarios that share the same unit status at the end of period 1 and the same pool prices throughout that period are forced to be equal by means of constraints (6.27y) and (6.27z).

6.3.8 Maximum Option Price

The proposed stochastic optimization problem (6.27) is designed to identify the optimal power to be traded through forward contracts and options to maximize the CVaR of the profit distribution of a power producer. To solve this problem, both the strike and the option price of the available options are assumed to be known at the beginning of the study horizon.

Notwithstanding this, the *maximum option price* that a power producer is willing to pay for a given option to sell or buy electricity can provide valuable information. To compute the maximum option price of a power producer given its risk aversion level as well as the characterization of the uncertainty affecting the market prices and the unit availability, we solve optimization problem (6.27) twice. Firstly, the optimal objective function value, denoted by $\mathrm{CVaR}^{\mathrm{P}}_{P_{(o)}=0}$, is calculated assuming that no option is available. Secondly, an option with option and strike prices equal to 0 and $\lambda^{\mathrm{S}}_{(o)}$, respectively, is considered and the optimal value of the CVaR in this second case is denoted $\mathrm{CVaR}^{\mathrm{P}}_{\lambda^{\mathrm{O}}_{(o)}=0}$.

Since the CVaR is a coherent measure of risk [140], an increase in the option price causes a proportional decrease in the CVaR. Therefore, it holds

$$\mathrm{CVaR}^{\mathrm{P}}(\lambda^{\mathrm{O}}_{(o)}) = \mathrm{CVaR}^{\mathrm{P}}_{\lambda^{\mathrm{O}}_{(o)}=0} - \lambda^{\mathrm{O}}_{(o)} P_{(o)} L_{(o)}, \tag{6.28}$$

on condition that the term $\lambda^{\mathrm{O}}_{(o)} P_{(o)} L_{(o)}$ is constant. However, the amount of power sold through the option, $P_{(o)}$, is indeed a continuous variable in optimization problem (6.27). Consequently, we limit ourselves to compute the maximum option price for specific values of this variables.

The maximum option price that the power producer is willing to pay for a given option is the one that makes the value of $\mathrm{CVaR}^{\mathrm{P}}(\lambda^{\mathrm{O}}_{(o)})$ equal to that obtained if the option is not available, i.e., $\mathrm{CVaR}^{\mathrm{P}}_{P_{(o)}=0}$. Thus, the maximum option price $\lambda^{\mathrm{O}*}_{(o)}$ is calculated as

$$\begin{aligned}
&\mathrm{CVaR}^{\mathrm{P}}(\lambda^{\mathrm{O}*}_{(o)}) = \mathrm{CVaR}^{\mathrm{P}}_{P_{(o)}=0} \\
&\mathrm{CVaR}^{\mathrm{P}}_{\lambda^{\mathrm{O}}_{(o)}=0} - \lambda^{\mathrm{O}*}_{(o)} P_{(o)} L_{(o)} = \mathrm{CVaR}^{\mathrm{P}}_{P_{(o)}=0} \\
&\lambda^{\mathrm{O}*}_{(o)} = \frac{\mathrm{CVaR}^{\mathrm{P}}_{\lambda^{\mathrm{O}}_{(o)}=0} - \mathrm{CVaR}^{\mathrm{P}}_{P_{(o)}=0}}{P_{(o)} L_{(o)}}.
\end{aligned} \tag{6.29}$$

For a put option, the value of the maximum option price provides information on how much the producer is willing to reduce the price of the energy it sells through the option in order to postpone some decisions until additional information of the uncertain parameters becomes available.

6.4 Option Contracting Examples

This section is devoted to explaining how put and call options reduce both the price and the availability risks faced by power producers using three illustrative examples. In the first one, a non-failing generating unit is considered to show the advantages of selling electricity through a put option, which allows the producer to adapt its decisions according to the evolution of the pool price throughout the study horizon, thus reducing the price risk. In the second example, we analyze how put options can be used by power producers to mitigate its availability risk. Finally, we explore the possibility of reducing the

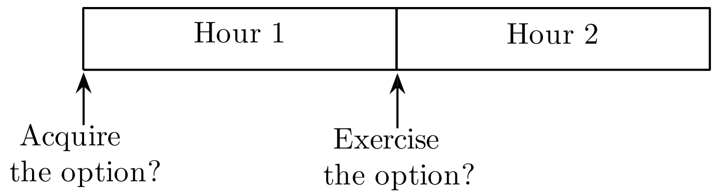


Figure 6.6: Decision framework for option contracting (Examples)

availability risk using a call option to buy electricity in the third example.

The study horizon covers two hours in all cases. Moreover, for both put and call options, the decision framework (depicted in Fig. 6.6) is identical. At the beginning of the study horizon and facing the uncertainty involved in the model for the following two hours, the power producer has to decide whether or not to acquire a given option whose delivery period spans the second hour. Then, depending on the realization of the uncertain parameters during the first hour, the power producer has to decide whether or not to exercise the option at the beginning of the second hour. As explained in Section 6.3.3, this decision framework has to be accounted for when generating the scenario set characterizing the uncertain parameters involved.

Finally, a power producer owning one 100-MW generating unit with a linear cost of 12 €/MWh and zero minimum power output is considered in the three illustrative examples that follow.

6.4.1 Example 1: Put Options Hedging Price Risk

The aim of this first example is to illustrate how put options can reduce the price risk faced by power producers. For the sake of simplicity, a non-failing generating unit is considered.

The variability of the pool price throughout the two-hour study horizon is characterized by the four scenarios depicted in Fig. 6.7, where the price realization is indicated in each branch followed by the associated probability in parentheses.

Note that the price scenario tree above takes into account the decision framework presented in Fig. 6.6 as follows. The pool price during the first hour can adopt two equiprobable values, 23 and 17 €/MWh. Since the power producer has to decide whether or not to exercise the option between the first and the second hour, scenarios representing the pool price during the second hour are generated considering the simulated outcome of the pool price during

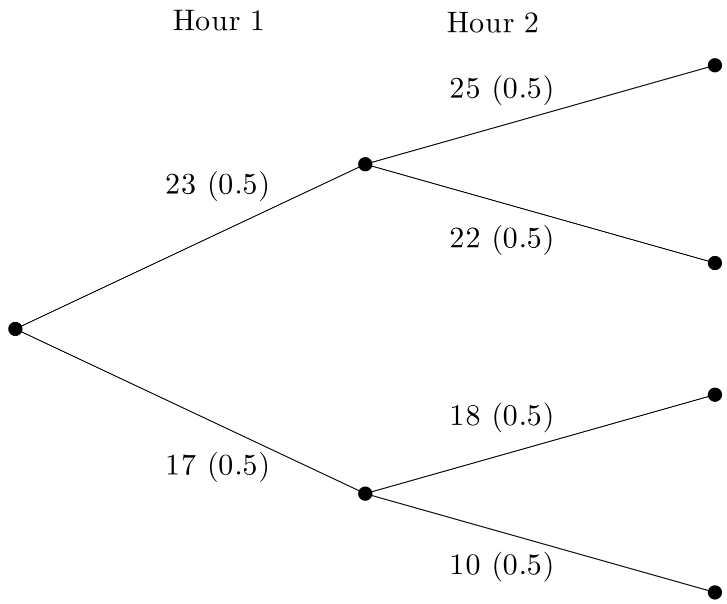


Figure 6.7: Option example 1: pool price scenario tree

Table 6.1: Option example 1: pool prices scenarios and their probabilities

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$(€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$\pi_{(\omega)}$
ω_1	23	25	0.25
ω_2	23	22	0.25
ω_3	17	18	0.25
ω_4	17	10	0.25

the first hour, thus incorporating this information into the scenario generation procedure. As a result, the realization of high/low prices during the first hour gives rise to high/low prices during the second hour, as observed in Fig. 6.7. Table 6.1 contains the four scenarios making up the pool price scenario tree along with their probabilities.

This example is divided into three parts each with a different purpose. In the first part of the example, we compare the profit distribution of the power producer for three different situations depending on whether the power producer sell all its production in the pool, through a forward contract, or through a put option, respectively. Note that, in this first part, the variables representing the energy sold in the pool, through forward contracts, and through options are fixed to appropriate values allowing us to directly determine the producer profit distribution. In the second part of the example, the abovementioned variables are not fixed anymore and optimization problem (6.27) is solved to determine their optimal values for different risk aversion levels. Finally, following the procedure explained in Section 6.3.8, we obtain the maximum option price of the power producer for different values of the risk aversion parameter α^{P}.

In order to highlight the major features of an option as a mechanism to hedge against price risk, we consider the three following cases:

- First, the power producer sells all its production in the pool at variable prices. For example, the profit associated with scenario ω_1 in this case is computed as

$$\Pi_{(\omega_1)} = (\lambda^{\mathrm{P}}_{(\omega_1,t_1)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) + (\lambda^{\mathrm{P}}_{(\omega_1,t_2)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) =$$
$$= (23 \cdot 100 - 12 \cdot 100) + (25 \cdot 100 - 12 \cdot 100) = €2400.$$

  The producer does not sell electricity during these hours in which the pool price happens to be lower than the production cost C. This occurs, for example, in the second hour under scenario ω_4 for which

$$\Pi_{(\omega_4)} = (\lambda^{\mathrm{P}}_{(\omega_4,t_1)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) + 0 = (17 \cdot 100 - 12 \cdot 100) + 0 = €500.$$

- Second, the producer sells its production through a forward contract that spans the second hour of the study horizon. During the first hour, the producer still has to sell its electricity in the pool at variable prices. In order to obtain unbiased results, the price of the forward contract is set to the average pool price during the second hour, i.e., $\lambda_{(c_1)} = 18.75$ €/MWh. Thus, the profit if scenario ω_3 realizes is

$$\Pi_{(\omega_3)} = (\lambda^{\mathrm{P}}_{(\omega_3,t_1)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) + (\lambda_{(c_1)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) =$$
$$= (23 \cdot 100 - 12 \cdot 100) + (18.75 \cdot 100 - 12 \cdot 100) = €1775.$$

- Third, the producer acquires a put option to sell electricity during the second hour. The strike price of the option is also set to the average pool price during the second hour, i.e., $\lambda^{\mathrm{S}}_{(o_1)} = 18.75$ €/MWh. However, regardless of whether or not the option is exercised, the producer has to pay the option price ($\lambda^{\mathrm{O}}_{(o_1)}$), which is assumed here to be equal to 1 €/MWh. For simplicity, the power sold through the put option $P_{(o_1)}$ is fixed to the total capacity of the unit, i.e., 100 MW.

Table 6.2: Option example 1: values of the binary variable $y_{(o,\omega)}$ for each scenario

# scenario	$y_{(o,\omega)}$
ω_1	0
ω_2	0
ω_3	1
ω_4	1

As opposed to selling electricity through forward contracts, which necessarily implies the delivery of the agreed power level, put options allow the producer to postpone its decisions related to the sale of its production until additional information becomes available. The optimal values of the binary variable $y_{(o,\omega)}$ after solving optimization problem (6.27) for $\alpha^{\mathrm{P}} = 0$ (risk-neutral case) if the producer acquires the put option are listed in Table 6.2. Note that if the price during the first hour turns out to be equal to 23 €/MWh, according to the scenario tree in Fig. 6.7, the pool price during the second hour will be necessarily higher than the strike price of the option (18.75 €/MWh) and therefore, the producer optimal decision is not to exercise the option but to sell its production in the pool instead. For example, if scenario ω_2 realizes and the option is not used, the producer profit is

$$\Pi_{(\omega_2)} = (\lambda^{\mathrm{P}}_{(\omega_2,t_1)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) + (\lambda^{\mathrm{P}}_{(\omega_2,t_2)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) - (\lambda^{\mathrm{O}}_{(o_1)} P^{\mathrm{Max}}) =$$
$$= (23 \cdot 100 - 12 \cdot 100) + (22 \cdot 100 - 12 \cdot 100) - (1 \cdot 100) = €2000.$$

Note that even if the option is not exercised, the producer has to pay the option price. In contrast, a pool price equal to 17 €/MWh during the first hour leads to a pool price during the second hour necessarily lower than the strike price of the option (see Fig. 6.7), thus the exercising the option becomes the optimal decision. For example, the profit associated with scenario ω_4 is

$$\Pi_{(\omega_4)} = (\lambda^{\mathrm{P}}_{(\omega_4,t_1)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) + (\lambda^{\mathrm{S}}_{(o_1)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) - (\lambda^{\mathrm{O}}_{(o_1)} P^{\mathrm{Max}}) =$$
$$= (17 \cdot 100 - 12 \cdot 100) + (18.75 \cdot 100 - 12 \cdot 100) - (1 \cdot 100) = €1075.$$

Table 6.3 provides the profit distribution as well as the expected profit of the producer depending on whether its production in the second hour of

Table 6.3: Option example 1: power profit distribution comparison

# scenario	Pool	Forward	Option	$\pi_{(\omega)}$
ω_1	2400	1775	2300	0.25
ω_2	2100	1775	2000	0.25
ω_3	1100	1175	1075	0.25
ω_4	500	1175	1075	0.25
Expected profit (€)	1525	1475	1612.5	

the study horizon is sold in the pool (column 2), through a forward contract (column 3), or through a put option (column 4). Observe that even though the strike price of the put option is equal to the average pool price and the price of the forward contract, the power producer obtains the highest expected profit if its production in the second hour is sold through the put option. This is so because if the put option is acquired and high pool prices are expected to realize (scenarios ω_1 and ω_2 in Table 6.1), the producer makes a profit almost as high as that obtained if the production is directly sold in the pool by not exercising the option. The difference between both profits is due to the cost of the option. On the other hand, if low pool prices are expected to occur (scenarios ω_3 and ω_4 in Table 6.1), the producer exercises the option, thus obtaining a profit almost as high as that achieved if its production is sold through the forward contract. Again, the difference between both profits stems from the cost of the option. Observe that selling electricity through a put option allows the power producer to include additional information (pool price during the first hour) into its decision process to obtain a higher expected profit. Therefore, we can conclude that, despite the additional cost related to the option price, selling electricity through put options may allow power producers to obtain high profits while reducing the price risk involved.

Note that the expected profit improvement achieved by the producer if the option is acquired is due to two reasons. The first reason is the fact that, unlike forward contracts, options themselves allow the producer to postpone its selling decisions. The second reason is that the procedure to generate scenarios that characterize the stochastic process involved (the pool price in this example) can use the information revealed during the first hour to generate more accurate price scenarios for the second hour, i.e., high/low prices during the first hour lead to high/low prices during the second hour. If this condition is not satisfied, postponed decisions would be made with no new information

Table 6.4: Option example 1: optimal values of $P_{(c_1)}$ and $P_{(o)}$ (MW) as a function of the risk aversion level without re-trading of forward contracts

α^{P}	$P_{(c_1)}$(MW)	$P_{(o)}$(MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$(€)	EP(€)
0	0	100	1713	1713
0.5	100	0	1175	1475
0.9	100	0	1175	1475

and therefore, the acquisition of the put option to sell electricity would be pointless.

In the second part of this example, we solve optimization problem (6.27) for the pool price scenario tree in Fig. 6.7 and for different risk aversion levels to determine the optimal power that the producer should sell through a put option with a strike price of 18.75 €/MWh and an option price 1 €/MWh. The price of the forward contract is also set to 18.75 €/MWh.

Table 6.4 provides the optimal power sold through the available forward contract (column 2) and put option (column 3), as well as the CVaR (column 4) and the expected value (column 5) of the producer profit distribution.

Note that the optimal decision for a risk-neutral producer ($\alpha^{\mathrm{P}} = 0$) consists in selling all its production in the second hour of the study horizon through the put option to obtain the highest possible expected profit. However, a risk-averse producer ($\alpha^{\mathrm{P}} = 0.5$ or $\alpha^{\mathrm{P}} = 0.9$), which focuses on maximizing its lowest profits, sells its production through the forward contract, whose price is equal to the strike price of the option. To properly understand this result, we have to be aware of the following two facts:

- The maximization of the Conditional Value-at-Risk of a profit probability distribution for a confidence level α^{P} higher than 0 only takes into account the profits of the $(1 - \alpha^{\mathrm{P}}) \times 100\%$ worst scenarios.

- The only reason why a power producer sells electricity through a put option, paying the additional option price, instead of through a forward contract at the same price is because of the *possibility* of exercising or not the option according to the scenario realization. Therefore, if an acquired put option will be exercised under any scenario realization accounted for in the determination of the CVaR, the power producer can save the option price by selling its production through the forward contract at the same price. Likewise, if the producer will not exercise the option

Table 6.5: Option example 1: maximum option price of the producer as a function of the risk aversion level

α^{P}	$\lambda_{(o)}^{\mathrm{O}*}$ (€/MWh)
0	1.875
0.5	0
0.9	0

under any scenario, its optimal decision consists in not signing such an option.

In this example, the maximization of the CVaR for $\alpha^{\mathrm{P}} = 0.5$ only takes into account the two worst scenarios of the tree in Fig. 6.7, which are characterized by the realization of low pool prices. Note that the pool price during the second hour in these two scenarios is lower than the strike price of the option (18 €/MWh) and therefore, the producer would exercise it under any of these two realizations of the pool price. For this reason, the power producer decides to sell its production through a forward contract at 18.75 €/MWh instead. Observe that this price is 1 €/MWh higher than the price at which its electricity production would be sold if the producer acquired the put option instead, which is computed as the strike price minus the option price, i.e., $18.75 - 1 = 17.75$ €/MWh. For the same reason, if the risk aversion level increases to 0.9, the producer also sells the electricity through the forward contract.

Finally, by means of the procedure presented in Section 6.3.8, we determine the maximum option price of a put option of 100 MW for the three considered risk aversion levels. These prices are provided in Table 6.5. Observe that while a risk-averse producer is not willing to pay any option price to acquire a put option because of the reason presented in the previous paragraph, the maximum option price of a risk-neutral producer rises up to 1.875 €/MWh.

6.4.2 Example 2: Put Options Hedging Availability Risk

In the previous section, we use a non-failing generating unit to illustrate how put options reduce the price risk faced by power producers. In this example, however, we consider that the producer owns a generating unit that is subject to failure with a certain probability distribution, with the aim of illustrating how put options can be used to reduce the availability risk associated with unexpected unit failures.

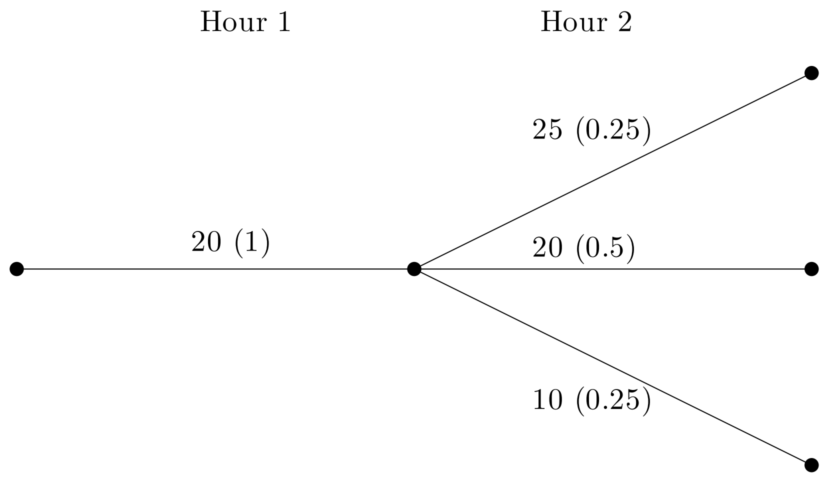


Figure 6.8: Option example 2: pool price scenario tree

Table 6.6: Option example 2: pool prices scenarios and their probabilities

# scenario	$\lambda^{\mathrm{P}}_{(\varpi,t_1)}$(€/MWh)	$\lambda^{\mathrm{P}}_{(\varpi,t_2)}$(€/MWh)	$\pi_{(\varpi)}$
ϖ_1	20	25	0.25
ϖ_2	20	20	0.5
ϖ_3	20	10	0.25

To focus on the availability risk faced by the power producer, for simplicity, the pool price during the first hour of the study horizon is assumed to be known and equal to 20 €/MWh. On the other hand, the pool price during the second hour is characterized by the three different scenarios depicted in Fig. 6.8. Note that unlike the tree in Fig.6.7, since just one scenario is considered to represent the pool price during the first hour, the pool price variability in the second hour that the decision maker observes at the beginning of the study horizon is the same than that observed once the pool price during the first hour realizes. The pool price scenarios and their associated probabilities are provided in Table 6.6.

The MTTF and the MTTR of the generating unit are equal to 2 h and 1 h, respectively. According to these values, Fig. 6.9 provides the availability scenario tree of the unit during the study horizon that results if the unit is assumed to be initially online. The probability of each availability scenario is computed using equation (3.3). A relevant observation that can be made from this figure is the effect of the unit status during the first hour on the probability that the unit is forced out during the second one. That is, the

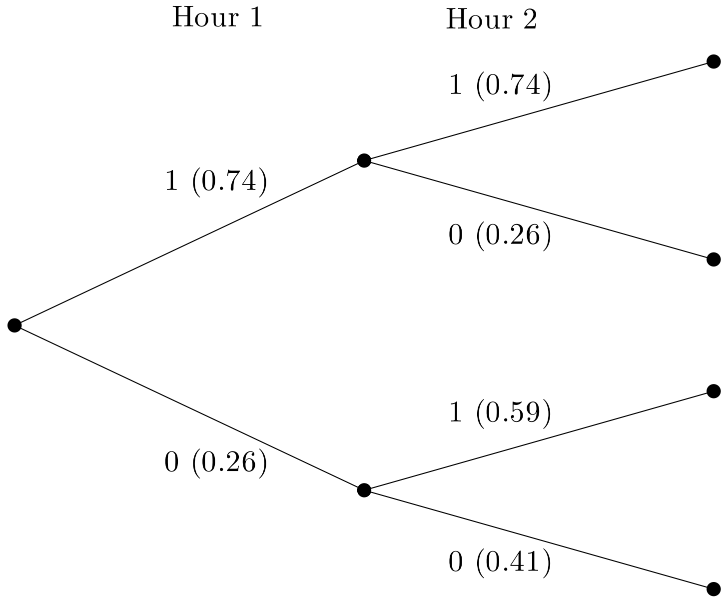


Figure 6.9: Option example 2: availability scenario tree

Table 6.7: Option example 2: availability scenarios and their probabilities

# scenario	$k_{(f,t_1)}$	$k_{(f,t_2)}$	$\pi_{(f)}$
f_1	1	1	0.5476
f_2	1	0	0.1924
f_3	0	1	0.1534
f_4	0	0	0.1066

probability that the unit is forced out during the second hour is equal to 0.26 or 0.41 depending on whether or not the generating unit is available during the first hour, respectively. The four scenarios characterizing the availability of the generating unit as well as their corresponding probabilities are given in Table 6.7.

Table 6.8 provides the scenario tree obtained by combining the three pool price scenarios with the four possible availability scenarios. In the first six scenarios the unit is available during the first hour, and it is not for the last six scenarios.

This example is also divided into three parts. In the first part we illustrate how put options can be used to hedge against the availability risk of the generating unit by considering three different cases. In the second one, we solve optimization problem (6.27) to determine the optimal power sold through forward contracts and options. Finally, the maximum option price that the power producer is willing to pay for a put option if unit failures are considered

Table 6.8: Option example 2: pool price and availability scenarios with their probabilities

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$(€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$k_{(\omega,t_1)}$	$k_{(\omega,t_2)}$	$\pi_{(\omega)}$
ω_1	20	25	1	1	0.13690
ω_2	20	25	1	0	0.04810
ω_3	20	20	1	1	0.27380
ω_4	20	20	1	0	0.09620
ω_5	20	10	1	1	0.13690
ω_6	20	10	1	0	0.04810
ω_7	20	25	0	1	0.03835
ω_8	20	25	0	0	0.02665
ω_9	20	20	0	1	0.07670
ω_{10}	20	20	0	0	0.05330
ω_{11}	20	10	0	1	0.03835
ω_{12}	20	10	0	0	0.02665

is determined.

Next, we describe the three different situations used to highlight the advantages of selling electricity through a put option to reduce the availability risk faced by a producer:

- In the first case, the producer sells its production exclusively in the pool. Therefore, to compute the profit obtained by the producer we have to take into account that no production can be sold in the pool if the generating unit suffers from a forced outage. As an example, the profit associated with scenario ω_7 is computed as

$$\Pi_{(\omega_7)} = (\lambda^{\mathrm{P}}_{(\omega_7,t_1)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) k_{(\omega_7,t_1)} + (\lambda^{\mathrm{P}}_{(\omega_7,t_2)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) k_{(\omega_7,t_2)} =$$
$$= (20 \cdot 100 - 12 \cdot 100) \cdot 0 + (25 \cdot 100 - 12 \cdot 100) \cdot 1 = €1300.$$

- In the second case, the producer sells its production through a forward contract that comes into effect in the second hour and whose fixed price is set to the average pool price during this hour, i.e., $\lambda_{(c_1)} = 18.75$ €/MWh. Then, if the unit is forced out during the second hour, the producer has to buy the energy in the pool to meet its contracting selling obligation.

Table 6.9: Option example 2: values of the binary variable $y_{(o,\omega)}$ for each scenario

# scenario	$y_{(o,\omega)}$
ω_1	1
ω_2	1
ω_3	1
ω_4	1
ω_5	1
ω_6	1
ω_7	0
ω_8	0
ω_9	0
ω_{10}	0
ω_{11}	0
ω_{12}	0

For example, the producer profit obtained if scenario ω_2 realizes is

$$\Pi_{(\omega_2)} = (\lambda^{\mathrm{P}}_{(\omega_2,t_1)}P^{\mathrm{Max}} - CP^{\mathrm{Max}})k_{(\omega_2,t_1)} + (\lambda_{(c_1)}P^{\mathrm{Max}} - ((1-k_{(\omega_2,t_2)})C+ \\ + k_{(\omega_2,t_2)} \cdot \lambda^{\mathrm{P}}_{(\omega_2,t_2)})P^{\mathrm{Max}}) = (20 \cdot 100 - 12 \cdot 100) \cdot 1 + (18.75 \cdot 100- \\ - 25 \cdot 100) = €175.$$

- In the third case, the producer acquires a put option to sell electricity during the second hour. The strike price of the option ($\lambda^{\mathrm{S}}_{(o_1)}$) is equal to the average pool price during the second hour, i.e., 18.75 €/MWh. Moreover, the option price ($\lambda^{\mathrm{O}}_{(o_1)}$) is set to 0.1 €/MWh.

  Since the pool price during the first hour is characterized by just one single scenario, the producer has to decide whether or not to exercise the option based solely on the unit status at the end of the first hour and its influence on the unit availability during the rest of the study horizon. By solving optimization problem (6.27) for $\alpha^{\mathrm{P}} = 0.5$, we obtain the optimal values of the binary variable $y_{(o,\omega)}$ provided in Table 6.9, which determines whether or not the put option is exercised depending on the scenario realization.

  Observe that the producer exercises the put option in scenarios $\omega_1, \ldots, \omega_6$. In these scenarios the unit is online during the first hour and as a result, the probability that the unit is also available during the second hour is higher than in scenarios $\omega_7, \ldots, \omega_{12}$. Thus, the profit obtained if, for example, scenario ω_5 realizes is

$$\Pi_{(\omega_5)} = (\lambda^{\mathrm{P}}_{(\omega_5,t_1)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) k_{(\omega_5,t_1)} + (\lambda^{\mathrm{S}}_{(o_1)} P^{\mathrm{Max}} - ((1 - k_{(\omega_5,t_2)}) C + \\ + k_{(\omega_5,t_2)} \cdot \lambda^{\mathrm{P}}_{(\omega_5,t_2)}) P^{\mathrm{Max}}) - \lambda^{\mathrm{O}}_{(o_1)} P^{\mathrm{Max}} = (20 \cdot 100 - 12 \cdot 100) \cdot 1 + \\ + (18.75 \cdot 100 - 12 \cdot 100) - 0.1 \cdot 100 = €1465.$$

If the option is exercised, note that the profit is equal to that obtained with the forward contract minus the option cost.

On the other hand, an unexpected failure happening during the first hour increases the probability that the unit is forced out during the second hour. Therefore, the producer does not exercise the option in these cases (scenarios $\omega_7, \ldots, \omega_{12}$) to reduce the probability of experiencing low profits if high pool prices realize and the unit is indeed forced out. As an example, the profit in scenario ω_8 is

$$\Pi_{(\omega_8)} = (\lambda^{\mathrm{P}}_{(\omega_8,t_1)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) k_{(\omega_8,t_1)} + (\lambda^{\mathrm{P}}_{(\omega_8,t_2)} P^{\mathrm{Max}} - CP^{\mathrm{Max}}) k_{(\omega_8,t_2)} - \\ - \lambda^{\mathrm{O}}_{(o_1)} P^{\mathrm{Max}} = (20 \cdot 100 - 12 \cdot 100) \cdot 0 + (25 \cdot 100 - 12 \cdot 100) \cdot 0 - \\ - 0.1 \cdot 100 = -€10.$$

Table 6.10 includes, for each scenario, the profit obtained if the production is sold in the pool (column 2), through a forward contract spanning the second hour (column 3), or through a put option also spanning the second hour (column 4). The last column contains the probability of each scenario. The two last rows provide the expected profit and the CVaR of the profit distribution, respectively, for a risk aversion level equal to 0.5.

From this table we observe that the highest expected profit is achieved if the producer sells its production in the pool. However, a risk-averse producer would sell its production through the put option in order to obtain the highest CVaR of the profit distribution. In doing so, the producer takes advantage of selling electricity at a fixed price if the probability that the unit is available is high, thus decreasing the price risk. Additionally, the producer reduces the availability risk by not exercising the option if the probability that the unit is forced out during the second hour is large enough.

For completeness, Fig. 6.10 shows the probability mass function of the producer if its production is sold in the pool (Fig. 6.10(a)), through the forward contract (Fig. 6.10(b)), or through the put option (Fig. 6.10(c)). By comparing

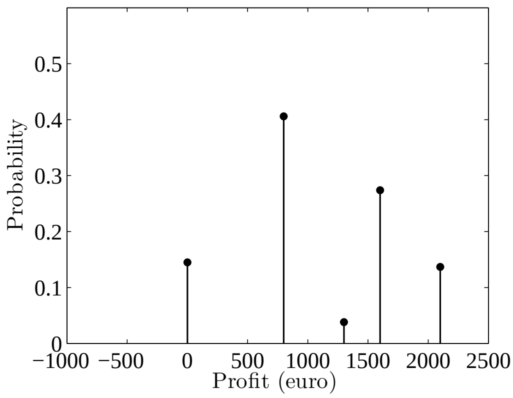
Table 6.10: Option example 2: power profit distribution comparison

# scenario	Pool	Forward	Option	$\pi_{(\varpi)}$
ω_1	2100	1475	1465	0.13690
ω_2	800	175	165	0.04810
ω_3	1600	1475	1465	0.27380
ω_4	800	675	665	0.09620
ω_5	800	1475	1465	0.13690
ω_6	800	1675	1665	0.04810
ω_7	1300	675	1290	0.03835
ω_8	0	−625	−10	0.02665
ω_9	800	675	790	0.07670
ω_{10}	0	−125	−10	0.05330
ω_{11}	0	675	−10	0.03835
ω_{12}	0	875	−10	0.02665
Expected profit (€)	**1100.2**	1065.2	1062.8	
$\text{CVaR}_{\alpha^\text{P}=0.5}$ (€)	568.1	636.1	**641.5**	

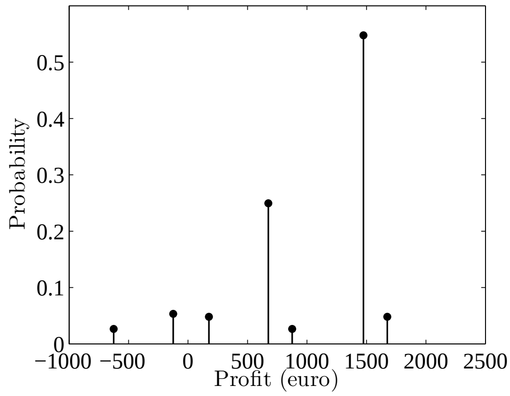
Figs.6.10(a) and 6.10(b) we infer that while selling electricity through a forward contract increases the probability of experiencing profits higher than €1000, it also involves the possibility of experiencing negative profits due to unit failures because the producer has the obligation to deliver a certain amount of electricity. In contrast, the acquisition of a put option to sell electricity allows the producer to eliminate the possibility of experiencing negative profits without reducing significantly the probability of achieving high profits. Note that selling electricity through the put option leads to the probability distribution with the lowest *risk* if understood as profit variability.

While the previous analysis is carried out considering that the producer sells all its production in the pool, through a forward contract or through a put option, in the following, we present the results obtained if optimization problem (6.27) is solved to determine the optimal mix of pool, forward contracts, and options. Table 6.11 provides the optimal power sold through the forward contract and the put option as well as the CVaR and the expected profit of the producer for three values of the risk aversion parameter.

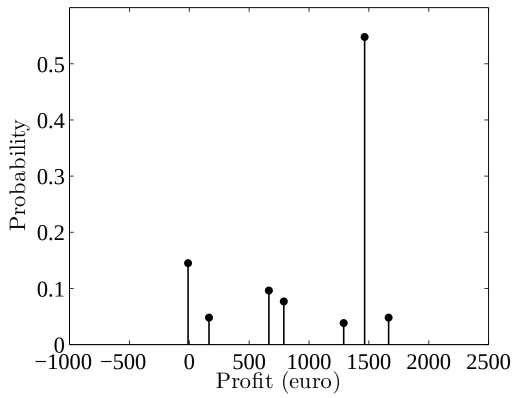
Due to the possibility of suffering from unexpected unit failures, observe that a risk-neutral producer sells all its production in the pool to maximize the expected profit. On the contrary, for $\alpha^\text{P} = 0.5$, the producer acquires the put option to decide, according to the status of its generating unit during the first hour, whether or not to sell all its production at a fixed price during the



(a) Electricity sold in the pool



(b) Electricity sold through a forward contract



(c) Electricity sold through a put option

Figure 6.10: Option example 2: probability mass function of the producer profit

Table 6.11: Option example 2: optimal values of $P_{(c_1)}$ and $P_{(o)}$ (MW) as a function of the risk aversion level

α^{P}	$P_{(c_1)}$(MW)	$P_{(o)}$(MW)	$\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$(€)	EP(€)
0	0	0	1100	1100
0.5	0	100	641	1063
0.9	0	0	0	1100

Table 6.12: Option example 2: maximum option price of the producer as a function of the risk aversion level

α^{P}	$\lambda^{\mathrm{O}*}_{(o)}$ (€/MWh)
0	0
0.5	0.153
0.9	0

second hour. Finally, a risk aversion level equal to 0.9 means that the power producer seeks to maximize the 10% of the lowest profits of the distribution, which in this case are caused by a generating unit failure spanning the whole study horizon. Therefore, and since the put option will not be exercised in any of these cases, the producer sells its production in the pool to avoid the possibility of having low profits caused by the availability risk.

For completeness, Table 6.12 contains the maximum option price that the producer is willing to pay for a put option of 100 MW for each considered risk aversion level. Note that both a risk-neutral ($\alpha^{\mathrm{P}} = 0$) and a highly risk-averse ($\alpha^{\mathrm{P}} = 0.9$) power producer are not willing to pay any quantity higher than zero for the put option. This is so because, even if the option price is equal to zero, both the expected profit and the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ of the profit distribution reach their maximum values if the electricity produced during the second hour is directly sold in the pool. In contrast, for $\alpha^{\mathrm{P}} = 0.5$, the maximum value of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.5}$ is obtained if the producer acquires the put option to sell electricity, then being its maximum option price higher than zero.

In summary, the example above shows how a put option to sell electricity allows the producer, in exchange for the option price, to update the forecast of its unit availability and to decide whether its production should be sold in the pool at uncertain prices or through an option at a fixed price, with the ultimate objective of maximizing the CVaR of its profit distribution.

6.4.3 Example 3: Call Options Hedging Availability Risk

In the previous two examples, put options to sell electricity are used by power producers to reduce both price and availability risks, respectively. In this example, however, we consider a call option that allows the power producer, in exchange for the option cost, to decide whether or not to buy electricity at the strike price of the option during the second hour of the study horizon. Exercising the option depends on the realization of the stochastic processes

Table 6.13: Option example 3: pool price and availability scenarios with their probabilities

# scenario	$\lambda^{\mathrm{P}}_{(\omega,t_1)}$(€/MWh)	$\lambda^{\mathrm{P}}_{(\omega,t_2)}$(€/MWh)	$k_{(\omega,t_1)}$	$k_{(\omega,t_2)}$	$\pi_{(\omega)}$
ω_1	23	25	1	1	0.13690
ω_2	23	25	1	0	0.04810
ω_3	23	22	1	1	0.13690
ω_4	23	22	1	0	0.04810
ω_5	17	18	1	1	0.13690
ω_6	17	18	1	0	0.04810
ω_7	17	10	1	1	0.13690
ω_8	17	10	1	0	0.04810
ω_9	23	25	0	1	0.03835
ω_{10}	23	25	0	0	0.02665
ω_{11}	23	22	0	1	0.03835
ω_{12}	23	22	0	0	0.02665
ω_{13}	17	18	0	1	0.03835
ω_{14}	17	18	0	0	0.02665
ω_{15}	17	10	0	1	0.03835
ω_{16}	17	10	0	0	0.02665

during the first hour, i.e., the pool price and the unit availability. In order to avoid the arbitrage between different markets and for clarity, the sum of the option power level (if exercised) plus the power produced by the generating unit is bounded to its maximum capacity. This way, we prevent that the producer can purchase and exercise a call option with the intention of making a profit by selling the agreed electricity in the pool or through a forward contract instead of using it to hedge against unexpected unit failures.

To characterize the pool price during the two-hour horizon, the four price scenarios depicted in Fig. 6.7 are considered. Likewise, the MTTF and the MTTR of the generating unit are equal to 2 h and 1 h, respectively, thus giving rise to the availability scenario set represented in Fig. 6.9. The complete scenario tree obtained by combining all price scenarios with all availability scenarios is characterized in Table 6.13.

Note that a producer that sells its production in the pool during the two hours of the study horizon does not have any contracting obligation to sell electricity. Therefore, a unit failure happening during the second hour only implies that the producer does not sell electricity in the pool, then obtaining a profit equal to €0 during that hour. In contrast, as previously discussed, selling electricity through a forward contract necessarily involves the purchase of the agreed energy in the pool during those time steps in which the generating unit is forced out. For this reason, in this example, we consider a power producer that sells its production through a forward contract spanning the second hour, whose price is fixed to the average pool price during this period (18.75 €/MWh), and that evaluates the possibility of acquiring a call option to reduce the financial losses associated with unexpected unit failures. In doing so, if scenarios characterized by unit failures and high pool prices are likely, the producer can exercise the option to buy electricity at a lower price. In this example, the strike price of the option is set to 15 €/MWh (a price higher than the production cost C and lower than the price of the forward contract) and its option price is equal to 0.1 €/MWh.

Firstly, we consider two different cases to highlight the main features of a call option to hedge against the availability risk. Secondly, we compute the maximum price that the power producer is willing to pay for a call option for different risk aversion levels.

The two cases considered in this example are described next in detail.

- In the first case, the producer sells its production during the second hour of the study horizon through a forward contract at 18.75 €/MWh. As an example of this situation, the profit of scenario ω_2 is

$$\begin{aligned}\Pi_{(\omega_2)} &= (\lambda^{\mathrm{P}}_{(\omega_2,t_1)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) k_{(\omega_2,t_1)} + (\lambda_{(c_1)} P^{\mathrm{Max}} - ((1 - k_{(\omega_2,t_2)}) C + \\ &+ k_{(\omega_2,t_2)} \cdot \lambda^{\mathrm{P}}_{(\omega_2,t_2)}) P^{\mathrm{Max}}) = (23 \cdot 100 - 12 \cdot 100) \cdot 1 + (18.75 \cdot 100 - \\ &- 25 \cdot 100) = €475.\end{aligned}$$

- In the second case, besides signing the same forward contract to sell electricity, the producer acquires a call option to have the possibility of buying electricity at 15 €/MWh during the second hour, with an option price equal to 0.1 €/MWh. Table 6.14 provides the values of the binary variable $y_{(o,\omega)}$ obtained by solving optimization problem (6.27) for three values of the risk aversion parameter.

  It is worth mentioning that in this second case the call option is exercised depending on the risk aversion level of the producer. In particular, if the producer is risk neutral the call option is only exercised if the pool price during the first hour is high and an unit failure occurs ($\omega_9, \ldots, \omega_{12}$). For $\alpha^{\mathrm{P}} = 0.5$, the producer exercises the call option if, independently of the unit status, high pool prices realize during the first hour ($\omega_1, \ldots, \omega_4$ and

Table 6.14: Option example 3: values of the binary variable $y_{(o,\omega)}$ for each scenario

# scenario	$\alpha^{\mathrm{P}} = 0$	$\alpha^{\mathrm{P}} = 0.5$	$\alpha^{\mathrm{P}} = 0.9$
ω_1	0	1	0
ω_2	0	1	0
ω_3	0	1	0
ω_4	0	1	0
ω_5	0	0	0
ω_6	0	0	0
ω_7	0	0	0
ω_8	0	0	0
ω_9	1	1	1
ω_{10}	1	1	1
ω_{11}	1	1	1
ω_{12}	1	1	1
ω_{13}	0	0	1
ω_{14}	0	0	1
ω_{15}	0	0	1
ω_{16}	0	0	1

$\omega_9, \ldots, \omega_{12}$). Finally, for $\alpha^{\mathrm{P}} = 0.9$, the call option is exercised if the generating unit suffers from an unexpected failure during the first hour ($\omega_8, \ldots, \omega_{16}$). As an example, we compute the profit in scenario ω_{12} since, for all risk aversion levels, the call option is exercised in this scenario, i.e.,

$$\Pi_{(\omega_{12})} = (\lambda^{\mathrm{P}}_{(\omega_{12},t_1)} P^{\mathrm{Max}} - C P^{\mathrm{Max}}) k_{(\omega_{12},t_1)} + (\lambda_{(c_1)} P^{\mathrm{Max}} - \lambda^{\mathrm{S}}_{(o_1)} P^{\mathrm{Max}}) -$$
$$- \lambda^{\mathrm{O}}_{(o_1)} P^{\mathrm{Max}} = (23 \cdot 100 - 12 \cdot 100) \cdot 0 + (18.75 \cdot 100 - 15 \cdot 100) -$$
$$- 0.1 \cdot 100 = €365.$$

Note that if the option is not exercised, the profit of the producer in the second case is equal to that obtained in the first case minus the cost of the option.

Table 6.15 provides the profit distribution of the producer depending on whether the producer acquires the option or not. Note that if the producer signs the forward contract to sell electricity and does not acquires the call option, its profit is independent of the risk aversion level. On the other hand, the acquisition of the call option allows the producer to adapt its profit distribution according to its risk aversion level. Therefore, Table 6.15 includes

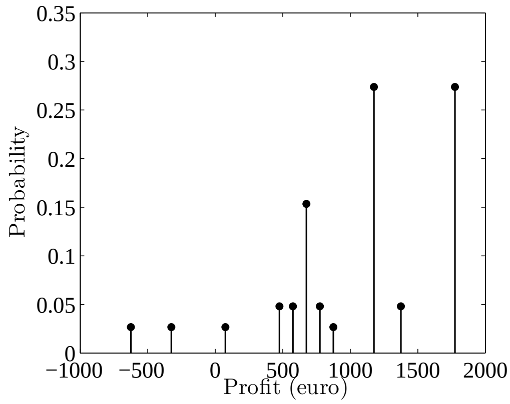
Table 6.15: Option example 3: power profit distribution comparison

# scenario	Forward	Option			$\pi_{(\varpi)}$
		$\alpha^{\mathrm{P}}=0$	$\alpha^{\mathrm{P}}=0.5$	$\alpha^{\mathrm{P}}=0.9$	
ω_1	1775	1765	1465	1765	0.13690
ω_2	475	465	1465	465	0.04810
ω_3	1775	1765	1465	1765	0.13690
ω_4	775	765	1465	765	0.04810
ω_5	1175	1165	1165	1165	0.13690
ω_6	575	565	565	565	0.04810
ω_7	1175	1165	1165	1165	0.13690
ω_8	1375	1365	1365	1365	0.04810
ω_9	675	365	365	365	0.03835
ω_{10}	-625	365	365	365	0.02665
ω_{11}	675	365	365	365	0.03835
ω_{12}	-325	365	365	365	0.02665
ω_{13}	675	665	665	365	0.03835
ω_{14}	75	65	65	365	0.02665
ω_{15}	675	665	665	365	0.03835
ω_{16}	875	865	865	365	0.02665
Expected profit (€)	1065.2	**1077.5**	1077.1	1049.1	
$\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.5}$ (€)	607.6	642.1	**747.9**	585.5	
$\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ (€)	-137.9	285.1	285.1	**365**	

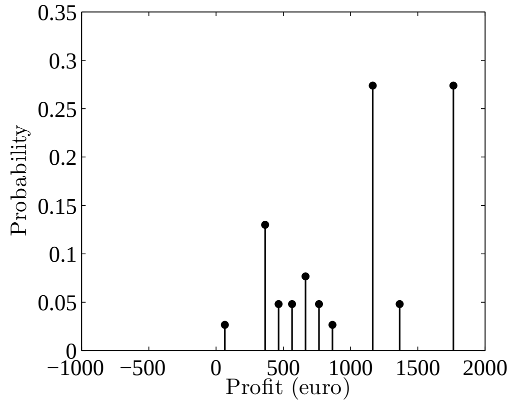
three additional profit distributions obtained by solving optimization problem (6.27) for three different risk aversion levels.

First, note that for $\alpha^{\mathrm{P}} = 0$ the average profit of the scenarios associated with high pool prices and the unit failure (ω_9, ω_{10}, ω_{11}, and ω_{12}) increases if compared with the situation in which the call option is not available. The profit for the rest of the scenarios is, however, equal to that obtained if the call option is not available minus the cost of the option, i.e., 0.1 €/MWh $\times$ 100 MW $\times$ 1 h. Likewise, if the risk aversion level increases up to 0.5, the exercise of the option for those scenarios characterized by high pool prices during period 1 allows the producer to obtain the highest $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.5}$. Finally, the maximum $\mathrm{CVaR}_{\alpha^{\mathrm{P}}=0.9}$ is obtained if the producer exercises the option based on the availability of its generating unit.

The probability mass functions of the producer profit for $\alpha^{\mathrm{P}} = 0$ with and without a call option to buy electricity are plotted in Fig. 6.11. We can observe that the call option eliminates the possibility of having negative profits without significantly changing the probabilities of occurrence of the highest profits of



(a) Electricity sold through a forward contract without call option



(b) Electricity sold through a forward contract with call option

Figure 6.11: Option example 3: probability mass function of the producer profit ($\alpha^{\mathrm{P}} = 0$)

Table 6.16: Option example 3: maximum option price of the producer as a function of the risk averion level

α^{P}	$\lambda^{\mathrm{O}*}_{(o)}$ (€/MWh)
0	0.223
0.5	1.504
0.9	5.130

the distribution.

Additionally, according to Section 6.3.8, we determine the maximum option price that the producer is willing to pay for a call option of 100 MW with a strike price of 15 €/MWh for three different risk aversion levels. This maximum option price is provided in Table 6.16. Observe that an increase in the risk aversion parameter translates into a higher concern of experiencing low profits caused by unexpected unit failures and therefore, the maximum option price of the producer increases.

In short, this example illustrates that the acquisition of a call option can reduce the availability risk of generating units by allowing a power producer with selling contracting obligations to buy electricity at a fixed price if there exists a high probability of suffering from unexpected unit failures.

Table 6.17: Option case study data: piecewise linear production cost function of the generating unit

# block	$P_{(b)}^{\text{Max}}$ (MW)	$\lambda_{(b)}$ (€/MWh)
1	140.0	10.08
2	227.5	10.66
3	280.0	11.09
4	350.0	11.72

Table 6.18: Option case study data: forced outage parameters

FOR (%)	MTTF (h)	MTTF (h)
5	950	50
10	450	50

6.5 Option Contracting Case Study

In this section, we test the proposed mid-term decision-making model on a 8-week case study that is divided into two parts and that allows us to analyze how put and call options, respectively, are used to manage the risk faced by power producers.

6.5.1 Data

We consider a power producer owning a single generating unit with the same technical and economic characteristics to those indicated in Chapters 4 and 5. This generating unit has maximum and minimum power outputs of 350 MW and 50 MW, respectively, and its production cost is characterized by the piecewise linear function described in Table 6.17.

The evaluation of electricity options is carried out considering three forced outage rates, namely, 0%, 5%, and 10%, whose parameters are provided in Table 6.18.

All the options considered in this case study are European options with a delivery period spanning the last four weeks of the decision horizon, i.e., the producer has to decide whether or not to acquire them at the beginning of the 8 weeks, while its exercise is decided at the end of the fourth week.

As discussed in Section 6.3.3, the decision pattern of the proposed multi-stage stochastic programming model needs to be accounted for in the scenario generation procedure. For example, to model unexpected unit failures, first we generate an initial set of 2000 scenarios representing the availability of the unit during the first four weeks according to the algorithm stated Section 3.5 of Chapter 3, which is subsequently and conveniently reduced to 10 availability scenarios. Then, for each one of these 10 scenarios, we generate a second group of 2000 availability scenarios for the last four weeks of the 8-week horizon using the unit status of the last hour of the fourth week as the initial status of the unit in this second phase of the scenario generation process. Likewise, each scenario set of 2000 scenarios is conveniently reduced to 3 scenarios, thus giving rise to a final tree of 30 availability scenarios.

Analogously, a set of 30 pool price scenarios is generated. First, historical pool price data of the electricity market of the Iberian Peninsula during the year 2000 are used to adjust the parameters of an ARIMA model following the procedure explained in Section 3.4 of Chapter 3. Then, an initial set of 500 scenarios representing the pool price during the first four weeks is first generated and then reduced to 10 using scenario reduction techniques. Next, for each one of these 10 scenarios, the "simulated" pool price outcomes during the first four weeks are included in the ARIMA model to generate a second set of 200 pool price scenarios for the last four weeks of the study horizon. Each one of these 10 scenario sets is reduced from 200 to 3 pool price scenarios to obtain a final set of 30 scenarios. Additionally, for tractability, hourly pool prices are aggregated in 3 prices per day using the procedure explained in Appendix A.

Two forward contracts that span the first four weeks and the last four weeks, respectively, are considered. Besides, the power producer can acquire either a put or a call option to sell electricity at their corresponding strike and option prices during the last four weeks. The price of the forward contract spanning the first four weeks is equal to 22 €/MWh. The price of the forward contract spanning the last four weeks at the beginning of the study horizon is set to the value of the strike price of the option in order to compare both derivatives on equal terms. Notwithstanding this, different strike and option prices are considered in the course of this case study in order to analyze all possible situations.

An approximate scenario set representing the forward prices at stage 2 is generated based on the fact that forward prices tend to equal pool prices as maturity date approaches. To this end, we first calculate the average value of

Table 6.19: Option case study data: forward price scenarios

$\lambda^2_{(c_2,\omega)}$ (€/MWh)	20.69	22.42	24.15
Probability	0.25	0.5	0.25

each one of the 30 generated pool price scenarios over the last four weeks as

$$\overline{\lambda^{\mathrm{P}}_{(\omega)}} = \frac{\sum_{t=N_{T_1+1}}^{N_T} \lambda^{\mathrm{P}}_{(\omega,t)} L_{(t)}}{N_T - N_{T_1}}. \tag{6.30}$$

Next, taking into account the probability of each scenario $\pi_{(\omega)}$, the expected value and the standard deviation of those average pool prices, denoted by ρ and ϑ respectively, are determined as follows

$$\rho = \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \overline{\lambda^{\mathrm{P}}_{(\omega)}} \tag{6.31}$$

$$\vartheta = \sqrt{\sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \left(\overline{\lambda^{\mathrm{P}}_{(\omega)}} - \rho\right)^2}. \tag{6.32}$$

Three forward prices are considered, namely, $\rho - \vartheta$, ρ, and $\rho + \vartheta$. Table 6.19 provides the forward price scenarios and their corresponding probabilities. If needed, forecasting techniques can be used for generating forward price scenarios in a more precise manner.

If the re-trading of forward contracts spanning the last four weeks is not allowed, the scenario tree is built by combining the 30 pool price scenarios with the 30 availability scenarios, then obtaining a set of 900 scenarios. On the other hand, if these forward contracts can be re-traded at stage 2 at uncertain prices, the final tree of 2700 scenarios is created by combining the 30 pool price scenarios with the 30 availability scenario and with the 3 forward price scenarios.

6.5.2 Case Study 1: Put Option

The purpose of this case study is twofold. Firstly, we compare the profit made by a power producer that sells electricity through a forward contract spanning the last four weeks with the profit obtained if a put option for the same period is acquired. The advantages of the put option vs. the forward contract are highlighted for different FOR and risk aversion levels (α^{P}). Secondly, the

Table 6.20: Option case study 1: values of the $\text{CVaR}_{\alpha^{\text{P}}}$ (€million) for different levels of FOR and risk aversion depending on whether the production is sold through a forward contract or through a put option

α^{P}	FOR = 0%		FOR = 5%		FOR = 10%	
	Case (a)	Case (b)	Case (a)	Case (b)	Case (a)	Case (b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499

maximum option price that the power producer is willing to pay for a given put option is calculated for different strike prices, FOR, and risk aversion levels.

6.5.2.1 Option vs. Forward Contract

In order to analyze the advantages of buying a put option to eventually sell electricity versus selling electricity through a forward contract, two different cases are considered below. In case (a), the power producer sells its production through a forward contract spanning the last four weeks at 21 €/MWh, i.e, we solve optimization problem (6.27) fixing variables $P^1_{(c2)}$ and $P_{(o)}$ to 350 MW and 0 MW, respectively. In case (b), the producer purchases a put option to sell its production during the last four weeks at a strike price equal to 21 €/MWh, i.e., optimization problem (6.27) is solved fixing variables $P^1_{(c2)}$ and $P_{(o)}$ to 0 MW and 350 MW, respectively. In this case, the option price is set to 0.1 €/MWh to highlight the advantages and disadvantages of selling electricity through either a forward contract or an option. Table 6.20 provides the $\text{CVaR}_{\alpha^{\text{P}}}$ of the producer profit distribution for cases (a) and (b), different FOR values, and risk aversion levels α^{P}. In this first analysis, for simplicity, re-trading of forward contracts at stage 2 is not allowed.

Observe that for eight out of the nine instances presented in Table 6.20, the $\text{CVaR}_{\alpha^{\text{P}}}$ of the power producer if the put option is purchased is higher than that obtained if the forward contract is signed. Note, however, that these CVaR values may significantly change as a function of the option price.

Next, we separately analyze these results depending on whether or not unit failures are taken into account. If a non-failing generating unit is considered (FOR = 0%, second column block of Table 6.20) and the power producer is risk neutral ($\alpha^{\text{P}} = 0$), the CVaR increases 6.5% from case (a) to (b) due to the fact that the producer can decide, at the end of the first four weeks, whether

Table 6.21: Option case study 1: option exercise for $\alpha^{\mathrm{P}} = 0$ and FOR = 0%

$E_2\{\lambda^{\mathrm{P}}_{(\omega,t)}\}$ (€/MWh)	22.41	22.58	22.64	20.97	24.39	21.85	22.35	20.28	26.01	22.04
$y_{(o,\omega)}$	0	0	0	1	0	0	0	1	0	0

or not to exercise the option according to the pool price forecasts for the last four weeks.

To give additional insight into the exercising of the put option by a risk-neutral power producer owning a non-failing generating unit (FOR = 0%), Table 6.21 indicates in the first row the average value of the pool price over the last four weeks depending on each possible realization of the pool price during the first four weeks (ten alternative values). These values are computed as

$$E_2\{\lambda^{\mathrm{P}}_{(\omega,t)}\} = \frac{1}{\sum_{\omega' \in S_{(\omega)}} \pi_{(\omega')}} \frac{1}{N_T - N_{T_1}} \sum_{\omega' \in S_{(\omega)}} \sum_{t=N_{T_1}+1}^{N_T} \pi_{(\omega')} \lambda^{\mathrm{P}}_{(\omega',t)} L_{(t)}.$$

The second row of this table provides the corresponding value of the binary variable $y_{(o,\omega)}$, which represents whether or not the producer exercises the option. Since a risk-neutral power producer seeks to maximize the expected profit, the option is only exercised in the cases in which the average pool price for the last four weeks ($E_2\{\lambda^{\mathrm{P}}_{(\omega,t)}\}$) is lower than the strike price (21 €/MWh). If the average pool price is higher than the strike price, the power producer makes a higher profit by selling its electricity directly in the pool and therefore, the option is not exercised.

To analyze the results provided in Table 6.20 for a risk-averse power producer ($\alpha^{\mathrm{P}} = 0.5$ and $\alpha^{\mathrm{P}} = 0.9$) owning a non-failing generating unit (FOR = 0%) we have to keep in mind two important points. One is that a risk-averse power producer pursues maximizing the lowest profits of its distribution, and the second one is that if unit failures are disregarded, the power producer only faces the uncertainty related to pool prices. Thus, from a sufficiently high risk-aversion level and a FOR equal to 0%, both the forward contract and the option become attractive to the power producer as financial instruments to hedge against this price risk. Observe that in that case, it may become more profitable just selling electricity through the forward contract rather than through the option so as to save the nonrefundable fee of the latter. In this line, note in Table 6.20 that for a value of α^{P} equal to 0.5, the CVaR obtained in case

Table 6.22: Option case study 1: values of the binary variable $y_{(o,\omega)}$ for $\alpha^{\mathrm{P}} = 0.5$ and FOR = 5%

$E_2\{\lambda^{\mathrm{P}}_{(\omega,t)}\}$ (€/MWh)	$k_{(\omega,N_{T_1})}$									
	1	0	1	1	1	1	1	1	1	1
26.01	0	0	0	0	0	0	0	0	0	0
20.97	1	0	1	1	1	1	1	1	1	1
20.28	1	1	1	1	1	1	1	1	1	1

(b) is only 0.8% higher than the CVaR of case (a). Moreover, for $\alpha^{\mathrm{P}} = 0.9$ the CVaR obtained if the option is purchased is indeed lower than that achieved if the electricity is sold through a forward contract during the last four weeks.

Once the results for non-failing generating units (FOR = 0%) have been analyzed, we next examine those cases in which unit failures are considered (FOR 5% and 10%, third and fourth column blocks of Table 6.20, respectively). Observe that the CVaR obtained buying a put option is higher than that achieved with a forward contract for all cases. The reason is that the put option allows the producer to avoid low profits due to both low pool prices and unexpected unit failures. In other words, while a forward contract reduces the price risk at the expense of increasing the risk associated with unit failures, selling electricity through a put option reduces both the price and the availability risk. In Table 6.22 the value of the binary variable $y_{(o,\omega)}$ is specified as a function of the average pool price over the last four weeks ($E_2\{\lambda^{\mathrm{P}}_{(\omega,t)}\}$) and the status of the unit at the end of the fourth week ($k_{(\omega,N_{T_1})}$). For clarity, only three values of the expected pool price are considered, namely, a high value (26.01 €/MWh), a value close to the strike price (20.97 €/MWh), and a low value (20.28 €/MWh).

Observe that for a low average pool price the option is exercised for all availability scenarios. On the contrary, for a high average pool price, the option is not exercised to take advantage of selling the produced electricity in the pool. If the average pool price is very close to the strike price, however, the power producer exercises the option just in those cases in which the unit is available at the end of the first four weeks because it is then more likely that the unit is also available during the last four weeks.

In the previous analysis, forward contracts spanning the last four weeks are only traded in stage 1 at known prices. Next, we include the possibility of re-trading forward contracts in stage 2 at the prices listed in Table 6.19.

Table 6.23: Option case study 1: values of the $\mathrm{CVaR}_{\alpha^{\mathrm{P}}}$ (€million) for different levels of FOR and risk aversion depending on whether the production is sold through a forward contract or through a put option, and considering the re-trading of forward contracts

α^{P}	FOR = 0%		FOR = 5%		FOR = 10%	
	Case (a)	Case (b)	Case (a)	Case (b)	Case (a)	Case (b)
0	5.245	5.418	5.141	5.314	5.035	5.209
0.5	5.078	5.117	4.919	4.970	4.756	4.860
0.9	5.078	5.055	4.599	4.649	4.429	4.499

Table 6.23 provides the optimal CVaR values obtained by solving optimization problem (6.27) for different FOR and risk aversion levels and two different cases. In case (a), the producer can re-trade in stage 2 the 350 MW sold in stage 1 at 21 €/MWh. In case (b), the producer buys an option with strike and option prices equal to 21 €/MWh and 0.1 €/MWh, respectively.

Depending on the updated forward price in stage 2 (see Table 6.19), if high pool prices realize during the first four weeks, the producer can repurchase the 350 MW previously sold in stage 1 trough the forward contract spanning the last four weeks to close its forward position and be able to sell all its production in the pool at high prices. Note that although the optimal CVaR obtained by purchasing the option is higher (case (b)), the re-trading of forward contracts in stage 2 allows the producer to increase its CVaR if compared with that provided in Table 6.20 (case (a)).

In the studies presented above, we have analyzed the objective function and the decisions of the power producer considering that all the electricity produced during the last four weeks is sold through either a forward contract (case (a)) or a put option (case (b)). Next, we solve optimization problem (6.27) allowing the producer to sell its production through both the forward contract and the put option. The price of the forward contract spanning the first four weeks is set to 22 €/MWh. The price of the forward contract spanning the last four weeks as well as the strike price of the option are set to 21 €/MWh, and the option price is equal to 0.1 €/MWh. For simplicity, re-trading of forward contract in stage 2 is not allowed in this study. Table 6.24 provides the optimal values of both $P^1_{(c_2)}$ and $P_{(o)}$ for different values of the risk aversion parameter and FOR.

To analyze the results in this table we have to take into account that the amount of electricity sold through the put option by the power producer basi-

Table 6.24: Option case study 1: optimal values of $P^1_{(c2)}$ and $P_{(o)}$ (MW) as a function of the risk aversion level and FOR

α^{P}	FOR = 0%		FOR = 5%		FOR = 10%	
	$P^1_{(c2)}$	$P_{(o)}$	$P^1_{(c2)}$	$P_{(o)}$	$P^1_{(c2)}$	$P_{(o)}$
0	0	0	0	0	0	0
0.5	0	72	0	146	0	280
0.9	350	0	96	209	71	192

cally depends on the proportion between the number of scenarios under which the option is exercised and the number of scenarios under which it is not. That is, if the put option is exercised under practically any scenario realization, the power producer sells its production through the forward contract to save the option price. Likewise, if the put option is not exercised under any circumstances, the producer sells the electricity directly in the pool. In contrast, the higher the dependence of the option exercising on the scenario realization, the higher the amount of power sold through the put option. That said, we can observe that, independently of its forced outage rate, a risk-neutral producer sells all its production in the pool, in which the clearing price during period 2 is expected to be higher than both the forward and the strike prices. Secondly, if the risk aversion parameter increases up to 0.5, the producer is willing to sell some of its production through the available put option to hedge against the price risk without increasing too much the probability of having low profits due to unit failures. Note that the quantity sold through the put option increases as the generating unit becomes more susceptible to failure to hedge against both the price and the availability risks. Lastly, a highly risk-averse producer increases the electricity it sells through forward contracts in order to decrease the payment pertaining to the option price.

To conclude this case study, optimization problem (6.27) is solved considering the re-trading of forward contract in stage 2. Table 6.25 provides the optimal values of the power sold through the forward contract spanning the last four weeks at stage 1 and the power sold through the put option. Note that in this case the producer decides, at stage 1, to sell all its production through the forward contract spanning period 2. This way, the producer may or may not close its forward position according to the pool price realization, the availability of the generating unit, and the forward contract price at the end of the fourth week.

Table 6.25: Option case study 1: optimal values of $P^1_{(c2)}$ and $P_{(o)}$ (MW) as a function of the risk aversion level and FOR, and considering the re-trading of forward contracts

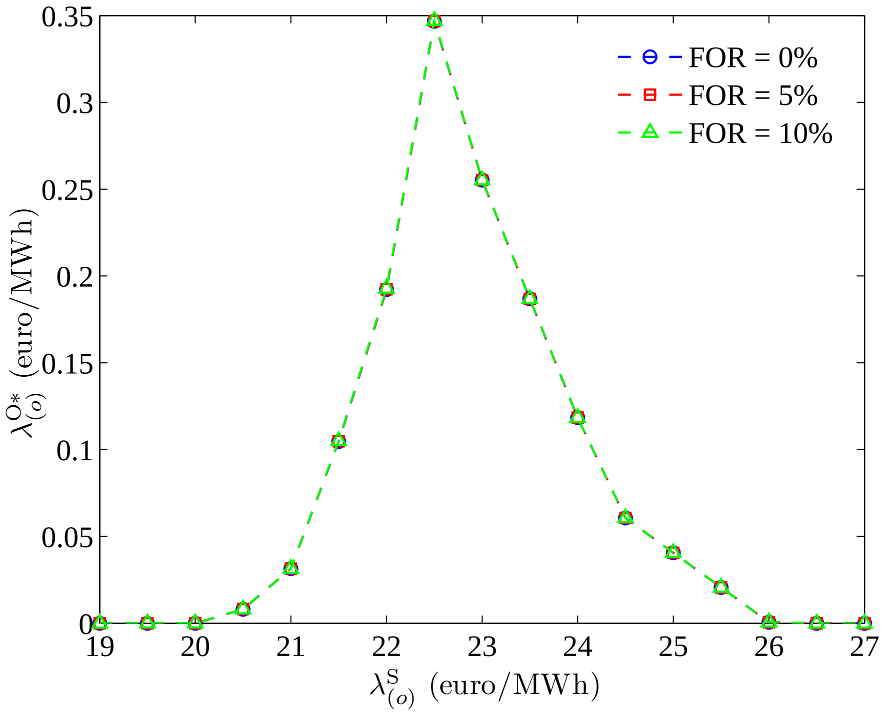
α^{P}	FOR = 0%		FOR = 5%		FOR = 10%	
	$P^1_{(c_2)}$	$P_{(o)}$	$P^1_{(c_2)}$	$P_{(o)}$	$P^1_{(c_2)}$	$P_{(o)}$
0	350	0	350	0	350	0
0.5	350	72	350	146	350	280
0.9	350	350	350	230	350	275

6.5.2.2 Maximum Option Price Calculation

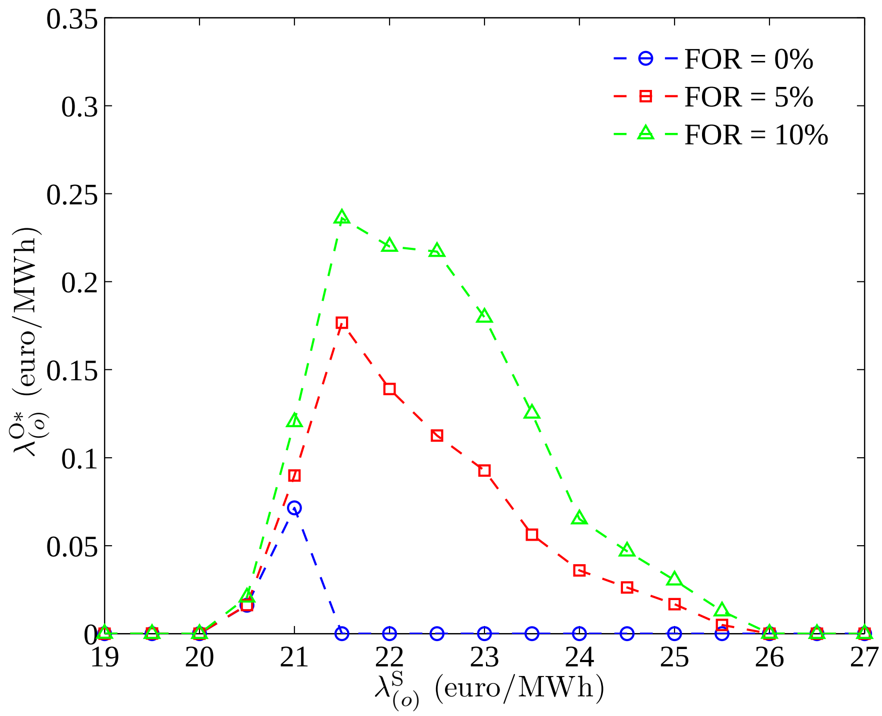
In this subsection, we calculate the maximum option price ($\lambda^{\mathrm{O}*}_{(o)}$) that the power producer is willing to pay for a given option as described in Section 6.3.8. Fig. 6.12 depicts the maximum option price as a function of the strike price of the option ($\lambda^{\mathrm{S}}_{(o)}$) for two values of the risk aversion parameter ($\alpha^{\mathrm{P}} = 0$ and $\alpha^{\mathrm{P}} = 0.5$) and three different forced outage rates of the generating unit, namely, 0, 5, and 10%. A put option of 350 MW is considered. For simplicity, forward contracts for periods 1 and 2 are assumed to be traded only at stage 1, i.e., no forward re-trading is allowed at the beginning of stage 2. Moreover, the price of the forward contract spanning period 1 is set to 22 €/MWh, while the price of the forward contract covering period 2 is considered to be equal to the strike price of the option.

First, observe that for both very low and very high values of the strike price, the maximum option price is equal to zero. If the strike price is very low, the option is not exercised for any possible realization of the pool price during the first four weeks and therefore, the power producer decides selling all its production in the pool. On the other hand, for high values of the strike price, the option would be exercised in all cases and therefore, the power producer prefers buying instead a forward contract at the same price to save the non-refundable fee of the option.

From the comparison of the plots corresponding to the non-failing case in Figs. 6.12(a) and 6.12(b), we can conclude that the maximum option price decreases if the risk aversion increases. The reason for this is that for sufficiently high risk aversion level, the option would be exercised in most of cases to hedge against pool price volatility, i.e., the price risk, and therefore, the producer would rather sell its production through forward contracts to save again the non-refundable option fee.



(a) $\alpha^{P} = 0$



(b) $\alpha^{P} = 0.5$

Figure 6.12: Option case study 1: maximum option price as a function of the strike price for different FOR and risk aversion levels

Table 6.26: Option case study 1: forced outage parameters revisited

FOR (%)	MTTF (h)	MTTF (h)
5	950	50
5	1900	100
10	450	50
10	900	100

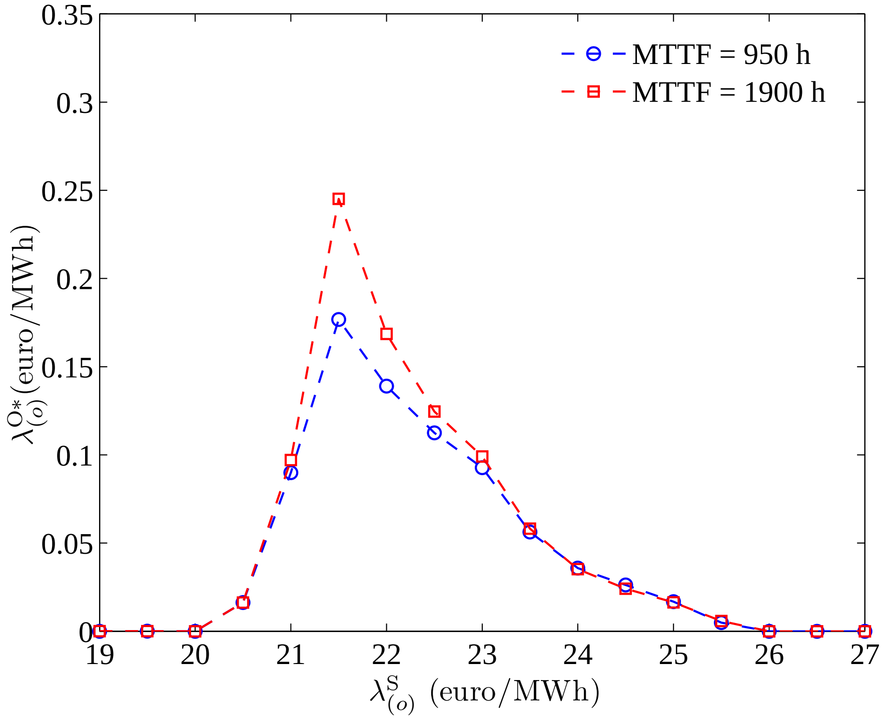
Observe that for $\alpha^{\mathrm{P}} = 0$ (Fig. 6.12(a)), the maximum option price is the same regardless of the FOR value. In other words, taking into account the low values of realistic forced outages rates, if the expected profit is maximized, the pool price uncertainty has the highest influence on option decisions, i.e., the option is exercised or not based only on the probability of experiencing low or high pool prices. However, in a risk-averse situation ($\alpha^{\mathrm{P}} = 0.5$), the higher the forced outage rate, the higher the maximum option price. This is so because the lowest profits are obtained when unit failures occur and consequently, a put option can be used to reduce the availability risk during the last four weeks of the study horizon.

As discussed in Section 3.5 of Chapter 3, for a given FOR value, the higher the value of the MTTF, the more important is the current status of the generating unit to predict its future availability. Therefore, a comparatively high value of the MTTF makes the put option a more attractive instrument to sell electricity as compared to a forward contract. To quantify this effect, we compute the maximum option price of the producer considering two different values of MTTF for each FOR value (5% and 10%). Table 6.26 provides the availability parameters considered in this analysis. The maximum option prices of the producer for these availability parameters and a risk aversion level equal to 0.5 are shown in Fig. 6.13.

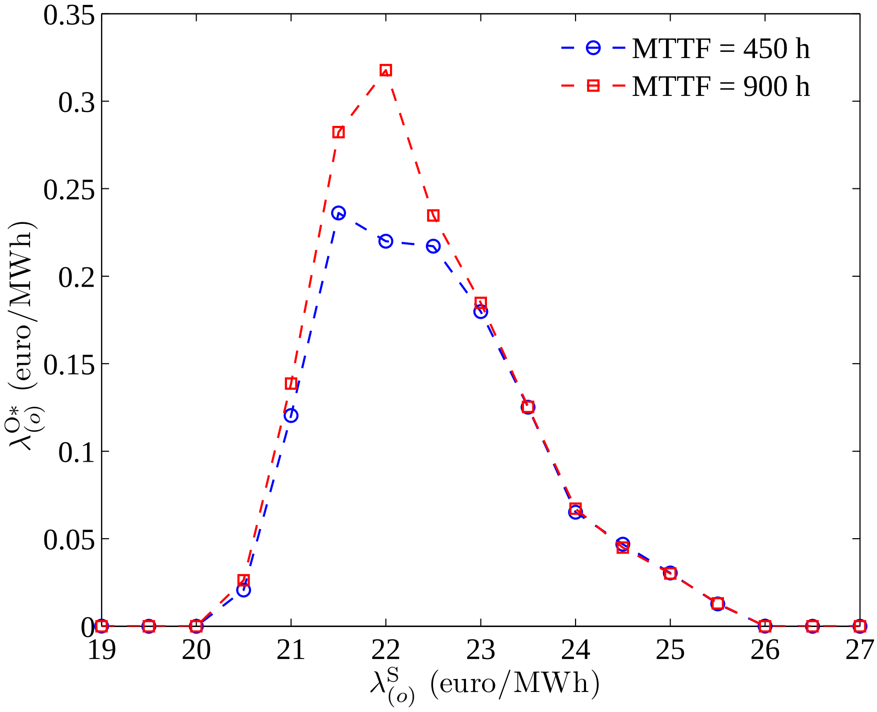
Note that irrespective of the forced outage rate of the generating unit, an increase in the MTTF brings about higher values of the maximum option price of the producer.

6.5.3 Case Study 2: Call Option

Similarly to the previous section, this second case study is divided into two parts. In the first one, we solve the stochastic optimization problem (6.27) to analyze how the acquisition of a call option to buy electricity during the last four weeks of the study horizon significantly reduces the availability risk faced



(a) FOR = 5%



(b) FOR = 10%

Figure 6.13: Option case study 1: impact of the MTTF on the maximum option price ($\alpha^{\mathrm{P}} = 0.5$)

Table 6.27: Option case study 2: values of the $\text{CVaR}_{\alpha^{\text{P}}}$ (€million) for different levels of FOR and risk aversion depending on whether the production is sold through a forward contract or through a put option

α^{P}	FOR = 0%		FOR = 5%		FOR = 10%	
	Case (a)	Case (b)	Case (a)	Case (b)	Case (a)	Case (b)
0	5.793	5.793	5.689	5.689	5.584	5.584
0.5	5.784	5.784	5.578	5.578	5.370	5.370
0.9	5.784	5.784	5.255	5.256	5.079	5.112

by the producer, especially if a forward contract to sell electricity during the same period is signed. In the second part, the maximum option price that the producer is willing to pay for a given call option to buy electricity is computed for different risk aversion values and forced outage rates.

6.5.3.1 Option vs. Forward Contract

We highlight the advantages of signing a call option to have the right to buy electricity by solving two optimization problems. In the first one (case (a)), we solve optimization problem (6.27) considering that the producer has signed a forward contract to sell electricity during the last four weeks at a price equal to 24 €/MWh. In the second problem (case (b)), the same optimization problem is solved but, besides selling electricity through the aforementioned forward contract, the producer can acquire a call option of 50 MW to have the right to buy electricity during the last four weeks of the study horizon. The strike price and the option price of the call option are 12.4 €/MWh and 0.05 €/MWh, respectively. Table 6.27 provides the $\text{CVaR}_{\alpha^{\text{P}}}$ of the producer profit distribution for cases (a) and (b), different FOR values, and risk aversion levels. We should point out that for the sake of simplicity the re-trading of forward contracts in stage 2 is not permitted.

Since the purpose of a call option consists in hedging against the risk caused by unexpected unit failures, if the generating unit does not fail (FOR = 0%), the producer does not acquire the call option for any risk aversion level and therefore, the objective values of cases (a) and (b) are the same. In contrast, if the generating unit can fail with a certain probability (FOR = 5% and FOR = 10%) and the risk aversion parameter is high enough ($\alpha^{\text{P}} = 0.9$) the CVaR obtained if the call option is signed is higher than that achieved in case (a).

Table 6.28, which provides the value of binary variable $y_{(o,\omega)}$ as a function

Table 6.28: Option case study 2: values of the binary variable $y_{(o,\omega)}$ for $\alpha^{\mathrm{P}} = 0.9$ and FOR = 5%

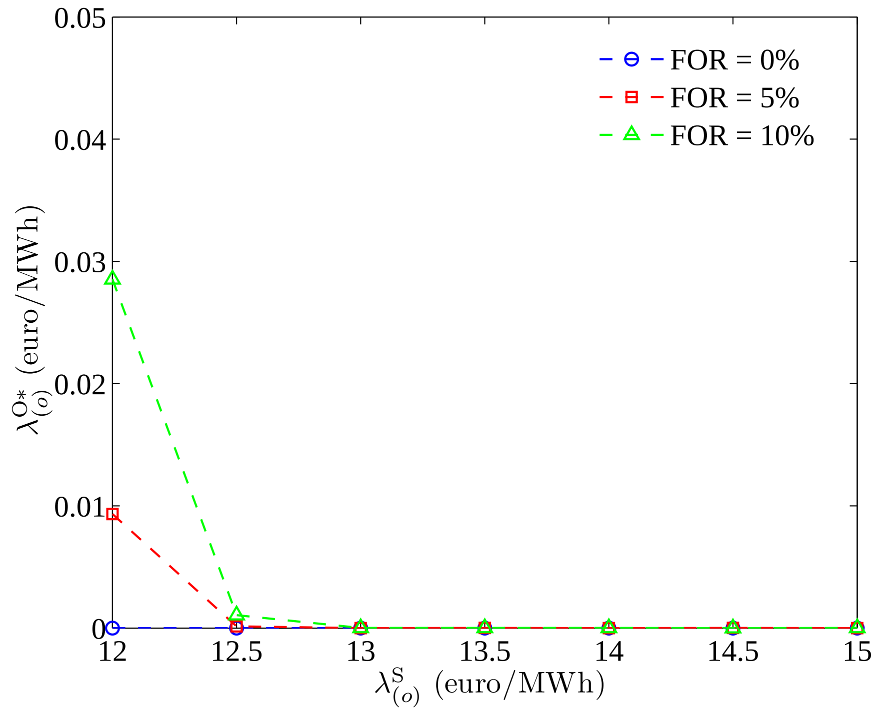
$k_{(\omega,N_{T_1})}$	$E_2\{\lambda^{\mathrm{P}}_{(\omega,t)}\}$ (€/MWh)									
	22.41	22.58	22.64	20.97	24.39	21.85	22.35	20.28	26.01	22.04
1	0	0	1	0	1	1	0	0	1	1
0	1	1	1	1	1	1	1	1	1	1

of the expected pool price during the last four weeks ($E_2\{\lambda^{\mathrm{P}}_{(\omega,t)}\}$) and the status of the generating unit at the end of the fourth week ($k_{(\omega,N_{T_1})}$), illustrates in which cases the call option is exercised by the producer. If the generating unit is forced out at the end of the fourth week ($k_{(\omega,N_{T_1})} = 0$), the producer exercises the call option for all pool price scenarios. This is due to the fact that the producer has signed a forward contract to sell electricity during the last four weeks of the study horizon, thus, if the generating unit is still unavailable during this period, the producer can buy some of the committed energy through the call option and reduce the financial losses caused by the forced outage. On the other hand, if the unit is available at the end of the fourth week ($k_{(\omega,N_{T_1})} = 1$), the option is exercised if the pool prices are expected to be high enough in order to reduce the financial losses incurred if the producer, due to an unexpected unit failure, has to buy electricity to fulfill its contracting selling obligations.

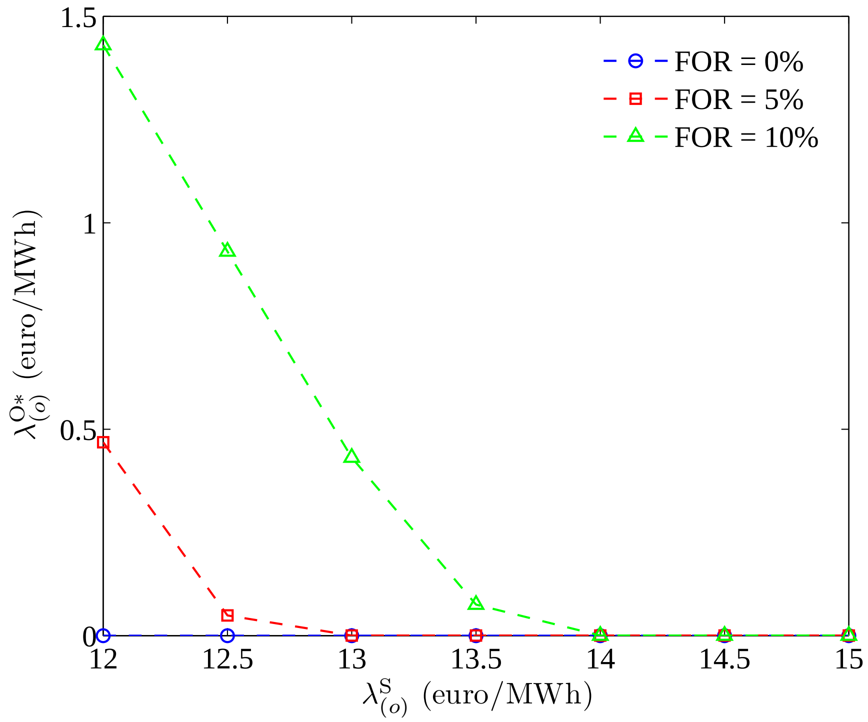
6.5.3.2 Maximum Option Price Calculation

According to the procedure presented in Section 6.3.8, we compute in this subsection the maximum option price that the producer is willing to pay for a call option of 50 MW. The price of the available forward contract spanning the last four weeks is set to 24 €/MWh. Fig. 6.14 depicts the maximum option price of the producer as a function of the strike price of the call option for different FOR values (0%, 5%, and 10%) and two risk aversion levels ($\alpha^{\mathrm{P}} = 0$ and $\alpha^{\mathrm{P}} = 0.9$).

In the previous section, we show that put options can be used to hedge against the uncertainty related to both pool prices and unit failures. However, since call options give the holder just the right to buy electricity, they are only effective to hedge against the risk associated to unexpected unit failures. For this reason, the maximum option price that the producer is willing to pay for a call option if it owns a non-failing generating unit is 0 €/MWh for all strike



(a) $\alpha^{P} = 0$



(b) $\alpha^{P} = 0.9$

Figure 6.14: Option case study 2: maximum option price as a function of the strike price for different FOR and risk aversion levels

prices and for both risk aversion levels.

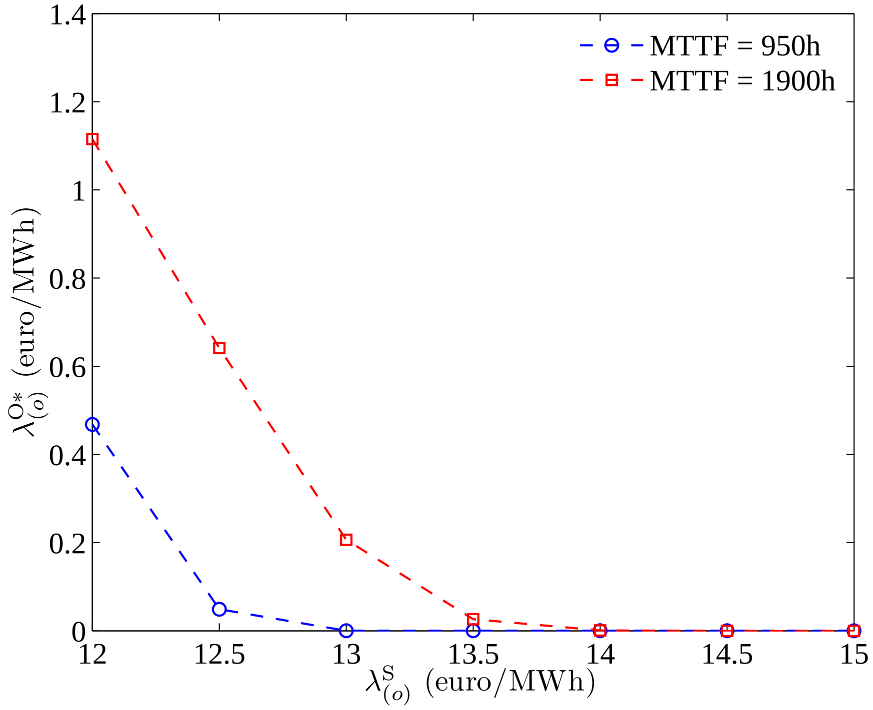
On the other hand, if the generating unit owned by the producer can fail with a certain probability distribution (FOR = 5% and FOR = 10%), the maximum option price reaches values higher than zero for the two considered risk aversion levels. Note that a low value of the strike price of the call option allows the producer to have the right to buy electricity at more competitive prices and therefore, the corresponding maximum option price increases.

In the case study involving put options, an increase in the risk aversion level causes a decrease in the maximum option price, because the producer can alternatively sign a forward contract to hedge against the price risk, with the advantage of saving the option price. On the contrary, in the analysis carried out in this case study, the call option is the only way to hedge against the availability risk faced by the power producer. This is the reason why an increase in the risk aversion parameter gives rise to higher maximum option prices as observed in Figs. 6.14(a) and 6.14(b). Moreover, note that for both risk aversion parameters, the higher the FOR value the higher the maximum option price that the producer is willing to pay for the call option.

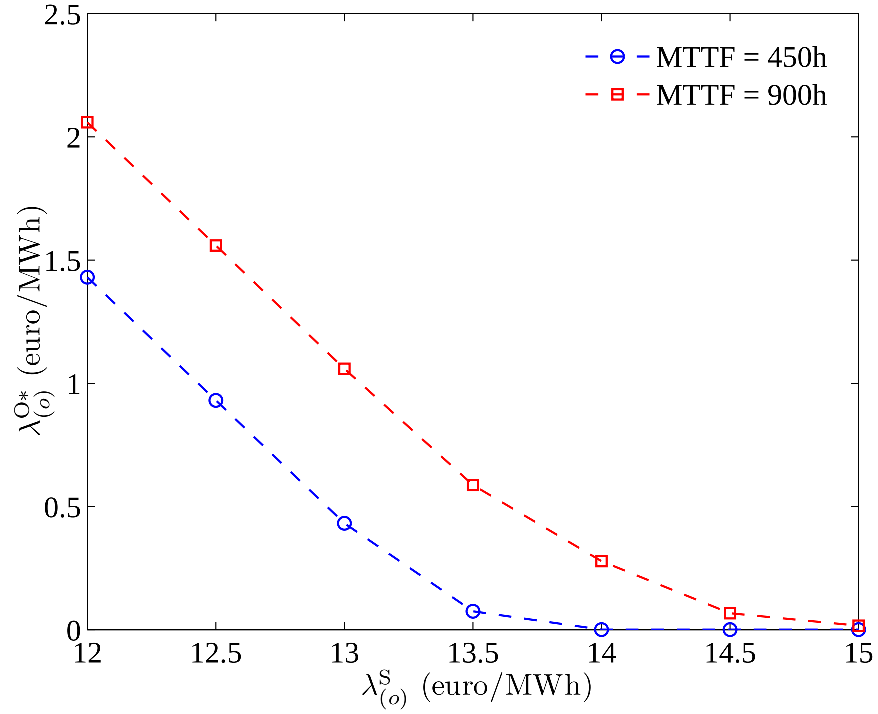
Finally, to analyze the impact of the MTTF value on the maximum price of the call option, Fig. 6.15 plots the maximum option price of the producer as a function of the strike price for the availability parameters indicated in Table 6.26. Note that an increase in the MTTF for the same FOR value lets the producer better forecast the availability of the generating unit during the last four weeks and the maximum option price increases in consequence.

6.5.4 Computational burden

To round off, we point out that the simulations results presented in the case studies reported in this chapter have been obtained using CPLEX 12.1.0 [6] under GAMS [5] on a Sun Fire X4600M2 with 8 Quad-Core processors running at 2.9 GHz and 256 GB of RAM. If a set of 900 scenarios is used to represent the uncertainty associated with the pool price and the unit availability, the computational time required to solve the three-stage optimization problem (6.27) is lower than 20 minutes.



(a) FOR = 5%



(b) FOR = 10%

Figure 6.15: Option case study 2: impact of the MTTF on the maximum option price ($\alpha^{\mathrm{P}} = 0.9$)

6.6 Summary

Power producers need to face the uncertainties related to price volatility and production availability when trading in electricity markets. In this sense, power producers must make their decisions not only to maximize the expected profit but also to reduce as much as possible the profit variability caused by the uncertainty involved. Forward contracts are available financial instruments that allow market participants to sell and buy electricity at fixed prices, which are agreed in advance, thus reducing their respective profit variabilities. The main disadvantage of a forward contract is that, once they are signed, the delivery of the agreed amount of electricity is compulsory. On the other hand, electricity options allow producers to put off decisions on selling or buying a given amount of electricity at a fixed price until the beginning of the delivery period of the option. This postponement lets the holder of the option use additional information about the uncertain parameters to make better informed decisions.

This chapter proposes a three-stage stochastic programming model to determine the optimal involvement in the derivatives and pool markets by a risk-averse power producer if put and call options are available. This model considers the uncertainty related to pool prices, unexpected unit failures, and forward prices, which are properly characterized via scenarios. The optimal decisions of the producer related to pool trading, forward contracts and options are derived to maximize the Conditional Value-at-Risk of the profit distribution for a given risk aversion level α^{P}. The proposed stochastic programming model results in a large-scale mixed-integer linear programming problem that is efficiently solved using commercially available software.

The main technical aspects of the proposed model, as well as the characteristics of the decisions derived from its use, are highlighted and discussed by means of a small-size example and a realistic case study based on the electricity market of the Iberian Peninsula.

Chapter 7

Closure

In this chapter, the thesis is summarized and the main conclusions are drawn. Additionally, the main contributions of the thesis are highlighted and proposals for future research are provided.

7.1 Summary and Conclusions

One of the major elements of the restructuring process of the electricity sector is the introduction of competition in the sale and purchase of electricity through a wholesale electricity market known as *pool*. Given the special features of electricity as a commodity, the electricity pool price volatility is usually significantly higher than that corresponding to the price of other commodities in energy markets. Alternatively to the pool, market agents can trade electricity at more stable prices through *derivatives* and financial contracts, which are being gradually introduced in electricity markets.

In this context, one of the main concerns of power producers is the identification and the measure of the risks affecting their profit probability distributions, as well as the determination of the best strategy to hedge against these risks. Within a mid-term decision framework, two of the main risks faced by power producers are:

- *price risk*, which is the risk related to the high variability of the electricity price in the pool.

- *availability risk*, which is caused by the adverse consequences of unexpected failures of the generating units owned by the power producer.

Electricity derivatives constitute one of the main instruments that allow power producers to hedge against price risk. Therefore, power producers have to determine the optimal quantities sold through the available derivatives considering the uncertainties related to the electricity pool price and the occurrence of unexpected failures of generating units. In this sense, *stochastic programming* is an adequate mathematical framework to identify optimal decisions under uncertainty. Moreover, the most commonly used risk measures can be easily included in a stochastic optimization problem. In this thesis, the Conditional Vale-at-Risk for a confidence level α is the measure used to quantify the risk associated with the profit probability distribution of a power producer.

7.1.1 Scenario Reduction

In a stochastic programming problem, the uncertain parameters are characterized via a set of possible outcomes called *scenarios*. The thorough generation and reduction of scenario sets representing uncertain parameters are two research topics within the field of stochastic programming. In this thesis, we use the ARIMA time series theory to explain and generate scenarios representing the uncertain behavior of the electricity pool price. Likewise, scenario sets characterizing the availability of generating units are built using exponential distributions.

The technical literature provides scenario reduction techniques that, based on the definition of a probability distance between two scenario sets, determine a reduced scenario set that retains, as much as possible, the statistical properties of the original set. In this thesis, two scenario reduction techniques are proposed to reduce the size of the scenario sets representing the electricity pool price and the availability of generating units. The motivation and contribution of each scenario reduction technique are provided below:

- The first technique proposed in this thesis accounts for information about the structure of the optimization problem to be solved in the scenario reduction process. To this end, we first solve as many single-scenario optimization problems as scenarios belonging to the initial set. Next, we reduce the scenario set based on how different the objective functions of the single-scenario problems are.

- In the second proposed technique we also take into account the risk aversion level of the decision maker in the scenario reduction process. This technique considers that the risk is quantified via the Conditional Value-at-Risk of the profit probability distribution. The aim of this technique is to describe, as accurately as possible, the scenarios that lead to low profits under the rationale that risk-averse decisions are made based on these scenarios.

A power producer decision-making model is used to compare the performance of the above two scenario reduction techniques with one reported in the technical literature, hereinafter referred to as the traditional scenario reduction technique. Numerical results indicate that:

1. The exact characterization of uncertain parameters is either impossible (in the case of continuous stochastic processes) or requires the generation of a number of scenarios that may make the associated stochastic optimization problem computationally intractable (in the case of discrete stochastic processes). Therefore, the selection of an appropriate scenario reduction technique becomes an important issue that needs to be tackled when solving stochastic optimization problems.

2. The traditional scenario reduction technique determines the distance between two scenarios as the norm of the difference between the two vectors corresponding to the two scenarios. Therefore, although this technique involves low computational burden to determine the matrix containing the distances between all scenarios of the original set, the reduced scenario set does not depend of the stochastic optimization problem to be solved.

3. The first scenario reduction technique proposed in this thesis advantageously compares with the existing one for reducing the scenario set characterizing the electricity pool price. Although it requires a significant computational time to solve the single-scenario problems, this technique embodies the special features of the stochastic optimization problem to be solved by selecting scenarios according with the profit objective of the power producer.

4. This first reduction technique can also be used to reduce scenario sets representing discrete stochastic processes. In this sense, this technique

can be used to efficiently reduce the scenario set characterizing the availability of the generating units owned by a power producer.

5. The risk aversion level of a power producer plays an important role in its decision-making process and therefore, its consideration within a scenario reduction procedure should be evaluated. In this sense, the second technique proposed in this thesis, which takes into account the risk aversion level in the reduction procedure, provides more accurate results than those obtained with the other two scenario reduction techniques if solving stochastic optimization problems for risk-averse power producers.

7.1.2 Forward Contracting

In order to avoid the possibility of experiencing low profits due to the realization of low electricity pool prices, a risk-averse power producer can sell part of its production through forward contracts at fixed prices. However, unlike selling in the pool, forward contracts involve an obligation to deliver the agreed electricity during every hour of the contracting time period. If some of the generating units owned by the power producer fail during the delivery period of a signed forward contract, the producer has to buy in the pool the electricity needed to meet its forward selling obligations. If the pool price during the outage periods is higher than the price of the forward contract, the economic losses of the power producer can be significant. Therefore, by taking into account the forced outage rates of its generating units as well as the volatility of electricity pool prices, a risk-averse power producer can identify the appropriate mix of pool and forward contract involvement to reduce the probability of having low profits. The model proposed in this thesis is based on a two-stage stochastic programming model with recourse in which the first-stage variables correspond to the quantity of electricity sold through each available forward contract. Likewise, the energy quantities sold in the pool are the second-stage variables. Both sets of variables are optimized to maximize the Conditional Value-at-Risk of the profit probability distribution of the power producer for a given risk aversion level. Mathematically, this decision-making tool is formulated as a mixed-integer linear optimization problem that can be solved using available commercial software.

Examples and realistic case studies are used to illustrate and test the proposed tool, concluding that:

1. Decision-making models able to include the uncertainty related to both electricity pool price and the availability of generating units are needed to determine the optimal quantities to be sold through forward contracts by a risk-averse power producer.

2. The risk aversion level of a power producer is efficiently modeled via the maximization of the Conditional Value-at-Risk of its profit probability distribution for a the confidence level α.

3. The occurrence of unexpected unit failures has two effects on the profit probability distribution of a power producer. On the one hand, average profit decreases in proportion to the time during which the unit is forced out. On the other, the probability of experiencing low profit increases as a result of buying in the pool during forced outage periods, thus causing a decrease in the Conditional Value-at-Risk of the profit distribution.

4. Considering that the value of the electricity pool price is unlikely to be lower than the production cost of the generating units, forward contract decisions for a risk-neutral producer with failing generating units are similar to those made by a risk-neutral power producer owing non-failing generating units.

5. In the case of a risk-averse power producer, the optimal involvement in forward contracting is significantly affected by the possibility of suffering from unexpected failures of its generating units. That is, in order to avoid the low profits that materialize if generating units fail when the pool price is high, a risk-averse producer reduces the quantity of electricity sold through forward contracts if the forced outage rates of its units are sufficiently high.

6. The evolution of the power sold through forward contracts with the risk aversion level of a power producer depends on the prevalence of the price risk vs. the availability risk. In other words, if the price risk is predominant over the availability risk, an increase in the risk aversion level leads to an increase in the power sold through forward contracts. On the other hand, if the risk associated with unit failures is the prevalent one, the higher the risk aversion level, the lower the power sold through forward contracts.

7. Out-of-sample analysis proves that deciding forward contracting without modeling the uncertainty related to unexpected unit failures may lead to underestimate the optimal value of the objective function. This estimation gap increases with the risk aversion level of the power producer.

8. Since unexpected failures of generating units are generally statistically independent, an increase in the number of units owned by a power producer reduces the availability risk and therefore, the percentage of energy sold through forward contracts increases.

7.1.3 Insurance Contracting

Alternatively to reducing the electricity sold through forward contracts, a power producer can hedge against its availability risk by signing an insurance contract. Through this type of insurance, a power producer receives, in exchange for an initial premium, an economic compensation if financial losses due to unexpected unit failures of its generating units occur. We analyze the convenience of signing a given insurance contract by a risk-averse power producer as well as the impact of such an insurance contract on its forward contracting decisions via a two-stage stochastic programming model. As in the case of considering forward contracts, the sources of uncertainty are the pool price of electricity and the availability of generating units. The proposed decision-making tool is formulated as a mixed-integer linear optimization problem in which the risk is measured by the Conditional Value-at-Risk of the profit distribution of the producer.

Results from an illustrative example and a realistic case study indicate that:

1. Stochastic programming provides an adequate framework for both modeling and making decisions regarding insurance contracts against unexpected forced outages of generating units owned by risk-averse power producers.

2. Signing an insurance contract increases the profit obtained by a power producer if a generating unit failure occurs when the pool price is high, thus efficiently reducing the availability risk faced by the producer. For this reason, the acquisition of an insurance against unit failures allows a power producer to increase the electricity sold through forward contracts.

On the other hand, signing an insurance contract reduces the expected profit of a power producer due to the premium cost.

3. The value of the premium is a relevant parameter to be accounted for when making decisions related to insurance contracts. Therefore, the calculation of the maximum premium that a risk-averse power producer is willing to pay for a given insurance contract constitutes a valuable information.

4. The maximum premium that a power producer is willing to pay for a given insurance contract grows if the risk aversion level of the producer also increases because its decisions are made to maximize the lowest profits of the distribution.

5. An increase in the forced outage rates of the generating units of a power producer entails a higher value of the maximum premium that the producer is willing to pay for a given insurance contract.

6. An insurance contract against unit failures by a power producer makes sense if the risk aversion level of the insurer is lower than the risk aversion level of the producer, thus corroborating the fact that an insurance contract is a financial mechanism to transfer risk from one agent to another in exchange for a premium.

7.1.4 Option Contracting

Electricity options provide another mechanism to hedge against the risks faced by power producers. Electricity options allow power producers, in exchange for a given quantity known as *option price*, to postpone decisions on the sale or purchase of electricity at fixed prices. This way, the realization of the uncertain parameters during the time period between the acquisition of the option and its expiration date can be included into the decision-making model to make more informed decisions. In this thesis, we propose a three-stage stochastic model which, taking into account both the price and the availability risk, determines the optimal selling strategy for a risk-averse power producer including options, forward contracts, and pool trading. The Conditional Value-at-Risk is the risk measure used in this model. Mathematically, this decision-making model is formulated as a mixed-integer linear optimization problem.

From the illustrate example and the detailed case study considered in this dissertation we conclude that:

1. Options to buy and sell electricity can be adequately modeled using multi-stage stochastic programming. Optimal decisions regarding option contracting for a risk-averse power producer can be determined using the model proposed in this dissertation.

2. Put electricity options can be used by risk-averse power producers to hedge against the pool price volatility. In this sense, if the evolution of the pool price since the acquisition of the option until its expiration date provides a forecast of low pool prices afterward, the option to sell electricity is exercised to avoid low profits for selling electricity in the pool. On the other hand, if the prices are expected to be high, the put option is not exercised to sell the electricity in the pool.

3. The financial consequences of the risk associated with the occurrence of unexpected failures of generating units can also be controlled using electricity put options. If a generating unit fails just before the expiration date of a put electricity option, a risk-averse power producer would not exercise it to avoid the low profits caused by the purchase of electricity in the pool.

4. Call options to buy electricity constitute an useful instrument to hedge against the availability risk faced by power producers. A producer can sign a call option to have the possibility of buying electricity at a fixed price to meet its selling contracting obligations if some of its generating units are forced out. Thus, the risk associated with unexpected unit failures is reduced.

5. The influence of the option price, which the producer has to pay even if the option is not exercised, on the decisions of a risk-averse power producer is particularly significant. The three-stage stochastic programming framework proposed in this thesis allows us to determine the maximum option price that a power producer is willing to pay for a given option.

6. The strike price of a given option contract significantly influences the value of the maximum option price for a power producer.

7. In the case of put options to sell electricity, an increase in the risk aversion level of the producer entails a decrease in the maximum option price of the producer, which prefers to sell its production through forward contracts to avoid paying the option price. In this case, note that the price risk is prevalent over the availability risk.

8. If the availability risk prevails over the price risk, an increase in the risk aversion level gives rise to an increase in the maximum option price that a power producer is willing to pay for a given call option to buy electricity at a fixed price. This way, the producer reduces the probability of experiencing low profits due to unexpected unit failures.

9. For both put and call electricity options, an increase in the forced outage rates of the generating units owned by a power producer, which translates into a higher availability risk, leads to an increase in the maximum option price for the producer.

10. For a generating unit with a given forced outage rate, an increase in the average time between two consecutive failures (MTTF) allows the producer that owns such an unit to better forecast its future availability knowing its current status and therefore, options become more attractive instruments than forward contracts to hedge against availability risk. For this reason, the higher the MTTF, the higher the maximum option price accepted by the producer.

7.2 Contributions

The main contributions of the work reported in this dissertation can be summarized as follows:

1. The use of the Conditional Value-at-Risk of a profit probability distribution as the objective function to be maximized by a power producer. This way, its risk aversion level can be modeled through a single parameter corresponding to the confidence level α.

2. The development of a procedure to generate scenarios representing the availability of a generating unit using the parameters of the two exponential distributions that characterize, respectively, the time between two consecutive unit failures and the time to repair a failure.

3. The proposal of a scenario reduction technique that, including the information embedded in the optimization problem to be solved, efficiently selects the scenarios belonging to the reduced scenario set.

4. The proposal of a second scenario reduction technique that accounts for the risk aversion level of the decision maker into the scenario reduction procedure.

5. The formulation of a medium-term forward contract selection problem for a power producer as a two-stage stochastic programming problem with recourse in which the risk is represented through the CVaR of the profit distribution.

6. The development of an equivalent mixed-integer linear formulation for the two-stage stochastic programming problem that determines the optimal forward contract decisions of a risk-averse power producer.

7. The modeling of an insurance contract against unexpected unit failures and the formulation of a two-stage stochastic optimization problem that allows a risk-averse power producer to make informed decisions on insurance contracts. The CVaR of the profit is used to measure the risk faced by the producer.

8. The development of an equivalent mixed-integer linear formulation for the two-stage stochastic programming problem that allows determining the optimal selection of insurance contracts against unit failures by a risk-averse power producer.

9. The formulation of the electricity option selection problem faced by a power producer as a multi-stage stochastic programming problem with recourse, in which the risk aversion is modeled through the CVaR of the producer profit.

10. The development of an equivalent mixed-integer linear formulation for the multi-stage stochastic programming problem that allows identifying the optimal contracting of electricity options by a risk-averse power producer.

11. The publication of four papers in relevant SCI journals and the submission of an additional one:

(a) S. Pineda, A. J. Conejo and M. Carrión, "Impact of unit failure on forward contracting", in *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1768–1775, November 2008. JCR 5-year impact factor: 2.746, position 42 of 229 in Engineering, Electrical and Electronic.

(b) J. M. Morales, S. Pineda, A. J. Conejo and M. Carrión, "Scenario reduction for futures market trading in electricity market", in *IEEE Transactions on Power Systems*, vol. 24, no. 2, pp. 878–888, May 2009. JCR 5-year impact factor: 2.992, position 31 of 245 in Engineering, Electrical and Electronic.

(c) S. Pineda and A. J. Conejo, "Scenario Reduction for Risk-Averse Electricity Trading", in *IET Generation, Transmission & Distribution*, vol. 4, no. 6, pp. 694–705, June 2010. JCR 5-year impact factor: 0.856, position 135 of 245 in Engineering, Electrical and Electronic.

(d) S. Pineda, A. J. Conejo and M. Carrión, "Insuring unit failures in electricity markets", in *Energy Economics*, vol. 32, no. 6, pp. 1268–1276, November 2010. JCR 5-year impact factor: 2.673, position 29 of 247 in Economics.

(e) S. Pineda and A. J. Conejo, "Managing producer risks using options", submitted to *Energy Economics*. JCR 5-year impact factor: 2.673, position 29 of 247 in Economics.

12. Additionally, part of the work developed in this dissertation has been included in the chapter entitled Futures Market Trading for Electricity Producers and Retailers by A. J. Conejo, R. García-Bertrand, M. Carrión and S. Pineda. This chapter in included in the book by S. Rebennack, P. M. Pardalos, M. V. F. Pereira and N. A. Iliadis denominated *Handbook of Power Systems*. Springer-Verlag, Heidelberg, 2010. ISBN: 978-3-642-02492-4.

7.3 Future Work

Suggestions for future work resulting from the study carried out in this thesis are listed below:

1. All the decision-making models described in this dissertation can only be used by price-taker power producers, whose decisions on the sale of electricity in the pool or in the derivatives market do not alter the resulting clearing prices. Therefore, one relevant improvement to the proposed models is their extension to consider power producers that may affect electricity prices in the different markets. Specifically:

   (a) The modeling of the relationships between the power quantity sold in the pool, through forward contracts, or through electricity options and the clearing prices in each one of these markets.

   (b) The modeling of the impact of generating unit failures on electricity pool prices by generating a joint scenario set that considers the statistical dependence between these two stochastic processes.

2. The consideration of technical constraints of generating units such as ramping limits and minimum up and down times, as well as the study of their influence on the optimal decisions made by power producers.

3. The consideration of the relationship between the prices of forward contracts and the pool price through the study and analysis of the risk premium in the electricity market.

4. The investigation of techniques to generate scenarios that characterize the evolution of the forward contract prices.

5. The properties of different risk measures for two-stage stochastic programming problems are well established in the technical literature. However, further research needs to be made regarding the adequate use of risk measures in multi-stage stochastic programming problems.

6. Although detailed simulation results show the ability of the proposed techniques in identifying a reduced number of scenarios that efficaciously represent the uncertainty involved in electricity-market decision-making problems, further research is required to develop a rigorous mathematical framework to generalize the use of these techniques or, at least, to state their range of application.

7. The scenario reduction techniques proposed in this thesis have been validated for two-stage stochastic programming problems and therefore, fur-

ther research is required to apply these procedures to multi-stage stochastic programming models.

8. Alternatively to scenario reduction, it is worth exploring other procedures such as decomposition techniques to make the resulting stochastic optimization problems tractable and solvable in reasonable computational times.

9. The study of more complex electricity derivatives such as American or exotic options.

10. The modeling of other type of derivatives relevant to power producers, like fuel or weather derivatives.

Appendix A

On Price Aggregation

This appendix is devoted to illustrate a procedure to efficiently aggregate the 24 hourly values representing the pool price evolution throughout one day into a smaller number of multi-hour price values. The duration in hours of the resulting time steps is determined so as to minimize the average error between the actual hourly prices and the aggregated ones over all possible scenarios.

Let $\lambda^P_{(\omega,t)}$ be the realization of the hourly pool price during time step t and scenario ω, with $t = 1, \ldots, 24$. $L_{(t)}$ represents the duration of time step t (in hours).

Consider that, due to the computational burden of solving a certain decision-making model, we need to aggregate the 24 hours of a given day into N_D periods. In that case, the aggregated pool prices are denoted by $\hat{\lambda}^P_{(\omega,\tau)}$, where ω and τ are indices in the set of scenarios and time steps, respectively. Note that the new time index τ varies from 1 to N_A. Moreover, $\hat{L}_{(\tau)}$ represents the duration in hours of the new time steps.

As an illustrative example, Fig. A.1 shows the 24 hourly pool prices and the three aggregated prices each covering a time interval of 8 hours. Each aggregated price is determined as the average value of its corresponding 8 hourly prices.

The question that this Appendix is intended to answer is: *Is there an efficient way of determining the duration of each of the three time steps to better represent the pool price during that day?* To answer this question, we need first to establish how to measure mathematically the closeness of the aggregated prices to the actual ones. In this sense, we use the summation of the squared errors defined as the difference between the hourly price during each time step t and the corresponding aggregated price for the same time step

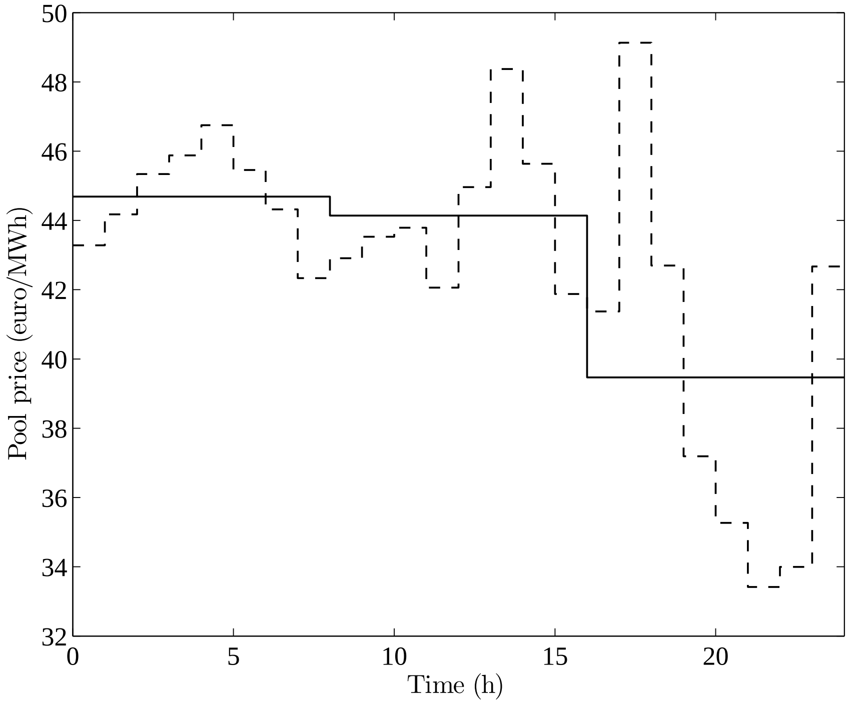


Figure A.1: Example of pool price aggregation

t. If only one scenario ω is considered, this error is calculated as follows

$$\mathrm{Error}_{(\omega)} = \sum_{t=1}^{24} (\lambda^P_{(\omega,t)} - \hat{\lambda}^P_{(\omega,t)})^2, \tag{A.1}$$

where $\hat{\lambda}^P_{(\omega,t)}$ is the aggregated pool price in time step t. If more than one scenario is considered, the average of the summation of the squared errors is

$$\mathrm{Error}_T = \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \Big(\sum_{t=1}^{24} \big(\lambda^P_{(\omega,t)} - \hat{\lambda}^P_{(\omega,t)}\big)^2 \Big), \tag{A.2}$$

where $\pi_{(\omega)}$ is the probability of occurrence of scenario ω.

An optimization model seeking to minimize the average of the summation of the squared errors if the 24 hourly prices are aggregated into N_D time steps is formulated as follows

$$\underset{\nu_{(t)},\hat{\lambda}^P_{(\omega,t)}}{\mathrm{Minimize}} \sum_{\omega=1}^{N_\Omega} \pi_{(\omega)} \Big(\sum_{t=1}^{24} \big(\lambda^P_{(\omega,t)} - \hat{\lambda}^P_{(\omega,t)}\big)^2 \Big) \tag{A.3}$$

subject to

$$\hat{\lambda}^P_{(\omega,t)}(1-\nu_{(t)}) = \hat{\lambda}^P_{(\omega,t-1)}(1-\nu_{(t)}), \qquad \forall t = 2, ..., 24 \qquad \text{(A.4)}$$

$$\sum_{t=1}^{24} \nu_{(t)} = N_D \qquad \text{(A.5)}$$

$$\nu_{(t=1)} = 1 \qquad \text{(A.6)}$$

$$\nu_{(t)} \in \{0, 1\}, \qquad \forall t. \qquad \text{(A.7)}$$

Objective function (A.3) is the average value over scenarios of the daily squared errors of the aggregated prices with respect to the hourly ones. Constraint (A.4) enforces that if $\nu_{(t)}$ is equal to 0, the aggregated pool prices of time step t and time step $t-1$ are the same. On the contrary, if $\nu_{(t)}$ is equal to 1, these aggregated prices can take different values. Constraint (A.5) forces the sum of $\nu_{(t)}$ to be equal to the final number of time steps, thus obtaining only N_D different aggregated prices to approximate the actual 24 hourly pool prices. Since the objective function minimizes the squared error, the aggregated values are guaranteed to be equal to the average pool price during their corresponding hours. Without loss of generality, constraint (A.6) fixes the value of $\nu_{(t_1)}$ to 1. Constraint (A.7) is a binary variable declaration.

The above optimization problem includes a non-linear constraint (A.4), in which the continuous variable $\hat{\lambda}^P_{(\omega,t)}$ is multiplied by the binary variable $\nu_{(t)}$. However, the product between a continuous and a binary variable can be easily linearized [185]. Thus, optimization model (A.3)–(A.7) turns into a mixed-integer quadratic programming problem that can be easily solved using commercial software.

Once optimization model (A.3)–(A.7) is solved, the duration of the new time steps $\hat{L}_{(\tau)}$ is determined based on the values of the binary variables $\nu_{(t)}$, which identify the hours in which the aggregated prices change.

Fig. A.2 illustrates the optimal aggregation of 24 hourly pool prices into 3 time steps according to the proposed procedure. Note that the third time step only spans the hour 24th.

If the study horizon spans more than one day, separate optimization problems have to be solved to determine the durations of the time steps corresponding to each day. Although optimization problem (A.3)–(A.7) can be formulated to find out the optimal time step durations for the whole study horizon ($t = t_1, ..., N_T$), by making the summation of $\nu_{(t)}$ over the study horizon equal to $\frac{N_T}{24} N_D$, this way to proceed may significantly increase the computational burden of the resulting optimization problem or make it intractable. In this

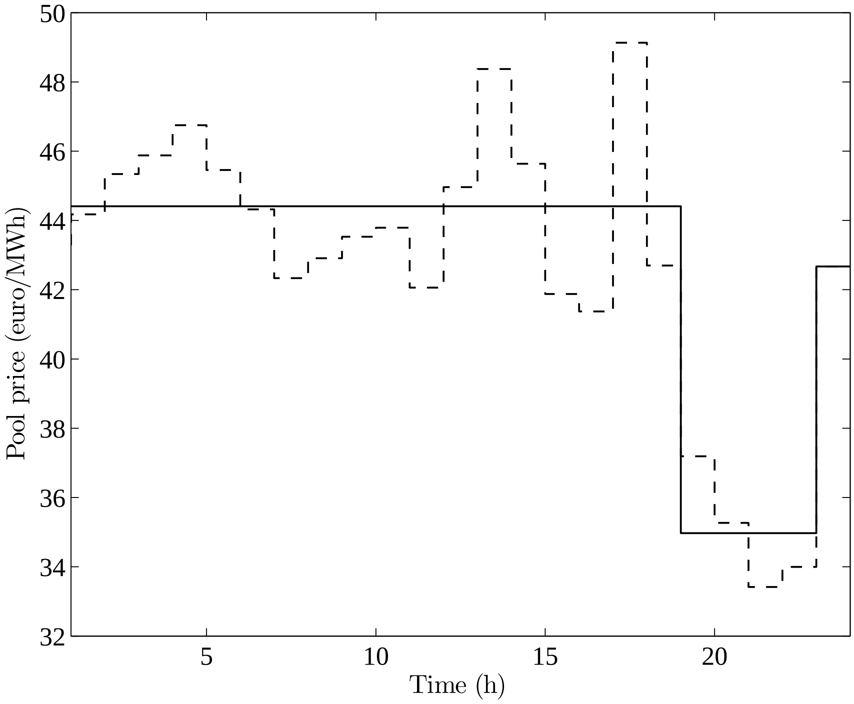


Figure A.2: Optimal pool price aggregation

thesis, we separately determine the optimal time step durations for each day of the considered study horizon.

Appendix B

Forward Selection Algorithm

The three scenario reduction techniques described in Chapter 3 are based on the *forward selection* algorithm provided in [83].

Denoting the original scenario set as Ω and the reduced scenario set, target of the search, as Ω^*_S, the forward selection algorithm works as follows:

- *Step 0*: This initial step consists in computing the function $c(\omega,\omega')$ defining the Kantorovich distance (3.6) for each pair of scenarios. For this, expression (3.7), (3.8) or (3.9) is used depending on the scenario reduction technique to be implemented, i.e., Technique A, Technique B, or Technique C, respectively.

- *Step 1*: The iterative process begins with the choice of a starting scenario, that is, with the scenario from which the reduced scenario set is built. Mathematically, the starting scenario (ω_1) is obtained from

$$\omega_1 = \arg\Big\{\min_{\omega'\in\Omega}\sum_{\omega\in\Omega}\pi_{(\omega)}\mathrm{c}(\omega,\omega')\Big\}. \tag{B.1}$$

  The starting scenario can be interpreted as the most equidistant one from the rest. In other words, this first scenario can be seen as the *average* scenario.

- *Step i*: Starting from the scenario selected in step 1, in each iteration a new scenario is added to the reduced scenario set until it is considered to be close enough to the original one. For this purpose, this selection is carried out using

$$\omega_i = \arg\left\{\min_{\omega' \in \Omega_J^{[i-1]}} \sum_{\omega \in \Omega_J^{[i-1]} \setminus \{\omega'\}} \pi_\omega \min_{\omega'' \in \Omega_S^{[i-1]} \bigcup \{\omega\}} \mathrm{c}(\omega, \omega'')\right\}, \tag{B.2}$$

where $\Omega_J^{[i]}$ represents the set including those scenarios which have not been selected in the first i steps of the algorithm and $\Omega_S^{[i]}$ symbolizes the set including the selected scenarios up to step i. Note that $\Omega_J^{[i]} \cup \Omega_S^{[i]} = \Omega$, $\Omega_J^{[0]} = \Omega$, $\Omega_S^{[0]} = \emptyset$, $\Omega_J^{[i]} = \Omega_J^{[i-1]} \setminus \{\omega_i\}$ and $\Omega_S^{[i]} = \Omega_S^{[i-1]} \cup \{\omega_i\}$.

This step is repeated $N_{\Omega_S^*} - 1$ times, where $N_{\Omega_S^*}$ is the number of scenarios comprising the reduced set Ω_S^*.

- *Step* $N_{\Omega_S^*} + 1$: In this step, an optimal redistribution of probabilities is carried out. It consists of adding the probabilities of those scenarios which have not been finally selected $\left(\omega \in \Omega_J^*\right)$ to the probabilities of those comprising the reduced set $(\omega \in \Omega_S^*)$.

  Mathematically, this redistribution of probabilities can be accomplished as follows:

$$\pi_{(\omega)}^* = \pi_{(\omega)} + \sum_{\omega' \in J(\omega)} \pi_{(\omega')}, \tag{B.3}$$

  where $J(\omega)$ is defined as the set of scenarios $\omega' \in \Omega_J^*$ such that $\omega = \arg \min_{\omega'' \in \Omega_S^*} \mathrm{c}(\omega'', \omega')$.

  In other words, the probability of each non-selected scenario is added to the probability of the closest selected scenario according to the function $c(\omega, \omega')$. Thus, the reduced scenario set is made up of the scenarios $\omega \in \Omega_S^*$ with associated probability $\pi_{(\omega)}^*$.

An efficient implementation of the algorithm described above is presented next and is referred to as *fast forward selection* algorithm, [77,83]. Specifically, the improvement of the fast forward selection is obtained by updating the cost function $c(\omega, \omega')$ each time a new scenario is selected, thus substantially reducing the computational burden associated with the search (B.2).

Fast Forward Selection Algorithm

Step 0) Compute the distances of scenario pairs $c(\omega, \omega')$ according to (3.7), (3.8), or (3.9).

Step 1) Compute $d_{(\omega)} = \sum_{\substack{\omega'=1 \\ \omega' \neq \omega}}^{N_\Omega} \pi_{(\omega')} c(\omega, \omega'); \;\; \forall \omega \in \Omega$

Select $\omega_1 \in \arg \min_{\omega \in \Omega} d_{(\omega)}$

Set $\Omega_J^{[1]} = \{1, \ldots, N_\Omega\} \setminus \omega_1$.

Step i) Compute

$c^{[i]}(\omega, \omega') = \min\Big\{ c^{[i-1]}(\omega, \omega'), c^{[i-1]}(\omega, \omega_{i-1}) \Big\}; \; \forall \omega, \omega' \in \Omega_J^{[i-1]}$

$d_{(\omega)}^{[i]} = \sum_{\omega' \in \Omega_J^{[i-1]} \setminus \{\omega\}} \pi_{(\omega')} c^{[i]}(\omega', \omega); \;\; \forall \omega \in \Omega_J^{[i-1]}$

Select $\omega_i \in \arg \min_{\omega \in \Omega_J^{[i-1]}} d_{(\omega)}^{[i]}$

Set $\Omega_J^{[i]} = \Omega_J^{[i-1]} \setminus \omega_i$

Repeat until the cardinal of $\Omega_S^{[i]}$ is equal to $N_{\Omega_S^*}$.

Step $N_{\Omega_S^*} + 1$) $\Omega_J^* = \Omega_J^{[N_{\Omega_J^*}]}$ is the final set of indices of discarded scenarios.

$\Omega_S^* = \Omega \setminus \Omega_J^*$ is the set of indices of selected scenarios.

The probabilities of selected scenarios $\omega \in \Omega_S^*$ are computed as:

$\pi_{(\omega)}^* \leftarrow \pi_{(\omega)} + \sum_{\omega' \in J(\omega)} \pi_{(\omega')}; \;\; \forall \omega \in \Omega_S^*$, where

$J(\omega) = \{\omega' \in \Omega_J^* | \omega = j(\omega')\}, \;\; j(\omega') \in \arg \min_{\omega'' \in \Omega_S^*} c(\omega'', \omega')$.

Bibliography

[1] Electricity Market of Norway, Denmark, Sweden and Finland, NordPool (2011) [Online]. Available at `http://www.nordpool.com/`.

[2] European Energy Exchange, EEX (2011) [Online]. Available at `http://www.eex.com/`.

[3] Futures Market of the Iberian Electricity Market, OMIP (2011) [Online]. Available at `http://www.omip.pt/`.

[4] Market Operator of the Electricity Market of Mainland Spain, OMEL (2011) [Online]. Available at `http://www.omel.es/`.

[5] The GAMS Development Corporation Website. (2011) [Online]. Available at `http://www.gams.com/`.

[6] The ILOG CPLEX Website. (2011) [Online]. Available at `http://www.ilog.com/products/cplex/` .

[7] G. Anders, R. Entriken, and P. Nitu. Risk assessment and financial management tutorial. In *Proceedings IEEE Power Engineering Society Winter Meeting*, 1999.

[8] E.J. Anderson and X. Hu. Forward contracts and market power in an electricity market. *International Journal of Industrial Organization*, 26(3):679–694, 2008.

[9] E.J. Anderson, X. Hu, and D. Winchester. Forward contracts in electricity markets: The Australian experience. *Energy Policy*, 35(5):3089–3103, 2007.

[10] E.J. Anderson and H. Xu. Optimal supply functions in electricity markets with option contracts and non-smooth costs. *Mathematical Methods of Operations Research*, 63(3):387–411, 2006.

[11] C.H. Antunes, A.G. Martins, and I.S. Brito. A multiple objective mixed integer linear programming model for power generation expansion planning. *Energy*, 29(4):613–627, 2004.

[12] JM Arroyo and AJ Conejo. Optimal response of a thermal unit to an electricity spot market. *IEEE Transactions on Power Systems*, 15(3):1098–1104, 2000.

[13] P. Artzner, F. Delbaen, J.M. Eber, and D. Heath. Coherent measures of risk. *Mathematical Finance*, 9(3):203–228, 1999.

[14] F. Azevedo, Z.A. Vale, P.B. Oliveira, and H.M. Khodr. A long-term risk management tool for electricity markets using swarm intelligence. *Electric Power Systems Research*, 80(4):380–389, 2010.

[15] A. Baillo, M. Ventosa, M. Rivier, and A. Ramos. Optimal offering strategies for generation companies operating in electricity spot markets. *IEEE Transactions on Power Systems*, 19(2):745–753, 2004.

[16] L.A. Barroso and A.J. Conejo. Decision making under uncertainty in electricity markets. In *Proceedings IEEE Power Engineering Society General Meeting*, 2006.

[17] G.N. Bathurst, J. Weatherill, and G. Strbac. Trading wind generation in short term energy markets. *IEEE Transactions on Power Systems*, 17(3):782–789, 2002.

[18] M. Beenstock. Generators and the cost of electricity outages. *Energy Economics*, 13(4):283–289, 1991.

[19] F.E. Benth, J.Š. Benth, and S. Koekebakker. *Stochastic modelling of electricity and related markets*. World Scientific Pub. Co. Inc., 2008.

[20] F.E. Benth, Á. Cartea, and R. Kiesel. Pricing forward contracts in power markets by the certainty equivalence principle: explaining the sign of the market risk premium. *Journal of Banking & Finance*, 32(10):2006–2021, 2008.

[21] F.E. Benth and S. Koekebakker. Stochastic modeling of financial electricity contracts. *Energy Economics*, 30(3):1116–1157, 2008.

[22] H. Bessembinder and M.L. Lemmon. Equilibrium pricing and optimal hedging in electricity forward markets. *The Journal of Finance*, 57(3):1347–1382, 2002.

[23] K. Bhanot. Value of an option to purchase electric power–the case of uncertain consumption. *Energy Economics*, 24(2):121–137, 2002.

[24] R. Billinton and R.N. Allan. *Reliability evaluation of power systems.* New York and London: Plenum Press, 1996.

[25] R. Billinton and R. Karki. Application of Monte Carlo simulation to generating system well-being analysis. *IEEE Transactions on Power Systems*, 14(3):1172–1177, 2002.

[26] R. Billinton and W. Li. *Reliability assessment of electric power systems using Monte Carlo methods.* Plenum Publishing Corporation, 1994.

[27] J.R. Birge. The value of the stochastic solution in stochastic linear programs with fixed recourse. *Mathematical Programming*, 24(1):314–325, 1982.

[28] J.R. Birge and F. Louveaux. *Introduction to stochastic programming.* Springer Verlag, 1997.

[29] R. Bjorgan, C.C. Liu, and J. Lawarree. Financial risk management in a competitive electricity market. *IEEE Transactions on Power Systems*, 14(4):1285–1291, 2002.

[30] T. Bjorkvoll, S.E. Fleten, M.P. Nowak, A. Tomasgard, and S.W. Wallace. Power generation planning and risk management in a liberalised market. In *Proceedings IEEE Porto Power Tech Conference*, 2001.

[31] A. Botterud, A.K. Bhattacharyya, and M. Ilic. Futures and spot prices– an analysis of the Scandinavian electricity market. In *Proceedings North American Power Symposium*, 2002.

[32] A. Botterud, M.D. Ilic, and I. Wangensteen. Optimal investments in power generation under centralized and decentralized decision making. *IEEE Transactions on Power Systems*, 20(1):254–263, 2005.

[33] G.E.P. Box and G.M. Jenkins. *Time series analysis: forecasting and control.* Prentice Hall PTR Upper Saddle River, NJ, USA, 1994.

[34] D.W. Bunn. *Modelling prices in competitive electricity markets*. J. Wiley, 2004.

[35] M. Burger, B. Klar, A. Müller, and G. Schindlmayr. A spot market model for pricing derivatives in electricity markets. *Quantitative Finance*, 4(1):109–122, 2004.

[36] Á. Capitán and C. Rodríguez. Analysis of the efficiency of the Iberian power futures market. *Energy Policy*, 37(9):3566–3579, 2009.

[37] M. Carrión, A.J. Conejo, and J.M. Arroyo. Forward contracting and selling price determination for a retailer. *IEEE Transactions on Power Systems*, 22(4):2105–2114, 2007.

[38] M. Carrión, A.B. Philpott, A.J. Conejo, and J.M. Arroyo. A stochastic programming approach to electric energy procurement for large consumers. *IEEE Transactions on Power Systems*, 22(2):744–754, 2007.

[39] E. Caruso, M. Dicorato, A. Minoia, and M. Ilic. The impact of physical bilateral agreements on the wholesale electricity price. In *Proc. IREP Symp. Bulk Power System Dynamics and Control–VI*, 2004.

[40] X. Chen, Y. He, Y.H. Song, Y. Nakanishi, C. Nakanishi, S. Takahashi, and Y. Sekine. Study of impacts of physical contracts and financial contracts on bidding strategies of GENCOs. *International Journal of Electrical Power & Energy Systems*, 26(9):715–723, 2004.

[41] T.S. Chung, S.H. Zhang, C.W. Yu, and K.P. Wong. Electricity market risk management using forward contracts with bilateral options. *Proceedings IEE Generation, Transmission and Distribution*, 150(5):588–594, 2003.

[42] A.J. Conejo and M. Carrión. Risk-constrained electricity procurement for a large consumer. *Proceedings IEE Generation, Transmission and Distribution*, 153(4):407–413, 2006.

[43] A.J. Conejo, J. Contreras, R. Espínola, and M.A. Plazas. Forecasting electricity prices for a day-ahead pool-based electric energy market. *International Journal of Forecasting*, 21(3):435–462, 2005.

[44] A.J. Conejo, R. García-Bertrand, M. Carrión, A. Caballero, and A. de Andrés. Optimal involvement in futures markets of a power producer. *IEEE Transactions on Power Systems*, 23(2):703–711, 2008.

[45] A.J. Conejo, F.J. Nogales, and J.M. Arroyo. Price-taker bidding strategy under price uncertainty. *IEEE Transactions on Power Systems*, 17(4):1081–1088, 2002.

[46] J. Contreras, R. Espínola, F.J. Nogales, and A.J. Conejo. ARIMA models to predict next-day electricity prices. *IEEE Transactions on Power Systems*, 18(3):1014–1020, 2003.

[47] R. Dahlgren, C.C. Liu, and J. Lawarree. Risk assessment in energy trading. *IEEE Transactions on Power Systems*, 18(2):503–511, 2003.

[48] G.B. Dantzig. Linear programming under uncertainty. *Management Science*, 1(3):197–206, 1955.

[49] D. Das and B.F. Wollenberg. Risk assessment of generators bidding in day-ahead market. *IEEE Transactions on Power Systems*, 20(1):416–424, 2005.

[50] D. Das and B.F. Wollenberg. Minimizing forced outage risk in generator bidding. *European Transactions on Electrical Power*, 17(4):347–357, 2007.

[51] S.J. Deng, B. Johnson, and A. Sogomonian. Exotic electricity options and the valuation of electricity generation and transmission assets. *Decision Support Systems*, 30(3):383–392, 2001.

[52] S.J. Deng and S.S. Oren. Electricity derivatives and risk management. *Energy*, 31(6-7):940–953, 2006.

[53] M. Denton, A. Palmer, R. Masiello, and P. Skantze. Managing market risk in energy. *IEEE Transactions on Power Systems*, 18(2):494–502, 2003.

[54] B.S. Dhillon. *Applied reliability and quality: fundamentals, methods and procedures*. Springer Verlag, 2007.

[55] P. Diko, S. Lawford, and V. Limpens. Risk premia in electricity forward prices. *Nonlinear Analysis*, 10(3):7, 2006.

[56] H.I. Doerpinghaus and W.T. Moore. Insurance contract valuation, experience rating, and asymmetric information. *Journal of Financial and Strategic Decisions*, 7(2):95–103, 1994.

[57] J. Dupačová, G. Consigli, and S.W. Wallace. Scenarios for multistage stochastic programs. *Annals of Operations Research*, 100(1):25–53, 2000.

[58] J. Dupačová, N. Gröwe-Kuska, and W. Römisch. Scenario reduction in stochastic programming. *Mathematical Programming*, 95(3):493–511, 2003.

[59] J. Dupačová and K. Sladký. Comparison of multistage stochastic programs with recourse and stochastic dynamic programs with discrete time. *Journal of Applied Mathematics and Mechanics*, 82(11-12):753–765, 2002.

[60] A. Eichhom and W. Römisch. Mean-risk optimization models for electricity portfolio management. In *International Conference on Probabilistic Methods Applied to Power Systems*, pages 1–7. IEEE, 2006.

[61] A. Eichhorn and W. Römisch. Polyhedral risk measures in stochastic programming. *SIAM Journal on Optimization*, 16(1):69–95, 2006.

[62] L.F. Escudero, A. Garín, M. Merino, and G. Pérez. The value of the stochastic solution in multistage problems. *Top*, 15(1):48–64, 2007.

[63] A. Eydeland and K. Wolyniec. *Energy and power risk management: New developments in modeling, pricing, and hedging.* John Wiley & Sons Inc, 2003.

[64] J.C. Felli and G.B. Hazen. Sensitivity analysis and the expected value of perfect information. *Medical Decision Making*, 18(1):95, 1998.

[65] S.E. Fleten and T.K. Kristoffersen. Short-term hydro-power production planning by stochastic programming. *Computers & Operations Research*, 35(8):2656–2671, 2008.

[66] S.E. Fleten and J. Lemming. Constructing forward price curves in electricity markets. *Energy Economics*, 25(5):409–424, 2003.

[67] S.E. Fleten, S.W. Wallace, and W.T. Ziemba. Hedging electricity portfolios via stochastic programming. *Decision Making under Uncertainty: Energy and Power*, 128:71–93, 2002.

[68] H. Föllmer and A. Schied. Convex measures of risk and trading constraints. *Finance and Stochastics*, 6(4):429–447, 2002.

[69] M. Frittelli and E. Rosazza Gianin. Putting order in risk measures. *Journal of Banking & Finance*, 26(7):1473–1486, 2002.

[70] E. Fumagalli, J.W. Black, I. Vogelsang, and M. Ilic. Quality of service provision in electric power distribution systems through reliability insurance. *IEEE Transactions on Power Systems*, 19(3):1286–1293, 2004.

[71] D. Furió and V. Meneu. Expectations and forward risk premium in the Spanish deregulated power market. *Energy Policy*, 38(2):784–793, 2010.

[72] H.I. Gassmann and A. Prékopa. On stages and consistency checks in stochastic programming. *Operations Research Letters*, 33(2):171–175, 2005.

[73] T.W. Gedra. Optional forward contracts for electric power markets. *IEEE Transactions on Power Systems*, 9(4):1766–1773, 2002.

[74] K. Ghosh and V.C. Ramesh. An options model for electric power markets. *International Journal of Electrical Power & Energy Systems*, 19(2):75–85, 1997.

[75] R. Green. The electricity contract market in England and Wales. *The Journal of Industrial Economics*, 47(1):107–124, 1999.

[76] C. Grigg, P. Wong, P. Albrecht, R. Allan, M. Bhavaraju, R. Billinton, Q. Chen, C. Fong, S. Haddad, S. Kuruganty, et al. The IEEE reliability test system-1996. A report prepared by the reliability test system task force of the application of probability methods subcommittee. *IEEE Transactions on Power Systems*, 14(3):1010–1020, 1999.

[77] N. Gröwe-Kuska, H. Heitsch, and W. Römisch. Scenario reduction and scenario tree construction for power management problems. In *Proceedings IEEE PowerTech*, 2003.

[78] X. Guan, J. Wu, F. Gao, and G. Sun. Optimal generation portfolio management for futures and spot market. In *Proceedings IEEE Power Engineering Society General Meeting*, 2006.

[79] N. Gülpinar, B. Rustem, and R. Settergren. Simulation and optimization approaches to scenario tree generation. *Journal of Economic Dynamics and Control*, 28(7):1291–1315, 2004.

[80] G. Haarbrücker and D. Kuhn. Valuation of electricity swing options by multistage stochastic programming. *Automatica*, 45(4):889–899, 2009.

[81] H. Haghighat, H. Seifi, and A.R. Kian. On the self-scheduling of a power producer in uncertain trading environments. *Electric Power Systems Research*, 78(3):311–317, 2008.

[82] L. Hai-feng and Z. Hao. Study on risk evasion in electricity market. In *Proceedings IEEE International Conference on Electric Utility Deregulation, Restructuring and Power Technologies*, volume 2, pages 495–499. IEEE, 2004.

[83] H. Heitsch and W. Römisch. Scenario reduction algorithms in stochastic programming. *Computational Optimization and Applications*, 24(2):187–206, 2003.

[84] H. Heitsch and W. Römisch. A note on scenario reduction for two-stage stochastic programs. *Operations Research Letters*, 35(6):731–738, 2007.

[85] H. Heitsch and W. Römisch. Scenario tree modeling for multistage stochastic programs. *Mathematical Programming*, 118(2):371–406, 2009.

[86] H. Heitsch and W. Römisch. Scenario tree reduction for multistage stochastic programs. *Computational Management Science*, 6(2):117–133, 2009.

[87] H. Heitsch, W. Römisch, and C. Strugarek. Stability of multistage stochastic programs. *SIAM Journal on Optimization*, 17(2):511–525, 2007.

[88] R. Henrion, C. Küchler, and W. Römisch. Scenario reduction in stochastic programming with respect to discrepancy distances. *Computational Optimization and Applications*, 43(1):67–93, 2009.

[89] J.L. Higle. Stochastic programming: Optimization when uncertainty matters. *Tutorials in Operations Research*, pages 1–24, 2005.

[90] J.L. Higle and S. Sen. Stochastic decomposition: An algorithm for two-stage linear programs with recourse. *Mathematics of Operations Research*, 16(3):650–669, 1991.

[91] K. Høyland, M. Kaut, and S.W. Wallace. A heuristic for moment-matching scenario generation. *Computational Optimization and Applications*, 24(2-3):169–185, 2003.

[92] K. Høyland and S.W. Wallace. Generating scenario trees for multistage decision problems. *Management Science*, 47(2):295–307, 2001.

[93] J. Hull. *Options, futures and other derivatives.* Pearson Prentice Hall, 2009.

[94] R.A. Jarrow and G.S. Oldfield. Forward contracts and futures contracts. *Journal of Financial Economics*, 9(4):373–382, 1981.

[95] J.N. Jiang, H. Chen, and M.L. Baughman. Evaluation of insurance on generation forced outages. In *Proceedings IEEE Power Engineering Society General Meeting*, page 7, 2006.

[96] N. Jiang. *Methods for market analysis, risk management and finance in the deregulated power industry.* PhD thesis, The University of Texas at Austin, 2005.

[97] P. Jirutitijaroen and C. Singh. Stochastic programming approach for unit availability consideration in multi-area generation expansion planning. In *Proceedings IEEE Power Engineering Society General Meeting*, pages 1–5, 2007.

[98] A.G. Kagiannas, D.T. Askounis, and J. Psarras. Power generation planning: a survey from monopoly to competition. *International Journal of Electrical Power & Energy Systems*, 26(6):413–421, 2004.

[99] P. Kall and J. Mayer. On solving stochastic linear programming problems. *Lectures Notes in Economics and Mathematical Systems*, pages 329–344, 1998.

[100] P. Kall and J. Mayer. *Stochastic linear programming: models, theory, and computation*. Springer, 2005.

[101] P. Kall and S.W. Wallace. *Stochastic programming*. Wiley, 1994.

[102] M. Kaut and S.W. Wallace. Evaluation of scenario-generation methods for stochastic programming. *SPEPS, Paper 14-2003. Available at* `http://edoc.hu-berlin.de/browsing/speps/`.

[103] R.J. Kaye, H.R. Outhred, and C.H. Bannister. Forward contracts for the operation of an electricity industry under spot pricing. *IEEE Transactions on Power Systems*, 5(1):46–52, 1990.

[104] J. Keppo. Pricing of electricity swing options. *The Journal of Derivatives*, 11(3):26–43, 2004.

[105] J. Kettunen. *Applications of Stochastic Modeling for Investment Decision-Making under Market Uncertainties*. PhD thesis, Helsinki University of Technology, 2009.

[106] S.E. Khatib and F.D. Galiana. Negotiating bilateral contracts in electricity markets. *IEEE Transactions on Power Systems*, 22(2):553–562, 2007.

[107] R.E. Kihlstrom and A.E. Roth. Risk aversion and the negotiation of insurance contracts. *Journal of Risk and Insurance*, 49(3):372–387, 1982.

[108] D.S. Kirschen. Demand-side view of electricity markets. *IEEE Transactions on Power Systems*, 18(2):520–527, 2003.

[109] D.S. Kirschen and G. Strbac. *Fundamentals of power system economics*. John Wiley & Sons, Ltd., Chichester, West Sussex, UK, 2004.

[110] R. Kouwenberg. Scenario generation and stochastic programming models for asset liability management. *European Journal of Operational Research*, 134(2):279–292, 2001.

[111] C. Küchler and S. Vigerske. Numerical evaluation of approximation methods in stochastic programming. *Optimization*, 59(3):401–415, 2010.

[112] D. Kuhn. Aggregation and discretization in multistage stochastic programming. *Mathematical Programming Series A*, 113(1):61–94, 2008.

[113] D.W. Lane, C.W. Richter Jr, and G.B. Sheblé. Modeling and evaluating electricity options markets with intelligent agents. In *Proceedings International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*, pages 203–208. IEEE, 2002.

[114] J. Lemming. *Risk and investment management in liberalized electricity markets*. PhD thesis, Technical University of Denmark, 2005.

[115] T. Li, M. Shahidehpour, and Z. Li. Risk-constrained bidding strategy with stochastic unit commitment. *IEEE Transactions on Power Systems*, 22(1):449–458, 2007.

[116] W. Li. *Risk assessment of power systems: models, methods, and applications*. Wiley-IEEE Press, 2005.

[117] D. Lien and Y.K. Tse. A survey on physical delivery versus cash settlement in futures contracts. *International Review of Economics & Finance*, 15(1):15–29, 2006.

[118] M. Liu and F.F. Wu. Risk management in a competitive electricity market. *International Journal of Electrical Power & Energy Systems*, 29(9):690–697, 2007.

[119] F.A. Longstaff and A.W. Wang. Electricity forward prices: a high-frequency empirical analysis. *The Journal of Finance*, 59(4):1877–1900, 2004.

[120] F.V. Louveaux and R. Schultz. Stochastic integer programming. *Handbooks in operations research and management science*, 10:213–266, 2003.

[121] Z. Lu and Z.Y. Dong. Electricity future market efficiency testing: The characteristics of electricity prices. In *Proceedings 7th International Power Engineering Conference*, pages 868–873. IEEE, 2005.

[122] J.J. Lucia and E.S. Schwartz. Electricity prices and power derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives Research*, 5(1):5–50, 2002.

[123] R. Mansini, W. Ogryczak, and M.G. Speranza. Conditional Value-at-Risk and related linear programming models for portfolio optimization. *Annals of Operations Research*, 152(1):227–256, 2007.

[124] H.M. Markowitz. *Portfolio Selection, Efficient Diversification of Investments*. John Wiley & Sons, New York, 1959.

[125] J.F. Marshall. *Dictionary of financial engineering.* John Wiley & Sons Inc, 2000.

[126] B. Mo, A. Gjelsvik, and A. Grundt. Integrated risk management of hydro power scheduling and contract management. *IEEE Transactions on Power Systems*, 16(2):216–221, 2002.

[127] J.M. Morales, A.J. Conejo, and J. Pérez-Ruiz. Short-term trading for a wind power producer. *IEEE Transactions on Power Systems*, 25(1):554–564, 2010.

[128] J.M. Morales, S. Pineda, A.J. Conejo, and M. Carrión. Scenario reduction for futures market trading in electricity markets. *IEEE Transactions on Power Systems*, 24(2):878–888, 2009.

[129] F.J. Nogales and A.J. Conejo. Electricity price forecasting through transfer function models. *Journal of the Operational Research Society*, 57(4):350–356, 2006.

[130] F.J. Nogales, J. Contreras, A.J. Conejo, and R. Espínola. Forecasting next-day electricity prices by time series models. *IEEE Transactions on Power Systems*, 17(2):342–348, 2002.

[131] M. Olsson and L. Soder. Generation of regulating power price scenarios. In *Proceedings International Conference on Probabilistic Methods Applied to Power Systems*, pages 26–31. IEEE, 2004.

[132] S.S. Oren. Integrating real and financial options in demand-side electricity contracts. *Decision Support Systems*, 30(3):279–288, 2001.

[133] S.S. Oren. Market based risk mitigation: Risk management vs. risk avoidance. In *Proceedings of a White House OSTP/NSF Workshop on Critical Infrastructure Interdependencies*, pages 14–15, 2001.

[134] S.S. Oren and J.A. Doucet. Interruption insurance for generation and distribution of electric power. *Journal of Regulatory Economics*, 2(1):5–19, 1990.

[135] Y. Oum and S.S. Oren. Optimal static hedging of volumetric risk in a competitive wholesale electricity market. *Decision Analysis*, 7(1):107–122, 2010.

[136] Y. Oum, S.S. Oren, and S.J. Deng. Hedging quantity risks with standard power options in a competitive wholesale electricity market. *Naval Research Logistics*, 53(7):697–712, 2006.

[137] H.T. Pao. Forecasting electricity market pricing using artificial neural networks. *Energy Conversion and Management*, 48(3):907–912, 2007.

[138] D. Paravan, G.B. Sheblé, and R. Golob. Price and volume risk management for power producers. In *Proceedings International Conference on Probabilistic Methods Applied to Power Systems*, pages 699–704. IEEE, 2004.

[139] D. Peña, G.C. Tiao, and R.S. Tsay. *A course in time series analysis*. Wiley New York, 2001.

[140] G.C. Pflug. Some remarks on the value-at-risk and the conditional value-at-risk. In S. Uryasev, editor, *Probabilistic Constrained Optimization: Methodology and Applications*. Kluwer Academic Publishers, Norwell, MA, 2000.

[141] G.C. Pflug. Scenario tree generation for multiperiod financial optimization by optimal discretization. *Mathematical Programming Series B*, 89(2):251–271, 2001.

[142] G.C. Pflug and W. Römisch. *Modeling, measuring and managing risk*. World Scientific Pub. Co. Inc., 2007.

[143] A.B. Philpott and E. Pettersen. Optimizing demand-side bids in day-ahead electricity markets. *IEEE Transactions on Power Systems*, 21(2):488–498, 2006.

[144] M. Pietz. Risk premia in the German electricity futures market. In *Proceedings International Conference on Energy and Environment*, pages 160–170. IEEE, 2009.

[145] S. Pineda and A.J. Conejo. Scenario reduction for risk-averse electricity trading. *IET Generation, Transmission & Distribution*, 4(6):694–705, 2010.

[146] P. Pinson, C. Chevallier, and G.N. Kariniotakis. Trading wind generation from short-term probabilistic forecasts of wind power. *IEEE Transactions on Power Systems*, 22(3):1148–1156, 2007.

[147] M.A. Plazas, A.J. Conejo, and F.J. Prieto. Multimarket optimal bidding for a power producer. *IEEE Transactions on Power Systems*, 20(4):2041–2050, 2005.

[148] M. Povh and S.E. Fleten. Modeling long-term electricity forward prices. *IEEE Transactions on Power Systems*, 24(4):1649–1656, 2009.

[149] A. Prékopa. *Stochastic programming.* Springer, 1995.

[150] S.T. Rachev. *Probability metrics and the stability of stochastic models.* Wiley New York, 1991.

[151] S.T. Rachev, S.V. Stoyanov, and F.J. Fabozzi. *Advanced stochastic models, risk assessment, and portfolio optimization: The ideal risk, uncertainty, and performance measures.* John Wiley & Sons Inc, 2008.

[152] C.W. Richter and G.B. Sheblé. Bidding strategies that minimize risk with options and futures contracts. In *Proceedings of the American Power Conference*, volume 60, pages 399–404, 1998.

[153] R.T. Rockafellar and S. Uryasev. Optimization of Conditional Value-at-Risk. *Journal of Risk*, 2(3):21–42, 2000.

[154] R.T. Rockafellar and S. Uryasev. Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance*, 26(7):1443–1471, 2002.

[155] T. Rockafellar. Coherent approaches to risk in optimization under uncertainty. *Tutorials in Operations Research: OR Tools and Applications. Glimpses of Future Technologies*, 2007.

[156] W. Römisch. Stability of stochastic programming problems. *Handbooks in Operations Research and Management Science*, 10:483–554, 2003.

[157] A.E. Roth. A note on risk aversion in a perfect equilibrium model of bargaining. *Econometrica: Journal of the Econometric Society*, 53(1):207–211, 1985.

[158] C. Ruiz and A.J. Conejo. Pool strategy of a producer with endogenous formation of locational marginal prices. *IEEE Transactions on Power Systems*, 24(4):1855–1866, 2009.

[159] A. Ruszczynski and A. Shapiro. Optimization of convex risk functions. *Mathematics of Operations Research*, 31(3):433–452, 2006.

[160] S. Sarykalin, G. Serraino, and S. Uryasev. Value-at-Risk vs. Conditional Value-at-risk in risk management and optimization. *2008 Tutorials in Operations Research: State-of-the-Art Decision-Making Tools in the Information-Intensive Age*, 2008.

[161] S. Sen and J.L. Higle. An introductory tutorial on stochastic linear programming models. *Interfaces*, 29(2):33–61, 1999.

[162] S. Sen, L. Yu, and T. Genc. A stochastic programming approach to power portfolio optimization. *Operations Research*, 54(1):55–72, 2006.

[163] M. Shahidehpour, H. Yamin, and Z. Li. *Market operations in electric power systems*. Wiley Online Library, 2002.

[164] H.A. Shawky, A. Marathe, and C.L. Barrett. A first look at the empirical relation between spot and futures electricity prices in the United States. *Journal of Futures Markets*, 23(10):931–955, 2003.

[165] G.B. Shrestha, B.K. Pokharel, T.T. Lie, and S.E. Fleten. Medium term power planning with bilateral contracts. *IEEE Transactions on Power Systems*, 20(2):627–633, 2005.

[166] S.N. Siddiqi. Project valuation and power portfolio management in a competitive market. *IEEE Transactions on Power Systems*, 15(1):116–121, 2002.

[167] H. Song, C.C. Liu, J. Lawarree, and R.W. Dahlgren. Optimal electricity supply bidding by Markov decision process. *IEEE Transactions on Power Systems*, 15(2):618–624, 2002.

[168] S. Spinler, A. Huchzermeier, and P. Kleindorfer. Risk hedging via options contracts for physical delivery. *OR Spectrum*, 25(3):379–395, 2003.

[169] M.C. Steinbach. Markowitz revisited: Mean-variance models in financial portfolio analysis. *Siam Review*, 43(1):31–85, 2001.

[170] B.R. Szkuta, L.A. Sanabria, and T.S. Dillon. Electricity price short-term forecasting using artificial neural networks. *IEEE Transactions on Power Systems*, 14(3):851–857, 2002.

[171] E. Tanlapco, J. Lawarree, and C.C. Liu. Hedging with futures contracts in a deregulated electricity industry. *IEEE Transactions on Power Systems*, 17(3):577–582, 2002.

[172] A. Tishler, I. Milstein, and C.K. Woo. Capacity commitment and price volatility in a competitive electricity market. *Energy Economics*, 30(4):1625–1647, 2008.

[173] I.A. Ushakov and R.A. Harrison. *Handbook of reliability engineering*. Wiley-Interscience, 1994.

[174] I. Vehvilainen. Basics of electricity derivative pricing in competitive markets. *Applied Mathematical Finance*, 9(1):45–60, 2002.

[175] S. Viaene, R. Veugelers, and G. Dedene. Insurance bargaining under risk aversion. *Economic Modelling*, 19(2):245–259, 2002.

[176] S. Vigerske and I. Nowak. Adaptive discretization of convex multistage stochastic programs. *Mathematical Methods of Operations Research*, 65(2):361–383, 2007.

[177] B.V. Vishnyakov and A.I. Kibzun. Deterministic equivalents for stochastic programming problems with probabilistic criteria. *Automation and Remote Control*, 67(6):945–961, 2006.

[178] S.W. Wallace and W.T. Ziemba. *Applications of stochastic programming*. SIAM Mathematical Programming Series on Optimization, 2005.

[179] C. Weber. *Uncertainty in the electric power industry: methods and models for decision support*. Springer Verlag, New York, 2005.

[180] E.U. Weber and O. Coskunoglu. Descriptive and prescriptive models of decision-making: Implications for the development of decision aids. *IEEE Transactions on Systems, Man and Cybernetics*, 20(2):310–317, 2002.

[181] R.J.B. Wets. Stochastic programs with fixed recourse: The equivalent deterministic program. *SIAM review*, 16(3):309–339, 1974.

[182] R.J.B. Wets. Stochastic programming models: wait-and-see versus here-and-now. In C. Greengard and A. Ruszczynski, editors, *Decision Making under Uncertainty: Energy and Power*. Springer-Verlag, New York, 2002.

[183] R.E. Whaley. *Derivatives: markets, valuation, and risk management*. John Wiley & Sons Inc, 2006.

[184] S. Wilkens and J. Wimschulte. The pricing of electricity futures: Evidence from the European Energy Exchange. *Journal of Futures Markets*, 27(4):387–410, 2007.

[185] H.P. Williams. *Model building in mathematical programming*, volume 4. Wiley New York, 1999.

[186] A.C. Worthington and H. Higgs. The relationship between energy spot and futures prices: Evidence from the Australian electricity market. *The IUP Journal of Applied Economics*, 3(4):65–82, 2004.

[187] H.C. Wu. Pricing European options based on the fuzzy pattern of Black-Scholes formula. *Computers & Operations Research*, 31(7):1069–1081, 2004.

[188] J. Xu, P.B. Luh, F.B. White, E. Ni, and K. Kasiviswanathan. Power portfolio optimization in deregulated electricity markets with risk management. *IEEE Transactions on Power Systems*, 21(4):1653–1662, 2006.

[189] L.Y. Yu, X.D. Ji, and S. Wang. Stochastic programming models in financial optimization: A survey. *Advanced Modeling and Optimization*, 5(1), 2003.

[190] Q. Zhang and H. Zhou. Analysis of forward option trades in electricity markets. In *Proceedings IEEE International Conference on Electric Utility Deregulation, Restructuring and Power Technologies*, volume 2, pages 500–504. IEEE, 2004.

Index