Impact of Unit Failure on Forward Contracting

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Abstract—A generating unit can sell its production in the pool facing the financial risk inherent to volatile pool prices but with no obligation to sell. Alternatively, it can sell its production through the futures market at stable but smaller average prices. Since forward contracts involve an obligation to sell during a specified time period, if the unit fails, it should buy energy from the pool to meet its futures contract commitments. Hence, taking into account its forced outage rate, the unit should determine the appropriate mix of pool and futures-market involvement so that its average profit is maximized for a pre-specified risk level on profit variability. Within this context, this paper analyzes the impact of the degree of unavailability of the generating unit on its forward contracting decisions. A detailed case study illustrates the analysis performed.

Index Terms—Forced Outage Rate (FOR), futures market, pool, power producer, risk, stochastic programming.

NOTATION

The notation used throughout the paper is stated below for quick reference.

A. Variables:

E_{tw}^{G} Energy generated by the unit during period t and scenario w (MWh).

E_{tw}^{P} Energy traded in the pool during period t and scenario w (MWh).

P_c Power sold through forward contract c (MW).

ζ Auxiliary variable used to compute the Conditional Value at Risk, CVaR ($).

η_w Auxiliary variable in scenario w used to compute the CVaR ($).

B. Stochastic variables:

λ_t^{P} Pool price in period t ($/MWh).

k_t Unit availability in period t.

u_1, u_2 Random variables uniformly distributed between 0 and 1.

t_F Time between two consecutive unit failures (h).

t_R Unit repair time (h).

C. Constants:

$\lambda_{tw}^{\mathrm{P}}$ Pool price in period t and scenario w ($/MWh).

k_{tw} Unit availability in period t and scenario w.

C Linear production cost of the unit ($/MWh).

L_{tw} Time duration of period t of scenario w (h).

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MTTF   Mean time to failure (h).

MTTR   Mean time to repair (h).

$P_c^{\mathrm{S,Max}}$ Maximum power (MW) that can be sold through contract c.

P^{Max} Capacity of the unit (MW).

T_c Duration of forward contract c (h).

α Per unit confidence level.

λ_c Energy price of forward contract c ($/MWh).

π_w Probability of occurrence of scenario w.

D. Numbers:

N_C Number of forward contracts.

N_T Number of time periods.

N_W Number of scenarios.

$N_{W,A}$ Number of availability scenarios.

$N_{W,P}$ Number of price scenarios.

E. Sets:

F_t Set of forward contracts available during period t.

I. INTRODUCTION

A generating unit can sell its production in the pool facing the financial risk on profit variability inherent to volatile pool prices. This alternative entails the obligation to sell just one day in advance. Thus, if the unit fails, no additional selling takes places in the pool in days other than the one after the failure. The financial consequences reduce to not being able to sell energy in the pool during the forced outage period.

Alternatively, the unit can sell its production through the futures market at stable but smaller average prices. In this arrangement, the unit acquires the obligation to sell over a specified time horizon (a month, a quarter, etc.). Under this arrangement, if the unit fails, it should buy energy in the pool to meet its contracting selling obligations. This may entail a reduced average profit and a higher financial risk.

Therefore, taking into account its forced outage rate, the unit should determine the optimal mix of pool and forward-market involvement so that its average profit is maximized for a pre-specified risk level on profit variability.

In addressing this problem, two independent sources of uncertainty have to be considered: the pool price and the availability of the generating unit. We address this problem using a stochastic programming model that incorporates the CVaR as risk measure [1], [2].

The aim of this paper is to analyze the impact of the unavailability of the generating unit, measured through its forced outage rate, FOR, on its forward contracting decisions in futures electricity markets.

A description of some relevant references on forward contracting in electricity markets is provided below. References

[3] and [4] illustrate the use of forward contracts to hedge against the risk of profit volatility. In [5], the author explores the effect of the strategic behavior of some market agents on forward contracting. Reference [6] provides a tutorial on risk management in electricity markets, emphasizing the unique features of these markets. Reference [7] provides statistical evidence of the reduction in risk obtained through forward contracts. [8] provides a framework to derive pool bidding strategies if producers have signed forward contracts. Related papers using stochastic programming to make decisions in electricity markets include [9] and [10]. No reference has been found that considers the effect of the forced outage rate of a unit on its involvement in forward markets.

Specifically, the contribution of this paper is to analyze how the forced outage rate of a generating unit affects its forward contracting decisions.

The remaining of this paper is organized as follows. Section II describes the model considered, including a characterization of the generating unit, the pool, the forward contracts and the treatment of scenarios. Section III provides the formulation of the considered problem. Section IV gives results from a comprehensive case study. Section V provides some relevant conclusions.

II. MODEL

Since, a medium-term study horizon (e.g., three months) is considered, appropriate models should be selected taking into account this study horizon.

The producer under study is considered to be a price-taker, both in the pool and the futures markets. For the sake of simplicity, just one single generating unit is considered.

A. Generating unit

The consider unit is characterized by a maximum output power in MW, P^{Max}, and a linear cost in \$/MWh, C. Since a medium-term study is carried out, its minimum power output is considered to be null.

The historical data of the time between two consecutive unit failures, as well as the time to repair a failure, follow exponential distributions [11]. In consequence, the time to failure of the unit is modeled as

$$t_F = -\text{MTTF} \times \ln(u_1) , \tag{1}$$

similarly, the time to repair is modeled as

$$t_R = -\text{MTTR} \times \ln(u_2) . \tag{2}$$

Note that t_F and t_R are stochastic variables that are modeled using the the auxiliary random variables u_1 and u_2, respectively.

Thus, the status of the generating unit is represented by a two state model as shown in Fig. 1.

A scenario of the availability of the unit is generated by evaluating successively equations (1) and (2) until the considered time horizon is completed. Fig. 2 illustrates a scenario of availability of the unit. In this figure, $t_F(i)$ indicates the time for failure i to occur while $t_R(i)$ indicates the time of
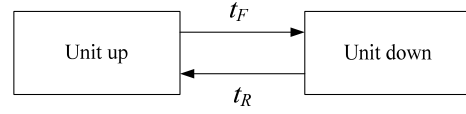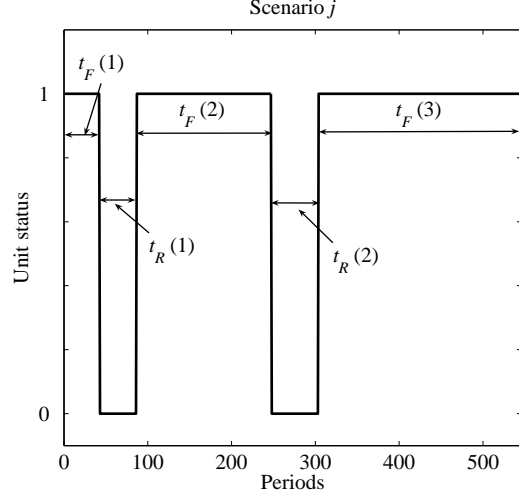

Fig. 1.   Two-state model of the generating unit.

Fig. 2.   Availability scenario of the unit.

repairing failure i. Additional details can be found in [11] and [12].

In order to represent all possible realizations of the availability of the unit throughout the considered study horizon a number of scenarios sufficiently large should be generated [13]. The appropriate number of availability scenarios depends on the time horizon and the value of the parameters MTTF and MTTR. For example, if the time horizon is one month and if the MTTF is one week, the number of scenarios to represent all possible realizations of the availability of the unit is larger than if the MTTF is equal to one year.

Since the number of scenarios required to represent all possible availability realizations of the unit can be large (e.g., a few thousands), a scenario reduction technique is needed [14]. This technique allows reducing significantly the number of scenarios while maintaining most of the structural properties of the stochastic variable being described via scenarios.

Different distance measures between two scenarios can be used to merge these two scenarios into a single one, thus achieving an appropriate scenario reduction. The one considered in this paper is the objective function of the mathematical programming problem formulated in Section III below, but considering just one single-availability scenario and the average pool-price scenario.

The first step of this scenario reduction technique is to solve the optimization problems associated with every single availability scenario. Then, two different scenarios are aggregated if their respective objective function values are similar. Since each optimization problem includes just a single-availability scenario, it is solved quickly, being the computational burden
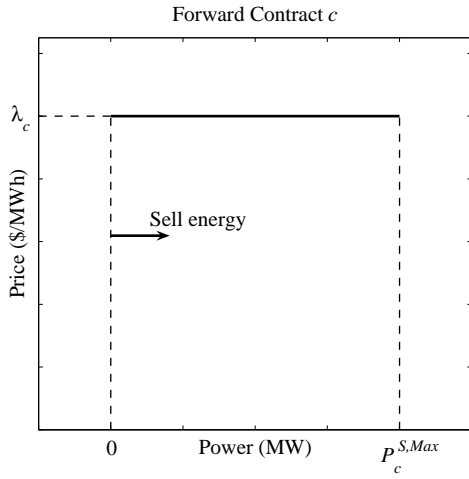
Fig. 3. Forward contract structure.

of the scenario-reduction procedure reasonable.

B. Pool

A forecasting procedure, based on time series or other suitable technique, can be used to generate price scenarios (e.g., see [15]–[17]).

Hourly pool prices throughout the considered study horizon are grouped so that the number of prices representing a given horizon is sufficiently small to be computational tractable, but reflecting accurately price variability.

In this paper, the 24 pool prices of each day have been aggregated in three periods. The number of hours and the pool price of each period is calculated in order to minimize the error between the actual prices and the prices of the three periods for each day and scenario. A simple optimization procedure is used to perform this bundling. Note that the number of hours of each one of the three periods can be different. Moreover, the number of hours of the first period of one day can be different than the number of hours of the same first period of another day.

As in the case of unit availability scenarios, a scenario reduction procedure is needed to make the number of price scenarios tractable while retaining most of its characteristics. The considered measure to compare two price scenarios is again the objective function of the mathematical programming problem formulated in Section III below but considering just one single price scenario and full unit availability.

C. Scenarios building

Once availability as well as price scenarios are generated and reduced, the final scenarios are created by combining all possible availability scenarios with all possible price scenarios. Let $N_{W,A}$ and $N_{W,P}$ be the numbers of availability and price scenarios after reducing them, respectively. Then, the final number of scenarios is $N_W = N_{W,A} \times N_{W,P}$. Each scenario is made up of three vectors. The first vector contains a 0 or a 1 for each time period representing the availability of the unit. The components of the second vector are the pool prices
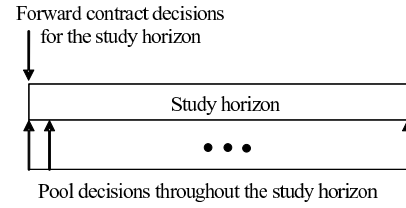

Fig. 4. Decision-making framework.

for the time periods of the study horizon. The third vector contains the duration in hours of each time period.

D. Forward contracts

From the viewpoint of a generating unit, a forward contract consists of a MW power quantity (up to a pre-specified maximum value) to be sold at a given price by the generating unit throughout a given time period, e.g., a week, a month or a quarter. Since the selling price pertaining to a forward contract is stable as it is pre-specified in advance, a forward contract is an appropriate financial instrument to hedge the risk of profit variability due to the inherent volatility plaguing pool prices. Needless to say that stable forward prices are lower on average than volatile pool prices. Fig. 3 illustrates the structure of the considered forward contract.
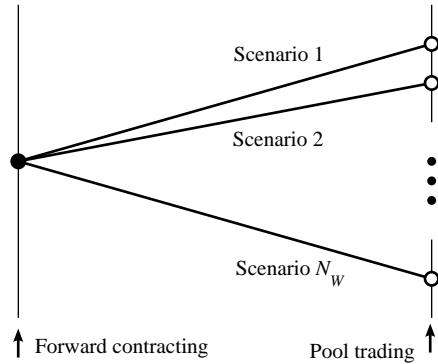
E. Decision Framework

Fig. 5. Scenario tree of the optimization problem.

The producer owning the generating unit has to make two decisions: forward contracting and pool trading. Forward contracting decisions are made on a weekly/monthly basis without knowing the realizations of the stochastic variables (pool price and unit availability), and they affect the whole study horizon. On the other hand, the set of decisions pertaining to the pool are made throughout the study horizon, that is, as a function of actual scenario realizations. Since the level of uncertainty involving day-ahead pool decisions is much smaller than that of making decisions in the futures market, we consider that

day-ahead pool decisions are made with perfect information. Fig. 4 illustrates the considered two-stage (forward market and pool) decision making framework.

Fig. 5 depicts the scenario tree of the stochastic optimization problem being analyzed. There exists a unique root node because the forward contracting decisions are the same regardless of pool price and unit availability. However, the tree has several final nodes (leaves) because the pool trading depends on the scenario realization.

F. Tool Usage

The proposed tool is run at the beginning of month one in order to know the optimal forward contracting decisions pertaining to the three-month study horizon. However, at the beginning of the month two the proposed tool is run again in order to find out forward contracting decisions pertaining to the following three months (months two to four), and so on. That is, the tool is run on a rolling window information basis once a month. Note that new forward contracting decisions may modify previous forward contracting decisions.

III. PROBLEM FORMULATION

A. Objective function

The objective of the considered stochastic programming problem is to

$$\text{maximize}_{E_{tw}^{G}, E_{tw}^{P}, P_c, \zeta, \eta_w}$$

$$
\begin{aligned}
z = &\sum_{w=1}^{N_W} \sum_{t=1}^{N_T} \pi_w (\lambda_{tw}^{P} E_{tw}^{P} - E_{tw}^{G} C) \\
&+ \sum_{c=1}^{N_C} \lambda_c P_c T_c \\
&+ \beta \Big[\zeta - \frac{1}{1-\alpha} \sum_{w=1}^{N_W} \pi_w \eta_w \Big]
\end{aligned}
\tag{3}
$$

Objective function (3) includes three terms, namely, (i) revenue from selling in the pool minus production cost, (ii) revenue from selling through forward contracts, and (iii) the CVaR of the profit multiplied by the weighting parameter β.

The pool revenue term is the summation over time periods and scenarios of the energy sold in the pool times the pool price times the scenario probability. The production cost term is the summation over time periods and scenarios of the energy produced times the production cost of the unit. The revenue term pertaining to forward contracting is the summation over forward contracts of the contracted power time the contract price times the time duration of the contract. The CVaR term is computed as indicated in [2].

B. Constraints

The problem constraints are:

$$\sum_{c \in F_t} P_c L_{tw} + E_{tw}^{P} - E_{tw}^{G} = 0, \forall t, \forall w \tag{4}$$

$$E_{tw}^{G} \leq k_{tw} P^{\text{Max}} L_{tw}, \forall t, \forall w \tag{5}$$

$$P_c \leq P_c^{\text{S,Max}}, \forall c \tag{6}$$

$$E_{tw}^{P} \leq k_{tw} P^{\text{Max}} L_{tw}, \forall t, \forall w \tag{7}$$

$$E_{tw}^{P} \geq (k_{tw} - 1) \sum_{c \in F_t} P_c L_{tw}, \forall t, \forall w \tag{8}$$

$$
\begin{aligned}
&-\sum_{t=1}^{N_T} (\lambda_{tw}^{P} E_{tw}^{P} - E_{tw}^{G} C) - \sum_{c=1}^{N_C} \lambda_c P_c T_c \\
&+ \zeta - \eta_w \leq 0, \forall w
\end{aligned}
\tag{9}
$$

$$E_{tw}^{G} \geq 0, \forall t, \forall w \tag{10}$$

$$P_c \geq 0, \forall c \tag{11}$$

$$\eta_w \geq 0, \forall w \tag{12}$$

Constraints (4) enforce the energy balance at each time period and for each scenario. In other words, for each time period and scenario, each constraint in (4) expresses that the energy produced by the unit is equal to the energy sold in the pool plus the energy sold through forward contracts. Constraints (5) enforce the capacity of the generating unit at each time period and for each scenario. Note that these constraints are enforced in terms of energy, not power. Note also that the parameter $k_{t,w}$ indicates the availability of the generating unit (1 if available and 0 otherwise) during period t and scenario w. Constraint (6) impose a cap over the power sold through each forward contract. In order to avoid arbitrage between the forward market and the pool, constraints (7) and (8) impose caps over the power bought and sold, respectively, in the pool. If and only if the unit is available, each constraint in (7) enforces for the corresponding period and scenario that the unit can sell in the pool at most its capacity times the duration of the corresponding time period. If and only if the unit is unavailable, each constraint in (8) enforces for the corresponding period and scenario that the unit can buy in the pool at most its contractual obligation during the corresponding period and scenario. Constraints (9) allow limiting financial risk through CVaR as thoroughly analyzed in [2]. Constraints (10)-(12) are positive variable declarations.

C. Problem size

The size of the linear programming problem (3)-(12) expressed as the number of continuous variables and constraints is provided in Table I.

TABLE I
PROBLEM SIZE

Continuous variables	$2N_T N_W + N_C + N_W + 1$
Constraints	$5N_T N_W + 2N_C + 2N_W$

IV. CASE STUDY

A. Data

The study reported below is carried out considering a 500 MW unit with a production cost of 20 $/MWh. The considered FOR values are three: 0, 0.04 and 0.06. The study horizon spans three months. Data for the four forward contracts available to the unit are provided in Tables II and III.

TABLE II
FORWARD CONTRACT QUANTITY AND PRICE DATA

Contract #	$P_c^{S,Max}$(MW)	λ_c($/MWh)
1	100	52.25
2	100	46.55
3	100	48.43
4	50	48.20

TABLE III
FORWARD CONTRACT DURATION

Month 1	Month 2	Month 3
Contract 1		
	Contract 2	
		Contract 3
Contract 4		

To describe pool price volatility, 288 scenarios are initially considered. Each scenario involves 273 price values calculated as indicated in Section II-B. These 288 scenarios are reduced to 25 using the scenario reduction technique also stated in Section II-B. Fig. 6 depicts the resulting 25 pool-price scenarios.
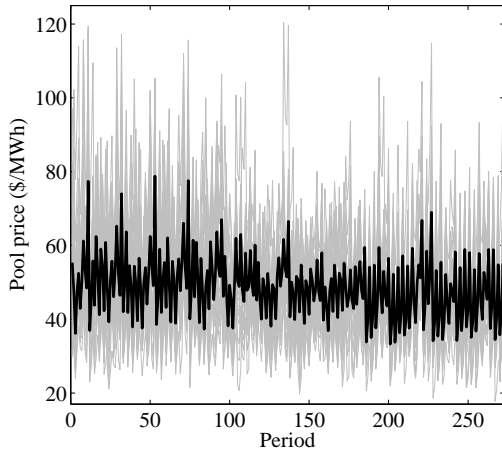

Fig. 6. Resulting pool-price scenarios.

Using equations (1)-(2), 1,000 unit availability scenarios are generated and then reduced to 20 through the scenario reduction technique explained in Section II-A. To ensure that the scenarios selected properly represent all possible avail-ability scenarios, the generation of 1,000 scenarios is repeated 10 times. The resulting 10 sets of 20 scenarios are merged

and finally reduced to 30. Comprehensive simulations show that this procedure drastically reduces the variability of the optimal value of the objective function with the set of selected scenarios. For the considered case study, this variability is below 0.5%.
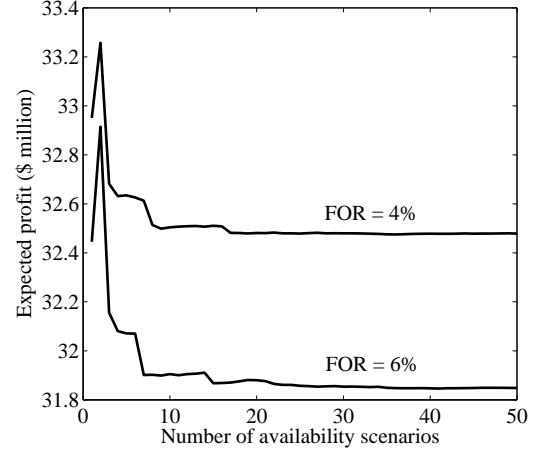

Fig. 7. Evolution of profit versus scenarios.

Fig. 7 plots the expected profits as a function of the number of considered availability scenarios for two FOR values, namely, 0.04 and 0.06. Observe that expected profit stabilizes after 30 scenarios for the two values of FOR.

The resulting number of (price plus availability) scenarios considered to solve problem (3)-(12) is finally obtained as $25 \times 30 = 750$.

B. Results and discussion

Fig. 8 plots the expected profit as a function of CVaR for three FOR values, 0, 0.04 and 0.06. Observe that expected profit decreases as FOR increases. Also, for a given FOR value, the expected profit decreases as CVaR increases. Note that increasing CVaR indicates decreasing standard deviation of the profit and decreasing risk of profit variability.
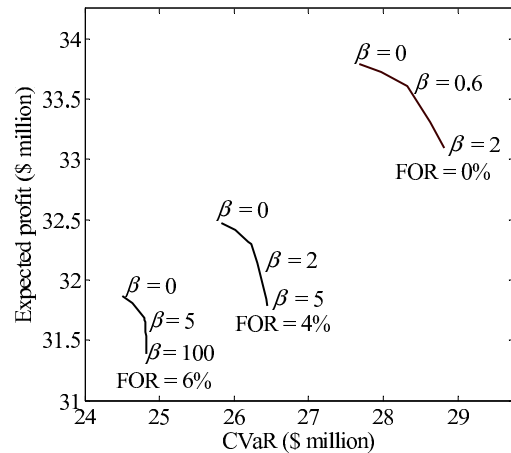

Fig. 8. Evolution of expected profit versus profit standard deviation.

Table IV and Fig. 9 illustrate contract usage by providing the actual power contracted for different values of FOR and the weighting factor β. Observe that the power sold through forward contracts decreases with FOR. As expected, the energy sold through forward contracts increases with β.

TABLE IV
POWER SOLD THROUGH FORWARD CONTRACT (MW)

	FC #	FOR		
		0%	4%	6%
$\beta = 0.6$	1	100	80.73	0
	2	0	0	0
	3	100	100	100
	4	0	0	0
$\beta = 2$	1	100	100	100
	2	100	72.28	0
	3	100	100	100
	4	50	0	0
$\beta = 5$	1	100	100	100
	2	100	100	8.17
	3	100	100	100
	4	50	50	0
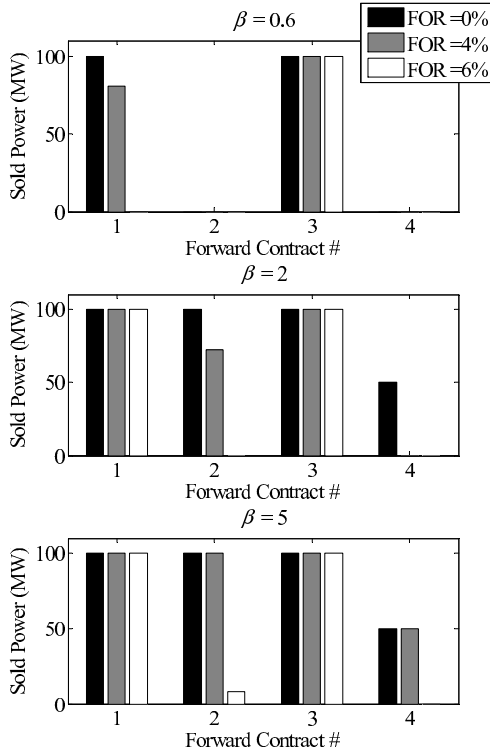
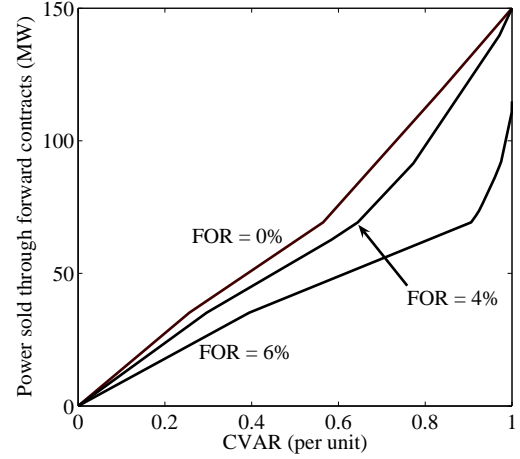Fig. 9. Forward contracts for different values of risk and three FORs.

Fig. 10. Evolution of sold power through forward contracts versus risk aversion.

the volatility of the expected profit increases as a consequence of buying in the pool during periods in which the unit is forced out.

C. Computational characterization

The size of the model solved in this case study is provided in Table V.

TABLE V
PROBLEM SIZE FOR THE CASE STUDY

	Number
Continuous variables	$410,255$
Constraints	$1,025,258$

All problems have been solved using CPLEX 10.2.0 under GAMS [18] on a Linux-based server with four processors clocking at 2.6 GHz and 32 GB of RAM. CPU time required to solve the stochastic problem with FOR = 0.04 if risk is not considered is 7 seconds, and to solve the stochastic programming problem considering risk with $\beta = 1$ is 185 seconds.

Since the number of constraints is always higher than the number of variables, the fastest technique to solve this linear programming problem is the dual Simplex method.

V. CONCLUSIONS

This paper shows that the optimal involvement in forward contracting of a generating unit is significantly affected by its forced outage rate. On one hand, average profit decreases in proportion to the time the unit is forced out. On the other hand, the volatility of the expected profit increases as a result of buying in the pool during forced outage periods. Thus, a producer must consider the FOR of its units while deciding its involvement in the futures market.

Finally, Fig. 10 shows the evolution of the total energy sold through forward contracts as a function of CVaR. To enhance clarity, the CVaR of each curve is normalized to lie within [0, 1]. Observe that the total energy sold decreases with FOR for each value of CVaR.

In summary, a FOR value different from 0 yield two consequences. On one hand, the expected profit decreases in proportion to the time the unit is forced out. On the other hand,

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