

# Day-ahead Operation of an Aggregator of Electric Vehicles via Optimization under Uncertainty

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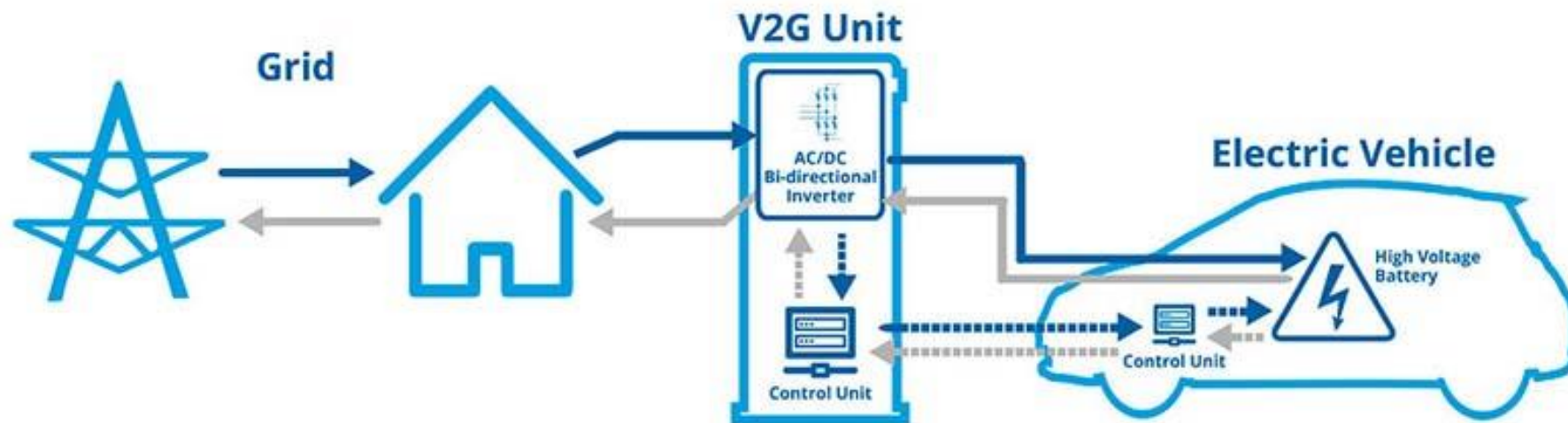
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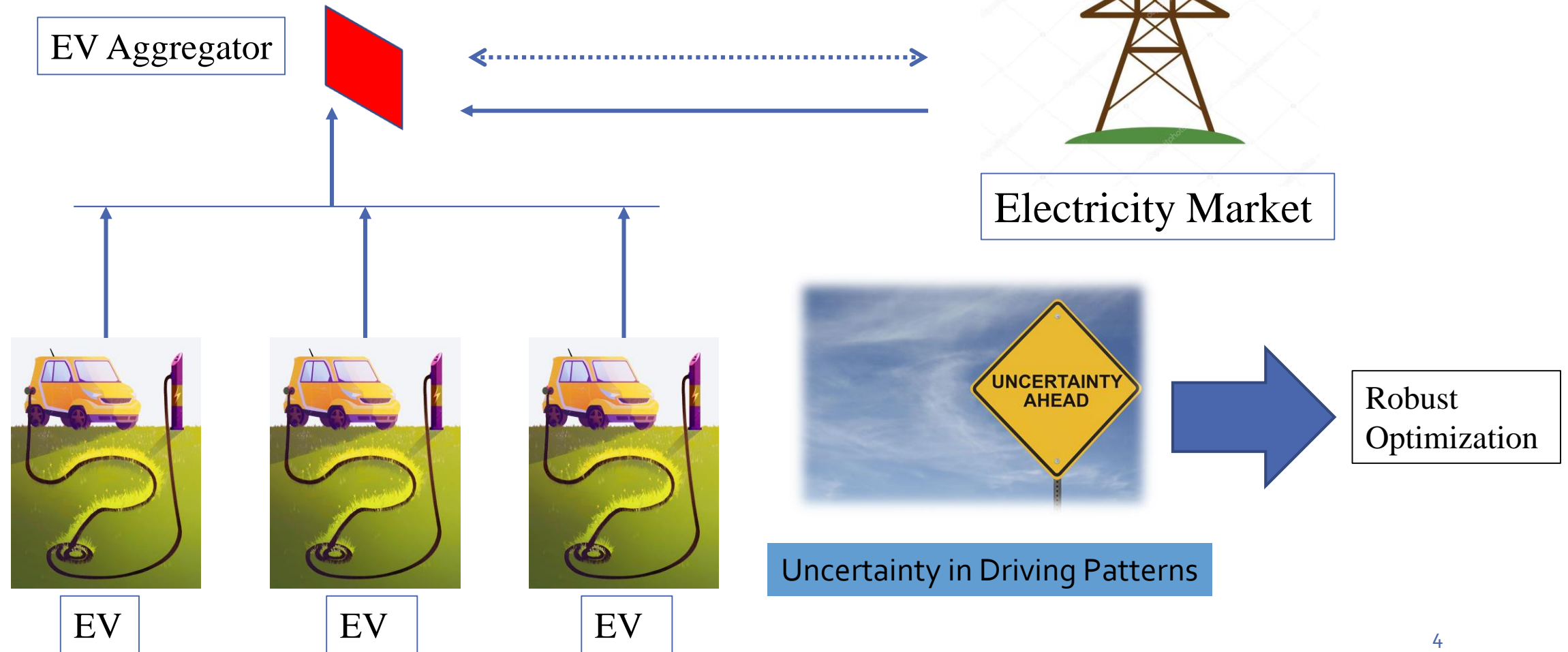


# Motivation

- Growing penetration electric vehicles (EV).
- Impact in operation and planning of electricity systems.



# Motivation



# Deterministic Formulation

- Objective function comprises two terms: energy bought and a penalty.
- Evolution of state of charge.
- Maximum rate of power charging.
- Maximum and minimum bounds on energy state of charge (ESOC).
- Boundary conditions for ESOC.

$$\min_{\Xi^{DO}} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} \lambda_t c_{v,t} \Delta t + \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} P (s_{v,t}^+ + s_{v,t}^-)$$

subject to:

$$e_{v,t} = e_{v,t-1} + \Delta t \eta_v c_{v,t} - \hat{\xi}_{v,t} + s_{v,t}^+ - s_{v,t}^-, \\ \forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$c_{v,t} \leq \bar{C}_v \hat{\alpha}_{v,t}, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$\underline{E}_v \leq e_{v,t} \leq \bar{E}_v, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$e_{v,N_T} = e_{v,0}, \quad \forall v \in \mathcal{V}$$

$$c_{v,t}, s_{v,t}^+, s_{v,t}^- \geq 0, \quad \forall v \in \mathcal{V}, t \in \mathcal{T},$$

$$\Xi^{DO} = (c_{v,t}, e_{v,t}, s_{v,t}^+, s_{v,t}^-)$$

# Deterministic Formulation

Energy required for transportation

Availability of each EV

$$\min_{\Xi^{DO}} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} \lambda_t c_{v,t} \Delta t + \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} P (s_{v,t}^+ + s_{v,t}^-)$$

subject to:

$$e_{v,t} = e_{v,t-1} + \Delta t \eta_v c_{v,t} - \hat{\xi}_{v,t} + s_{v,t}^+ - s_{v,t}^-,$$

$$\forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$c_{v,t} \leq \bar{C}_v \hat{\alpha}_{v,t}, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

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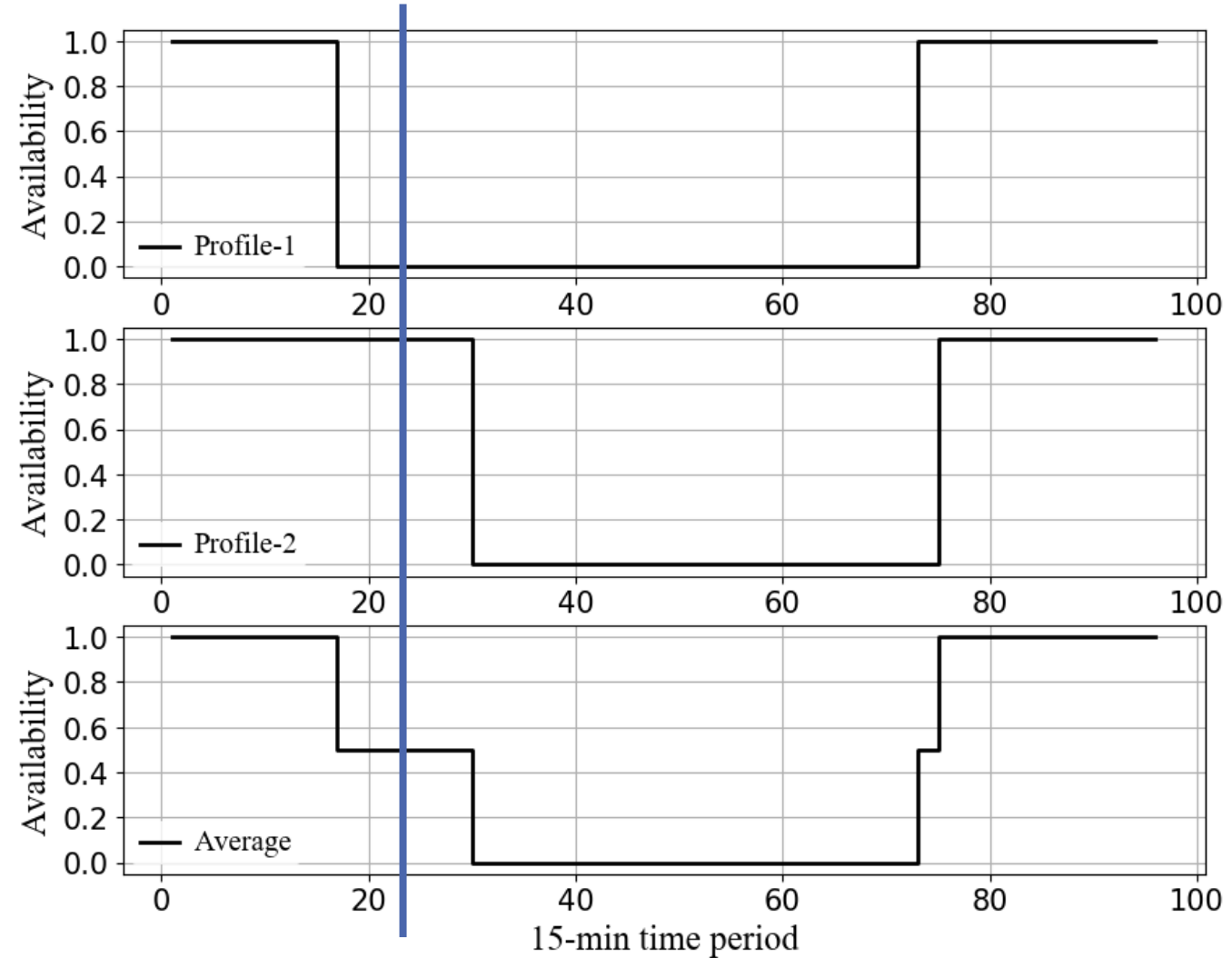
# Deterministic Formulation

## Availability Profiles

$$\alpha_1 = 0$$

$$\alpha_2 = 1$$

$$\alpha_a = 0.5$$



# Robust Formulation

- Where  $\alpha_{v,t}$  is not expected value now (1 EV is available to charge, 0 is unavailable).
- $\alpha_{v,t}$  depends on an uncertain set.

Uncertainty in Driving Patterns

$$\min_{\Xi^{RB}} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} \lambda_t c_{v,t} \Delta t + \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} P(s_{v,t}^+ + s_{v,t}^-)$$

subject to:

$$e_{v,t} = e_{v,t-1} + \Delta t \eta_v c_{v,t} \alpha_{v,t} - \hat{\xi}_{v,t} + s_{v,t}^+ - s_{v,t}^-, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$c_{v,t} \leq \bar{C}_v, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$\underline{E}_v \leq e_{v,t} \leq \bar{E}_v, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$e_{v,N_T} = e_{v,0}, \quad \forall v \in \mathcal{V}$$

$$c_{v,t}, s_{v,t}^+, s_{v,t}^- \geq 0, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

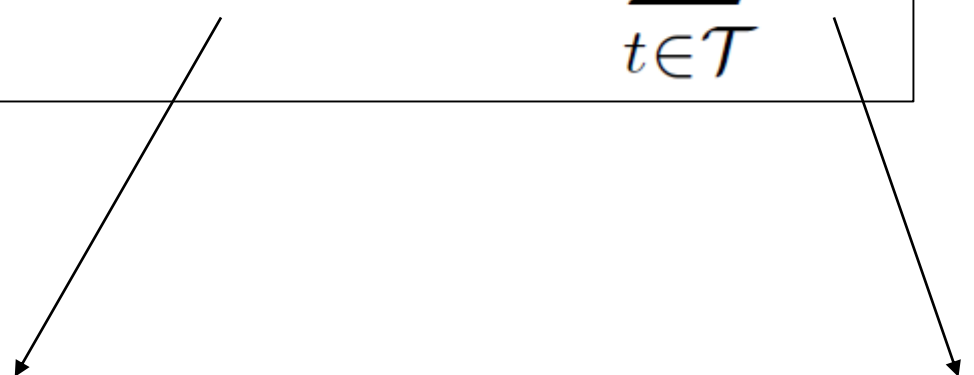
$$\min_{\alpha_{v,t}} \sum_{t \in \mathcal{T}} \Delta t \eta_v c_{v,t} \alpha_{v,t} \geq \sum_{t \in \mathcal{T}} \hat{\xi}_{v,t}, \quad \forall v \in \mathcal{V}$$

$$\alpha_{v,t} \in \{0, 1\}, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$



# Robust Formulation

It turns into a bilevel problem

$$\min_{\alpha_{v,t}} \sum_{t \in \mathcal{T}} \Delta t \eta_v c_{v,t} \alpha_{v,t} \geq \sum_{t \in \mathcal{T}} \hat{\xi}_{v,t}$$
Two arrows originate from the equation. One arrow points from the left-hand side of the inequality to the text 'Optimization problem'. The other arrow points from the right-hand side of the inequality to the text 'Total demand for transportation for each EV'.

Optimization problem

Total demand for transportation for each EV

Satisfied the demand of each EV for the worst case

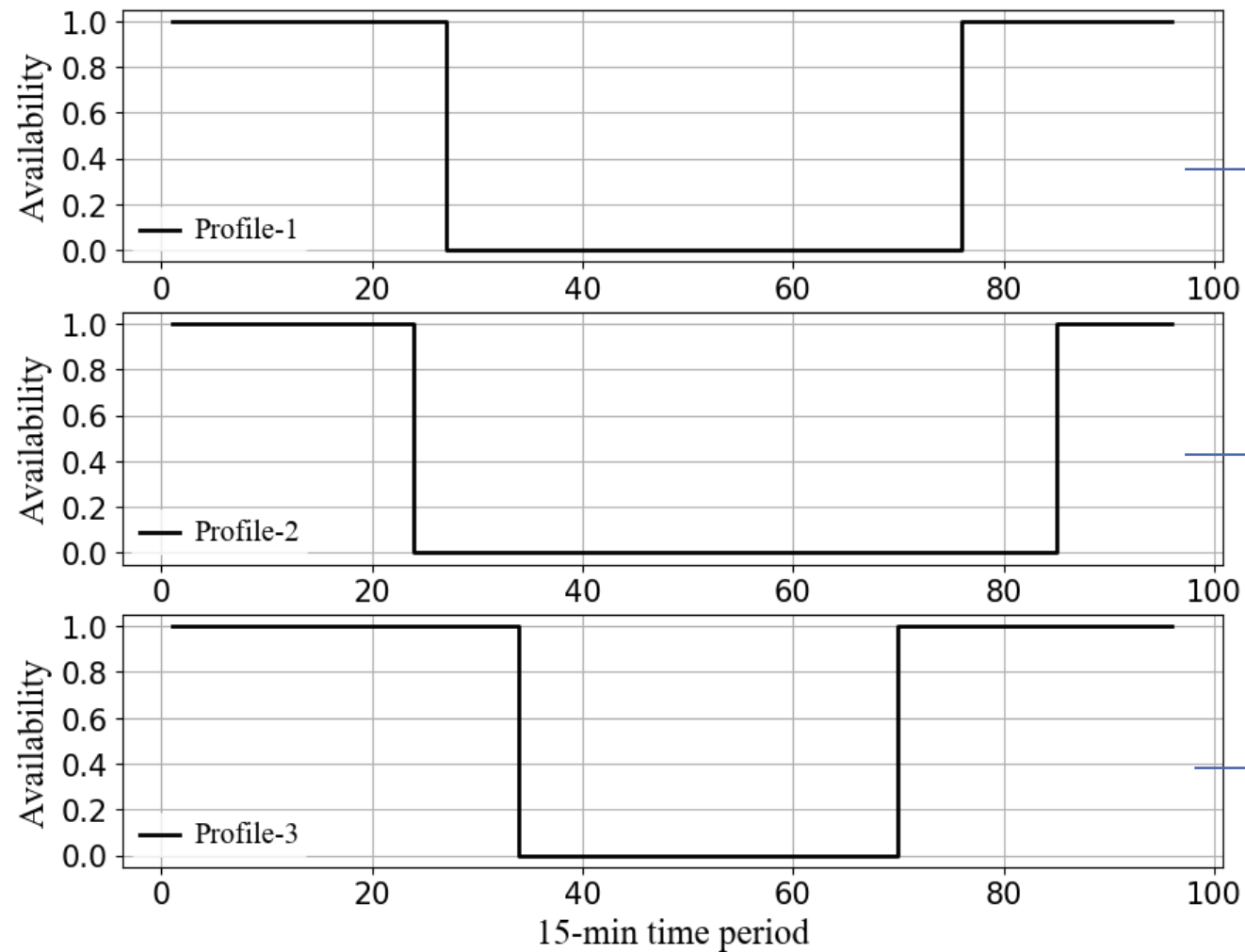
# Robust Formulation

- Uncertain set estimated based on historical records.

$$\sum_{t \in \mathcal{T}} \alpha_{v,t} \geq K_v : (\zeta_v)$$
$$\underline{\alpha}_{v,t} \leq \alpha_{v,t} \leq \bar{\alpha}_{v,t} : (\underline{\beta}_{v,t}, \bar{\beta}_{v,t}), \quad \forall t \in \mathcal{T}$$

Uncertainty in Driving Patterns

# Robust Formulation



$$K_v = 65$$

57

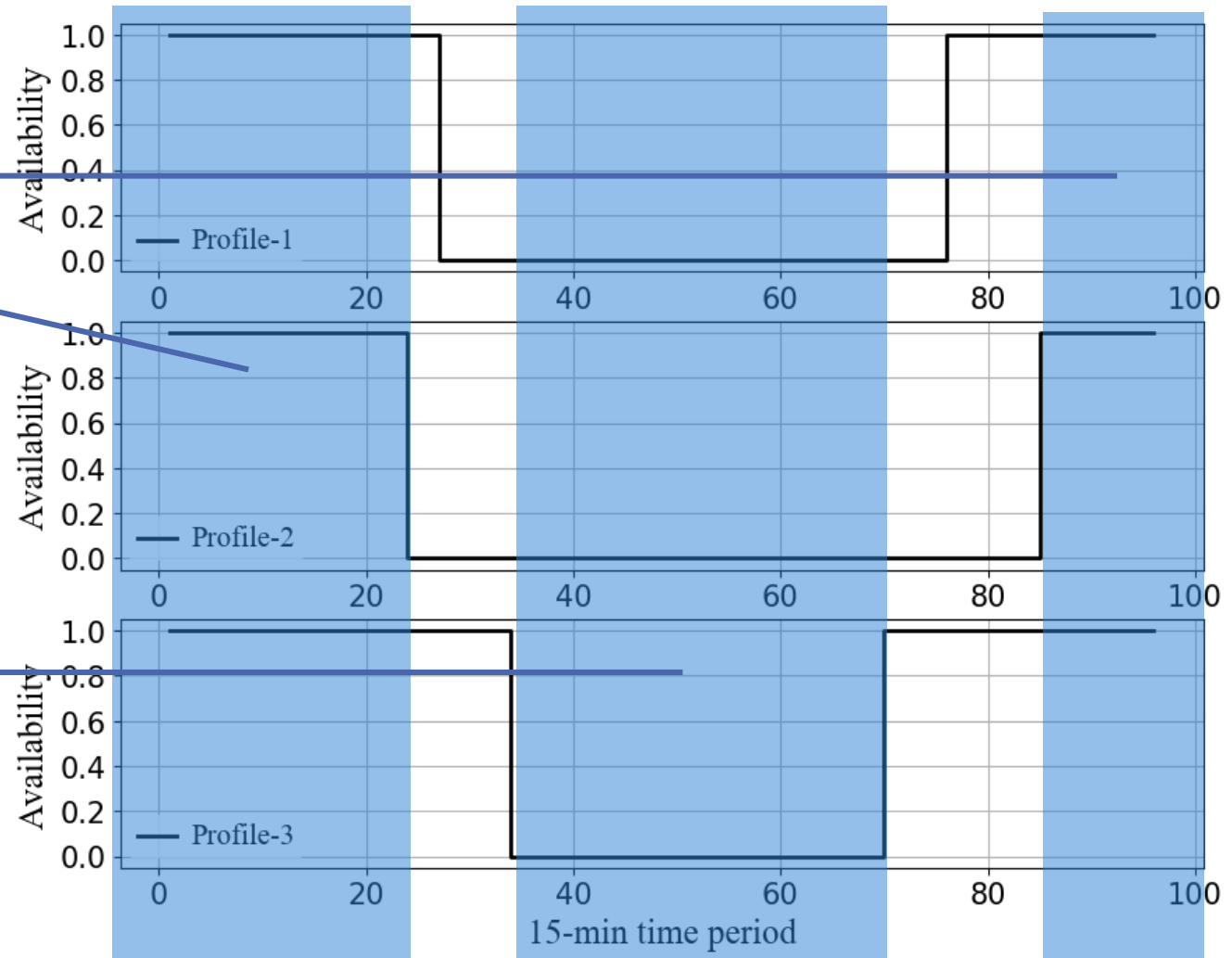
64

72

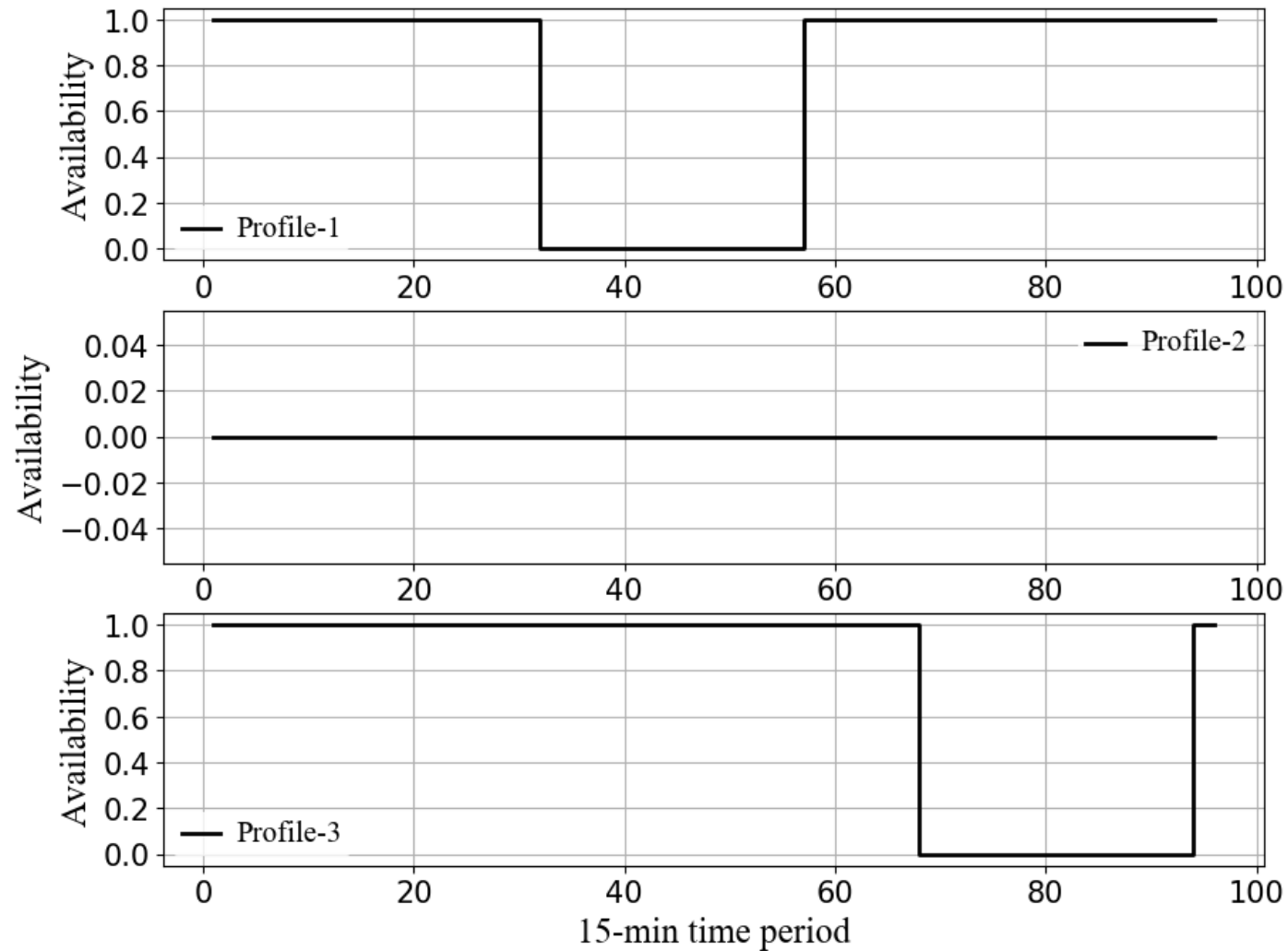
# Robust Formulation

$$\underline{\alpha}_{v,t} = 1$$

$$\bar{\alpha}_{v,t} = 0$$



# Robust Formulation



# Robust Formulation



$$\begin{aligned} \xi_v^{wc} &\geq \sum_{t \in \mathcal{T}} \hat{\xi}_{v,t}, \quad \forall v \in \mathcal{V} \\ \xi_v^{wc} &= \min_{\alpha_{v,t}} \left\{ \sum_{t \in \mathcal{T}} \Delta t \eta_v c_{v,t} \alpha_{v,t} \right. \\ &\quad \left. \sum_{t \in \mathcal{T}} \alpha_{v,t} \geq K_v : (\zeta_v) \right. \\ &\quad \underline{\alpha}_{v,t} \leq \alpha_{v,t} \leq \bar{\alpha}_{v,t} : (\underline{\beta}_{v,t}, \bar{\beta}_{v,t}), \quad \forall t \in \mathcal{T} \\ &\quad \left. \right\}, \forall v \in \mathcal{V} \\ \alpha_{v,t} &\in \{0, 1\}, \quad \forall t \in \mathcal{T}, v \in \mathcal{V} \end{aligned}$$

Non-Convex

# Robust Formulation



- Constraint matrix is totally unimodular.
- Parameters  $(K_v, \bar{\alpha}_{v,t}, \underline{\alpha}_{v,t})$  are integer.
- Under these conditions, variables take integer values.

$$\xi_v^{wc} \geq \sum_{t \in \mathcal{T}} \hat{\xi}_{v,t}, \quad \forall v \in \mathcal{V}$$

$$\xi_v^{wc} = \min_{\alpha_{v,t}} \left\{ \sum_{t \in \mathcal{T}} \Delta t \eta_v c_{v,t} \alpha_{v,t} \right.$$

$$\left. \sum_{t \in \mathcal{T}} \alpha_{v,t} \geq K_v : (\zeta_v) \right.$$

$$\left. \underline{\alpha}_{v,t} \leq \alpha_{v,t} \leq \bar{\alpha}_{v,t} : (\underline{\beta}_{v,t}, \bar{\beta}_{v,t}), \quad \forall t \in \mathcal{T} \right\}, \forall v \in \mathcal{V},$$

# Robust Formulation

- Lower-level problem is a linear problem.
- Original bilevel problem is transformed into an equivalent single level problem by using duality theory linear programming.

$$\min_{\alpha_{v,t}} \sum_{t \in \mathcal{T}} \Delta t \eta_v c_{v,t} \alpha_{v,t}$$

subject to:

$$\sum_{t \in \mathcal{T}} \alpha_{v,t} \geq K_v : (\zeta_v)$$

$$\underline{\alpha}_{v,t} \leq \alpha_{v,t} \leq \bar{\alpha}_{v,t} : (\underline{\beta}_{v,t}, \bar{\beta}_{v,t}), \quad \forall t \in \mathcal{T}$$



DUAL

$$\max_{\zeta_v, \underline{\beta}_{v,t}, \bar{\beta}_{v,t}} K_v \zeta_v + \sum_{t \in \mathcal{T}} (\underline{\alpha}_{v,t} \underline{\beta}_{v,t} + \bar{\alpha}_{v,t} \bar{\beta}_{v,t})$$

subject to:

$$\zeta_v + \underline{\beta}_{v,t} + \bar{\beta}_{v,t} \leq \Delta t \eta_v c_{v,t} \alpha_{v,t}, \quad \forall t \in \mathcal{T}$$

$$\underline{\beta}_{v,t} \geq 0, \bar{\beta}_{v,t} \leq 0, \quad \forall t \in \mathcal{T}$$

$$\zeta_v \geq 0.$$



# Robust Formulation

- Single-level equivalent problem is formulated as:

Upper-Level Problem

Primal Lower Level Feasibility Constraint

Dual Lower Level Feasibility Constraint

Strong Duality

Linearization

$$\min_{\Xi_{RB}} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} \lambda_t c_{v,t} \Delta t + \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} P(s_{v,t}^+ + s_{v,t}^-)$$

subject to:

$$e_{v,t} = e_{v,t-1} + \Delta t \eta_v z_{v,t} - \hat{\xi}_{v,t} + s_{v,t}^+ - s_{v,t}^-, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$c_{v,t} \leq \bar{C}_v, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

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$$e_{v,N_T} = e_{v,0}, \quad \forall v \in \mathcal{V}$$

$$c_{v,t}, s_{v,t}^+, s_{v,t}^- \geq 0, \quad \forall v \in \mathcal{V}, t \in \mathcal{T}$$

$$K_v \zeta_v + \sum_{t \in \mathcal{T}} (\underline{\alpha}_{v,t} \underline{\beta}_{v,t} + \bar{\alpha}_{v,t} \bar{\beta}_{v,t}) \geq \sum_{t \in \mathcal{T}} \hat{\xi}_{v,t}, \quad \forall v \in \mathcal{V}$$

$$\sum_{t \in \mathcal{T}} \alpha_{v,t} \geq K_v, \quad \forall v \in \mathcal{V}$$

$$\underline{\alpha}_{v,t} \leq \alpha_{v,t} \leq \bar{\alpha}_{v,t}, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$\zeta_v + \underline{\beta}_{v,t} + \bar{\beta}_{v,t} \leq \Delta t \eta_v c_{v,t} \alpha_{v,t}, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$K_v \zeta_v + \sum_{t \in \mathcal{T}} (\underline{\alpha}_{v,t} \underline{\beta}_{v,t} + \bar{\alpha}_{v,t} \bar{\beta}_{v,t}) = \sum_{t \in \mathcal{T}} \Delta t \eta_v z_{v,t}, \quad \forall v \in \mathcal{V}$$

$$0 \leq c_{v,t} - z_{v,t} \leq (1 - \alpha_{v,t}) \bar{C}_v, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$0 \leq z_{v,t} \leq \alpha_{v,t} \bar{C}_v, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$\underline{\beta}_{v,t} \geq 0, \bar{\beta}_{v,t} \leq 0, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$

$$\zeta_v \geq 0, \quad \forall v \in \mathcal{V}$$

$$\alpha_{v,t} \in \{0, 1\}, \quad \forall t \in \mathcal{T}, v \in \mathcal{V}$$

# Case Study

- 29 days of simulation.
- Prices from ENTSO-e Transparency Platform (2018).
- Data from National Household Travel Survey (NHTS) 2017.
- The technical parameters associated with each EV are identical (Renault ZOE)



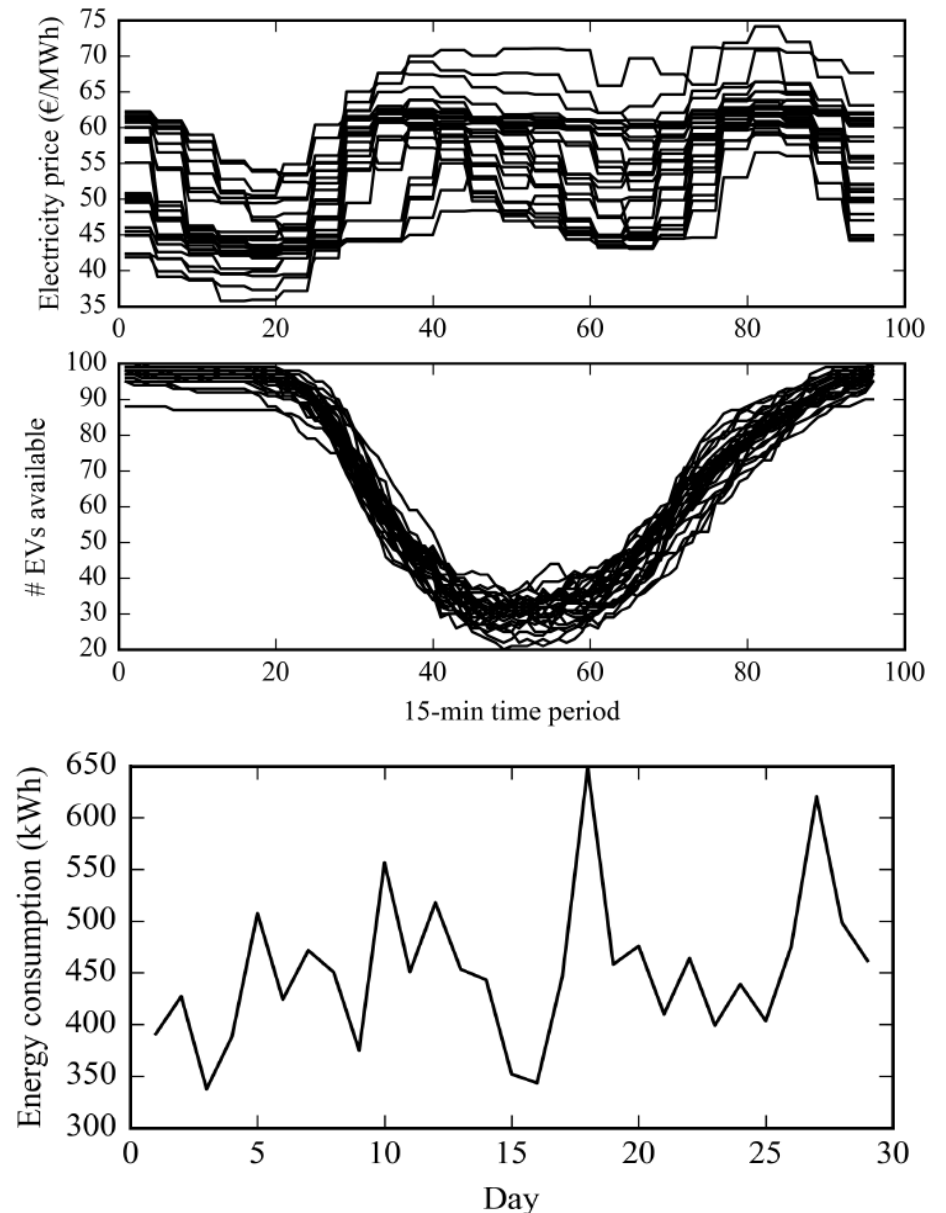
**National Household  
Travel Survey**

Understanding How People Get from  
Place to Place



# Case Study

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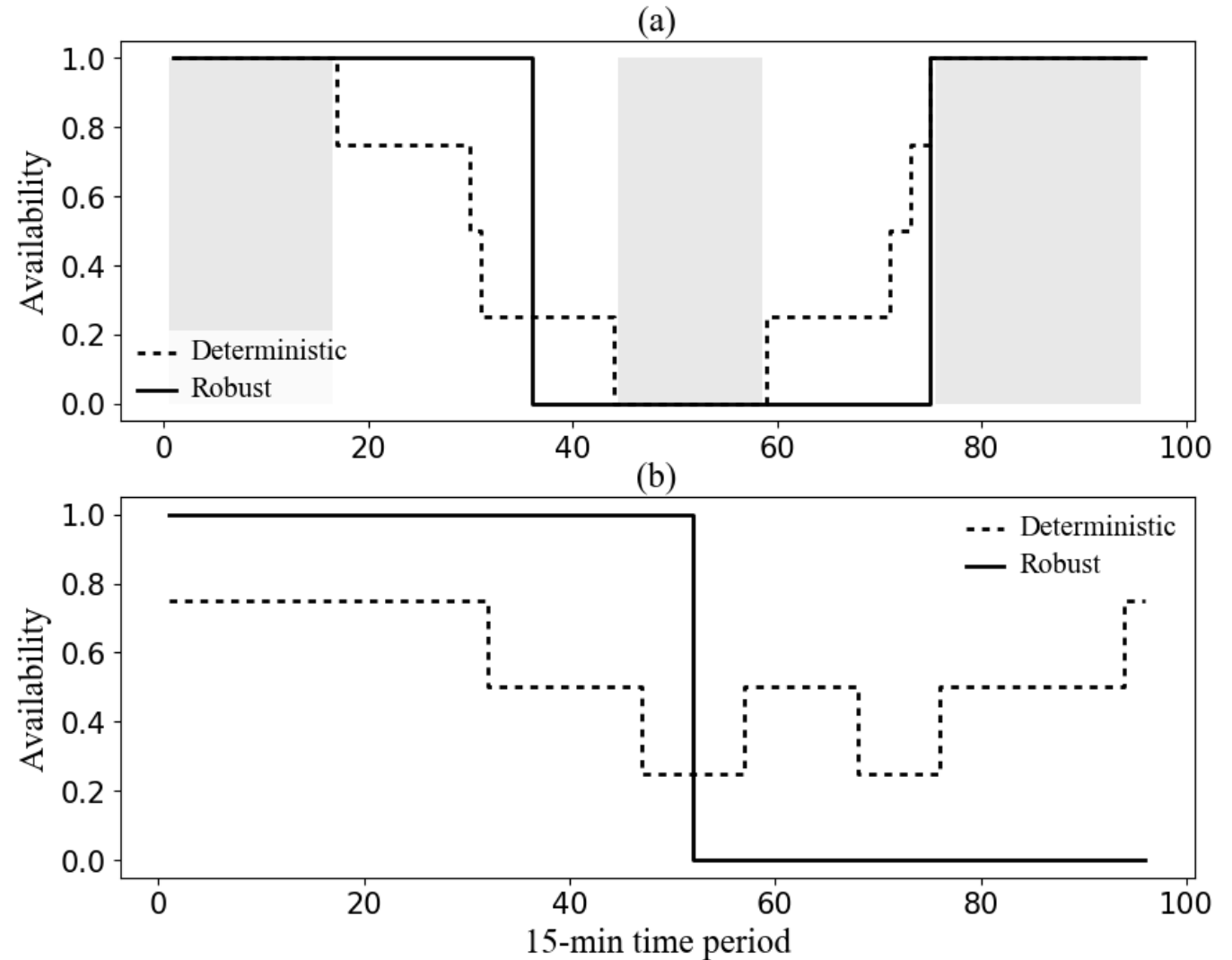


# Case Study

(a) predictable pattern.

(b) non-predictable pattern.

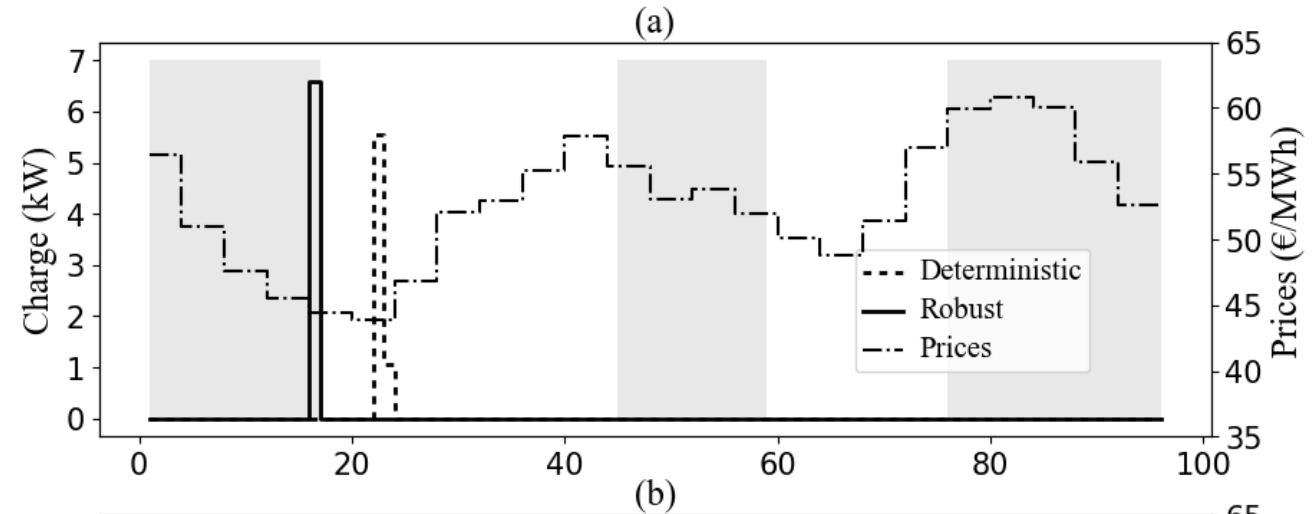
## Day-ahead Operation of an Individual Electric Vehicle



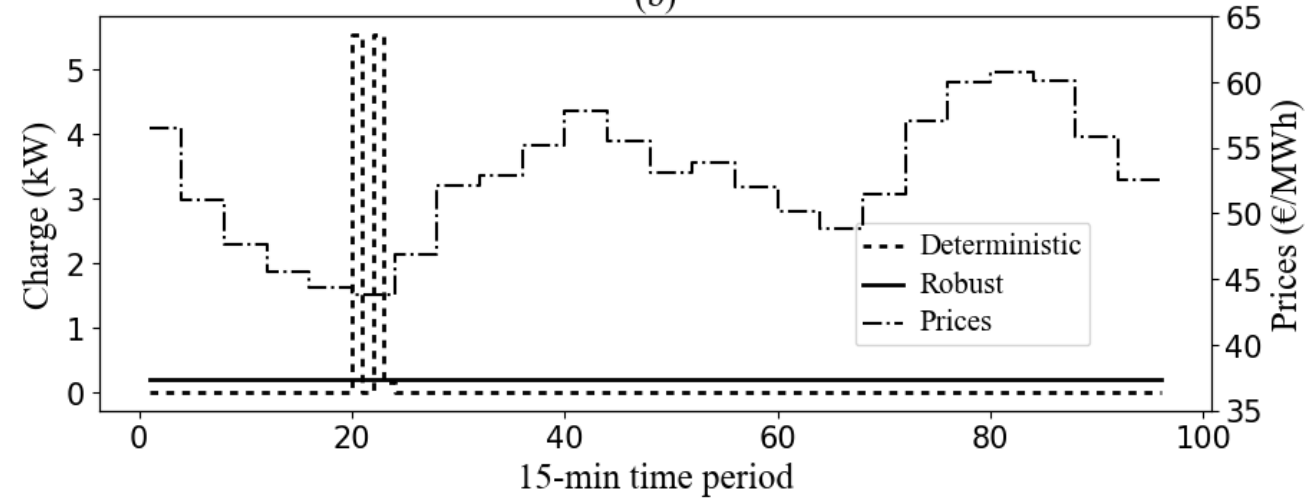
# Case Study

(a) predictable pattern.

## Day-ahead Operation of an Individual Electric Vehicle



(b) non-predictable pattern.



# Case Study

## Operation of the Fleet of Electric Vehicles

- To compare deterministic and robust optimization we solve the real-time operation of the aggregator for one month by fixing the energy bought from the day-ahead market.
- The goal of the real-time problem is to minimize the deviations from the energy balance in the EVs' batteries for the fleet of EVs.

Table 1: Results for Day 21

Metric	Deterministic	Robust
Cost in the day-ahead (€)	31.0	32.8
Purchased power in the day-ahead (kW)	1745.6	1851.2
Deviation in real time (kWh)	305.7	123.4

# Case Study

## Monthly Operation of the Fleet of Electric Vehicles

- RO-EV reduces the deviations up to 50% over 29 days.

Table 2: Monthly Results

Metric		Deterministic	Robust
Cost in the day-ahead (€)		586.6 ↓	643.0 ↑
Deviations in real time (kWh)	Max.	305.7	190.5
	Mean	156.9	82.9
	Min.	77.5	24.3
	Total	4548.9 ↑	2404.3 ↓

# Conclusion

- A novel and computationally efficient model for the day-ahead operation of an aggregator of EVs in a residential district.
- The uncertainty on the availability of an EV is modelled via robust optimization.
- Robust approach leads to a reduction of the deviations from the energy balance of the vehicles' batteries up to 50%.





# Any questions?



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