

New indices of market power in transmission-constrained electricity markets

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Abstract—In this paper we introduce four new indices of market power in transmission-constrained electricity markets that are based on economic principles together with faithful representation of the effects of Kirchhoff’s laws.

Keywords: Transmission constraints, electricity markets, market power.

I. INTRODUCTION

Market power continues to be a problematic issue in restructured electricity markets, with the interaction between tight supply conditions and transmission limits posing serious difficulties for market operations, particularly in the context of “energy-only” markets such as the Electric Reliability Council of Texas (ERCOT) balancing market and the upcoming ERCOT “nodal” (that is, “locational marginal pricing,” (LMP)) market, to be introduced in late 2010 [1]. In energy-only markets, since there is no installed capacity payment mechanism as in the Northeast US restructured markets [2], competitive energy prices must occasionally rise above the highest typical marginal costs in the system in order for generation asset owners to recover their investments and to encourage new investment to ensure resource adequacy to cope with demand growth.

Because the occasions of high competitive prices are also times when market participants have market power—defined as the ability to profitably shift market prices away from competitive prices [3, §4-1.1]—it can be extremely difficult to distinguish high competitive prices from the exercise of market power. Normatively, market power mitigation rules should be aimed at restoring prices towards competitive levels; however, when supply is tight, the competitive level of electricity prices depends on demand willingness-to-pay for electricity, in principle exhibited to the market by price-responsive demands or, under curtailment conditions, specified through a proxy to the “value of lost load” [3].

In restructured electricity markets today, the paucity of price-responsive demands and the lack of experience in North America with energy-only markets makes the determination of competitive price levels problematic under tight supply conditions. Moreover, the very lack of price-responsive demand contributes to the market power of market participants.

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This issue is problematic in both energy-only markets such as ERCOT and in the markets in the Northeast US that have installed capacity markets.

Consequently, in the face of market power issues, restructured markets have adopted *ad hoc* market power mitigation rules that have only tenuous connection to:

- the fundamental economic drivers of the exercise of market power, such as the “residual demand” faced by market participants [4], and
- the fundamental goal of making prices more competitive.

The significance of this issue is that in an energy-only market, such as in ERCOT, deviation from competitive prices is problematic. On the one hand, “under-mitigation,” allowing market prices to rise excessively above competitive prices, would cause unacceptable transfers of wealth from consumers to generators. Market power mitigation should be aimed at preventing such unacceptable transfers; however, on the other hand, *ad hoc* approaches to market power assessment and mitigation also run the risk of “over-mitigating,” driving prices below competitive levels. Prices below competitive levels will result in inadequate investment in generation in an energy-only market.

To summarize, in an energy-only market, it is critical to avoid both under- and over-mitigation of market power. *Ad hoc* approaches to analyzing market power are unlikely to result in appropriate mitigation of market power in an energy-only market, but such approaches are prevalent in North American markets today.

In this paper, we develop four related indices of market power in a transmission-constrained electricity market that are based on economic fundamentals as well as a representation of the transmission network that is faithful to Kirchhoff’s laws. The indices involve the transmission-constrained residual demand faced by a market participant as derived in [5]. Calculation and visualization of the indices developed in this paper provides the basis for a screening tool for market power that can be of particular use to regulators and independent system operators (ISOs) in their assessment of market power in LMP markets.

The rest of this paper is organized as follows. In section II there is a literature survey of current approaches to analyzing market power. Then, in section III, the indices are developed. Implementation of the indices is discussed in section IV. Section V presents case studies to illustrate the indices. Section VI concludes and describes future work.

II. CURRENT APPROACHES TO ANALYZING MARKET POWER

Current market power assessment approaches and screening tools fall into two broad categories:

- 1) principles-based approaches that explicitly examine incentives to deviate from competitive prices, and
- 2) *ad hoc* approaches based on indices such as HHI.

As an example of the first category, [3], [4] use the derivative of the residual demand faced by a generator to assess the incentives of a hypothetical profit-maximizing firm to mark-up prices above marginal; however, the analysis does not explicitly treat transmission constraints. An example of this kind of assessment is outlined in section II-A in order to provide background and basis for extension to the case of transmission constraints.

As another example of the first category, [6] considers two zones joined by a single radial transmission line in a Cournot framework. While a two zone model is appropriate for some particular cases, such as modelling of California represented as two zones, LMP markets have more complicated interactions due to the meshed nature of transmission. To summarize, the approaches to assessing market power that are rooted in economic analysis have not been extended to consider the implications of Kirchhoff's laws in a realistic meshed network. The limitations of a model of transmission that is limited to a single radial transmission line will be explored in section II-B.

The second category, of *ad hoc* approaches, includes application to transmission-constrained markets with meshed networks. However, because the foundation is *ad hoc*, the results can be unreliable or even misleading. (See, for example, the discussion in [3, Chapter 4-5].) An example of such an *ad hoc* screening tool, based on the ERCOT nodal market design, is outlined and critically evaluated in section II-C.

A. Principles-based analysis of market power in the absence of transmission constraints

As discussed above, [3], [4] describe an approach to assessing the incentives for a generator to mark-up its offer price above marginal costs in the absence of transmission constraints. The basic analysis considers the residual demand, that is, actual demand minus the supply of all the other participants in the market, and asks how a hypothetical profit-maximizing market participant would have offered in response to this residual demand. If that offer would involve significant mark-up of price above marginal cost and the generator is not operating at its maximum capacity, then the market participant has market power. Such a finding could be used in a subsequent market power mitigation process.

To understand the residual demand, suppose that the demand in a particular pricing interval was D . (We ignore price-responsiveness of demand here, but it can be incorporated into the analysis.) Furthermore, consider a particular market participant k and suppose that the total offered generation of all the *other* market participants besides k was specified by the function $q_{-k} : \mathbb{R} \rightarrow \mathbb{R}$. In particular, at the price P , the total offered generation of all the other market participants is $q_{-k}(P)$. (We will follow the "economics" convention of using

the symbol P for price and the symbol q for quantity produced, in this case quantity of electricity.) The *residual demand* faced by market participant k is simply $D - q_{-k}(P)$. The inverse of the function $(D - q_{-k})$ is the inverse residual demand function faced by participant k , $p_{-k} : \mathbb{R} \rightarrow \mathbb{R}$.

Following, for example, [3], [4], to analyze the incentives faced by market participant k , we consider the conditions for market participant k to maximize its profit. Profit, $\pi_k : \mathbb{R} \rightarrow \mathbb{R}$, in this context, is defined as operating profit, meaning revenue minus costs. Ignoring forward contracts, revenue equals the product of:

- quantity, q_k , of electricity produced by the generator, multiplied by
- the resulting price $p_{-k}(q_k)$,

noting that the definition of inverse residual demand is the resulting market clearing price in the market given that the generator produces the quantity q_k . We assume that the production cost function of participant k is specified by the function $c_k : \mathbb{R} \rightarrow \mathbb{R}$.

Summarizing, profit for market participant k is:

$$\forall q_k \in \mathbb{R}, \pi_k(q_k) = q_k p_{-k}(q_k) - c_k(q_k).$$

Assuming that sufficient conditions for maximization are satisfied, that p_{-k} and c_k are differentiable, and that generation capacity constraints are not binding at the profit maximizing condition, we can find the maximum of profit by setting its derivative to zero:

$$0 = \frac{\partial \pi_k}{\partial q_k}(q_k) = p_{-k}(q_k) + q_k \frac{\partial p_{-k}}{\partial q_k}(q_k) - c'_k(q_k),$$

where $c'_k = \frac{\partial c_k}{\partial q_k}$ is the marginal costs. Re-arranging, we obtain the price-cost mark-up of price above marginal cost under the assumptions above and the hypothesis that the generator was maximizing its profits:

$$p_{-k}(q_k) - c'_k(q_k) = -q_k \frac{\partial p_{-k}}{\partial q_k}(q_k). \quad (1)$$

(Since the market clearing price is non-increasing in increasing generation by market participant k , we have that $\frac{\partial p_{-k}}{\partial q_k}(q_k) \leq 0$ and the right-hand side of (1) is non-negative.) As a basic measure of market power, if the right-hand side of (1) is "large" then a profit-maximizing generator has incentives to drive up prices by withholding (at least in the absence of a forward contract, to be discussed below).

The estimate of mark-up (1) provides one basic index of market power that could be utilized by a market monitor. For example, a threshold could be established of, say, 10% above marginal cost or \$20/MWh above marginal cost. Any generator that is not at full production but such that the right-hand side of (1) is above the threshold would be flagged as of concern. Such generators might then be subject to market power mitigation such as limits on offer prices.

Equation (1) is sometimes re-arranged to obtain an expression for the Lerner index $(p_{-k}(q_k) - c'_k(q_k)) / p_{-k}(q_k)$ in terms of the price elasticity [3, §4-3.4]. However, as argued in [3, §4-5.2], particularly in the context of average measures over

time, mark-up above competitive prices (and mark-up above marginal costs) is more useful than the Lerner index. In any case, the subsequent analysis in this paper is most easily written in terms of the price mark-up, so the rest of the analysis will focus on mark-up rather than Lerner index.

To the extent that marginal costs roughly represent the level of competitive prices, the estimated mark-up approximates the excess transfer of wealth, over and above competitive levels, from consumers to producers, per MW of production. Multiplying by production q_k , the following can be used as an index of estimation of excess wealth transfer to participant k :

$$-(q_k)^2 \frac{\partial p_{-k}}{\partial q_k}(q_k). \quad (2)$$

However, this index of estimation of excess wealth transfer should be used with caution since the marginal cost $c'_k(q_k)$ of participant k at its production level may be below the competitive price [3] and, consequently, the estimated excess wealth transfer may be less than implied by (2).

So far the analysis has not considered forward contracts. Forward contracts change the competitive situation somewhat [3, §4-4.3]. In the presence of a forward sale of quantity q_k^f , a similar derivation results in the following estimate of price-cost mark-up:

$$p_{-k}(q_k) - c'_k(q_k) = -(q_k - q_k^f) \frac{\partial p_{-k}}{\partial q_k}(q_k). \quad (3)$$

Similarly, excess wealth transfer can be estimated by:

$$-(q_k - q_k^f)^2 \frac{\partial p_{-k}}{\partial q_k}(q_k). \quad (4)$$

To the extent that a significant fraction of the production is forward contracted, these expressions show that the short-term incentives for mark-up are reduced [3, §4-4.3]. (However, in general, a more complicated analysis involving the interaction between incentives in the forward and “spot” markets is necessary to fully elucidate incentives [7], [8], [9], [10]. Although forward contracts do contribute to reducing incentives for exercising market power, the results are not completely understood in the context of supply offers in an electricity market, however, and we will ignore this interaction in the discussion below by simply assuming that the expression on the right-hand side of (3) is appropriate as an index for assessing market power.)

Paralleling the previous argument, the right-hand side of (3) provides an index for assessing the incentives to exercise market power. However, it relies on knowledge of forward market positions. In some cases, forward contract positions are available to the market monitor. For example, in the context of a real-time market, the positions from the day-ahead market constitute forward financial positions.

To summarize, given the derivative of the inverse residual demand faced by a market participant, it is possible to evaluate the incentives to mark up price above marginal cost. This incentive can be used as an index of market power that, together with a threshold value of “large” for the index, constitutes a screening tool for market power. Unlike other indices such as HHI as applied to electricity markets (particularly “HHI”

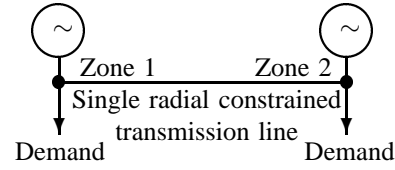


Fig. 1. Two zone network joined by radial transmission.

based on *capacity* rather than market share [3, §4-3.3]) this index is based on a principled analysis of the underlying economic incentives.

In addition to estimating the price-cost mark-up for generator k , three further indices of market power that can be useful are:

- the inverse residual demand derivative itself faced by generator k , $\frac{\partial p_{-k}}{\partial q_k}(q_k)$,
- the derivative of price with respect to injection at bus k , which can be calculated from the inverse residual demand derivative by adding to it the slope of the offer by generator k , and
- the estimated excess wealth transfer to generator k , (2) or (4).

The analysis so far does not consider the issue of transmission constraints. To the best of the authors' knowledge, no extensions of first principles analysis to the case of transmission constraints in *meshed* systems have been reported in the literature. In section III, an approach will be proposed to extending this principled analysis to a meshed transmission system.

Before discussing the proposed approach to meshed systems, however, we will first discuss the case of a single *radial* transmission constraint in section II-B. Although a single radial transmission constraint is an unrealistic situation in the context of an LMP market, it will help to explain and motivate the more complex and realistic context of a meshed system. Then, in section II-C, we will discuss an example of an *ad hoc* approach to treating market power in the presence of transmission constraints in a meshed system.

B. Principled analysis of radial transmission constraints

In a system with a single radial transmission constraint joining two zones, as shown in Figure 1, whenever there is transmission congestion between the zones, the two zones are separated into two markets. In this case, analysis of residual demand involves considering each zone separately. Consequently, it is straightforward to extend the approach in [3], [4] to analyze the residual demand in each zone when the single constraint is binding. The basic approach simply divides the market participants into the two zones and considers each zone as a separate market.

Such analysis is possible in radial systems because of a particular feature of the “shift factors,” that is, the fraction of power flowing on a line due to injection at one zone and withdrawal at another zone. For a radial line, the shift factors are always either zero or one. Consequently, given that the constraint is binding, the two zone system can validly be analyzed as two separate markets.

However, in more typical meshed systems, the shift factors are typically between minus and plus one, so that market participants *cannot* be unequivocally divided into being in one “zone” or another. The market participants interact in more complex ways due to the shift factors having values other than zero and one, requiring additional analysis. The complexity of meshed systems has prompted *ad hoc* approaches that will be discussed in the next section.

C. Ad hoc analyses of market power with transmission constraints

In contrast to the discussion in section II-A, which started with an underlying economic model of profit maximization to define an index of market power, in this section we will consider an example of an *ad hoc* analysis that attempts to analyze market power in the presence of transmission constraints in a meshed system.

In the upcoming ERCOT nodal market there are several *ad hoc* methods to assess market power. One of them, the “Element Competitiveness Index” (ECI) is styled as an *ex ante* test of competitiveness in the face of transmission constraints. In particular, it is performed to “[d]etermine if there is sufficient competition to resolve the [transmission] constraint on the import and export side” [1, §3.19.1]. There are several parts to this complicated test involving capacity shares and DC shift factors for potentially binding transmission constraints.

At its heart, the ECI test is an HHI test based on capacity. As Stoft points out, despite the use of capacity-based HHI measures by various regulatory authorities, there is no theoretical justification for capacity-based HHIs as a measure of market power [3, §4-3.3]. When HHIs are based on market shares instead of capacities, there is a connection to the Cournot model; however, even such share-based HHIs are unreliable as a measure of market power since they omit consideration of supply and demand elasticity and of forward contract positions, which are essential determinants of market power [3, §4-5.1][11].

Although the ECI test incorporates, through the shift factors, a proxy for the geographical extent of the market, it fundamentally omits the drivers of market power: the “residual demand” faced by market participants and the forward contract positions. In particular, the ECI test is not based on any offer information [1, §3.19].

Moreover, the ECI test considers each line separately and therefore does not consider the effect of *interactions* between constraints on market power. As will be illustrated in the case studies in Section V, multiple constraints are typically binding in a large system.

The focus in the ECI and other tests such as PJM’s “three pivotal supplier test” [12, Appendix J] is on particular *lines*. However, in fact, the key economic issue is the incentives to market participants at particular *buses* due to potentially multiple interacting constraints. The ECI test obscures the locus of the fundamental economic incentives.

To summarize, the approaches that consider Kirchhoff’s laws in the assessment of market power have not considered the fundamental economic incentives. As will be discussed in

the next section, the indices we develop are based on analysis of incentives provided to market participants at each bus.

III. TRANSMISSION-CONSTRAINED MARKET POWER INDICES

In previous work, [5] describes calculation of the derivative of the inverse residual demand faced by a given generator at a single bus, assuming that offers are differentiable. Information about the binding constraints and injections at the market clearing conditions based on solution of an optimal power flow (OPF), together with the generator offers, provide the information needed to evaluate the derivative. The calculation is computationally efficient given the market clearing results.

To be concrete, consider generator k located at bus k . (The case where there are multiple generators owned by different firms at a single bus is similar. We briefly mention the case where a firm owns multiple generators at different buses in section VI.) The calculation of inverse residual demand derivative at bus k considers the effects of binding transmission constraints and quantitatively evaluates the decrease in residual demand elasticity faced by generator k when transmission constraints are binding.

For simplicity, assume that only transmission constraints are binding and suppose that $H \in \mathbb{R}^{b \times (n-1)}$ is the matrix of shift factors for injection at generators $j \neq k$ for the b binding constraints in the system at the market clearing conditions, with bus k chosen to be the (price) reference bus and where there are n generators in total. That is, $\forall \ell, \forall j \neq k, H_{\ell j}$ is the amount of power flowing on constrained line ℓ given a unit injection at bus j and withdrawal at bus k . Furthermore, let $\Lambda \in \mathbb{R}^{(n-1) \times (n-1)}$ be a diagonal matrix whose diagonal entries are the inverses of the derivatives of the offers at buses other than bus k , evaluated at the market clearing conditions. Then, from [5], the inverse residual demand derivative faced by generator k evaluated at the market clearing injection q_k^* is:

$$\frac{\partial p_k}{\partial q_k}(q_k^*) = \left[-\mathbf{1}^\dagger \Lambda \mathbf{1} + \mathbf{1}^\dagger \Lambda H^\dagger (H \Lambda H^\dagger)^{-1} H \Lambda \mathbf{1} \right]^{-1}, \quad (5)$$

where $\mathbf{1} \in \mathbb{R}^{n-1}$ is the vector of all ones and superscript \dagger means transpose. That is, to calculate the inverse residual demand derivative for generator k , we need the following:

- The slope of the offer of each generator $j \neq k$ evaluated at the market clearing quantity, and
- The shift factor to the binding transmission lines for injection at each generator $j \neq k$ and withdrawal at generator k .

The inverse residual demand derivative provides the first of four indices of transmission-constrained market power and shows the level of price responsiveness faced by the generator at bus k due to the combination of offers and transmission constraints in the rest of the system. A second index is the derivative of price with respect to injection. This second index shows the level of price responsiveness at bus k including all offers and transmission constraints in the system, including the offer at bus k . It shows the level of price responsiveness faced by, for example, a new entrant at bus k .

We develop two further indices of the market conditions faced by the generator at bus k that depend on the transmission-constrained inverse residual demand derivative. These indices are conceptually straightforward extensions of the development in Section II-A to the transmission-constrained case. That is, they are based on analyzing the incentives to a profit maximizing market participant, assuming that it owns generation at only a single bus.

In particular, substitute into the right-hand side of (3):

- the market clearing injection at bus k , $q_k = q_k^*$;
- the forward contract quantity, q_k^f ; and
- the inverse residual demand derivative evaluated at the market clearing injection, $\frac{\partial p_{-k}}{\partial q_k}(q_k^*)$.

This expression provides the estimated price-cost mark-up, again assuming that sufficient conditions for profit maximization are satisfied, that p_{-k} and the cost c_k are differentiable, and that generator capacity constraints are not binding at the profit maximum. As a fourth index of transmission-constrained market power, excess wealth transfer can be estimated using (4).

In contrast to, for example, ECI, the index (3) has a concrete interpretation in terms of market power: it estimates the mark-up of price above marginal cost for a hypothetical profit maximizing generator. If forward contract information was not available then $-q_k \frac{\partial p_{-k}}{\partial q_k}(q_k)$ could be used as an index instead; however, any subsequent market power mitigation should be sensitive to the implications of forward contracting on market power. The index (4) also has a concrete interpretation: it estimates the excess transfer above competitive transfer levels.

To summarize, four indices of transmission-constrained market power developed in this section are:

- the transmission-constrained inverse residual demand derivative,
- the derivative of price with respect to injection,
- the estimated mark-up, and
- the estimated wealth transfer over competitive levels.

Calculation and visualization of these indices is described in the next section.

IV. IMPLEMENTATION

We implemented the prototype calculation of the indices in a convenient graphical tool using Powerworld and Matlab. The flow chart of the tool design is shown in Figure 2. Given generation offer cost data, load data, and transmission data, the OPF solution is generated by Powerworld. Using results from the OPF solution, the market power index is evaluated using Matlab. We ignore forward contracts. The calculated market power index is then visualized with Powerworld. The following section describes these issues in detail.

A. Powerworld OPF

The Powerworld OPF is capable of considering various representations of cost curves. In the context of an electricity market, the *cost* data used by the OPF corresponds

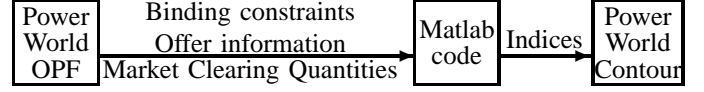


Fig. 2. Flow chart of tool design.

to the integral of the offers. We utilized the representation in Powerworld that allows for cubic (or lower degree) cost curves (corresponding to quadratic or lower degree offers). This guarantees that the offers are differentiable, consistent with the assumption in the analysis of residual demand as developed in [5], but requires that piecewise constant and other non-smooth offers be smoothed for use in the OPF.¹

Generator data including cost data, transmission data, and load data are utilized by PowerWorld to calculate the OPF. From the solution of the OPF, several other files are produced in order to calculate the market power index as described in more detail in the next section.

B. Market power indices calculation in Matlab

The files produced by Powerworld are passed to Matlab and the calculation of the indices is implemented with Matlab. To calculate the residual demand derivative faced by each generator, we evaluate (5). However, using this formula literally would involve matrix inversion, which has various drawbacks. In order to avoid matrix inversion and to avoid ill-conditioning issues, QR factorization and forwards and backwards substitution is utilized. The residual demand derivative calculation is performed for each generator.

As well as the residual demand derivative, the three other indices are also calculated:

- Derivative of price with respect to injection,
- Estimated price-cost mark-up.
- Estimated wealth transfer.

For each index, the calculated values for each generator are then visualized in Powerworld, as discussed in the next section.

C. Visualization

In order to better illustrate the market power indices, the contouring function of the Powerworld simulator is utilized. The contouring visualization function can significantly improve understanding of the calculated market power indices and help operators to evaluate market power of a large interconnected system. As indicated in Figure 2, the market power indices are passed to the contour module of the simulator as a customized market power index field.

D. Piece-wise constant offer functions

In many electricity markets, the offers are specified by piece-wise constant functions, which poses difficulties for calculating the indices since the offer slopes at the market clearing quantities might be 0, causing the matrix Λ to be

¹A DC OPF based on linear programming (LP) solution is used in Powerworld, implying that the polynomial cost curves are themselves piecewise linearized for calculations within Powerworld.

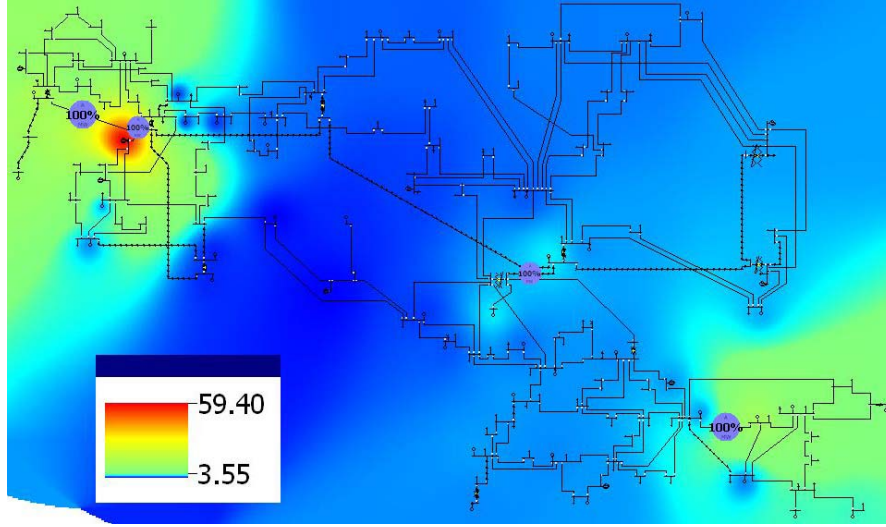


Fig. 3. Contour map of absolute value of inverse residual demand derivative for 118 bus system. The units are $(\$/\text{MWh})/\text{GW}$.

ill-defined. This issue can be dealt with by fitting a smooth curve to the piece-wise constant offer functions, as discussed in [?], so that the average slopes in the vicinity of the market clearing conditions are estimated using the whole offer function information. In our indices calculator, quadratic curves are utilized to fit the piece-wise constant offers and the offer slopes are evaluated based on the quadratic curves. Note that this post-processing step for the offer functions is only required for calculating the residual demand derivatives, while the OPF solver still uses piece-wise constant offers to clear the market.

V. CASE STUDIES

In this section, we will demonstrate the indices in the context of the 118 bus reliability test system and the ERCOT system.

A. Reliability test system

Power flow data for the 118 bus Reliability Test System [13] together with generic cost data were used to create a data set for Powerworld. An OPF was solved for this system with all pre-contingency thermal limits imposed. Figure 3 shows the contours of the inverse residual demand derivative for this system. Note that three transmission constraints (shown by the three light blue “pie charts” labelled as 100% flow) are binding in this system.

Figure 3 indicates the competitive situation faced by generators at the buses, as determined by the shift factors to binding constraints, market clearing quantities, and the offers. Many buses in the system are such that the residual demand faced at that bus is not significantly affected by the transmission constraints. These buses are shown in deep to light blue. Other buses are somewhat affected by the transmission constraints, as shown by the green contours.

However, one bus in this system has a significantly greater magnitude for its inverse residual demand derivative, as shown

by the red contours in the top left of the system near to two of the binding transmission constraints. This indicates that changes in injection at this bus would significantly affect prices. Generation at this bus would potentially be flagged for further market power analysis.

The contours of the absolute value of derivative of price with respect to injection for the 118 bus system are qualitatively similar to those in Figure 3 and, for brevity, we do not show this plot. The main difference compared to Figure 3 is that the magnitude of the entries is smaller, reflecting the inclusion of the effect of the offers at each bus in the derivative of price with respect to injection.

Figure 4 shows the estimate of the price-cost mark-up according to (1). Many buses have a very low estimate, as shown by the dark to light blue contours. However, it is interesting to note that even in the regions of low magnitude of inverse residual demand derivative, the estimated mark-ups can still be moderately high, as shown by the green contours. Unsurprisingly, the bus that had a high value of magnitude for its inverse residual demand derivative also has a high estimate for the price-cost mark-up, as indicated by the red contours.

The contours of the estimate of wealth transfer according to (2) are qualitatively similar to those in Figure 4, although there are some detailed differences. For brevity, we do not show this plot.

B. ERCOT system

Power flow data for the ERCOT system together with generic cost data were used to create a data set for Powerworld. A security-constrained OPF was solved for this system with all thermal pre- and single post-contingency constraints enforced except for those post-contingency constraints that could not be satisfied, such as those involving:

- radially connected generators,
- radially connected loads,
- certain radial loops, and

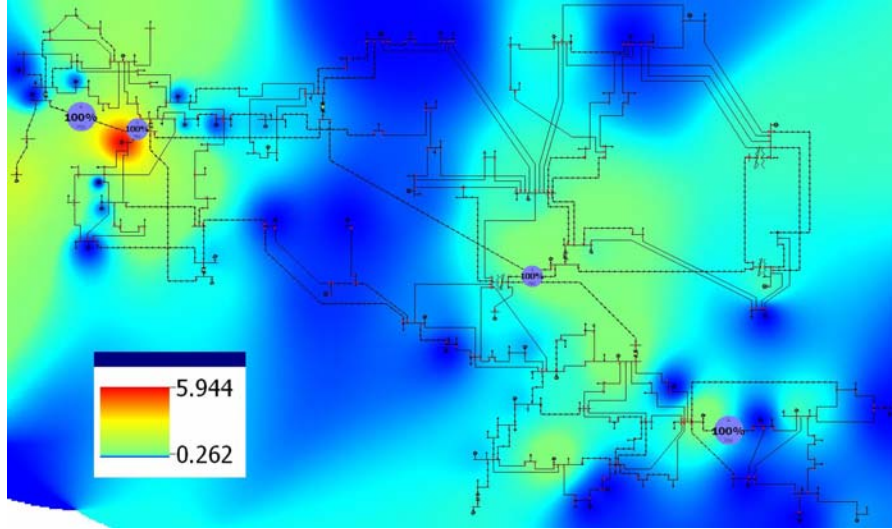


Fig. 4. Contour map of estimated price-cost mark-up for 118 bus system. The units are \$/MWh.

- nearby normally open circuit breakers that could be closed in the event of the contingency to mitigate an overload.

We did not enforce voltage constraints.

There were three binding pre-contingency constraints and sixteen binding post-contingency constraints at the OPF solution, indicating the importance of considering multiple interacting constraints. The inverse residual demand was calculated from the solution of the security-constrained OPF according to (5). For brevity, only the index of estimated price-cost mark-up according to (1) will be shown.

Figure 5 shows the estimate of the price-cost mark-up (on a common log scale) according to (1) for buses in the ERCOT system. Most of ERCOT has relatively low values for the estimates of the price-cost mark-up. However there are several regions of moderate estimated mark-up and one region with very large estimated mark-up nearby to Austin due to several generators having large shift factors to a binding transmission constraint, which results in large inverse residual demand derivatives. Estimated price-cost mark-ups of more than 5 million \$/MWh are unrealistic given the (unmodeled) levels of forward contract cover and offer caps. If this information were known, more realistic estimates could be obtained that would more faithfully reveal the incentives for generators to drive up the prices.

VI. CONCLUSION AND FUTURE WORK

In this paper we have developed four indices of market power in the context of transmission-constrained electricity markets and shown an effective visualization of the indices. The indices could be evaluated in several contexts, such as:

- part of an *ex ante* simulation of market operation over pricing intervals in a time horizon using a production cost simulator, either based on competitive offers or based on some assumed strategic offers;
- alongside the clearing of the actual market based on actual offers; and

- part of an *ex post* analysis by a market monitor, based on historical offer information.

In future work, we plan to consider a number of additional issues. For example, the profit maximizing condition in Section III was derived under the assumption that the cost function and, more importantly, the inverse residual demand function is differentiable. In fact, this assumption does not always hold. In particular, if a transmission or generator capacity constraint is only “just” binding at the market clearing conditions (or if a marginal offer is at a point of non-differentiability of the offer) then the left-hand and right-hand derivatives of the inverse residual demand are not equal. We will consider this issue in more detail in future work.

As another example, the development in this paper has assumed that any particular firm owns a generator or generators at only one bus, bus k . However, in reality, in LMP markets, assets will be owned at multiple buses and the LMPs at these different buses are related. We will consider the issue of ownership at multiple buses in future work using the concept of the residual demand *Jacobian*.

As a third example, the analysis in this paper considers the “small signal” issue of whether the slope of the residual demand is such that the first-order necessary conditions for profit maximization imply “economic” withholding that would significantly increase price over marginal costs. At high levels of demand, particularly, another concern is that the “large signal” action of “physically” withholding capacity would lead to infeasibility. In this case, the generator is sometimes said to be “pivotal” [14], [15], [16]. If a generator is pivotal then, in principle, it can increase the price arbitrarily.

In the absence of transmission constraints, [14] discusses an index, the “residual supply index” (RSI), that reflects the degree to which a firm’s offer is necessary to meet demand. However, binding transmission constraints may make firms pivotal in particular geographic areas even if from a region-wide perspective there is no pivotal firm. In future work, we will consider generalizations of RSI that apply in the

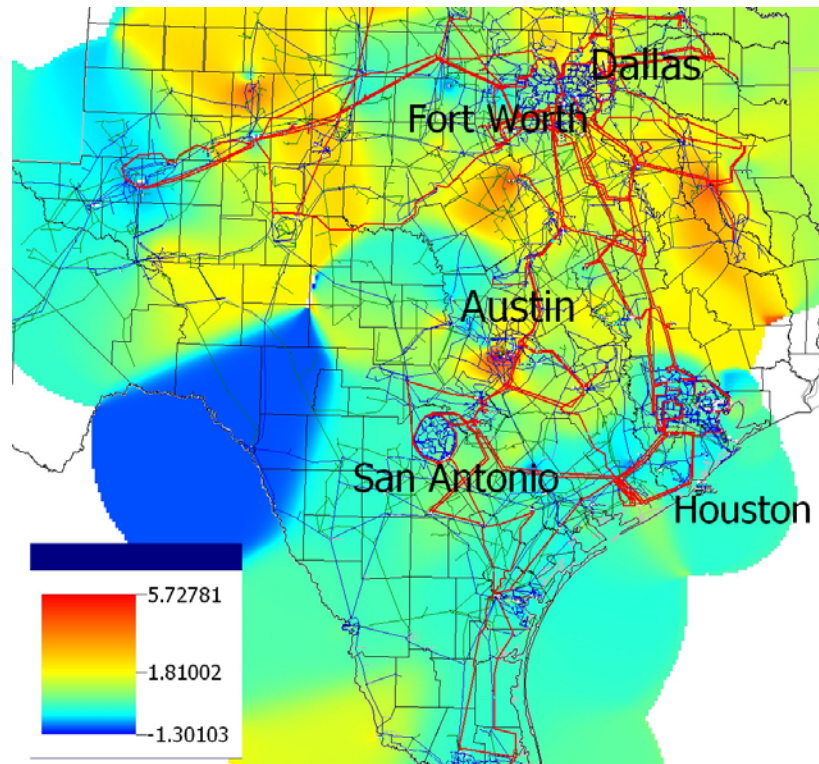


Fig. 5. Contour map of common log of estimated price-cost mark-up (in \$/MWh) for the ERCOT system.

transmission-constrained case, generalizing tests such as the “three pivotal supplier test” used in PJM [12, Appendix J].

As a final example, in future work we also plan to include contract cover in the estimates.

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Response to reviewer 1:

Q: The reviewer suggested using “notional price cost mark-up” instead of “price cost mark-up” to refer to the index.

A: We agree that our indices make behavioral assumptions. Therefore, we use “estimated price-cost mark-up” to distinguish it from the actual price-cost mark-up.

Q: It is not clear the precise meaning of “type of analysis” in the sentence starting “To the best of the author’s knowledge...” in page 3.

A: “type of analysis” means principles-based analysis. To be more clear, we had rephrased the sentence.

Q: The claim that HHI is misleading at worst is too harsh.

A: We agree that “ad hoc” approaches, such as HHI, have distinct value. We have re-phrased the description.

Response to reviewer 2:

Q: Assuming differentiability of offer functions is not realistic.

A: We agree that most electricity markets use non-differentiable offers. We added a paragraph Section IV-D to describe how we cope with piece-wise constant offer functions and how this was implemented in our indices calculator.

Response to reviewer 3:

Q: Re-run the model with different contract coverage scenarios might be interesting.

A: Due to the limited space, we just demonstrated the basic calculation of indices for the ERCOT system to show that our analysis is feasible for large scale system. We agree with the reviewer that it would be interesting to do more analysis with different scenarios, which could be part of our future work.

Q: Some mark-up values of the ERCOT system seem to be unreasonably large.

A: We double-checked the numerical values and for the assumed case study conditions it yields the correct numerical values. The unrealism is due to the effect of contract cover and offer caps. We added some description to explain this issue.

Q: The range of values takes peculiarly unappealing values.

A: For the 118 bus system, the color map can display the index taking on moderate inverse residual demand derivatives. For the ERCOT system, we replaced the graph with one having a logarithmic scale to show the variation more clearly (see Figure 5).