

# Managing the financial risks of electricity producers using options

S. Pineda<sup>a</sup>, A.J. Conejo<sup>a,\*</sup>

<sup>a</sup>*Department of Electrical Engineering, Univ. Castilla-La Mancha  
Campus Universitario s/n, 13071 Ciudad Real, Spain*

---

## Abstract

Electricity producers participating in electricity markets face risks pertaining to both selling prices and the availability of the production units. Among electricity derivatives, options represent an adequate instrument to manage these risks. In this paper, we propose a multi-stage stochastic model to determine the optimal selling strategy of a risk-averse electricity producer including options, forward contracts, and pool trading. A detailed case study highlights the advantages of an option vs. a forward contract to hedge against the financial risks related to pool prices and unexpected unit failures.

*Keywords:* Electricity market, option, forward contract, electricity producer, risk, multi-stage stochastic programming.

---

## 1. Introduction

### 1.1. Motivation and purpose

Over the last decade, electricity energy systems worldwide have undergone major restructuring, introducing competition among electricity suppliers. In this environment, electricity is commonly traded in a pool. Due to the non-storability of electricity, the uncertain and inelastic demand, and the physical capacity of transmission limits, electricity pool prices are highly volatile. Thus, electricity producers need to manage the risk associated with the volatility of the pool price to avoid financial losses. Additionally, unexpected failures of production units

---

\*Corresponding author. Phone: +34 926 295433; Fax: +34 926 295361.

Email addresses: Salvador.Pineda@uclm.es (S. Pineda), Antonio.Conejo@uclm.es (A.J. Conejo)

entail a financial risk because of the impossibility of delivering the energy pertaining to contract obligations during those time steps in which some of the production units are forced out. Forward contracts and options are the main derivatives for risk management in electricity markets (Deng and Oren, 2006; Liu and Wu, 2007).

Forward contracts are agreements to buy/sell a fixed amount of electricity at a given price throughout a certain time span in the future. Selling electricity through a forward contract at a fixed price allows electricity producers to hedge against the risk due to pool price volatility. On the other hand, the main disadvantage of a forward contract is that its delivery is mandatory, i.e., if the electricity producer is unable to deliver the agreed amount of energy, it must buy the corresponding energy in the pool to indirectly deliver it. If the pool price is high during these time steps, financial losses may occur. Although usually neglected in long-term trading, forced outages of production units may have significant effects on short and medium-term trading (Haghighat et al., 2008). For example, considering the Generating Availability Data System of NERC (NERC, 2010), the average forced outage rates of coal, oil and gas production units between 2002 and 2006 were 4.1%, 7.8%, and 2.6%, respectively. Therefore, a risk-averse producer has to decide its optimal forward contract portfolio taking into account the volatility of the pool price (price risk) and the possibility of experiencing production unit failures (availability risk) (Pineda et al., 2008; Beenstock, 1991).

Alternately to sell electricity through forward contracts, a producer can sell its production through options. An option is a contract which gives the holder of the option the right (not the obligation) to buy/sell a specified amount of energy during a certain future time at the so-called *strike price* (Hull, 2003). Therefore, an option provides more flexibility than a forward contract since the holder can decide whether or not the option is exercised at a future time depending on the availability of its production units and/or the pool price. On the other hand, whereas signing a forward contract entails no cost, there is a non-refundable cost to acquire an option, which is called *option price*. In this paper, we consider options on physical forward contracts, i.e., the exercising of the option necessarily implies the physical delivery of the agreed amount of electricity (Deng and Oren, 2006; Oum et al., 2006).

### *1.2. Literature review and contribution*

Although the technical literature is not broad, several works analyze the different types of risks faced by electricity producers as well as the derivatives to manage these risks (e.g., Deng and Oren (2006), Liu and Wu (2007), Tan et al. (2005), and Paravan et al. (2004)). References Kaye et al. (1990) and Tanlapco et al. (2002) analyze forward contracts as derivatives to manage price risk in electricity markets. On the other hand, reference Quan and Hao (2004) shows that options reduce the price risk and allow market participants to increase their potential profits. Since electricity cannot be stored, the well-known Black-Scholes' equation (Wu, 2004) is not generally a good method for pricing electricity derivatives. In this context, reference Lane et al. (2000) proposes a heuristic algorithm to value electricity options. Reference Richter and Sheblé (1998) analyzes the impact of options and forward contracts on the offering strategies of electricity market agents. References Oum et al. (2006) and Xu et al. (2006) discuss the possibility of mitigating the risks faced by load serving entities using electricity options.

Within the context above, the contribution of this paper consists in analyzing electricity options as instruments to manage the two main risks faced by price-taker electricity producers: price and production-availability risks. This is achieved through a multi-stage stochastic programming model (Benth, 2008; Fleten et al., 2008), which is used to decide the optimal portfolio of forward contracts and options for a risk-averse electricity producer taking into account the pool price volatility, the unexpected production unit failures, and the uncertain forward prices. Although contributions to manage price risk through options have been reported in the literature, contributions to manage availability risk have not.

### *1.3. Paper organization*

The rest of this paper is organized as follows. Section 2 describes the model proposed, including a characterization of production units, pool prices, forward contracts, and options. The corresponding multi-stage stochastic optimization problem is explained in Section 3. Section 4 reports a case study whose results highlight the advantages of managing risk using options. Relevant conclusions are stated in Section 6. The modeling of the Conditional

Value-at-Risk (CVaR) is briefly presented in Appendix A. Appendix B provides the notation used in this paper.

## **2. Model characterization**

### *2.1. Model assumptions*

The following modeling assumptions are made:

1. The production units owned by the electricity producer are thermal units, being each one a dispatchable source of electricity whose cost is modeled by a piecewise linear function and whose power output is limited by minimum and maximum bounds. Ramp limits and minimum up and down times are short-term operating constraints disregarded in the proposed model.
2. The electricity producer can sell its production in the pool at volatile prices, or at fixed prices through forward contracts or options. For the sake of clarity, the arbitrage (understood as the practice of making profit by a simultaneous purchase and sale of the same commodity) between these markets is avoided in the proposed model.
3. The prices of forward contracts and options are not affected by the decisions made by the electricity producer, which is assumed to behave as a price-taker.
4. Three uncorrelated sources of uncertainty are considered in the proposed model: the pool price, the availability of the production units, and the forward prices.
5. Although both physical and financial options are available in electricity markets, due to the energy-oriented approach of the work reported in this paper, all the considered options imply the physical delivery of the option power.
6. Only European put options are considered in this paper.
7. Options can be traded any time up to their expiration date. However, for the sake of simplicity, the electricity producer is only allowed to trade options at the beginning of the decision horizon for a certain option price.
8. The time throughout the decision horizon is measured in hourly steps.

## 2.2. Production unit availability

In this section we present the availability characterization of a single production unit. Taking into account that the failure and repair rates of this unit are constant, the probability that the unit is available in time  $t$  is (Dhillon, 2007):

$$p(k_t = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu \cdot (k_0 - 1) + \lambda}{\lambda + \mu} e^{-(\lambda + \mu)(t - t_0)}, \quad (1)$$

where  $k_0$  is equal to 1 if the unit is available at  $t_0$  and 0 otherwise,  $\lambda$  is the production unit failure rate, and  $\mu$  is the production unit repair rate. The probability that the production unit is unavailable in  $t$  is equal to  $1 - p(k_t = 1)$ .

The failure rate ( $\lambda$ ) and repair rate ( $\mu$ ) are equal to the inverse of the mean time to failure (MTTF) and mean time to repair (MTTR), respectively, i.e.,  $\lambda = \frac{1}{\text{MTTF}}$  and  $\mu = \frac{1}{\text{MTTR}}$ . The values of the MTTF and MTTR are determined based on historical data. The forced outage rate (FOR) is the percentage of time that the production unit is unavailable, i.e.,  $\text{FOR} = \frac{\text{MTTR}}{\text{MTTF} + \text{MTTR}}$ , (Billinton, 1984).

As stated in Billinton (1984), if the unit is initially available ( $k_0 = 1$ ), the probability that the unit is available in the next hour is equal to

$$p(k_1 = 1) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu) \cdot 1}. \quad (2)$$

Randomly simulating a Bernoulli distribution with success probability  $p(k_1 = 1)$ , we obtain a possible realization of the availability of the unit for the first hour of the study horizon. Repeating the same process for each hour of this study horizon and updating the success probability of the Bernoulli distribution as

$$p(k_t = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu \cdot (k_{t-1} - 1) + \lambda}{\lambda + \mu} e^{-(\lambda + \mu) \cdot 1}, \quad (3)$$

we obtain one availability scenario of the unit for the whole study horizon. In order to characterize the uncertainty associated with unit failures, a sufficiently large number of availability scenarios has to be generated.

Considering (1), note that for  $t$  high enough, the probability that the unit is available is independent of its initial status  $k_0$  and is equal to  $\frac{\mu}{\lambda + \mu}$ . On the other hand, for a small  $t$ , this

probability strongly depends on the initial status of the unit. In order to visualize this relevant effect, Fig. 1 depicts the probability that a production unit with FOR equal to 10% is available during one month depending on its MTTF and its initial status. In Fig. 1(a), the MTTF of the production unit is much smaller than the time horizon  $N_T$  and therefore, the probability that the unit is available depends on its initial status only for the first few time steps of the time horizon. For time steps higher than 100, the probability that the unit is available is practically the same whether or not the unit was initially available. On the other hand, in Fig. 1(b), the MTTF of the production unit is much larger than  $N_T$  and therefore, the probability that the unit is available depends on its initial status during the whole time horizon.

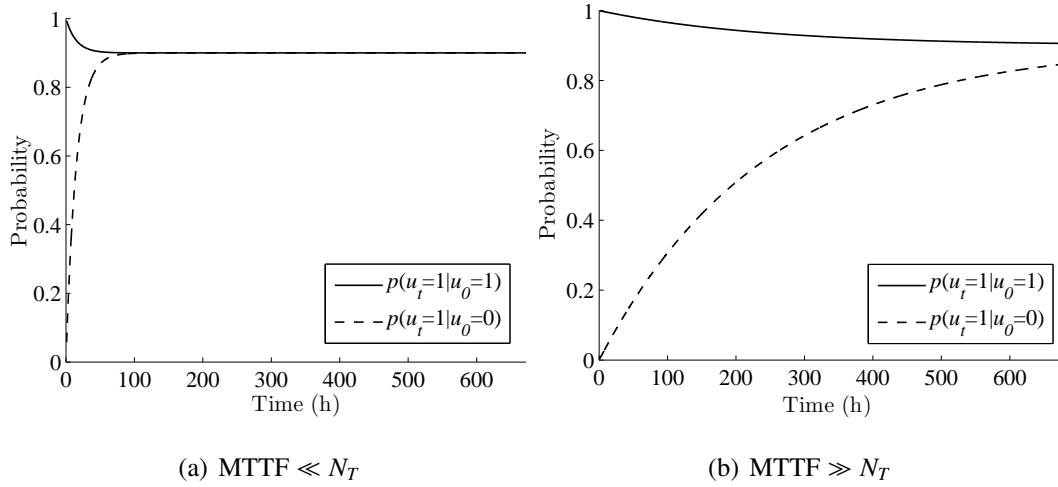


Figure 1: Availability probability functions.

Consequently, for a given time horizon and FOR, the higher the MTTF, the more importance has the current status of the unit to predict its future availability.

Finally, it is relevant to note that if the electricity producer owns several units, availability scenarios are formed by combining the individual availability scenarios of the different units, which are generated according to the above procedure.

### 2.3. Pool price

Uncertainty related to pool prices is characterized via a set of scenarios. Historical pool price data are used to adjust the parameters of an ARIMA model (Conejo et al., 2005). Then,

hourly pool price scenarios are generated randomly simulating the innovation term of the ARIMA model.

Since a price-taker producer is considered, i.e., a producer that has no influence on the market clearing prices, production unit failures do not affect pool prices and thus, the possible correlation between these stochastic processes (price and unit failures) is neglected. Moreover, to avoid arbitrage, the electricity producer is not allowed to buy electricity from the pool, except during these time steps in which the production unit is forced out.

In medium-term decision-making models, the level of uncertainty involved in pool decisions is much smaller than that affecting other type of decisions such as the energy to be sold through forward or option contracts. For this reason, decisions on pool trading are assumed to be made with perfect information. Within a multi-stage stochastic programming model, this assumption is mathematically stated by considering that pool decisions affecting a given time period depend on each scenario realization.

#### *2.4. Forward contracts and options*

Forward contracts in electricity market can be settled either physically or financially (Lien and Tse, 2006). However, and given that the model proposed in this paper is energy-oriented, we consider that all power traded is within physical limits, in other words, that all forward contracts imply the physical delivery of the electricity. Therefore, each forward contract is characterized by a given quantity, a fixed forward price, and a delivery period. Knowing this, the producer has to decide how much energy to sell through each available forward contract.

A put/call option is an agreement which gives the buyer the right, but not the obligation, to sell/buy a particular amount of electricity during a certain future time period and at a fixed price called strike price. Needless to say, buying an option has an additional cost (compared with a forward contract) called option price, which has to be paid even if the option is not exercised. Depending on whether the option can be exercised at any time up to the expiration date or only on the expiration date itself, options are classified into American or European options, respectively (Hull, 2003). In this paper, only European put options are considered. Moreover, all options analyzed in this paper are option on physical forward contracts, i.e., the

exercise of the option implies the physical delivery of the agreed option power (Spinler et al., 2003).

Fig. 2 illustrates the differences between a forward contract and an option to hedge against price risk. In this figure, the profit/loss of a electricity producer selling its electricity through either a forward contract with a price equal to  $\lambda_c$  or a put option with a strike price equal to  $\lambda_o^S$  and an option price equal to  $\lambda_o^O$  are compared for different values of the pool price  $\lambda^P$ . The profit/loss related to the forward contract depends on experiencing low/high pool prices, and moreover, it is unlimited. On the other hand, if the producer has purchased a put option, the loss associated with high pool prices are reduced to the option price by not exercising the option. Note, however, that the profit obtained if low pool realize is lower than that associated with the forward contract because of the option price.

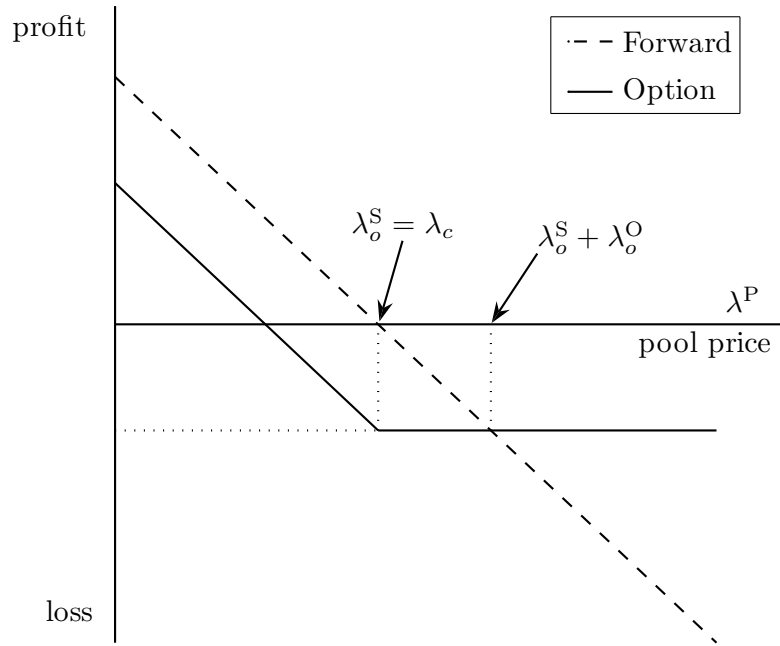


Figure 2: Forward contract vs. Put option.

It is also important to note that the producer considered in this study acts as a price-taker in the trading of derivatives, i.e., producer decisions to acquire forward contracts or options to sell electricity do not affect the prices of these derivatives.



## 2.5. Decision framework

Decisions related to electricity options are made in different stages. First, the producer has to decide whether or not to sell electricity through a given option, and later, and provided that the option has been purchased, the producer has to determine whether or not it is exercised. Since we consider European options, this second decision has to be made on the expiration date. Therefore, the study horizon is divided in two periods and three stages (Fig. 3).

As an example, we consider that the producer can sell its production at fixed prices through two forward contracts and an option. The first forward contract spans period 1 (comprising time steps  $t = 1, \dots, N_{T_1}$ ), which implies that the producer must deliver the agreed energy during each hour of period 1. Likewise, a second forward contract spanning period 2 (comprising time steps  $t = N_{T_1} + 1, \dots, N_T$ ) is considered. Moreover, the producer has the possibility of selling its production through an option spanning period 2 (comprising time steps  $t = N_{T_1} + 1, \dots, N_T$ ). This option can be bought at the beginning of period 1 and exercised at the beginning of period 2. Therefore, the sequence of decisions is the following:

- Stage 1. Determination of the energy to be sold through the available forward contracts (whose delivery periods are within either period 1 or 2) and the energy to be sold through the available option for period 2. These decisions are made considering all plausible realizations of the stochastic processes involved, i.e., knowing neither the pool price, nor the forward prices, nor the unit availability throughout the study horizon.
- Stage 2. Once the pool price and the unit availability throughout period 1 are known, both the amount of energy that the producer sells in the pool and its production planning during this period, i.e., from  $t = 1$  to  $t = N_{T_1}$  are determined. Additionally, the producer can modify the energy sold through the forward contract spanning period 2 depending on the updated forward and pool prices. Finally, if the put option has been purchased, the producer decides whether or not to exercise it.
- Stage 3. Once pool prices and the unit availability during period 2 ( $t = N_{T_1} + 1, \dots, N_T$ )

become certain, the producer determines the energy to sell in the pool during period 2 as well as the produced energy by the unit from  $t = N_{T_1} + 1$  to  $t = N_T$ .

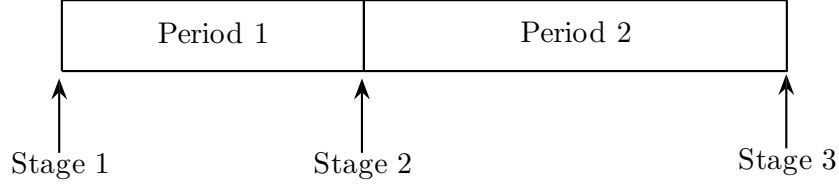


Figure 3: Time horizon and stages.

Observe that although the option spanning period 2 can be traded any time up to the beginning of this period, we have simplified its trading by allowing the producer to sign options only at the beginning of period 1. Additionally, note that there is no incentive to acquire an option close to its maturity since almost no new information becomes available between its purchase and its exercise.

It is important to highlight that other arrangements involving a larger number of forward contracts and options and a different time-horizon setting can be modeled similarly as in the example described above.

In two-stage stochastic problems, some decisions (here-and-now decisions) are made before the uncertain parameters are known, whereas other decisions (wait-and-see decisions) are delayed until the stochastic processes realize in particular values. Multi-stage recourse problems deal with models in which this “decide-observe-decide” pattern is repeated more than once and therefore, they turn out to be an adequate mathematical model to determine option purchases in electricity markets (Birge, 1997; Fleten et al., 2002).

Fig. 4 depicts an illustrative scenario tree for a three-stage stochastic problem. Each branch represents the realization of the stochastic processes between two consecutive stages. For example, each branch between stages 1 and 2 corresponds to a possible realization of the pool price and the unit availability during period 1. Likewise, each node corresponds to the decisions to be made in each stage. In this sense, decisions in stage 1 are made facing uncertainty affecting period 1 and 2; decisions made in stage 2 depend on the realization of

the stochastic processes during period 1, but they are independent of the possible scenario realizations during period 2; and third-stage decisions are selected depending on each realization of the stochastic processes throughout the study horizon. In Section 3.1, we present the constraints that impose the *non-anticipativity* conditions implied by the above decide-observe-decide process. The duration of each period and the decisions in each stage are specified in Fig. 4.

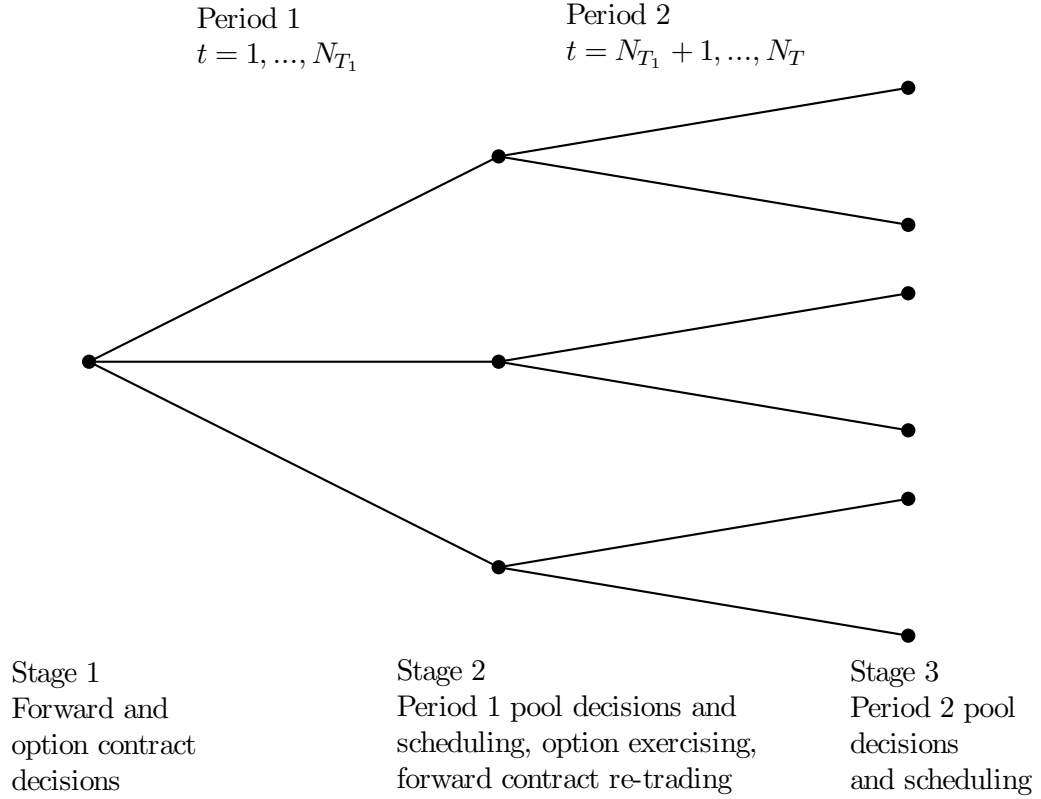


Figure 4: Three-stage scenario tree.

## 2.6. Scenario generation

In order to appropriately characterize the uncertainty of the parameters involved in the proposed model, the sequence of decisions of the multi-stage stochastic problem has to be taken into account to generate the scenario set. In other words, the knowledge of the realization of a stochastic process during period 1 has to be properly taken into account to generate

the scenario set representing the uncertainty of the same process during period 2.

Availability scenarios are built as follows. First, a scenario set representing the availability of the production unit during period 1 is generated according to the method explained in Section 2.2. Then, and for each scenario generated for period 1, a new availability scenario set for period 2 is generated considering the initial status of the unit ( $k_0$ ) as its availability in the last hour of period 1.

Pool price scenarios are built as follows. A scenario set representing the pool price during period 1 is generated by simulating the innovation term of the ARIMA model (Section 2.3). Then, the values of each pool price scenario for period 1 are taken as certain and included in the ARIMA model to generate each scenario set for period 2. In this way, a scenario of high/low pool prices during period 1 gives rise generally to a scenario set of high/low prices during period 2.

Forward contracts with power delivery during period 2 can be traded in both stages 1 and 2, at known and uncertain prices, respectively. Thus, a scenario set representing forward prices at the end of period 1 needs to be generated.

### *2.7. Risk treatment*

Electricity producers are generally risk-averse, i.e., they make their decisions based not only on the expected profit but also on the probability of suffering low profits. The risk measure considered in this paper is the Conditional Value-at-Risk (CVaR) of the profit distribution with a confidence level  $\alpha$  (Rockafellar and Uryasev, 2000; Pflug, 2000). If  $\alpha$  is equal to zero, the CVaR coincides with the expected value of the whole profit distribution. On the other hand, a value of  $\alpha$  close to 1 makes the CVaR equal to the lowest profit. Therefore, the risk aversion of the producer is simulated by increasing the confidence level  $\alpha$  from 0 (risk-neutral case) to 1 (highly risk-averse case). The modeling of the CVaR is presented in Appendix A.

## **3. Formulation**

The proposed model is formulated and discussed below.

### 3.1. General formulation

The deterministic equivalent of the considered multi-stage stochastic programming problem is

$$\text{Maximize}_{P_{t\omega}^P, P_{it\omega}^G, P_o, y_{o\omega}, P_{c_1}^1, P_{c_2}^1, P_{c_2\omega}^2, \zeta, \eta_\omega} \quad \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \quad (4a)$$

subject to

$$\sum_{i=1}^{N_I} P_{it\omega}^G = \sum_{c_1 \in F_t^1} P_{c_1}^1 + \sum_{c_2 \in F_t^2} (P_{c_2}^1 + P_{c_2\omega}^2) + \sum_{o \in O_t} y_{o\omega} P_o + P_{t\omega}^P, \quad \forall t, \forall \omega \quad (4b)$$

$$P_{it\omega}^G \leq u_{it\omega} k_{it\omega} P_i^{\max}, \quad \forall i, \forall t, \forall \omega \quad (4c)$$

$$P_{it\omega}^G \geq u_{it\omega} k_{it\omega} P_i^{\min}, \quad \forall i, \forall t, \forall \omega \quad (4d)$$

$$\Pi_\omega = \sum_{t=1}^{N_T} \lambda_{t\omega}^P P_{t\omega}^P L_t - \sum_{t=1}^{N_T} \sum_{i=1}^{N_I} C_i P_{it\omega}^G L_t + \sum_{c_1=1}^{N_{C_1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \quad (4e)$$

$$+ \sum_{c_2=1}^{N_{C_2}} (\lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2\omega}^2 P_{c_2\omega}^2) T_{c_2} + \sum_{o=1}^{N_O} (-\lambda_o^O + y_{o\omega} \lambda_o^S) P_o T_o, \quad \forall \omega \quad (4f)$$

$$-\Pi_\omega + \zeta - \eta_\omega \leq 0, \quad \forall \omega \quad (4g)$$

$$P_{it\omega}^G = P_{it\omega'}^G, \quad \forall i, \forall t = 1, \dots, N_{T_1}, \forall \omega, \omega' : \omega' \in S_\omega \quad (4h)$$

$$P_{t\omega}^P = P_{t\omega'}^P, \quad \forall t = 1, \dots, N_{T_1}; \forall \omega, \omega' : \omega' \in S_\omega \quad (4i)$$

$$u_{it\omega} = u_{it\omega'}, \quad \forall i, \forall t = 1, \dots, N_{T_1}; \forall \omega, \omega' : \omega' \in S_\omega \quad (4j)$$

$$P_{c_2\omega}^2 = P_{c_2\omega'}^2, \quad \forall c_2, \forall \omega, \omega' : \omega' \in S_\omega \quad (4k)$$

$$y_{o\omega} = y_{o\omega'}, \quad \forall o, \forall \omega, \omega' : \omega' \in S_\omega \quad (4l)$$

$$P_{c_2\omega}^2 = P_{c_2\omega'}^2, \quad \forall c_2, \forall \omega, \omega' : (k_{iN_{T_1}\omega} = k_{iN_{T_1}\omega'}, \forall i, \text{ and } \lambda_{t\omega}^P = \lambda_{t\omega'}^P, \forall t = 1, \dots, N_{T_1}) \quad (4m)$$

$$y_{o\omega} = y_{o\omega'}, \quad \forall o, \forall \omega, \omega' : (k_{iN_{T_1}\omega} = k_{iN_{T_1}\omega'}, \forall i, \text{ and } \lambda_{t\omega}^P = \lambda_{t\omega'}^P, \forall t = 1, \dots, N_{T_1}) \quad (4n)$$

$$P_{c_1}^1 \geq 0, \quad \forall c_1 \quad (4o)$$

$$P_{c_2}^1 \geq 0, \quad \forall c_2 \quad (4p)$$

$$\eta_\omega \geq 0, \quad \forall \omega \quad (4q)$$

$$P_o \geq 0, \quad \forall o \quad (4r)$$

$$P_{t\omega}^P \geq \sum_{i=1}^{N_I} (k_{it\omega} - 1)P_i^{\text{Max}}, \quad \forall t, \forall \omega \quad (4s)$$

$$P_{c_2}^1 + P_{c_2\omega}^2 \geq 0, \quad \forall c_2, \forall \omega \quad (4t)$$

$$u_{it\omega} \in \{0, 1\}, \quad \forall i, \forall t, \forall \omega \quad (4u)$$

$$y_{o\omega} \in \{0, 1\}, \quad \forall o, \forall \omega \quad (4v)$$

where

$$S_\omega = \{\omega' : (\lambda_{t\omega'}^P = \lambda_{t\omega}^P, k_{it\omega'} = k_{it\omega}, \forall i, \forall t = 1, \dots, N_{T_1}) \text{ and } (\lambda_{c_2\omega'}^2 = \lambda_{c_2\omega}^2, \forall c_2)\}.$$

Objective function (4a) corresponds to the CVaR of the probability distribution of the profit over all stages. The modeling of the CVaR is explained in Appendix A.

Constraints (4b) make the power generated in each time step and scenario equal to the power sold through forward contracts, through options (only put options are considered), and in the pool. Note that if this constraint is removed from the model, the power traded from forward and option contracts are not constrained by the physical production of the units and therefore, both contracts can be understood as financial derivatives instead of physical contracts. Constraints (4c) and (4d) bound above and below, respectively, the generated power. Constraints (4f) define the profit obtained in each scenario  $\omega$ . The first term represents the sum over all time steps of the revenue obtained in the pool; the second term is the production cost; the third one, the revenue associated to forward contracts spanning period 1; the fourth one, the profit of the energy traded through forward contract spanning period 2; and the last one, the revenue pertaining to the signed options. Constraints (4g) impose that the optimal value of each  $\eta_\omega$  associated with a scenario with a profit lower than the VaR (Value-at-Risk, see Appendix A) is equal to the difference between the VaR and that profit, and 0 otherwise (Pflug, 2000). Constraints (4h)–(4l) are the nonanticipativity constraints that impose that second-stage decisions are made knowing the corresponding scenario realization during period 1, but still facing the uncertainty related to period 2 (Higle, 2005). Two sets of availability scenarios for period 2 associated with two availability scenarios for period 1 with the same final unit status are generated using the same probability distribution. Therefore, if

a sufficiently high number of scenarios is generated to represent the availability uncertainty during period 2, two-stage decision variables associated with all the availability scenario sets for period 2 sharing the same final unit status in period 1 will be naturally the same. However, since a relatively small number of scenarios must be considered to make the stochastic optimization problem computationally tractable, these availability scenarios sets for period 2 that are generated with the same final unit status in period 1 may be comprised of very different availability scenarios, thus probably leading to different two-stage decision variables. In order to *artificially* eliminate the possibility of obtaining results that contradict the previous statement, constraints (4m) and (4n) are included in the stochastic optimization model. Constraints (4o)–(4r) are nonnegativity declarations. Constraints (4s) prevent arbitrage between futures and pool markets by allowing the electricity producer to buy energy in the pool only in those time steps and scenarios in which some production units are forced out. If such constraint is removed, arbitrage does occur. Although the power re-traded through each forward contract  $c_2$  at stage 2 can be positive or negative, constraints (4t) impose that the producer can only sell electricity through each forward contract  $c_2$  to avoid arbitrage (for the sake of simplicity). Constraints (4u)–(4v) are binary variable declarations.

The above optimization problem contains two non-linear constraints (4b) and (4f), in which the continuous variable  $P_o$  is multiplied by the binary variable  $y_{ow}$ . However, this product can be easily linearized (Williams, 1999).

Note that only options to sell electricity are included in the stochastic optimization model (4). However, other types of options can be modeled within the proposed multi-stage stochastic framework.

### 3.2. Maximum option price

In the optimization model described in the previous section, the option price of the available option is assumed to be known at the beginning of the study horizon. However, the maximum option price that a electricity producer is willing to pay for a given option can also be calculated by solving two optimization problems in which the strike price and the option power sold through the option,  $\lambda_{(o)}^S$  and  $P_o$ , are given parameters. The value of the CVaR

in the first problem ( $\text{CVaR}^1$ ) is calculated considering that there is no available option. In the second problem, an option is considered with option and strike prices equal to 0 and  $\lambda_o^S$ , respectively. The value of the CVaR in this case is denoted  $\text{CVaR}^2$ . Since the CVaR is a coherent measure of risk (Pflug, 2000), an increase of the option price in problem 2 originates a proportional decrease in CVaR, i.e.,

$$\text{CVaR} = \text{CVaR}^2 - \lambda_o^O P_o T_o. \quad (5)$$

where  $T_o$  is the duration of the delivery horizon.

Note that although  $P_o$  is a continuous variable, the above property needs  $P_o$  to be constant and therefore, the maximum option price is calculated for a fixed value of the power sold through the option.

The maximum option price that the electricity producer is willing to pay for the option is the one that makes the value of the CVaR equal to that obtained if the option is not available, i.e.,  $\text{CVaR}^1$ . Therefore, the maximum option price  $\lambda_o^{O*}$  is calculated as

$$\lambda_o^{O*} = \frac{\text{CVaR}^1 - \text{CVaR}^2}{P_o T_o}. \quad (6)$$

The value of the maximum option price provides information on how much the producer is willing to reduce the price of the energy it sells through the option in order to postpone some decisions until more information of the uncertain parameters become known.

It is worth mentioning that although the optimization model (4) can be solved considering several options to sell electricity, the procedure presented in this section to identify the maximum option price that a electricity producer is willing to pay considers that just one option is available.

#### 4. Case study

For simplicity, and in order to highlight the main features of options, the electricity producer of this case study is assumed to own only one production unit and can sing just one option to sell electricity. The extension to the multi-unit and multi-option case is straight forward.



The purpose of this case study is threefold. Firstly, we compare the profit made by a electricity producer that sells electricity through a forward contract spanning period 2 with the profit obtained if a put option for the same period is acquired. The situations in which selling electricity through options, instead of through forward contracts, entails advantages for the electricity producer are highlighted for different FOR and risk-aversion levels ( $\alpha$ ). Secondly, the optimal power to be sold through an option is determined by solving optimization problem (4). Finally, the maximum option price that the electricity producer is willing to pay for a given option is calculated for different strike prices, FOR and risk-aversion levels.

#### 4.1. Data

The planning horizon of this case study comprises 2 months. Periods 1 and 2 span the first and the second month, respectively. Historical pool price data from EEX (EEX, 2009) during three years (2004, 2005, and 2006) are used to adjust the parameters of an ARIMA model (Conejo et al., 2005). Hourly pool price scenarios are generated randomly simulating the innovation term of the ARIMA model, which consists in a Gaussian random variable of zero mean and standard deviation equal to 0.16. An initial set of 500 scenarios representing the pool price during the first month is generated and reduced to 10 using a scenario reduction technique that, roughly speaking, consist in solving one optimization problem per scenario of the original tree and to merge those scenarios with *similar enough* objective function optimal values (Morales et al., 2009; Pineda and Conejo, 2010). Then, and for each one of these 10 scenarios, the values of the pool price during the first month are included in the ARIMA model to generate a set of 200 pool price scenarios for the second month. Each one of these 10 scenario sets are reduced from 200 to 3 pool price scenarios in order to obtain a final set of 30 scenarios. This number of scenarios is appropriate since considering a higher number does not significantly change producer decisions. Additionally, and for the sake of tractability, hourly pool prices are aggregated in 3 prices per day.

An unit with a maximum and minimum power output of 450 and 50 MW, respectively, is considered. The linear cost function of this unit is equal to 12 €/MWh. We consider three different (realistic) forced outage rates (RTS, 1999):

- FOR = 0%. Unit failures negligible.
- FOR = 5%. MTTF = 950 and MTTR = 50.
- FOR = 10%. MTTF = 450 and MTTR = 50.

A set of 30 availability scenarios is generated for each value of FOR as explained in Sections 2.2 and 2.5. Note that only one scenario is needed for the no failure case.

Two forward contracts that span the first and the second month, respectively, are considered. Additionally, the electricity producer can acquire an option to sell electricity during the second month. The price of the forward contract spanning the first month is equal 52.5 €/MWh. The price of the forward contract spanning the second month at the beginning of the first month is set to the value of the strike price of the option in order to compare both derivatives on equal terms. Different strike and option prices are considered throughout the case study in order to analyze all possible alternatives.

An approximate scenario set representing the forward prices at stage 2 is generated based on the fact that forward prices tend to equal pool prices as maturity date approaches. To this end, we first calculate the average value of each one of the 30 generated pool price scenarios over the last four weeks as

$$\overline{\lambda_{(\omega)}^P} = \frac{\sum_{t=N_{T_1}+1}^{N_T} \lambda_{(\omega,t)}^P L(t)}{N_T - N_{T_1}}. \quad (7)$$

Next, taking into account the probability of each scenario  $\pi_{(\omega)}$ , the expected value and the standard deviation of those average pool prices, denoted by  $\rho$  and  $\vartheta$  respectively, are determined as follows

$$\rho = \sum_{\omega=1}^{N_{\Omega}} \pi_{(\omega)} \overline{\lambda_{(\omega)}^P} \quad \vartheta = \sqrt{\sum_{\omega=1}^{N_{\Omega}} \pi_{(\omega)} (\overline{\lambda_{(\omega)}^P} - \rho)^2}.$$

Three forward prices are then considered, namely,  $\rho - \vartheta$ ,  $\rho$ , and  $\rho + \vartheta$ . Table 1 provides the forward price scenarios and their corresponding probabilities. If needed, forecasting techniques can be used for generating forward price scenarios in a more precise manner.

It should be noted that if forward contracts spanning the second month can be re-traded at stage 2, the final scenario tree is obtained by combining all pool price scenarios, all availability scenario, and the three scenarios representing the uncertainty of the forward prices.

Table 1: Forward price scenarios.

$\lambda_{c_2 \omega}^2$	30.9	36.6	42.4
Probability	0.25	0.5	0.25

#### 4.2. Forward contract vs. option

In order to analyze the advantages of buying an option to eventually sell electricity versus selling electricity through a forward contract, two different cases are considered below. In case (a), the electricity producer sells 200 MW through a forward contract spanning the second month at 32 €/MWh, i.e, the variables  $P_{c_1}^1$  and  $P_o$  are fixed to 200 and 0, respectively. In case (b), the producer purchases a put option of 200 MW to sell electricity during the second month at a strike price equal to 32 €/MWh, i.e, the variables  $P_{c_1}^1$  and  $P_o$  are fixed to 0 and 200, respectively. The option price is set to 0.3 €/MWh to highlight the advantages and disadvantages of selling electricity through either a forward contract or an option. Table 2 contains the CVaR of the producer profit distribution for cases (a) and (b), different FOR values, and risk-aversion levels  $\alpha$ . In this first analysis, for simplicity, re-trading of forward contracts in stage 2 is not considered. The values in Table 2 are obtained solving different instances of problem (4).

Table 2: CVaR comparison considering either a forward contract or an option (no re-trading).

$\alpha$	FOR = 0%		FOR = 5%		FOR = 10%	
	Case (a)	Case (b)	Case (a)	Case (b)	Case (a)	Case (b)
0	19.07	19.68	18.78	19.39	18.51	19.11
0.5	18.32	18.34	17.92	18.03	17.60	17.75
0.9	17.75	17.71	17.13	17.16	16.76	16.78

First, note that for eight of the nine cases presented in Table 2, the CVaR of the electricity producer if the option is purchased is higher than that obtained if the forward contract is signed. Observe, however, that these CVaR values may significantly change as a function of the option price.

Next, we separately analyze these results depending on whether or not unit failures are considered. If the unit failures are neglected (FOR = 0%, second column block of Table 2) and the electricity producer is risk neutral ( $\alpha = 0$ ), the CVaR increases a 3.2 % from case (a) to (b) due to the fact that the producer can decide, at the end of the first month, whether or not to exercise the option according to the pool price forecasts for the second month.

To provide further insight into the exercising of the option by a risk-neutral electricity producer ( $\alpha = 0$ ) owning a non-failure production unit (FOR = 0%), Table 3 indicates in the first row the average value of the pool price over the last four weeks depending on each possible realization of the pool prices during the first four weeks (ten alternative values). These values are computed as

$$E_2\{\lambda_{(\omega,t)}^P\} = \frac{1}{\sum_{\omega' \in S_{(\omega)}} \pi_{(\omega')}} \frac{1}{N_T - N_{T_1}} \sum_{\omega' \in S_{(\omega)}} \sum_{t=N_{T_1}+1}^{N_T} \pi_{(\omega')} \lambda_{(\omega',t)}^P L_{(t)}.$$

The second row of this table provides the corresponding value of the binary variable  $y_{(o,\omega)}$ , which represents whether or not the producer exercises the option. Since a risk-neutral electricity producer seeks to maximize the expected profit, the option is only exercised in the cases in which the average pool price for the last four weeks ( $E_2\{\lambda_{(\omega,t)}^P\}$ ) is lower than the strike price (32 €/MWh). If the average pool price is higher than the strike price, the electricity producer makes a higher profit by selling its electricity directly in the pool and therefore, the option is not exercised.

Table 3: Option exercise depending on the average pool price during the second month for  $\alpha = 0$  and FOR = 0%.

$E_2\{\lambda_{t\omega}^P\}$	37.0	31.6	37.8	43.7	39.7	37.3	31.2	44.4	39.7	40.0
$y_{o\omega}$	0	1	0	0	0	0	1	0	0	0

To analyze the results provided in Table 3 for a risk-averse electricity producer ( $\alpha^P = 0.5$  and  $\alpha^P = 0.9$ ) owning a non-failing production unit (FOR = 0%) we have to keep in mind two important points. One is that a risk-averse electricity producer focuses on the lowest profits of its distribution, and the second one is that if unit failures are disregarded, the electricity

producer only faces the uncertainty related to pool prices. Thus, from a sufficiently high risk-aversion level and a FOR equal to 0%, both the forward contract and the option become attractive to the electricity producer as instruments to hedge against this price risk. Observe that in that case, it may become more profitable just selling electricity through the forward contract rather than through the option so as to save the nonrefundable fee of the latter. In this vein, note in Table 2 that for a value of  $\alpha^P$  equal to 0.5, the CVaR obtained in case (b) is only 0.1% higher than the CVaR of case (a). Moreover, for  $\alpha^P = 0.9$  the CVaR obtained if the option is purchased is indeed lower than that achieved if the electricity is sold through a forward contract during the last month.

If unit failures are considered (FOR 5% and 10%, third and fourth column blocks of Table 2, respectively), the CVaR obtained buying an option is higher than that achieved with a forward contract for all risk-aversion levels. The reason is that the option allows the producer to avoid low profits due to both low pool prices and unexpected unit failures.

Table 4 contains the value of the binary variable  $y_{o\omega}$  as a function of the average pool price during the second month ( $E_2\{\lambda_{t\omega}^P\}$ ) and the status of the unit at the end of the first month ( $k_{N_{T_1}\omega}$ ). For clarity, only three values of the expected pool price are presented: a high value (44.39 €/MWh), a medium value (39.73 €/MWh), and a low value (31.57 €/MWh). Observe in Table 4 that for a low average pool price the option is exercised for all availability scenarios. On the other hand, for a high average pool price, the option is not exercised to take advantage of selling the electricity in the pool. However, for a medium average pool price, the electricity producer exercises the option in these cases in which the unit is available at the end of the first month because, in such a case, it is more likely that the unit is also available during the second month. Note that in this case and due to the high risk aversion of the producer, the option is exercised even in the case in which the strike price is lower than the expected pool price during period 2. On the other hand, for a producer with a comparatively smaller risk-aversion level (Table 5), the option is only exercised if its strike price is higher than the expected pool price during period 2.

In the previous analysis, forward contracts spanning the second month are only traded in stage 1 at known prices. Next, we include the possibility of re-trading forward contracts

Table 4: Option exercise depending on the average pool price during the second month and the status of the unit at the end of the first month for  $\alpha = 0.9$  and FOR = 10%.

$E_2\{\lambda_{t\omega}^P\}$	$k_{N_{T_1}\omega}$									
	1	0	1	1	1	1	1	1	0	1
44.4	0	0	0	0	0	0	0	0	0	0
39.7	1	0	1	1	1	1	1	1	0	1
31.6	1	1	1	1	1	1	1	1	1	1

Table 5: Option exercise depending on the average pool price during the second month and the status of the unit at the end of the first month for  $\alpha = 0.5$  and FOR = 10%.

$E_2\{\lambda_{t\omega}^P\}$	$k_{N_{T_1}\omega}$									
	1	0	1	1	1	1	1	1	0	1
44.4	0	0	0	0	0	0	0	0	0	0
39.7	0	0	0	0	0	0	0	0	0	0
31.6	1	1	1	1	1	1	1	1	1	1

in stage 2 at the prices contained in Table 1. Table 6 provides the optimal CVaR values for different FOR and risk-aversion levels. In case (a), the producer can re-trade in stage 2 the 200 MW sold in stage 1 at 32 €/MWh. In case (b), the producer buys an option with a strike and option price equal to 32 €/MWh and 0.3 €/MWh, respectively. The values in Table 6 are obtained solving different instances of problem (4).

Depending on the updated forward price, if high pool prices occur during the first month, the producer may buy 200 MW through the forward contract spanning the second month to close its position and be able to sell all its production in the pool. Note that although the optimal CVaR obtained by purchasing the option is higher (case (b)), the re-trading of forward contracts in stage 2 allows the producer to increase its CVaR if compared with that contained in Table 2 (case (a)).

Table 6: CVaR comparison considering either the purchase of a forward contract or an option (with re-trading).

$\alpha$	FOR = 0%		FOR = 5%		FOR = 10%	
	Case (a)	Case (b)	Case (a)	Case (b)	Case (a)	Case (b)
0	19.38	19.68	19.09	19.39	18.82	19.11
0.5	18.34	18.34	17.97	18.03	17.66	17.75
0.9	17.75	17.71	17.15	17.16	16.78	16.78

#### 4.3. Optimal $P_o$

In the previous analysis, the variable  $P_o$  is fixed to certain values in order to highlight the differences between selling a given quantity of electricity either through a forward contract or an option. In this section, however, we solve the optimization model (4) to identify the optimal amount of energy sold through the forward contracts spanning either period 1 ( $P_{c_1}^1$ ) or period 2 ( $P_{c_2}^1$ ), as well as the optimal quantity to be sold through the option contract spanning period 2 ( $P_o$ ). Table 7 contains the optimal values of both  $P_{c_2}^1$  and  $P_o$  for different values of the risk aversion parameter and FOR. The forward and strike price are equal to 32 €/MWh, and the option price is 0.3 €/MWh.

Table 7: Optimal values of  $P_{c_2}^1$  and  $P_o$  (MW).

$\alpha$	FOR = 0%		FOR = 5%		FOR = 10%	
	$P_{c_2}^1$	$P_o$	$P_{c_2}^1$	$P_o$	$P_{c_2}^1$	$P_o$
0	0	0	0	0	0	0
0.5	0	145	0	450	0	450
0.9	450	0	61	389	2	372

To analyze the results in Table 7 we have to take into account that the amount of electricity sold through the option by the electricity producer basically depends on the proportion between the number of scenarios under which the option is exercised and the number of scenarios under which it is not. That is, if the put option is exercised under practically any

scenario realization, the electricity producer sells its production through the forward contract to save the option price. Likewise, if the put option is not exercised under any circumstances, the producer sells the electricity directly in the pool. In contrast, as the exercising of the option increasingly depends on the scenario realization, the amount of power sold through the option increases. That said, we can observe that, independently of its forced outage rate, a risk-neutral producer sells all its production in the pool, in which the clearing price during period 2 is expected to be higher than both the forward and the strike prices. Secondly, if the risk aversion parameter increases up to 0.5, the producer is willing to sell some of its production through the available option to hedge against the price risk without increasing too much the probability of having low profits due to unit failures. Note that the quantity sold through the option increases as the production unit becomes more susceptible to failure to hedge against both the price and the availability risks. Lastly, a highly risk-averse producer increases the electricity it sells through forward contracts in order to avoid paying the option price.

#### 4.4. Maximum option price calculation

In this subsection, we calculate the maximum option price ( $\lambda_o^{O*}$ ) that the electricity producer is willing to pay for a given option as described in Section 3.2. Fig. 5 depicts the maximum option price as a function of the strike price of the option ( $\lambda_o^S$ ) for two values of the risk aversion parameter and three different forced outage rates of the production unit. A put option of 200 MW is considered. Forward contracts for periods 1 and 2 are considered to be traded only at stage 1, i.e., no forward re-trading is allowed at the beginning of stage 2. Moreover, the price of the forward contract spanning period 1 is set to 52.5 €/MWh, while the price of the forward contract covering period 2 is considered to be equal to the strike price of the option.

First, observe that for both very low and very high values of the strike price, the maximum option price is equal to zero. If the strike price is very low, the option is not exercised for any possible realization of the pool price during the first month and therefore, the electricity producer decides selling all its production in the pool. On the other hand, for high values



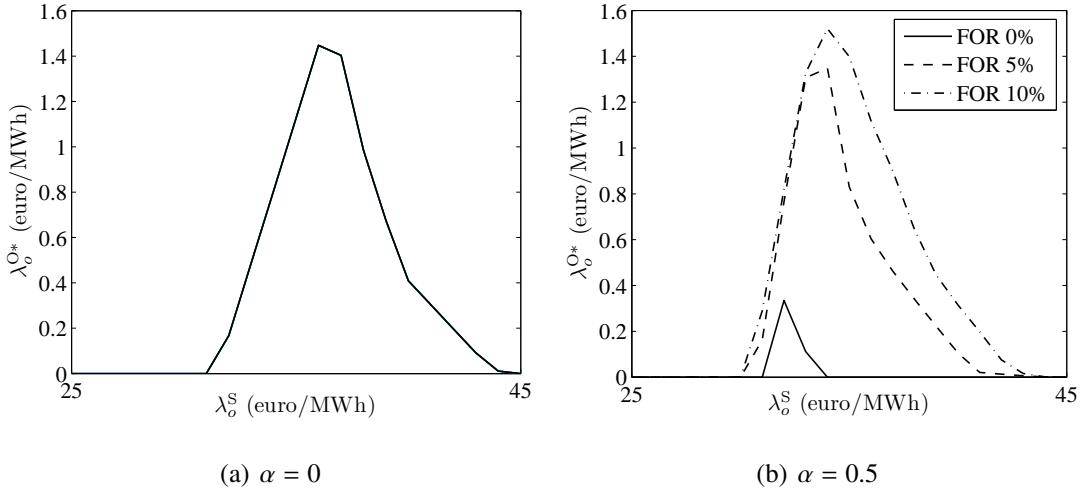


Figure 5: Maximum option price as a function of the strike price for different forced outage rates and risk-aversion levels.

of the strike price, the option is exercised in all cases and therefore, the electricity producer prefers buying a forward contract at the strike price to save the non-refundable fee of the option.

Considering the no failure case (FOR = 0%), and comparing the two risk-aversion levels (Figs. 5(a) and 5(b), respectively), we can conclude that the maximum option price decreases if the risk aversion increases. The reason is that for a risk-averse electricity producer, the option is exercised in all cases to hedge against pool price volatility (even for low values of the strike price) and therefore, the producer would rather sell its production through forward contracts to save the non-refundable option fee.

Observe that for  $\alpha = 0$  (Fig. 5(a)), the maximum option price is the same regardless of the value of FOR. In other words, considering the low values of realistic forced outages rates, if the expected profit is maximized, the pool price uncertainty has the highest influence on option decisions, i.e., the option is exercised or not based only on the probability of experiencing low or high pool prices. On the other hand, for a risk-averse situation ( $\alpha = 0.5$ ), the higher the forced outage rate, the higher the maximum option price. The reason for this is that the lowest profits are obtained when unit failures occur and therefore, a put option can be used to reduce the availability risk during the second month.

A case of particular interest arises if the strike price of the option is equal to the expected

pool price during period 2 (36.6 €/MWh), i.e., the case of a risk premium equal to 0. Table 8 contains the maximum option price if the strike price is equal to 36.6 €/MWh. Note that even in this case both risk-neutral and risk-averse producers are willing to pay an option price in order to postpone their respective decisions until it is known whether or not high pool prices realize or forced outages occur during period 1.

Table 8: Maximum option price for a strike price equal to the expected pool price during period 2 (36.6 €/MWh).

$\alpha$	FOR = 0%	FOR = 5%	FOR = 10%
0	1.42	1.42	1.42
0.5	0.98	0.51	0

As indicated in Section 3.2, the maximum option price is an index to measure the maximum increase of the objective function of problem (4) due to the purchase of an option. This increase is caused by the additional information of the uncertain parameters that become known between the purchase of the option and its exercising. In Fig. 5, the maximum option price is calculated considering that the realization of both the pool price and the unit availability during the first month have influence on the option exercising. In order to isolate the effect of the unexpected failures, we replace constraints (4k) and (4l) with the following ones:

$$P_{c_2\omega}^2 = P_{c_2\omega'}^2 \quad \forall c_2, \forall \omega, \omega' : \omega' \in Q_\omega \quad (8)$$

$$y_{o\omega} = y_{o\omega'} \quad \forall o, \forall \omega, \omega' : \omega' \in Q_\omega \quad (9)$$

where  $Q_\omega = \{\omega' : (k_{t\omega'} = k_{t\omega}, \forall t = 1, \dots, N_{T_1})\}$ . This way, the maximum option price depicted in Fig. 6 as a function of the strike price is calculated considering that second-stage decisions only depend on the unit availability at the end of the first month. A risk-aversion level equal to 0.5, two values of FOR (5 and 10%), and different values of the mean time between two consecutive failures (MTTF) are considered.

Comparing Figs. 5 and 6, we observe that the maximum option price is lower if the additional information associated with the pool price realization is not taken into account (Fig. 6).

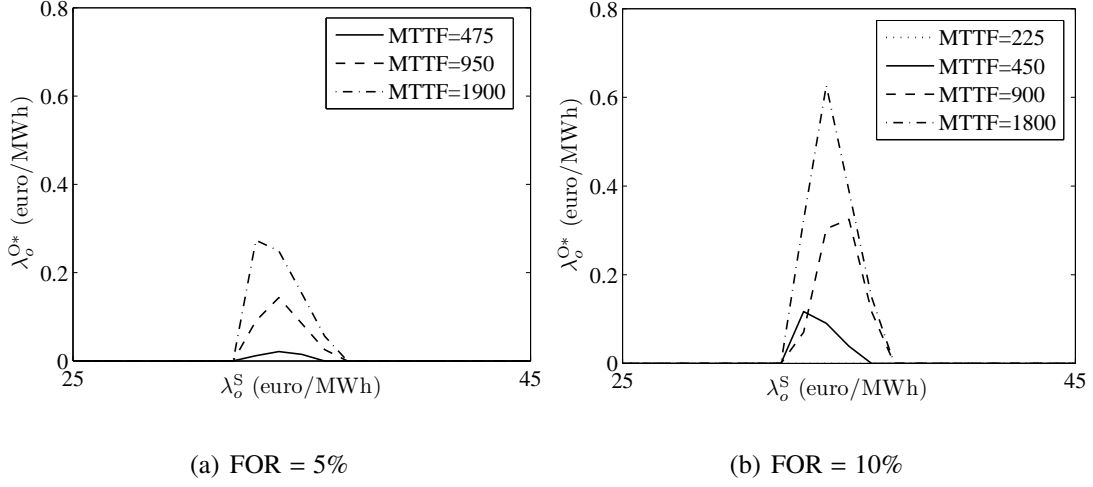


Figure 6: Maximum option price produced only by unit availability for different forced outage rates and MTTF's.

Secondly, note that a higher forced outage rate implies a higher maximum option price since in such cases the option becomes essential to avoid low profits. Moreover, for the same value of FOR, the higher the MTTF, the higher the maximum option price since the reduction of the availability uncertainty during the second month is more significant (see Fig. 1).

## 5. Computational burden

The simulations presented in this paper have been carried out using CPLEX 12.1.0 (CPLEX, 2011) under GAMS (GAMS, 2011) on a Sun Fire X4600M2 with 8 Quad-Core processors running at 2.9 GHz and 256 GB of RAM. If a set of 900 scenarios is used to represent the uncertainty related to the pool price and the unit availability, the computational time required to solve any of the instances considered of the three-stage optimization problem (4) is lower than 20 minutes.

## 6. Conclusions

This paper proposes a multi-stage stochastic programming model to determine the optimal involvement in the futures and pool markets by a risk-averse electricity producer if options to sell electricity are available. It is shown that, despite the non-refundable cost of

an option, the reduction of the uncertainty faced by the producer at the time when it has to decide whether or not to exercise the option diminishes the probability of having low profits. In this sense, we have verified that purchasing an option reduces the risks faced by a electricity producer that are related to both pool price and unit availability uncertainties. Moreover, quantitative tools based on multi-stage stochastic programming are developed and reported for optimal decision making pertaining to forward contracts and options.

### A. Conditional Value-at-Risk

This appendix provides a brief mathematical description of the Conditional Value-at-Risk (CVaR) of a probability distribution.

Let  $\chi$  be a random variable whose probability distribution is depicted in Fig. 7(a) and whose cumulative distribution function  $F_\chi(\cdot)$  is represented in Fig. 7(b). The Value-at-Risk of a probability distribution for a confidence level  $\alpha$  is equal to the  $\alpha$ -percentile of such distribution and is calculated as follows

$$\text{VaR}_\alpha(\chi) = q_\alpha(\chi) = \min\{\tau : F_\chi(\tau) \geq 1 - \alpha\}. \quad (10)$$

Since the VaR does not have some desirable properties as a risk measure, the CVaR is commonly used as a coherent risk measure in decision-making optimization models (Pflug, 2000). The CVaR is defined as the average of those values of the probability distribution lower than the value of the VaR (shadow area in Fig. 7(a)), i.e.,

$$\text{CVaR}_\alpha(\chi) = \text{Exp}\{\tau : \tau \leq \text{VaR}_\alpha(\chi)\}. \quad (11)$$

Within a stochastic programming framework in which uncertainty is characterized via a scenario set, the CVaR can be calculated by solving the following linear optimization problem

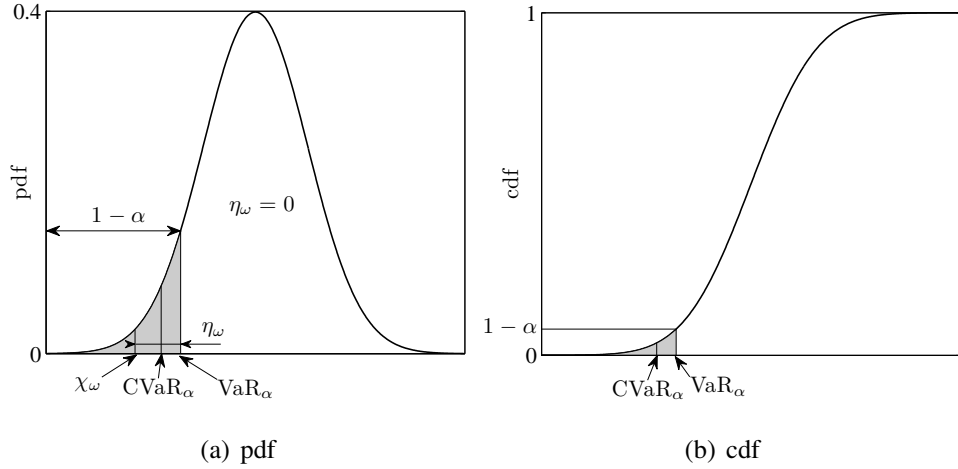


Figure 7: Conditional Value-at-Risk calculation.

(Rockafellar and Uryasev , 2000):

$$\text{CVaR}_\alpha = \underset{\zeta, \eta_\omega}{\text{Maximize}} \quad \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \quad (12a)$$

subject to

$$-\chi_\omega + \zeta - \eta_\omega \leq 0 \quad \forall \omega \quad (12b)$$

$$\eta_\omega \geq 0 \quad \forall \omega, \quad (12c)$$

where  $\chi_\omega$  corresponds to each discrete realization of the stochastic variable  $\chi$ , whose probability is equal to  $\pi_\omega$ .  $\zeta$  is an auxiliary variable whose optimal value is equal to the value of the VaR, and the optimal value of  $\eta_\omega$  is the difference between the VaR and the value of the profit for those scenarios with a profit lower than the VaR, and 0 otherwise. Therefore, the optimal value of the linear expression (12a) corresponds to the average of all the profits smaller than the VaR.

## B. Notation

The notation used in this paper is presented below for quick reference.

### *Indices and Numbers*

$c_1$  Index of forward contracts spanning period 1 (from 1 to  $N_{C_1}$ ).

$c_2$  Index of forward contracts spanning period 2 (from 1 to  $N_{C_1}$ ).

$i$  Index of production units (from 1 to  $N_I$ ).

$o$  Index of option contracts (from 1 to  $N_O$ ).

$t$  Index of time steps (period 1:  $t$  varies from 1 to  $N_{T_1}$  ; period 2:  $t$  varies from  $N_{T_1} + 1$  to  $N_T$ ).

$\omega$  Index of scenarios (from 1 to  $N_\Omega$ ).

#### *Continuous variables*

$P_{it\omega}^G$  Generated power by unit  $i$  in time step  $t$  and scenario  $\omega$  (MW).

$P_{t\omega}^P$  Power sold in the pool in time step  $t$  and scenario  $\omega$  (MW).

$P_{c_1}^1$  Power sold through forward contract  $c_1$  at stage 1 (MW).

$P_{c_2}^1$  Power sold through forward contract  $c_2$  at stage 1 (MW).

$P_{c_2\omega}^2$  Power re-traded in stage 2 through forward contract  $c_2$  in scenario  $\omega$  (MW).

$P_o$  Power sold through option contract  $o$  (MW).

$\zeta$  Auxiliary variable related to the CVaR (€).

$\eta_\omega$  Auxiliary variable in scenario  $\omega$  related to the CVaR (€).

$\Pi_\omega$  Producer profit in scenario  $\omega$  spanning the whole study horizon (€).

#### *Binary variables*

$u_{it\omega}$  Binary variable that is equal to 1 if production unit  $i$  is online during time step  $t$  and scenario  $\omega$ , and 0 otherwise.

$y_{o\omega}$  Binary variable that is equal to 1 if the producer exercises option  $o$  under scenario  $\omega$ , and 0 otherwise.

*Random variables*

$\lambda_t^p$  Pool price in time  $t$  (€/MWh).

$k_{it}$  Availability of production unit  $i$  in time  $t$  (1 if available and 0 otherwise).

$\lambda_{c_2}$  Price of forward contract  $c_2$  at stage 2 (€/MWh).

*Constants*

$C_i$  Linear production cost of production unit  $i$  (€/MWh).

$k_{it\omega}$  Availability of production unit  $i$  in time step  $t$  and scenario  $\omega$ .

$L_t$  Duration of time step  $t$  (h).

$P_i^{\max}$  Capacity of production unit  $i$  (MW).

$P_i^{\min}$  Minimum power output of production unit  $i$  (MW).

$T_{c_1}$  Duration of contract  $c_1$  (h).

$T_{c_2}$  Duration of contract  $c_2$  (h).

$T_o$  Duration of option  $o$  (h).

$\alpha$  Per unit confidence level (risk aversion parameter).

$\lambda_{c_1}^1$  Energy price of forward contract  $c_1$  at stage 1 (€/MWh).

$\lambda_{c_2}^1$  Energy price of forward contract  $c_2$  at stage 1 (€/MWh).

$\lambda_{c_2\omega}^2$  Energy price of forward contract  $c_2$  at stage 2 and scenario  $\omega$  (€/MWh).

$\lambda_o^S$  Strike price of option contract  $o$  (€/MWh).

$\lambda_o^O$  Option price of option contract  $o$  (€/MWh).

$\lambda_{t\omega}^p$  Pool price in time step  $t$  and scenario  $\omega$  (€/MWh).

$\pi_\omega$  Probability of occurrence of scenario  $\omega$ .

*Sets*

$F_t^1$  Set of forward contracts  $c_1$  available during time  $t$ .

$F_t^2$  Set of forward contracts  $c_2$  available during time  $t$ .

$O_t$  Set of option contracts available during time  $t$ .

$S_\omega$  Set of scenarios whose uncertain parameter values during period 1 are equal to those associated with scenario  $\omega$ , i.e.,

$$S_\omega = \{\omega' : (\lambda_{t\omega'}^P = \lambda_{t\omega}^P, k_{it\omega'} = k_{it\omega}, \forall i, \forall t = 1, \dots, N_{T_1}) \text{ and } (\lambda_{c_2\omega'}^2 = \lambda_{c_2\omega}^2, \forall c_2)\}.$$

## Acknowledgements

S. Pineda and A.J. Conejo are partly supported by the Junta de Comunidades de Castilla-La Mancha through project PCI-08-0102 and by the Ministry of Education and Science of Spain through CICYT Project DPI2009-09573.

## References

- Beenstock M. Generators and the cost of electricity outages. *Energy Econ.* 1991; 13(4); 283–289.
- Benth F E, Koekebakker S. Stochastic modeling of financial electricity contracts. *Energy Econ.* 2008; 30(3); 1116–1157.
- Billinton R, Allan R N. Reliability evaluation of power systems. New York and London: Plenum Press; 1984.
- Birge J R, Louveaux F. Introduction to stochastic programming. New York: Springer-Verlag; 1997.
- The ILOG CPLEX Website (2011). [Online]. Available in <http://www.ilog.com/products/cplex/>



- Conejo A J, Contreras J, Espínola R, Plazas M A. Forecasting electricity prices for a day-ahead pool-based electric energy market. *Int. J. Forecast* 2005; 21(3); 435–462.
- Conejo A J, García-Bertrand R, Carrión M, Caballero Á, de Andrés A. Optimal Involvement in Forward Markets of a Power Producer. *IEEE Trans. Power Syst.* 2008; 23(2); 703–711.
- Deng S J, Oren S. Electricity derivatives and risk management. *Energy* 2006; 31(6–7); 940–953.
- Dhillon B S. *Applied reliability and quality*. London: Springer-Verlag; 2007.
- European Energy Exchange, EEX (2009). [Online]. Available in <http://www.eex.com/>
- Fleten S-E, Wallace S W, Ziemba W T. Hedging electricity portfolios via stochastic programming. In *Decision making under uncertainty*; 1st ed.; New York: Springer-Verlag; 71–93; 2002.
- Fleten S-E, Kristoffersen T K. Short-term hydropower production planning by stochastic programming. *Comput. Oper. Res.* 2008; 35(8); 2656–2671.
- The GAMS Development Corporation Website (2011). [Online]. Available in <http://www.gams.com/>
- Gedra T W. Optional forward contracts for electric power markets. *IEEE Trans. Power Syst.* 1994; 9(4); 1766–1773.
- Haghighat H, Seifi H, Kian A R. On the self-scheduling of a electricity producer in uncertain trading environments. *Electr. Power Syst. Res.* 2008; 78(3); 311–317.
- Higle J L. Stochastic programming: Optimization when uncertainty matters. In Greenberg HJ (ed) *TutORials in operations research 2005*, INFORMS, Baltimore; 30–53.
- Hull J C. *Options, futures, and other derivatives*. 5th ed. New Jersey: Prentice Hall; 2003.
- Kaye R J, Outhred H R, Bannister C H. Forward contracts for the operation of an electricity industry under spot pricing. *IEEE Trans. Power Syst.* 1990; 5(1); 46–52.

- Lane D W, Richter C W, Sheblé G B. Modeling and evaluating electricity options markets with intelligent agents. In proceedings International Conference on Electric Utility Deregulation and Restructuring and Power Technologies, 2000, London, UK, Apr. 4-7.
- Lien D, Tse Y K. A survey on physical delivery versus cash settlement in futures contracts. *Int. Rev. Econ. Finance* 2006; 15(1); 15–29.
- Liu M, Wu F F. Risk management in a competitive electricity market. *Int. J. Electr. Power Energy Syst.* 2007; 29(9); 690–697.
- Morales J M, Pineda S, Conejo A J, Carrión M. Scenario Reduction for Futures Market Trading in Electricity Markets. *IEEE Trans. Power Syst.* 2009; 24(2); 878–888.
- North American Electric Reliability Corporation, NERC (2010). [Online]. Available in <http://www.nerc.com/>
- Oum Y, Oren S, Deng S J. Hedging quantity risks with standard power options in a competitive wholesale electricity market. *Nav. Res. Logist.* 2006; 53(7); 697–712.
- Paravan D, Sheblé G B, Golob R. Price and volume risk management for electricity producers. In Proceedings International Conference Probabilistic Methods Applied Power Systems, PMAPS 2004, Ames, USA, Sep. 12–16; 699-704.
- Pflug G Ch. Some remarks on the value-at-risk and the conditional value-at-risk. In Probabilistic constrained optimization: methodology and applications, Ed. S. Uryasev, Kluwer Academic Publishers; 272–281; 2000.
- Pineda S, Conejo A J, Carrión M. Impact of unit failure on forward contracting. *IEEE Trans. Power Syst.* 2008; 23(4); 1768–1775.
- Pineda S, Conejo A J. Scenario reduction for risk-averse electricity trading. *IET Gener. Transm. Distrib.* 2010; 4(6); 694–705.

- Quan Z, Hao Z. Analysis of forward option trades in electricity markets. In proceedings of the 2004 IEEE International Conference on Electric Utility Deregulation, Restructuring and Power Technologies, 2004. (DRPT 2004), Hong Kong, China, Apr. 5–8.
- Richter C W, Sheblé G B. Bidding strategies that minimize risk with options and futures contracts. In Proceedings Amer. Power Conf. Rel. Econ. Technol. Focus for Competition and Globalization 1998, Chicago, USA.
- Rockafellar R T, Uryasev S. Optimization of conditional value-at-risk. *Journal of Risk* 2000; 2(3); 21–41.
- Reliability Test System Task Force. The IEEE reliability test system—1996. *IEEE Transactions on Power Systems* 1999; 14(3); 1010–1020.
- Spinler S, Huchzermeier A, Kleindorfer P. Risk hedging via options contracts for physical delivery. *OR Spectrum* 2003; 25(3); 379–395.
- Tan B J, Lu Z, Xu Z, Song J, Dong Z Y, Tang W, Cai H W, Feng Z X. Risk hedging in electricity generation planning. In proceedings 7th International Power Engineering Conference, 2005. IPEC 2005, Singapore, Singapore, Nov. 29–Dec. 2.
- Tanlapco E, Lawarree J, Liu C C. Hedging with futures contracts in a deregulated electricity industry. *IEEE Trans. Power Syst.* 2002; 17(3); 577–582.
- Williams H P. Model building in mathematical programming. John Wiley and Sons Inc.: New York; 1999.
- Wu H-C. Pricing European options based on the fuzzy pattern of Black-Scholes formula. *Oper. Res.* 2004; 31(7); 1069–1081.
- Xu J, Luh P B, White F B, Ernan N, Kasiviswanathan K. Power portfolio optimization in deregulated electricity markets with risk management. *IEEE Trans. Power Syst.* 2006; 21(4); 1653–1662.