

Insuring Unit Failures in Electricity Markets

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Abstract

An electric energy producer participates in futures markets in the hope of hedging the risk of trading in the pool. However, this producer is required to supply the energy associated with all its signed forward contracts even if some of its units are forced out due to unexpected failures. In this case, the producer must purchase in the pool some of the energy needed to meet its futures market commitments, which may result in high losses if the pool prices happen to be higher than the forward contract prices. To mitigate these losses, the producer can take out insurance against the forced outages of its units. Using a stochastic programming model, this paper analyzes the convenience of signing an insurance against unit failure by an electric energy producer and its impact on forward contracting decisions. Results from a realistic case study are provided and analyzed.

Key words: Insurance contract, electricity markets, stochastic programming, unit forced outage rate.

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1. Introduction

A power producer can sell all its production in the pool facing the financial risk inherent to pool price volatility. On the other hand, it can sell its energy through forward contracts in order to hedge against price variability. However, if some of the production units fail, it should buy in the pool the energy to meet its contracting selling obligations, Pineda et al. (2008). If the pool price is high when some production units are forced out, the financial losses can be significant. Hence, there exists a risk associated with the possibility of unit failure. In order to hedge against this risk, the producer can take out an insurance contract.

If an insurance contract is signed, the insurer (seller of the insurance) receives a certain premium (an amount agreed beforehand) at the beginning of the contract period, but later it should pay the insured (buyer of the insurance) an amount equal to the difference between the pool price and the strike price (a price agreed beforehand) times the insured power times the number of hours in the periods in which the pool price is higher than the strike price and one or several units are forced out.

Therefore, the producer should both determine the optimal mix of pool and futures market participation and decide whether or not to sign the insurance contract, taking into account the pool price volatility, the forced outage rate of its units and the characteristics of the forward and insurance contracts. The forced outage rate (FOR) is defined as the percentage of hours that a production unit is nonfunctional due to unplanned failures.

In order to address this problem a two-stage stochastic programming model is proposed to take into account the uncertainty associated with pool prices and unit availability. The objective function of the model is to maximize the Conditional Value at Risk (CVaR) of the profit distribution with a confidence level α^P , which represents the risk aversion of the producer.

In references Kihlstrom (1982) and Wang and Yang (2006), insurance contracts are

analyzed as financial instruments. In these papers, game theory and equilibrium models are used to calculate the parameters defining the insurance contract between insurer and insured. Within an electricity market framework, in Braun and Lai (2006) the authors summarize the risks that energy companies face and analyze which ones of these risks can be covered by insurances. References Torrey and Russell (2001) and Fumagalli et al. (2004) propose the use of a certain type of insurance for enhancing the reliability of supply to consumers, whose aim is to allocate the risk of forced outages to the distribution provider, rather than to the consumer. In Jiang et al. (2006) an insurance against financial losses caused by generation forced outages is presented.

Within the above context, the aim of this paper is to analyze the effect of the possibility of signing an insurance contract on the decisions of an electric energy producer. In this regard, this paper presents a novel stochastic programming model to calculate the maximum premium that the producer is willing to pay for a given insurance contract and for different values of risk aversion and forced outage rates of the production units. This critical premium is compared with the minimum premium that the insurer is willing to receive for the same insurance contract.

The remaining of this paper is organized as follows. Section 2 describes the model considered, including a characterization of the energy production units, the pool, the forward contracts, the insurance contract and the treatment of scenarios. Section 3 provides the formulation of the problem for a producer, a discussion about the critical premium and the formulation of the problem of an insurer. Section 4 gives results from a comprehensive case study. Section 5 provides some relevant conclusions. Finally, the notation used throughout the paper is provided in the appendix.

2. Model

2.1. *Generating unit*

For the sake of simplicity, an electric energy producer owning a single production unit is considered. This generating unit is characterized by a production capacity and a quadratic cost function which is approximated by a set of piecewise linear blocks, Arroyo and Conejo (2000). The availability scenarios of the unit are generated simulating the time between two consecutive failures and the repair time using exponential distributions with mean values of MTTF (mean time to failure) and MTTR (mean time to repair), respectively, Pineda et al. (2008) and Billinton and Karki (1999). Each availability scenario is a vector which contains a 0 or a 1 in the position corresponding to each time period representing whether or not the unit is forced out.

2.2. *Pool*

An ARIMA model is used to generate pool-price scenarios as in Conejo et al. (2005). Hourly pool prices throughout the considered study horizon are grouped in periods of different durations, which results in a reduced number of periods to characterize the pool prices in each scenario, Pineda et al. (2008). This constitutes an adequate tradeoff between computational tractability and model accuracy.

2.3. *Scenario building*

In order to represent all possible realizations of the random variables, a sufficiently large number of availability and pool price scenarios is generated. Then, a scenario-reduction technique is used to achieve tractability while maintaining most of the characteristics of the original scenario set, Pineda et al. (2008).

Firstly, pool price scenarios are reduced considering that the generation unit does not fail and secondly, availability scenarios are reduced. Then, all pool price scenarios per-

taining to the first reduced set are combining with all availability scenarios of the second reduced set. Finally, this set of combined scenarios is reduced again.

Each scenario is made up of three vectors. The first one contains a 0 or a 1 for each time period and represents the availability of the unit. It should be noted that the availability of each period is computed as the average value of the hourly availability values pertaining to that period. If this average is different from 0 or 1, it is rounded to one of these two values. The second vector are the pool prices throughout the study horizon and the third vector contains the duration in hours of each time period.

2.4. Futures market

A producer can participate in the futures market selling energy through forward contracts. From a producer perspective, a forward contract consists in selling a certain power quantity (e.g., 100 MW) during a given time period (e.g., next month) at a constant price (e.g., 20 €/MWh). For the sake of simplicity, it is assumed that the price of each forward contract does not depend on the amount of power sold and that there is a cap on the power sold through each forward contract. Note that a forward contract is a hedging instrument against pool price volatility since it has a stable energy price, Huisman et al. (2008). However, selling all the production through forward contracts entails a risk associated with the possibility of unit failures, Pineda et al. (2008).

2.5. Insurance contracts

An insurance contract is a financial instrument whereby the insured receives a certain quantity from the insurer if financial losses associated with production forced outages occur. In exchange, the insured pays a certain premium to the insurer at the beginning of the time period covered by the insurance. Usually two conditions must be met so that the insurer has the obligation to pay to the insured: the production unit is forced out and the pool price is higher than a strike price, which is agreed in advance.

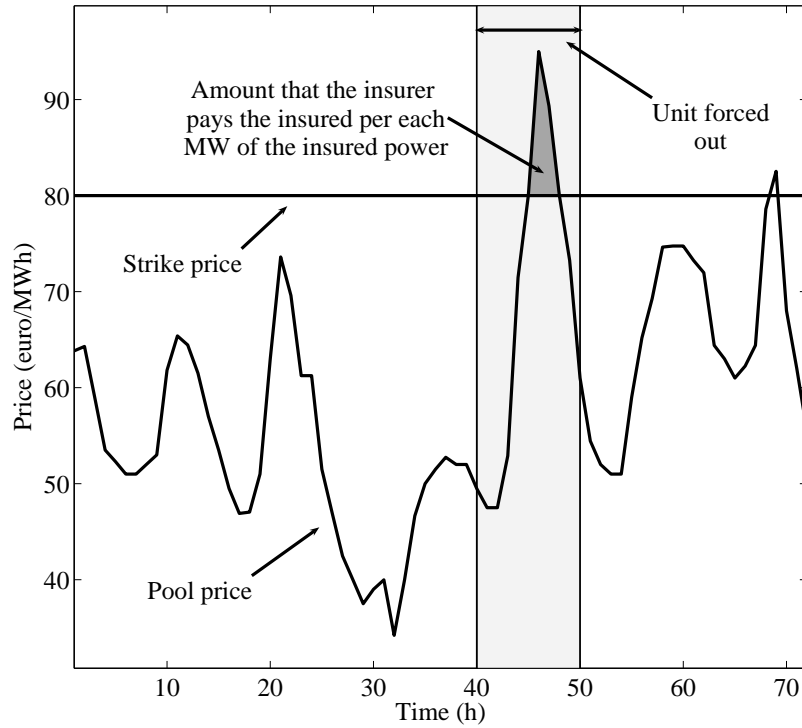


Figure 1: Insurance Contract.

For further clarity, Figure 1 presents an illustrative example of an insurance contract. In this figure pool prices during three days are depicted. During this time horizon the unit is forced out from hour 40 to hour 50 (light gray area). If the producer has signed a forward contract for these periods and its production unit fails, it has to buy the energy in the pool to meet its selling obligations. However, if the producer has signed an insurance contract, the insurer has to pay to the producer the difference between the pool price and the strike price (80 €/MWh in this case) in all the periods in which the two conditions below are met: the pool price is higher than the strike price and the production unit is forced out. In Figure 1 the darker gray area represents the amount per each MW of the insured power

that the insurer should pay to the producer. Mathematically, the amount that the insurer pays to the producer in each scenario is

$$D_{\omega} = P_1 \sum_{t \in G_{\omega}} L_{t\omega}(\lambda_{t\omega}^P - \lambda_1). \quad (1)$$

The insurance contract used in this study is defined by a premium, a strike price and a insured power level. There exist more complex insurance arrangements that include other conditions like a maximum payment by the insurer or a deductible capacity, that is, the insurer does not pay if the total outage capacity is smaller than this deductible capacity, Jiang et al. (2006).

The decisions made by the producer and the insurer depend on non-negotiable and negotiable variables, Jiang et al. (2006). In this study, non-negotiable variables are pool prices and the availability of the production unit because they are out of the control of both the insurer and the insured. On the other hand, the strike price, the premium and the insured power level are negotiable variables because both parties have to agree in their values for the insurance contract.

2.6. Decision Framework

The producer has to make two decisions: forward and insurance contracting, which are made at the beginning of the time horizon considered, and pool trading, which is decided throughout the study horizon, that is, knowing the realizations of the stochastic processes. Day-ahead decisions are considered to be made with perfect information with respect to pool prices and unit availability since the level of uncertainty involving day-ahead pool decisions is much smaller than that of futures market and insurance decisions.

The proposed tool is run at the beginning of the time horizon in order to decide the forward and insurance contracting. However, if new forward or insurance contracts are available in the future, the proposed tool can be run again for the new time horizon, and

so on. Needless to say, optimal decisions for new contracts should be made taking into account all previous contracting decisions.

3. Problem formulation

Firstly, the problem formulation of the power producer is presented and explained. Then, the calculation of the maximum premium that this producer is willing to pay for a given insurance contract is discussed. Finally, the problem of the insurer and the calculation of its minimum premium is presented.

3.1. Producer problem

The stochastic programming problem for the producer owning only one generating unit is:

Maximize $P_{t\omega b}^G, P_{t\omega}^P, P_c, u_{t\omega}, s_I, \zeta, \eta_\omega$

$$\zeta - \frac{1}{1 - \alpha^P} \sum_{\omega=1}^{N_W} \pi_\omega \eta_\omega \quad (2)$$

subject to

$$P_{t\omega}^G = P_{t\omega}^{\min} u_{t\omega} + \sum_{b=1}^{N_B} P_{t\omega b}^G, \quad \forall t, \forall \omega \quad (3)$$

$$P_{t\omega 1}^G \leq P_1^{\max} - P_{t\omega}^{\min}, \quad \forall t, \forall \omega \quad (4)$$

$$P_{t\omega b}^G \leq P_b^{\max} - P_{b-1}^{\max}, \quad \forall t, \forall \omega, \forall b = 2, \dots, N_B \quad (5)$$

$$P_{t\omega}^G L_{t\omega} = \sum_{c \in F_t} P_c L_{t\omega} + P_{t\omega}^P L_{t\omega}, \quad \forall t, \forall \omega \quad (6)$$

$$P_{t\omega}^G \leq u_{t\omega} k_{t\omega} P^{\max}, \quad \forall t, \forall \omega \quad (7)$$

$$P_{t\omega}^G \geq u_{t\omega} k_{t\omega} P^{\min}, \quad \forall t, \forall \omega \quad (8)$$

$$P_{t\omega}^P \geq (k_{t\omega} - 1) \sum_{c \in F_t} P_c, \quad \forall t, \forall \omega \quad (9)$$

$$P_c \leq P_c^{\max}, \quad \forall c \quad (10)$$

$$\begin{aligned} & - \sum_{t=1}^{N_T} (\lambda_{t\omega}^P P_{t\omega}^P L_{t\omega} - A u_{t\omega} L_{t\omega} - \sum_{b=1}^{N_B} \lambda_b P_{t\omega b}^G L_{t\omega}) - \\ & - \sum_{c=1}^{N_C} \lambda_c P_c T_c - s_I (D_\omega - M_I) + \zeta - \eta_\omega \leq 0, \quad \forall \omega \end{aligned} \quad (11)$$

$$\eta_\omega \geq 0, \quad \forall \omega \quad (12)$$

$$P_{t\omega b}^G \geq 0, \quad \forall t, \forall \omega, \forall b \quad (13)$$

$$P_c \geq 0, \quad \forall c \quad (14)$$

$$u_{t\omega} \in \{0, 1\}, \quad \forall t, \forall \omega \quad (15)$$

$$s_I \in \{0, 1\} \quad (16)$$

The objective function (2) consists in maximizing the CVaR, Rockafellar and Uryasev (2000). Different risk aversion situations can be simulated by changing the value of the confidence level α^P . If $\alpha^P = 0$ the value of the CVaR is equal to the expected profit over all scenarios, leading to the risk neutral case. On the other hand, the most risk averse situation is achieved for a value of α^P approaching 1, because this case corresponds to maximizing the profit for the worst possible scenario. Therefore, the higher the value of the confidence level α^P , the higher the risk aversion.

Constraints (3) define the total power produced by the unit at each time period and for each scenario. To better understand previous constraint, Figure 2 shows a piecewise linear production cost of a generating unit. Constraints (4) and (5) enforce the production limit of each block of the piecewise linear production cost that is considered. For each time period

and scenario, each constraint (6) expresses that the energy produced by the unit is equal to the energy sold in the pool plus the energy sold through forward contracts. Constraints (7) and (8) enforce the generating limits of the unit at each time period and for each scenario. Through constraints (9) the arbitrage between the pool and the futures market is explicitly eliminated for the sake of clarity. Constraints (10) impose a cap over the power sold through each forward contract. Constraints (11) allow limiting the financial risk through the CVaR. Constraints (12)-(14) are positive variable declarations and constraints (15)-(16) are binary variable declarations.

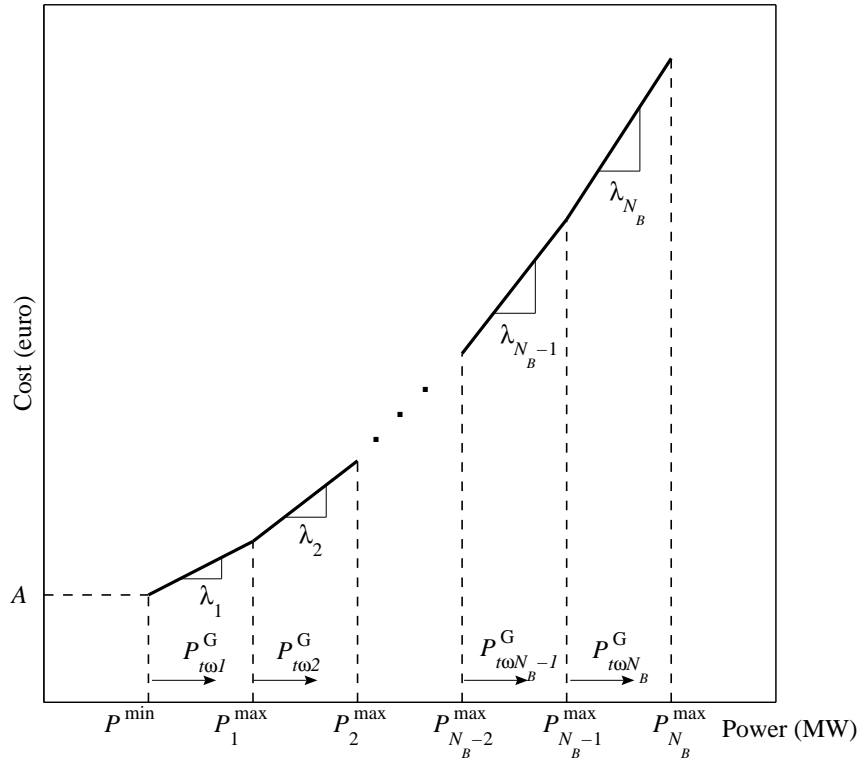


Figure 2: Piecewise linear production cost of a generating unit.

If variable s_I is equal to 1, the producer is willing to pay M_I for the insurance contract. In this case, the premium is deducted from the profit of each scenario, while the devolution defined by (1) is added to the profit of each scenario.

Note that problem (2)–(16) is a mixed-integer linear programming model that can be solved using commercial branch and cut software.

3.2. Critical premium

The strike price, the insured power level and the premium are negotiable parameters of an insurance contract. In this respect, the producer is willing to pay up to a maximum amount (*critical premium*) in exchange for an insurance contract with a given strike price and insured power level. The producer buys the insurance if its premium is lower than the critical premium, and does not otherwise. If the producer is risk neutral, the value of the critical premium is equal to the expected repayment due to the insurance contract ($\sum_{\omega=1}^{N_W} \pi_{\omega} D_{\omega}$). However, if the producer is risk averse, this value cannot be calculated directly. A novel technique to determine the critical premium via stochastic programming is presented in this paper.

In order to illustrate how the objective function (CVaR) changes with the value of the premium of the insurance contract, two situations are considered depending on whether or not the producer signs the insurance contract. Consider first that the producer signs the insurance contract for all values of the premium, i.e., s_I is fixed to 1. For this case, the profit distribution of all scenarios can be decomposed into one stochastic term, independent of the premium, and a constant (the premium of the insurance contract):

$$\text{Profit}_{\omega} = \text{Profit}'_{\omega} - M_I. \quad (17)$$

Note that the CVaR is a coherent measure of risk, Pflug (2000), and therefore is positively

homogeneous, i.e.,

$$\text{CVaR}_\alpha(Y + c) = \text{CVaR}_\alpha(Y) + c, \quad (18)$$

where Y is a random variable and c a constant.

Therefore, taking into account (17) and (18), a variation of the premium equal to ΔM_I involves a variation of $-\Delta M_I$ in the value of the CVaR of the profit distribution, i.e.,

$$\text{CVaR}_\alpha(\text{Profit}) = \text{CVaR}_\alpha(\text{Profit}' - M_I) = \text{CVaR}_\alpha(\text{Profit}') - M_I.$$

Consequently, if the insurance contract is signed, the variation of the CVaR with the premium is linear with a slope equal to -1 (CVaR_1^P in Figure 3).

On the other hand, consider that the producer does not sign the insurance contract for any premium. In this case the value of the CVaR is independent of the value of the premium (CVaR_2^P in Figure 3). Since the objective of the producer is to maximize the value of the CVaR, the producer signs the insurance contract if the premium is lower than the value of the premium at the intersection of both lines because the value of the CVaR of case 1 is higher than the value of the CVaR of case 2. For the same reason, if the premium is higher than the value of the premium at the intersection, the producer prefers the value of the CVaR of case 2, that is, not to sign the insurance contract. Therefore, the value of the premium corresponding with the intersection of both lines is the critical premium, i.e., if the premium is lower than this value the producer signs the insurance contract and it does not otherwise.

Mathematically, the critical premium of the producer (M_I^{P*}) is calculated solving two problems. For problem 1 it is assumed that the producer signs the insurance contract ($s_I = 1$). If the premium is equal to zero, the corresponding objective function of problem 1 is denoted by CVaR_0^P . As the premium increases while enforcing $s_I = 1$, the objective

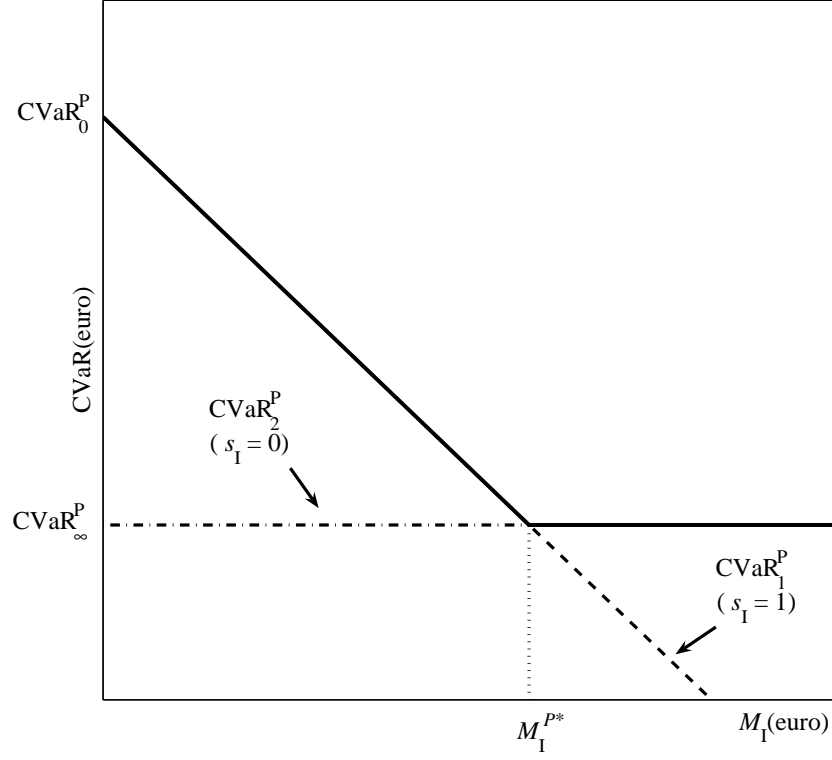


Figure 3: CVaR of the producer versus the premium of the insurance contract.

function of problem 1 decreases as follows:

$$\text{CVaR}_1^P = \text{CVaR}_0^P - M_I. \quad (19)$$

For problem 2 it is considered that the producer does not sign the insurance contract ($s_I = 0$). In this case the objective function is denoted by CVaR_∞^P and is independent of the premium, that is:

$$\text{CVaR}_2^P = \text{CVaR}_\infty^P. \quad (20)$$

The critical premium is the value of the premium that makes equal the values of CVaR_1^P and CVaR_2^P , that is:

$$\begin{aligned}\text{CVaR}_1^P &= \text{CVaR}_2^P \\ \text{CVaR}_0^P - M_I^{P*} &= \text{CVaR}_\infty^P \\ M_I^{P*} &= \text{CVaR}_0^P - \text{CVaR}_\infty^P.\end{aligned}\tag{21}$$

Note that CVaR_0^P is always higher than or equal to CVaR_∞^P .

3.3. Insurer

In addition to the decision making model for the producer, a model is proposed below to maximize the utility of the insurer for a given insurance contract, pool prices, and availability scenarios of the insured unit. The utility of the insurer is modeled as the value of the CVaR for a given risk aversion level represented by the parameter α^S . To calculate the CVaR of the profit distribution of the insurer considering that the insurance is signed, the following maximization problem is solved:

Maximize $_{\xi, \rho_\omega}$

$$\xi - \frac{1}{1 - \alpha^S} \sum_{\omega=1}^{N_W} \pi_\omega \rho_\omega\tag{22}$$

subject to

$$-M_I + D_\omega + \xi - \rho_\omega \leq 0, \quad \forall \omega\tag{23}$$

$$\rho_\omega \geq 0, \quad \forall \omega\tag{24}$$

Note that in problem (22)–(24) the insurer does not make any decision. The aim of this optimization problem is to calculate the CVaR of the profit distribution of the insurer.

The profit of the insurer in each scenario ω is equal to the value of the premium minus the devolution that the insurer has to pay to the producer during the periods in which the unit is unavailable and the pool price is higher than the strike price.

Similarly as the critical premium for the producer, the insurer has a minimum premium for a given insurance contract. Figure 4 shows CVaR_1^S and CVaR_2^S representing the variation of the CVaR of the insurer with the premium if the insurance contract is signed or not, respectively. If the insurance contract is signed the change of the CVaR with the premium is linear, with slope equal to 1 (CVaR_1^S in Figure 4), because the CVaR is a positively homogeneous metric. If the insurer signs an insurance contract with a premium equal to 0, the profit of the insurer in each scenario is negative because the insurer has to pay to the producer when its unit fails and the pool price is higher than the strike price, but the insurer does not receive any income in exchange. For this reason, the CVaR of the distribution of the profit has a negative value. If the insurance is not signed, the value of the CVaR is always equal to 0 because there is no profit associated with any scenario (CVaR_2^S in Figure 4).

Likewise to the case of the producer, the objective of the insurer is to maximize the CVaR of its profit distribution. Therefore, if the premium is lower than the value of the premium at the intersection of both lines, the insurer does not sign the insurance contract because the CVaR of case 2 is higher than the CVaR of case 1. On the other hand, if the premium is higher than this value, the insurer signs the insurance. Thus, the intersection between these two lines defines the value of the critical premium of the insurer.

In order to calculate mathematically the critical premium, a single problem is solved. In this problem the premium of the insurance contract is equal to 0 and the value of the objective function is called CVaR_0^S . If the insurance is signed, the variation of the CVaR

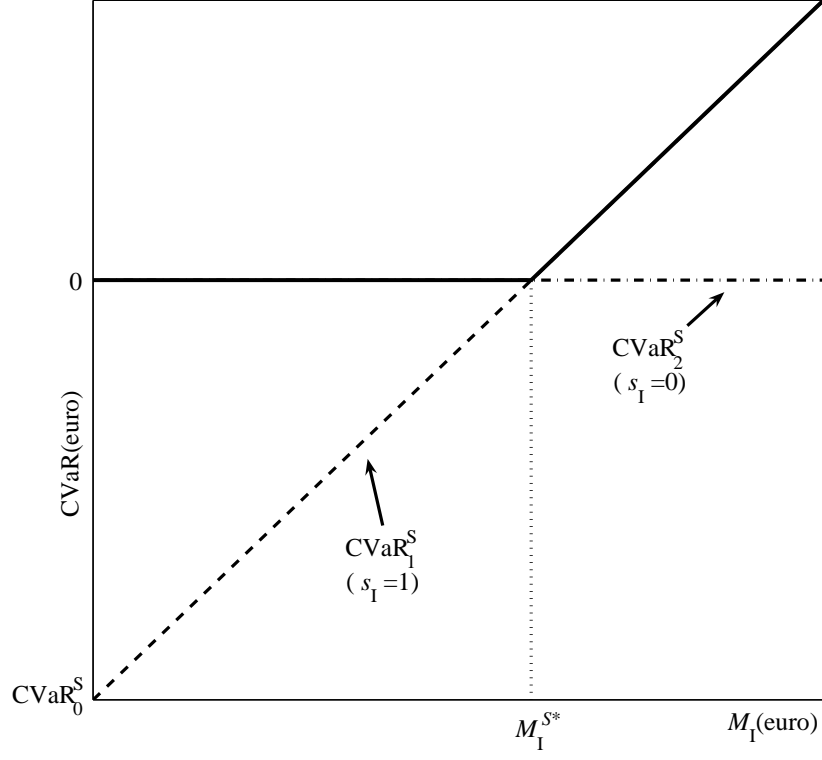


Figure 4: CVaR of the insurer versus the premium of the insurance contract.

with the premium is calculated as:

$$\text{CVaR}_1^S = \text{CVaR}_0^S + M_I. \quad (25)$$

On the other hand, if the insurance is not signed the value of the CVaR of the insurer is equal to 0: $\text{CVaR}_2^S = 0$.

The critical premium is equal to the value of the premium that make equal the values

of CVaR_1^S and CVaR_2^S , that is:

$$\begin{aligned}\text{CVaR}_1^S &= \text{CVaR}_2^S \\ \text{CVaR}_0^S + M_1^{S*} &= 0 \\ M_1^{S*} &= -\text{CVaR}_0^S.\end{aligned}\tag{26}$$

Note that the value of CVaR_0^S is always lower than or equal to 0.

4. Case study

A detailed case study is presented in this section.

4.1. Data

The study reported below is carried out considering a 500 MW unit with a minimum power output of 50 MW. The quadratic production cost of the unit is approximated by the set of four piecewise blocks characterized in Table 1. Three FOR values for this unit are considered: 0, 4 and 8%.

Table 1: Piecewise linear production cost

b	1	2	3	4
P_b^{\max}	125	250	375	500
λ_b	12.0	12.5	13.0	13.5

The study horizon spans three months, in which 4 forward contracts are available to the producer. The forward contract data are provided in Table 2. The contracts 1, 2 and 3 expand the first, second and third month respectively. The contract 4 expands the whole quarter.

Table 2: Forward contract data		
Contract #	P_c^{\max} (MW)	λ_c (€/MWh)
1	100	21.76
2	100	21.61
3	100	21.25
4	50	21.15

The considered insurance contract is defined with a strike price of 10 €/MWh, an insured power level of 75 MW and a premium of €100,000.

To describe pool price volatility, an ARIMA model is used to generate 300 scenarios. Each scenario consists of 273 values which describes the pool price during the three months. These 300 scenarios are reduced to 50 using scenario reduction techniques, Heitsch and Römisch (2003). These 50 reduced scenarios are depicted in Figure 5, where the bold line represents the average pool price.

Two groups of 10,000 availability scenarios, for FOR equal to 4 and 8% respectively, are generated. Using scenario reduction techniques, each set of availability scenarios is reduced to 200 scenarios. The total number of scenarios after combining all pool price scenarios with all availability scenarios is 10,000 for each value of FOR. Finally, this set of combined scenarios is further reduced to 200 final scenarios in order to solve the optimization problem in a reasonable time.

4.2. Insurance contract decision

If the possibility of signing an insurance contract does not exist, Table 3 provides the power sold through each forward contract, the value of the objective function (CVaR), and the expected profit (EP) for different values of the forced outage rate (FOR) and risk aversion (α^P). Observe that the power sold through forward contracts increases with the

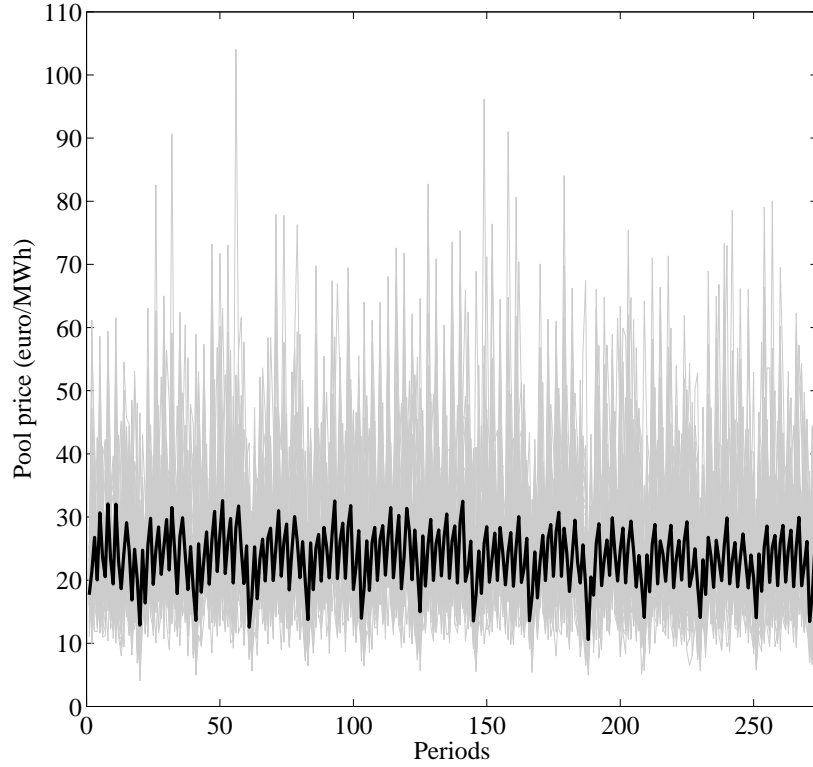


Figure 5: Pool price scenarios.

risk aversion for each value of FOR to hedge against pool price variability. Moreover, the power signed through forward contracts decreases with FOR for the same value of α^P to reduce the financial losses associated with unit failures.

In Table 4 similar results as in Table 3 are provided but considering that the producer has the possibility of signing an insurance contract ($\lambda_I = 10$ €/MWh, $P_I = 75$ MW and $M_I = \text{€}100,000$). Notice that an extra column has been added containing the value of the binary variable s_I . This variable is equal to 1 if the producer signs the insurance contract and 0 otherwise.

Table 3: Results without an available insurance contract							
	α^P	P_1	P_2	P_3	P_4	CVaR	EP
		(MW)	(MW)	(MW)	(MW)	(€ million)	(€ million)
FOR 0%	0	0	0	0	0	11.7	11.7
	0.5	0	0	0	0	10.1	11.7
	0.8	100	100	100	50	9.2	11.0
	0.9	100	100	100	50	8.8	11.0
	0.95	100	100	100	50	8.5	11.0
FOR 4%	0	0	0	0	0	11.2	11.2
	0.5	0	0	0	0	9.7	11.2
	0.8	100	100	100	50	8.7	10.6
	0.9	100	100	100	50	8.3	10.6
	0.95	100	100	100	50	7.9	10.6
FOR 8%	0	0	0	0	0	10.7	10.7
	0.5	0	0	0	0	9.1	10.7
	0.8	0	0	100	0	8.1	10.6
	0.9	100	97	100	0	7.5	10.3
	0.95	100	100	100	25	7.0	10.2

Note that for all the cases in which the value of s_1 is equal to 0, the results are identical in Tables 3 and 4. If the probability of unit failure is null, the producer does not sign the insurance contract for any level of risk aversion. On the other hand, if the generation unit can fail (e.g., FOR = 4%) and the risk aversion level is high enough ($\alpha^P \geq 0.9$), the producer signs the insurance contract. If the FOR of the unit is high enough, e.g. 8%, the producer signs the insurance contract for all values of risk aversion. For the cases in which

Table 4: Results with an available insurance contract

	α^P	P_1	P_2	P_3	P_4	CVaR	EP	s_I
		(MW)	(MW)	(MW)	(MW)	(€ million)	(€ million)	
FOR 0%	0	0	0	0	0	11.7	11.7	0
	0.5	0	0	0	0	10.1	11.7	0
	0.8	100	100	100	50	9.2	11.0	0
	0.9	100	100	100	50	8.8	11.0	0
	0.95	100	100	100	50	8.5	11.0	0
FOR 4%	0	0	0	0	0	11.2	11.2	0
	0.5	0	0	0	0	9.7	11.2	0
	0.8	100	100	100	50	8.7	10.6	0
	0.9	100	100	100	50	8.3	10.5	1
	0.95	100	100	100	50	8.0	10.5	1
FOR 8%	0	0	0	0	0	10.8	10.8	1
	0.5	0	0	0	0	9.3	10.8	1
	0.8	0	81	100	0	8.3	10.5	1
	0.9	100	100	100	50	7.8	10.2	1
	0.95	100	100	100	50	7.4	10.2	1

the insurance contract is signed, the value of the objective function (CVaR) is higher than that without insurance contract.

Observe that the total power sold through forward contracts is higher if the insurance contract is signed. This fact is analyzed below. If no insurance contract is available, financial losses occur if the unit fails and the pool price is high. These losses are due to the fact that the producer has to buy energy in the pool to meet its selling obligations. Thus,

in this case, the higher the power sold through forward contracts, the higher the possible financial losses associated with unplanned unit failures. For this reason, the power sold through forward contract decreases as the forced outage rate increases. On the other hand, if an insurance contract is signed, the financial losses associated with an unplanned outage decrease because the insurer pays the producer if the unit is forced out and the pool price is higher than the strike price. Therefore, the power sold through forward contracts is higher if an insurance contract is signed.

Note that in some cases the insurance contract is signed even if no energy is sold through forward contracts.

4.3. Calculation of the critical premium for the producer

An example of the calculation of the critical premium as explained in Section 3.2 is presented below. Data and results for cases 1 and 2 are provided in Table 5. In this example, first stage decisions are made solving the stochastic model with a reduced scenario tree. However, once these decisions are known, the second stage decision are calculated maximizing the profit of each scenario of the original tree. Therefore, the values of the CVaR and the expected profit are calculated considering the initial 10,000 scenarios.

The critical premium of the producer according to (21) is equal to:

$$M_I^{P*} = 8.3582 \times 10^6 - 8.0751 \times 10^6 = \text{€}283,100 \quad (27)$$

Therefore, if the premium of the insurance contract is higher than this value the producer does not sign it.

In a similar way, the critical premium of the producer can be calculated for different values of risk aversion, forced outage rate, strike price and insured power level. Figure 6 depicts the variation of the critical premium that the producer is willing to pay for the insurance contract for different values of strike price, risk aversion level and FOR. The

Table 5: Calculation of the critical premium for the producer		
	Case 1 ($M_I = 0$)	Case 2 ($M_I \rightarrow \infty$)
FOR (%)	8	8
α^P	0.8	0.8
λ_I (€/MWh)	10	10
P_I (MW)	75	75
s_I	1	0
Expected profit(€)	1.0640×10^7	1.0595×10^7
CVaR(€)	8.3582×10^6	8.0751×10^6

value of the insured power level has been fixed to 75 MW. Observe that the value of M_I^{P*} decreases with the strike price due to the fact that the devolution that the insurer has to pay to the producer also decreases. Note that the producer is willing to pay a higher premium for a given level of risk aversion and strike price if the probability of an unexpected failure increases (higher FOR). Moreover, the higher the risk aversion, the higher the critical premium.

Figure 7 shows the variation of the critical premium of the producer with the insured power. In this case the strike price is fixed to 10 €/MWh. Note that the higher the insured power level, the higher the amount that the producer is willing to pay for the insurance contract.

4.4. Calculation of the critical premium of the insurer

Data and results of the problem to be solved to calculate the critical premium of the insurer are provided in Table 6. Note that the premium of the insurance contract is equal to €0. Considering (26) the critical premium of the insurer is equal to $-CVaR$, that is, €302,970.

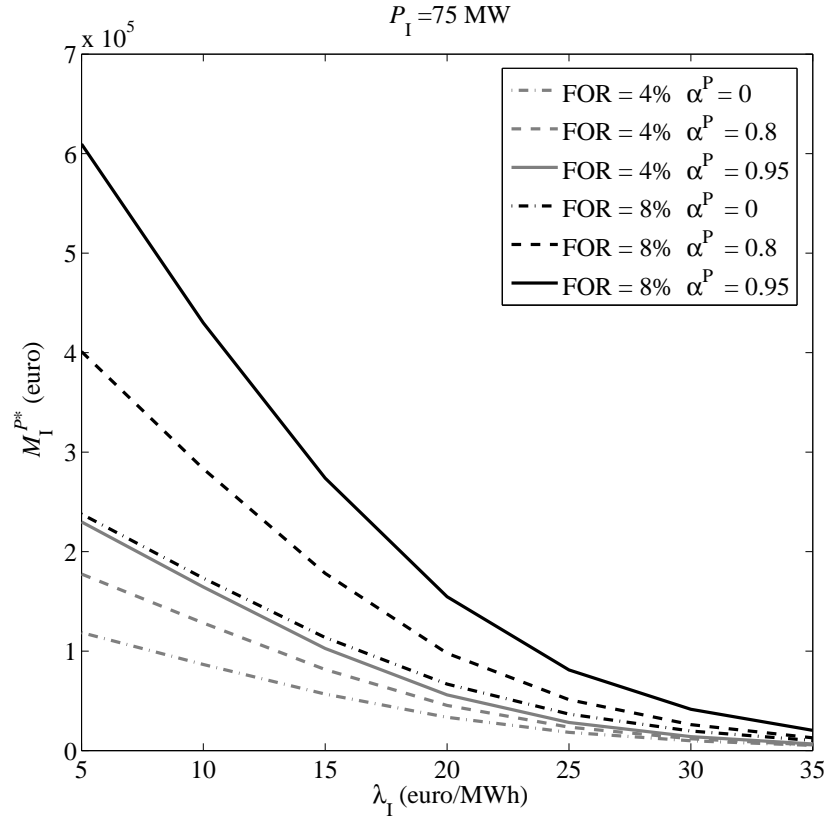


Figure 6: Producer critical premium versus the strike price for different values of FOR and risk aversion ($P_I = 75 \text{ MW}$).

In Figure 8 the critical premiums of the insurer for different values of the risk aversion (α^S) and FOR values are depicted for a value of insured power equal to 25 MW. Observe that the critical premium decreases with the strike price and increases with FOR as in the case of the producer. Moreover, the higher the risk aversion level, the higher the critical premium of the insurer.

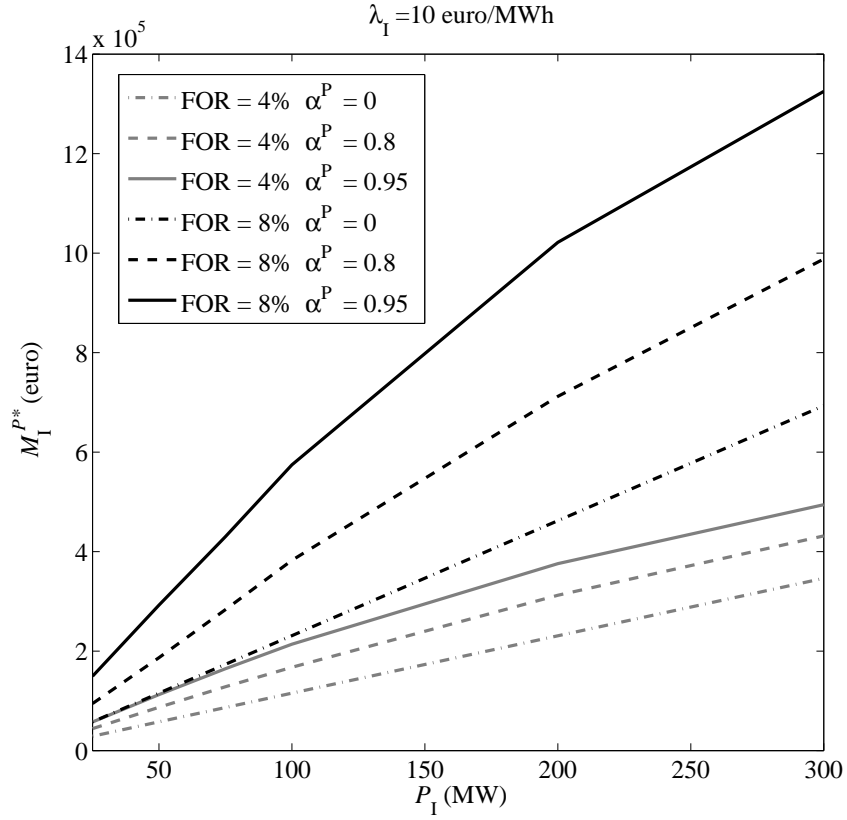


Figure 7: Producer critical premium versus the insured power level for different values of FOR and risk aversion ($\lambda_I = 10 \text{ €/MWh}$).

4.5. On insurance bargaining

Two critical premiums are calculated: the maximum premium that the producer is willing to pay for a given insurance contract and the minimum premium at which the insurer is willing to sell an insurance contract. In Table 7 the critical premiums of the producer are provided for a strike price equal to 10 €/MWh, an insured power level equal to 75 MW, FOR equal to 8% and several risk aversion levels. Table 8 provides the corresponding critical premiums of the insurer.

Table 6: Calculation of the critical premium for the insurer

Case 1 ($M_I = 0$)	
FOR (%)	8
α^S	0.5
λ_I (€/MWh)	10
P_I (MW)	75
s_I	1
Expected profit(€)	-1.7328×10^5
CVaR(€)	-3.0297×10^5

Table 7: Producer critical premium for $\lambda_I = 10\text{€/MWh}$, $P_I = 75\text{ MW}$ and FOR = 8 %

α^P	0	0.1	0.3	0.5	0.8	0.9	0.95
$M_I^{P*}(\text{€})$	173,275	181,649	196,166	214,615	283,115	358,210	429,855

A given insurance contract is signed if the critical premium of the insurer is lower than the critical premium of the producer. It is not possible that both agents make a deal if the maximum amount that the producer is willing to pay for an insurance contract is lower than the minimum amount imposed by the insurer. Observe that if the value of the risk aversion of the producer (α^P) and the risk aversion of the insurer (α^S) are equal, the insurance contract is not signed in any case. On the other hand, if $\alpha^P = 0.9$ and $\alpha^S = 0.5$,

Table 8: Critical premium of the insurer for $\lambda_I = 10\text{€/MWh}$, $P_I = 75\text{ MW}$ and FOR = 8 %

α^S	0	0.1	0.3	0.5	0.8	0.9	0.95
$M_I^{S*}(\text{€})$	173,275	192,528	242,071	302,974	454,577	563,779	667,665

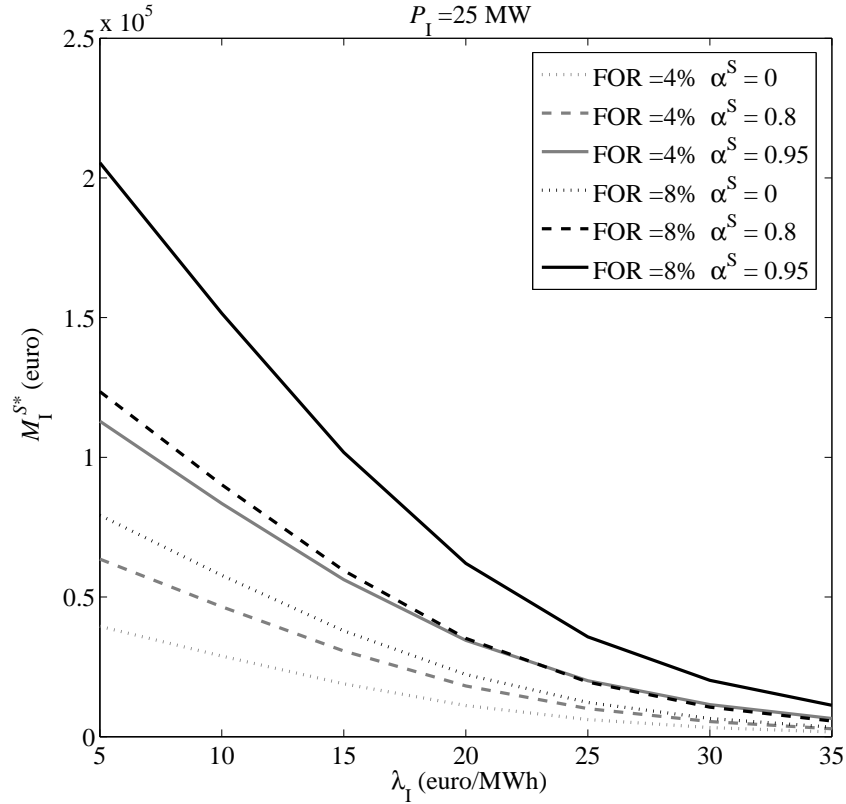


Figure 8: Critical premium of the insurer for different values of FOR and risk aversion levels.

for example, the critical premium of the producer (358,210 €) is higher than the critical premium of the insurer (302,974 €), so both parties can reach a deal.

In Table 9 the differences between M_I^{P*} and M_I^{S*} are represented for all considered values of α^P and α^S . If this difference is positive, the insurance contract can be signed. Likewise, the higher the difference between M_I^{P*} and M_I^{S*} , the more likely the agreement is. If M_I^{S*} is higher than M_I^{P*} , the corresponding entry in the table is 0, i.e., the insurance contract is not signed.

Observe that the insurance contract is signed for values of the risk aversion of the

Table 9: $M_I^{P*} - M_I^{S*}$ for $\lambda_I = 10\text{€/MWh}$, $P_I = 75\text{ MW}$ and $\text{FOR} = 8\%$

	α^P						
	0	0.1	0.3	0.5	0.8	0.9	0.95
0	0	8,374	22,891	41,339	109,840	184,935	256,580
0.1	0	0	3,638	22,087	90,587	165,682	237,327
0.3	0	0	0	0	41,044	116,139	187,785
α^S 0.5	0	0	0	0	0	55,236	126,882
0.8	0	0	0	0	0	0	0
0.9	0	0	0	0	0	0	0
0.95	0	0	0	0	0	0	0

insurer lower than those of the risk aversion of the producer. This corroborates the fact that an insurance contract is a mechanism whose aim is to transfer risk from one party (the producer) to another one (the insurer) in exchange for a certain amount of money (the premium). Note that the lower the risk aversion of the insurer, the higher the difference $M_I^{P*} - M_I^{S*}$, and therefore the higher the possibility that a deal is reached between both parties.

5. Conclusions

This paper presents a methodology to study the convenience of signing an insurance contract by a power producer to reduce the financial risk associated with failures of the production units when trading in electricity futures and pool markets.

Contracting an insurance reduces the financial risk associated with the failures of the production units and allows the producer to sell a higher amount of energy in the futures market. It is shown that the higher the risk aversion or the FOR of the producer, the higher

the amount that the producer is willing to pay for a given insurance contract.

Additionally, it is shown that the insurance contract is signed if the risk aversion of the insurer is lower than the risk aversion of the producer, corroborating that an insurance contract is a financial mechanism to transfer risk from one agent to another.

A. Notation

Continuous variables

$P_{t\omega}^G$ Power generated by the unit during period t and scenario ω (MW).

$P_{t\omega b}^G$ Power generated with the b -th power block of the unit during period t and scenario ω (MW).

$P_{t\omega}^P$ Power traded in the pool during period t and scenario ω (MW).

P_c Power sold through forward contract c (MW).

ζ Auxiliary variable related to the Conditional Value at Risk (CVaR) of the profit distribution of the producer (€).

η_ω Auxiliary variable in scenario ω related to the CVaR of the profit distribution of the producer (€).

ξ Auxiliary variable related to the CVaR of the profit distribution of the insurer (€).

ρ_ω Auxiliary variable in scenario ω related to the CVaR of the profit distribution of the insurer (€).

Random variables

k_t Unit status in period t , 1 if available and 0 otherwise.

λ_t^P Pool price in period t (€/MWh).

Binary variables

s_I Binary variable that is equal to 1 if the producer signs the insurance contract, and 0 otherwise.

$u_{t\omega}$ Binary variable that is equal to 1 if the unit is online during period t and scenario ω , and 0 otherwise.

Constants

A Coefficient of the piecewise linear production cost function of the unit (€/h).

D_ω Amount that the insurer pays to the producer in scenario ω (€).

$k_{t\omega}$ Unit status in period t and scenario ω , 1 if available and 0 if forced out.

$L_{t\omega}$ Duration of time period t in scenario ω (h).

M_I Premium of the insurance contract (€).

M_I^{P*} Maximum premium that the producer is willing to pay for a given insurance contract (€).

M_I^{S*} Minimum premium at which the insurer is willing to sell a given insurance contract (€).

P^{\max} Capacity of the unit (MW).

P^{\min} Minimum power output of the unit (MW).

P_b^{\max} Size of the b -th power block of the piecewise linear production cost function (MW).

P_c^{\max} Maximum power that can be sold through contract c (MW).

P_I Insured power (MW).

- T_c Duration of contract c (h).
- α^P Per unit confidence level of the producer.
- α^S Per unit confidence level of the insurer.
- λ_b Slope of the b -th power block of the piecewise linear production cost function of the generating unit (€/MWh).
- λ_c Energy price of forward contract c (€/MWh).
- λ_I Strike price of the insurance contract (€/MWh).
- $\lambda_{t\omega}^P$ Pool price in period t and scenario ω (€/MWh).
- π_ω Probability of occurrence of scenario ω .

Numbers

- N_C Number of forward contracts.
- N_B Number of blocks of the piecewise linear cost function of the production unit.
- N_T Number of time periods.
- N_W Number of scenarios.

Sets

- F_t Set of forward contracts available during period t .
- G_ω Set of periods in scenario ω in which the pool price is higher than the strike price of the insurance contract ($\lambda_{t\omega}^P > \lambda_I$) and the unit is forced out ($k_{t\omega} = 0$).

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