

On improving MILP performance via Machine Learning tools

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Outline

- 1 Motivation
- 2 Methodology
- 3 Computational Experience
- 4 Conclusions and Further Research

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Mixed Integer
Linear Programs
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Machine
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Combine knowledge from both worlds

Recent reviews: *Bengio et al. [2021]*; *Gambella et al. [2021]*

Literature review

- Branch-and-bound methods: *Karapetyan et al. [2017]; Kruber et al. [2017]; Liberto et al. [2016]; Lodi and Zarpellon [2017]*.
- End-to-end approaches: *Kool et al. [2019]; Larsen et al. [2018]*.

Literature review

- Branch-and-bound methods: *Karapetyan et al. [2017]*; *Kruber et al. [2017]*; *Liberto et al. [2016]*; *Lodi and Zarpellon [2017]*.
 - ✓ Optimality guarantees.
 - × Slow.
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 - ✓ Fast.
 - × Suboptimal/infeasible solutions.

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Our approach

- Computational gains.
- Reduce risk of infeasible problems.

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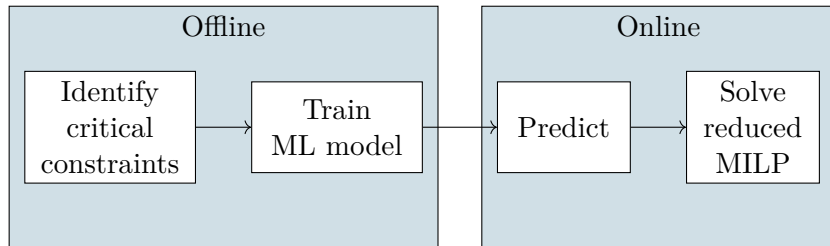
Our approach

- Computational gains.
- Reduce risk of infeasible problems.

Simpler problems

- Remove non-critical constraints.
- Solve a reduced optimization problem.

Our methodology in a nutshell



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MILP

$$(P_{\theta}[\mathcal{J}]) \quad \begin{cases} \min_{\mathbf{z} \in \mathbb{R}^n \times \mathbb{Z}^q} \mathbf{c}^T \mathbf{z} \\ \text{s.t. } \mathbf{a}_j^T \mathbf{z} \leq b_j, \quad \forall j \in \mathcal{J} \end{cases}$$

- $\theta = \{\mathbf{c}, \mathbf{a}_j, b_j, \forall j \in \mathcal{J}\}$.
- $P_{\theta}[\mathcal{J}]$ bounded and feasible.
- Optimal solution $\mathbf{z}_{\theta}^*[\mathcal{J}]$ is a singleton.

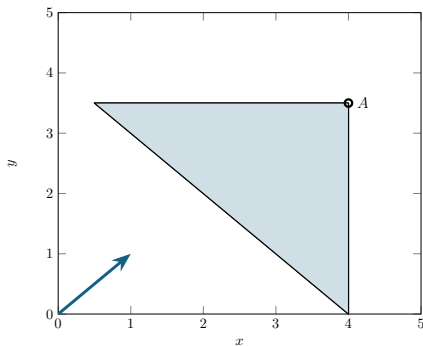
Invariant Constraint Set, \mathcal{S}

According to *Calafiore [2010]*:

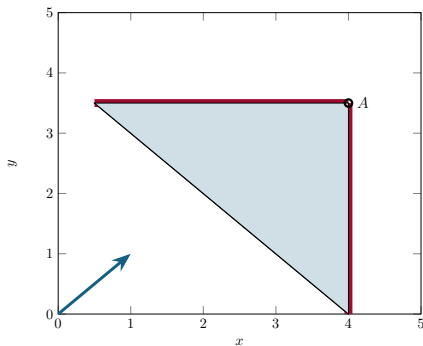
$$\mathcal{S} \subset \mathcal{J} \text{ s.t. } \mathbf{c}^T \mathbf{z}_{\theta}^*[\mathcal{S}] = \mathbf{c}^T \mathbf{z}_{\theta}^*[\mathcal{J}]$$

The integrality of the decision variables is crucial to find out which constraints belong to \mathcal{S} .

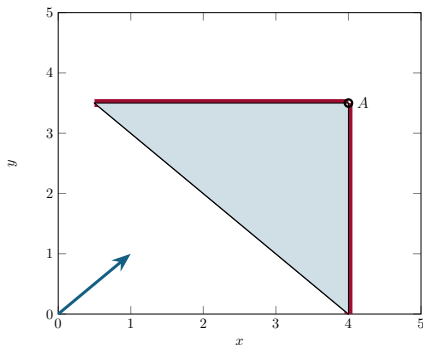
LP vs MILP (Example taken from *Pineda et al. [2020]*)



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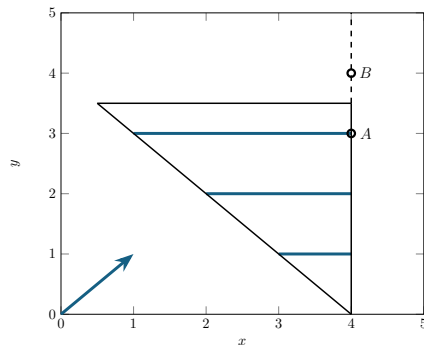
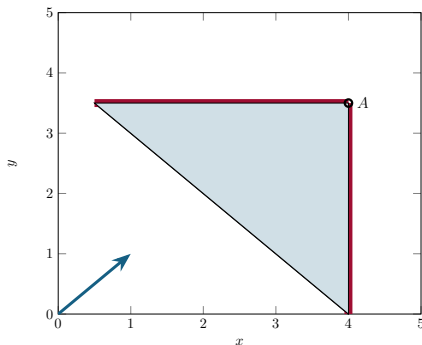


Binding constraints

$$\mathcal{B} = \{j \in \mathcal{J} : \mathbf{a}_j^T \mathbf{z}_\theta^*[\mathcal{J}] = b_j\}$$

$$\mathcal{S} = \mathcal{B}$$

LP vs MILP (Example taken from *Pineda et al. [2020]*)

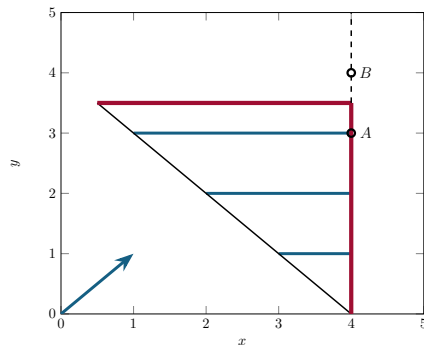
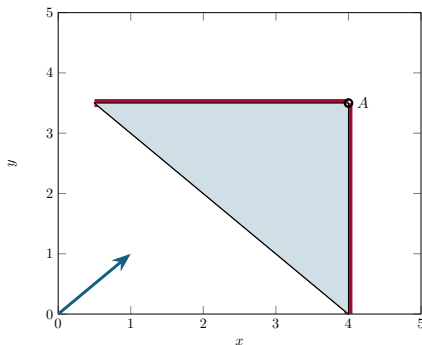


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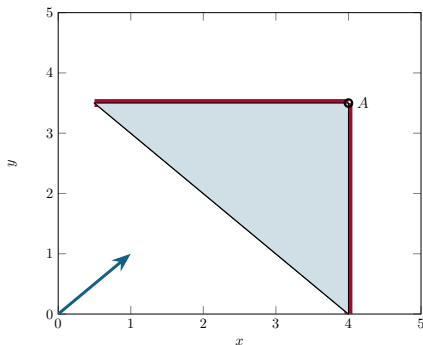


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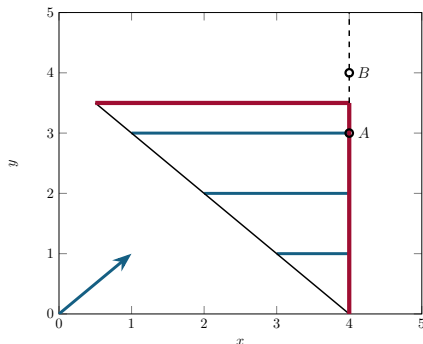
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Binding constraints

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Some non-binding constraints

also belong to \mathcal{S} .

$$\mathcal{S} \supset \mathcal{B}$$

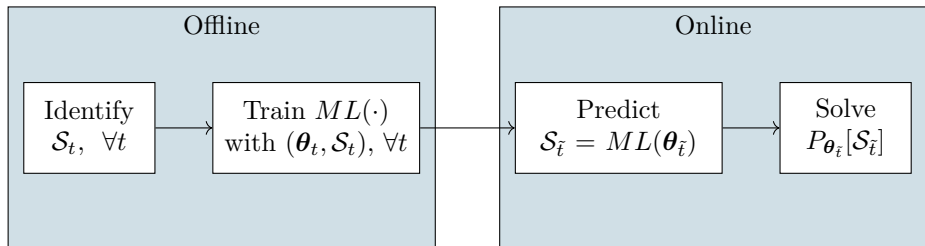
- Finding \mathcal{S} is challenging in MILPs.
- For each train instance t , we look for \mathcal{S}_t , including some of the non-binding constraints.
- And reduced MILP $P_{\theta_t}[\mathcal{S}_t]$ is solved.

How to find \mathcal{S}_t ?

Algorithm 1 Identifying an invariant constraint set for each instance t

- 0) Initialize $\mathcal{S}_t = \mathcal{B}_t$.
 - 1) Solve $P_{\theta_t}[\mathcal{S}_t]$ with solution $\mathbf{z}_{\theta_t}^*[\mathcal{S}_t]$.
 - 2) If $\max_{j \in \mathcal{J} \setminus \mathcal{S}_t} \left\{ \mathbf{a}_j^T \mathbf{z}_{\theta_t}^*[\mathcal{S}_t] - b_j \right\} > 0$, go to step 3).
Otherwise, stop.
 - 3) $\mathcal{S}_t := \mathcal{S}_t \cup \left\{ \arg \max_{j \in \mathcal{J} \setminus \mathcal{S}_t} \left\{ \mathbf{a}_j^T \mathbf{z}_{\theta_t}^*[\mathcal{S}_t] - b_j \right\} \right\}$, go to step 1).
-

Recap



Advantages

- Based on constraint generation. Crucial non-binding constraints are guaranteed to be included.
- Reduce risk of infeasible problems.
- Independent on the ML method used.
- Identifying S and training ML is performed offline.

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Experimental Setup

- Binary classification problem. *knn*.
- Label $s_j^t = \pm 1$ depending on inclusion on \mathcal{S}_t .
- Two approaches: \mathcal{B} -learner and \mathcal{S} -learner.
- Synthetic and real-world applications.

Unit Commitment problem

$$\left\{ \begin{array}{ll} \min_{\mathbf{x} \in \mathbb{R}^n, \mathbf{y} \in \{0,1\}^n} & \sum_{i=1}^n c_i x_i \\ \text{s.t.} & \sum_{i=1}^n x_i = \sum_{i=1}^n d_i, \\ & -f_j \leq \sum_{i=1}^n a_{ij}(x_i - d_i) \leq f_j, \quad j = 1, \dots, m \\ & l_i y_i \leq x_i \leq u_i y_i, \quad i = 1, \dots, n \end{array} \right.$$

- $\theta = \mathbf{d}$.
- $n = 96$.
- $m = 120$ (240 constraints).
- $T = 8640$ (Leave-one-out).

		k					
		Bench.	5	10	20	50	100
\mathcal{B} -learner	constraints	240	[0, 23]	[0, 23]	[0, 26]	[0, 26]	[0, 28]
	# infeasible	0%	45.71%	38.02%	31.73%	23.11%	16.29%
	MILP time	100%	24.59%	27.05%	28.72%	31.31%	33.43%
\mathcal{S} -learner	constraints	240	[0, 26]	[0, 28]	[0, 29]	[0, 30]	[0, 32]
	# infeasible	0%	7.38%	2.78%	1.20%	0.55%	0.28%
	MILP time	100%	33.96%	34.97%	36.45%	36.14%	37.79%

- Larger values of k imply more constraints (more conservative).
- Computational gains in both approaches.
- Few extra constraints in \mathcal{S} -learner.
- Large improvements with regard to infeasible problems in \mathcal{S} -learner.
- Adding constraints is not enough ($k = 5$ vs $k = 50$).

More details

Offline constraint screening for online mixed-integer optimization

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Abstract

Mixed Integer Linear Programs (MILP) are well known to be NP-hard problems in general, and therefore, tackling and using their solution in online applications is a great challenge. Some Machine Learning techniques have been proposed in the literature to alleviate their computational burden. Unfortunately, these techniques report that a non-negligible percentage of the resulting machine-learning-based solutions are infeasible.

By linking Mathematical Optimization and Machine Learning, this paper proposes an offline-online strategy to solve MILPs in a fast and reliable manner. The offline step seeks to identify the so-called support constraints of past instances of the target MILP and uses them to train a Machine Learning model of our choice. This model is subsequently used online to generate a reduced MILP that is significantly easier to solve.

Through synthetic and real-life MILPs, we show that our approach dramatically decreases the risk of obtaining solutions from the reduced MILP that are infeasible in the original formulation without any extra cost in terms of computational time. Hence, our methodology is appropriate for online applications where feasibility is of particular importance.

Keywords: Machine Learning, Mixed Integer Linear Programming, constraint screening, offline-online strategy

Available at:

[https://www.researchgate.net/publication/
350371853_Offline_constraint_screening_for_
online_mixed-integer_optimization](https://www.researchgate.net/publication/350371853_Offline_constraint_screening_for_online_mixed-integer_optimization)



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Conclusions

- Approach which combines MILPs and ML.
- Offline-online strategy.
- Reduce risk of infeasible problems.
- Reduce computational burden.
- Tested on synthetic and real-world applications.

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Further research

- Other input parameters.
- Introduce expert-knowledge information.

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Thank you very much for your attention!



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