Scenario Reduction for Futures Market Trading in Electricity Markets

S. Pineda, *Student Member, IEEE*, J. M. Morales, *Student Member, IEEE*, A. J. Conejo, *Fellow, IEEE*, M. Carrión, *Student Member, IEEE*

***Abstract*— To make informed decisions in futures markets of electric energy, stochastic programming models are commonly used. Such models treat stochastic variables via a set of scenarios, which are plausible realizations throughout the decision-making horizon of the stochastic variables. The number of scenarios needed to accurately represent the uncertainty involved is generally large, which renders the associated stochastic programming problem hard to solve. Hence, scenario reduction techniques are needed to trim down the number of scenarios while keeping most of the stochastic information embedded in such scenarios. This paper proposes a novel scenario-reduction procedure that advantageously compares with existing ones.**

***Index Terms*— Electricity markets, decision making, scenario reduction, stochastic programming.**

NOTATION

The main notation used throughout the paper is stated below for quick reference. Other symbols are defined as required.

A. Variables:

- $C^{\mathrm{G}}_{t\omega}$ Generation cost during period t and scenario ω ($).
- $C^{\mathrm{U}}_{t\omega}$ Start-up cost incurred by the unit in period t and scenario ω ($).
- $C^{\mathrm{D}}_{t\omega}$ Shut-down cost incurred by the unit in period t and scenario ω ($).
- $P^{\mathrm{G}}_{t\omega}$ Total power generated during period t and scenario w (MW).
- $P^{\mathrm{G}}_{t\omega b}$ Power produced from the b-th block of the piecewise linear production cost function during period t and scenario w (MW).
- $P^{\mathrm{P}}_{t\omega}$ Power traded in the pool in period t and scenario w (MW).
- P_c Power sold through forward contract c (MW).
- $u_{t\omega}$ Binary variable which is equal to 1 if the unit is online during period t and scenario ω, and 0 otherwise.

B. Stochastic variables:

- λ^{P}_t Pool price in period t ($/MWh).

S. Pineda, J. M. Morales, A. J. Conejo and M. Carrión are partly supported by Junta de Comunidades de Castilla – La Mancha through project PCI-08-0102 and by the Ministry of Education and Science of Spain through CICYT Project DPI2006-08001.

S. Pineda, J. M. Morales, A. J. Conejo and M. Carrión are with University of Castilla-La Mancha, Ciudad Real, Spain (e-mails: Salvador.Pineda@uclm.es, JuanMiguel.Morales@uclm.es, Antonio.Conejo@uclm.es, Miguel.Carrion@uclm.es)

C. Constants:

- λ_b Slope of the b-th block of the piecewise linear production cost function ($/MW).
- λ_c Energy price of forward contract c ($/MWh).
- $\lambda^{\mathrm{P}}_{t\omega}$ Pool price in period t and scenario w ($/MWh).
- π_ω Probability of occurrence of scenario w.
- A Coefficient of the piecewise linear production cost function of the unit ($).
- C^{SU} Start-up cost ($).
- C^{SD} Shut-down cost ($).
- D^{T} Minimum down time (h).
- L_t Time duration of period t (h).
- P^{G}_0 Power production in the period before the first period of the study horizon (MW).
- $P^{\mathrm{S,Max}}_c$ Maximum power that can be sold through forward contract c (MW).
- P^{Max} Capacity of the unit (MW).
- P^{Max}_b Upper limit of the b-th block of the piecewise linear production cost function (MW).
- P^{Min} Minimum power output of the unit (MW).
- R^{U} Ramp-up limit (MW/h).
- R^{D} Ramp-down limit (MW/h).
- S^{U} Start-up ramp limit (MW/h).
- S^{D} Shut-down ramp limit (MW/h).
- T_0 Number of periods that the unit has been online/offline prior to the first period of the study horizon (h).
- T_c Duration of the forward contract c (h).
- u_0 Commitment state of the unit in the period before the first period of the study horizon.
- U^{T} Minimum up time (h).

D. Numbers:

- N_Ω Number of scenarios.
- N_B Number of blocks of the piecewise linear production cost.
- N_C Number of forward contracts.
- N_T Number of time periods.

E. Set:

- F_t Set of forward contracts available during period t.

I. INTRODUCTION

To make informed decisions in electricity markets, stochastic programming models are commonly used [1]–[10].

In particular, some of these models are used by producers to determine the sales of energy through forward contracts in futures markets and in the pool, and by retailers and consumers to decide purchases in futures markets through forward contracts and in the pool.

Stochastic programming models [11] represent stochastic variables (e.g., electricity pool prices or electricity demands) via a set of scenarios, which are plausible realizations of the stochastic variables throughout the decision-making horizon. For example, if the decision-making horizon is one week, pool price uncertainty can be represented by a number of vectors (e.g., 1,000), each one containing a possible realization of hourly pool prices throughout the week (i.e., 168 values).

The number of scenarios needed to accurately represent the most plausible realizations of the stochastic variables is generally very large, which may render the associated stochastic programming problem untractable. For instance, a decision-making problem considering 1,000 scenarios, each one represented by a vector of 168 values, may be untractable or may require an unreasonably high solution time.

Therefore, scenario reduction techniques are needed to trim down significantly the number of scenarios while keeping as intact as possible the stochastic information embedded in such scenarios.

This paper proposes a novel scenario-reduction procedure that advantageously compares with existing ones. The proposed technique allows further reducing the number of scenarios determined by other techniques, while retaining the same level of stochastic information as the scenario set provided by these techniques.

There are available several methods to generate and reduce scenarios to be used in stochastic programming. A short overview of the most common methods is provided in [12]. An easy way to generate price scenarios is just the sampling method, proposed in [13]. In [14], the author proposes a moment matching method to generate a limited number of scenarios that satisfy specified statistical properties. [15] describes an optimal discretization method that seeks to find an approximation of the initial scenario set that minimizes an error based on the objective function. Traditional scenario reduction techniques are described in [16], [17] and [18]. These methods seek to obtain a reduced number of scenarios that best retain the essential features of a given original scenario set according to a *probability distance*. The generation of scenarios for multistage stochastic problems is described in [19].

The contribution of this paper is providing an efficacious scenario reduction technique. Moderately increasing the computational burden required by other procedures, while retaining the same level of stochastic information, the proposed technique results in a significantly smaller number of scenarios than the number provided by alternative procedures.

The remaining of this paper is organized as follows. Section II describes, as a motivating example, the weekly decision-making problem of a power producer to decide optimally its sales through forward contracts and in the pool. Section III describes the mathematical structure of decision-making problems pertaining to electric energy futures markets. Section IV describes a commonly used and the proposed scenario reduction techniques. Section V provides a realistic case study that illustrates the proposed scenario reduction technique, and compares it with a commonly used technique. Section VI provides some relevant conclusions that can be drawn from the study reported.

II. Motivating model

As a motivating example, we consider the problem of a power producer owning a single generating unit that has to decide at the beginning of the week which forward contracts should be signed. Within a futures market framework, a forward contract consists in a constant-power quantity to be sold during the whole week (e.g., 100 MW) at a given energy price (e.g., 50 $/MWh).

In addition to selling energy through forward contracts, the power producer may sell the energy in the pool at uncertain selling prices. As previously mentioned, uncertainty on pool prices throughout the week can be modeled using a set of pool price scenarios, i.e., a set of vectors each one containing 168 values representing the hourly pool prices of the considered week.

Specifically, the problem faced by the producer is to decide which forward contracts should be signed through the futures market and, also, the specific involvement in the pool for each price-scenario realization. The futures market decisions are made prior to the knowledge of which price scenario materializes, and, therefore, they are *here-and-now* decisions. Decisions pertaining to pool involvement can be deferred in time until a given price scenario materializes, and, therefore, they are *wait-and-see* decisions. More precisely, these decisions are made once a day with good forecasts of the pool prices for the following day, and, hence, it is considered that these decisions are made with certainty.

For the sake of simplicity, no risk metric is considered in the formulation of the problem above. However, the proposed scenario reduction technique is equally applicable to problems that do include risk treatment.

The problem above can be mathematically formulated as

Maximize$_{P^{\mathrm{P}}_{t\omega}, P^{\mathrm{G}}_{t\omega b}, P_c, u_{t\omega}}$

$$z = \sum_{\omega=1}^{N_\Omega} \sum_{t=1}^{N_T} \pi_\omega (\lambda^{\mathrm{P}}_{t\omega} P^{\mathrm{P}}_{t\omega} L_t - C^{\mathrm{G}}_{t\omega} - C^{\mathrm{U}}_{t\omega} - C^{\mathrm{D}}_{t\omega}) + \sum_{c=1}^{N_C} \lambda_c P_c T_c \quad (1)$$

subject to

$$C^{\mathrm{G}}_{t\omega} = A u_{t\omega} + \sum_{b=1}^{N_B} \lambda_b P^{\mathrm{G}}_{t\omega b} L_t, \quad \forall t, \forall \omega \quad (2)$$

$$P^{\mathrm{G}}_{t\omega} = P^{\mathrm{Min}} u_{t\omega} + \sum_{b=1}^{N_B} P^{\mathrm{G}}_{t\omega b}, \quad \forall t, \forall \omega \quad (3)$$

$$P^{\mathrm{G}}_{t\omega 1} \leq P^{\mathrm{Max}}_1 - P^{\mathrm{Min}}, \quad \forall t, \forall \omega \quad (4)$$

$$P^{\mathrm{G}}_{t\omega b} \leq P^{\mathrm{Max}}_b - P^{\mathrm{Max}}_{b-1}, \quad \forall t, \forall \omega, \forall b = 2, ..., N_B \quad (5)$$

$$P^{\mathrm{G}}_{t\omega b} \geq 0, \quad \forall t, \forall \omega, \forall b \quad (6)$$

$$C^{\mathrm{U}}_{t\omega} \geq C^{\mathrm{SU}}(u_{t\omega} - u_{t-1\omega}), \quad \forall t, \forall \omega \tag{7}$$
$$C^{\mathrm{U}}_{t\omega} \geq 0, \quad \forall t, \forall \omega \tag{8}$$
$$C^{\mathrm{D}}_{t\omega} \geq C^{\mathrm{SD}}(u_{t-1\omega} - u_{t\omega}), \quad \forall t, \forall \omega \tag{9}$$
$$C^{\mathrm{D}}_{t\omega} \geq 0, \quad \forall t, \forall \omega \tag{10}$$

$$\sum_{c \in F_t} P_c + P^{\mathrm{P}}_{t\omega} - P^{\mathrm{G}}_{t\omega} = 0, \quad \forall t, \forall \omega \tag{11}$$

$$P^{\mathrm{G}}_{t\omega} \geq P^{\mathrm{Min}} u_{t\omega}, \quad \forall t, \forall \omega \tag{12}$$
$$P^{\mathrm{G}}_{t\omega} \leq P^{\mathrm{Max}} u_{t\omega}, \quad \forall t, \forall \omega \tag{13}$$

$$P_c \leq P^{\mathrm{S,Max}}_c, \quad \forall c \tag{14}$$

$$P_c \geq 0, \quad \forall c \tag{15}$$
$$P^{\mathrm{P}}_{t\omega} \geq 0, \quad \forall t, \forall \omega \tag{16}$$

$$\begin{aligned} P^{\mathrm{G}}_{t\omega} \leq {} & P^{\mathrm{G}}_{t-1\omega} + R^{\mathrm{U}} u_{t-1\omega} + S^{\mathrm{U}}(u_{t\omega} - u_{t-1\omega}) + \\ & + P^{\mathrm{Max}}(1 - u_{t\omega}), \quad \forall t = 2, ..., N_T, \forall \omega \end{aligned} \tag{17}$$
$$\begin{aligned} P^{\mathrm{G}}_{t\omega} \geq {} & P^{\mathrm{G}}_{t-1\omega} - R^{\mathrm{D}} u_{t\omega} - S^{\mathrm{D}}(u_{t-1\omega} - u_{t\omega}) - \\ & - P^{\mathrm{Max}}(1 - u_{t-1\omega}), \quad \forall t = 2, ..., N_T, \forall \omega \end{aligned} \tag{18}$$

$$\sum_{t=1}^{G} (1 - u_{t\omega}) = 0, \quad \forall \omega \tag{19}$$
$$\sum_{k=t}^{\mathrm{t+U^T-1}} u_{k\omega} \geq U^{\mathrm{T}}(u_{t\omega} - u_{t-1\omega}), \quad \forall t = G+1, ..., N_T - U^{\mathrm{T}} + 1, \forall \omega \tag{20}$$
$$\sum_{k=t}^{N_T} \{u_{k\omega} - (u_{t\omega} - u_{t-1\omega})\} \geq 0, \quad \forall t = N_T - U^{\mathrm{T}} + 2, ..., N_T, \forall \omega \tag{21}$$

$$\sum_{t=1}^{\mathrm{F}} u_{t\omega} = 0, \quad \forall \omega \tag{22}$$
$$\sum_{k=t}^{t+D^T-1} (1 - u_{k\omega}) \geq D^{\mathrm{T}}(u_{t-1\omega} - u_{t\omega}), \quad \forall t = F+1, ..., N_T - D^{\mathrm{T}} + 1, \forall \omega \tag{23}$$
$$\sum_{k=t}^{N_T} \{1 - u_{k\omega} - (u_{t-1\omega} - u_{t\omega})\} \geq 0, \quad \forall t = N_T - D^{\mathrm{T}} + 2, ..., N_T, \forall \omega \tag{24}$$

Objective function (1) includes two terms, namely, (i) expected revenue from selling in the pool minus expected production cost of the unit, and (ii) the revenue from selling through forward contracts.

Constraints (2)–(6) approximate the quadratic production cost function of the unit by a set of piecewise blocks as stated in [20]. Constraints (7)–(10) constitute a mixed-integer linear formulation to model the start-up and shut-down costs. Constraints (11) enforce the power balance at each time period and for each scenario. Constraints (12) and (13) enforce, respectively, the minimum power output and the capacity of the generating unit at each time period and for each scenario. Constraints (14) impose a cap over the power sold through each forward contract. Constraints (15) and (16) avoid the arbitrage between the pool and the futures market. Constraints (17) and (18) enforce the ramping limitations of the unit. Lastly, minimum up and down times are enforced through constraints (19)–(21) and (22)–(24) respectively, where

$$G = \min\{N_T, (U^{\mathrm{T}} - T_0)u_0\} \tag{25}$$

and

$$F = \min\{N_T, (D^{\mathrm{T}} - T_0)(1 - u_0)\}. \tag{26}$$

Further details about the constraints pertaining to the feasible operational region of a generating unit are provided in [20].

III. Mathematical structure

Problems similar to the one discussed in Section II have the general mathematical structure described in the following. $\boldsymbol{\lambda_\omega}$ represents the different scenario realizations of the vector of electricity pool prices (stochastic variable). Variable $\boldsymbol{x}$ represents the vector defining the energy sold in the futures market through forward contracts, and $\boldsymbol{y_\omega}$ represents the variable vector defining the operating decisions and the energy sold in the pool. Note that pool decisions depend on price scenario realizations while futures market decisions do not. $R^F(\cdot)$ is the revenue associated with selling energy in the futures market, while $R^P(\cdot)$ is the revenue associated with selling in the pool. $C^O(\cdot)$ is the total production cost of the producer. $\mathcal{E}_\omega\{\cdot\}$ is the expectation operator over the stochastic variables represented by scenarios indexed by ω. Sets Ω^F, Ω^P and Ω^O represent, respectively, the feasibility region of forward contracting, the feasibility region of pool operation and the operating constraints of the producer. This problem is formulated as

$$\underset{\boldsymbol{x}, \boldsymbol{y_\omega}}{\text{maximize}} \quad R^F(\boldsymbol{x}) + S(\boldsymbol{x}) \tag{27}$$

subject to

$$\boldsymbol{x} \in \Omega^F, \tag{28}$$

where $S(\boldsymbol{x}) =$

$$\mathcal{E}_\omega \left\{ \underset{\boldsymbol{y_\omega}}{\text{maximize}} \quad \left(R^P(\boldsymbol{\lambda_\omega}, \boldsymbol{y_\omega}) - C^O(\boldsymbol{x}, \boldsymbol{y_\omega}) \right) \right\} \tag{29}$$

subject to

$$\boldsymbol{y_\omega} \in \Omega^P, \forall \omega; \quad (\boldsymbol{x}, \boldsymbol{y_\omega}) \in \Omega^O, \forall \omega. \tag{30}$$

Objective function (27) represents the expected profit of the producer, which is the sum of the revenue from selling in the futures market and, as expressed by (29), the expected revenue from selling in the pool minus the expected production cost. Constraints (28) impose the feasibility conditions pertaining to the futures market, and constraints (30) enforce the feasibility conditions pertaining to the pool and the operating constraints of the producer.

Under rather general assumptions [11], the maximization and expectation operators can be swapped in (29). Then, the two-stage stochastic programming problem (27)-(30) is conveniently formulated as the deterministic mathematical programming problem stated below,

$$\underset{\boldsymbol{x}, \boldsymbol{y_\omega}}{\text{maximize}} \quad z = R^F(\boldsymbol{x}) + \mathcal{E}_\omega \left\{ R^P(\boldsymbol{\lambda_\omega}, \boldsymbol{y_\omega}) - C^O(\boldsymbol{x}, \boldsymbol{y_\omega}) \right\} \tag{31}$$

subject to

$$\boldsymbol{x} \in \Omega^F; \;\; \boldsymbol{y_\omega} \in \Omega^P, \forall \omega; \;\; (\boldsymbol{x}, \boldsymbol{y_\omega}) \in \Omega^O, \forall \omega. \tag{32}$$

If just one single scenario is considered, e.g., scenario $\hat{\omega}$, the problem above becomes

$$\underset{\boldsymbol{x}, \boldsymbol{y_{\hat{\omega}}}}{\text{maximize}} \quad z_{\hat{\omega}} = R^F(\boldsymbol{x}) + R^P(\boldsymbol{\lambda_{\hat{\omega}}}, \boldsymbol{y_{\hat{\omega}}}) - C^O(\boldsymbol{x}, \boldsymbol{y_{\hat{\omega}}}) \tag{33}$$

subject to

$$\boldsymbol{x} \in \Omega^F; \;\; \boldsymbol{y_{\hat{\omega}}} \in \Omega^P; \;\; (\boldsymbol{x}, \boldsymbol{y_{\hat{\omega}}}) \in \Omega^O. \tag{34}$$

The single-scenario problem (33)-(34) is much easier to solve than the multi-scenario problem (31)-(32) as its number of variables and constraints is comparatively much smaller.

Finally, observe that decision-making problems for retailers and consumers are mathematically similar to problem (27)-(30) above and, therefore, the scenario reduction technique explained below can also be applied to these problems.

IV. Scenario tree reduction

Two scenario reduction techniques are explained in this section. The first one is proposed in this paper while the second one is commonly used [18]. Both are based on the *fast-forward* algorithm provided in [18], whose mathematical formulation is described in detail in the Appendix.

The *fast-forward* procedure is a heuristic technique whose aim is to reduce a large scenario set to a smaller one that is close to the original set if measured by the *Kantorovich metric*. The Kantorovich metric is a probability distance widely used in stochastic optimization. For finite distributions, the Kantorovich distance, $D_K(\cdot)$, between a reduced scenario set Λ and the original one Ω is as follows:

$$D_K(\Omega, \Lambda) = \sum_{\omega \in \Omega \setminus \Lambda} \pi_\omega \min_{\omega' \in \Lambda} c(\omega, \omega') \tag{35}$$

where $c(\omega, \omega')$ is a nonnegative, continuous and symmetric function defined for each pair of scenarios belonging to the original scenario set. This function is evaluated for each pair of scenarios at the beginning of both algorithms and the results are conveniently stored as a matrix.

As stated below, the key factor that distinguishes one algorithm from the other is the definition of the function $c(\cdot)$. In simple words, this function allows establishing how different two scenarios are.

A. Algorithm 1. Reduction based on the objective function of a single-scenario problem

The rationale for scenario reduction in this technique is as follows:

1) Solve the single-scenario problem corresponding to each scenario (Problem (33)-(34)). The optimal objective function value of this problem characterizes the scenario.
2) Select the scenario which minimizes the distance between the reduced scenario tree (Λ) and the original one (Ω) according to the function $c(\cdot)$, which is defined from the optimal objective function values computed in 1) as follows:
$$c(\omega, \omega') = \left| z_\omega - z_{\omega'} \right| \tag{36}$$
where z_ω is the value taken by the objective function (33) if the optimization problem (33)–(34) is solved for the single scenario ω. Note that $c(\omega, \omega) = 0, \forall w \in \Omega$.
It should be noted that (36) characterizes the novel variant of the fast-forward algorithm proposed in this paper. Note also that (36) does not require any assumption on the nature of the stochastic variable whose scenarios are reduced. Particularly, it allows reducing scenarios pertaining to discrete stochastic variables.
3) Repeat 2) until a sufficient number of scenarios is selected.

Therefore, the algorithm based on the objective function of a single-scenario problem (Algorithm 1) seeks to minimize the Kantorovich distance (35), which is computed in this case as:

$$D_K(\Omega, \Lambda) = \sum_{\omega \in \Omega \setminus \Lambda} \pi_\omega \min_{\omega' \in \Lambda} \left(|z_\omega - z_{\omega'}| \right). \tag{37}$$

B. Algorithm 2. Reduction based on a norm of each price difference scenario vector

The rationale for scenario reduction in this technique is as follows:

1) Select the scenario which minimizes the distance between the reduced scenario tree (Λ) and the original one (Ω) according to the function $c(\cdot)$, which is defined from an appropriate norm of the difference between each pair of price scenarios as follows:
$$c(\omega, \omega') = \| \lambda^{\mathrm{P}}_\omega - \lambda^{\mathrm{P}}_{\omega'} \|^r \tag{38}$$
where $\lambda^P_\omega = \{\lambda^P_{1\omega}, \ldots, \lambda^P_{N_T\omega}\}$ and $\| \cdot \|^r$ refers to the Wasserstein metric of order *r* (e.g., the quadratic norm), i.e., the appropriate norm previously mentioned. Again, note that $c(\omega, \omega) = 0, \forall w \in \Omega$.
2) Repeat 2) until a sufficient number of scenarios is selected.

Therefore, the algorithm based on a norm of each price difference scenario vector (Algorithm 2) seeks to minimize the Kantorovich distance (35), which is computed in this case as:

$$D_K(\Omega, \Lambda) = \sum_{\omega \in \Omega \setminus \Lambda} \pi_\omega \min_{\omega' \in \Lambda} \left(\| \lambda^{\mathrm{P}}_\omega - \lambda^{\mathrm{P}}_{\omega'} \|^r \right). \tag{39}$$

C. Comparison

The computational burden of Algorithm 2 is low as it requires just calculating an appropriate norm of the difference between each pair of price scenario vectors. On the other hand, Algorithm 1 requires the solution of a single-scenario optimization problem per scenario; thus, its computational burden is higher than that pertaining to Algorithm 2. However, the ability of Algorithm 1 to locate the key scenarios that stabilize the expected value of the objective function (as well as its standard deviation) is much higher than that of Algorithm 2. This is so because Algorithm 1 operates on the target measure, the problem objective function, while Algorithm 2 operates on a measure of input parameters, an appropriate norm of the difference between each pair of price scenario vectors. The performances of both algorithms are comprehensively compared in the case study below.

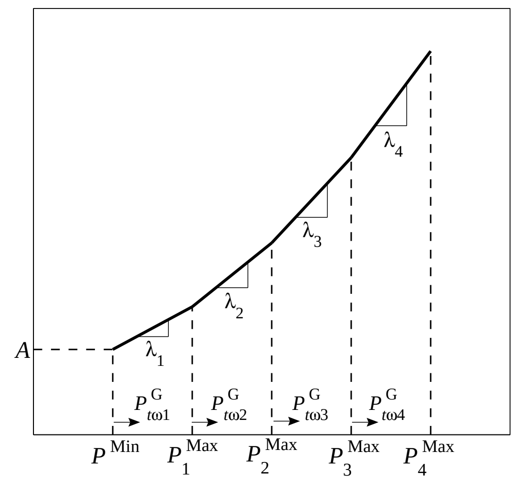

Fig. 1. Piecewise linear production cost.

V. Case Study

A. Data

The study reported below is carried out considering a generating unit with the characteristics provided in Table I.

TABLE I
GENERATION UNIT CHARACTERISTICS

Technical data		Cost data	
P^{Max}(MW)	450	C^{SU} ($)	7000
P^{Min}(MW)	50	C^{SD} ($)	350
R^{U} (MW/h)	300	a ($)	1000
R^{D} (MW/h)	300	b ($/MW)	22.19
U^{T} (h)	6	c ($/MW^2)	0.02
D^{T} (h)	6		

The parameters a, b and c are the coefficients of the quadratic production cost function of the unit $C^{\mathrm{G}}_{t\omega} = au_{t\omega} + bP^{\mathrm{G}}_{t\omega} + c(P^{\mathrm{G}}_{t\omega})^2$. This cost function is approximated by four piecewise blocks whose characteristics are provided in Table II. For the purpose of illustration, this piecewise linear approximation is depicted in Fig. 1. The start-up and shut-down ramp limits of the unit are considered equal to its ramp-up and ramp-down limits, respectively. The unit is assumed to be on-line at the beginning of the study horizon ($u_0 = 1$) with a power output of 250 MW (P^{G}_0 = 250 MW). Furthermore, the unit has been online during 4 hours ($T_0 = 4$).

TABLE II
DATA FOR THE PIECEWISE LINEAR PRODUCTION COST FUNCTION

Block #	P^{Max}_b	λ_b
1	150	26.19
2	250	30.19
3	350	34.19
4	450	38.19

The study horizon spans one week. Data for the 3 forward contracts available for the unit are provided in Table III. The duration of the three forward contracts is the whole week, but they are available during off-peak periods (hours 0:00 a.m. – 7:00 a.m. and 9:00 p.m. – 0.00 a.m.), peak periods (hours 8:00 a.m. – 8:00 p.m.) and all periods, depending on contract type being A, B or C respectively.

TABLE III
FORWARD CONTRACT PRICE AND QUANTITY DATA

Contract #	$P^{\mathrm{S,Max}}_c$	λ_c	Contract type
1	100	42.54	A
2	100	45.82	B
3	100	43.81	C

To describe pool price volatility through the whole week, 288 equiprobable scenarios are initially considered (see Fig. 2). Each scenario involves 168 prices values, one for each hour of the week. The bold line in Fig. 2 represents the average pool-price for each hour.

B. Results and discussion

Fig. 3 provides the expected profit evolution with the number of scenarios for Algorithms 1 and 2, while Fig. 4 provides the evolution of the standard deviation of the profit with the number of scenarios for Algorithms 1 and 2.

From Fig. 3 and Fig. 4 it can be concluded that Algorithm 1 stabilizes both the expected profit and its standard deviation much faster than Algorithm 2. Moreover, Algorithm 1 achieves a good solution in terms of expected profit and profit standard deviation with just a few scenarios (e.g., 30), and does not exhibit an oscillatory behavior. On the contrary, Algorithm 2 requires a rather high number of scenarios to achieve a good solution (e.g., 100) and suffers an unpredictable oscillatory behavior.

Fig. 5 provides different probability mass functions (pmf) of the profit for different number of scenarios and Algorithms 1 and 2. The profit is given in per unit with respect to the

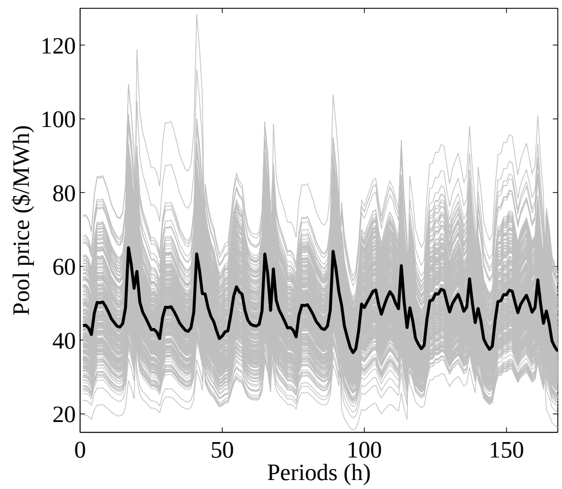

Fig. 2. Pool price scenarios.

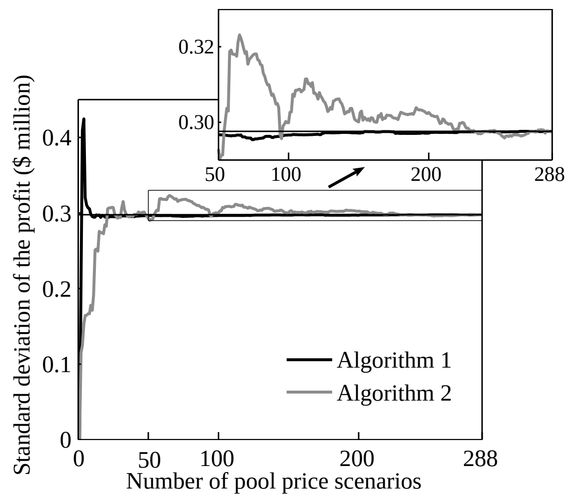

Fig. 4. Profit standard deviation as a function of the number of scenarios.

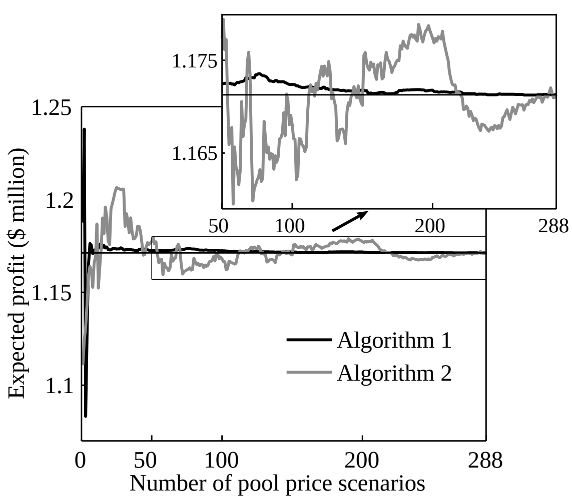

Fig. 3. Expected profit as a function of the number of scenarios.

maximum profit, and to represent the pmf, profit intervals of 1/15 per unit are used. Each bar provides the sum of the probabilities of the scenarios yielding a profit within the interval defined by the base of the bar. The upper subplot provides the profit pmf considering all scenarios (288). The two central subplots are similar to the upper one but considering 30 and 100 scenarios, respectively, obtained using Algorithm 1. The two lower subplots provide analogous information as the central subplots but using Algorithm 2 to reduce the number of scenarios. Note that the superiority of Algorithm 1 with respect to Algorithm 2 in reproducing the shape of the profit pmf is apparent. Note also that the scale of the vertical axis of all plots are identical but the left bottom one.

Fig. 6 shows the evolution of the distance, in per unit with respect to the maximum distance, of the reduced scenario set to the original scenario set as a function of the number of scenarios for Algorithms 1 and 2, respectively, according to expressions (37) and (39). Algorithm 1 results in a much faster distance reduction than Algorithm 2. Algorithm 1 reaches 20% distance between the original and the reduced sets with fewer than 10 scenarios and 10% with less than 20 scenarios, while Algorithm 2 requires more than 150 and 200 scenarios, respectively, to reach these distances.

Using a Linux-based server with two processors clocking at 2.4 GHz and 8 GB of RAM and CPLEX 10.2.0 under GAMS [21], the CPU time required by Algorithm 1 to calculate the function $c(\cdot)$ defined by (36) for all pairs of scenarios is 241 seconds. The corresponding time by Algorithm 2 according to (38) is negligible. Although the computational burden of Algorithm 1 is higher than that of Algorithm 2, CPU time required by Algorithm 1 is very reasonable for the problem addressed. Furthermore, note that the computation time required by Algorithm 1 can be reduced as much as needed by using parallel computation to solve the set of mono-scenario problems.

VI. Conclusions

This paper proposes a procedure to reduce efficiently the number of scenarios required to represent adequately the uncertainty in trading problems pertaining to electricity markets. The technique is valid for scenarios involving both continuous and discrete variables. An example related to a producer involved in both the futures market and the pool is used for the purpose of illustration.

Exhaustive simulation results show the superiority of the proposed technique with respect to previous techniques in locating a reduced number of scenarios that efficaciously represent the uncertainty involved. The technique proposed has a higher computational burden than previously reported techniques, but this burden is kept under reasonable levels.

Appendix

Fast forward algorithm [18]

The scenario reduction techniques previously described are both based on the *fast-forward* algorithm provided in [18].

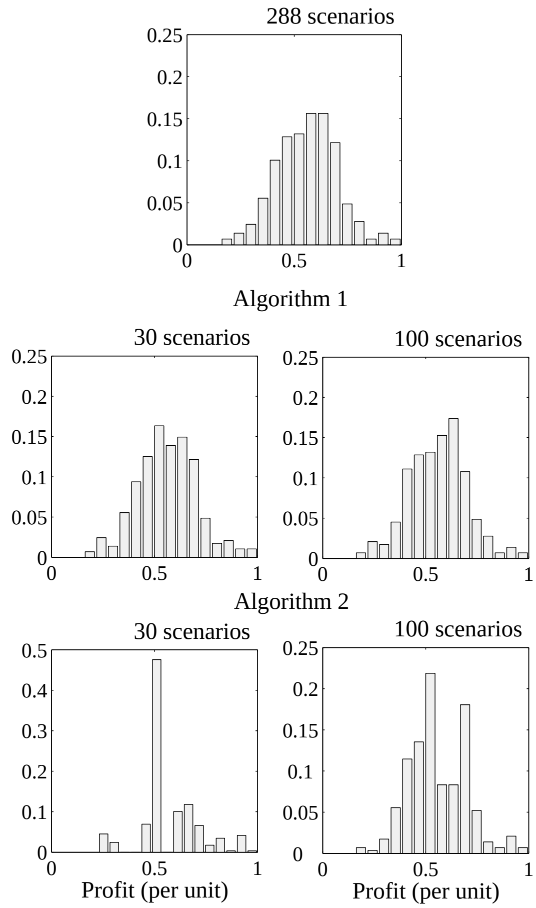

Fig. 5. Probability mass function of the profit for different number of scenarios.

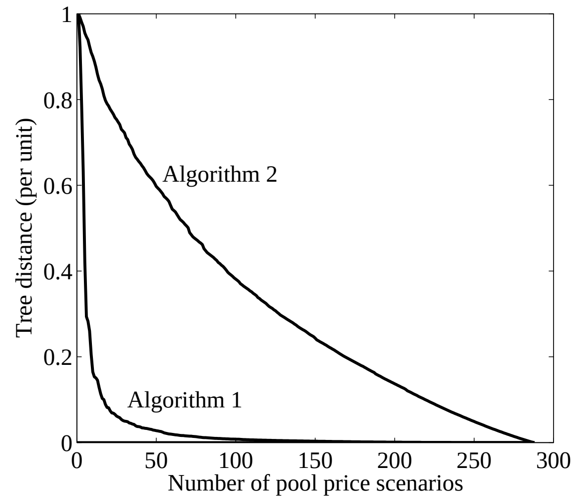

Fig. 6. Distances between the original and the reduced scenario sets for Algorithms 1 (expression (37)) and 2 (expression (39)) as a function of the number of scenarios.

Denoting the original scenario set as Ω and the reduced scenario set, target of the search, as Ω_S^*, the fast-forward algorithm works as follows:

- *Step 0*: The iterative process begins with the choice of a starting scenario, that is, with the scenario from which the reduced scenario set is built.
  Mathematically, the starting scenario (ω_0) is obtained from

$$\omega_0 = \arg\{\min_{\omega' \in \Omega} \sum_{\omega \in \Omega} \pi_\omega \mathrm{c}(\omega, \omega')\}. \tag{40}$$

  The starting scenario can be interpreted as the most equidistant one from the rest. In other words, this first scenario can be seen as the *average* scenario.

- *Step i*: Starting from the scenario selected in step 0, in each iteration a new scenario is added to the reduced scenario set until it is considered to be close enough to the original one. For this purpose, this selection is carried out using

$$\omega_i = \arg\left\{ \min_{\omega' \in \Omega_J^{[i-1]}} \sum_{\omega \in \Omega_J^{[i-1]} \setminus \{\omega'\}} \pi_\omega \min_{\omega'' \in \Omega_S^{[i-1]}} \mathrm{c}(\omega, \omega'') \right\}, \tag{41}$$

  where $\Omega_J^{[i]}$ represents the set composed of those scenarios which have not been selected in the first i steps of the algorithm and $\Omega_S^{[i]}$ symbolizes the set comprising the selected scenarios until step i. Note that $\Omega_J^{[i]} \cup \Omega_S^{[i]} = \Omega$, $\Omega_J^{[0]} = \Omega$, $\Omega_S^{[0]} = \emptyset$, $\Omega_J^{[i]} = \Omega_J^{[i-1]} \setminus \{\omega_i\}$ and $\Omega_S^{[i]} = \Omega_S^{[i-1]} \cup \{\omega_i\}$.
  This step is repeated $N_{\Omega_S^*} - 1$ times, where $N_{\Omega_S^*}$ is the number of scenarios comprising the reduced set Ω_S^*.

- *Step* $N_{\Omega_S^*} + 1$: In this step, an optimal redistribution of probabilities is carried out. It consists of adding the probabilities of those scenarios which have not been finally selected $\left(\omega \in \Omega_J^*, \text{ with } \Omega_J^* \cup \Omega_S^* = \Omega\right)$ to the probabilities of those comprising the reduced set ($\omega \in \Omega_S^*$).
  Mathematically, this redistribution of probabilities can be accomplished as follows:

$$\pi_\omega^* = \pi_\omega + \sum_{\omega' \in J(\omega)} \pi_{\omega'}, \tag{42}$$

  where $J(\omega)$ is defined as the set of scenarios $\omega' \in \Omega_J^*$ such that $\omega = \arg \min_{\omega'' \in \Omega_S^*} \mathrm{c}(\omega'', \omega')$.
  In other words, the probability of each non-selected scenario is aggregated to the probability of the closest selected scenario according to the function $c(\cdot)$. Thus, the reduced scenario set is made up of the scenarios $w \in \Omega_S^*$ with associated probability π_ω^*.

ACKNOWLEDGMENT

We are thankful to A. Canoyra, Á. Caballero and A. de Andrés, from UNION FENOSA Generación for insightful comments and relevant observations linking our models to the actual world.

REFERENCES

[1] A. J. Conejo, R. García-Bertrand, M. Carrión, A. Caballero and A. de Andrés, "Optimal Involvement in Forward Markets of a Power Producer," to appear in *IEEE Trans. Power Syst.*

[2] L. Wu, M. Shahidehpour and T. Li, "GENCO's Risk-Based Maintenance Outage Scheduling," *IEEE Trans. Power Syst.*, vol. 23, no. 1, pp. 127–136, Feb 2008.

[3] M. Carrión, A. J. Conejo, and J. M. Arroyo, "Forward contracting and selling price determination for a retailer," *IEEE Trans. Power Syst.*, vol. 22, pp. 2105–2114, Nov 2007.

[4] T. Li and M. Shahidehpour, "Risk-Constrained Generation Asset Arbitrage in Power Systems," *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp. 1130–1139, Aug 2007.

[5] R. Hochreiter and G. C. Pflug, "Financial scenario generation for stochastic multi-stage decision processes as facility location problems," in *Ann. Oper. Res.*, vol. 152, pp. 257–272, Jul 2007.

[6] M. Carrión, A. B. Philpott, A. J. Conejo, and J. M. Arroyo, "A stochastic programming approach to electric energy procurement for large consumers," *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 744–754, May 2007.

[7] T. Li, M. Shahidehpour and Z. Li, "Risk-Constrained Bidding Strategy With Stochastic Unit Commitment," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 449–458, Feb 2007.

[8] S. Sen, L. Yu and T. Genc, "A Stochastic Programming Approach to Power Portfolio Optimization," in *Operations Research*, vol. 54, pp. 55–72, Jan 2006.

[9] M. A. Plazas, A. Conejo and F. J. Prieto, "Multimarket Optimal Bidding for a Power Producer," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 2041–2050, Nov 2005.

[10] J. Cabero, A. Baíllo, S. Cerisola, M. Ventosa, A. García-Alcalde, F. Perán and G. Relano, "A Medium-Term Integrated Risk Management Model for a Hydrothermal Generation Company ," *IEEE Trans. Power Syst.*, vol. 20, pp. 1379–1388, Aug 2005.

[11] J. R. Birge and F. Louveaux, *Introduction to stochastic programming*. New York: Springer-Verlag, 1997.

[12] M. Kaut and S. W. Wallace, "Evaluation of scenario-generation methods for stochastic programming," in *SPEPS*, Paper 14–2003, available in http://edoc.hu-berlin.de/browsing/speps/.

[13] M. Olsson and L. Söder, "Generation of regulating power price scenarios," in *Proceedings of the 8th International Conference Probabilistic Methods Applied to Power Systems PMAPS 2004*, pp. 26–31, 2004.

[14] K. Høyland and S. W. Wallace, "Generating Scenario Trees for Multistage Decision Problems," in *Management Science*, vol. 47 n. 2, pp.295–307, Feb 2001.

[15] G. C. Pflug, "Scenario tree generation for multiperiod financial optimization by optimal discretization," in *Mathematical Programming*, pp. 251–271, 2001.

[16] H. Heitsch and W. Römisch, "Scenario reduction algorithms in stochastic programming," in *Computational Optimization and Applications*, vol. 24, pp. 187-206, 2003.

[17] J. Dupačová, N. Gröwe-Kuska and W. Römisch, "Scenario reduction in stochastic programming: An approach using probability metrics," in *Mathematical Programming*, Ser. A 95, pp. 493–511, 2003.

[18] N. Gröwe-Kuska, H. Heitsch and W. Römisch, "Scenario reduction and scenario tree construction for power management problems," in *Power Tech Conference Proceeding IEEE*, Bologna, June 2003.

[19] J. Dupačová, G. Consigli and S. W. Wallace, "Scenarios for multistage stochastic programs," in *Annals of Operations Research*, vol. 100, pp. 25–53, 2000.

[20] M. Carrión and J. M. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1371–1378, Aug 2006.

[21] A. Brooke, D. Kendrick, A. Meeraus, R. Raman, and R. E. Rosenthal, *GAMS, a user's guide*. Washington: GAMS Development Corporation, 1998.

Salvador Pineda (S'07) received the Ingeniero Industrial degree from the Universidad de Malaga, Malaga, Spain, in 2006.

He is currently working toward the Ph.D. degree at the Universidad de Castilla-La Mancha. His research interests include stochastic programming and electricity markets.

Juan M. Morales (S'07) received the Ingeniero Industrial degree from the Universidad de Málaga, Spain, in 2006. He is currently working toward the Ph.D. degree at the Universidad de Castilla-La Mancha.

His research interests are in the fields of power systems economics, reliability, stochastic programming and electricity markets.

Antonio J. Conejo (F'04) received the M.S. degree from MIT, Cambridge, MA, in 1987, and a Ph.D. degree from the Royal Institute of Technology, Stockholm, Sweden in 1990. He is currently a full Professor at the Universidad de Castilla – La Mancha, Ciudad Real, Spain.

His research interests include control, operations, planning and economics of electric energy systems, as well as statistics and optimization theory and its applications.

Miguel Carrión (S'04) received the Ingeniero Industrial degree from the Universidad de Castilla-La Mancha, Ciudad Real, Spain, in 2003.

He is currently working toward the Ph.D. degree at the Universidad de Castilla-La Mancha. His research interests are in the fields of power systems economics, stochastic programming and electricity markets.