

Options to Hedge Against Producer Risks in Electricity Markets

**Salvador Pineda
Antonio Conejo
November 2010**



**UNIVERSIDAD DE
CASTILLA-LA MANCHA**

- **Introduction**
- **Model**
- **Case study**
- **Conclusions**

➤ **Introduction**

➤ **Model**

➤ **Case study**

➤ **Conclusions**

Introduction

- How do electricity options work?
- How can electricity options be modeled?
- How do electricity options reduce price risk?
- How do electricity options reduce availability risk?
- When is an option contract more profitable than a forward contract?

Introduction



Pool market
(price risk)



Introduction



Pool market
(price risk)



Futures market
(fixed price)

Introduction



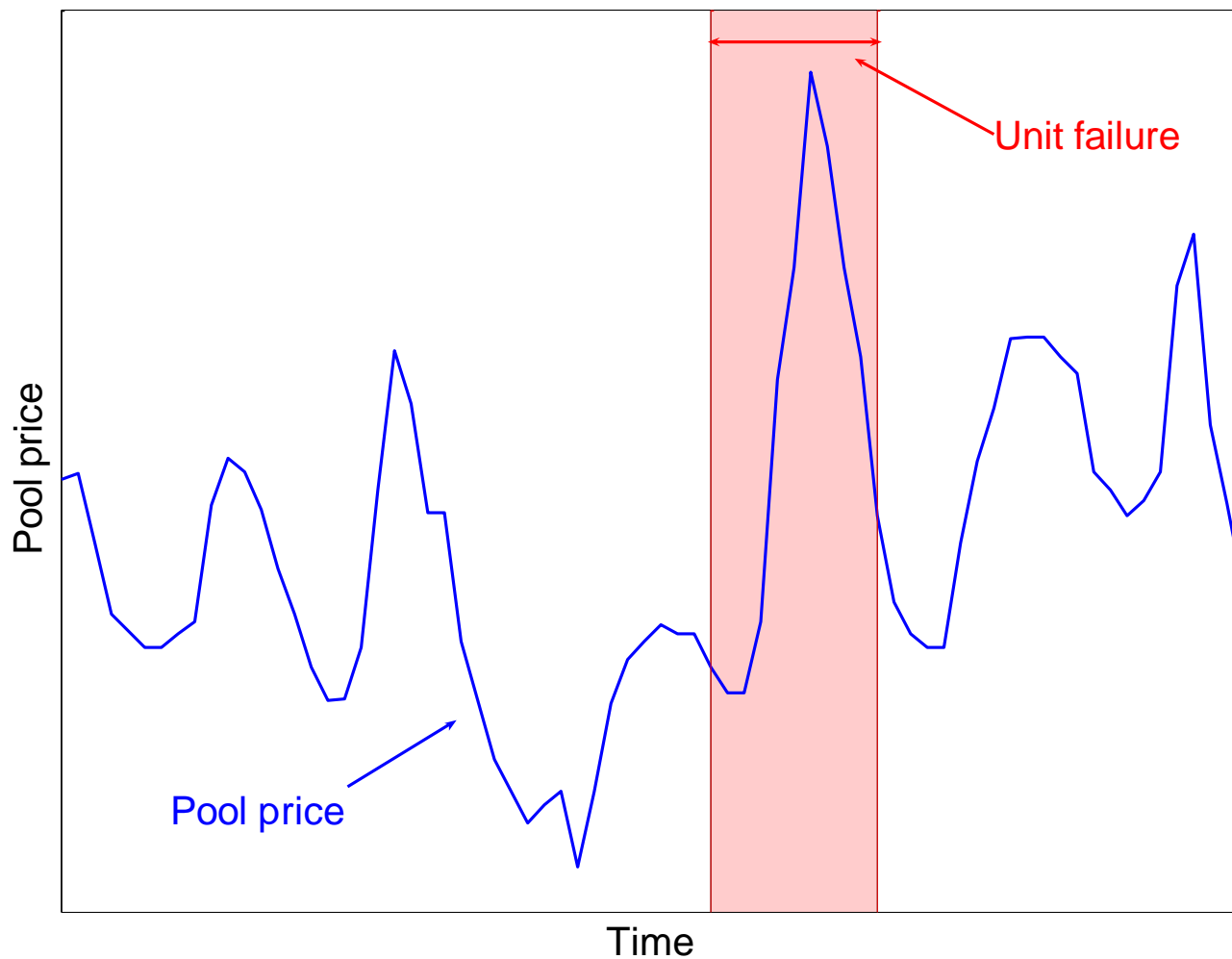
Pool market
(price risk)



Production unit
(availability risk)



Introduction



Introduction



Pool market
(price risk)



Production unit
(availability risk)



Futures market
(fixed price)



Introduction

Forward contract

- Fixed price
- Obligation to buy/sell
- No cost

Option contract

- Fixed price
- Right to buy/sell
- Option price

Introduction

Option contract

- **Two positions (buyer and seller of the option)**
- **Put options (right to sell)**
- **Call options (right to buy)**
- **European options (exercised at expiration)**
- **American options (exercised any time until expiration)**

Introduction

Option contract

- Two positions (**buyer** and seller of the option)
- **Put options** (right to sell)
- Call options (right to buy)
- **European options** (exercised at expiration)
- American options (exercised any time until expiration)

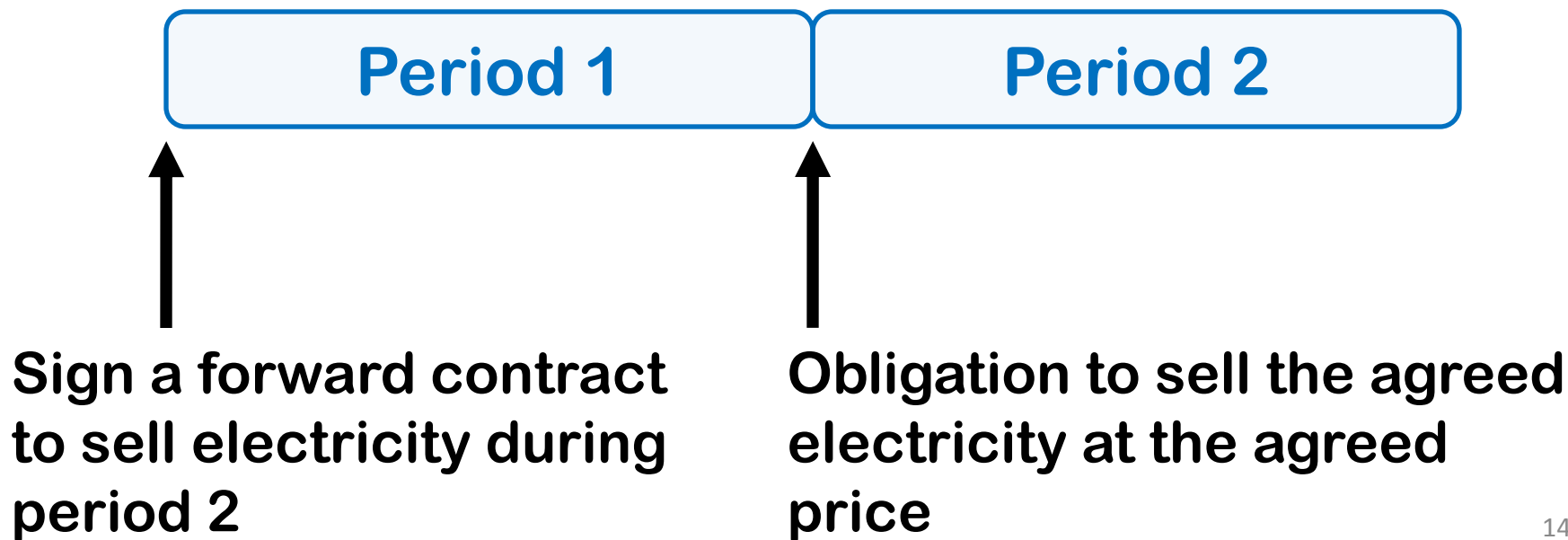
Introduction

Option contract

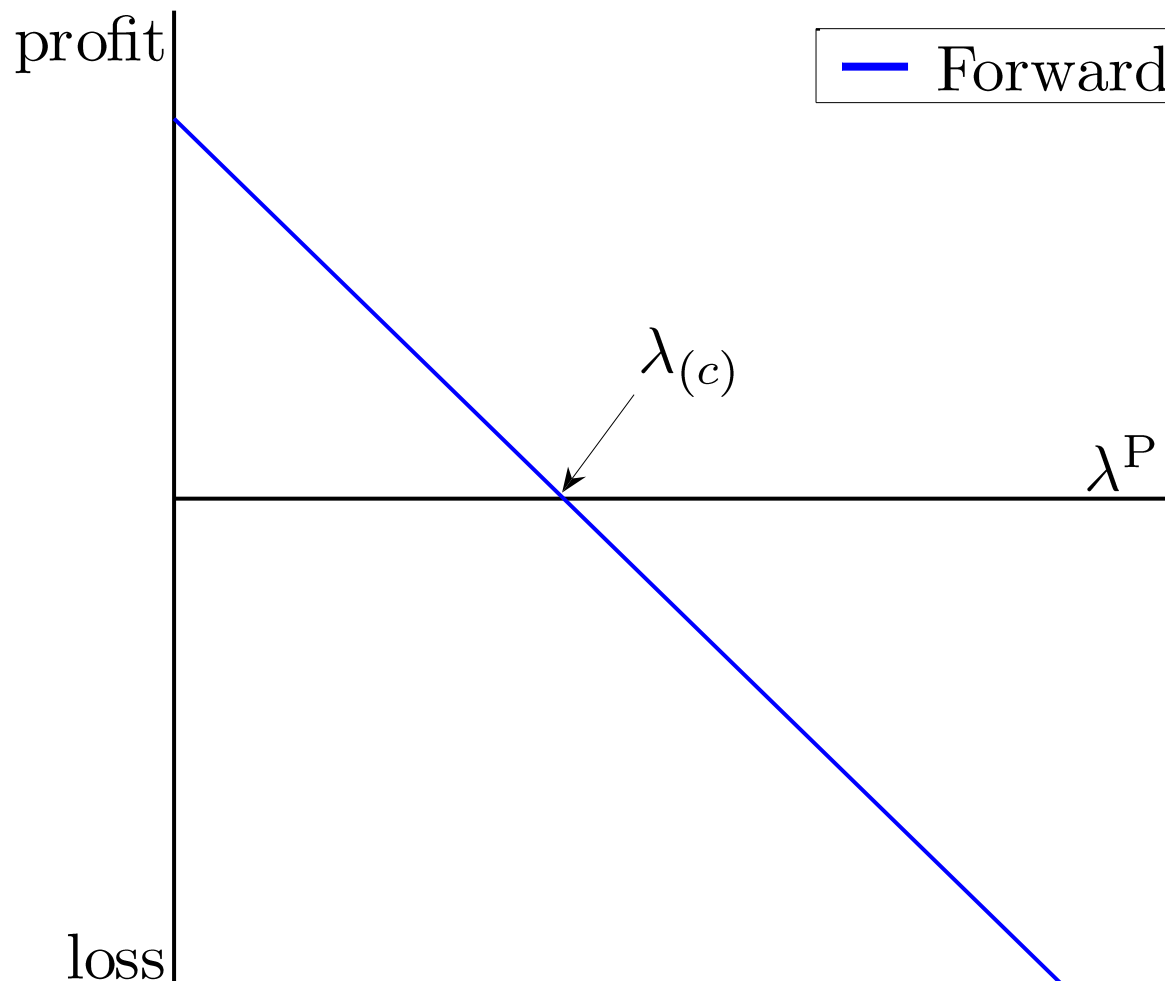
Name	Best Bid	Best Ask	No. of Contr.	Last Price	Abs. Change	Last Time	Last Vol.	Settl. Price
<u>Nov-10</u>	-	-	-	-	-	-	-	48.52
▼ <u>Dec-10</u>	47.00	47.15	85	-	-	-	-	47.07
P 4300	-	-	-	-	-	-	-	0.058
P 4400	-	-	-	-	-	-	-	0.134
P 4500	-	-	-	-	-	-	-	0.281
P 4600	-	-	-	-	-	-	-	0.542
P 4700	-	-	-	-	-	-	-	0.955
P 4800	-	-	-	-	-	-	-	1.536
P 4900	-	-	-	-	-	-	-	2.267
P 5000	-	-	-	-	-	-	-	3.110
P 5100	-	-	-	-	-	-	-	4.024

Introduction

Basic idea

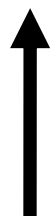


Introduction



Introduction

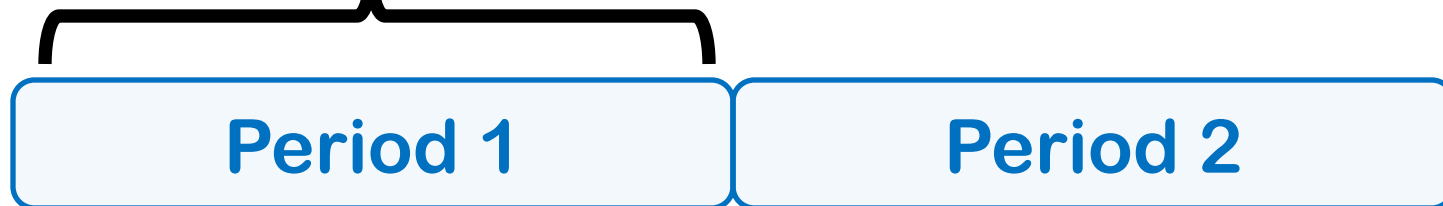
Basic idea



**Sign a put option
to sell electricity
during period 2**

Introduction

Basic idea

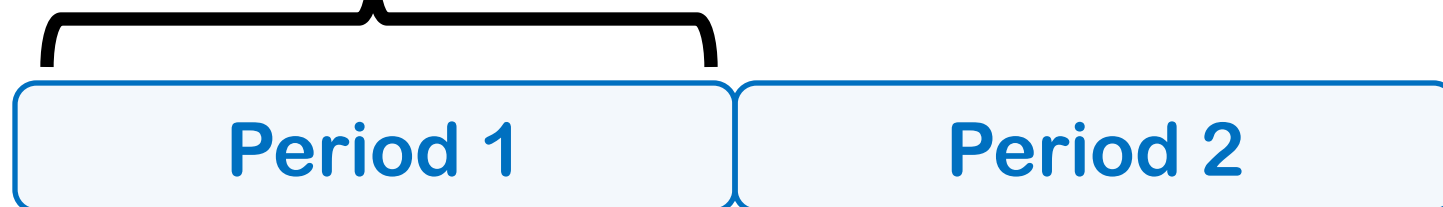


Sign a put option to sell electricity during period 2

Option is exercised to hedge against low prices

Introduction

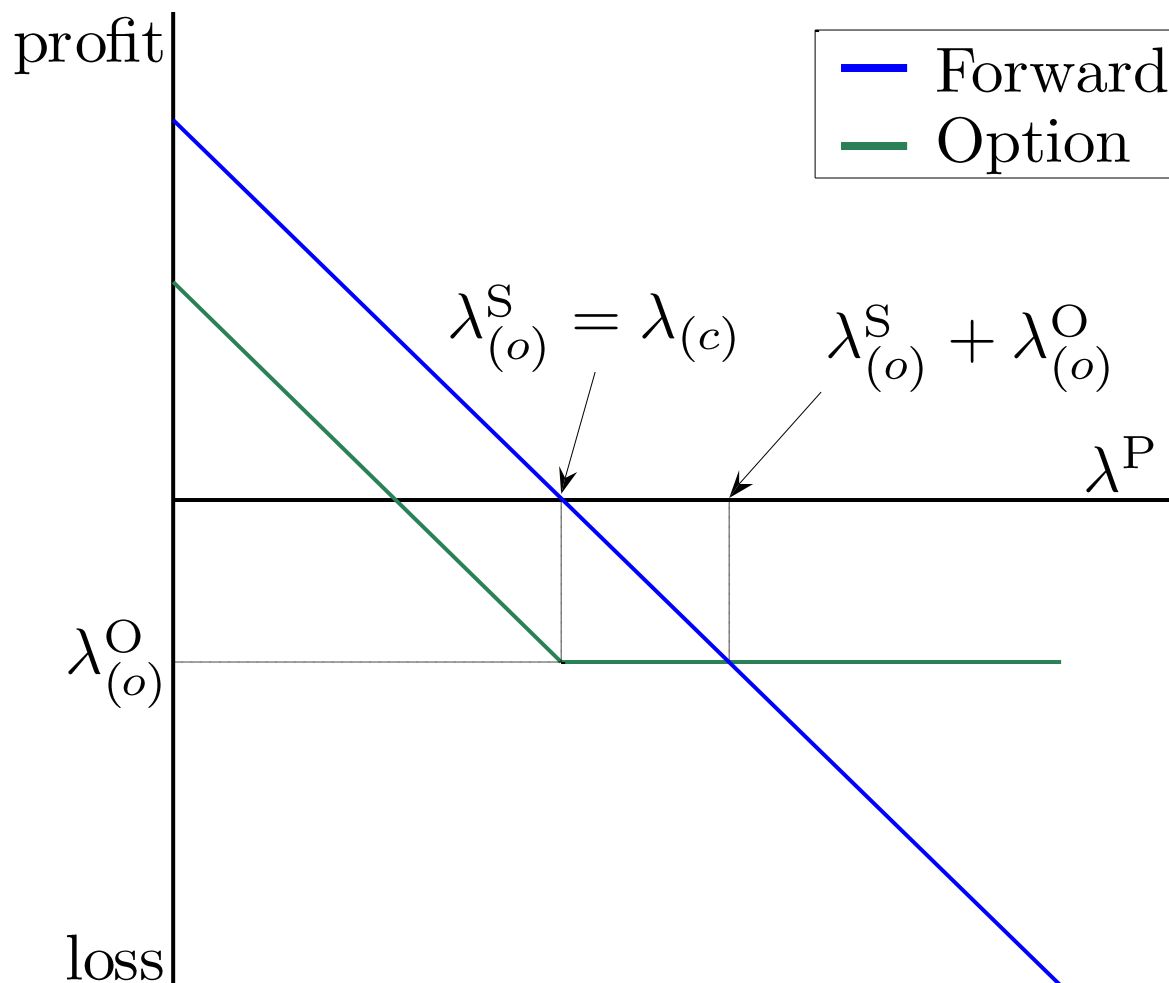
Basic idea



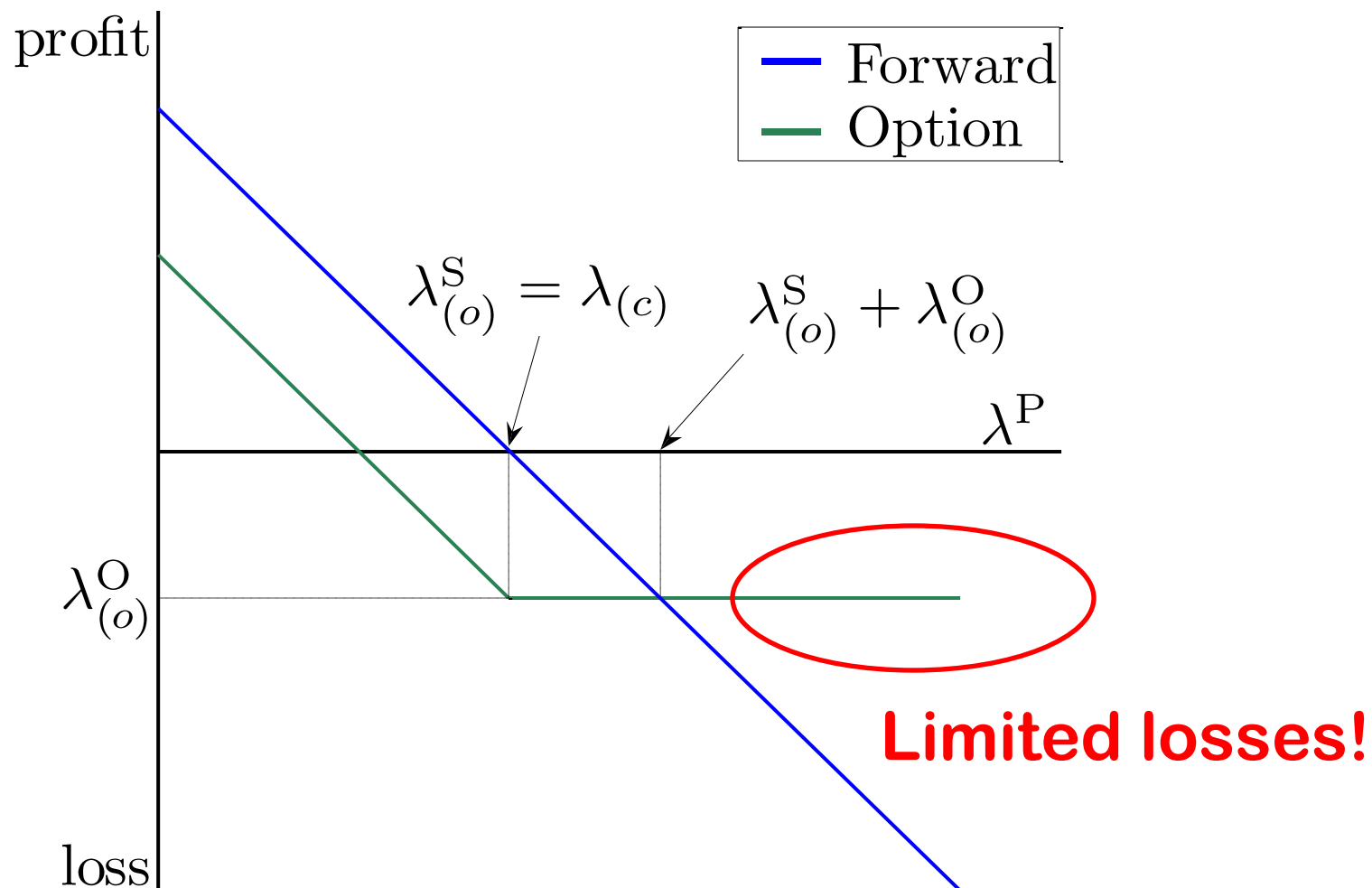
Sign a put option
to sell electricity
during period 2

Option is not exercised
to obtain high profits

Introduction

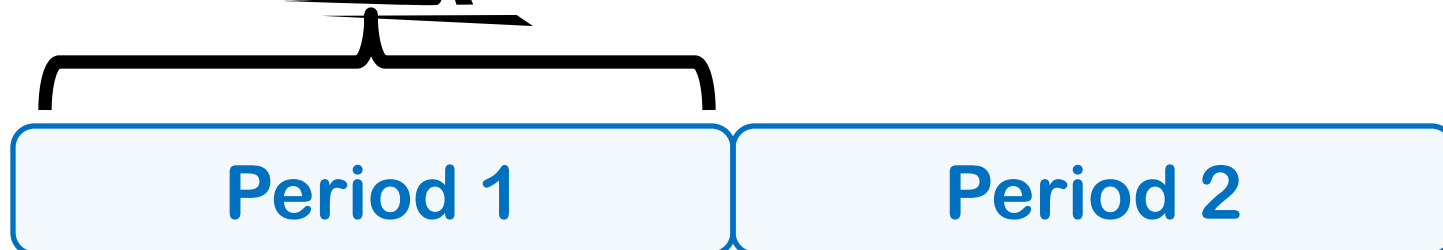


Introduction



Introduction

Basic idea

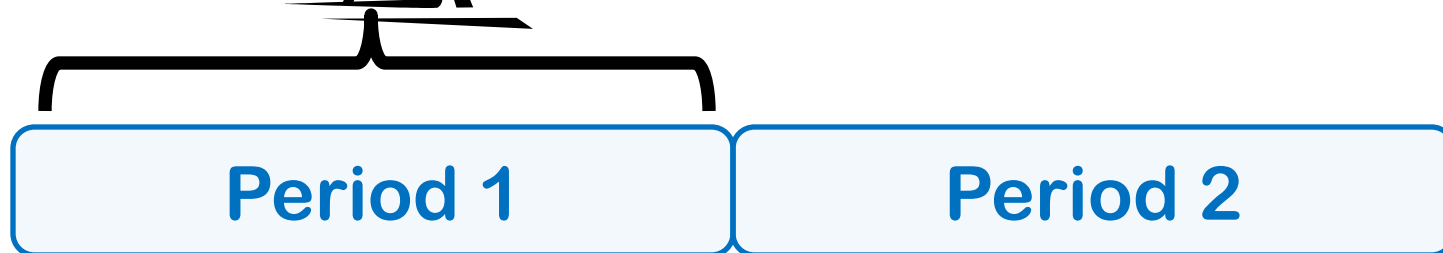


Sign a put option
to sell electricity
during period 2

Option is not exercised
to hedge against unit
failures

Introduction


Basic idea



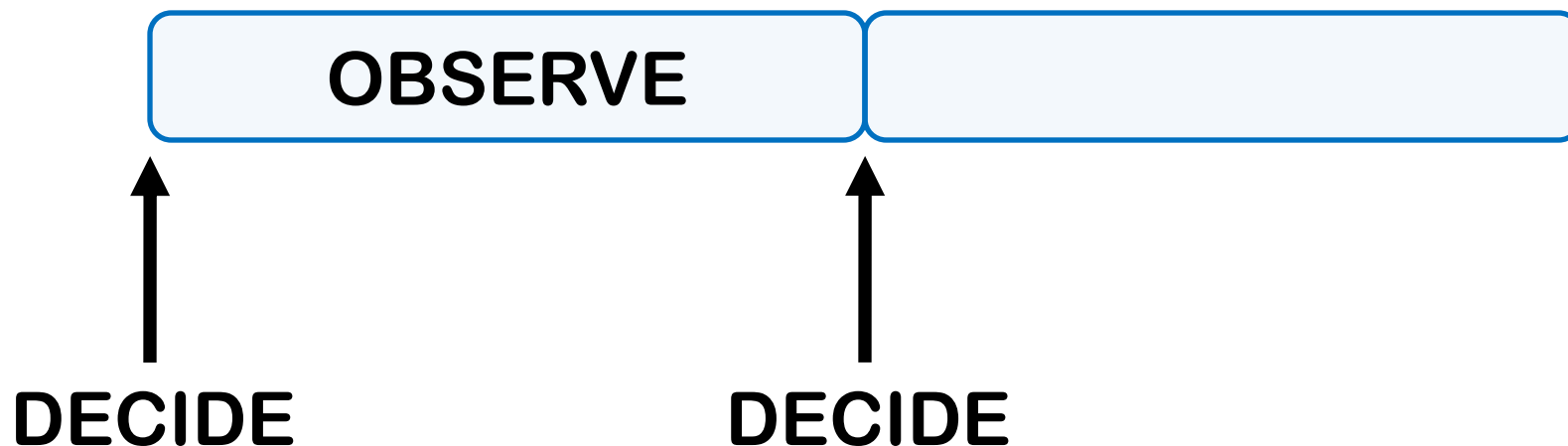
Sign a put option
to sell electricity
during period 2

Option is exercised

Introduction

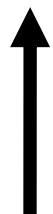
- How do electricity options work? 
- How can electricity options be modeled?
- How do electricity options reduce price risk?
- How do electricity options reduce availability risk?
- When is an option contract more profitable than a forward contract?

Introduction



**Multi-stage
stochastic
programming**

Introduction



First-stage decisions:

- Option purchase
- Forward contracting

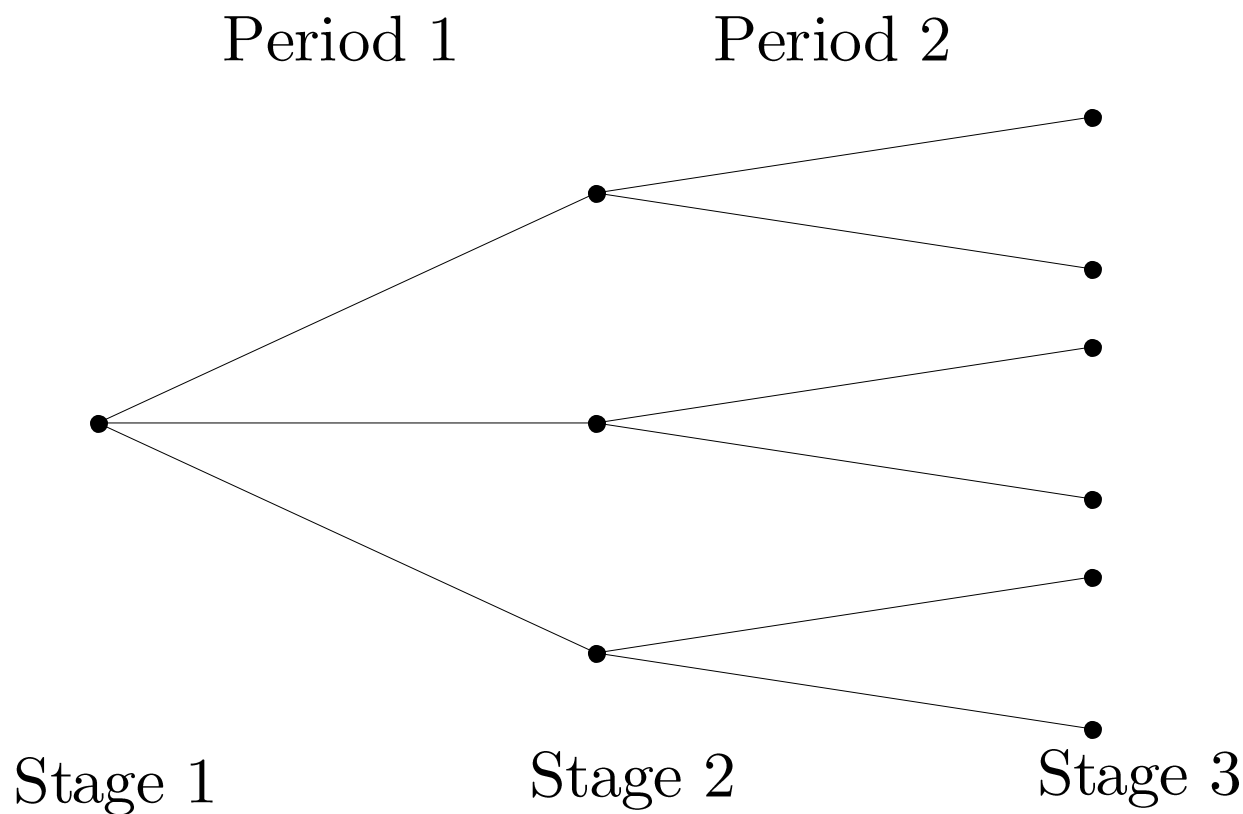


Second-stage decision:

- Option exercise

Introduction

Scenario tree



Introduction

- How do electricity options work? ✓
- How can electricity options be modeled? ✓
- How do electricity options reduce price risk?
- How do electricity options reduce availability risk?
- When is an option contract more profitable than a forward contract?

Analyze electricity options to manage the two main risks faced by power producers: price and availability risks.

- **Introduction**
- **Model**
- **Case study**
- **Conclusions**

➤ Introduction

➤ **Model**

➤ Case study

➤ Conclusions

Model

Sources of uncertainty

Pool prices

Unit availability

**Forward
contracts**

**Option
contracts**

Model

Sources of uncertainty

Pool prices

Unit availability

Forward
contracts

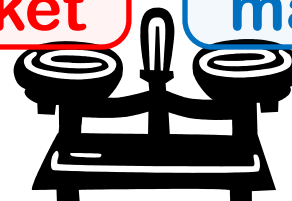
Option
contracts



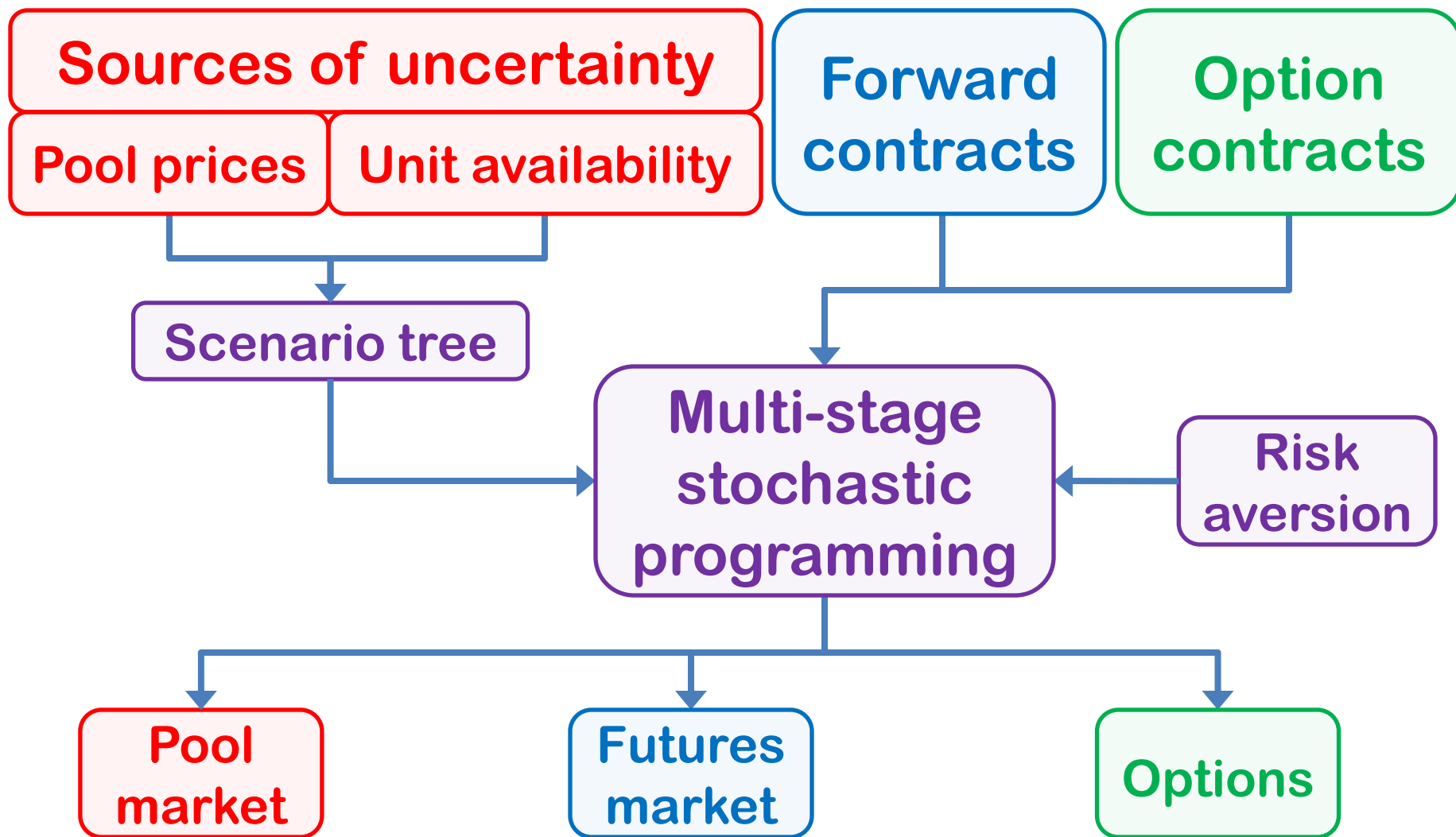
Pool
market

Options

Futures
market



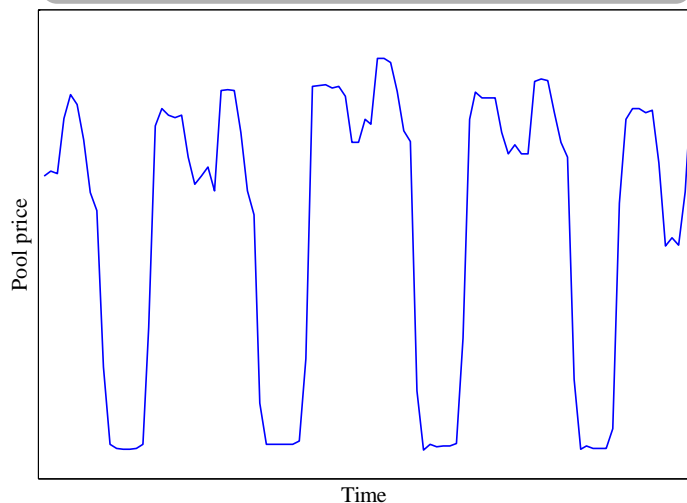
Model



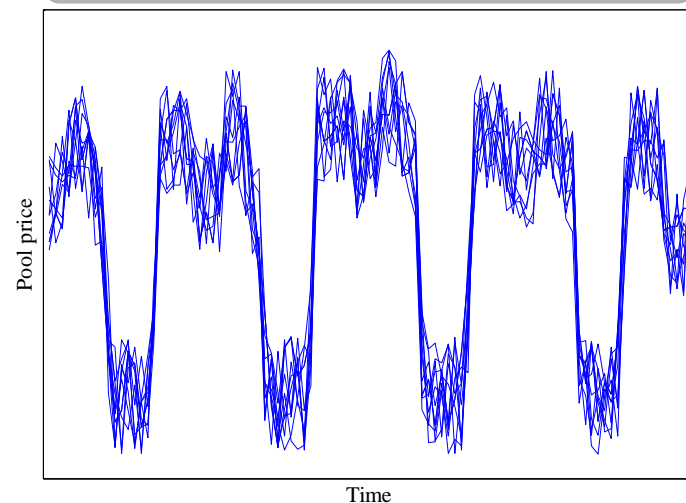
Model

Pool prices

Historical data



Pool price scenarios

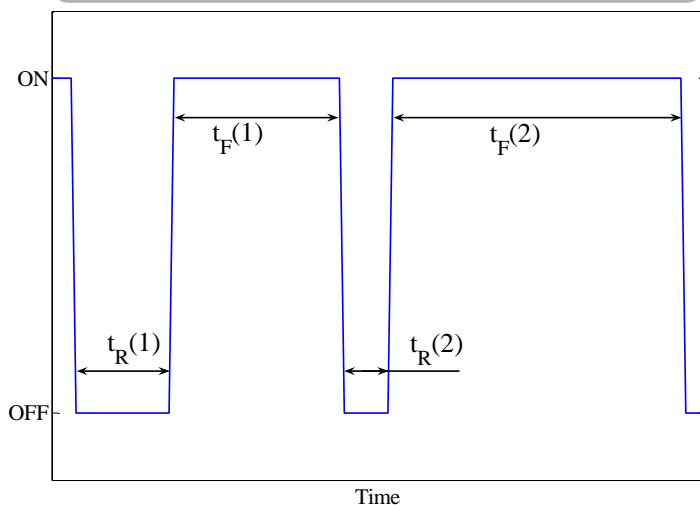


Time series
model

Model

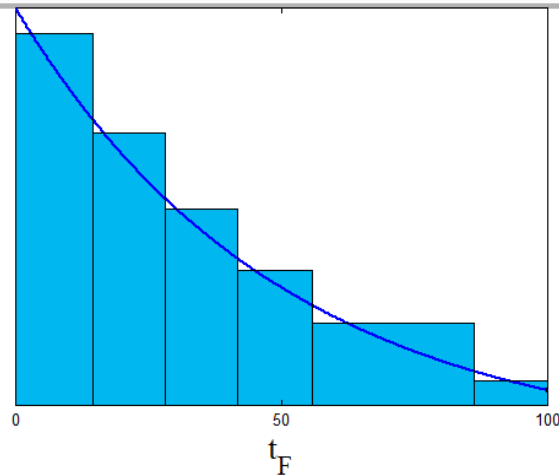
Unit availability

Historical data



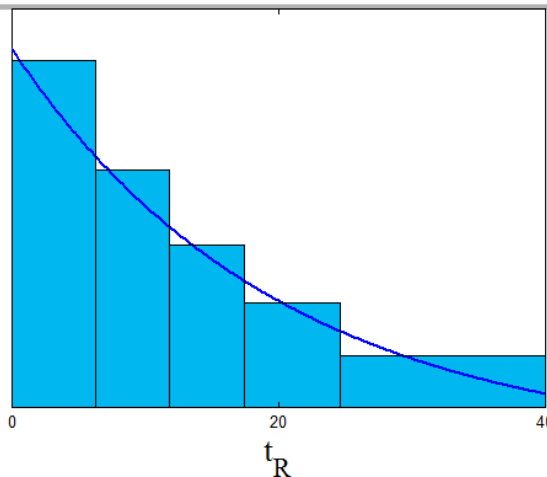
$$\text{FOR}(\%) = \frac{\text{MTTR}}{\text{MTTR} + \text{MTTF}}$$

Failure time series = 20, 35,



$\sim \exp(\text{MTTF})$

Repair time series = 12, 8,



$\sim \exp(\text{MTTR})$

Model

Unit availability

$$\left. \begin{array}{l} t_F \sim \exp(MTTF) \\ t_R \sim \exp(MTTR) \end{array} \right\} p(u_t = \mathbf{1}) = \frac{\mu}{\lambda + \mu} + \frac{\mu(u_0 - \mathbf{1}) + \lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

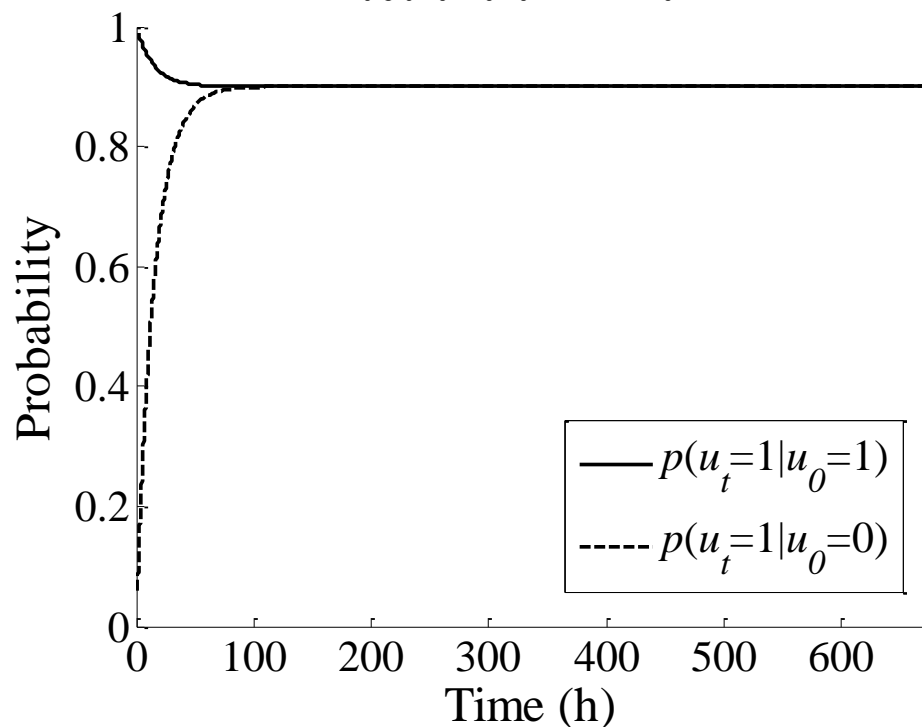
$$\lambda = \frac{1}{MTTF} \quad \mu = \frac{1}{MTTR}$$

Model

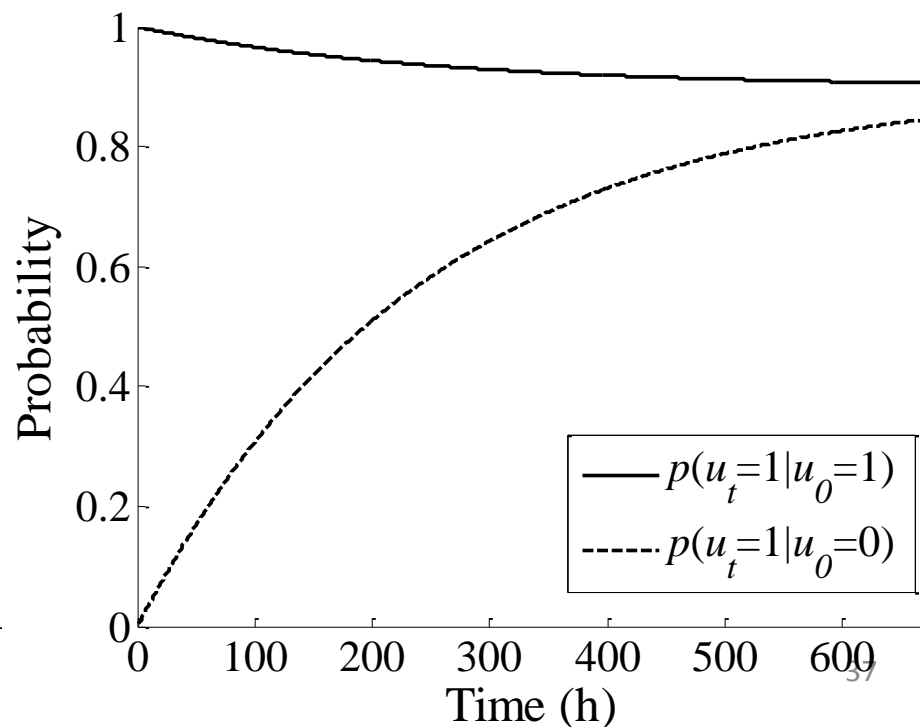
Unit availability

$$\left. \begin{array}{l} t_F \sim \exp(\text{MTTF}) \\ t_R \sim \exp(\text{MTTR}) \end{array} \right\} p(u_t = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu(u_0 - 1) + \lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

MTTF << T



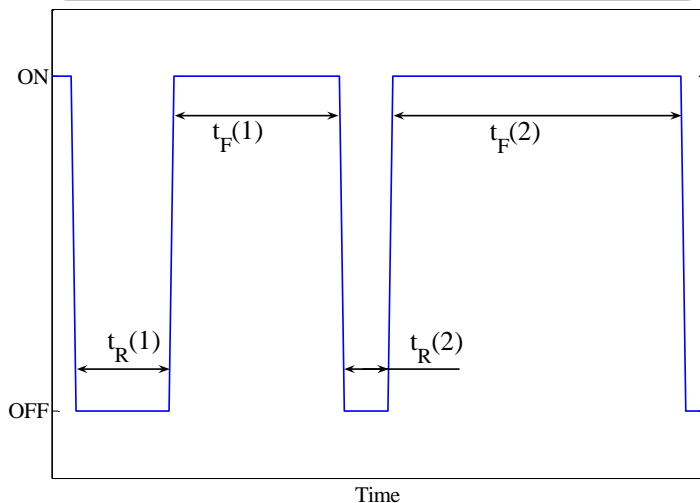
MTTF >> T



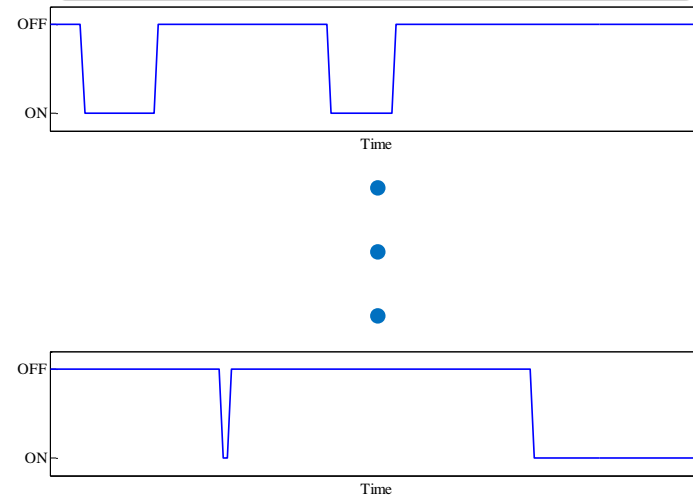
Model

Unit availability

Historical data



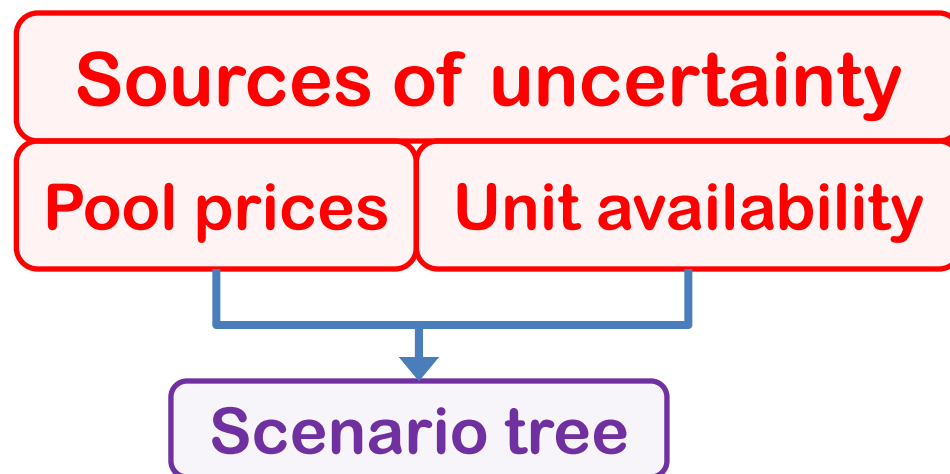
Availability scenarios



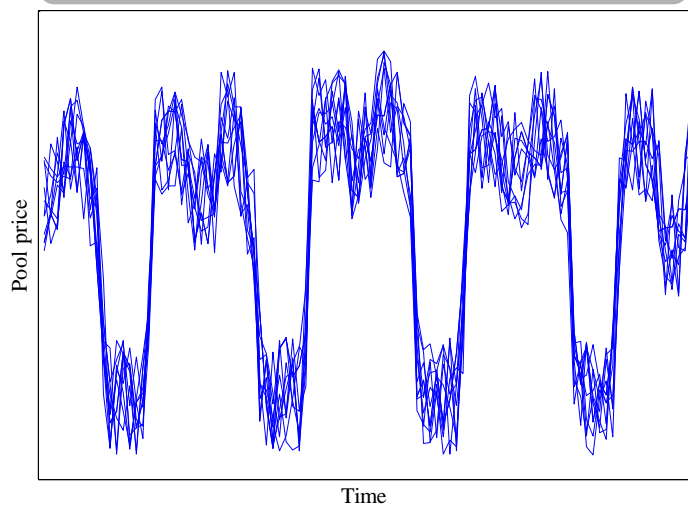
$$t_F \sim \exp(\text{MTTF})$$

$$t_R \sim \exp(\text{MTTR})$$

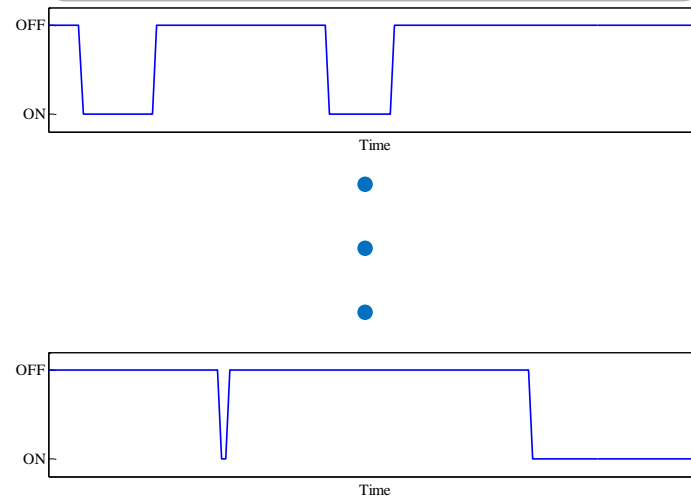
Model



Pool price scenarios



Availability scenarios



Forward contracts

- Specified quantity (MW)
- Fixed price
- Future delivery period

Option contract

- Specified quantity (physical delivery)
- Strike price
- Option price
- Time period covered
- Time to decide whether it is exercised

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

Constraints

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints

Model

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

$$CVaR_{\alpha} = \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_W} \pi_{\omega} \eta_{\omega}$$

$$- \text{profit}_{\omega} + \zeta - \eta_{\omega} \leq 0$$

$$\eta_{\omega} \geq 0$$

Constraints

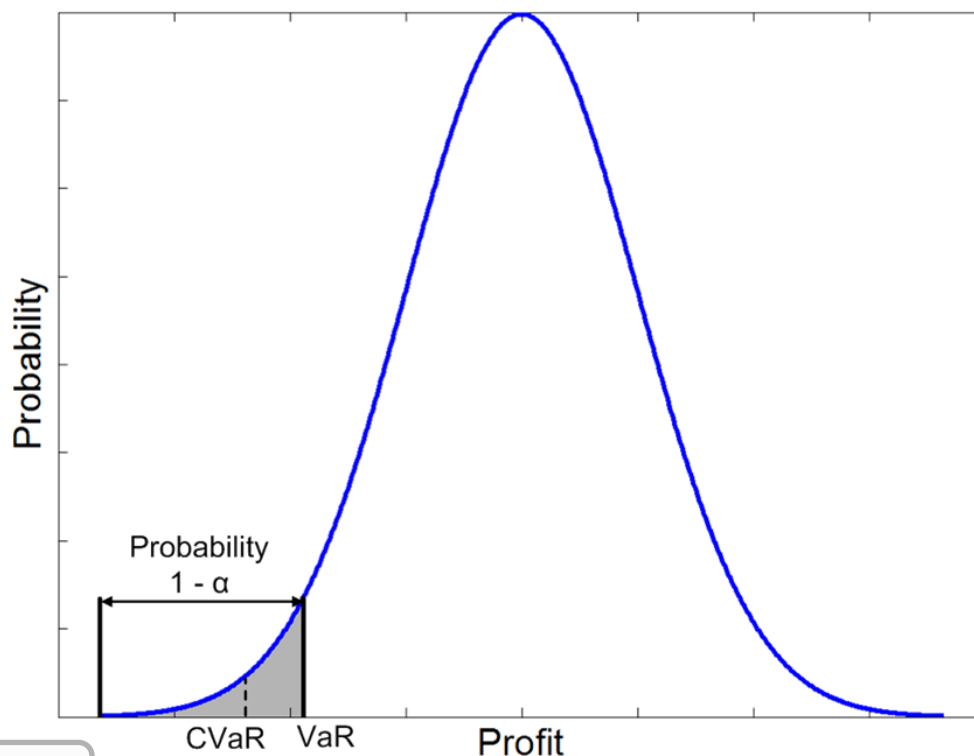
Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints

Risk aversion



Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

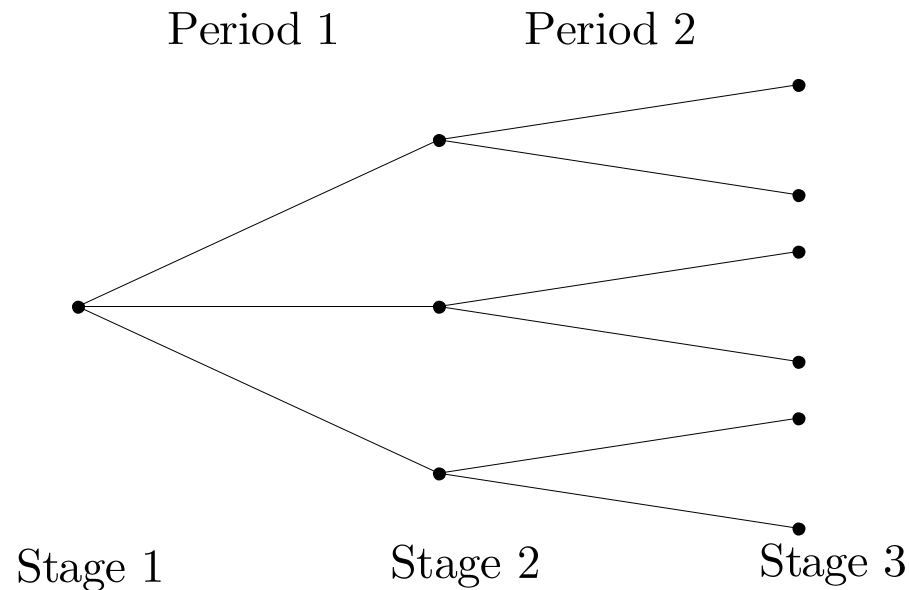
Constraints

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints



Second-stage decisions are made knowing the scenario realization during period 1 but still facing uncertainty related to period 2.

- **Introduction**
- **Model**
- **Case study**
- **Conclusions**

Outline

➤ Introduction

➤ Model

➤ Case study

➤ Conclusions

Case study

- 2 months (P1 = first month, P2 = second month)
- Generating unit
 - $P_{\max} = 350$ MW, $P_{\min} = 50$ MW, $C = 12$ €/MWh (linear)
 - Three FOR values: 0, 5 and 10%
- 30 pool-price scenarios (ARIMA)
- 30 availability scenarios for each value of FOR
- 2 forward contracts, one for each month
- 1 put option spanning the second month

Case study

- (a) sell 350MW during the second month at 21€/MWh. No re-trading in stage 2.
- (b) buy a put option to sell 350MW during the second month at 21€/MWh. Option price = 0.1€/MWh

Case study

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499

- (a) sell 350MW during the second month at 21€/MWh. No re-trading in stage 2.
- (b) buy a put option to sell 350MW during the second month at 21€/MWh. Option price = 0.1€/MWh

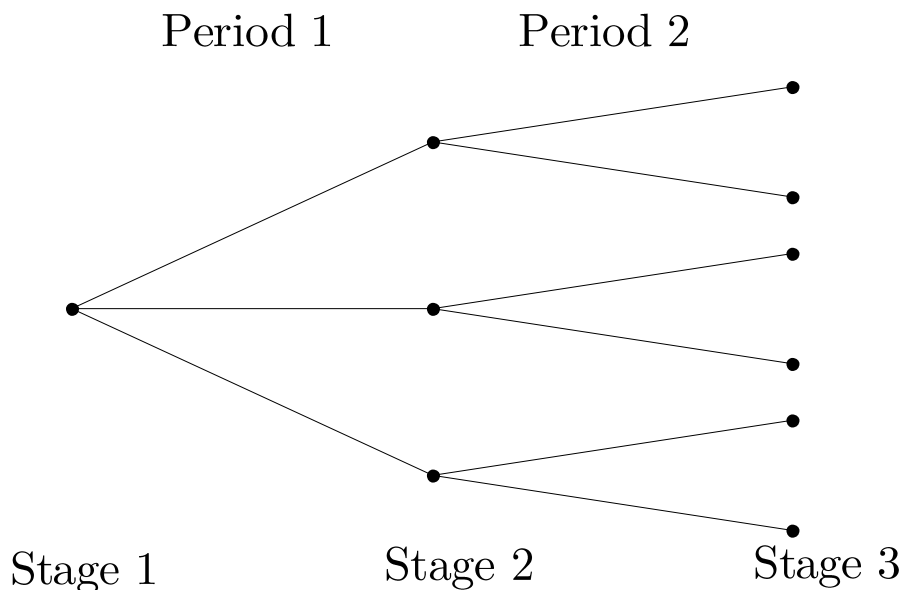
Case study

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499

OPTION > FORWARD

Case study

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499



$E_2\{\lambda^P\}$	y_{ow}
22.41	0
22.58	0
22.64	0
20.97	1
24.39	0
21.85	0
22.35	0
20.28	1
26.01	0
22.04	0

Case study

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499

OPTIONS reduce price risk

Case study

- How do electricity options work? ✓
- How can electricity options be modeled? ✓
- How do electricity options reduce price risk? ✓
- How do electricity options reduce availability risk?
- When is an option contract more profitable than a forward contract?

Case study

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499

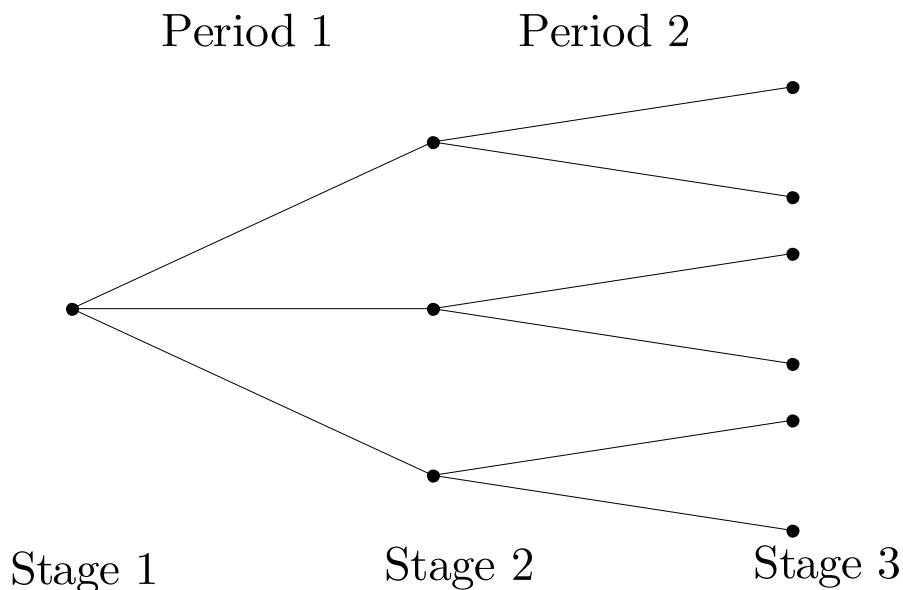
↑ risk aversion → **OPTION \approx FORWARD**

Case study

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499

Case study

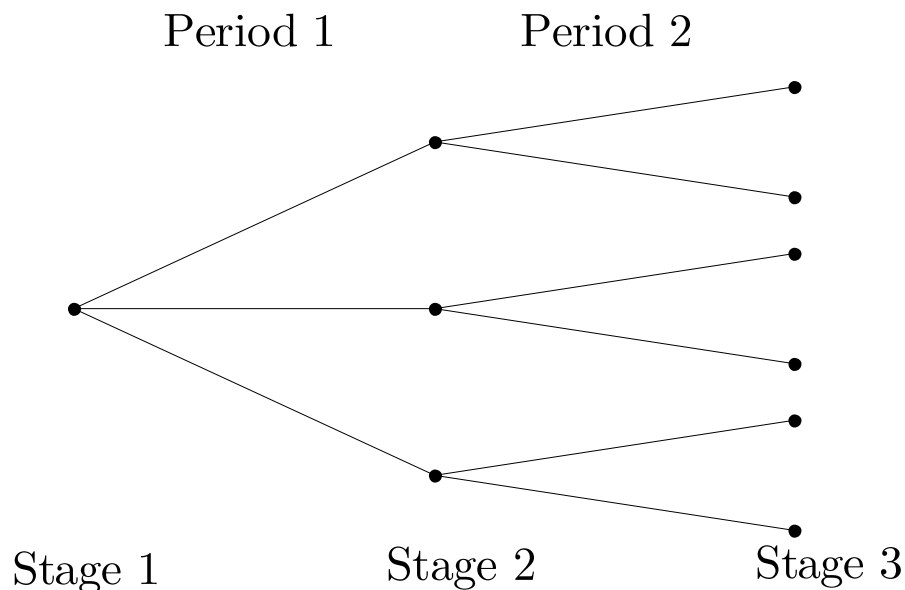
α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499



$E_2\{\lambda^P\}$	26.01	20.97	20.28
k_{NT1}			
1	0	1	1
0	0	0	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1

Case study

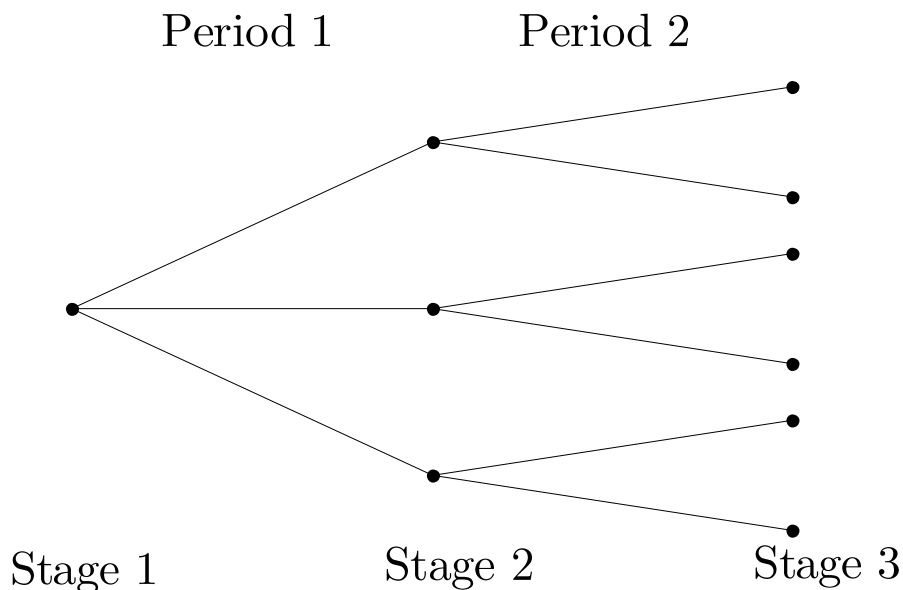
α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499



$E_2\{\lambda^P\}$	26.01	20.97	20.28
k_{NT1}			
1	0	1	1
0	0	0	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1

Case study

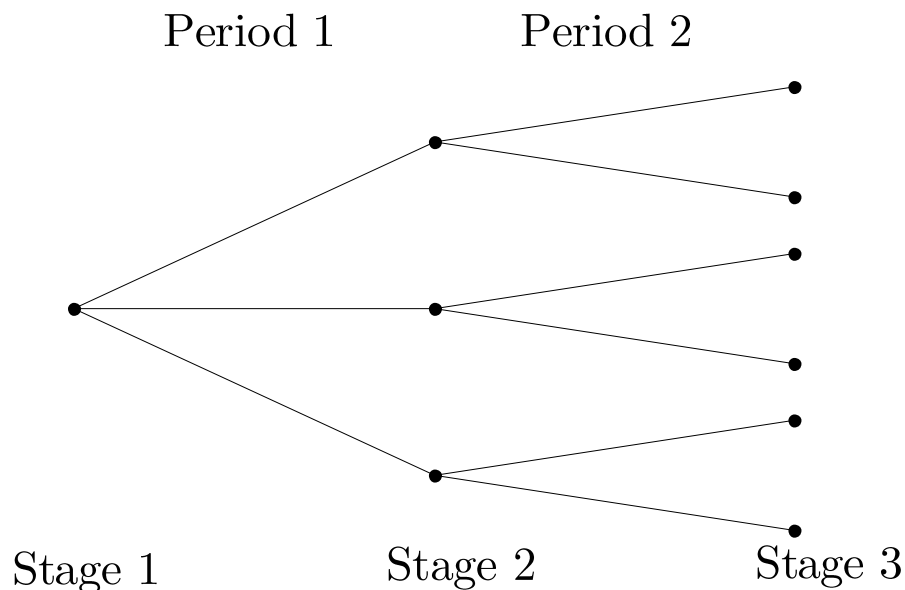
α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499



$E_2\{\lambda^P\}$	26.01	20.97	20.28
k_{NT1}			
1	0	1	1
0	0	0	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1

Case study

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499



$E_2\{\lambda^P\}$	26.01	20.97	20.28
k_{NT1}			
1	0	1	1
0	0	0	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1

Case study

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.087	5.418	4.984	5.314	4.878	5.209
0.5	5.078	5.117	4.872	4.970	4.664	4.860
0.9	5.078	5.055	4.549	4.649	4.374	4.499

OPTIONS reduce availability risk

Case study

- How do electricity options work? ✓
- How can electricity options be modeled? ✓
- How do electricity options reduce price risk? ✓
- How do electricity options reduce availability risk? ✓
- When is an option contract more profitable than a forward contract?

Case study

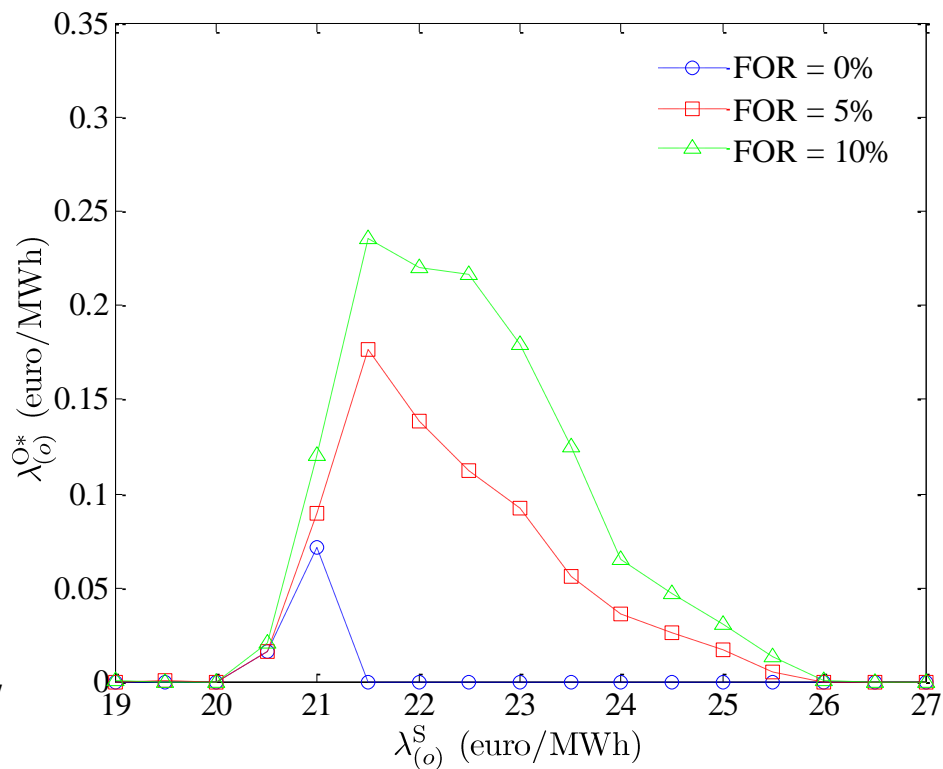
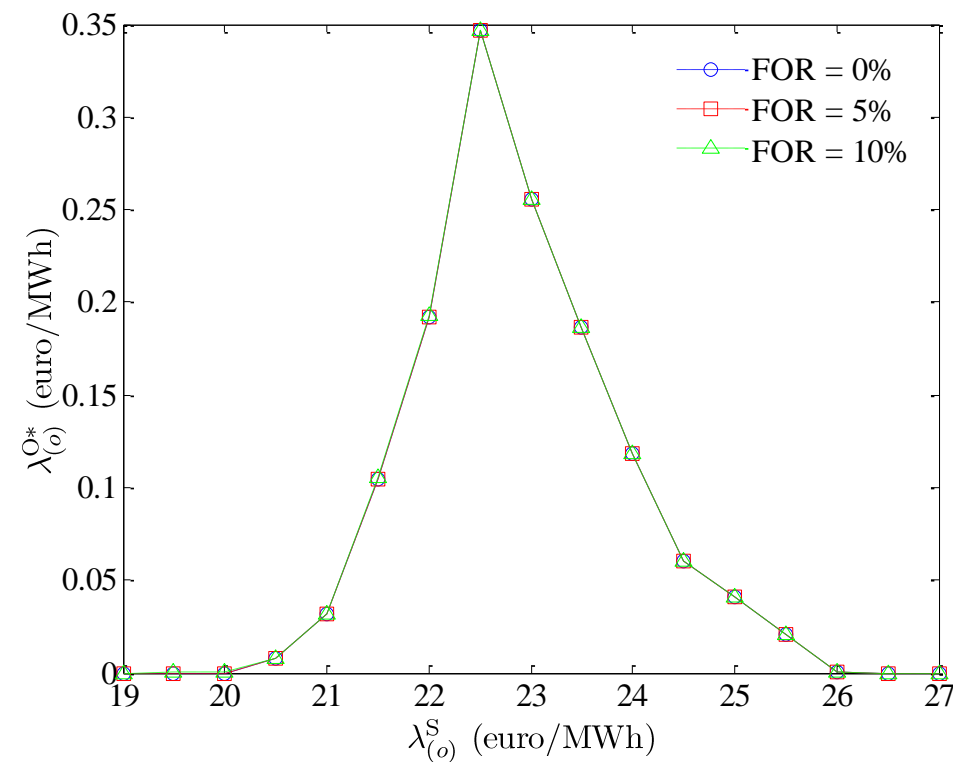
- **MAXIMUM OPTION PRICE** that the power producer is willing to pay for a given option contract.

Case study

➤ MAXIMUM OPTION PRICE

$\alpha = 0$

$\alpha = 0.5$

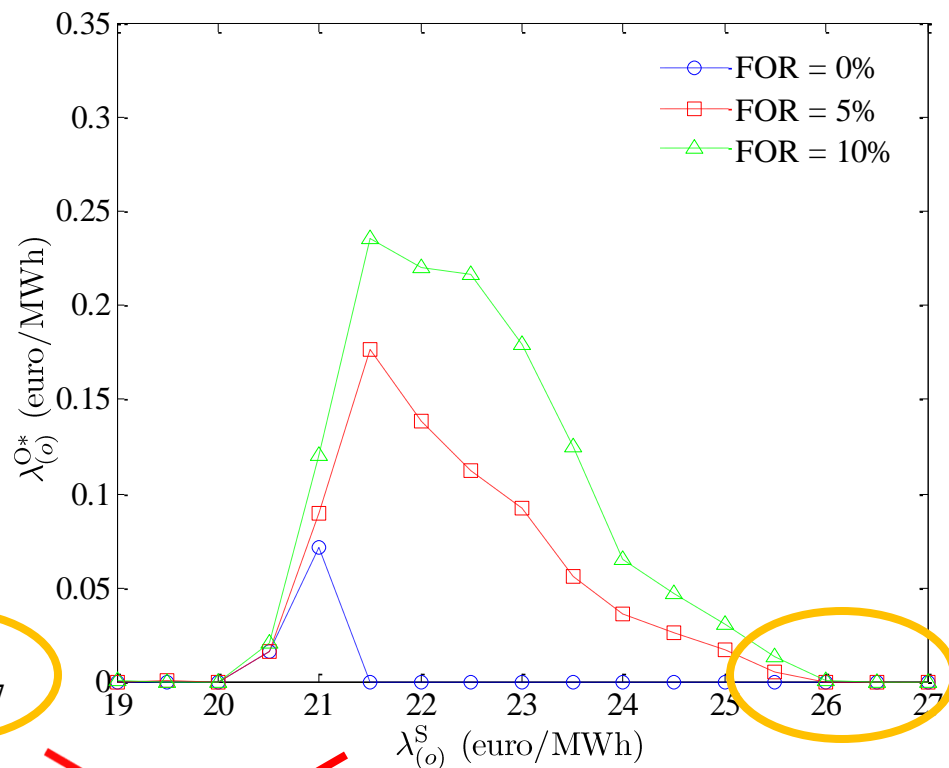
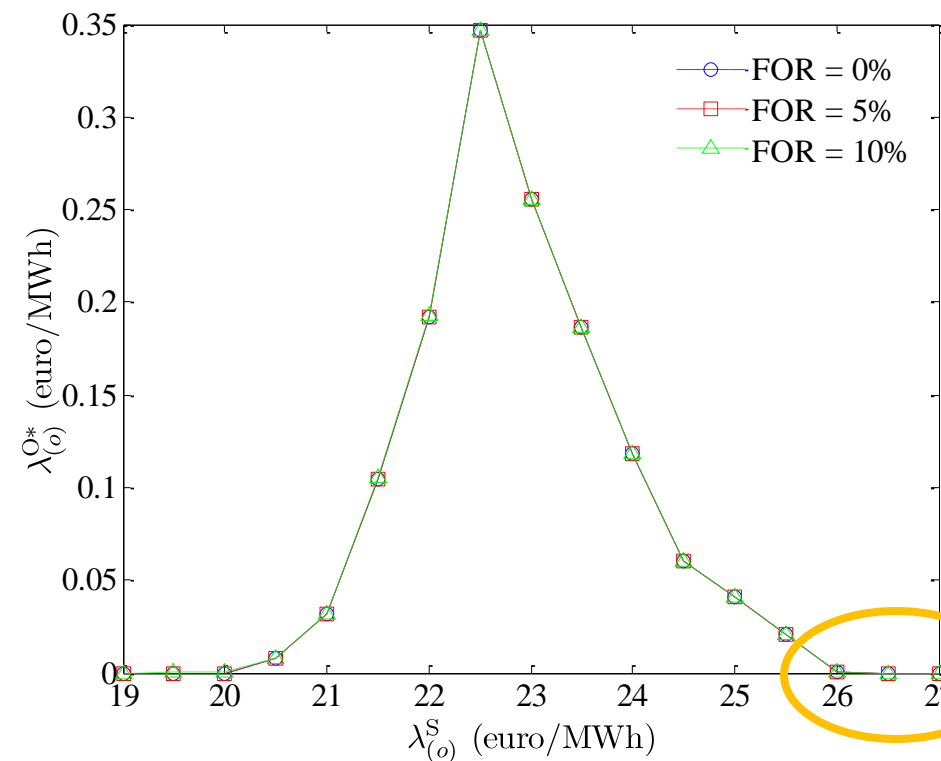


Case study

➤ MAXIMUM OPTION PRICE

$\alpha = 0$

$\alpha = 0.5$



Strike price >> Spot

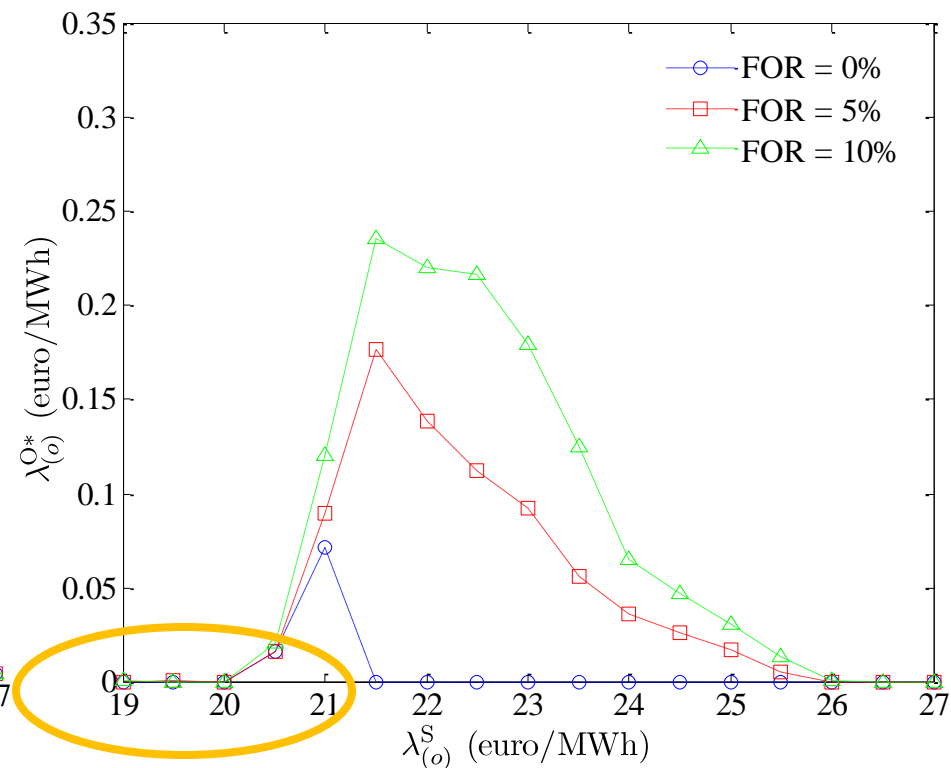
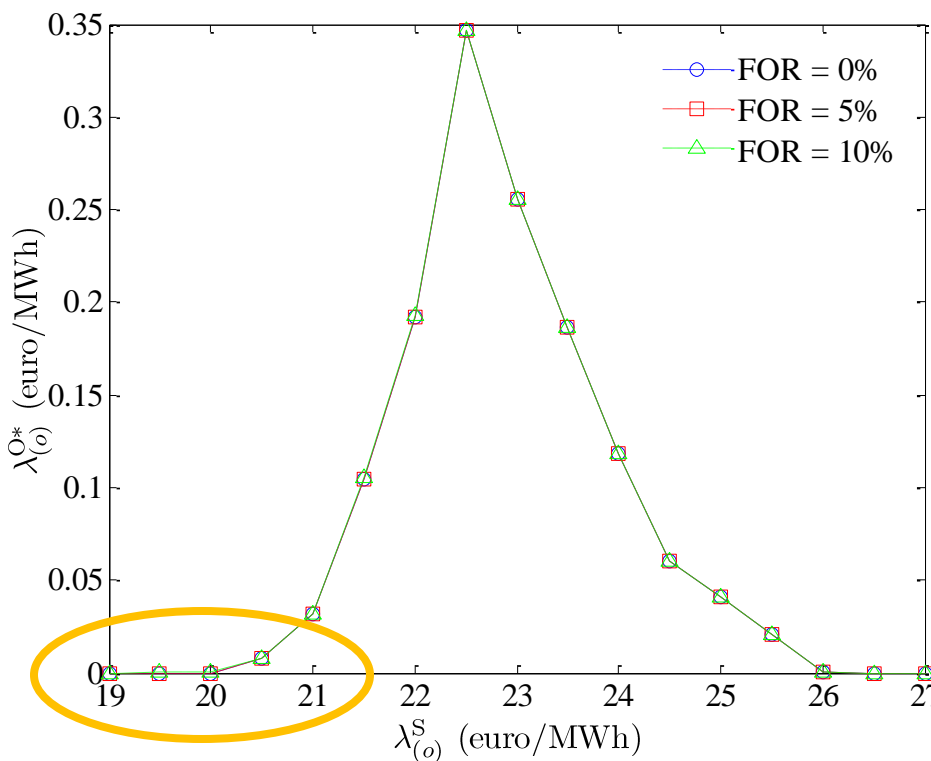
~~OPTION FORWARD~~

Case study

➤ MAXIMUM OPTION PRICE

$\alpha = 0$

$\alpha = 0.5$



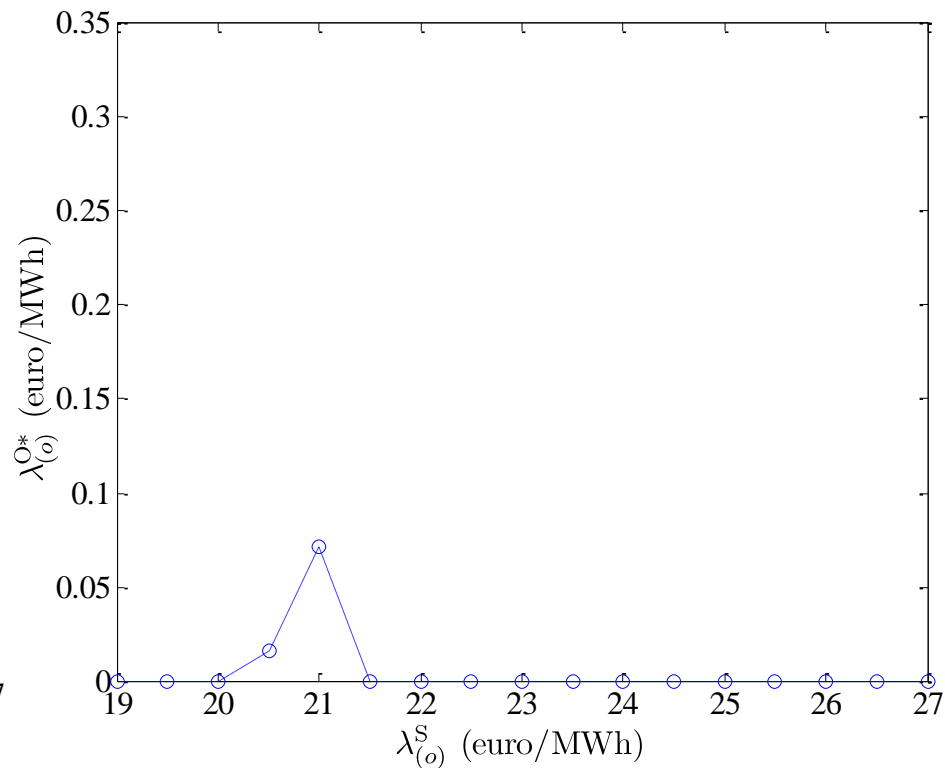
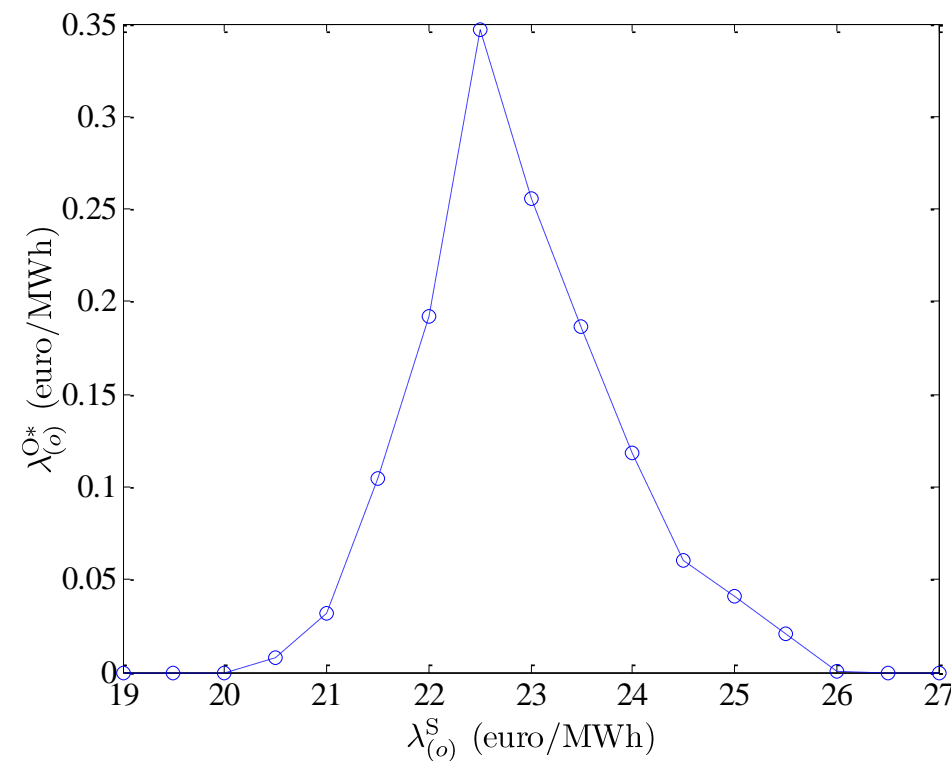
Strike price \ll Spot ~~OPTION FORWARD~~

Case study

➤ MAXIMUM OPTION PRICE

$\alpha = 0$

$\alpha = 0.5$



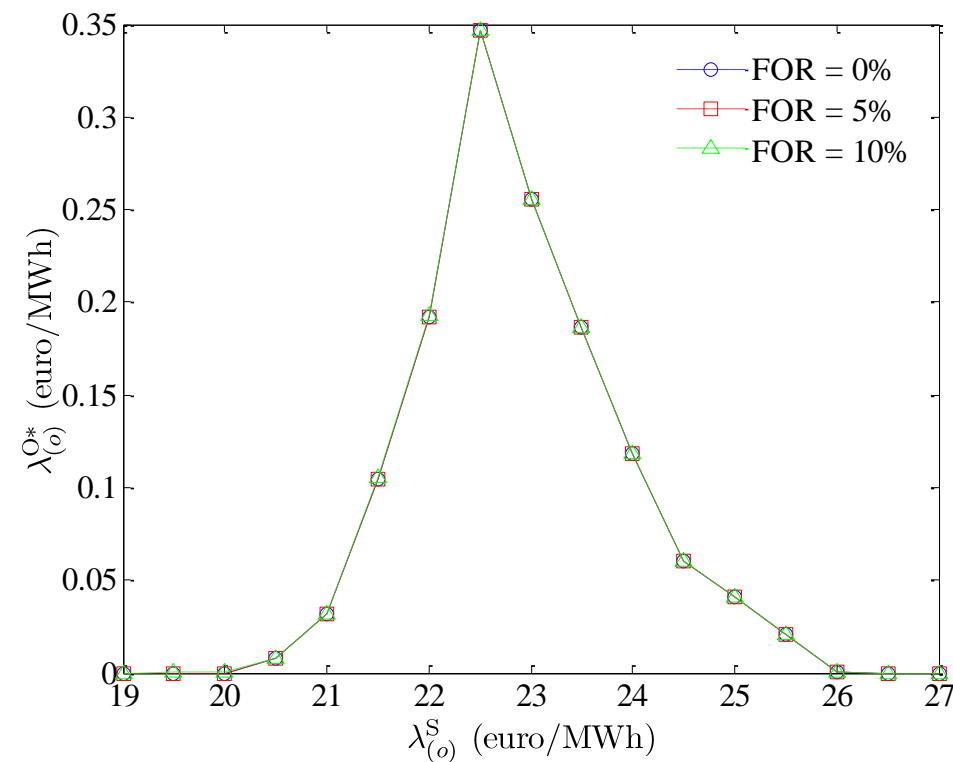
↑ risk aversion → **OPTION \approx FORWARD**

Case study

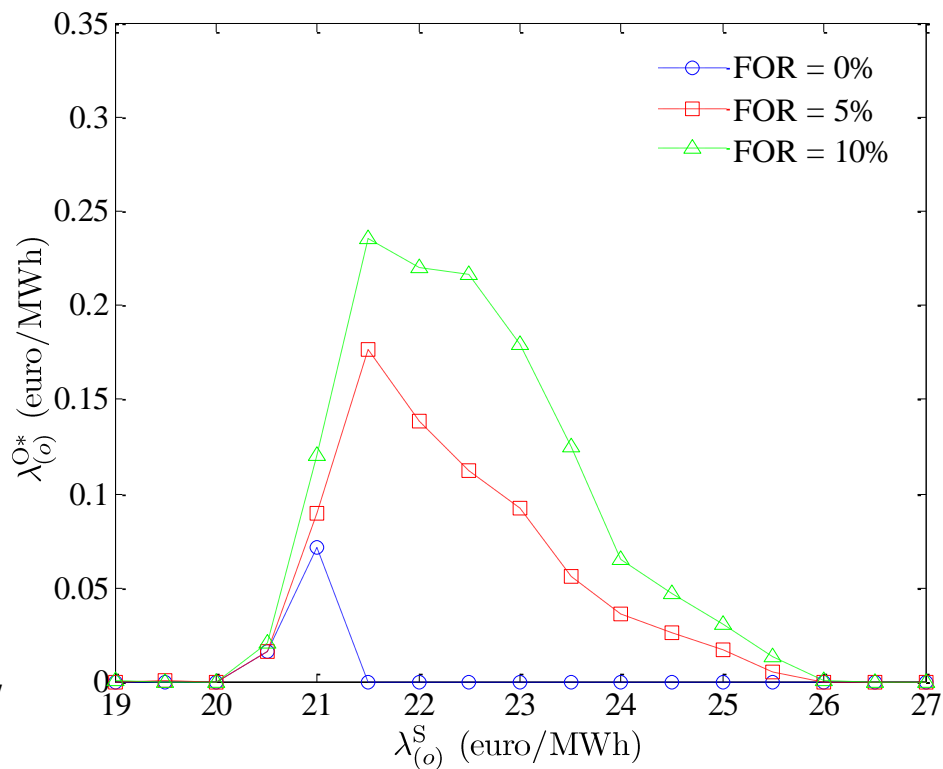
➤ MAXIMUM OPTION PRICE

$\alpha = 0$

$\alpha = 0.5$



FOR not relevant



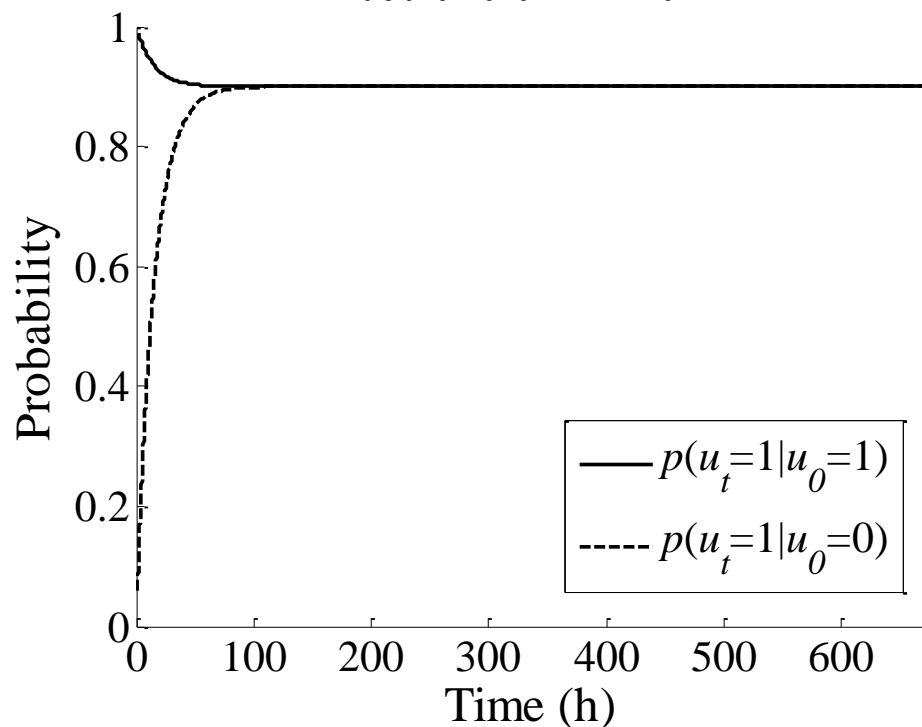
↑ FOR → OPTION

Model

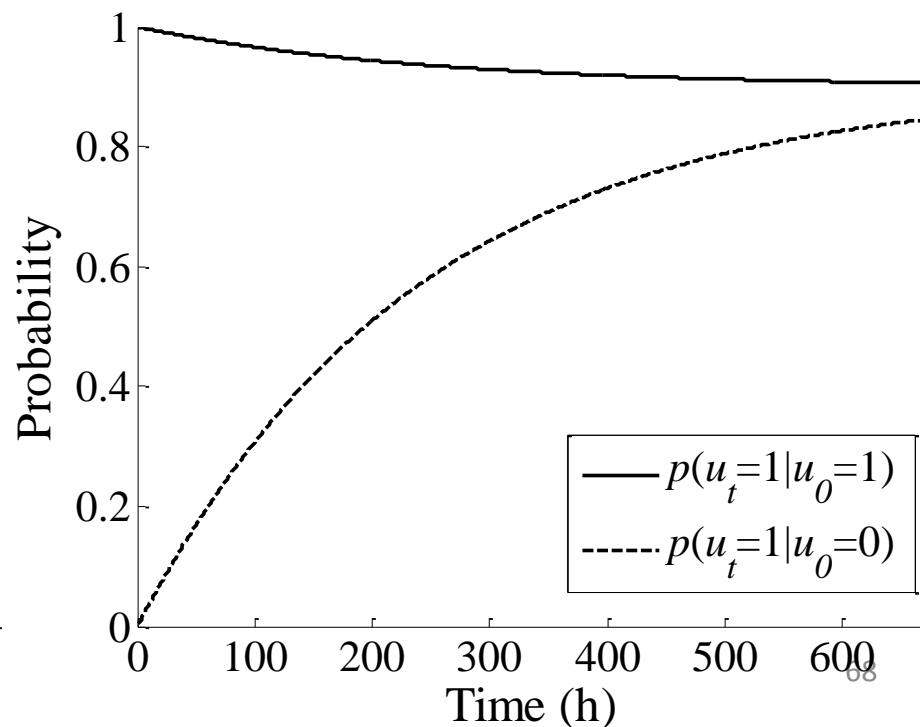
Unit availability

$$\left. \begin{array}{l} t_F \sim \exp(\text{MTTF}) \\ t_R \sim \exp(\text{MTTR}) \end{array} \right\} p(u_t = 1) = \frac{\mu}{\lambda + \mu} + \frac{\mu(u_0 - 1) + \lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

MTTF << T



MTTF >> T

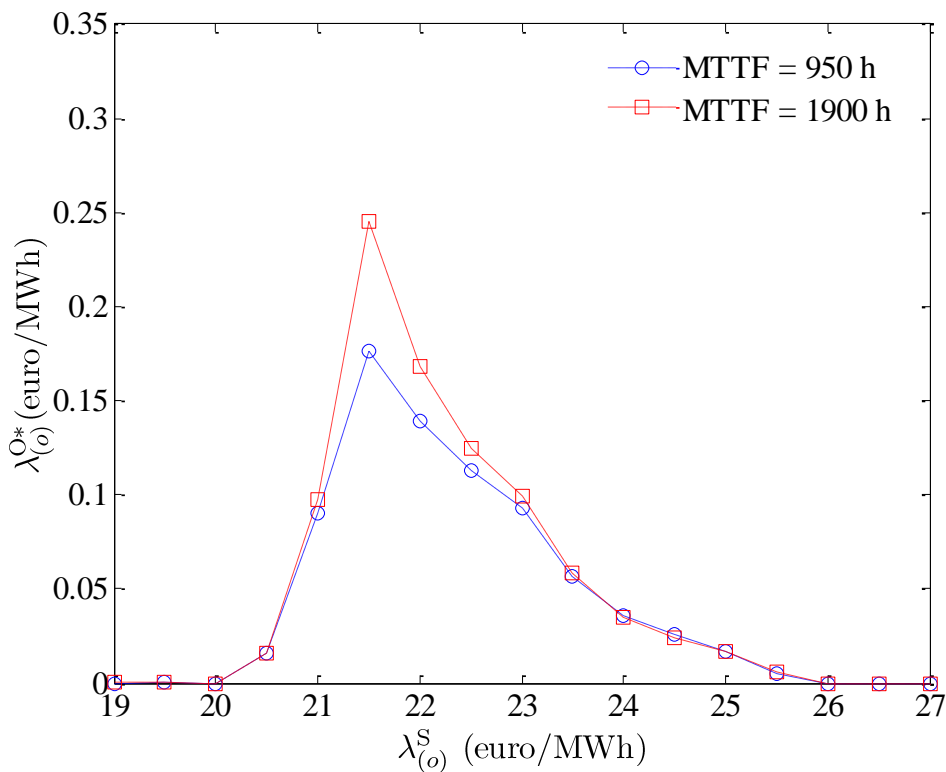


Case study

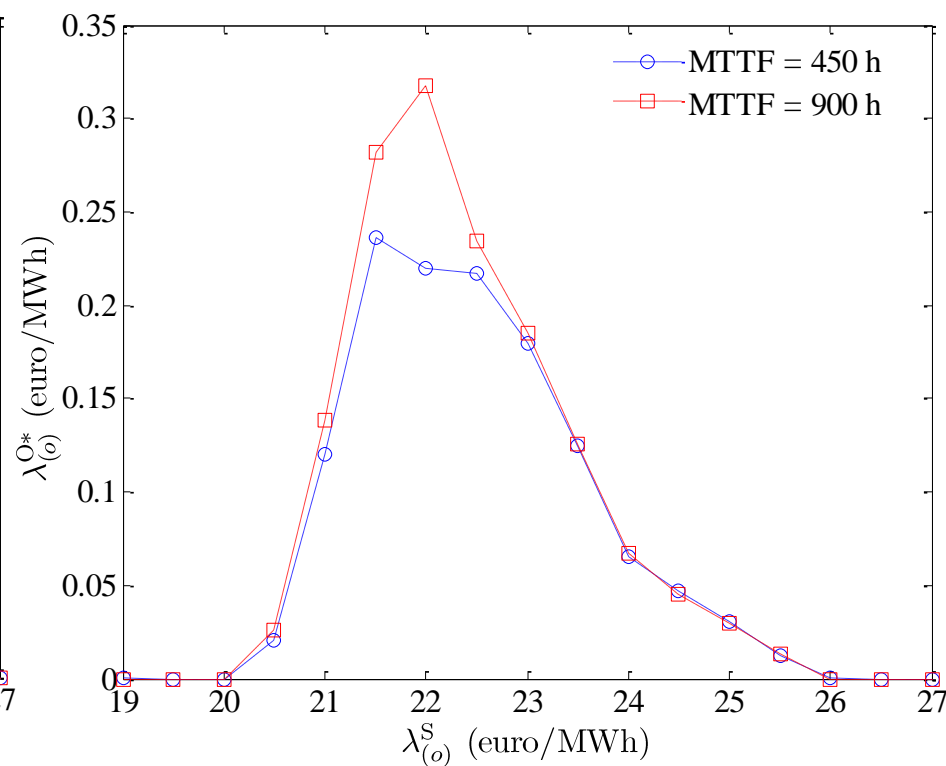
➤ MAXIMUM OPTION PRICE (impact of MTTF)

FOR = 5%

FOR = 10%



↑ **FOR → OPTION**



↑ **MTTF → OPTION**

- **Introduction**
- **Model**
- **Case study**
- **Conclusions**

➤ Introduction

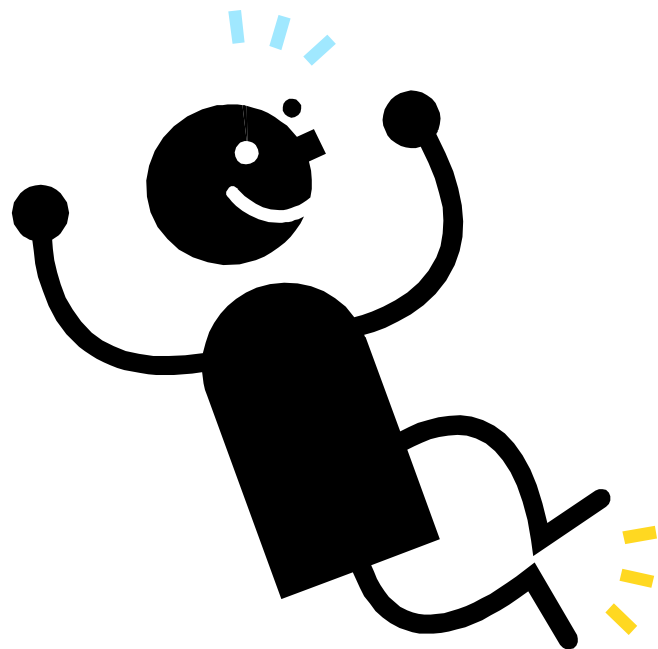
➤ Model

➤ Case study

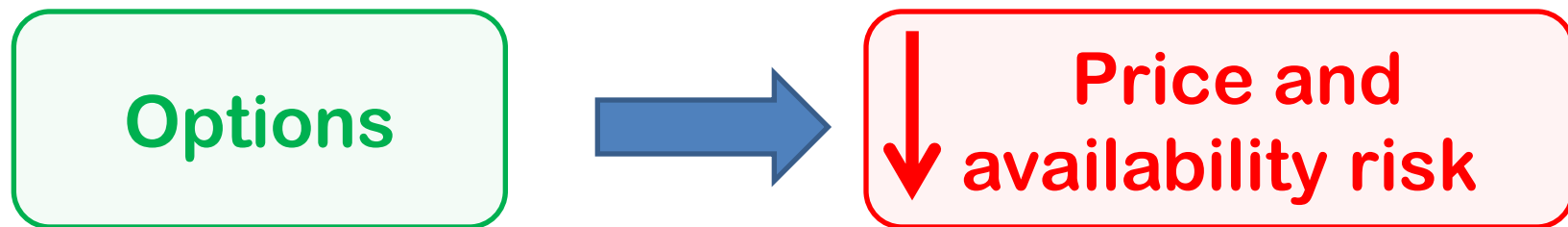
➤ Conclusions

Conclusions

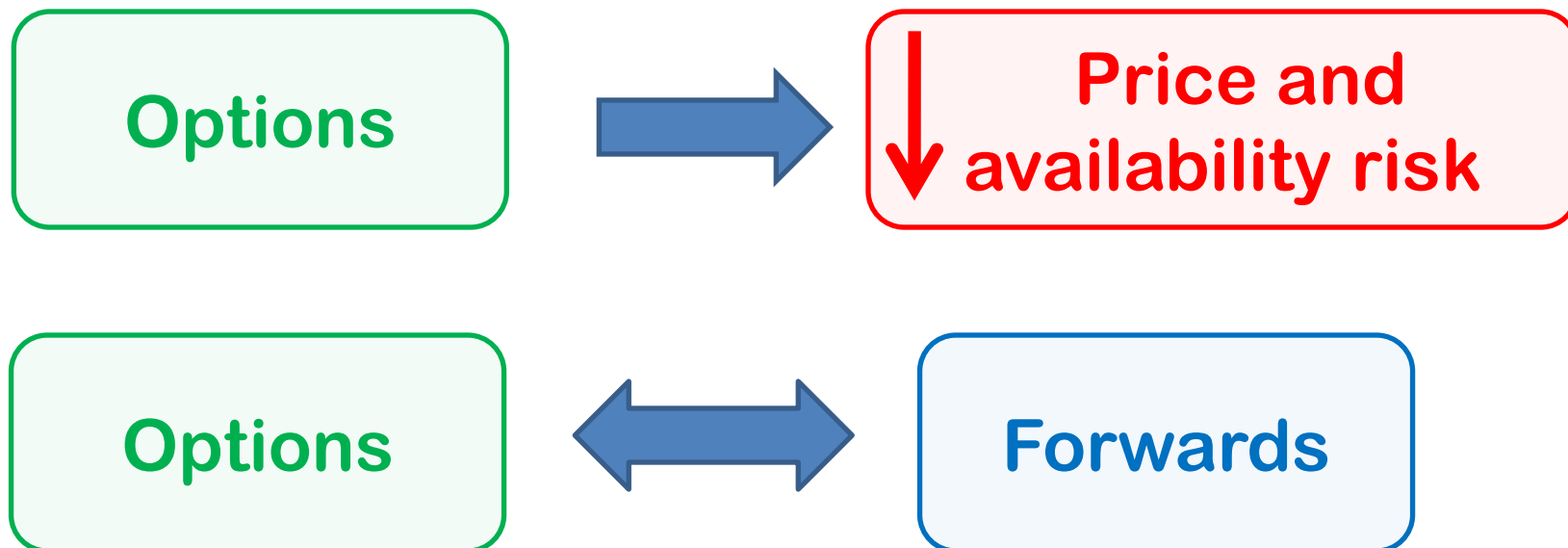
Multi-stage
stochastic
programming



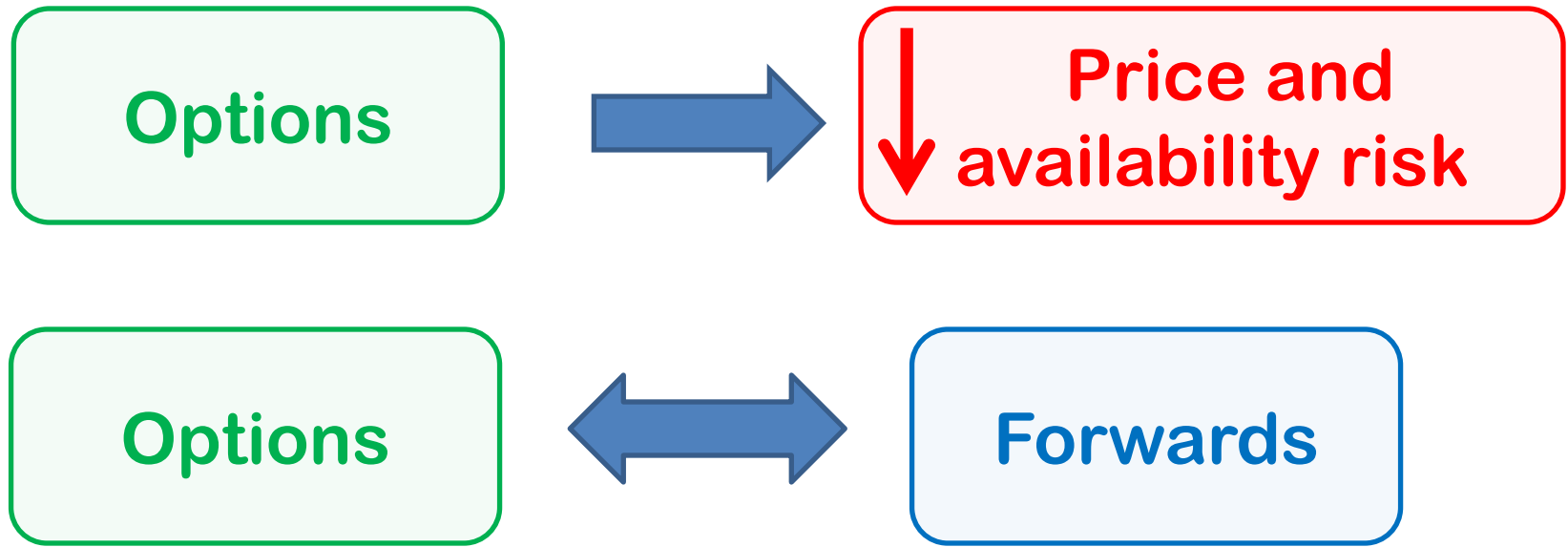
Conclusions



Conclusions

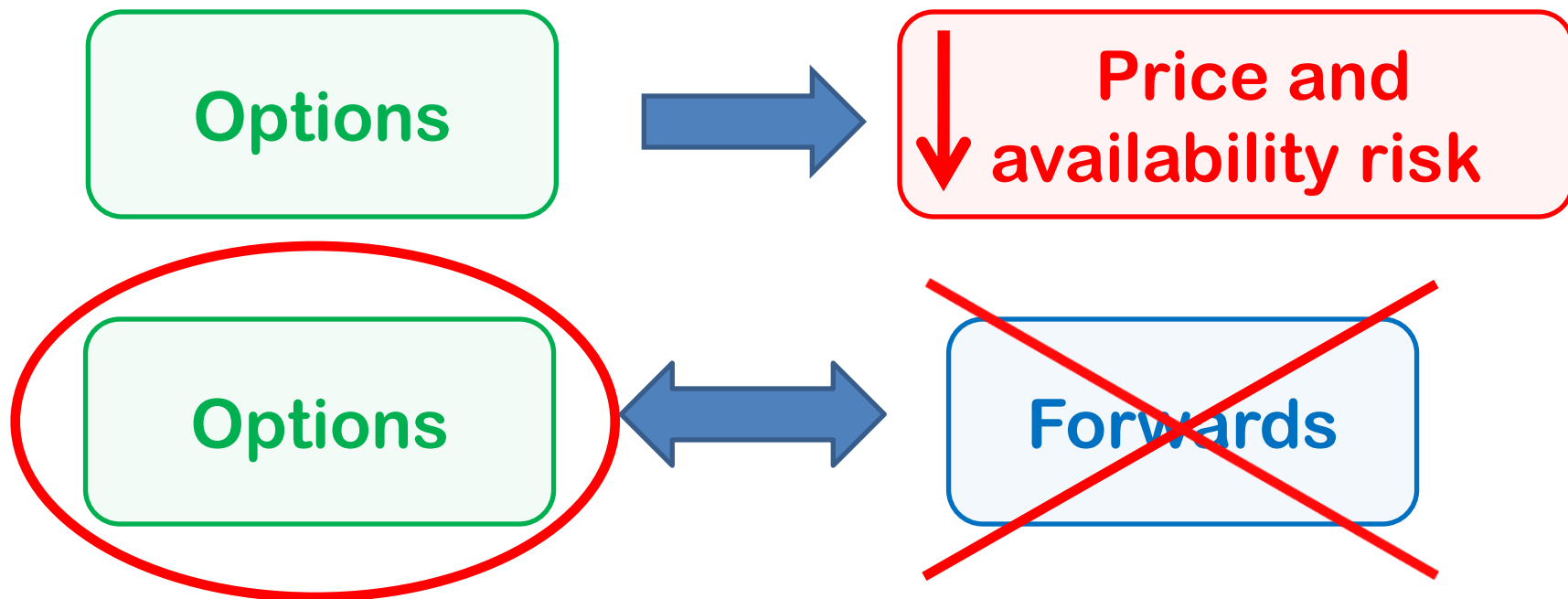


Conclusions



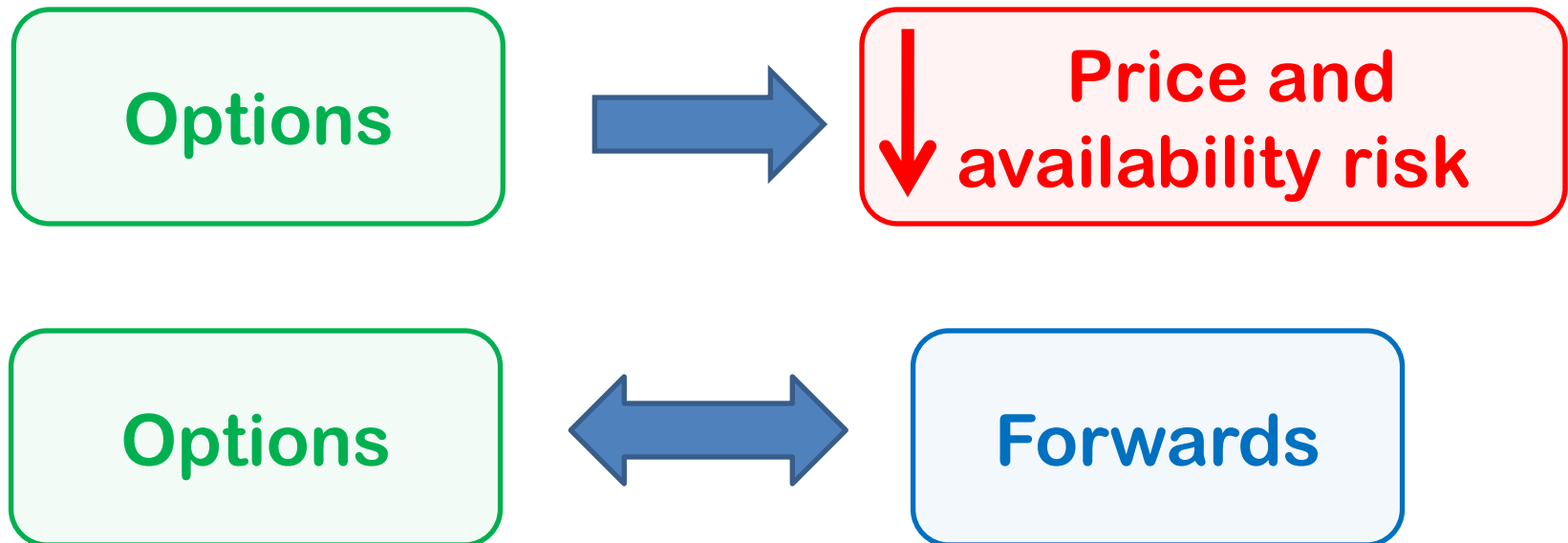
- Strike price \approx Spot
- FOR with \uparrow MTTF
- Option price $<$ Max. Option

Conclusions



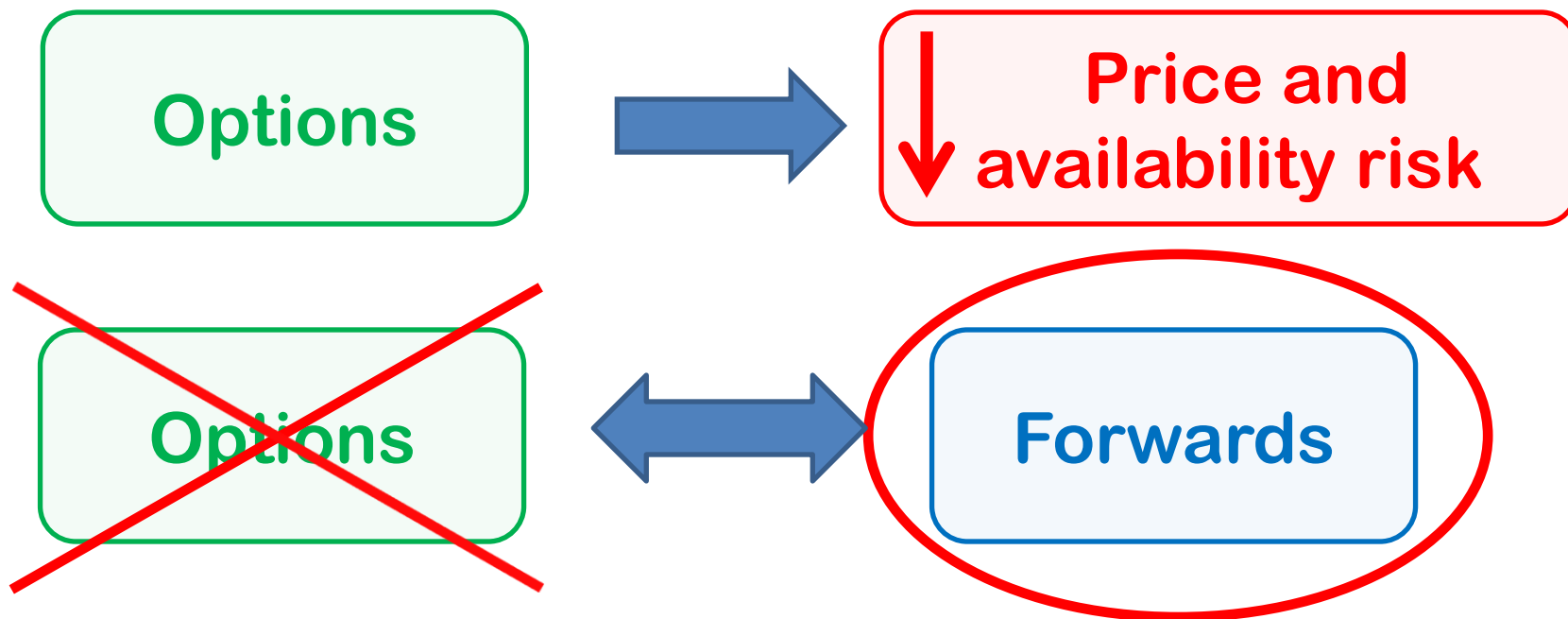
- Strike price \approx Spot
- FOR with \uparrow MTTF
- Option price $<$ Max. Option

Conclusions



- Forward price > Spot
- ↓FOR
- ↓MTTF

Conclusions



- Forward price > Spot
- ↓FOR
- ↓MTTF

Conclusions

- How do electricity options work? ✓
- How can electricity options be modeled? ✓
- How do electricity options reduce price risk? ✓
- How do electricity options reduce availability risk? ✓
- When is an option contract more profitable than a forward contract? ✓

Thank you!

Questions?

www.uclm.es/area/gsee

Model

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

$$\text{profit}_{\omega} = \sum_{t=1}^{N_T} \left(\lambda_{t\omega}^P P_{t\omega}^P T_t - C(P_{t\omega}^G) \right) + \sum_{c_1=1}^{N_{C1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{C2}} (\lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2\omega}^2 P_{c_2\omega}^2) T_{c_2} + \sum_{o=1}^{N_O} v_o (-\lambda_o^O P_o + y_{o\omega} \lambda_o^S P_o) T_o$$

$\lambda_{t\omega}^P \rightarrow$ Pool price
 $P_{t\omega}^P \rightarrow$ Power sold in the pool
 $T_t \rightarrow$ Duration of time period

Constraints

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints

Model

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

$$\text{profit}_{\omega} = \sum_{t=1}^{N_T} \left(\lambda_{t\omega}^P P_{t\omega}^P T_t - C(P_{t\omega}^G) \right) + \sum_{c_1=1}^{N_{C1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{C2}} (\lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2\omega}^2 P_{c_2\omega}^2) T_{c_2} + \sum_{o=1}^{N_O} v_o (-\lambda_o^O P_o + y_{o\omega} \lambda_o^S P_o) T_o$$

$C(\cdot) \rightarrow$ Cost function
 $P_{t\omega}^G \rightarrow$ Generated power

Constraints

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints

Model

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

Contracts spanning period 1

$\lambda_{c_1}^1 \rightarrow$ Forward price

$P_{c_1}^1 \rightarrow$ Sold power

$T_{c_1} \rightarrow$ Forward contract duration

$$\text{profit}_{\omega} = \sum_{t=1}^{N_T} \left(\lambda_{t\omega}^P P_{t\omega}^P T_t - C(P_{t\omega}^G) \right) + \sum_{c_1=1}^{N_{C_1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{C_2}} (\lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2\omega}^2 P_{c_2\omega}^2) T_{c_2} + \sum_{o=1}^{N_O} v_o (-\lambda_o^O P_o + y_{o\omega} \lambda_o^S P_o) T_o$$

Constraints

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints

Model

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

Contracts spanning period 2

$\lambda_{c_2}^1, \lambda_{c_2\omega}^2 \rightarrow$ Forward price in stage 1/2

$P_{c_2}^1, P_{c_2\omega}^2 \rightarrow$ Sold power in stage 1/2

$T_{c_2} \rightarrow$ Forward contract duration

$$\text{profit}_{\omega} = \sum_{t=1}^{N_T} \left(\lambda_{t\omega}^P P_{t\omega}^P T_t - C(P_{t\omega}^G) \right) + \sum_{c_1=1}^{N_{C1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{C2}} (\lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2\omega}^2 P_{c_2\omega}^2) T_{c_2} + \sum_{o=1}^{N_O} v_o (-\lambda_o^O P_o + y_{o\omega} \lambda_o^S P_o) T_o$$

Constraints

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints

Model

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

$$\text{profit}_{\omega} = \sum_{t=1}^{N_T} \left(\lambda_{t\omega}^P P_{t\omega}^P T_t - C(P_{t\omega}^G) \right) + \sum_{c_1=1}^{N_{C1}} \lambda_{c_1}^1 P_{c_1}^1 T_{c_1} + \sum_{c_2=1}^{N_{C2}} (\lambda_{c_2}^1 P_{c_2}^1 + \lambda_{c_2\omega}^2 P_{c_2\omega}^2) T_{c_2} + \sum_{o=1}^{N_O} v_o (-\lambda_o^O P_o + y_{o\omega} \lambda_o^S P_o) T_o$$

Pool
Cost
Forward 1
Forward 2
Option

Constraints

Production unit bounds

Energy balances

Forward and option constraints

Nonanticipativity constraints

$v_o \rightarrow$ Option purchase (1/0)

$\lambda_o^O, \lambda_o^S \rightarrow$ Option and strike price

$y_{o\omega} \rightarrow$ Option exercise (1/0)

$P_o \rightarrow$ Sold power

$T_o \rightarrow$ Option duration

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

Constraints

Production unit bounds

$$u_{t\omega} k_{t\omega} P_{\max} \geq P_{t\omega}^G \geq u_{t\omega} k_{t\omega} P_{\min}$$

\rightarrow Constant (availability scenario)
 \rightarrow Binary variable (on/off)

Energy balances

Forward and option constraints

Nonanticipativity constraints

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

Constraints

Production unit bounds

Energy balances

$$P_{t\omega}^G = \sum_{c_1 \in F_t^1} P_{c_1}^1 + \sum_{c_2 \in F_t^2} (P_{c_2}^1 + P_{c_2\omega}^2) + \sum_{o \in O_t} v_o y_{o\omega} P_o + P_{t\omega}^P$$

$$P_{t\omega}^P \geq 0, \forall k_{t\omega} = 1 \leftarrow$$

Forward and option constraints

Nonanticipativity constraints

Stochastic programming

Objective function

Maximize $CVaR_{\alpha}(\text{profit}_{\omega})$

Constraints

Production unit bounds

Energy balances

Forward and option constraints

$$P_{c_1}^1 \geq 0$$

$$P_{c_2}^1 + P_{c_2\omega}^2 \geq 0$$

$$P_o \geq 0$$

Nonanticipativity constraints

Case study

- (a) sell 350MW during the second month at 21€/MWh. Re-trading in stage 2 at the following prices:

$\lambda^2_{c2\omega}$	20.69	22.42	24.15
Probability	0.25	0.5	0.25

- (b) buy a put option to sell 350MW during the second month at 21€/MWh. Option price = 0.1€/MWh

α	FOR = 0%		FOR = 5%		FOR = 10%	
	(a)	(b)	(a)	(b)	(a)	(b)
0	5.245	5.418	5.141	5.314	5.035	5.209
0.5	5.078	5.117	4.919	4.970	4.756	4.860
0.9	5.078	5.055	4.599	4.649	4.429	4.499