Scenario Reduction for risk-averse Electricity Trading

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Abstract

Stochastic optimization models used to identify risk-averse decisions in electricity futures markets are usually hard to solve due to the large number of scenarios representing the uncertain parameters involved. In this paper, a novel scenario reduction technique is proposed to select those scenarios that, considering the risk aversion of the decision maker, best represent the original scenario set and make the optimization problem tractable. Two case studies illustrate the performance of the proposed technique to reduce scenarios pertaining to both continuous and discrete uncertain parameters. The advantage of the proposed technique versus the existing ones is apparent in highly risk-averse cases.

Keywords: Scenario reduction, stochastic programming, Conditional Value at Risk (CVaR), risk aversion.

1 Introduction

1.1 Motivation

Nowadays, a wide range of electricity derivatives (futures market products) are available in current electricity markets, allowing market agents to hedge their profit risks due to the high volatility of the pool prices [1]. Among these electricity derivatives, the one considered in this paper is the forward contract, which is by far the commonest in electricity markets. Roughly speaking, a forward contract is a financial instrument that allows selling/buying electricity throughout a future period at a certain price. Since decisions related to forward contracts should be made without knowing the actual realizations of some uncertain parameters affecting the time period covered by these contracts, stochastic optimization is an appropriate mathematical tool to made these decisions.

In stochastic programming, plausible realizations of the stochastic processes describing uncertain parameters throughout the decision-making horizon are represented via a set of scenarios [2]. Therefore, the higher the number of scenarios representing the stochastic processes involved, the more accurate the solution of the optimization problem. However, a large number of scenarios may result in high computational times or even intractability. For this reason, a scenario reduction technique is needed to identify a reduced scenario set which keeps, as much as possible, the stochastic properties of the original one.

The stochastic optimization models used by market agents to determine their optimal portfolio of contracts in the futures market include an appropriate model for the risk of profit/cost. Basically, a risk-averse agent makes its decisions trying to avoid low profit/high cost scenarios. Consequently, the required scenario reduction technique should take into account the risk aversion of the decision maker.

1.2 Literature review

In the technical literature, the most used scenario reduction technique is based on the norm of the difference between pairs of input vectors that contain the realizations of the stochastic process throughout the decision-making horizon, [3–6]. This technique is used in [7–12] to reduce the scenario set representing the uncertainty of the considered models, thus making the corresponding stochastic optimization problems computationally tractable. An alternative scenario reduction technique based on the values of the objective functions of single-scenario optimization problems is proposed in [13] for risk-neutral models. However, most electricity market agents are risk averse, i.e., they make their decisions taking into account not only the expected profit, but also the profit variability. Recognizing this fact, we propose a scenario reduction technique tailored to risk-averse electricity trading.

1.3 Contribution

The contributions of this paper are twofold:

1. To provide an efficient scenario reduction technique for risk constrained

stochastic programming problems involving both continuous and discrete uncertain parameters.

2. To show the good computational performance of the proposed technique using two realistic case studies.

1.4 Paper organization

The rest of this document is organized as follows. In Section 2, a generic stochastic optimization model to identify market agent decisions is presented. Based on this model, the proposed scenario reduction algorithm is explained, emphasizing its differences with existing algorithms. Two different case studies related with the risk-averse electricity trading of a producer are used to analyze the performance of the proposed technique in Section 3 and 4. In Section 3, a large scenario set involving uncertain pool prices throughout one month is reduced using the proposed scenario reduction technique and alternative ones. In Section 4, the proposed scenario reduction technique and others are applied to reduce a scenario set representing the discrete availability of a production unit. Section 5 provides some relevant conclusions.

2 Proposed Methodology

2.1 Mathematical formulation

In stochastic optimization models, the uncertain parameters (e.g., the pool price or the availability of a production unit) are represented via a set of scenarios $\boldsymbol{\lambda_\omega}$. In stochastic recourse models, some of the decision must be made before information relevant to the uncertainties is available (here-and-now or first-stage decisions), while some other decisions can be delayed until uncertain parameter values realize (wait-and-see or second-stage decisions). Therefore, these models are the best choice to decide the optimal portfolio in electricity futures markets, since the energy traded through forward contracts should be determined in advance, i.e., without knowing the values of the pool prices; whereas the decisions related with the energy traded in the pool are made with precise information on pool prices. In the following, we consider a profit maximization market agent. Note that the case of a cost minimization agent can be addressed similarly as the case of a profit maximization agent.

Decision makers are generally risk averse, i.e., they make futures market decisions based not only on the expected profit, but also on the possibility of experimenting low profits. The Conditional Value at Risk (CVaR) is a commonly used risk measure in portfolio optimization models, [14] and [15]. The CVaR of a probability distribution with a confidence level α is defined as the mean value of these scenarios with the lowest profits and with an accumulated probability equal to $1-\alpha$ (see Fig. 1). The Value at Risk (VaR) is equal to the profit such that the probability for a profit being less than this value is equal to $1-\alpha$. Therefore, the CVaR can be also defined as the mean value of that profits lower than the VaR, and it is calculated as follows:

$$\underset{\zeta,\,\eta_\omega}{\text{maximize}} \quad \text{CVaR}_\alpha \;= \zeta - \frac{1}{1-\alpha}\sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \tag{1}$$

subject to

$$-\text{profit}_\omega + \zeta - \eta_\omega \leq 0; \quad \forall \omega \tag{2}$$

$$\eta_\omega \geq 0; \quad \forall \omega \tag{3}$$

where profit_ω is the profit in scenario ω with an associated probability equal to π_ω. ζ is an auxiliar variable whose optimal value corresponds to the value of the VaR. The optimal value of the auxiliar variable η_ω represents the difference between the VaR and the value of the profit in scenario ω for those scenarios with a profit lower than the VaR (shadow area of Fig. 1), and is equal to 0 otherwise. Consequently, only profit values smaller than the VaR are needed to calculate the value of the CVaR.

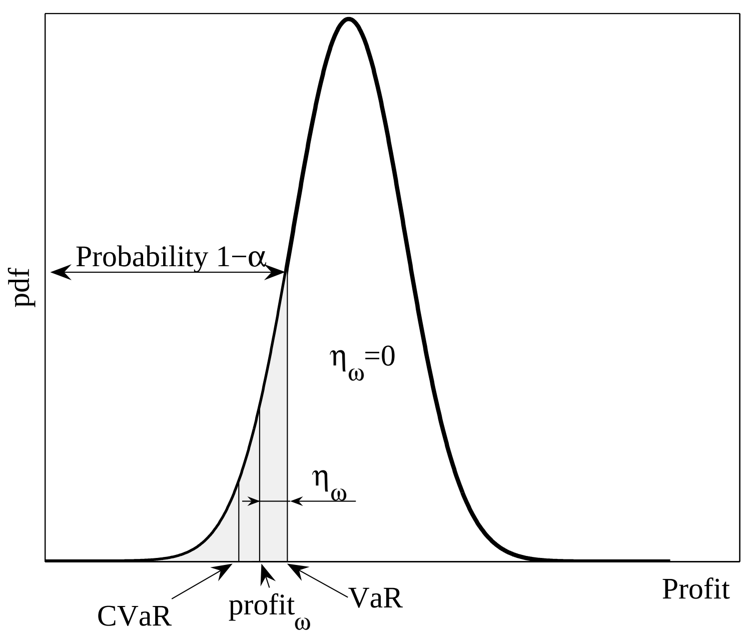

Figure 1: Conditional Value at Risk definition.

Considering a two-stage stochastic programming model, variable $\boldsymbol{x}$ represents the first-stage decisions, i.e., the energy traded in the futures market throughout forward contracts; and the variable $\boldsymbol{y_\omega}$ represents the second-stage decisions, i.e., the energy traded in the pool as a function of the realization of the stochastic process $\boldsymbol{\lambda_\omega}$. $\Pi^F(\cdot)$ and $\Pi^P(\cdot)$ represent the profit obtained from the futures and pool trading, respectively. Therefore, the stochastic optimization problem including the risk aversion of the decision maker to be solved is [15]:

$$\underset{\boldsymbol{x}, \boldsymbol{y_\omega}, \zeta, \eta_\omega}{\text{maximize}} \quad z = \Pi^F(\boldsymbol{x}) + \text{CVaR}_\alpha \left(\Pi^P(\boldsymbol{\lambda_\omega}, \boldsymbol{x}, \boldsymbol{y_\omega}), \zeta, \eta_\omega\right) \tag{4}$$

subject to

$$(\boldsymbol{x}, \boldsymbol{y_\omega}) \in \Omega, \forall \omega. \tag{5}$$

$$(\zeta, \eta_\omega) \in \Xi, \forall \omega. \tag{6}$$

Objective function (4) represents the forward contract revenue plus the conditional value at risk of the pool profit for a confidence level α, calculated according to (1). Equations (5) determine the feasibility region of first and second-stage variables according to the constraints of the futures and pool markets. Equations (6) impose limits on the auxiliar variables needed to calculate the CVaR (see Equations (2) and (3)).

As the objective function of the optimization model is to maximize the value of the CVaR, the higher the value of α, the higher the risk aversion of the market agent. Note that maximizing the CVaR of a profit distribution for $\alpha = 1$ is equivalent to make the decisions based on the worst case scenario. On the other hand, the risk-neutral case is achieved for $\alpha = 0$ since the corresponding value of the CVaR is equal to the expected value of the probability distribution of profit.

2.2 Scenario reduction algorithm

The time and memory resources needed to solve model (4)–(6) may increase exponentially if a large set of scenarios is considered to represent the stochastic processes involved. For this reason, a scenario reduction algorithm is needed.

Since only scenarios with a profit lower than the VaR are needed to calculate CVaR, the idea of the proposed scenario reduction technique consists in reducing the original scenario set considering the values of the variable η_ω. This way, all scenarios with a profit higher than the VaR ($\eta_\omega = 0$) are merged into one single scenario with a probability equal to α; while the scenarios with a profit lower than VaR are reduced based on the values of η_ω.

Let $\boldsymbol{\lambda_\omega}$ be the set of scenarios representing the realizations of a stochastic process $\boldsymbol{\lambda}$. The algorithm to reduce the original scenario set according to the proposed method is:

1. Calculate the average scenario ($\overline{\boldsymbol{\lambda}} = \mathcal{E}\{\boldsymbol{\lambda_\omega}\}$) and solve the deterministic problem associated with it to compute the values for the first-stage variables ($\overline{\boldsymbol{x}}$).

2. Fixing the first-stage variables to those values calculated in the previous step ($\boldsymbol{x} = \overline{\boldsymbol{x}}$), solve as many single-scenario optimization problems as scenarios in the original set ($\boldsymbol{\lambda_\omega}$), obtaining the corresponding values of the objective functions z_ω. The set of pairs (z_ω,π_ω) determines the probability distribution of the profit obtained through the set of

deterministic problems.

3. Calculate the value of the CVaR_α of the profit probability distribution obtained in step 2, and compute the values of the auxiliary variables η_ω associated with all scenarios of the original set.

4. Compute the distance matrix, $c(\omega, \omega')$. Each component of this matrix is equal to the absolute value of the difference between the variables η_ω and η'_ω calculated in the previous step, i.e.,

$$c(\omega, \omega') = |\eta_\omega - \eta_{\omega'}|.$$

   Note that the distance between any two scenarios with a profit higher than the VaR is equal to 0.

5. Use the fast forward algorithm in [3] to select those scenarios for the reduced set and update their probabilities. Roughly speaking, the fast forward algorithm is a heuristic procedure to build a reduced scenario set minimizing its so-called probability distance with respect to the original set. In each step of the algorithm, the scenario which minimizes the probability distance between the reduced and the original set is added to the former until the required number of selected scenarios is achieved. Once the reduced scenario set is determined, the probability of each non-selected scenario is added to its "closest" selected scenario according to the probability distance. A detailed mathematical description of the fast forward algorithm can be found in [3].

It is important to note that the proposed scenario reduction technique is based on solving single-scenario problems with first-stage decisions fixed to those obtained solving a deterministic average-value problem (step 1 of the proposed algorithm). Thus, the probability distribution function (pdf) of the profit corresponding to the single-scenario problems may differ from the pdf of the profit obtained solving the stochastic optimization with the original scenario set. The closer these pdfs are, the higher the accuracy of the proposed algorithm.

Note also that the proposed technique ignores all scenarios with a profit higher than the value of the VaR. Consequently, non-exact values for the first-stage variables may cause some relevant scenarios for the calculation of the CVaR to be neglected. For this reason, in the calculation of the variables η_ω (step 3 of the algorithm), the value of the parameter α is reduced a quantity $\Delta\alpha$ to decrease the possibility of neglecting potentially relevant scenarios (see Fig. 1). The higher the value of $\Delta\alpha$, the lower the probability of ignoring important scenarios. However, a high value of $\Delta\alpha$ implies a value of α close to 0, and therefore, the risk aversion of the decision maker is neglected in the scenario reduction process. Note that the modification of the parameter α only affects the scenario reduction stage, while its original value is maintained to solve the stochastic problem with the reduced scenario set.

2.3 Comparison

The proposed scenario reduction technique is compared with other two techniques. The main difference between these techniques and the proposed one is the definition of the distance between two scenarios, i.e., the calculation of the distance matrix described in the step 4 of the proposed algorithm in

Section 2.2.

- **Technique 1**. In [3]– [6] the distance between two scenarios is defined as the norm of the difference between the two vectors containing the values of the stochastic process for these two scenarios, i.e.,

$$c(\omega, \omega') = \|\boldsymbol{\lambda}_{\omega} - \boldsymbol{\lambda}_{\omega'}\|.$$

- **Technique 2**. The algorithm proposed in [13] considers the distance between two scenarios as the absolute value of the difference between the objective function of the corresponding single-scenario optimization problems if the first-stage decisions are fixed to those obtained for the deterministic average-value problem, i.e.,

$$c(\omega, \omega') = |z_{\omega} - z_{\omega'}|.$$

- **Technique 3**. The algorithm proposed in this paper computes the distance between two scenarios using the auxiliar variable η_{ω}, i.e.,

$$c(\omega, \omega') = |\eta_{\omega} - \eta_{\omega'}|.$$

The main purpose of these three techniques is to build a reduced scenario set that results in a CVaR value as close as possible to that obtained with the original set. For this reason, these three techniques are compared based on the evolution of the CVaR error with the number of scenarios in the reduced scenario set. The lower the CVaR error value, the more efficient the scenario reduction technique. The evolution of the CVaR error with the number of scenarios is determined according to the following steps:

1. Select the number of scenarios in the reduced scenario set (n). This number is changed between one and the total number of scenarios in the original set to obtain the evolution of the CVaR error with the number of scenarios.

2. Use one of the considered scenario reduction techniques to select n scenarios according to its corresponding distance definition and the fast forward selection algorithm.

3. Solve the considered stochastic optimization problem taking into account only the n scenarios selected in the previous step, and obtain the value of first-stage decisions.

4. Fix first-stage decisions to those calculated in the previous step and solve one single-scenario optimization problem for each scenario of the original set.

5. Compute the value of the CVaR_α of the profit distribution determined in the previous step.

6. Calculate the relative error between the CVaR obtained in the previous step and the CVaR corresponding to the stochastic model solved with the total number of scenarios. Note that the later value of the CVaR is independent of the scenario reduction technique, and thus it is used for comparison.

It is relevant to note that in the simulations presented below, the number of scenarios pertaining to the original set is small enough for the stochastic optimization problem to be tractable considering all scenarios, and therefore, the CVaR error can be calculated to compare the different scenario reduction techniques. However, in most cases, the stochastic optimization problem with the original scenario set is intractable.

3 Application 1: Producer trading model

In this section, we present the formulation of a stochastic optimization model to determine the optimal forward contract decisions of a power producer [10]. For a producer, a forward contract is an agreement to sell a power quantity throughout a certain future time period and for a certain price agreed at the time of signing the forward contract.

In this first case study the forced outage rate of the production unit is neglected. Therefore, the proposed scenario reduction technique is used to reduce the original scenario set representing the stochastic pool price throughout the planing horizon. Some conclusions about the performance of the proposed algorithm are presented at the end of this section.

3.1 Formulation

The considered stochastic programming problem is

$$\underset{P_c,\, P^{\mathrm{P}}_{t\omega},\, \zeta,\, \eta_\omega}{\text{maximize}} \quad \sum_{c=1}^{N_C} \lambda_c P_c T_c + \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_W} \pi_\omega \eta_\omega \tag{7}$$

subject to

$$P^{\mathrm{G}}_{t\omega}L_t = \sum_{c\in F_t} P_c L_t + P^{\mathrm{P}}_{t\omega}L_t, \quad \forall t, \forall \omega \tag{8}$$

$$P^{\mathrm{G}}_{t\omega} \leq u_{t\omega}P^{\max}, \quad \forall t, \forall \omega \tag{9}$$

$$P^{\mathrm{G}}_{t\omega} \geq u_{t\omega}P^{\min}, \quad \forall t, \forall \omega \tag{10}$$

$$P_c \leq P^{\max}_c, \quad \forall c \tag{11}$$

$$\begin{aligned} &-\sum_{t=1}^{N_T}(\lambda^{\mathrm{P}}_{t\omega}P^{\mathrm{P}}_{t\omega}L_t - (au_{t\omega} + bP^{\mathrm{G}}_{t\omega} + c(P^{\mathrm{G}}_{t\omega})^2)L_t)+ \\ &+\zeta - \eta_\omega \leq 0, \quad \forall \omega \end{aligned} \tag{12}$$

$$\eta_\omega \geq 0, \quad \forall \omega \tag{13}$$

$$P_c \geq 0, \quad \forall c \tag{14}$$

$$P^{\mathrm{P}}_{t\omega} \geq 0, \quad \forall t, \forall \omega \tag{15}$$

$$u_{t\omega} \in \{0, 1\}, \quad \forall t, \forall \omega \tag{16}$$

The notation used in the stochastic optimization model above is stated next. Continuous variables are described in the following. $P^{\mathrm{G}}_{t\omega}$ and $P^{\mathrm{P}}_{t\omega}$ are, respectively, the power generated by the unit and the power traded in the pool during period t and scenario ω (MW), P_c is the power sold through forward contract c (MW), and ζ and η_ω are the auxiliary variables related to the Conditional Value at Risk (€).

The random variable considered is the pool price in period t, λ^{P}_t (€/MWh).

$u_{t\omega}$ represents a binary variable that is equal to 1 if the unit is online during period t and scenario ω, and 0 otherwise.

Constants are described in the following. a, b, c are the coefficients of the quadratic production cost function of the unit, L_t is the duration of time period t (h), $P^{\max}$ and $P^{\min}$ are the maximum and minimum power output of the unit (MW), $P_c^{\max}$ is the maximum power that can be sold through contract c (MW), T_c is the duration of contract c (h), α is the per unit confidence level, λ_c is the energy price of forward contract c (€/MWh), $\lambda_{t\omega}^{\mathrm{P}}$ is the pool price in period t and scenario ω (€/MWh), and π_ω is the probability of occurrence of scenario ω.

N_C, N_T and N_W are the number of forward contracts, time periods and scenarios, respectively.

F_t represents the set of forward contracts available during period t.

The objective function (7) consists of maximizing the CVaR of the profit probability distribution. The risk aversion of the decision maker is modeled via the confidence level α.

Constraints (8) imposes the balance between the energy produced by the unit and energy sold both in the pool and through forward contracts. Constraints (9) and (10) enforce the generating limits of the production unit in each time period and for each scenario. The maximum power sold through each forward contract is imposed with constraints (11). Constraints (12) are needed to calculate the value of the CVaR of the profit distribution. Constraints (13)-(15) are nonnegativity variable declarations and constraints (16) are binary variable declarations.

In this stochastic model, the first-stage variables are the powers sold through the available forward contracts. Each one of these first-stage variables depends mainly on the value of each forward price compared with the pool prices throughout its delivery period. For this reason, a set of prices for each forward contract are considered to analyze the behavior of the scenario reduction techniques for high, medium and low forward contract prices. For such a purpose, a critical price is calculated as the price for which the optimal objective function value remains invariant whether or not the contract is signed. Therefore, if the price of the contract is much higher than the critical price, the contract would be signed by the producer, and it would not if the price is significantly smaller. However, if the forward price is close to its critical price, the pool price scenarios belonging to the reduced set can significantly change first-stage decisions, and therefore, the scenario reduction technique becomes a very outstanding issue.

To calculate the value of the critical price of a given forward contract two optimization problems are solved. In the first one, the power sold through this contract is fixed to 0 MW ($P_c = 0$). The value of the objective function is obtained and denoted CVaR1. In the second one, the forward contract price is set to 0 €/MWh ($\lambda_c = 0$) and the power sold fixed to the maximum power of this particular forward contract ($P_c = P_c^{\max}$). The value of the objective functions of this problem is denoted CVaR2. Taking into account that the CVaR is a coherent risk measure [14], and therefore positively homogeneous, if the price of the contract is increased a given value and the power sold through the contract is maintained to its maximum value, CVaR2 increases a quantity equal to the new price times the maximum power sold times the contract duration. Therefore, the price for which the CVaR remains invariant whether or not the contract is signed can be computed as follows:

$$\mathrm{CVaR1} = \mathrm{CVaR2} + \lambda_c^* P_c^{\max} T_c \tag{17}$$

and

$$\lambda_c^* = \frac{\mathrm{CVaR1} - \mathrm{CVaR2}}{P_c^{\max} T_c} \tag{18}$$

where $P_c^{\max}$, λ_c^* and T_c are the maximum power to be sold, the critical price and the duration of the forward contract c, respectively.

3.2 Problem Data

The case study reported below considers a planning horizon of one month. Historical data from the Spain zone of the electricity market of the Iberian Peninsula in year 2000 are used to generate 200 pool price scenarios as described in [16] using ARIMA models. For the sake of tractability, the 24 pool prices of each day are aggregated in six values per day.

The considered production unit has a maximum and minimum power outputs of 450 and 50 MW, respectively. The quadratic production cost of this unit is approximated by the set of four linear blocks characterized in Table 1 [17].

Table 1: Piecewise linear production cost for the production unit

Block #	1	2	3	4
Power block (MW)	150	250	350	450
Price (€/MWh)	12.0	12.5	13.0	13.5

The risk aversion parameter (α) of the producer varies from 0 (neutral-risk case) to 0.9 (highly risk-averse case).

Five forward contracts are available during the month: one spans the whole month and the other four span each one of the weeks of the month. The maximum quantity that can be sold through each forward contract is 200 MW for the monthly contract and 150 MW for each weekly one. The forward contract characteristics are given in Table 2.

Table 2: Forward contract quantity and duration

Contract #	Power (MW)	Duration
1	200	Month
2	150	Week 1
3	150	Week 2
4	150	Week 3
5	150	Week 4

Table 3 provides the critical price for the five available contracts for the neutral-risk case ($\alpha = 0$) and for a highly risk-averse case ($\alpha = 0.9$). Note that the higher the risk aversion of the producer, the lower the critical price of the forward contract. The reason is that a very risk-averse producer is willing to hedge its profits through a forward contract even if its price is low.

Table 3: Critical price of forward contracts (€/MWh)

Contract #	1	2	3	4	5
$\alpha = 0$	26.88	24.52	26.98	28.04	27.90
$\alpha = 0.9$	20.23	20.77	20.58	19.67	20.40

For each forward contract, ten different uniformly distributed prices between the critical prices for $\alpha = 0.9$ and $\alpha = 0$ are considered. These ten sets of contract prices are presented in Table 4.

The aim of solving ten different optimization problems (one for each set

Table 4: Price scenario reduction. Sets of prices for forward contracts (€/MWh)

	Contract #				
Set	1	2	3	4	5
1	19.73	20.27	20.08	19.17	19.90
2	20.58	20.80	20.90	20.21	20.84
3	21.43	21.33	21.72	21.25	21.79
4	22.28	21.85	22.55	22.29	22.73
5	23.13	22.38	23.37	23.33	23.68
6	23.98	22.91	24.19	24.38	24.62
7	24.83	23.44	25.01	25.42	25.57
8	25.68	23.96	25.84	26.46	26.51
9	26.53	24.49	26.66	27.50	27.46
10	27.38	25.02	27.48	28.54	28.40

of contract prices) for each value of the risk aversion α is to consider a variety of relevant cases. Although the critical prices are calculated only for $\alpha = 0$ and $\alpha = 0.9$ (extreme cases), there exists a critical price for each value of α and each contract. However, these values are between the two critical prices in Table 3, and thus, they are adequately represented using the ten price sets of Table 4.

Six values for the risk aversion parameter ($\alpha = 0, 0.2, 0.4, 0.6, 0.8, 0.9$) and ten values for contract prices (Table 4) are considered. Therefore, the three scenario reduction techniques described in Section 2.3 are used for solving sixty optimization problems. As explained in Section 2.2, the value of the risk aversion parameter α is reduced to calculate the values of the auxiliary variables η_ω used in the proposed scenario reduction algorithm. The values of $\Delta\alpha$ used in this case study are:

$$\Delta\alpha = \begin{cases} 0 & \text{if} \quad 0 < \alpha \leq 0.1 \\ 0.1 & \text{if} \quad 0.1 < \alpha \leq 0.5 \\ 0.2 & \text{if} \quad 0.5 < \alpha \leq 0.7 \\ 0.3 & \text{if} \quad 0.7 < \alpha \leq 1. \end{cases}$$

3.3 Results and discussion

Fig. 2 compares the evolution of the CVaR error with the number of scenarios obtained for each scenario reduction technique, two values of the risk aversion parameter(α) and two sets of contract prices. Techniques 2 and 3 outperform Technique 1 for both low and high values of the risk aversion parameter ($\alpha = 0.2$ and $\alpha = 0.8$). Note that for a low value of α, Techniques 2 and 3 perform similarly, whereas for the highly risk-averse case, Technique 3 reduces the error of the CVaR faster than Technique 2. The reason is that the former considers the risk aversion parameter α in the algorithm to select the reduced scenario set.

To analyze the performance of the algorithms over the 60 considered problems, a mean value of the CVaR error over the contract price sets is computed for each value of α. Fig. 3 depicts this mean value for three values of the risk aversion parameter ($\alpha = 0, 0.6, 0.9$). Techniques 2 and 3 produce reduced scenario sets that yield CVaR values closer to that obtained with the original scenario set (200 scenarios). Moreover, the higher the risk aversion parameter, the more effective Technique 3 is as compared with Technique 2.

Fig. 4 represents the mean value of the CVaR error over all values of the risk aversion parameter and forward prices for the three techniques. It

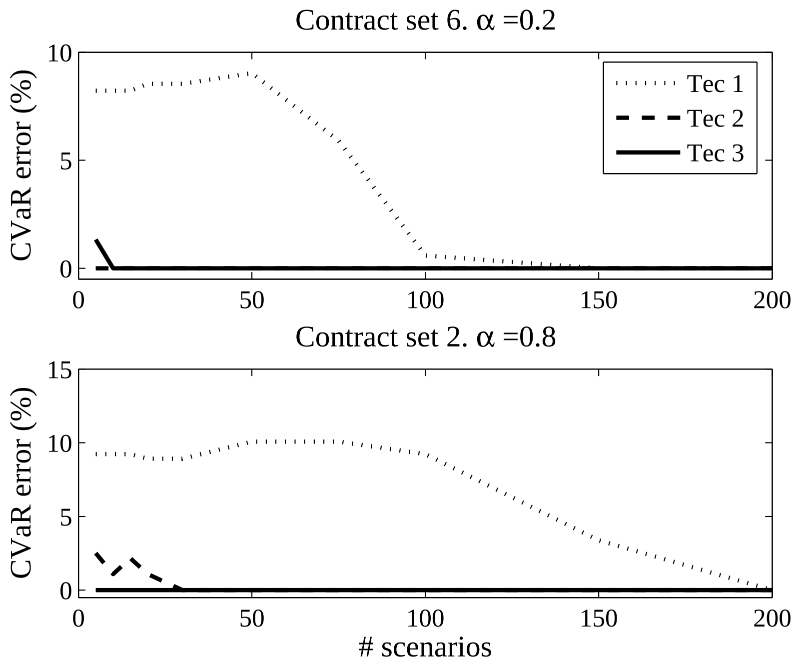

Figure 2: CVaR error evolution with the number of scenarios and selected α and contract price values.

can be concluded that if risk aversion is considered, Technique 3 results in significantly smaller CVaR errors than those obtained using the other two techniques.

4 Application 2: Producer trading model revisited

In this section, the unexpected forced outages of the production unit are considered by the producer to make its futures market decisions. The availability of the production unit is represented via a large number of 1/0 scenarios, which are reduced using the proposed scenario reduction technique.

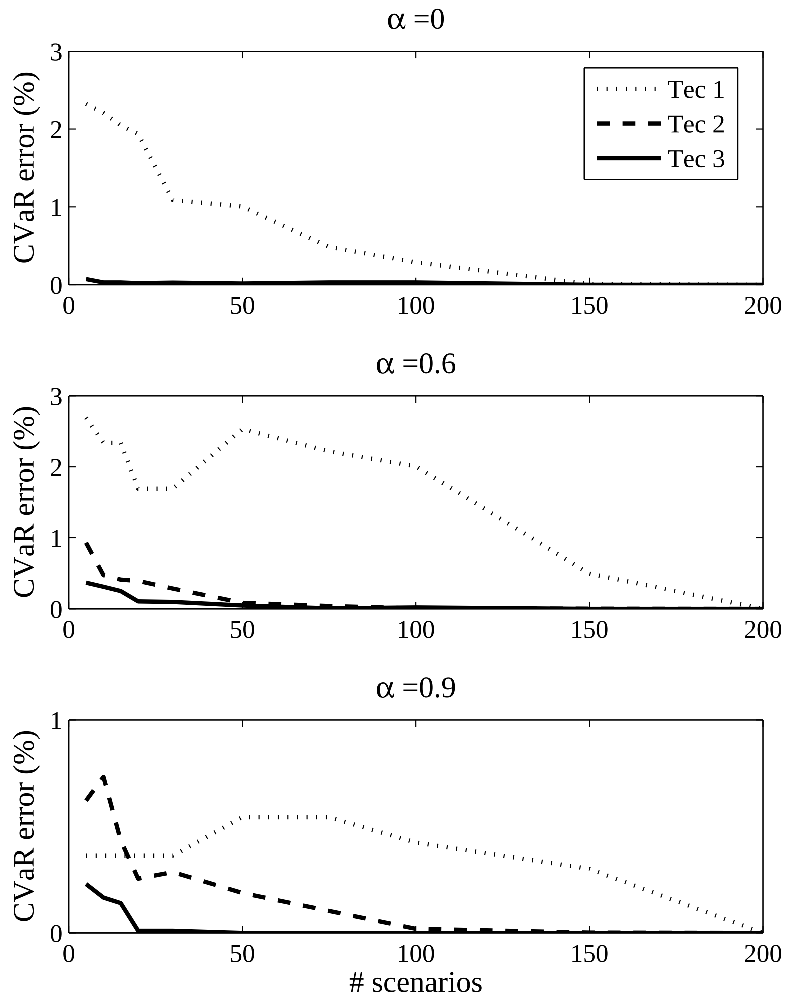

Figure 3: Average CVaR error evolution with the number of scenarios and selected α values.

4.1 Formulation

The 1/0 scenarios representing the availability of the production unit are generated based on two random variables: the time to the next failure and the time to repair. Historical data of the availability of production units prove that these random variables follow exponential distributions with mean val-

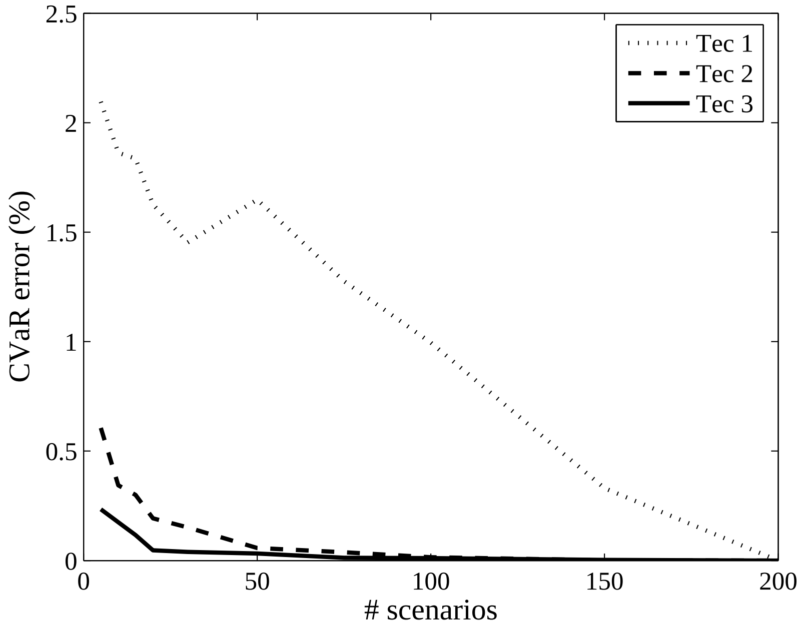

Figure 4: Average CVaR error evolution with the number of price scenarios and the three scenario reduction techniques.

ues equal to the Mean Time to Failure (MTTF) and the Mean Time to Repair (MTTR), respectively. Therefore, each availability scenario is generated simulating consecutively the time until the next failure occurs (t_{F}), and the time to repair this failure (t_{F}) until the planning horizon is covered according to the following equations:

$$t_F = -\mathrm{MTTF} \times \ln(u_1) \tag{19}$$

$$t_R = -\mathrm{MTTR} \times \ln(u_2), \tag{20}$$

where u_1 and u_2 are random variables uniformly distributed between 0 and 1. The percentage of the time that the production unit is unavailable is the Forced Outage Rate (FOR) that can be calculated as:

$$\mathrm{FOR}(\%) = \frac{\mathrm{MTTR}}{\mathrm{MTTR} + \mathrm{MTTF}} 100. \tag{21}$$

The random variable k_t represents the availability of the unit, i.e., the value of $k_{t\omega}$ is equal to 0 if the production unit has an unexpected failure during the time period t and scenario ω, and 1 otherwise. Therefore, equations (9), (10) and (15) are modified as follows:

$$P^{\mathrm{G}}_{t\omega} \leq k_{t\omega} u_{t\omega} P^{\max}, \quad \forall t, \forall \omega \tag{22}$$

$$P^{\mathrm{G}}_{t\omega} \geq k_{t\omega} u_{t\omega} P^{\min}, \quad \forall t, \forall \omega \tag{23}$$

$$P^{\mathrm{P}}_{t\omega} \geq 0, \quad \forall t, \forall \omega : k_{t\omega} = 1 \tag{24}$$

Note that constraints (22) and (23) impose that if the production unit has an unexpected failure ($k_{t\omega} = 0$), the power output is necessarily equal to 0 MW. However, if the production unit is available ($k_{t\omega} = 1$), the power output can take any value between $P^{\min}$ and $P^{\max}$ ($u_{t\omega} = 1$) or be equal to 0 MW ($u_{t\omega} = 0$). Constraints (24) allows the producer to buy energy in the pool during those periods and scenarios in which the unit is unavailable.

The stochastic optimization model (7)–(8), (11)–(14), (16), and (22)–(24) is used by a risk-averse producer to determine the power to be sold through forward contracts taking into account the unexpected failures of the production unit [18].

An important issue to apply the proposed algorithm to reduce discrete availability scenarios is the definition of the average scenario used to compute the first-stage decisions associated with the deterministic problem. Selecting an adequate average value scenario is important for the good performance of the proposed technique, because the single-scenario problems which determine the value of η_ω are solved fixing the first-stage decision to those obtained for the deterministic problem associated with the average scenario. An appropriate manner to generate the average availability scenario is to consider that the time between two consecutive failures and the time to repair a failure are equal to their mean values, i.e., MTTF and MTTR, respectively. Therefore, the average scenario used to solve the deterministic problem is obtained as:

$$\overline{\boldsymbol{\lambda}} = [\,\underbrace{1\,1\,1\,...\,1\,1}_{\text{MTTF ones}}\;\underbrace{0\,0\,0\,...\,0\,0}_{\text{MTTR zeros}}\;\ldots\;\underbrace{1\,1\,1\,...\,1\,1}_{\text{MTTF ones}}\,].$$

4.2 Problem Data

As in the previous case study, the considered planning horizon expands one month. The scenario reduction technique is used to reduce the set of scenarios representing the availability of one production unit. For this case study, 20 pool price scenarios are used in all cases.

Three different values of the Forced Outage Rate (FOR) are considered: 5, 10 y 20%. The data presented in Table 5 are used to generate 50 availability scenarios for each value of FOR.

Table 5: Unit availability data

FOR (%)	MTTF (h)	MTTR (h)
5	500	26
10	400	45
20	300	75

Once pool price as well as availability scenarios are generated, the final set of scenarios is obtained by combining all possible availability scenarios with all possible price scenarios. Since the 24 hourly pool prices of each day are aggregated in six periods per day, the availability of the production unit in each period and scenario is calculated as the average value of the corresponding hourly availabilities. Note that this average value has to be rounded to 0 or 1. Besides the value of FOR, the rest of characteristic of the production unit are identical to those presented in Section 3.2.

The same five forward contracts as in Section 3.2 are considered in this case study. However, the values of the critical prices change due to the forced outage rate of the production unit. Ten uniformly distributed contract prices are determined for each contract between its highest critical price ($\alpha = 0$ and FOR = 5%) and its lowest one ($\alpha = 0.9$ and FOR = 20%). Table 6 contains the resulting 10 forward prices sets.

Table 6: Availability scenario reduction. Sets of prices for forward contracts (€/MWh)

	Contract #				
Set	1	2	3	4	5
1	17.0	15.0	16.8	18.5	19.0
2	18.0	16.0	17.8	19.5	20.0
3	19.0	17.0	18.8	20.5	21.0
4	20.0	18.0	19.8	21.5	22.0
5	21.0	19.0	20.8	22.5	23.0
6	22.0	20.0	21.8	23.5	24.0
7	23.0	21.0	22.8	24.5	25.0
8	24.0	22.0	23.8	25.5	26.0
9	25.0	23.0	24.8	26.5	27.0
10	26.0	24.0	25.8	27.5	28.0

The three scenario reduction techniques are used to reduce the set of scenarios representing the availability of the unit and to compute the CVaR error in 180 optimization problems (3 values of FOR, 6 risk aversion parameters and 10 sets of forward prices). Note that to calculate the first-stage decisions associated with a reduced availability scenario set, the original 20 pool price scenario set is always considered.

4.3 Results and discussion

The evolution of the CVaR error obtained with the three scenario reduction techniques and for three values of forced outage rate, risk aversion parameters and forward price sets are depicted in Fig. 5. Techniques 2 and 3 outperform Technique 1 in all cases. However, as the risk aversion parameter α becomes higher, Technique 3 performs more efficiently than Technique 2.

Fig. 6 represents for each value of FOR the evolution of the average CVaR error over all values of forward prices and risk aversion parameters. For a forced outage rate equal to 5%, the probability of a unexpected failure is so small that the CVaR error is low even for a small set of availability scenarios (note that the scale of the vertical axis of all plots are identical). Therefore, no conclusions can be drawn regarding the scenario reduction techniques. On the other hand, for higher FOR values (10% and 20%) the CVaR error associated with the reduced availability scenario sets is high enough to compare the three scenario reduction techniques. In this regard, Techniques 2 and 3 yield notably lower values of the CVaR error than Technique 1.

Fig. 7 contains, for some values of the risk aversion parameter α, the average CVaR error over all forward prices and forced outage rates. As

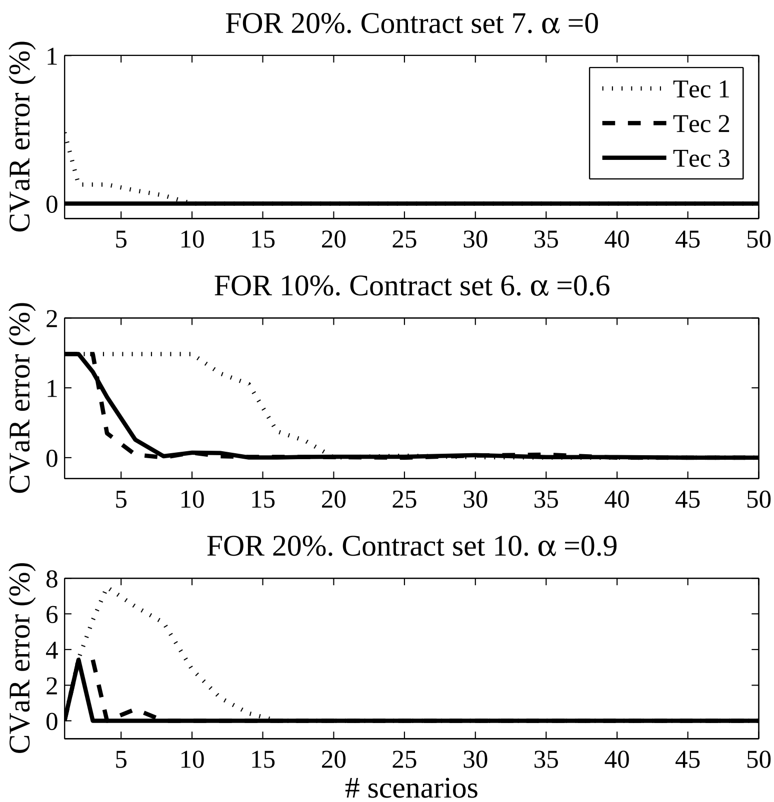

Figure 5: CVaR error evolution with the number of availability scenarios and selected α, FOR and contract price values.

explained in Section 2.2, Techniques 2 and 3 are equivalent in the neutral-risk case ($\alpha = 0$). For $\alpha = 0.6$, Techniques 2 and 3 result in similar values for the CVaR error, presenting a better performance than Technique 1. However, the proposed technique outperforms the other techniques for high values of the risk aversion parameter, as $\alpha = 0.9$.

An average value of the CVaR error is calculated considering the 180 optimization problems considered. Fig. 8 depicts the evolution of this mean CVaR error with respect to the cardinality of the reduced availability scenario set. Note that the scenario reduction technique proposed in this paper

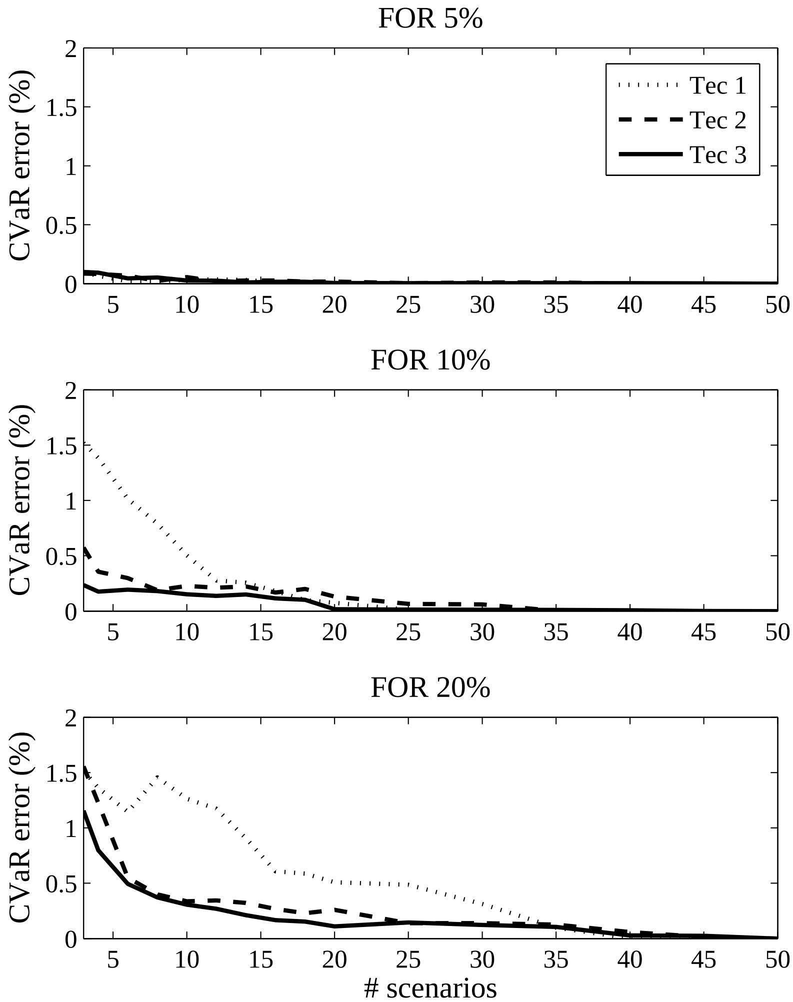

Figure 6: Average CVaR error evolution with the number of availability scenarios for selected FOR values.

reduces the original availability scenario set more effectively than the other two techniques described in the literature.

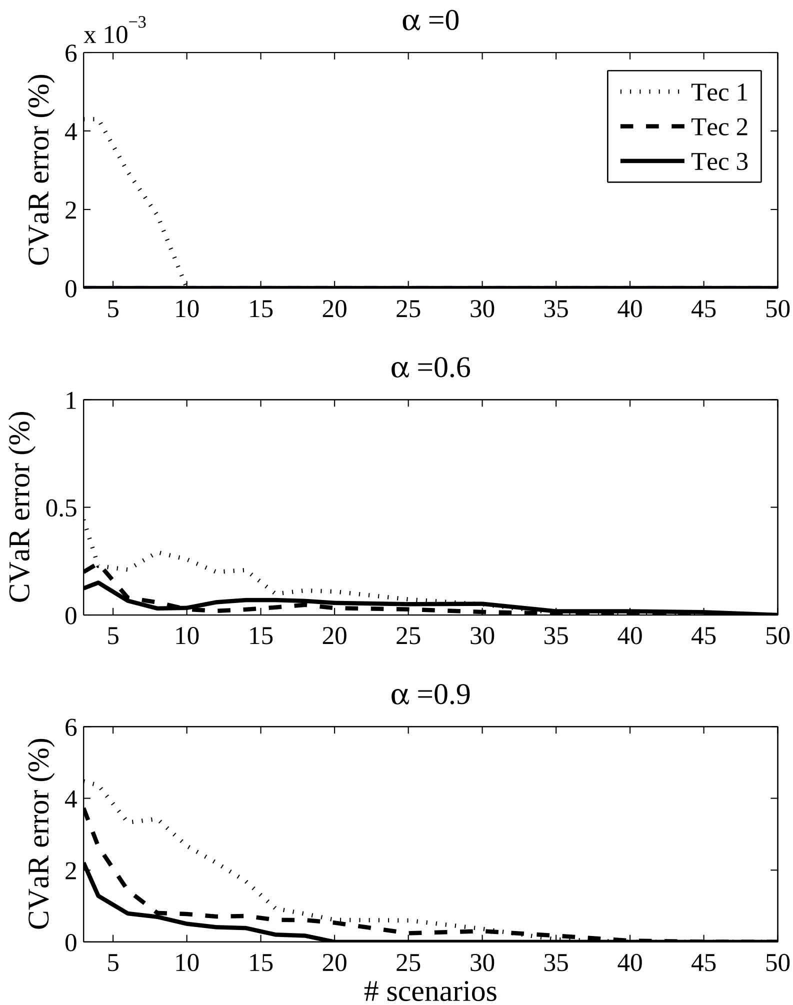

Figure 7: Average CVaR error evolution with the number of availability scenarios for selected α values.

5 Conclusion

Electricity market agents use stochastic optimization models to make informed trading decisions. These models describe uncertain parameters through scenarios.

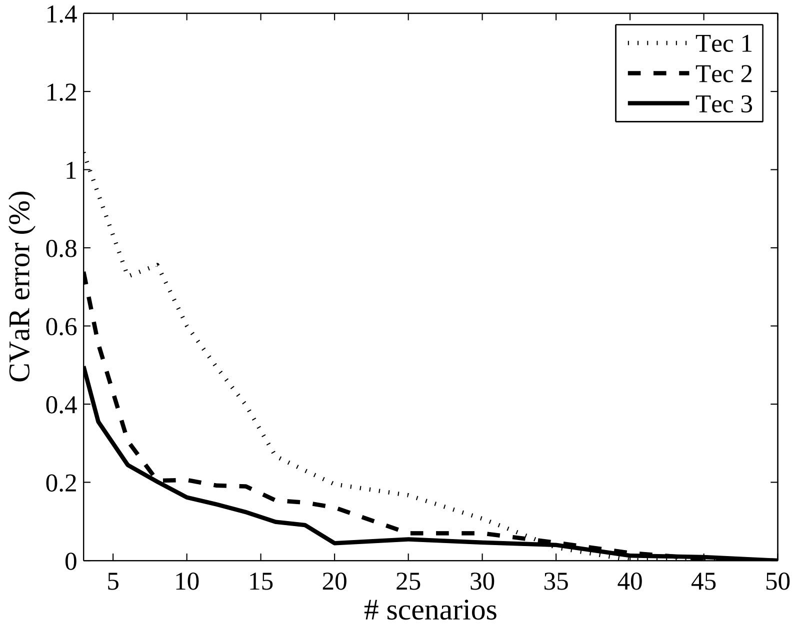

Figure 8: Average CVaR error evolution with the number of availability scenarios for the three scenario reduction techniques.

Due to the large number of scenario required to represent the uncertain parameters of stochastic optimization models, a scenario reduction technique is required to make the corresponding problems tractable while keeping as much as possible the information contained in the original set of scenarios.

This paper proposes a scenario reduction technique which explicitly considers the risk aversion of the decision maker in reducing the number of scenarios. The proposed technique proves effective to reduce scenarios pertaining to both continuous and discrete stochastic processes.

Computational simulations involving decision making problems in electricity markets allow concluding that the proposed technique is computationally efficient and able to produce reduced scenario sets that result in decisions identical or very close to those pertaining to the original scenario set.

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