

Impact of Equipment Failures and Wind Correlation on Generation Expansion Planning

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Abstract

Generation expansion planning has become a complex problem within a deregulated electricity market environment due to all the uncertainties affecting the profitability of a given investment. Current expansion models usually overlook some of these uncertainties in order to reduce the computational burden. In this paper, we raise a flag on the importance of both equipment failures (units and lines) and wind power correlation on generation expansion decisions. For this purpose, we use a bilevel stochastic optimization problem, which models the sequential and noncooperative game between the generating company (GENCO) and the system operator. The upper-level problem maximizes the GENCO's expected profit, while the lower-level problem simulates an hourly market-clearing procedure, through which LMPs are determined. The uncertainty pertaining to failures and wind power correlation are characterized by a scenario set, and their impact on generation expansion decisions are quantified and discussed for a 24-bus power system.

Keywords: generation expansion, optimal location, bilevel programming, stochastic programming, market clearing.

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1. Notation

1.1. Indexes and sets

b	Index of load blocks.
g/g'	Index of existing/new conventional generating units.
n, m	Index of buses.
s	Index of scenarios.
w/w'	Index of existing/new wind farms.
Γ	Set of existing wind farms owned by GENCO.
Θ_n	Set of wind farms connected to bus n .
Φ	Set of existing conventional generating units owned by GENCO.
Ψ_n	Set of conventional generating units connected to bus n .
Ω_n	Set of buses connected to bus n .

1.2. Constants

B_{nm}	Susceptance of line connecting buses n and m (p.u.).
$C_{g/g'}$	Production cost of conventional generating unit g/g' (\$/MWh).
F_{nm}^{max}	Capacity of line connecting buses n and m (MW).
$k_{g/g's}$	Status of unit g/g' in scenario s (1 if available, 0 otherwise).
k_{nms}	Status of line $n - m$ in scenario s (1 if available, 0 otherwise).
L_b	Load percentage (with respect to peak load) of block b (p.u.).
L_{nb}	Load at bus n corresponding to block b (MW).
L_n^{peak}	Peak load at bus n (MW).
$N_{w/w'}^T$	Number of wind turbines of wind farm w/w' .

$P_{g/g'}^{max}$	Capacity of conventional generating unit g/g' (MW).
$Q_{g'/w'n}$	Annualized investment cost of building unit g'/w' at bus n (\$).
T_b	Duration of load block b (h).
W_{ws}	Power output of wind farm w in scenario s (MW).
W_{ws}^{ST}	Power output of a single turbine of wind farm w in scenario s (MW).
$W'_{w'ns}$	Power output of wind farm w' in scenario s if placed at bus n (MW).
$W_{w'ns}^{ST}$	Power output of a single turbine within wind farm w' in scenario s if located at bus n (MW).
V^L	Value of shed load (\$/MWh).
π_s	Probability of scenario s .

1.3. Variables

L_{nsb}^S	Amount of load shed from load block b at bus n in scenario s (MW).
$P_{g/g'sb}$	Power generated by unit g/g' in scenario s and load block b (MW).
$u_{g'/w'n}$	Binary variable equal to 1 if unit g'/w' is placed at bus n .
$W_{w/w'sb}^S$	Wind spillage of farm w/w' in scenario s and load block b (MW).
δ_{nsb}	Voltage angle at bus n , scenario s , and load block b (rad).
λ_{nsb}	Locational marginal price at bus n , scenario s , and load block b (\$/MWh).
Π_{sb}	GENCO's profit in scenario s and load block b (\$).

2. Introduction

The rapid growth of electricity demand in developed countries have turned the generation and transmission expansion planning into a key component in the long-term operation of power systems. In a *vertically integrated* electricity supply industry, both generation and transmission expansion decisions are *centrally undertaken to minimize the total cost*, including investment and operation cost, and/or maximize the reliability and security of the network. A review of the main methods for generation and transmission planning is presented in [1] and [2], respectively. Most of these models are formulated as one-level optimization problems involving a relatively low computational burden, even if uncertainties pertaining to demand level, wind power production, and equipment availability are accounted for [3, 4, 5, 6].

On the other hand, since the *liberalization* of the electricity sector in many countries around the world, new generation expansion models are required to determine GENCO's investment decisions according to *profit maximization criteria* [7]. In this framework, GENCOs have to evaluate their expansion decisions according to the profits they make in the wholesale electricity market, which is cleared by an independent system operator (ISO) aiming, in turn, at minimizing the generation supply cost to satisfy electricity demand. Moreover, market outcomes are directly influenced by GENCOs' expansion decisions. It is easy to see that this problem falls within the framework of bilevel programming, in which two decision makers (GENCO and ISO), each with their individual objectives (maximize GENCO's profit and minimize system cost, respectively), act and react in a noncooperative sequential manner [8].

In this line, the bilevel model proposed in [9] calculates the optimal investment decisions for a producer to maximize its expected profit using a Benders decomposition approach. Similarly, reference [10] proposes a bilevel model that determines the expansion decisions of a producer via a conjectured-price response formulation. References [11] and [12] analyze the generation capacity expansion including renewable energy sources (RES). In [11], RES incentives

such as feed-in tariff and quota obligation systems are considered. The stochastic bilevel model proposed in [12] aims at identifying the new wind farms to be built to maximize the wind producer's profit. Equilibrium models that analyze the generation expansion competitive behavior among GENCOs are discussed in [13] and [14]. Finally, reference [15] proposes an iterative algorithm to analyze the coordination transmission and generation expansion made by the ISO and market agents.

Conversely to models that centrally determine the optimal expansion strategy, previously mentioned bilevel expansion models for GENCOs participating in electricity markets usually involve a significantly high computational complexity. In fact, although some of them include uncertainties corresponding to rival decisions [10, 14], demand variation [16], or wind power production [17], all of them disregard, to the best of our knowledge, the effect of equipment availability and wind correlation.

In this paper, we prove that a GENCO's optimal generation expansion decisions may be significantly influenced by the uncertainty corresponding to equipment failures and wind power correlation at different geographical locations. To this end, we use a bilevel formulation in which the upper-level problem maximizes the GENCO's expected profit including both pool revenue and investment cost. Pool revenue is computed according to the locational marginal prices (LMP) and dispatched quantities resulting from the lower-level market-clearing problem which, in turn, aims at minimizing expected operational cost while ensuring the continuous balance between production and consumption and the fulfillment of the network constraints. Using the primal-dual theory, a single-level mixed-integer equivalent formulation that can be readily solved by off-the-shelf optimization software is obtained.

The main contributions of this paper are thus twofold:

1. To quantify the impact of unit and line failures on optimal generation expansion decisions of a GENCO.
2. To quantify the impact of wind power correlation at different sites on optimal

generation expansion decisions of a GENCO.

This paper is organized as follows. Section 3 elaborates on the model assumptions. The bilevel optimization problem and its equivalent MILP formulation are described in Section 4. In Section 5, the results of a 24-bus case study are provided. The main conclusions of this work are discussed in Section 6. Finally, Appendix A contains linearization technicalities.

3. Model assumptions

Alternatively to dynamic expansion planning, static models are computationally more manageable because expansion decisions are determined for a single future target year. One-year static expansion models have been used for generation [9, 12, 14, 18], transmission [19, 20], as well as combined generation and transmission planning [5]. Since this paper primarily focuses on quantifying the effect of equipment failures and wind speed correlation on GENCO's expansion planning, a single future target year is considered. A dynamic approach would drastically increase the computational complexity of the investment model and add little value to the purpose of this paper.

In order to take advantage of economies of scale, typical constructions of new units are given in discrete and relatively large amounts. We model this fact by just considering a finite set of possible new investments and modeling expansion decisions through the binary variables $u_{g'n}$ and $u_{w'n}$, which are equal to 1 if unit g' and wind farm w' , respectively, are placed at bus n , and 0 otherwise. $P_{g'}^{max}$ and $C_{g'}$ represent the capacity and marginal cost of new units. Likewise, new wind farms are only characterized by its capacity $P_{w'}^{max}$, being their marginal cost equal to 0. Annualized investment cost of new units and wind farms are denoted by $Q_{g'n}$ and $Q_{w'n}$, in that order.

Assuming a perfectly competitive market, selling offers submitted by producers exactly represent their corresponding marginal costs [21]. Moreover, for the sake of simplicity, we suppose that all the revenues produced by new and old units come solely from the sale of electrical energy in the wholesale market, then

disregarding additional revenues associated with ancillary services or capacity payments.

In order to render the subsequent analysis and discussion more intuitive, the modeling of uncertainties different from equipment availability and wind speed have not been considered here.

Wind speed stochastic behavior is modeled using historical data at different geographical locations. Once the wind speed scenarios are selected, the power production of a single wind turbine (W_{ws}^{ST} and $W_{w'ns}^{ST}$) is computed according to its power curve. Thus, the power production of existing and new wind farms is determined as

$$W_{ws} = N_w^T W_{ws}^{ST}; \quad W_{w'ns} = N_{w'}^T W_{w'ns}^{ST}. \quad (1)$$

The availability status of existing/new units and transmission lines are modeled through parameters k_{gs} , $k_{g's}$, and k_{nms} , respectively, being equal to 1 if the corresponding device is available in scenario s , and 0 otherwise. In order to keep the model computationally tractable, only single-element contingencies are considered [21, 22]. As an example, the probability that unit g_1 fails is determined as

$$\text{FOR}_{g_1} \cdot \prod_{g \neq g_1} (1 - \text{FOR}_g) \cdot \prod_{g'} (1 - \text{FOR}_{g'}) \cdot \prod_{nm: m \in \Omega_n} (1 - \text{FOR}_{nm}) \quad (2)$$

where $\text{FOR}_{g/g'/nm}$ represents the probability of having an individual unexpected failure of device $g/g'/nm$. Since not all the possible contingencies are considered, the probability calculated in (2) has to be normalized. Note, however, that including multi-device failure scenarios in the analysis is straightforward. For simplicity, unexpected failures of wind farms are not considered in this analysis.

The lower-level optimization problem consists of an hourly market-clearing algorithm that minimizes the expected operational cost including DC power flow equations as well as maximum capacity constraints for both generating units and transmission lines. The system reliability cost is modeled through the value of shed load V^L . Electricity consumption is considered to be known,

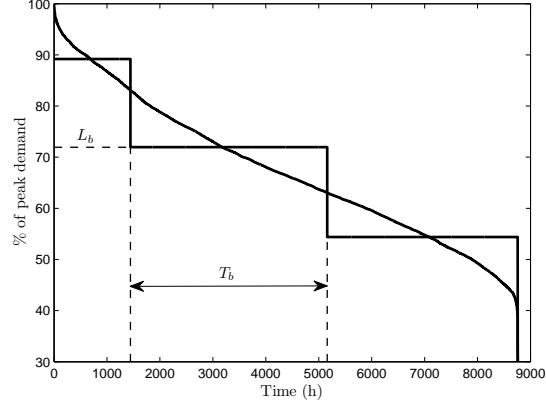


Figure 1: Load duration curve approximation

inelastic, and uncorrelated with the wind power production. The hourly load duration curve is approximated by N_B blocks (as depicted in Fig.1) in order to account for the load variability throughout the target year. Assuming that the load is proportionally distributed among all buses, the load at each bus n and block b is computed as

$$L_{nb} = L_b \cdot L_n^{peak}, \quad \forall n, \forall b, \quad (3)$$

where L_n^{peak} is the peak load at each bus of the network.

4. Model Formulation

The bilevel stochastic optimization problem used to determine the impact of equipment failures and wind correlation on the GENCO's expansion planning decisions is presented below.

$$\begin{aligned} & \text{Maximize}_{u_{g'n}, u_{w'n}} \\ & \sum_{sb} \pi_s T_b \Pi_{sb} - \sum_n \left(\sum_{g'} u_{g'n} Q_{g'n} + \sum_{w'} u_{w'n} Q_{w'n} \right) \end{aligned} \quad (4a)$$

subject to

$$\sum_n u_{g'n} \leq 1, \quad \forall g' \quad (4b)$$

$$\sum_n u_{w'n} \leq 1, \quad \forall w' \quad (4c)$$

$$\begin{aligned} \Pi_{sb} = & \sum_{g \in \Phi} (\lambda_{nsb: g \in \Psi_n} - C_g) P_{gsb} + \sum_{w \in \Gamma} \lambda_{nsb: w \in \Theta_n} (W_{ws} - W_{wsb}^S) + \\ & + \sum_{g'n} u_{g'n} (\lambda_{nsb} - C_{g'}) P_{g'sb} + \sum_{w'n} u_{w'n} \lambda_{nsb} (W'_{w'ns} - W_{w'sb}^S), \quad \forall s, \forall b \end{aligned} \quad (4d)$$

$$(P_{g/g'sb}, W_{w/w'sb}^S, \lambda_{nsb}) \in \arg \left\{ \text{Min} \sum_g C_g P_{gsb} + \sum_{g'} C_{g'} P_{g'sb} + \sum_n V^L L_{nsb}^S \right. \quad (4e)$$

subject to

$$0 \leq P_{gsb} \leq k_{gs} P_g^{max} : \phi_{gsb}^{min}, \phi_{gsb}^{max}, \quad \forall g \quad (4f)$$

$$0 \leq P_{g'sb} \leq k_{g's} P_{g'}^{max} : \phi_{g'sb}^{min}, \phi_{g'sb}^{max}, \quad \forall g' \quad (4g)$$

$$0 \leq L_{nsb}^S \leq L_{nb} : \beta_{nsb}^{min}, \beta_{nsb}^{max}, \quad \forall n \quad (4h)$$

$$0 \leq W_{wsb}^S \leq W_{ws} : \gamma_{wsb}^{min}, \gamma_{wsb}^{max}, \quad \forall w \quad (4i)$$

$$0 \leq W_{w'sb}^S \leq \sum_n u_{w'n} W'_{w'ns} : \gamma_{w'sb}^{min}, \gamma_{w'sb}^{max}, \quad \forall w' \quad (4j)$$

$$\begin{aligned} & \sum_{g \in \Psi_n} P_{gsb} + \sum_{g'} u_{g'n} P_{g'sb} + \sum_{w \in \Theta_n} (W_{ws} - W_{wsb}^S) + \sum_{w'} u_{w'n} (W'_{w'ns} - W_{w'sb}^S) = \\ & = L_{nb} - L_{nsb}^S + \sum_{m \in \Omega_n} B_{nm} k_{nms} (\delta_{nsb} - \delta_{msb}) : \lambda_{nsb}, \quad \forall n \end{aligned} \quad (4k)$$

$$B_{nm} k_{nms} (\delta_{nsb} - \delta_{msb}) \leq k_{nms} F_{nm}^{max} : \theta_{nmsb}^{max}, \forall n, m \quad (4l)$$

$$\delta_{n_1 sb} = 0 : \xi_{n_1 sb} \left. \vphantom{\sum} \right\}, \quad \forall s, \forall b. \quad (4m)$$

Model (4a)–(4m) is a bilevel optimization problem. The upper-level objective function (4a) aims at maximizing the GENCO's expected profit subject to a set of lower-level optimization problems, (4e)–(4m), representing an hourly pool-based market clearing, one for each scenario s and load block b . Objective function (4a) includes the profit from selling electricity in the pool (first term) and the investment cost of new units and wind farms (second term). Equations (4b) and (4c) impose that each unit and wind farm can be either placed at one single bus or not built at all. Equation (4d) computes the pool profit for each

scenario s and load block b as the sum of the profit corresponding to existing units (first term), existing wind farms (second term), new units (third term), and new wind farms (fourth term).

The lower-level objective function (4e) minimizes the system cost for each scenario s and load block b . Constraints (4f) and (4g) limit the output of each unit. Constraints (4h), (4i) and (4j) limit, respectively, the load shedding at each bus and the wind spillage of existing and new wind farms. Constraint (4k) is the power balance equation at each bus. Equation (4l) imposes the maximum power flow through the lines. Equation (4m) arbitrarily sets the value of angle δ_{n_1} to 0.

To solve the bilevel optimization problem (4a)–(4m) we replace each lower-level problem by its corresponding primal and dual constraints plus the strong duality theorem [20, 23].

$$\begin{aligned} & \text{Maximize}_{u_{g'n}, u_{w'n}} \\ & \sum_{sb} \pi_s T_b \Pi_{sb} - \sum_n \left(\sum_{g'} u_{g'n} Q_{g'n} + \sum_{w'} u_{w'n} Q_{w'n} \right) \end{aligned} \quad (5a)$$

subject to

$$(4b) - (4d)$$

$$(4f) - (4m)$$

$$\lambda_{nsb: g \in \Psi_n} + \phi_{gsb}^{max} + \phi_{gsb}^{min} = C_g, \quad \forall g, \forall s, \forall b \quad (5b)$$

$$\sum_n u_{g'n} \lambda_{nsb} + \phi_{g'sb}^{max} + \phi_{g'sb}^{min} = C_{g'}, \quad \forall g', \forall s, \forall b \quad (5c)$$

$$\lambda_{nsb} + \beta_{nsb}^{max} + \beta_{nsb}^{min} = V^L, \quad \forall n, \forall s, \forall b \quad (5d)$$

$$-\lambda_{nsb: w \in \Theta_n} + \gamma_{wsb}^{max} + \gamma_{wsb}^{min} = 0, \quad \forall w, \forall s, \forall b \quad (5e)$$

$$-\sum_n u_{w'n} \lambda_{nsb} + \gamma_{w'sb}^{max} + \gamma_{w'sb}^{min} = 0, \quad \forall w', \forall s, \forall b \quad (5f)$$

$$\sum_{m \in \Omega_n} B_{nm} (\lambda_{msb} - \lambda_{nsb} + \theta_{nmsb}^{max} - \theta_{mnsb}^{max}) + \xi_{n_1 sb} = 0, \forall nsb \quad (5g)$$

$$\phi_{gsb}^{max}, \phi_{g'sb}^{max}, \beta_{nsb}^{max}, \theta_{mnsb}^{max}, \gamma_{wsb}^{max}, \gamma_{w'sb}^{max} \leq 0 \quad (5h)$$

$$\phi_{gsb}^{min}, \phi_{g'sb}^{min}, \beta_{nsb}^{min}, \gamma_{wsb}^{min}, \gamma_{w'sb}^{min} \geq 0 \quad (5i)$$

$$\begin{aligned}
& \sum_g C_g P_{gsb} + \sum_{g'} C_{g'} P_{g'sb} + \sum_n V^L L_{nsb}^S = \sum_g \phi_{gsb}^{max} k_{gs} P_g^{max} + \\
& + \sum_{g'} \phi_{g'sb}^{max} k_{g's} P_{g'}^{max} + \sum_n \beta_{nsb}^{max} L_{nb} + \sum_w \gamma_{wsb}^{max} W_{ws} + \\
& + \sum_{w'n} \gamma_{w'sb}^{max} u_{w'n} W_{w'ns} + \sum_{n,m \in \Omega_n} k_{nms} F_{nm}^{max} \theta_{nmsb}^{max} + \\
& + \sum_n \lambda_{nsb} \left(L_{nb} - \sum_{w \in \Theta_n} W_{ws} - \sum_{w'n} u_{w'n} W_{w'ns} \right), \forall s, \forall b. \tag{5j}
\end{aligned}$$

Equations (4b)–(4d) are the constraints corresponding to the upper-level problem. Equations (4f)–(4m) represent the primal constraints of the lower-level problem. Likewise, equations (5b)–(5i) correspond to the constraints of the dual formulation of the lower-level problem. Finally, constraints (5j) ensure that the primal and dual formulation of the lower-level problems reach the same objective function at the optimal solution. The formulation above contains several non-linear terms:

1. Product of binary variables and continuous variables: $u_{g'n} \lambda_{nsb}$ in equation (5c), $u_{g'n} P_{g'sb}$ and $u_{w'n} W_{w'sb}^S$ in equation (4k), $u_{w'n} \lambda_{nsb}$ in equation (5f), and $u_{w'n} \gamma_{nsb}^{max}$ in equation (5j). Note that these expressions can be linearized as explained in Appendix A [24].
2. Product of two continuous variables: $\lambda_{nsb:g \in \Psi_n} P_{gsb}$, $\lambda_{nsb:w \in \Theta_n} W_{wsb}^S$, $u_{g'n} \lambda_{nsb} P_{g'sb}$ and $u_{w'n} \lambda_{nsb} W_{w'sb}^S$, all of them in equation (4d). The procedure to linearize these terms using KKT conditions is explained in Appendix A.

In doing so, optimization model (5a)–(5j) can be equivalently formulated as a mixed-integer linear optimization problem that can be solved using commercial software.

5. Case study

5.1. Data

The IEEE RTS 24-bus system is analyzed in this section [25]. The data corresponding to already existing units and transmission lines are provided in

Table 1 and Table 2, respectively. Table 3 provides the peak load at each bus. For the sake of simplicity, only two load blocks are considered: $L_{b_1} = 0.95$, $T_{b_1} = 4380\text{h}$ and $L_{b_2} = 0.85$, $T_{b_2} = 4380\text{h}$. It is worth mentioning that under any single-device failure scenario, no load shedding occurs in the system, i.e., the power system satisfies the N-1 reliability criterion. The planning horizon spans one year for all case studies.

Table 1: Generating unit data

n	$\#units$	P_g^{max}	C_g	FOR_g	n	$\#units$	P_g^{max}	C_g	FOR_g
n_1	2	20	43.5	10	n_{15}	1	155	11.5	4
n_1	2	76	14.4	2	n_{16}	1	155	11.5	4
n_2	2	20	43.5	10	n_{18}	1	400	6.0	12
n_2	2	76	14.4	2	n_{21}	1	400	6.0	12
n_7	3	100	23.0	4	n_{22}	6	50	0.0	1
n_{13}	3	197	22.1	5	n_{23}	2	155	11.5	4
n_{15}	5	12	27.6	2	n_{23}	1	350	11.4	8

5.2. Impact of unit and line failures on capacity expansion

The impact of unit and line failures on the expansion of conventional units is analyzed and discussed next. For simplicity, no wind farms are included in the network. Unavailability rates of generating units are provided in Table 1, while the unavailability rate of all transmission lines is considered equal to 2%. Assuming that new units can be built at any bus of the transmission grid, the effect of failures on GENCO's profit is computed as follows:

1. Optimization problem (5a)–(5j) is solved considering that all units and lines are available, denoting the locations of the new units as B^{NF} .
2. GENCO's expected profit if expansion decisions are made without considering failures, $\bar{\Pi}^{NF}$, is determined by solving (5a)–(5j) with the complete availability scenario set and the new unit locations fixed to B^{NF} .

Table 2: Network data

nm	B_{nm}	F_{nm}^{max}	nm	B_{nm}	F_{nm}^{max}	nm	B_{nm}	F_{nm}^{max}	nm	B_{nm}	F_{nm}^{max}
n_1n_2	68.5	50	n_6n_{10}	15.6	150	$n_{11}n_{14}$	23.5	200	$n_{16}n_{17}$	38.0	200
n_1n_3	4.4	75	n_7n_8	15.3	200	$n_{12}n_{13}$	20.5	225	$n_{16}n_{19}$	42.7	300
n_1n_5	11.0	100	n_8n_9	5.7	100	$n_{13}n_{23}$	10.2	200	$n_{17}n_{18}$	69.9	125
n_2n_4	7.4	75	n_8n_{10}	5.7	100	$n_{13}n_{23}$	11.3	125	$n_{17}n_{22}$	9.4	75
n_2n_6	4.9	150	n_9n_{11}	11.9	150	$n_{14}n_{16}$	16.8	250	$n_{18}n_{21}$	75.8	275
n_3n_9	7.9	125	n_9n_{12}	11.9	175	$n_{15}n_{16}$	58.1	200	$n_{19}n_{20}$	49.3	275
n_3n_{24}	11.9	175	$n_{10}n_{11}$	11.9	200	$n_{15}n_{21}$	40.2	275	$n_{20}n_{23}$	89.3	400
n_4n_9	9.0	75	$n_{10}n_{12}$	11.9	225	$n_{15}n_{24}$	18.9	175	$n_{21}n_{22}$	14.5	75
n_5n_{10}	10.6	75	$n_{11}n_{13}$	20.5	325						

Table 3: Peak load data

n	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	n_{13}	n_{14}	n_{15}	n_{16}	n_{18}	n_{19}	n_{20}
L_n^{peak}	108	97	180	74	71	137	125	171	174	194	265	194	316	100	334	182	128

3. Optimization problem (5a)–(5j) is solved again including all the availability scenarios. The optimal location and expected profit are denoted by B^F and $\bar{\Pi}^F$, respectively.
4. The impact of failures on expansion decisions is measured through the difference between $\bar{\Pi}^{NF}$ and $\bar{\Pi}^F$, which is denoted as $\Delta\bar{\Pi}$.

The above procedure was carried out for a high number of different generating units to be built. In a significant number of the analyzed cases, GENCO's expected profit substantially decreases if equipment failures are disregarded. A sample of these cases is provided in Table 4. For simplicity, the expected profit (expressed in \$million) is computed assuming that the GENCO only owns the new unit and the annualized investment cost is proportional to its capacity (\$400/kW to be paid in 40 years) and independent of its location.

Table 4: Impact of failures on expansion planning (case study)

$P_{g'}^{max}$	$C_{g'}$	$Q_{g'n}$	B^{NF}	$\bar{\Pi}^{NF}$	B^F	$\bar{\Pi}^F$	$\Delta\bar{\Pi}$
55	19	0.55	n_7	1.3524	n_8	1.6018	18.4%
55	20	0.55	n_7	0.8768	n_8	1.1292	28.8%
55	21	0.55	n_7	0.4012	n_8	0.6567	63.7%
60	20	0.60	n_7	0.9494	n_8	1.0763	13.4%
68	20	0.68	n_{13}	0.6605	n_8	1.0932	65.5%

Note that in spite of the low values of FORs, GENCO's expected profit can decrease up to 65% if unexpected failures are disregarded. Moreover, observe how small variations of $P_{g'}^{max}$ or $C_{g'}$ significantly impact the magnitude of the failures effect.

5.3. Impact of wind speed correlation on expansion planning

We analyze below the influence of wind speed correlation on the optimal location of new wind farms owned by a GENCO. For the sake of simplicity, equipment failures are not accounted for. Moreover, new wind farms can only be located at certain nodes of the network, namely, n_1 , n_2 , n_7 , and n_8 . Wind speed data of 2006 provided by the National Renewable Energy Laboratory (NREL) at four different locations is employed in this analysis. The coordinates of the selected sites are shown in Table 5. These data can be freely downloaded from [26], and further information on the software employed to simulate these wind power data can be found in [27]. New wind farms will be comprised of 2.5-MW wind generators, model Nordex N80/2500 with a hub height of 105m. The power curve of this turbine model can be found in [28].

A representative set of 200 wind speed scenarios is used to characterize the correlated wind speed at the four selected locations, being the correlation coefficients those provided in Table 6. On the other hand, to isolate the effect of wind correlation on expansion decisions, the marginal distribution of the wind speed at each site should be maintained. For this reason, an uncorrelated

Table 5: Site coordinates for wind data

n	Coordinates	n	Coordinates
n_1	42°21' N, 95°45' W	n_7	42°26' N, 95°04' W
n_2	42°21' N, 95°20' W	n_8	43°20' N, 95°18' W

scenario set is generated by simply repeating the values of the correlated one as if they were randomly generated, thereby obtaining almost negligible correlation coefficients [29]. Similarly to the previous section, the impact of wind correlation on expansion outcomes is determined as follows:

1. Model (5a)–(5j) is solved for the uncorrelated scenario set, denoting the wind farm location as B^{NC} .
2. The expected profit if decisions are made disregarding wind speed correlation, $\bar{\Pi}^{\text{NC}}$, is computed by solving (5a)–(5j) for the correlated scenario set and fixing the wind farm allocation to B^{NC} .
3. The optimal wind farm expansion planning, B^{C} , and GENCO's expected profit, $\bar{\Pi}^{\text{C}}$, is calculated by solving model (5a)–(5j) for the correlated scenario set.
4. $\Delta\bar{\Pi}$, defined as the difference between the profits obtained in 2) and 3), evaluates the impact of wind correlation on expansion decisions.

Table 6: Wind speed correlation parameters

(n, m)	(n_1, n_2)	(n_1, n_7)	(n_1, n_8)	(n_2, n_7)	(n_2, n_8)	(n_7, n_8)
ρ_{nm}	0.94	0.82	0.80	0.84	0.83	0.98

Table 7 includes the results regarding the optimal location of two wind farms of $N_{w'}^T$ wind turbines each. The investment cost of each wind farm is assumed to be proportional to its capacity with a rate of \$1000/kW and independent of

Table 7: Impact of wind correlation on expansion planning (case study)

$N_{w'}^T$	$Q_{w'n}$	B^{NC}	$\bar{\Pi}^{\text{NC}}$	B^{C}	$\bar{\Pi}^{\text{C}}$	$\Delta\bar{\Pi}$
100	12.50	n_7, n_8	11.83	n_1, n_8	14.16	19.8%
110	13.75	n_7, n_8	9.38	n_2, n_8	13.99	49.2%
120	15.00	n_7, n_8	7.39	n_1, n_8	13.98	89.1%
130	16.25	n_7, n_8	6.21	n_1, n_8	13.37	115.5%

its location, being the payback period equal to 40 years. Profits and investment costs are expressed in \$million.

Note that if wind speed correlation is neglected, wind farms are located at buses n_7 and n_8 with comparatively higher LPMs. However, the remarkably high wind speed correlation at these two locations ($\rho_{n_7 n_8} = 0.98$) and the relatively low capacity of the line connecting these two buses ($F_{n_7 n_8}^{\text{max}} = 200$ MW) contributes to high levels of wind spillage. Consequently, if wind speed correlation is taken into account, this strategy is avoided to reduce wind spillage and increase the profit. Observe that, in some cases, the impact of wind correlation to determine the optimal location of wind farms is of such relevance that GENCO's profit can be increased by 115%.

5.4. Computational performance

The simulations presented in this paper are solved using CPLEX 12.3.0 under GAMS on a Windows-based server with eight processors clocking at 3.0 GHz and 30 GB of RAM. The duality gap is set to 0% in all cases. The average computational times required to solve optimization model (5a)–(5j) for the cases presented in sections 5.2 and 5.3 are around 60s and 1500s, respectively. Note that these computational times can be significantly reduced by applying different resolution procedures or scenario reduction techniques. However, these issues are out of the scope of this paper and further research is required in this regard.

6. Conclusions

Driven by the fact that current generation expansion models for GENCOs do not take into account some uncertainties affecting the profitability of an investment, we quantify in this paper the impact of both equipment failures and wind power production correlation on the expected profit of a GENCO. To do so, we use a bilevel stochastic optimization problem that accounts for the interaction between the producer and the system operator. The variability of the demand throughout the target year, as well as the uncertainty related to both wind speed and unexpected failures of units and transmission lines are accounted for. Wind speed correlation among different geographical locations is also considered. The use of the primal-dual theory allow us to formulate a single-level mixed-integer equivalent formulation that can be readily solved by off-the-shelf optimization software.

Results provided by the 24-bus case study bring to light the remarkably high sensitivity of generating expansion decisions to both equipment failures and wind power correlation. If unit and line failures are not modeled, GENCO's expected profit may decrease up to 65%. Likewise, a expected profit drop of 115% may be incurred if wind speed correlation is disregarded. The reason behind these results is that both equipment failures and wind power correlation may cause network congestions with the consequent impact on the locational marginal prices and hence on the GENCO's profit.

Appendix A. Linearization

- *Linearization of $\lambda_{nsb:g \in \Psi_n} P_{gsb}$:*

The partial derivative of the Lagrangian function of the lower-level problem with respect to P_{gsb} is equal to 0, i.e.,

$$\frac{\partial \mathcal{L}}{\partial P_{gsb}} = 0 \Rightarrow \lambda_{nsb:g \in \Psi_n} = C_g - \phi_{gsb}^{min} - \phi_{gsb}^{max}. \quad (\text{A.1})$$

The two complementarity conditions corresponding to (4f) are

$$\phi_{gsb}^{min} P_{gsb} = 0 \quad (\text{A.2a})$$

$$(P_{gsb} - k_{gs} P_g^{max}) \phi_{gsb}^{max} = 0. \quad (\text{A.2b})$$

Multiplying (A.1) times P_{gsb} and using (A.2a) and (A.2b), we obtain

$$\lambda_{nsb:g \in \Psi_n} P_{gsb} = C_g P_{gsb} - k_{gs} \phi_{gsb}^{max} P_g^{max}, \quad (\text{A.3})$$

which is a linear term.

- *Linearization of $\lambda_{nsb:w \in \Theta_n} W_{wsb}^S$:*

The partial derivative of the Lagrangian with respect to W_{wsb}^S is equal to 0, i.e.,

$$\frac{\partial \mathcal{L}}{\partial W_{wsb}^S} = 0 \Rightarrow -\lambda_{nsb:w \in \Theta_n} + \gamma_{wsb}^{max} + \gamma_{wsb}^{min} = 0. \quad (\text{A.4})$$

The complementarity conditions corresponding to (4i) are

$$W_{wsb}^S \gamma_{wsb}^{min} = 0 \quad (\text{A.5a})$$

$$(W_{wsb}^S - W_{ws}) \gamma_{wsb}^{max} = 0. \quad (\text{A.5b})$$

Multiplying (A.4) times W_{wsb}^S and using (A.5a) and (A.5b), we have

$$\lambda_{nsb:w \in \Theta_n} W_{nsb}^S = \gamma_{wsb}^{max} W_{ws}. \quad (\text{A.6})$$

- *Linearization of $u_{g'n} \lambda_{nsb} P_{g'sb}$:*

Similarly,

$$\frac{\partial \mathcal{L}}{\partial P_{g'sb}} = 0 \Rightarrow \sum_n u_{g'n} (\lambda_{nsb} - C_{g'}) = \phi_{g'sb}^{min} - \phi_{g'sb}^{max}. \quad (\text{A.7})$$

The complementarity conditions corresponding to (4g) are

$$\phi_{g'sb}^{min} P_{g'sb} = 0 \quad (\text{A.8a})$$

$$(P_{g'sb} - k_{g's} P_{g'}^{max}) \phi_{g'sb}^{max} = 0. \quad (\text{A.8b})$$

Multiplying (A.7) times $P_{g'sb}$ and using (A.8a) and (A.8b), we arrive at

$$\sum_n u_{g'n} (\lambda_{nsb} - C_{g'}) P_{g'sb} = -k_{g's} P_{g'}^{max} \phi_{g'sb}^{max}. \quad (\text{A.9})$$

- *Linearization of $u_{w'n}\lambda_{nsb}W_{w'sb}^S$:*

The partial derivative of the Lagrangian with respect to $W_{w'sb}^S$ is expressed as

$$\frac{\partial \mathcal{L}}{\partial W_{w'sb}^S} = 0 \Rightarrow -\sum_n u_{w'n}\lambda_{nsb} + \gamma_{w'sb}^{max} + \gamma_{w'sb}^{min} = 0. \quad (\text{A.10})$$

The complementarity conditions corresponding to (4j) are

$$W_{w'sb}^S \gamma_{w'sb}^{min} = 0 \quad (\text{A.11a})$$

$$(W_{w'sb}^S - \sum_n u_{w'n}W'_{w'ns})\gamma_{w'sb}^{max} = 0. \quad (\text{A.11b})$$

Multiplying (A.10) times $W_{w'sb}^S$ and using (A.11a) and (A.11b), we obtain

$$\sum_n u_{w'n}\lambda_{nsb}W_{w'sb}^S = \sum_n u_{w'n}\gamma_{w'sb}^{max}W'_{w'ns}. \quad (\text{A.12})$$

- *Linearization of the product of binary and continuous variables:*

Let χ be a binary variable and p a continuous one bounded by p^{min} and p^{max} . Then, the product $z = \chi \cdot p$ is equivalent to the following set of mixed-integer linear expressions:

$$z = p - r \quad (\text{A.13a})$$

$$\chi \cdot p^{min} \leq z \leq \chi \cdot p^{max} \quad (\text{A.13b})$$

$$(1 - \chi)p^{min} \leq r \leq (1 - \chi)p^{max}, \quad (\text{A.13c})$$

where r is an auxiliary continuous variable.

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