

Tightening big-M for Optimal Transmission Switching

PSCC 2024 (June 6th 2024)

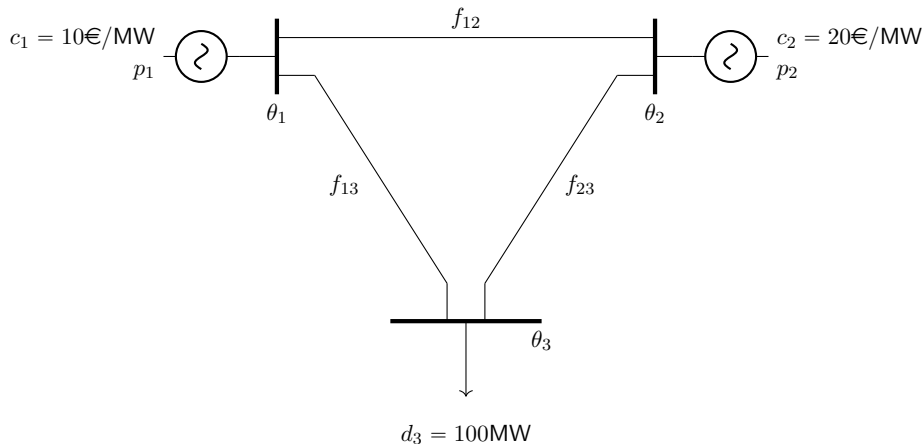
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Motivating example

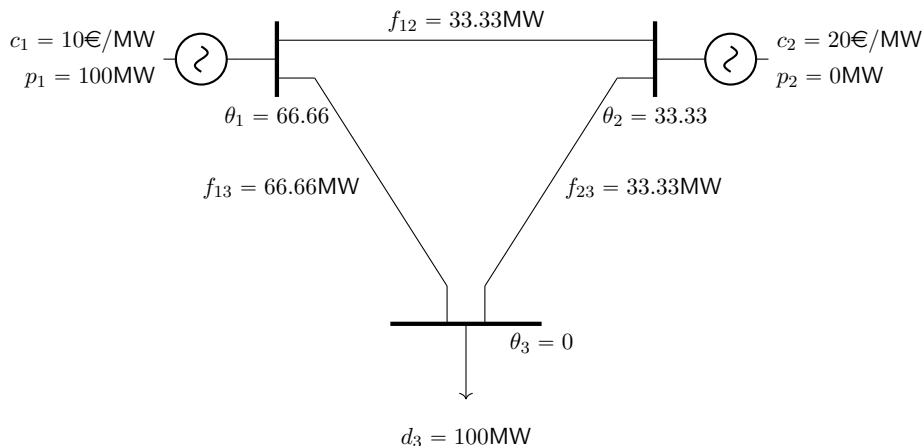
Optimal power flow (OPF): Determine the power generation and power flows to satisfy the demand at the minimum cost



The cheap unit satisfies the demand

Motivating example

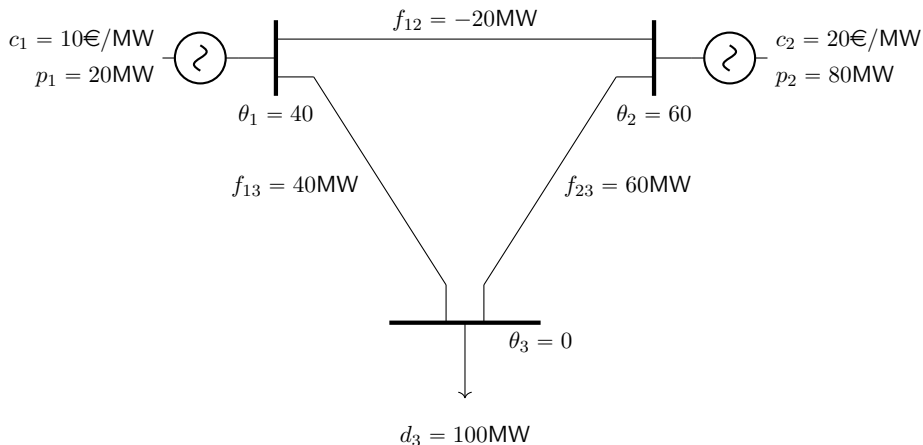
Optimal solution: generate all with cheapest unit (cost = 1000€)



Electrons are not potatoes!!!

Motivating example

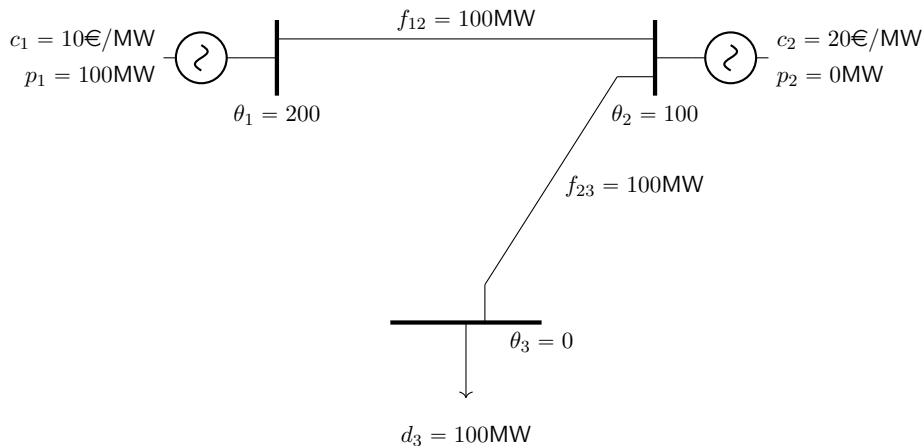
If $f_{13} \leq 40$, the expensive unit also generates (cost=1800€)



Network limits increase the cost!!!

Motivating example

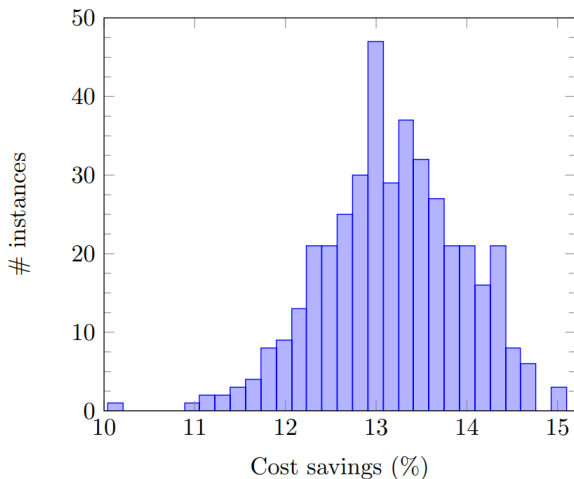
If line 13 is disconnected, cost = 1000€



Disconnecting lines can reduce cost!!!

Motivating example

In the 118-bus system, the average cost saving is 13.2%



The optimal power flow (OPF) is formulated as a linear optimization problem

$$\min_{p_i, f_{ij}, \theta_i} \sum_i c_i p_i \quad (1a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i \quad (1b)$$

$$f_{ij} = b_{ij}(\theta_i - \theta_j), \quad \forall (i, j) \in \mathcal{L} \quad (1c)$$

$$\underline{p}_i \leq p_i \leq \bar{p}_i, \quad \forall i \quad (1d)$$

$$-\underline{f}_{ij} \leq f_{ij} \leq \bar{f}_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (1e)$$

The optimal transmission switching (OTS) requires binary variables x_{ij} and is formulated as a mixed-integer non-linear problem ...

$$\min_{p_i, f_{ij}, \theta_i, x_{ij}} \sum_i c_i p_i \quad (2a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i \quad (2b)$$

$$f_{ij} = x_{ij} b_{ij} (\theta_i - \theta_j), \quad \forall (i, j) \in \mathcal{L} \quad (2c)$$

$$\underline{p}_i \leq p_i \leq \bar{p}_i, \quad \forall i \quad (2d)$$

$$-x_{ij} \underline{f}_{ij} \leq f_{ij} \leq x_{ij} \bar{f}_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (2e)$$

$$x_{ij} \in \{0, 1\}, \quad \forall (i, j) \in \mathcal{L} \quad (2f)$$

... that can be directly solved using optimization solvers such as Gurobi.

Methodology

To avoid the non-linear terms in

$$f_{ij} = x_{ij} b_{ij}(\theta_i - \theta_j)$$

We replace it by

$$b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leq f_{ij} \leq b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij})$$

Together with equation

$$-x_{ij} \underline{f}_{ij} \leq f_{ij} \leq x_{ij} \overline{f}_{ij}$$

We have that:

- If $x_{ij} = 1 \Rightarrow b_{ij}(\theta_i - \theta_j) \leq f_{ij} \leq b_{ij}(\theta_i - \theta_j)$ and $-\underline{f}_{ij} \leq f_{ij} \leq \overline{f}_{ij}$
- If $x_{ij} = 0 \Rightarrow f_{ij} = 0$ and $\underline{M}_{ij} \leq b_{ij}(\theta_i - \theta_j) \leq \overline{M}_{ij}$

The OTS is reformulated as a mixed-integer linear problem

$$\min_{p_i, f_{ij}, \theta_i, x_{ij}} \sum_i c_i p_i \quad (3a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i \quad (3b)$$

$$b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leq f_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (3c)$$

$$f_{ij} \leq b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i, j) \in \mathcal{L} \quad (3d)$$

$$\underline{p}_i \leq p_i \leq \overline{p}_i, \quad \forall i \quad (3e)$$

$$-x_{ij}\underline{f}_{ij} \leq f_{ij} \leq x_{ij}\overline{f}_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (3f)$$

$$x_{ij} \in \{0, 1\}, \quad \forall (i, j) \in \mathcal{L} \quad (3g)$$

\underline{M}_{ij} and \overline{M}_{ij} must be valid bounds for $b_{ij}(\theta_i - \theta_j)$ if line is open

\underline{M}_{ij} and \overline{M}_{ij} must be small enough to avoid computational issues

Methodology

Since $\underline{M}_{ij} \leq b_{ij}(\theta_i - \theta_j) \leq \overline{M}_{ij}$ when $x_{ij} = 0$, we can compute these bounds for a particular switchable line (i', j') as

$$\overline{M}_{i'j'}^{\text{OPT}} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} b_{i'j'}(\theta_{i'} - \theta_{j'}) \quad (4a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i \quad (4b)$$

$$b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leq f_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (4c)$$

$$f_{ij} \leq b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i, j) \in \mathcal{L} \quad (4d)$$

$$\underline{p}_i \leq p_i \leq \overline{p}_i, \quad \forall i \quad (4e)$$

$$-x_{ij}\underline{f}_{ij} \leq f_{ij} \leq x_{ij}\overline{f}_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (4f)$$

$$x_{ij} \in \{0, 1\}, \quad \forall (i, j) \in \mathcal{L} \quad (4g)$$

$$x_{i'j'} = 0 \quad (\text{Line is forced to be open}) \quad (4h)$$

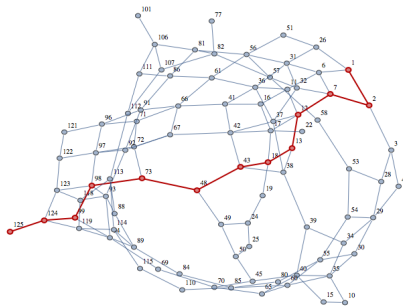
However, this problem is as difficult as the original one!!

Methodology

Fattahi et. al (2019) find a bound on $\overline{M}_{ij}^{\text{OPT}}$ if there exists a connected spanning subgraph of the network with non-switchable lines

$$\overline{M}_{i'j'}^{\text{OPT}} \leq b_{i'j'} \sum_{(k,l) \in SP_{i'j'}} \frac{\overline{f}_{kl}}{b_{kl}}$$

where $SP_{i'j'}$ is the shortest path between nodes i' and j' (very easy to compute using Dijkstra's algorithm)



Methodology

Instead of solving the original bounding problem to compute $\overline{M}_{ij}^{\text{opt}}$, we solve the following linear relaxation

$$\overline{M}_{i'j'}^{\text{LR}} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} b_{i'j'}(\theta_{i'} - \theta_{j'}) \quad (5a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i \quad (5b)$$

$$b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leq f_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (5c)$$

$$f_{ij} \leq b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i, j) \in \mathcal{L} \quad (5d)$$

$$\underline{p}_i \leq p_i \leq \overline{p}_i, \quad \forall i \quad (5e)$$

$$-x_{ij}\underline{f}_{ij} \leq f_{ij} \leq x_{ij}\overline{f}_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (5f)$$

$$0 \leq x_{ij} \leq 1, \quad \forall (i, j) \in \mathcal{L} \quad (5g)$$

$$x_{i'j'} = 0 \quad (\text{Line is forced to be open}) \quad (5h)$$

This problem can be “too relaxed” and provide poor bounds...

Inspired by Porras et. at (2022), we include a bound on the cost using a “reasonable good” feasible solution.

$$\sum_i c_i p_i \leq \text{cost}$$

A **naive approach** is to satisfy the demand with the most expensive units. We denote this cost as cost^{NAI}

Another option is using **greedy heuristics**. For instance, we can start with all lines connected and disconnect the line that involves highest savings by solving several OPF problem. We repeat the process until any disconnection increases the cost. We denote this as cost^{HEU}

If the bound on the cost is considered, we solve the following problem

$$\overline{M}_{i'j'}^{\text{LR}} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} b_{i'j'}(\theta_{i'} - \theta_{j'}) \quad (6a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i \quad (6b)$$

$$b_{ij}(\theta_i - \theta_j) - \overline{M}_{ij}(1 - x_{ij}) \leq f_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (6c)$$

$$f_{ij} \leq b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i, j) \in \mathcal{L} \quad (6d)$$

$$\underline{p}_i \leq p_i \leq \overline{p}_i, \quad \forall i \quad (6e)$$

$$-x_{ij}\underline{f}_{ij} \leq f_{ij} \leq x_{ij}\overline{f}_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (6f)$$

$$0 \leq x_{ij} \leq 1, \quad \forall (i, j) \in \mathcal{L} \quad (6g)$$

$$x_{i'j'} = 0 \quad (6h)$$

$$\sum_i c_i p_i \leq \text{cost}^{\text{NAI/HEU}} \quad (6i)$$

We also solve bounding problems to improve \underline{f}_{ij} and \bar{f}_{ij} as follows

$$\bar{f}_{i'j'}^{\text{LR}} = \max_{p_i, f_{ij}, \theta_i, x_{ij}} b_{i'j'}(\theta_{i'} - \theta_{j'}) \quad (7a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}_i^-} f_{ij} - \sum_{(i,j) \in \mathcal{L}_i^+} f_{ij} = p_i - d_i, \quad \forall i \quad (7b)$$

$$b_{ij}(\theta_i - \theta_j) - \bar{M}_{ij}(1 - x_{ij}) \leq f_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (7c)$$

$$f_{ij} \leq b_{ij}(\theta_i - \theta_j) - \underline{M}_{ij}(1 - x_{ij}), \quad \forall (i, j) \in \mathcal{L} \quad (7d)$$

$$\underline{p}_i \leq p_i \leq \bar{p}_i, \quad \forall i \quad (7e)$$

$$-x_{ij}\underline{f}_{ij} \leq f_{ij} \leq x_{ij}\bar{f}_{ij}, \quad \forall (i, j) \in \mathcal{L} \quad (7f)$$

$$0 \leq x_{ij} \leq 1, \quad \forall (i, j) \in \mathcal{L} \quad (7g)$$

$$x_{i'j'} = 1 \quad (7h)$$

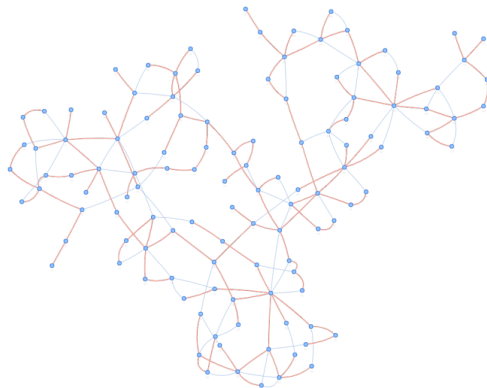
$$\sum_i c_i p_i \leq \text{cost}^{\text{NAI/HEU}} \quad (7i)$$

The proposed method runs as follows:

- 1 Set $\underline{f}_{ij}, \bar{f}_{ij}$ to original capacities
- 2 Compute $\underline{M}_{ij}, \bar{M}_{ij}$ using Fattahi's method
- 3 Compute the cost bound cost^{NAI} or cost^{HEU}
- 4 Solve bounding problems (LP) to adjust $\underline{f}_{ij}, \bar{f}_{ij}, \underline{M}_{ij}, \bar{M}_{ij}$
- 5 Repeat step 4 if needed
- 6 Solve the OTS problem with improved bounds and maximum cost

Numerical simulations

- 118-bus system with 186-lines (69 switchable lines)
- 100 instances with different demands and connected subgraphs
- Total time = bounding problems (LPs) and final OTS problem (MIP)
- GAP at 0.01% and maximum time 1 hour



Numerical simulations

- 1 round of bounding problems
- Δ^M : Relative bigM values with respect to Fattahi
- Δ^L : Relative line capacities with respect to original

Method	Δ^M	Δ^L	Time (s)	Unsolved	Max gap
Fattahi	100%	100%	672	14	0.69%
cost ^{NAI}	68%	74%	487	8	1.04%
cost ^{HEU}	64%	68%	170	1	0.03%

- Even without the cost constraint, the bounding problems improve computational performance
- Using the cost of a feasible solution close to the optimal further improves the performance of the proposed methodology

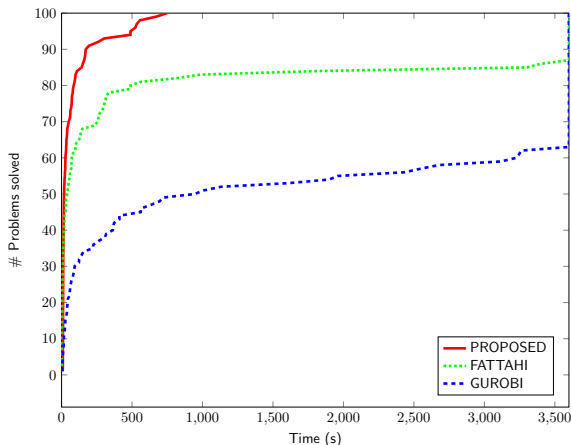
Numerical simulations

- Upper bound cost^{HEU}
- More rounds of bounding problems

Method	Δ^M	Δ^L	Time (s)	Unsolved	Max gap
Fattahi	100%	100%	672	14	0.69%
1 round	64%	68%	170	1	0.03%
2 rounds	56%	63%	87	0	-
3 rounds	52%	61%	81	0	-
4 rounds	50%	59%	80	0	-

- The bounds \underline{f}_{ij} , \bar{f}_{ij} , \underline{M}_{ij} , \bar{M}_{ij} are tightened in each iteration
- With four rounds the average time is reduced by 88%

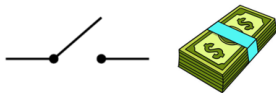
Numerical simulations



- General-purpose solvers perform poorly (38 unsolved problems)
- Fattahi uses knowledge about the problem to adjust bigMs
- Our bound tightening method is the winner (max time 13 minutes)

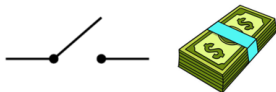
Conclusions

- The optimal transmission switching (OTS) determines the lines that can be disconnected to reduce the operating cost.



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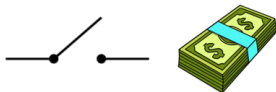


- The OTS is formulated as a mixed-integer linear problem with bigMs that is computationally difficult to solve.



Conclusions

- The optimal transmission switching (OTS) determines the lines that can be disconnected to reduce the operating cost.



- The OTS is formulated as a mixed-integer linear problem with bigMs that is computationally difficult to solve.



- We propose a methodology that solves relaxed bounding problems to find tight bigMs and reduce the computational burden of the OTS.



Thanks for the attention!! Questions??



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Tight Big-Ms for Optimal Transmission Switching

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